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ABSTRACT

Several empirical and theoretical analyses related to scientific problem-solving are reviewed, including: detailed studies of individuals at different levels of expertise, and computer models simulating some aspects of human information processing during problem solving. Analysis of these studies has revealed many facets about the nature of the knowledge required for solving problems in complex subject-matter domains. In addition, the analyses have provided detailed descriptions of the performance of problem solvers at "novice" and "expert" levels. Although these descriptive analyses are intended to document and explain naturally occurring performance, a prescriptive approach could be used to identify effective problem-solving methods that might be "prescribed" to students. The nature of and major steps in such a prescriptive model and results of empirical tests of the model are discussed. Steps include specification of applicability, formulation of a model of good performance, elaboration of the model, measures to ensure implementability, and controlled experiments. Relevance of the model as well as instructional implications (reasonable conclusions about both what to teach and how that information should be taught) are addressed. Making tacit processes explicit, getting students to talk about processes, providing guiding practice, understanding/reasoning processes are among the strategies suggested. (JN)

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# TOWARD THEORY-BASED INSTRUCTION IN SCIENTIFIC PROBLEM SOLVING

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A major goal of instruction is to improve students' abilities to perform tasks they were not able to perform well prior to instruction. Such improvement can only come about if students learn something as a result of their exposure to instruction. Put another way, the students' knowledge for performing particular tasks must undergo some changes during the instructional process. The primary task facing educators, then, is to set up the instructional conditions under which desired changes in students' knowledge will occur. Ideally, the design of instruction should be guided by general theoretical principles about the relationship between instructional conditions and changes in student knowledge. Without such general principles, the development of effective instruction remains heavily dependent on the skills, experience, and frequently the personality of the particular educator designing or delivering the instruction.

There are a great many variables which affect this relationship between instructional conditions and learning, and many of these need to be understood for principled or theory-based instruction to become possible. These include cognitive, affective, and social aspects of the educational process; structure and content of instructional materials; medium of delivery of the instruction; the nature and structure of the subject matter being taught; and a myriad of student variables such as aptitudes, prior knowledge, and personality. A comprehensive instructional theory would explain the ways in which all of these factors interact during instruction. Progress toward this end is being made by gradually accumulating insights into each factor, insights which need to be synthesized eventually into a complete account.

In this paper, we review selected findings in one area, recent analyses of cognition during problem solving, and explore the instructional implications of this piece of the complex puzzle described above. The work we will discuss has evolved from several decades of interest in the psychology of problem solving. Early psychological analyses of problem solving considered fairly general aspects of problem-solving activities. Gestalt theorists such as Duncker (1945), Köhler (1927), and Wertheimer (1945/1959) saw as most important for problem solution the achievement of understanding of the problem as a whole. They were interested in sudden "insights" during problem solving and saw problem solving as requiring the integration of previously learned responses in novel ways. Beginning around 1950, theorists taking a behaviorist approach (e.g., Maltzman, 1955) concerned themselves primarily with general connections between actions performed by the problem solver and conditions under which actions are performed. More recent "information-processing" analyses (e.g., Newell & Simon, 1972) incorporate both the Gestalt psychologists' interest in internal mental states involved in understanding problems, and the Behaviorists' emphasis on actions performed in response to specifiable stimuli. The aim of these more recent analyses has been to characterize the knowledge required for solving problems in considerably greater detail than was attempted in earlier work.

Central to the "information-processing" approach is an interest in the ways humans store and process information to perform complex intellectual tasks. The aim is to build models of the conceptual knowledge structures, procedures, and general strategies required for understanding and solving problems. These theories are implemented in the form of programs which are often run on a computer because they tend to be quite complex. The models are intended to be hypotheses about limited aspects of cognition. They are tested by assessing the match between a "trace" of the processes performed by the model as it solves a problem and the data from human subjects performing the same task. The claim is that the existence of these programs and their ability to simulate human behavior demonstrate that the theories "are operational and do not depend on vague, mentalistic concepts" (Larkin, McDermott, Simon, and Simon, 1980a).

While early information-processing studies focussed on the study of puzzle-like problems in areas such as cryptarithmetic, chess, and symbolic logic, interest has turned toward analyzing solution of problems in complex subject-matter domains such as physics (Simon & Simon, 1978; Larkin, McDermott, Simon, & Simon, 1980a), geometry (Greeno, 1978), and arithmetic word problems (Riley, Greeno, & Heller, 1983). The problems looked at in this work have been fairly standard textbook problems of the kind students typically encounter in school (see example in Figure 1).

Figure 1 about here

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These problems are well-structured--they provide a clearly specified problem situation containing a small amount of given information and a specific goal which can be determined from the given information. Such problems are used throughout mathematics and science curricula both to teach content material and to assess students' abilities to apply the concepts and principles taught in courses. Despite their high degree of structure, these problems generally require very complex knowledge for their solution, and most students find them quite difficult. Effective methods for teaching students how to solve such problems continue to elude educators. They remain, therefore, a serious educational challenge.

As we mentioned above, it is useful to think about instruction as a process which involves modification of students' knowledge for performing tasks. This process can be viewed as a series of transitions from one knowledge state to another; a theory of instruction would specify how to bring about these transitions between knowledge states. The information-processing analyses have mostly involved concepts and methods for characterizing individual knowledge states, although some progress has begun to be made in specifying the mechanisms involved in changes in knowledge, i.e., learning (Anderson, 1981).

Our contribution to this symposium is to identify some of the implications these cognitive analyses have for the design of instruction in scientific problem solving. We begin by reviewing descriptive analyses of problem solving in scientific domains, and then describe in some detail an analysis which takes a more prescriptive approach to the identification of knowledge needed for solving problems. We conclude with a discussion of the instructional implications of this work.

DESCRIPTIVE ANALYSES OF PROBLEM-SOLVING PERFORMANCE

## Studies of Problem-Solving Performance

One method of discovering the kinds of knowledge required for solving scientific problems is to compare solutions by highly skilled, "expert" problem solvers with those by "novices." Such analyses can provide theoretical insights into the nature of skilled performance as well as into the kinds of difficulties beginning problem solvers experience. Once the components of skilled performance have been identified, it becomes possible to consider teaching this knowledge to novices (Greeno, 1976; 1980). Furthermore, explication of the prior knowledge of students could guide the design of instruction specifically intended to address their difficulties. In this section we will review the major findings of research exploring differences between the scientific problem-solving performance of individuals at different levels of expertise.

Simon and Simon (1978) analyzed the differences between expert and novice solutions of 19 kinematics problems. Their expert had extensive experience solving mechanics problems; their novice had only recently studied the kinematics chapter of an elementary physics textbook, and had never solved problems of this kind before. The problems involved situations in which objects moved with uniform acceleration, such as the following.

> A bullet leaves the muzzle of a gun at a speed of 400 m/sec. The length of the gun barrel is 0.5 m. Assuming that the bullet is uniformly accelerated, what is the average speed within the barrel?

There were major differences in the overall solution strategies used by these two problem solvers. Not surprisingly, the expert solved the problems in less than one quarter of the time needed by the novice and made fewer errors. But of more interest was the fact that he seemed to solve the problems almost automatically. Immediately upon reading the problem statement, the expert generated equations into which known values had already been substituted. That is, he spontaneously and almost effortlessly combined information given in the problem statement with his knowledge of physical laws to produce already instantiated equations. He than easily solved these equations.

The novice's solution was in sharp contrast to the expert's smoothly executed sequences of identifying, instantiating, and solving equations. The novice had to ask herself at each step what to do next, and frequently expressed little confidence in her abilities. She had to search through pages of the textbook for formulas that might apply, and would explicitly mention which formulas she was considering.

Simon and Simon found some evidence that the expert used what they referred to as "physical intuition." They defined physical intuition in information-processing terms as the construction of a cognitive representation of the physical situation, and the use of that representation to guide the generation and application of equations. (This notion of understanding as the construction of a mental representation of the elements and relations in a situation is central to recent cognitive theories of language understanding

(Anderson, 1976; Norman & Rumelhart, 1975; Schank & Abelson, 1977).) The expert's thorough qualitative understanding of the problem situation, in the form of such a representation, allowed him to solve the problems efficiently and quickly. Simon and Simon contrasted this "physical" approach of the expert (moving from the problem statement to a cognitive representation of the physical situation, and from that representation to equations) with the novice's "algebraic" approach (going directly from the problem statements to the equations). They asserted that knowledge of the physical laws or equations needed for solving these problems comprises only the "algebra of kinematics." To "know physics," an individual must be able to understand complex problem situations in terms of the laws of physics.

Simon and Simon also observed differences in the sequence in which equations were generated by the expert and novice. The expert seemed to use a "working-forward" strategy, operating from the givens of the problem by successively applying equations until the desired values were found. The novice's strategy was nearly opposite to the expert's--she used a "working-backward" approach, going from the unknowns to the givens. She would look for formulas which contained variables that were wanted in the problem, and apply those formulas. If those formulas contained other variables for which the values were unknown, she would apply the same procedure to find values for these new variables.

The hypothesis was that these two strategies accounted for the differences between the sequences in which equations were used by the expert and novice. To test this hypothesis, Simon and Simon created two very simple production systems to try to model the observed performance. Productions are condition-action pairs which are more sophisticated versions of the stimulus-response pairs of classical behaviorist psychology. Whenever the conditions in one of the productions are satisfied, the action associated with that condition is performed. (One can think of these as similar to "if-then" statements.) Production systems are lists of these condition-action pairs which constitute a theory about the contents and organization of some portion of memory. Many of the recent information-processing theories are in the form of production systems.

Simon and Simon created two separate lists of productions, one representing the working-forward strategy of the expert, and the other representing the working-backward strategy of the novice. In both cases, the action part of each production was one of the kinematic equations and the condition part was a list of variables in that equation. The systems only differed in one respect. In the expert system the rule was: If the equation contains known variables (condition) then try to solve that equation (action). In the novice system the rule was: If the equation contains one or more desired variables (condition) then try to solve that equation (action).

Simon and Simon compared the sequences of equations which each of these systems generated with the sequences produced by their two subjects. They found that this simple model matched very well the equation-generating behavior of the expert and novice, thus confirming that the working-forward and working-backward strategies constituted reasonable explanations of some differences between the observed solutions.

A substantially expanded, and computer-implemented, version of these models was developed to account for the performance of a larger group of subjects solving kinematics and dynamics problems (Larkin, McDermott, Simon and Simon, 1980b). Two models were created, one simulating naive performance and the other competent performance. The naive system used "means-ends analysis" (ME), which corresponds to Simon and Simon's "working-backward" strategy. The other used a "knowledge-development" (KD) strategy, a method corresponding to Simon and Simon's "working-forward" procedure. The KD model also had knowledge of problem-solving "methods" (such as an "energy" method or "force" method) comprised of clusters of physics principles.

These models were run on Simon and Simon's 19 kinematics problems plus two new problems requiring different principles. The resulting traces were compared with the performance of eight novices (undergraduate students taking their first physics course) and eleven experts (physics professors and advanced graduate students), solving the two new problems, plus the data from Simon and Simon's expert and novice solving the other 19 problems. The models predicted well the order in which the human subjects applied principles during their solutions. They also reflected the observed differences in the automaticity with which principles are used. In the ME model, the selection and instantiation of equations are done in separate steps, while in the KD model, they are performed in a single step.

Larkin (1981a, b) also explored the question of how a less-skilled problem solver might improve through practice. She created a production system, ABLE, which "learns" from experience solving problems. This model, which initially uses an algebraic, means-ends strategy in its "barely ABLE" state, acquires new knowledge each time it solves a problem, until it becomes "more ABLE." The process by which it learns is to notice, whenever it successfully applies a principle, how and under what problem conditions that principle was applied. In its final state, the program uses the KD strategy to generate equations automatically.

In the studies thus far discussed, the experts were solving what were for them very easy problems. To obtain more information about experts' capabilities, Larkin (1977a, 1977b) observed physicists solving more complex problems in mechanics. These studies revealed a rich sequence of problem representations which the experts used at different points in their solutions.

The experts' solutions were comprised of an initial phase of "qualitative analysis" which was performed before any equations were generated. In this preliminary phase, the experts first drew sketches representing the physical situation described in the problem, then drew more abstract diagrams representing the problem in terms of concepts from their knowledge of physics (such as force and energy). The information in these abstract representations was not mentioned in the problem statement, yet this information apparently was required for understanding the problem.

During this qualitative phase, the experts also explored alternative ways to solve the problem. This planning was done at a very general level. The experts referred to a small set of candidate solution methods, which consisted of clusters or "chunks" of physics principles to be applied as a group. The experts would consider these alternatives, exploring the utility of one or more before selecting an approach to use. After this exploratory phase was completed, as signaled by their writing of an equation, none of the experts subsequently changed their approach to a problem.

Novices, in contrast, generated equations shortly after reading the problem, without this intervening step of checking the usefulness of coherent solution methods. The novices did frequently sketch physical situations described in the problems, and would sometimes draw abstract representations, especially force diagrams. However, they always went directly to and stayed at a single detailed level during the rest of their solutions, writing and manipulating equations.

Larkin (1977b) modeled some of the experts' planning processes in a production system, and McDermott and Larkin (1978) developed a model known as PH632 (for the number of the most advanced mechanics course at Carnegie-Mellon University) which simulates experts' reliance upon a sequence of problem representations. Four types of representations are used by these systems: the verbal statement of the problem, a sketch of the physical situation in the problem, a more abstract representation including conceptual entities in physics, and a quantitative representation in the form of equations (referred to as verbal, naive, scientific, and mathematical representations by Larkin, 1982a). Larkin (1982b) has also modeled the important role of spatial knowledge in construction of the naive and scientific representations.

Problem Perception

Knowledge for constructing problem representations has also received attention in studies of human subjects performing tasks other than solving problems. Chi and her colleagues (Chi, Feltovich, & Glaser, 1981; Chi, Glaser, & Rees, 1981) asked expert physicists and novice physics students to sort mechanics problems, and analyzed the categories constructed and justifications for these categories. Novices were found to sort problems on the basis of physical objects (such as pulleys and inclined planes), and physics concepts (such as friction), mentioned in the problem statements. Experts sorted problems on the basis of more abstract physics principles (such as Conservation of Energy) that are applicable in the problems but are not mentioned in the problem statements. (Very similar results have been found in mathematics by Silver, 1979, and Schoenfeld and Herrmann, 1982.) These data are quite consistent with the findings that experts represent and plan solutions for problems in terms of underlying physics principles, whereas novices' solutions are not guided by this kind of understanding, since they do not "see" the physics principles underlying the problem statements.

## Knowledge of Basic Concepts

Not only do novices lack this ability to understand problems in terms of scientific principles, they also have been found to misinterpret individual concepts. A veritable catalog of students' non-scientific conceptions is being accumulated by researchers around the world. For example, a number of studies document students' inability to predict or explain correctly the motion of objects (Champagne, Klopfer, & Anderson, 1980; Clement, 1982; diSessa, 1982; McCloskey, 1983; McCloskey, Caramazza, & Green, 1980; McCloskey & Kohl, 1983; Minstrell, 1982a; Shanon, 1976; Trowbridge & McDermott, 1980; Trowbridge & McDermott, 1981; Viennot, 1979), and basic concepts like gravity and heat (Albert, 1978; Gunstone & White; 1981; Mali & Howe, 1979). It has been shown that students' naive conceptions are extremely widespread and are very resistent to change, persisting after considerable exposure to scientifically correct explanations in traditional instruction and in experimental laboratory settings.

## Artificial-Intelligence Programs

Processes for building and using problem representations have also been explored in artificial-intelligence programs. (Although these programs were not intended to model human cognition, they can identify processes for which functionally equivalent mechanisms might exist in human performance.) Novak (1977), for example, has demonstrated the process of translating verbal problem statements (for statics problems) into representations of the objects and relations in the problems. This program, ISAAC, can read, understand, draw pictures of, and solve a set of problems stated in English. A program called NEWTON, by de Kleer (1975, 1977), models qualitative analysis involving "envisionment" of how objects will move in problem situations. For some simple problems, this analysis is sufficient for reaching a solution. When it is not, NEWTON uses means-ends analysis to select formulas to apply. Another program, MECHO (Bundy, 1978; Bundy, Byrd, Luger, Mellish, & Palmer, 1979; Byrd & Borning, 1980), models several levels of representation in solution of statics and dynamics problems. All of these programs model the critical role of problem understanding and representation in achieving solutions to complex problems.

Summary

We have briefly reviewed several empirical and theoretical analyses related to scientific problem solving. These include detailed studies of individuals at different levels of expertise; computer models simulating some aspects of human information processing during problem solving; and artificial-intelligence models of problem solving. We have learned a good deal from these analyses about the nature of the knowledge required for solving problems in complex subject-matter domains.

First, knowledge for understanding and representing problems frequently is critical for reaching correct, or even reasonable, problem solutions. Understanding is viewed as a process of creating a representation of the problem. This representation mediates between the problem text and its solution, guiding expert human and computer systems in the selection of methods for solving problems. Novices tend to be quite deficient with respect to understanding or perceiving problems in terms of fundamental principles or concepts. They cannot and/or do not construct problem representations that are helpful in achieving solutions.

Strategic knowledge governs the approach problem solvers take to the task. Experts solve problems using a process of successive refinements--unless they are faced with a simple problem for which they can immediately recall a specific solution method, the strategy experts use is to perform high-level planning and qualitative analysis before beginning to generate equations. Novices do not have the knowledge required to approach problems in this way, and tend to go directly from the problem text to equations.

Problem solving also requires extensive knowledge of basic concepts and principles. Experts have a great amount of such domain-specific factual knowledge which is both technically correct and well-organized. Experts also have knowledge about when concepts and principles are applicable and useful, and have procedures for interpreting and applying their factual knowledge. Novices are lacking much of this knowledge, do not have their knowledge well organized, and frequently exhibit naive preconceptions rather than scientifically correct ideas.

Finally, repertoires of familiar patterns and known procedures are necessary for reliable performance. Experts have such repertoires, including knowledge of familiar problems and known solution methods, which novices have not yet developed.

## THE PRESCRIPTIVE APPROACH

The psychological analyses discussed thus far have provided detailed descriptions of the performance of problem solvers at different levels of expertise. These analyses have revealed that the knowledge required for solving problems is much richer, more extensive, and more complex than we had realized. This complexity had not been recognized earlier because so much knowledge is implicit in skilled performance and remains "tacit" (Polyani, 1967) for the person who had the knowledge. Since the expert problem solvers themselves are unaware of this tacit knowledge, those who teach problem solving have not been able to make explicit for students the knowledge needed to achieve good performance. In turn, since this tacit knowledge has rarely been taught explicitly, it is not surprising that novices differ so much from experts, even after the novices have completed courses which cover the problem-solving material.

It might appear that we should now begin to remedy this problem by teaching students directly the knowledge possessed by experts. While this approach has merit, we believe it is not always the one to follow for several reasons. First, we may not always want to teach beginning students this expert knowledge. For one thing, not all students need to reach the very advanced level of performance exhibited by the experts in these studies. The students vary widely in their professional goals, and extensive expertise in a scientific domain is not reasonable to expect of students enrolled in each course in that domain. There is also no reason to assume that individuals we somewhat arbitrarily dub "experts" on the basis of their position or title always perform optimally. We want students to achieve at least minimally competent levels of performance, and competent performance is not necessarily synonymous with expert performance.

Even if we wanted to teach novices to be experts, it is not clear that we could. Experts acquire their knowledge through years of experience. They have large repertoires of familiar patterns and highly automatized procedures which gradually develop through repeated exposure to problems and which would be extremely difficult to teach directly. We are also well aware that novices do not begin instruction as "blank slates." They enter courses with a great deal of prior knowledge, including strong preconceptions which are frequently incompatible with the ideas and language of the science to be learned. Learning is not a process of filling empty spaces in students' heads--existing knowledge must be reorganized and restructured to accommodate and assimilate the new knowledge, and some of the prior ideas must be relinquished and replaced. As demonstrated by Minstrell (1982b, 1983), it may be necessary to engage students in extended periods of intense discussion before they accept "expert" scientific explanations and definitions which differ from the students' earlier intuitions.

Instead of teaching novices to be experts, then, a more realistic goal might be what we could call "expert novice" performance. An individual at this level would be able to solve problems competently, but not necessarily using the same processes as experts. In order to design instruction for this purpose, we need to generate theories of the knowledge novices could rely on to achieve good performance. This approach, which has been referred to as "prescriptive" (Bruner, 1964; Reif, 1979), involves identification of effective problem-solving methods that we might wish to "prescribe" for students to learn. Purely descriptive analyses, in contrast, are intended to document and explain naturally occurring performance, whether or not that performance is effective.

A prescriptive theory of problem-solving performance can incorporate components of descriptive theories of expert performance, but it differs from the latter in important ways. It would substitute for the highly automatized procedures of the expert, alternative procedures which draw on the knowledge available to novices. For example, experts recognize types of problems, and rapidly retrieve and apply solution methods which are associated with problem types, but novices neither have knowledge of problem types nor stores of known methods. A prescriptive theory would explicate procedures novices could use to recognize features of problems and decide which solution method to use for which problem. Such a theory would also take into consideration the preconceptions and knowledge deficiencies of novices which lead to common errors, and would include preventive or compensatory procedures to block or catch these errors. For example, a variety of powerful checks not necessarily evident in experts' solutions could be included throughout the solution procedure.

It should be noted that the "expert novice" level of performance is not necessarily where individuals would remain for long periods of time after instruction. It encompasses a set of minimal, reliably effective procedures intended for conscious and deliberate use by individuals who have not yet acquired years of experience. For those individuals who continue in the field, problem-solving procedures would be expected to evolve naturally, becoming less explicit, and more automatized and efficient. As Resnick (1976) has suggested, efficient instruction is not necessarily direct instruction in skilled performance. Rather, the aim can be to teach routines "that put learners in a good position to discover or invent efficient strategies for themselves."

As described by Heller and Reif (in press), the formulation and testing of a prescriptive model involves the following major steps:

-Specification of applicability: Specify the conditions under which the proposed model is supposed to be applicable. These conditions include the characteristics of the tasks to be performed and the characteristics (capabilities and limitations) of the persons who are to perform these tasks.

-Formulation of a model of good performance: Formulate a prescriptive model specifying explicitly the procedures and associated factual knowledge whereby a person with the previously specified characteristics can perform effectively the specified kinds of tasks.

-Elaboration of the model: Translate the model into a detailed "program" which makes explicit the knowledge specified by the model. This program consists of a sequence of specific steps and associated facts.

-<u>Measures to ensure implementability</u>: Pilot test and modify the program until it is specified at an optimal level of detail and all steps in the program can be readily interpreted and reliably executed.

-<u>Controlled experiments</u>: Carry out controlled experiments in which individuals are induced to act in accordance with the program for the model. Observe in detail the resulting performance. (Experiments can also compare the performance which results when different subjects work in accordance with alternative models. Comparison between models which differ in specific ways allows one to ascertain which particular features of these models are necessary or sufficient for good performance.)

This approach for formulating and testing a prescriptive model is analogous to that used in artificial intelligence for models of effective performance by computers: One develops a program which embodies a theory of how a task can be performed by an information-processing system, and tests that program by running it on the system for which it was intended. If the task is performed well, the model is judged to be sufficient; if the system fails to perform the task correctly, the model is not sufficient and needs modification.

The use of this approach with human problem solvers is, however, motivated by very different goals from those generally pursued by artificial-intelligence theorists. In general, an effective AI program is an end in itself. In contrast, development of a sound model of human performance is only a means for identifying potentially teachable problem-solving methods. Once a prescriptive model is shown to lead to good problem-solving performance by human subjects, there still remains the question of whether to teach people to use that model. The answer to this question depends on factors such as how easily learnable the knowledge in the model is. Teaching experiments would be required to evaluate these aspects of a validated model of performance.

Heller and Reif (in press) applied the prescriptive approach to an important aspect of scientific problem solving, namely the initial qualitative description of problems. Because this approach may be very useful as a bridge between descriptive studies and the design of instruction, we present this work in considerable detail here. In the following sections we discuss the prescriptive model, and the results of empirical tests of the model.

## Prescriptive Model of Problem Description

### Specification of Applicability

The tasks in this study were problems in basic college-level physics, specifically in the field of mechanics. The subjects were presumed to have not only typical human limitations (e.g., of short-term memory), but also relatively complex human capabilities (e.g., the ability to understand English, to draw diagrams, to do algebra, and to interpret individual principles in basic physics).

### Formulation of a Model of Good Performance

Reif and Heller (1982) formulated a prescriptive model of effective human problem solving in the domain of physics. This model specifies some general procedures to be used in conjunction with a knowledge base about a particular scientific domain. The general procedures subdivide the problem-solving process into three major stages: (a) the generation of an initial problem description; (b) the generation of the actual solution; and (c) the assessment and improvement of this solution. The domain-specific knowledge base contains declarative knowledge of concepts and principles, together with specific procedures facilitating their use, and is organized hierarchically.

Heller and Reif (in press) studied one component of this model, the process for generating a useful initial description (or "representation") of any problem. According to their model, the generation of this initial description can conveniently be decomposed into two stages. In the first of these, a person starts from an originally presented problem and uses general domain-independent knowledge to generate a "basic description" of the problem. This basic description transforms the problem into a readily interpretable form. It summarizes explicitly the information specified and wanted in the problem, identifies relevant time-dependent processes and decomposes them into distinct subprocesses, introduces useful symbols, and expresses the relevant information in convenient symbolic representations (in pictorial as well as verbal forms). Although generation of this basic description is non-trivial, Heller and Reif restricted their attention to the second, more complex and interesting stage of the description process, the generation of a "theoretical description" of a problem.

A "theoretical description" of a problem is a description deliberately expressed in terms of special concepts and properties in the knowledge base for the problem domain (corresponding to the "abstract" or "scientific" problem representations in descriptive analyses). The procedures for generating theoretical descriptions of problems are determined largely by the contents and organization of the knowledge base. Therefore, the delineation of the knowledge base for any domain is an extremely important part of a prescriptive model of problem solving in that domain.

Reif and Heller (1982) suggest that the knowledge base for any scientific domain should specify the entities of interest in this domain, concepts for describing these entities, properties of these concepts, and principles and rules expressed in terms of these concepts.

In mechanics, for example, the knowledge base includes the following information:

-The entities of interest are particles or systems consisting of many such particles.

-Concepts for describing particles are of two kinds, according to Reif and Heller: concepts to describe <u>individual</u> particles (e.g., "mass" and "velocity"), and concepts to describe the <u>interaction</u> between particles (e.g., "force").

-Properties of these concepts include "interaction laws" which specify how concepts describing interaction are related to concepts describing motion. These interaction laws are specified for "short-range" interactions which occur when particles "touch" each other, and "long-range" interactions (such as gravitational interaction) which occur even if the interacting particles are separated by some distance.

-Finally, the knowledge base for mechanics includes important "motion principles" (such as Newton's famous "second law" $m\vec{a}=\vec{F}$) which specify how the motion of particles changes as a result of interactions between them.

The declarative knowledge in the knowledge base implicitly indicates the ingredients needed for the theoretical description of any problem in this domain: entities of interest must be described in terms of the concepts in the knowledge base, taking into consideration the properties of those concepts. This description must also conform with the constraints imposed by known principles in the domain. However, these general statements are too vague to be useful for an inexperienced problem solver who is asked to generate a particular description. The prescriptive model therefore specifies the <u>exact</u> steps by which the declarative knowledge should be applied to generate a theoretical description. That is, it specifies how to identify the particular entities to be described, how to apply the concepts to describe these entities, how to exploit properties of these concepts, and how to apply principles in the knowledge base to check that the description is self-consistent and correct.

The procedure for generating theoretical descriptions of any problem in mechanics is summarized in Table 1. (The procedure is preceded by identification of relevant times and systems, which will not be discussed here.) Descriptions are constructed separately for each relevant system, until all such systems have been described. For each system, two diagrams are drawn: one describing its motion and the other its interaction with other systems (that is, a diagram of the forces exerted on the system by all other systems, commonly referred to as a "free-body diagram"). An algorithm is provided for identifying all short- and long-range forces on a system. This algorithm stresses identification of systems responsible for exerting those forces. An example helps to make clear how this procedure is used to describe a problem.

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Table 1 about here

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Consider the problem in Figure 1. The description procedure would be applied to both block A and block B, yielding motion and interaction diagrams of each. For example, for block B, first a motion diagram is drawn to describe its velocity and acceleration (see Figure 2). We have been told that block B is pulled to the left with constant speed--hence a labelled arrow is drawn on a simple sketch of block B to indicate the velocity is to the left, and a note is made that the acceleration of the block is zero (it moves "with constant speed"). Then the interaction diagram is drawn to describe all forces on block B. To do this, first all objects which touch the block are identified, and all of the corresponding short-range forces exerted on B by these objects are indicated. As shown in Figure 2, these objects are the system pulling block B to the left, which exerts an applied force $\underset{\sim}{F}_a$; the string, which exerts a tension force $\underset{\sim}{T}$; the floor, which exerts normal and friction forces $\underset{\sim}{N}$ and $\underset{\sim}{f}$; and block A, which exerts normal and friction forces $\underset{\sim}{N}'$ and $\underset{\sim}{f}'$. Then the procedure identifies the long-range force $\underset{\sim}{F}_g$ exerted on block B by the earth.

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Figure 2 about here.

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Despite its seeming simplicity, this procedure is far from trivial. Application of the procedure ensures, in the following ways, that highly important declarative knowledge is systematically and correctly incorporated in the initial description of any problem, and that common errors are prevented.

The relation between motion and interaction is central to the science of mechanics. Accordingly, the description procedure requires that <u>both</u> the motion and the interaction of any particle be carefully described. By contrast, most physics textbooks (e.g., Resnick & Halliday, 1977) emphasize the need to describe forces, but not the corresponding need to describe motion. Expert problem solvers also tend to describe explicitly only forces, leaving motion description implicit. However, novices frequently neglect motion information (Heller & Reif, in press) which contributes to their inadequate performance on these problems.

In the science of mechanics, the concept of "force" is introduced to describe the interaction between objects. Correspondingly, the description procedure requires that interacting objects should be identified before the specification of particular forces describing their interaction. Hence the procedure guards against errors caused by students' prescientific conceptions (e.g., Viennot, 1979; Clement, 1982) in which forces are viewed as ultimate causal agents, producing effects independently of the existence of other objects.

The description procedure also incorporates the distinction between short-range and long-range interactions. It requires the enumeration of all "touching objects" in order to identify short-range forces--every object which touches another object must exert a force on that object. This procedure, while not evident in experts' solutions, comprises a very easy method for novices to use because the identification of objects touching a given system is trivial for human problem solvers. It can thus help prevent novices' common error of omitting short-range forces acting on a system.

Finally, the procedure exploits the motion principles in mechanics to check the correctness of problem descriptions. One such check requires that the descriptions of the motion and interaction of each system be qualitatively consistent with the fundamental motion principle $m\underset{\sim}{a}=\underset{\sim}{F}$, i.e., that the acceleration of a particle have the same direction as the total force on it. In order to perform this check, both the motion and interaction of each system must have been described explicitly, as required by the model.

The power of this checking procedure can be illustrated in the case of the problem of Figure 3. It is quite easy for subjects to determine that the acceleration of block C is directed to the right. However, subjects very frequently claim that the friction force on this block is directed to the left, because "friction opposes the motion" of an object and block C "moves" to the right. (In fact, friction opposes the <u>relative</u> motion of objects, and block C would move to the left relative to block A, in the absence of friction.) The checking procedure would immediately reveal that the direction of this force is inconsistent with that of the acceleration and must therefore be incorrect. Thus this check provides a reliable method for detecting and correcting the common error of ascribing the wrong direction to the friction force in this problem.

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Figure 3 about here

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## <u>Testing the Model of Problem Description</u>

The basic paradigm for testing a prescriptive model of human performance is to induce subjects to act in accordance with the model, and to observe whether their resulting performance is effective in the predicted ways. In the experimental procedure used by Heller and Reif, subjects were asked to carry out the description and subsequent solution of various problems by executing "external-control" directions that were successively read to them according to the program specified by the model. For purposes of comparison, a modified model was formulated which lacks selected features of the proposed model of good performance. The experiment could then reveal whether the particular features omitted from the original model were actually necessary for good performance.

It should be emphasized that the aim of such external-control experiments is to ascertain the merits of a proposed model of good performance, but not to teach. Subjects may, of course, learn incidentally while working under conditions of external control. However, such learning need not occur, because no effort is made to have the subjects internalize the directions. A subject, performing very well while working under external control, might revert to poor performance if that control were to be removed.

In the next sections we discuss the method and results of the Heller and Reif study. We first discuss the elaboration of the proposed model M, and of the modified model M*, into detailed programs. We then discuss the actual experiment which compared the performance of three groups of subjects: a group M guided by external-control directions based on the full model M, a group M* guided by similar directions based on the modified model M*, and a comparison group C working without any external guidance.

Elaboration of the models

The procedures in model M were elaborated into detailed external-control directions. The relevant factual knowledge was summarized in written form so that subjects could refer to it during problem-solving sessions.

The external-control directions for generating problem descriptions were supplemented with some additional directions to guide subjects' subsequent solutions of the described problems. These directions provided minimal guidance for generating and combining equations. They will not be discussed further.

The elaboration of the model (which was summarized in Table 1) into detailed directions is shown in the Appendix. The following activities were required to elaborate the model into directions which met a set of implementability criteria proposed by Heller and Reif.

To achieve interpretability, the steps in the description procedure were first expressed as easily comprehensible and natural-sounding directions. The steps in Table 1 thus yielded corresponding directions, such as to construct separate motion and interaction descriptions for each relevant system (steps 3, 4, 7, and 11 in the Appendix) and directions to check that these descriptions are qualitatively consistent with known mechanics principles (steps 17, 18, and 25).

Additional steps had to be introduced to ensure that steps would be executed in proper sequence. For example, the procedure of Table 1 describes that motion and interaction descriptions be constructed for each system, one system at a time, before the solution of a problem is attempted. Correspondingly, directions in the Appendix include specific steps to coordinate these activities in cycles of choosing a particular system (steps 1 and 2), describing the motion and interaction of this system, determining whether any other systems remain to be described (step 19), and branching accordingly (step 20).

Further steps were needed to provide adequate control over subjects' use of the declarative knowledge available to them. Pretesting revealed that subjects, even when prompted, would sometimes fail to use factual knowledge readily available in the summary provided to them. Accordingly, the directions included explicit mention of some especially important elements of declarative knowledge (steps 6, 9, and 10). These steps ensure that a general procedure is used in conjunction with appropriate declarative knowledge, e.g., that the procedure for describing forces is coupled with domain-specific knowledge about the types of forces exerted by various kinds of systems.

Finally, several additional steps were added as checks to ensure that previous steps had been performed completely and correctly. Such checks are necessary since human subjects tend to be fallible and distractable, prone to forget steps in a procedure or to disregard available information. These checks (steps 12-16 and 21-24) are in addition to the more general checks mentioned in Table 1 (corresponding to steps 17, 18, and 25).

A modified model, M*, was also developed to test whether selected procedures in model M were necessary for good performance. The modified model, designed to simulate somewhat the descriptive advice commonly found in physics textbooks, is considerably less complete and explicit than the proposed model M. The detailed differences between the elaborated versions of model M and model M* are exhibited in the Appendix. The differences between these versions are the following: (a) The full model M includes descriptions of both motion and interaction for every system, while model M* includes only a description of interactions. (b) The model M includes a detailed procedure specifying how to enumerate all forces on a system, while model M* includes only a direction to enumerate all forces on the system by other objects. (c) The full model M contains some explicit references to elements in the knowledge base (to some particular properties of motion and interaction), while such explicit references are omitted in model M*. (d) The full model M includes some powerful checks based on general physics principles, while the modified model M* does not include such checks.

Measures to Ensure Implementability

Since testing of the prescriptive model depends heavily on the comparison of performance guided by external-control directions, it is extremely important that these directions are implementable by the subject (i.e., that they are readily interpretable, actually executed, and implemented correctly). To ensure such implementability, the directions were pretested extensively with pilot subjects, and practice activities were designed to familiarize experimental subjects with the directions.

## Experimental Method

Problem tasks for assessing performance. Three approximately matching pairs of mechanics problems were selected from commonly used introductory physics texts (French, 1971; Resnick & Halliday, 1977; Symon, 1971) and reworded slightly for clarity. All of the problems could be solved by application of one fundamental motion principle, Newton's second law ($ma=F$).

The pairs of problems were split into two roughly equivalent sets. Half of the subjects in each group received one set as a pretest and the other set during the experimental treatment sessions; the other half of the subjects received these sets in the opposite order.

Subjects. The subjects in the experiment were 24 paid volunteers, all undergraduate students currently enrolled in the second course of an introductory physics sequence at the University of California, Berkeley. All subjects had received a grade of B- or better in the first course of this sequence, in which the physics principles of mechanics were studied. These subjects were randomly assigned to the three experimental groups, eight subjects to each group.

Procedure. Problems were individually administered to each subject in all sessions. All subjects solved one set of problems in a pretest session while working without external guidance. Before solving the second set of problems, subjects in groups M and M* were introduced to the experimental procedure of working under external control. These subjects then solved the problems under the guidance of directions read to them by the experimenter, while subjects in group C worked without such guidance. Subjects were asked to talk aloud about what they were thinking while solving the problems, and their verbalized statements were recorded with their permission. All subjects, during all sessions, had access to the printed summary of mechanics principles.

Subjects working with external guidance were read the standard directions from a script, one step at a time. Each direction had to be performed by the subject before the next one was read. As long as the subject implemented the direction, the experimenter considered that direction executed, regardless of whether it had been done correctly. (For example, during construction of a force diagram, subjects were directed to indicate all forces exerted on a system by other objects. If a subject described such a force in the wrong direction, the step was nevertheless considered executed. As another example, subjects in group M were asked whether any object, other than those already named, touched the system of interest. Even if the subject responded in the negative when there was another touching object, the step was nevertheless considered executed.) However, if instead of answering the question or performing the step, the subjects prematurely skipped to a later step, they were stopped and the direction was repeated until an appropriate response was made.

The methodology of external control was not always initially accepted by the subjects. A frequently encountered difficulty was the subjects' resistance to surrendering control to the experimenter. Many seemed determined to solve the problems in their own way, rather than to follow the directions given to them. They were then reminded that their solutions during the first session had already provided us with information about their own methods, and that we now needed their help to assess our methods. For the most part, this reminder was sufficient to enlist the subjects' cooperation.

Once their initial resistance had been overcome, many subjects became overtly positive in their response to the directions. Several remarked, with notable surprise, that these steps "really work" and that the problems seemed suddenly "easy" to solve.

Results

The adequacy of every solution was assessed with respect to four performance criteria: the adequacy of subjects' analyses of motion (as judged by assessing the information about acceleration in subjects' equations), the adequacy of subjects' analyses of interaction (also judged by examining equations), the adequacy of the set of equations generated (assessed by determining both whether the solution contained the minimal set of equations needed to determine a value for the wanted quantity, and whether all individual equations were correct), and the correctness of the final answer obtained. The data summarized in Figure 4 show the mean number of each subject's solutions (on the three problems solved during pretest or treatment sessions) that were correct on each of these measures. The data are summarized for subjects in each of the three treatment groups, and for all 24 subjects on the pretest. There were no significant differences between the various groups on this pretest, nor between these pretest results and the performance of the comparison group C in the treatment.

Figure 4 about here.

Are the procedures proposed by the model sufficient for producing successful solutions? The performance of subjects in group M, working under external control, indicates that they are. As is apparent from Figure 4, these students performed nearly perfectly: All of their solutions contained every required equation, and all of their equations contained correct and complete information about motion and interaction.

But could the subjects have performed just as well on the basis of their prior knowledge? The subjects' performance on the pretest, and the performance of the comparison group C, indicate that the subjects' prior knowledge was definitely not sufficient to solve these kinds of problems adequately. On the average, the subjects solved correctly less than one third of the pretest problems. Furthermore, less than one third of their solutions contained enough correct equations to achieve a solution, and less than one half of these solutions incorporated correct information about both the motion and interaction of the relevant systems. These results indicate that the knowledge students acquire as a result of ordinary instruction in an introductory mechanics course is not sufficient to endow them with the ability to solve fairly standard mechanics problems at the level of this course. (Or at least the knowledge is lost within two months of taking the course!)

Are all of the components of this model actually necessary? This question can be partially answered by comparing the performance of group M with the performance of group M*. As mentioned previously, if the knowledge components omitted from M* were in fact necessary for good performance, the observed performance of group M* should be less adequate than that of group M. In particular, since the differences between the models lay in the completeness and explicitness of procedures for constructing initial problem descriptions, the descriptions of motion and interaction by group M* would be expected to be inferior to those by group M. Correspondingly, the equations generated by subjects in group M*, and hence also the final problem answers obtained by them, should be less often correct than those generated by subjects in group M.

The data in Figure 4 reveal essentially this pattern of results. All results are statistically significant (using Kruskal-Wallis Test, $p < .01$), except in the case of motion description, where the performance of group M* was not significantly poorer than the perfect performance of group M. It thus appears that at least some of the components, deleted from model M to create the modified model M*, are indeed necessary for achieving good problem solutions.

## Relevance of the Prescriptive Approach

In this work, Heller and Reif formulated and validated a prescriptive theoretical model of some of the knowledge and procedures needed for good problem solving in mechanics. Particular attention was given to the generation of initial problem descriptions which facilitate the solution of such problems. The model included very specific procedures for generating "theoretical problem descriptions." These procedures specified how to describe the motion and interaction of systems in terms of special concepts in the knowledge base for mechanics.

The model was tested by a method of inducing human subjects to solve problems using the prescribed procedures. Results indicated that the model does lead to explicit and correct problem descriptions, and that these descriptions markedly facilitate the construction of correct problem solutions.

Descriptive analyses have shown that novices typically lack the kinds of strategic knowledge that were included in Heller and Reif's model--i.e., the meta-knowledge that it is important to describe a problem with care before attempting to solve it, explicit knowledge about what types of information should be included in an effective description, and explicit procedures for generating such descriptions. Experts usually possess these kinds of knowledge, although predominantly in tacit form, and the knowledge is rarely taught explicitly in physics courses. The work discussed here shows that such knowledge can be made more explicit and that, if used by students, it can strikingly improve their problem-solving performance.

What is the relevance of this work for problem-solving instruction? A prescriptive model can be thought of as a specification of the knowledge students should acquire as a result of instruction or, in Greeno's (1976) terminology, the "cognitive objectives" of instruction. Once such a model has been validated, showing that it does successfully lead to good performance, it is then appropriate to raise questions such as whether and how to teach students to internalize the knowledge in the model.

## INSTRUCTIONAL IMPLICATIONS

We will now outline some of the general instructional implications of the cognitive analyses of problem solving which we have discussed. By "instructional implications," we refer to reasonable conclusions about both what to teach, and how that information should be taught. As we will discuss, the research has more direct implications for the former than the latter, but the two questions are not entirely independent.

### General Remarks

The studies of novice performance in scientific domains have shown that, even after instruction, many students lack real qualitative understanding of the concepts and problems they encounter in science courses. They also do not know how to approach problems, or when to apply which formulas, and why. At the risk of stating the obvious, we would say that a first, very general instructional implication of these findings is simply that there is a strong need for more effective instruction, particularly to prepare students to solve problems with understanding.

Such instruction should be specifically tailored to prepare students to solve the kinds of problems they will encounter. For the design of such instruction, an understanding of the knowledge required for solving problems is of paramount importance. Greeno (1980) summarized this point well:

> To teach students how to solve a class of problems, first analyze the knowledge that they need in order to solve that class of problems, and then carry out instruction that will result in their acquisition of the

required knowledge. (p. 13)

This plan has begun to be implemented, and at this point both descriptive and prescriptive efforts have contributed powerful methods for identifying this knowledge. The application of these methods has revealed insights into the nature of the knowledge that is required in selected scientific and mathematical domains. In order to design effective problem-solving instruction, we should continue to apply these methods in different problem domains to identify the specific knowledge that would need to be taught in each of those domains.

Thirdly, this literature demonstrates the impressive quantity, level of detail, and complexity of the knowledge we must transmit to our students. It is somewhat overwhelming in this respect; it begins to seem amazing that some people do acquire and coordinate the diverse kinds of knowledge required for solving even fairly standard problems in technical domains. The studies and theoretical models consistently show that one cannot solve problems correctly without a large amount of domain-specific knowledge. When even a small amount of this knowledge is missing or removed from descriptive or prescriptive models, performance deteriorates dramatically. Instruction must somehow supply students with the extensive body of knowledge needed to solve problems, and must do so at a very fine level of detail.

Finally, the scientific problem-solving literature has brought to our attention, or underscored the importance of, particular kinds of knowledge which are needed for solving problems and thus need to be considered in the design of problem-solving instruction. We have learned that knowledge for understanding problems and concepts qualitatively is extremely important in problem solving yet is not learned by students in science classes. This has also been shown in studies of mathematical problem solving (see, for example, Paige & Simon, 1966, and Hinsley, Hayes, & Simon, 1978). Students are not learning to appreciate the importance of qualitative processes during problem solving. In fact, they seem instead to hold the serious misconception that mathematics and science are entirely quantitative fields, hence only quantitative reasoning is respectable (Patterson, 1983). We need to communicate better the great value of seemingly vague qualitative exploration, including construction of sketches and general solution plans prior to generation of equations. And we must teach specific procedures for accomplishing these aspects of problem solving, as well as for constructing solutions by selecting and applying formulas or principles. We have been shown, too, that knowledge about <u>when</u> to perform procedures is extremely important, and must be made explicit to students along with knowledge about <u>how to</u> perform them. Finally, the research shows that problem solving cannot be accomplished without an extensive, well-organized knowledge base of domain-specific facts. We need to find ways to communicate not only the contents of domain-specific knowledge, but also the form or organization of that knowledge that would facilitate its retention and retrieval.

We now turn to some more speculative remarks about how this knowledge might be taught.

## What Should Be Taught?

We have stressed in this paper the importance of identifying the knowledge which students need in order to be able to describe and solve problems. It is evident that there is a tremendous amount to know, hence a great deal to teach. The body of required knowledge includes knowledge for understanding problems, for constructing useful problem descriptions, and for selecting and applying principles or concepts to solve the problem. The knowledge for performing these activities includes meta-knowledge about the activities, and knowledge about when and how to perform them.

The answer to the question of what should be taught (or at least learned) is, as Greeno indicated, the knowledge required for solving the problem. This knowledge can, as we have shown, be explicated through descriptive and prescriptive analyses.

## How Should Problem-Solving Knowledge Be Taught?

Research has told us much less about how to teach problem solving than about what to teach and about how to determine what to teach. But these questions are not unrelated. We will now briefly consider some specific ideas about how to improve students' problem-solving skills, ideas which are consistent with our current knowledge about problem-solving knowledge.

Students need to become better able to reason qualitatively about problems, and to know when and how to perform the many component procedures required for solving problems. In order to encourage this learning, the students' attention should be turned to these particular activities. The following are some ways in which this might be accomplished.

Make tacit processes explicit. Traditionally, class lectures have been ineffective means for developing problem-solving skills. This is in part because problem solving is heavily procedural, thus is probably best learned "by doing" (cf. Anzai & Simon, 1979). However, we know that under certain conditions learning can occur through observation of a model performing the activity (Bandura, 1977. Instructors should be able to communicate some aspects of problem-solving knowledge in a large group setting, but only if they model in sufficient detail the process of building a solution. Too often, instructors jump from reading a problem statement to writing on the board an already completed solution, skipping the qualitative analyses, strategic decisions, and explanations of how and why each step was done. Missing too are the mistakes, tentativeness, and exploration that are all parts of problem solving. If these aspects of problem solving are modelled by instructors in greater detail, students would have at least a chance of learning some of these processes by observation.

Get students talking about processes. While observing an instructor modelling solution processes should be beneficial, more active involvement on the part of the student is very important. Students should be encouraged to generate those processes themselves, and to think about their own thought processes. One technique which has been found useful is to have the students solve a problem aloud, examine or observe a model solving the same problem using desired processes, and then discuss among themselves differences between their own and the model's procedures (Bloom & Broder, 1950). Repeated activities of this type should help develop in students explicit awareness of the processes involved in describing and solving problems.

Provide guided practice. Students typically have an opportunity to practice solving problems, usually on homework. But these experiences are rarely structured in a way that ensures that the right activities are being practiced. We suggest that guided practice is needed, such as could be provided by intelligent coaches (human or computer) which oversee the students' performance, interject comments when the student is performing in less than optimal ways, and provide suggestions for alternative methods. We use coaching to teach activities involving motor skills such as athletics and playing musical instruments--it is ridiculous to think of someone learning to shoot baskets or play the violin only by watching a skilled performer, and the close scrutiny and feedback on performance of a knowledgeable observer is known to be extremely useful for improving performance. It seems very likely that students would benefit from this kind of guidance when learning complex intellectual procedures as well.

Ensure that component procedures are well learned.

Carefully structured exercises could be used to help students develop the component procedures required for solving problems. What might such exercises look like? A promising approach is to provide students with problems which are partially solved, and have them practice performing selected subsets of the entire solution procedure (cf. Vygotsky, 1978). For example, with respect to generating equations for the physics problems we discussed, we could provide students with problem descriptions and have them practice applying principles to generate equations. By carefully designing these tasks, we could ensure that the students are exposed to the main kinds of situations which they should be able to handle. And by providing explicit guidance and prompt feedback on the basis of prescriptive analyses of knowledge, we could ensure that they were indeed practicing effective procedures.

Similarly, we could pose a variety of problems and ask only that the students draw diagrams which are appropriate for problems of each type. Again, with guidance and feedback, such exercises could develop specific components of the problem-solving knowledge students need to acquire.

Emphasize both qualitative understanding and specific procedures. We have stressed that instruction in problem solving should differ from traditional educational efforts in two main ways: There should be much more attention paid to qualitative reasoning, and there should be direct and explicit instruction of the component procedures and knowledge structures that are required for competent performance. But there are various ways in which these kinds of knowledge might be taught. For example, we could try to develop qualitative understanding of problems and concepts prior to attempting instruction in specific problem-solving procedures. Or we could first provide practice using such procedures in the hope that such experience would naturally bring about understanding of the problems. As Resnick (1983) explains,

> Research has not yet told us whether it is better to first become skillful at a procedure and then analyze it, or to allow procedures to grow out of understanding a situation... But research has made it clear that procedures must take on meaning and make sense, or they are unlikely to be used in any situation that is at all different from the exact ones in which they were taught.

Careful instructional research is needed to inform us about how to teach in a way that communicates the meanings of procedures to our students. However, we can reasonably conclude that both understanding and procedures need to be emphasized, and therefore recommend an iterative instructional strategy in which attention would shift frequently between practice of specific procedures and qualitative analysis of underlying concepts. Neither can be very meaningful or useful in the absence of the other, hence both must be stressed. We might, for example, carefully guide students through the performance of specific problem-solving procedures, but also intersperse frequent discussions about why these particular procedures are useful in this problem context, and how they relate to earlier and later procedures. By performing detailed procedures for interpreting and applying principles and concepts, it is possible that students will begin to understand better the general concepts involved in the problems. If they understand the general nature of the problems, students would have more reason to perform the procedures and may in fact be better at remembering how to perform them.

This combination of attention to qualitative reasoning and to performance of procedures is also needed to help students resolve conflicts between their conceptions and technically correct definitions and explanations. Specific feedback on details of their performance could flag such contradictions. Extended discussions may be needed to force students to confront the contradictions between their notions and scientific or mathematically correct conceptions (see Minstrell, 1982b, 1983).

Test for understanding and reasoning processes. Unfortunately, students are very resistant to giving attention to anything which is not "going to be on the test." While we may be convinced that the kinds of knowledge we have been discussing are crucial to performing well on tests involving problem solving, it is difficult to convince students that this is the case. An alternative would be to accept students' preoccupation with exams, as distasteful as it may be, and use this concern to motivate attention to the problem-solving skills we know are important. This would mean designing examinations to assess these kinds of knowledge rather than rewarding only quantitative performance. Taken to an extreme, this would mean grading only the reasoning underlying the process of solving the problem, or at least giving no "credit" for achieving correct answers unless evidence of qualitative understanding is included in the problem solutions. Test items could ask only for qualitative problem descriptions, or for identification and justification of a solution method, rather than an answer to the problem. We would stress, in effect, that the problem solution is not synonymous with the answer, and it is the solution with which we are most concerned.

## Some Pragmatic Comments

We have suggested a variety of techniques and approaches for effective instruction. Some of these techniques are currently practiced on a limited basis--for example, some instructors place strong emphasis upon both qualitative and quantitative aspects of subject matter, and some examinations are designed and graded for solutions, not just right answers. However, even these occasional efforts have probably been based on insufficiently detailed analyses of underlying knowledge.

The basic approach of analyzing knowledge and tailoring instruction to teach that knowledge would be extremely time-consuming and seems indeed overwhelming. It requires expertise and energy on the part of teachers and instructional designers, considerable effort by students, and perhaps even serious restructuring of curricula and courses. Such restructuring may be necessary for educational systems to place the goal of teaching for mastery over that of screening out all but the most able students (as frequently seems to occur at the undergraduate level, especially). Less material would be taught in greater depth, and the focus would be on students' progress, not on the quantity of material "covered." As an alternative to restructuring current courses, another option would be to introduce special problem-solving courses or labs, or to design supplementary materials for individual use (such as computer-based tutors). Special measures such as these might allow students to continue to be exposed to the large amount of basic factual information traditionally considered important in each discipline, while also learning how to solve problems that involve application of those facts and concepts.

Achieving a balance between traditionally stressed domain-specific facts, and the large body of procedures, strategic knowledge, and qualitative understanding called for by recent research, may indeed be a major pragmatic problem for the field of education. We know now that these kinds of knowledge are all critically important for competent technical problem solving. By both theoretical and pragmatic efforts, we need to find ways to bring them to our students.

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# APPENDIX

## External-Control Directions for Problem Description

The following are the detailed external-control directions used in our experiment to elaborate the proposed description model M and modified model M*. Alternative directions used for model M or M* are marked by the letters M or M*, respectively. Directions marked "M only" occur only for model M, but are omitted in model M*. Directions not specifically marked are common to both models. The letters E and S refer to actions by the experimenter or subject, respectively, with statements made by either put between quotes.

THEORETICAL DESCRIPTION OF SYSTEMS

M [ E: "Let's now draw diagrams describing each system of interest."

M* [ E: "Let's now draw diagrams describing the forces on each system of interest."

CHOICE OF PARTICULAR SYSTEM

E: "Which system . . . do you wish to consider (first)/(next)?" (1)

S: Names a system '<u>X</u>'.

<u>IF</u> **<u>X</u>** is a string or is not affected by interactions with other systems:

E: "There is no need to describe <u>X</u>." (2)
Return to step 1.

<u>ELSE</u> continue:

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1 MOTION DESCRIPTION

nly

E: "First draw a motion diagram of X, including any available information about its position, velocity, and acceleration relative to a convenient reference frame. If the velocity or acceleration is zero, indicate that on your diagram." (3)

S: Draws motion diagram of X.

E: "It is also useful to include on this diagram any known properties of the system, such as mass." (4)

IF previous systems have been described:

E: "Be sure to use convenient symbols and to relate them to those you've used previously." (5)

IF X has circular motion:

E: "Remember, the acceleration of a system in circular motion ordinarily, although not always, has two components, one tangential and the other toward the center of the circle. Check to be sure whether both components exist in this case." (6)

INTERACTION DESCRIPTION

E: "Now let's draw an interaction diagram for X, using the method I've suggested."

SHORT-RANGE FORCES

1 E: "First name each system that touches X, including those that exert applied forces. As you identify each system, indicate all external contact forces exerted on X by that system." (7)

1* E: "Draw a force diagram indicating the forces exerted on X by all other systems." (7')

IF previous systems have been described:

E: "Be sure to use convenient symbols and to relate them to those you've used previously." (8)

S: Names interacting systems ('Y') and/or indicates forces

1 only IF interaction with surface:

E: "Remember, the force exerted by a surface ordinarily, although not always, has two components, the normal force and friction force. Check to be sure whether both components exist in this case. (9)

The normal force is perpendicular to the surface and directed away from it. The friction force opposes the relative motion of the contact points--here it opposes the motion of X relative to Y." (10)

LONG-RANGE FORCES

E: "Name all external systems that directly interact with X without touching it or through any other physical contact. Then indicate the long-range forces exerted on X by each such system." (11)

S: Names system and/or indicates force.

CHECK: MISSING OR EXTRANEOUS FORCES

1 E: "Are there any other systems touching X?" (12)

1* E: "Are there any other forces on X by anything else?" (12')

S: "Yes" or "no".

IF yes:

1 E: "Draw the forces exerted by that (those system(s)." (13)

1* E: "Draw the forces." (13')

Return to step 12.

1 only ELSE continue:

E: "Are there any other systems directly interacting with X by long-range forces?" (14

S: "Yes" or "no".

IF yes:

E: "Draw the force exerted by that system." Return to step 14. (15)

ELSE continue:

E: "If not, you are finished describing all forces on X. Do not add any others. (16)

CHECK: CONSISTENCY BETWEEN MOTION AND INTERACTION

E: "The motion and interaction of the system must be consistent. In your diagrams, are the forces on X such that, with proper magnitudes, their vector sum can have the same direction as X's acceleration? Show me how you determine this. (You might want to check whether this is true by comparing components along convenient directions.)" (17)

S: Checks consistency; responds "yes" or "no" with explanation. Modifies description(s) if necessary.

E: "What would have to be true about the relative magnitudes of the forces on X for the acceleration and resultant force to have the same direction?" (18)

S: Describes required relative magnitudes of forces.

REPETITION OF DESCRIPTION FOR EACH SYSTEM

E: "Have all systems of interest been described yet?" (19)

S" "Yes" or "no".

IF no:

Repeat theoretical description procedure, beginning at step 1. (20)

ELSE continue:

CHECK OF ENTIRE DESCRIPTION

E: "After describing all systems, it's useful to double-check your work. Let's run through a checklist to make sure you haven't missed anything."

CHECK: CHOICE OF USEFUL SYMBOLS

E: "All arrows should be labeled." (21)

S: Checks arrows.

E: "Except for the gravitational force (which may be expressed as "mg"), or any magnitudes actually given in the problem statement, the values of quantities should not be evaluated at this time. Symbols like "F," "T," and "N," with subscripts, should be used instead." (22)

S: Checks symbols.

E: "Look at the symbols in all of your diagrams. Wherever different symbols have been used, the values of these quantities should actually be unrelated. If values are the same or simple multiples, use the same symbol. If values are unrelated, different symbols should be used." (23)

S: Checks symbols.

CHECK: USE OF ALL INFORMATION IN PROBLEM

E: "All information specified in the problem should be incorporated in your analysis. Please reread the problem carefully to make sure you have considered all the given information. In particular, make sure you've obtained from the problem all available information about the magnitude and direction of the velocity and acceleration of each system." (24)

S: Rereads problem statement. Modifies descriptions if needed.

CHECK: EXPLOITATION OF CONSTRAINTS (MUTUAL FORCES)

M only

E: "Check to make sure that all action-reaction pairs of forces are described as equal in magnitude and opposite in direction. For example, if systems A and B interact, the force of A on B in your diagram of B should be opposite in direction but should have the same magnitude as the force of B on A in your diagram of A. Look for forces between each pair of systems and check that they are described right." (25)

S: Checks forces.

Table 1

Procedure Generating a Theoretical Problem Description in Mechanics

---

* Description of relevant systems: At each relevant time, describe in the following way each relevant system (if simple enough to be considered a single particle), introducing convenient symbols and expressing simply related quantities in terms of the same symbol:
  * Description of motion: Draw a "motion diagram" indicating available information about the position, velocity, and acceleration of the system.
  * Description of forces: Draw a "force diagram" indicating available information about all external forces on the system. Identify these forces as follows:
    * Short-range forces: Identify every object which touches the given system and thus interacts with it by short-range interaction. For each such interaction, indicate on the diagram the corresponding force and all available information about it.
    * Long-range forces: Identify all objects interacting with the given system by long-range interactions. (Ordinarily this is just the earth interacting by gravitational interaction.) For each such interaction, indicate on the diagram the corresponding force and all available information about it.
* Checks of description: Check that the descriptions of motion and interaction are qualitatively consistent with known motion principles (e.g., that the acceleration of each particle has the same direction as the total force on it, as required by Newton's motion principle $m\underset{\sim}{a} = \underset{\sim}{F}$).

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## Figure Captions

Figure 1. Sample mechanics problem used in introductory physics courses.

Figure 2. Problem involving two sliding blocks connected by a string, with motion and force descriptions of block B. (Forces frequently omitted by subjects are indicated by dashed arrows.)

Figure 3. Problem involving three blocks, with motion and force descriptions of block C. (The friction force f, indicated by a dashed arrow, is frequently ascribed the wrong direction, i.e., to the left.)

Figure 4. Graph of the mean number of solutions (out of three) with correct performance on specified measures.

Figure 5. Integration problem involving finding the area bounded by curves.

Figure 6. Examples of student errors drawing graphs for problem in Figure 5.

## Sliding Blocks Problem

Two blocks A and B are connected by a light flexible string passing around a frictionless pulley of negligible mass. Block A has a mass $m_A$ and block B has a mass $m_B$. The coefficient of sliding friction between the two blocks, and also between block B and the horizontal table below it, has a value u. What is the magnitude $F_0$ of the force necessary to pull block B to the left at constant speed?

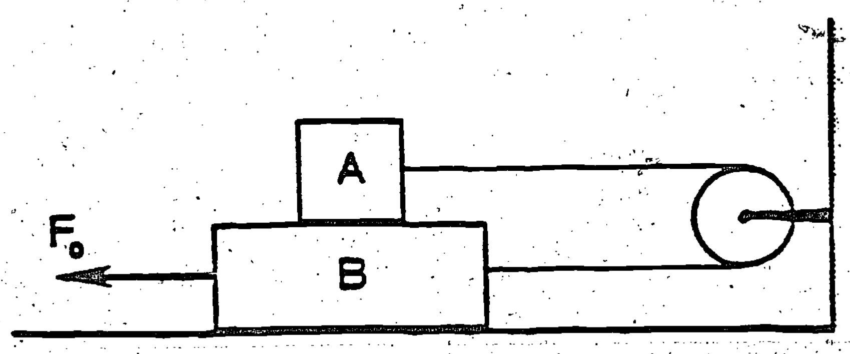



Figure 1

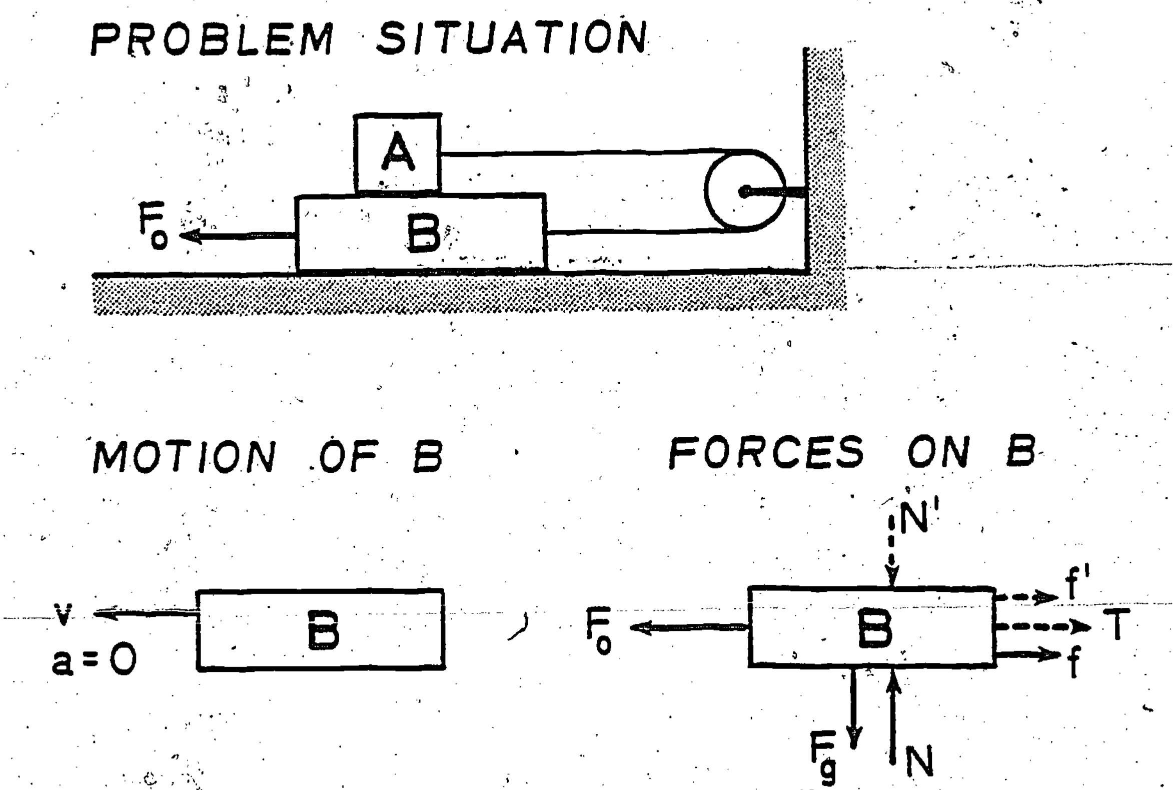
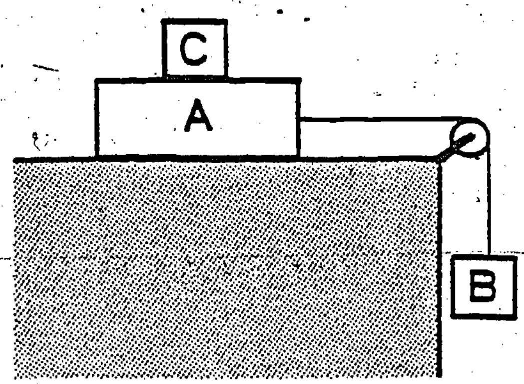
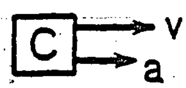

PROBLEM SITUATION
A
B
$F_o$
MOTION OF B
B
v
a=0
FORCES ON B
B
N'
f'
T
f
$F_o$
$F_g$
N

Figure 2

PROBLEM SITUATION




MOTION OF C




FORCES ON C

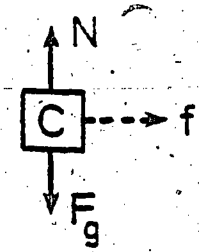



Figure 3

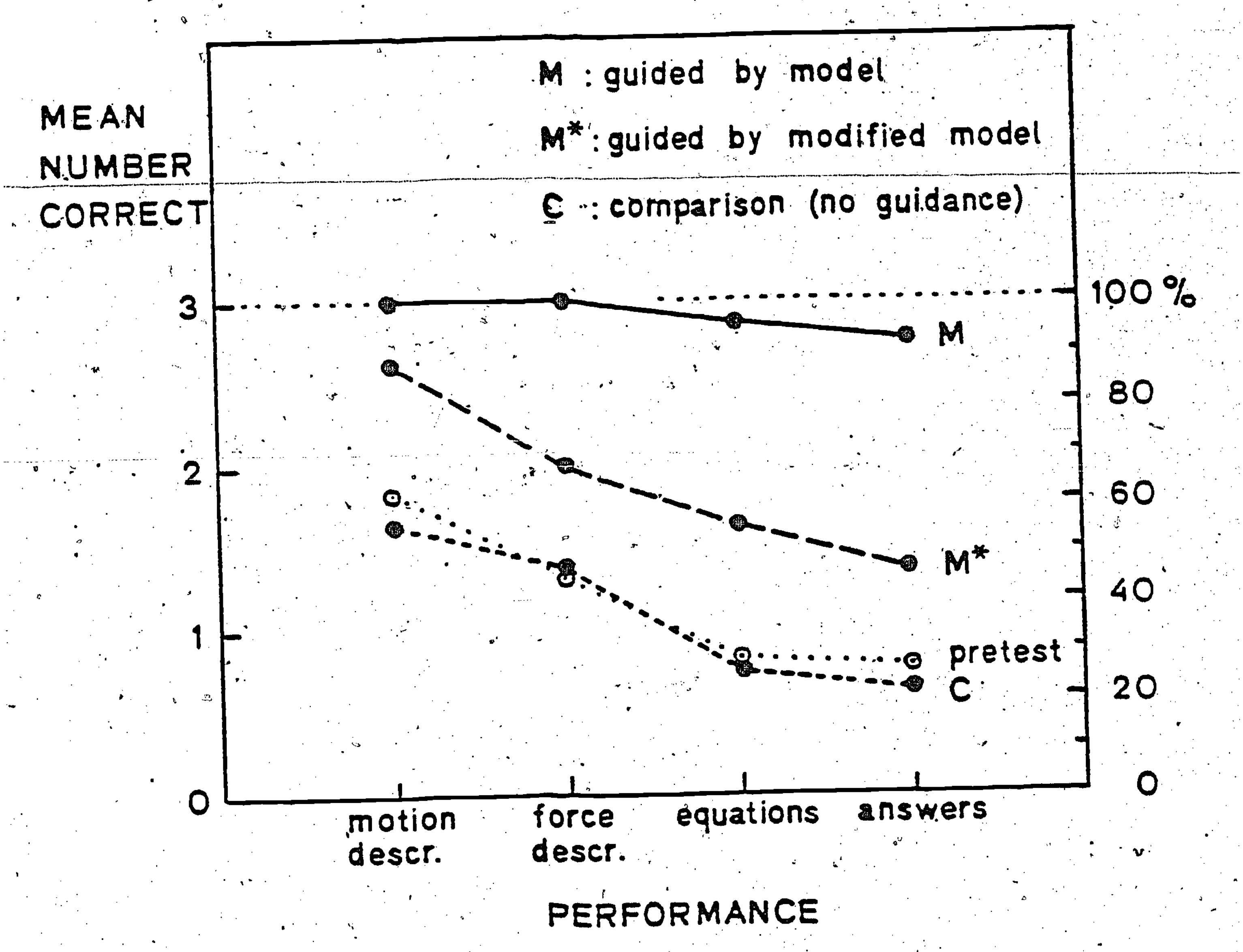

M : guided by model
M*: guided by modified model
C : comparison (no guidance)
MEAN
NUMBER
CORRECT
3
2
1
0
100 %
80
60
40
20
0
M
M*
pretest
C
motion descr.
force descr.
equations
answers
PERFORMANCE



Figure 4