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ABSTRACT

The data analysis problem posed by a repeated measures design that includes a single observation on a covariate for each subject is considered. The current paper discusses how best to capture a possible dependence of the effect of the within-subject factor on the level of the covariate. Procedures originally explicated by Rogosa (1980) for dealing with heterogeneity of regression in the between-subjects case are extended to this repeated measures situation. We conclude that such pick-a-point and simultaneous inferential procedures not only provide more powerful overall tests of the within-subject effects but also permit a thorough analysis of the attribute-by-treatment interaction implied by a significant regression of the within effect on the covariate. (Author)

ANCOVA and Repeated Measures:

Dealing with Heterogeneity of Regression

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Abstract

The data analysis problem posed by a repeated measures design that includes a single observation on a covariate for each subject is considered. The current paper discusses how best to capture a possible dependence of the effect of the within-subject factor on the level of the covariate. Procedures originally explicated by Rogosa (1980) for dealing with heterogeneity of regression in the between-subjects case are extended to this repeated measures situation. We conclude that such pick-a-point and simultaneous inferential procedures not only provide more powerful overall tests of the within-subject effects but also permit a thorough analysis of the attribute-by-treatment interaction implied by a significant regression of the within effect on the covariate.

ANCOVA and Repeated Measures: Dealing with Heterogeneity of Regression

Since almost by definition educational researchers are interested in examining progress, it is not surprising that repeated measures designs are among the most commonly used in educational research. In addition, individual differences are frequently a component if not the focus of educational studies. It is therefore important that researchers be facile at analyzing designs that incorporate both repeated measures and individual difference variables. Unfortunately, this is an area in which there has been confusion and controversy regarding appropriate methods of analysis.

Although it is well known that the repeated measures design itself "controls" for individual differences in level of performance on the dependent variable, it has not generally been viewed as a vehicle for specifically examining individual differences. It is possible however to combine a repeated measures analysis with an analysis of covariance as a means of incorporating individual difference variables. Recent papers on the subject by Ceurvorst and Stock (1978), Delaney and Maxwell (1981), and Algina (1982) have clarified some of the issues involved in using such an approach. But these papers have not made clear how best to capture the possible dependence of the effect of the repeated measures factor on the characteristics of the individual taking part in the study. Thus, our principal concern in the current paper is with describing how one can most effectively utilize the information available when an individual difference variable or covariate has been included in a repeated measures design.

Naturally, the conclusion one draws about what analysis is most appropriate depends upon the assumptions made at the beginning about the structure of the problem. In the traditional univariate approach to analyzing repeated measures designs (Keppel, 1982, p.367 ff), in addition to the restrictive assumptions made about interrelationships among the dependent variables, it is also typically assumed that the slope of the regression of the dependent measure on the covariate is the same for all measures (cf. Ceurvorst and Stock, 1978). Thus when computing "adjusted effects" the same adjustment would be made on each of the dependent measures, and in contrasts assessing the within-subject effect such adjustments would drop out entirely.

An advantage of the multivariate approach to repeated measures designs (e.g. McCall & Appelbaum, 1974) is that the interrelationships among the dependent variables as well as between each dependent variable and the covariate are not constrained. One implication of this assumption is that it is possible to make adjustments, via analysis of covariance (ANCOVA), of effects involving the within-subjects factor (Delaney & Maxwell, 1981). Such effects are assessed in the multivariate approach by forming combinations of the dependent variables to represent contrasts of interest in the levels of the within-subject factor(s).

Testing whether the grand mean(s) of the new variable(s) equals zero assesses the main effect of the within-subject factor. Further, a covariate can be included in the model to remove variability in the trend variable(s) predictable by these previously observed individual differences among subjects. Delaney and Maxwell (1981) showed that, under the typical ANCOVA assumptions, one can thereby achieve a valid test of the within-subjects factor that will generally be more powerful than the unadjusted test. However, it was noted that the proportional reduction in error variance will usually not be as great as in the between-subjects design, in part because of the unreliability common to difference scores. In order to correct an erroneous example of this type of analysis offered by Ceurvorst and Stock (1978), the technical point was also noted that the covariate needed to be "centered" or expressed in deviation score form in order for the estimate of the intercept in the ANCOVA situation to correspond to the estimate of the grand mean in the unadjusted test.

In our previous paper we mentioned the fact that the regression of the trend variable on the covariate could be viewed as an indication of an attribute-by-treatment interaction (ATI). However, we did not develop this point in detail, and now see the need for procedures to specify the nature of the ATI. Fortunately, a set of analytic procedures developed for dealing with heterogeneity of regression in between-groups designs can be fruitfully extended to the repeated measures case. It is the primary purpose of the current paper to detail how this can be done. The paper will begin with consideration of the simplest possible within-subjects design with a covariate, and then deal with more complex designs.

Throughout the paper we will be making the assumption typically made in ANCOVA that the covariate, X, is fixed. This does not mean that the values of X must be specified by the experimenter in advance, but rather that the inferences are made to subpopulations of subjects having the same values on X as those observed. If this assumption is not made, then certain of the parameter estimates of interest will be less precise (see Algina, 1982). The only distributional assumptions we require are that the residual errors associated with the trend variable be independently and identically distributed as normal random variables, or in the case of multiple trend variables, that they jointly follow a multivariate normal distribution.

Totally Within Design

Consider first the simplest possible repeated measurement design with a covariate. Assume a single score is available for each of a group of subjects on a covariate, X, as well as on each of two dependent variables, Y_1 and Y_2. Y_1 and Y_2 must at least be commensurate and will typically be scores on the same conceptual variable assessed at two different points in time. For example, an investigator might assume a linear relationship between a child's age (X) and performance on a problem-solving task, with problem-solving being assessed before and after instruction to yield two scores, Y_1 and Y_2. The primary questions of interest in such a design would likely be whether there is an effect of instruction (and/or practice) and whether this effect depends on the subject's age. That is, is there a main effect of instruction, and is there an interaction of instruction with age?

One might view the problem as involving the comparison of two regression lines, that of Y_2 on X and that of Y_1 on X. Denote these regression models as follows:

$$Y_2 = a_2 + b_2x + e_2;$$
$$Y_1 = a_1 + b_1x + e_1. \qquad (1)$$

where we use a lower case x as a reminder that the covariate should be expressed in deviation score form. Here of course the subscript designates the dependent variable; were it to designate groups of subjects then a test of the interaction of age with instructions would correspond to the test of heterogeneity of regression in a between-groups ANCOVA. In the present context however since the same subjects serve in both conditions the errors, e_1 and e_2, will be correlated and any test must take this into account. The appropriate correction is easily accomplished by simply computing differences between the observations in the two conditions, i.e. $Y_D = Y_2 - Y_1$, and using this as the dependent variable. Use of such a variable is prototypical of the "multivariate approach" to repeated measures (McCall & Appelbaum, 1974). Then we have the single regression model

$$Y_D = a_D + b_D x + e_D \qquad (2)$$

Note that $b_D = b_2 - b_1$ and $a_D = a_2 - a_1$. Further, since X is such that $\bar{x} = 0$, $\hat{a}_D = \bar{Y}_D = \bar{Y}_2 - \bar{Y}_1$. Since $\Sigma e_D^2 = (1 - r_{xY_D}^2) \Sigma (Y_D - \bar{Y}_D)^2$, one can perform a test of the within-subject main effect (by testing for whether a_D is equal to zero) that will likely have greater power than would the unadjusted test. Admittedly, because of the unreliability of difference scores and the fact that typically covariates will predict even true change considerably less well than final level of perfomance, the gain in power resulting from using the covariate will typically be less than in between-subject designs with the same variables. Nonetheless, there are conditions where ρ_{xY_D}, and hence the gain in power in the within-subjects analysis, will be substantial (Delaney & Maxwell, 1981).

Two points about such a test are deserving of comment. First, the test involves a conditional inference. Statistical inferences are restricted to subpopulations having the particular same values of X. The test just described which concerned the mean of the subpopulation of Y_D values having an X score equal to $\overline{X}$ is illustrative of such a conditional inference. This test of the significance of the change for a typical individual will usually be of most interest. However, as Rogosa (1980) has made clear one could "pick-a-point" other than $\overline{X}$ at which to perform the significance test. For some investigators this flexibility will not be an attraction - simply the benefit of increased power of testing the within-subject factor will more than offset the cost of restricting the inference to a conditional one. This would almost certainly be the case in situations like the example given above where the ages of the children would in fact likely be chosen in advance, and thus making an inference conditional upon those ages would be the intent of the investigator.

The second point concerns an issue about which varying opinions have been expressed by methodologists. We have said that the regression of the difference score on the covariate, e.g., as indicated by a test of b_D in (2) above, needs to be substantial in order for there to be substantial benefit from using the covariate. However, a substantial regression here means that there is violation of "the assumption of homogeneity of regression" in that it indicates a difference between b_1 and b_2. Some have argued in this context, e.g., Algina (1982), that the unadjusted test of the within main effect is preferable to the adjusted test, in part because the latter goes against the principle that the main effect of a factor is meaningful only when it does not interact with other factors. While a world with no interactions would be simpler (and duller), ignoring them when they exist is not the optimal strategy. We would opt instead for conducting conditional tests in this situation, and agree with Rogosa (1980, p. 308) that one can meaningfully interpret the average distance between two regressio.. .ines even when the slopes differ.

Following Rogosa's notation, let the difference between the population regression lines at a specific value of X be denoted $\Delta(x_i)$ which may be written

$$\Delta(x_i) = (a_2 - a_1) + (b_2 - b_1)x_i = a_D + b_D x_i$$

and which is estimated by

$$D(x_i) = \hat{a}_D + \hat{b}_D x_i.$$

Denoting the residual variance in model (2) as

$$\sigma^2 = (1 - \rho^2_{XY_D})\sigma^2_{Y_D} = (1 - \rho^2_{XY_D})(\sigma^2_1 + \sigma^2_2 - 2\rho_{12}\sigma_1\sigma_2),$$

where the numerical subscripts refer to the original variables Y_1 and Y_2, then the variance of the sampling distribution of $D(x_i)$ may be expressed here as

$$\sigma^2_{D(x)} = \sigma^2 \left[\frac{1}{n} + \frac{x_i^2}{\Sigma x_i^2}\right] \qquad (3)$$

and may be estimated simply by substituting for σ^2 the sample mean square error from model (2), i.e.

$$\hat{\sigma}^2 = s^2 = \frac{\Sigma e_D^2}{n - 2} = \frac{\Sigma\,(Y_D - \hat{a}_D - \hat{b}_D x)^2}{n-2}$$

Since (3) will be a minimum when x is at its mean of 0, one can interpret the test of the intercept of model (2), as being an evaluation of the difference between the two regression lines in (1) at the point where the estimate of that difference has the greatest precision.

Although one could perform and meaningfully interpret such a test as one of the treatment effect for the average individual, typically one would want to pursue further the nature of the attribute-by-treatment interaction implied by the regression of Y_D on x. Our earlier discussion of this general problem (Delaney & Maxwell, 1981) did not give sufficient attention to this point. One would like to perform the equivalent of tests of the simple main effects of the treatment that are frequently employed as follow-up tests of interactions in completely crossed factorial designs. This could be done here by constructing confidence intervals around the conditional means, $\hat{\mu}_{Y_D|x_i}$, for the X values observed in the study. The $1 - \alpha$ interval for this mean is bounded by

$$D(x_i) \pm t^{\alpha}_{N-2}\, \hat{\sigma}_{D(x)} \qquad (4)$$

Thus, analogously to the nonsimultaneous region of significance typically used in Johnson-Neyman analyses of ATI's in between-subjects designs, one can define a region composed of points on the X-axis for which a Y_D value of zero is outside the confidence interval at that x_i. This would allow one to make statements about one's confidence of the presence of the treatment effect for a particular value of X. Note that because of the dependence of $\sigma_{D(x)}$ on how far the particular X score is from $\bar{X}$ it is possible that estimated values of $\mu_{Y_D|x}$ which are larger in absolute value than a significant $\hat{a}_D$ may be judged nonsignificant because of the widening confidence bands as you move away from $\bar{X}$.

Alternatively, one could opt for a simultaneous inferential procedure which would allow one to make statements about the reliability of the difference between $\mu_{Y_D|x}$ and 0 for a whole set of X values. The Working-Hotelling formula for the confidence band for the entire regression line may be used so that there will be a known level of assurance that all estimates of the conditional means are correct (Neter & Wasserman, 1974, pp. 149-154). Here the 1-α simultaneous confidence band would consist of the concatenation of the intervals each of which is bounded by

$$D(x_i) \pm W \hat{\sigma}_{D(x)}$$

where

$$W^2 = 2F^{\alpha}_{2,n-2}.$$

Between X Within Case

Now consider a repeated measures design that also involves a between-subjects factor as well as a covariate. Again we will for simplicity restrict the discrete factors to two levels each. The previously cited example could easily be expanded for this case. Assume that children of various ages are randomly assigned either to a treatment condition (T) designed to motivate them to do well in the upcoming task, or to a control condition (C) where the initial instruction is neutral. As before, all children's baseline and final performance are then observed on a problem solving task.

Now one might view the problem as involving four regression lines

$$Y_{T2} = a_{T2} + b_{T2}x + e_{T2}$$

$$Y_{T1} = a_{T1} + b_{T1}x + e_{T1}$$

$$Y_{C2} = a_{C2} + b_{C2}x + e_{C2}$$

$$Y_{C1} = a_{C1} + b_{C1}x + e_{C1}$$

As before the numerical subscripts refer to levels of the within-subjects factor, and the letter subscripts now designate levels of the between-subjects factor. And, as before, it is convenient to work with transformed scores to obtain the tests of interest. First consider the between-subjects effects. Letting $Y_S = Y_1 + Y_2$, then the between-subjects effects could be assessed in the context of a model which allows for heterogeneous slopes:

$$Y_{ST} = a_{ST} + b_{ST}x + e_{ST}$$
$$Y_{SC} = a_{SC} + b_{SC}x + e_{SC}$$

where $b_{ST} = b_{T2} + b_{T1}$; $b_{SC} = b_{C1} + b_{C2}$; $a_{ST} = a_{S2} + a_{S1}$; and $a_{SC} = a_{C1} + a_{C2}$. Rogosa (1980) has fully developed such tests (these are illustrated below for the within-effects) and the arguments for them, which include the fact that any heterogeneity of slopes across the levels of the between-subject factor results in the test statistic for the conventional ANCOVA being distributed as a non-central F. If the evidence for heterogeneity is sufficiently weak that you choose to assume it non-existent, then the typical ANCOVA using a pooled within group slope estimate could be used:

$$Y_{ST} = a_{ST} + b_S x + e_{ST}$$
$$Y_{SC} = a_{SC} + b_S x + e_{SC}$$

The analysis of effects involving the within-subjects factor would as before utilize difference scores. Two possible outcomes indicating different kinds of heterogeneity are possible here. First a significant regression of the difference scores on the covariate would indicate, as we discussed in our treatment of the totally within design, an ATI involving the covariate and the within subjects factor. In this event, we would suggest evaluating the effect of the within factor, not only at $\overline{X}$ but also at different points along the X dimension, as outlined above. The only difference would be in the degrees of freedom used to estimate residual error variance and hence determine the critical values of the test statistics for the confidence intervals.

Secondly, a difference across levels of the between-subjects factor in the slopes of the regression lines would indicate a three-way interaction involving the between factor, the within factor and the covariate. Not surprisingly, there are a variety of ways one could proceed to specify the locus of the three-way interaction. Perhaps most straightforward would be to use Rogosa's methods for testing the vertical difference between two regression lines, keeping in mind that the dependent variable for the analysis is itself a difference score across levels of the within-subject factor. Thus one would be examining the two within-group regression lines:

$$Y_{DT} = a_{DT} + b_{DT}X + e_{DT}$$
$$Y_{DC} = a_{DC} + b_{DC}X + e_{DC} \qquad (5)$$

The difference between the sample regression lines at any point on X, $D(x_i)$, would here equal

$$D(x_i) = (\hat{a}_{DT} - \hat{a}_{DC}) + (\hat{b}_{DT} - \hat{b}_{DC})x \qquad (6)$$

Non-simultaneous inference procedures would be used to assess the difference at a particular value of x, such as the grand mean on x. Here one would estimate the variance of the sampling distribution of $D(x_i)$ by using the following expression:

$$\hat{\sigma}^2_{D(x_i)} = s^2\left[\frac{1}{n_T} + \frac{1}{n_C} + \frac{(x_i - \bar{x}_T)^2}{\Sigma(x_i - \bar{x}_T)^2} + \frac{(x_i - \bar{x}_C)^2}{\Sigma(x_i - \bar{x}_C)^2}\right]$$

where s^2 is the pooled residual error variance estimate from the model in (5). The critical value of a t test of $D(x_i)$ would now be based on N-4 degrees of freedom. A significant difference would be interpreted to mean that for the subpopulation of people having that X score, the within-subject effect in the treatment condition would be reliably different from the effect of the within factor in the control condition. Simultaneous inferences for a range of values of X could be made (Rogosa, 1980) by constructing confidence intervals centered at $D(x_i)$ and bounded by

$$D(x_i) \pm W \hat{\sigma}_{D(x)} \qquad (7)$$

where

$$W^2 = 2F^{\alpha}_{2,n-4}.$$

The method just described for analyzing the three-way interaction essentially examined the simple two-way interaction of the between and within factors at particular levels of the covariate. Alternatively, one could look at simple effects within the levels of a different factor.

For example, one might proceed by examining the simple interaction of the within-factor and the covariate at each level of the between-subjects factor. This would involve tests made separately on the two equations in (5), using the techniques for the totally within design separately for each. This would allow specification of the particular X values for which a significant effect of the within factor was observed for the treatment subjects, and a different set of X values for which the within effect was observed for controls.

Finally, one might look at tests within levels of the repeated measures factor. This would imply analysis of the regression of the original dependent measures (instead of their sum or some other linear combination) on the covariate. Tests of heterogeneity of these regression lines across groups would then be performed separately for each repeated measure, and would be interpreted as simple interactions of the between subjects factor with the covariate at levels of the within subjects factor. The choice among these methods of analyzing the three-way interaction would be guided by considerations of what results are most interpretable in the particular context of any study.

Extensions

Although we will not develop them here, the methods discussed in this paper can be extended to larger designs. With just two levels of the within-subjects factor, univariate tests similar to those we have discussed can be used with designs involving more between-subjects factors. The true multivariate situation arises when there are more than two levels of the within-subject factor. In that case, methods utilizing confidence intervals for predictions in multivariate multiple regression (see Finn, 1974, p. 121ff.) can be used to generate analogous procedures to those discussed here.

Examples

Table 1 contains two sets of hypothetical data that will be used to illustrate the procedures we have described. The data for Group C correspond to the example of the totally within design we discussed initially, i.e. two problem solving scores, Y_1 and Y_2, are available for each of a set of children of different ages, X. The equivalent of a matched-pairs t-test comparing the means of Y_1 and Y_2 does not reach significance, $F(1,9) = 3.807$, $MS_e = 34.044$, $p > .05$. However, when the test of this within-subjects factor is made more sensitive by including

$X-\bar{X}_C$ as a covariate in the model, the adjusted test of the within effect is significant, $F(1,8) = 7.834$, $MS_e = 16.544$, $p < .025$. The sample regression lines, which are shown in Figure 1 along with a scatterplot of the data, indicate the form of the ATI that results in the conditional test of the mean change being more powerful. The effect of the within-subjects factor in Group C is seen to decrease as age increases. Thus, further tests of the effect of the within-subjects factor at particular points on X would be of interest and can be conducted by forming confidence intervals, as in equation (4) above, around the difference in regression lines. These intervals are sketched in Figure 2 where the solid lines indicate the boundaries of the non-simultaneous confidence intervals and the dashed lines indicate the boundaries of the simultaneous intervals. Using non-simultaneous intervals one would conclude a significant within effect at X values of 3, 4 and 5. Using simultaneous bounds, so that assertions can be made at a specified α about the conditional means for all values of X, results in being able to conclude a significant within effect only at ages 3 and 4.

We may now illustrate the between x within analysis by combining the data just analyzed with that labelled Group T in Table 1. Scatterplots and regression lines for the Group T data are shown in Figure 3. The regressions of the difference scores on X for the two groups are shown in Figure 4. A standard ANCOVA of these difference scores would have resulted in less sensitive tests of the within effects than the unadjusted test, because there is no regression overall of the difference scores on X, $F(1,17) = 0$, i.e. there is no overall interaction of the covariate and the within effect. However, this obtains because the simple interactions of the covariate with the within factor differ across levels of the between-groups factor. A test for heterogeneity of regression is highly significant, $F(1,16) = 28.559$, $MS_e = 11.699$, $p < .001$, indicating there is a prominent three-way interaction. As we have indicated, a number of different approaches to further analyses are possible in this situation. We illustrate here the method of examining the difference between the regression lines at different points on X. The difference between these regressions (of the form of equation (6) above) is indicated by the dashed line in Figure 4. Both simultaneous confidence intervals (cf. equation (7)) and non-simultaneous confidence intervals around this line would lead one to conclude here that the effect of the within factor is significantly greater in the treatment condition than the control condition for ages 5, 6 and 7. The simple interaction of the between and within factors is non-significant for ages 3 and 4. Finally, one might wish to follow up these tests with still further analyses, e.g. of the "simple simple" effects of instructions at particular ages within levels of the treatment factor. These could be carried out by constructing the appropriate confidence intervals around the regressions of the difference scores on X, as illustrated in Figure 2.

Conclusion

We have discussed a method of analyzing repeated measures designs involving a covariate. Essentially, the method involves viewing the effect of the within-subject factor as a linear function of the covariate. We conclude that this approach to repeated measures designs permits not only more sensitive overall tests of the effect of the within-subject factor but also a thorough analysis of the ATI implied by a significant regression of the within effect on the covariate.

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Table 1

Data, Summary Statistics, and Sample Regression Equations

	Group C				Group T			
	X	Y_1	Y_2	Y_D	X	Y_1	Y_2	Y_D
	3	10	20	10	3	10	17	7
	3	12	22	10	3	13	17	4
	4	21	32	11	4	16	23	7
	4	25	30	5	5	20	25	5
	5	30	35	5	5	28	39	11
	5	36	30	-6	5	29	36	7
	6	38	40	2	6	40	52	12
	6	40	43	3	7	37	50	13
	7	51	51	0	7	53	70	17
	7	59	55	-4	7	62	80	18
Mean	5.0	32.6	35.8	3.6	5.2	30.8	40.9	10.1
S.D.	1.5	15.9	11.5	5.8	1.5	17.3	21.9	4.9

Regression Equation (Group C)	SS_{error}	Regression Equation (Group T)	SS_{error}
$Y_2 = 35.8 + 7.45\ (X - \bar{X}_C)$	81.6	$Y_2 = 40.9 + 12.93\ (X - \bar{X}_T)$	716.0
$Y_1 = 32.2 + 10.40\ (X - \bar{X}_C)$	100.4	$Y_1 = 30.8 + 10.20\ (X - \bar{X}_T)$	436.7
$Y_D = 3.6 - 2.95\ (X - \bar{X}_C)$	132.4	$Y_D = 10.1 + 2.72\ (X - \bar{X}_T)$	54.8

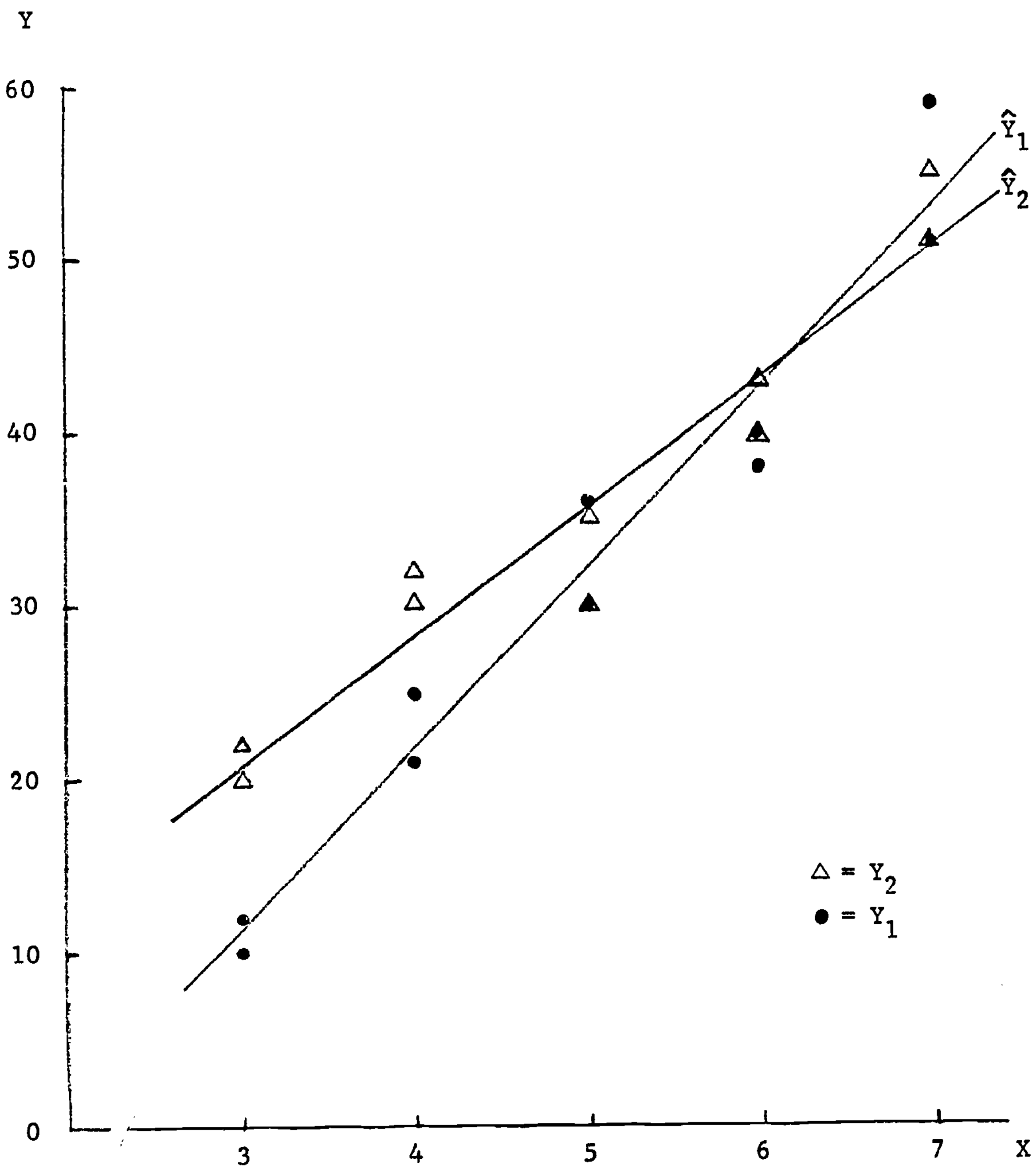

Figure 1. Regression of Y_1 and Y_2 on X in Group C.

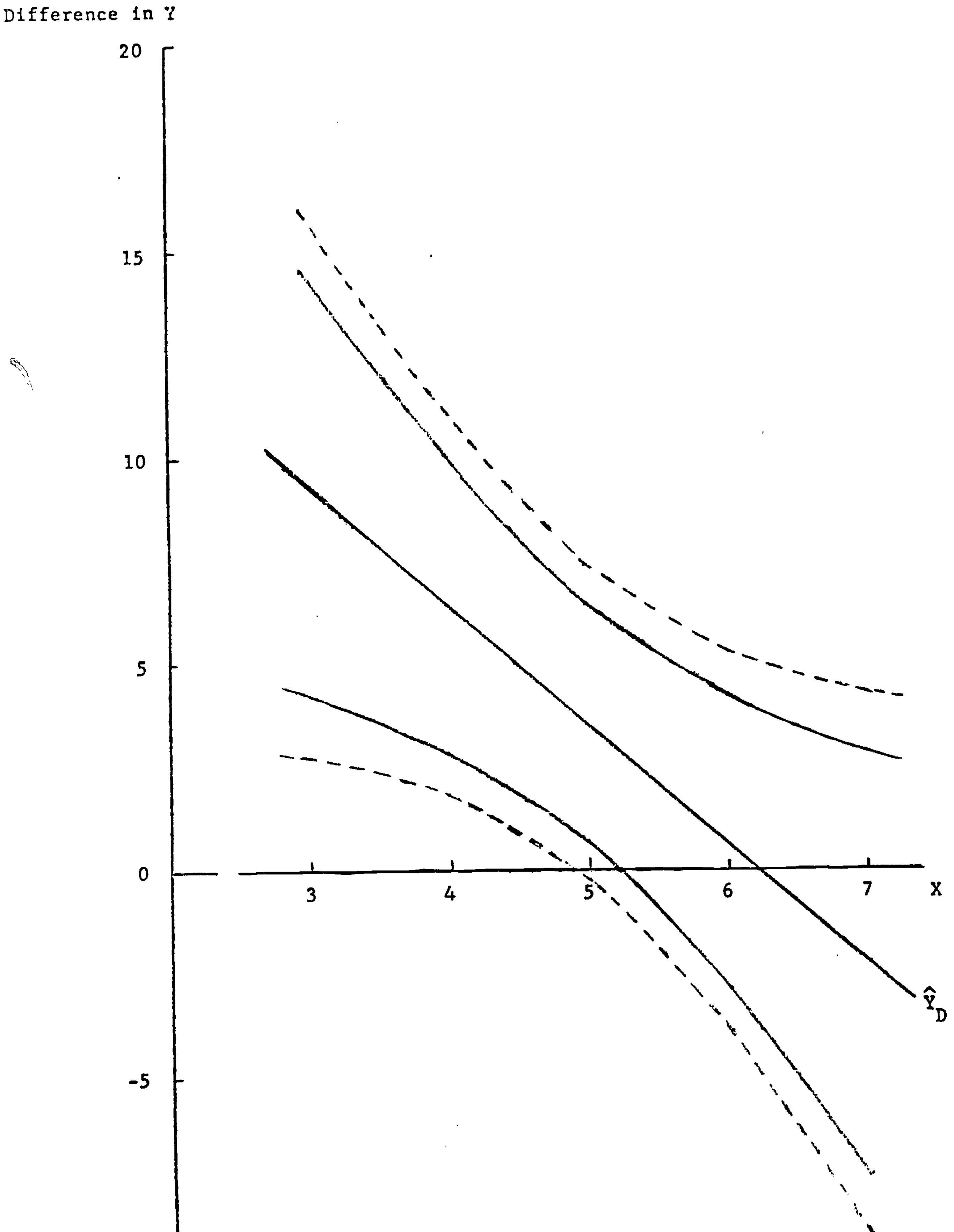

Figure 2. Regression of Y_D on X in Group C along with the simultaneous (dashed lines) and non-simultaneous (solid lines) confidence bands for the population regression line.

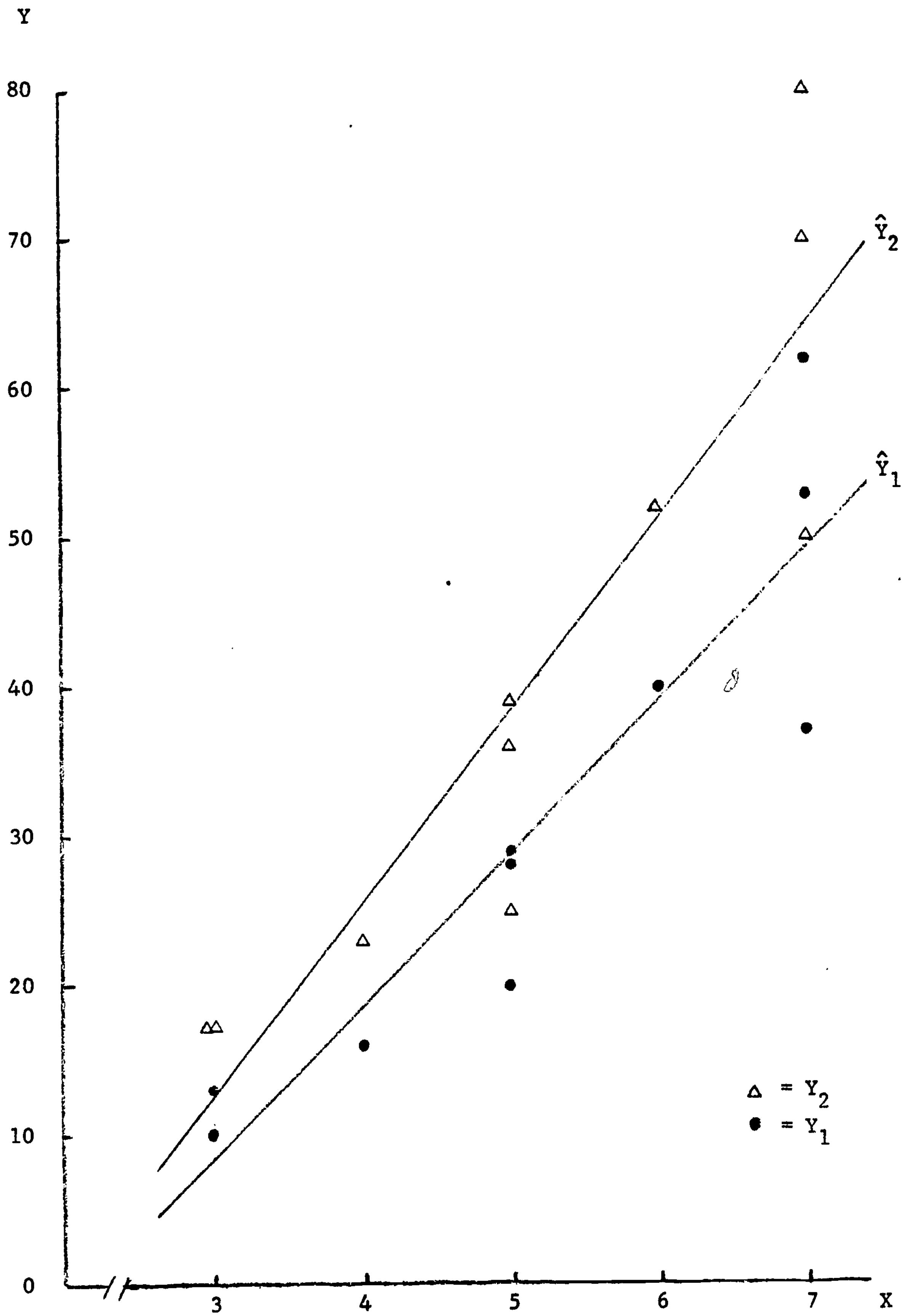

Figure 3. Regression of Y_1 and Y_2 on X in Group T.

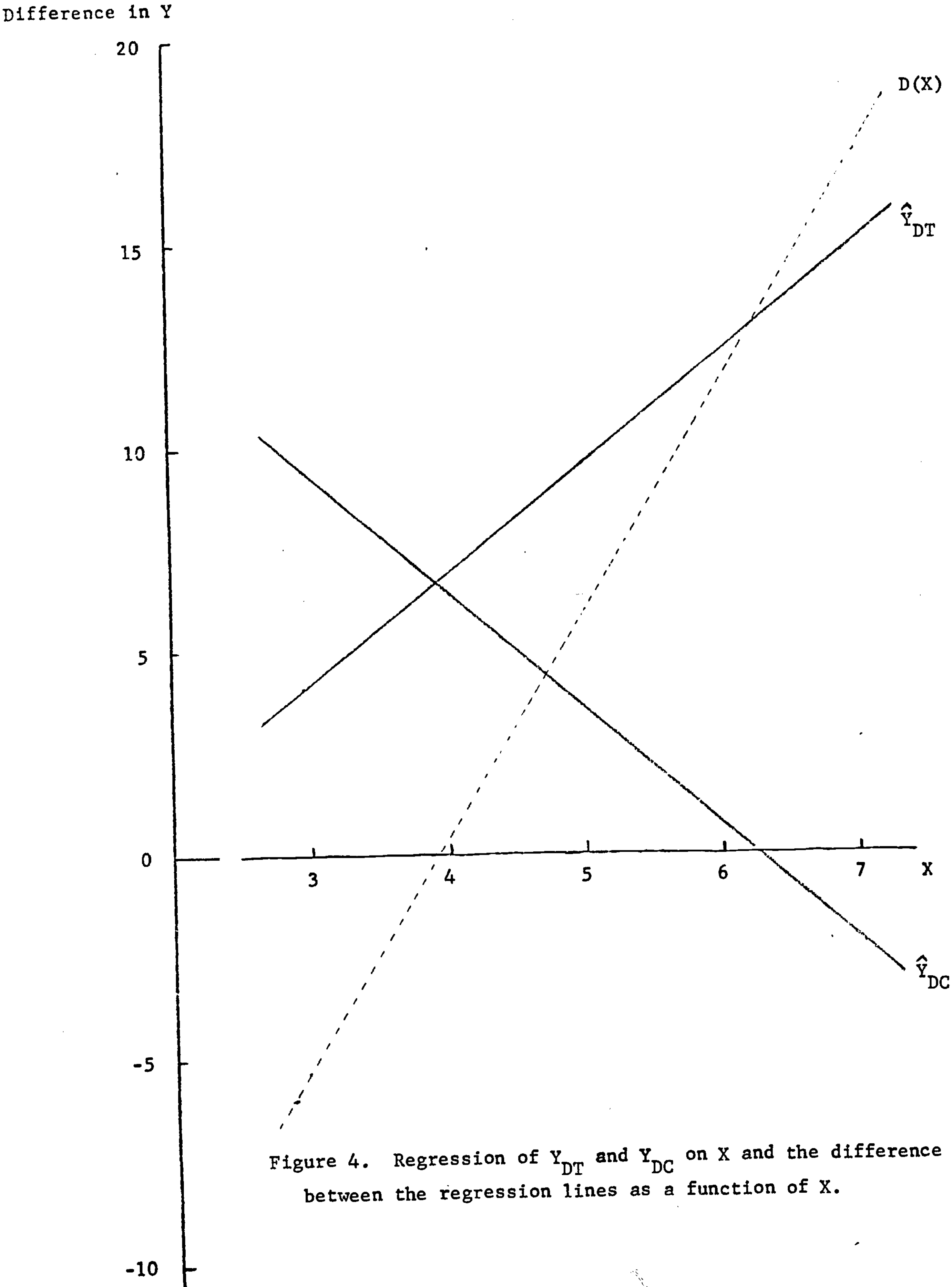

Figure 4. Regression of Y_{DT} and Y_{DC} on X and the difference between the regression lines as a function of X.