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ABSTRACT

This paper presents three different ways of computing the internal consistency coefficient alpha for a same set of data. The main objective of the paper is the illustration of a method for maximizing coefficient alpha. The maximization of alpha can be achieved with the aid of a principal component analysis. The relation between alpha max. and the first eigenvalue of a principal component analysis is presented. Alpha reaches its optimum value for a given set of items when each standardized item is weighted by its respective saturation coefficient under the first principal component. An example shows how one can calculate alpha, alpha standardized and alpha max. Though statistical weighting of the items offers the possibility of increasing the internal consistency of a scale, the different importances given to each item should not be too far from a rational weighting based the nature of the items or other theoretical considerations. (Author)

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A Method for Maximizing the Internal

Consistency Coefficient Alpha

by

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Abstract

This paper presents three different ways of computing the internal consistency coefficient alpha for a same set of data. The main objective of the paper is the illustration of a method for maximizing coefficient alpha. The maximization of alpha can be achieve with the aid of a principal component analysis. The relation between alpha max. and the first eigenvalue of a principal component analysis is presented. Alpha reaches its optimum value for a given set of items when each standardized item is weighted by its respective saturation coefficient under the first principal component. An example shows how one can calculate alpha, alpha standardized and alpha max. Though statistical weighting of the items offers the possibility of increasing the internal consistency of a scale, the different importances given to each item should not be too far from a rational weighting based the nature of the items or other theoretical considerations (Author).

Suggested descriptors: coefficient alpha /maximization of alpha/.

Cronbach alpha coefficient is a well known statistic for estimating the internal consistency (reliability) of a scale or a test. Our paper presents three different ways of computing alpha for a same set of data. Generally these three ways or methods do not produce a unique value for alpha. Two of these methods are well known; the first consists of calculating alpha from the covariance matrix; the second method deals with the calculation of alpha from the correlation matrix. Finally, the third method, based on the weighting of the items, is a new one and we will demonstrate that this method will result in the optimum value of alpha for any set of items.

There are many situations in education and psychology where it is almost impossible to measure a trait by asking only one question to our subjects. We usually refer to many items that are all related in some ways to the trait we are interested to measure. The alpha coefficient evaluates the degree of homogeneity of such a set of items. When alpha is satisfactory, .80 or more, we give more reliability to the total score on the scale and therefore we consider this total more representative of the underlying trait than any particular items of the scale.

Considere first equation (1) which is the classical formula where alpha is calculated from the covariance matrix.

$$\text{Alpha} = \frac{p}{p-1} \left[\frac{\sum_{i=1}^{p} \sum_{\substack{j=1 \\ i \neq j}}^{p} \text{COV}(X_i, X_j)}{\text{VAR}(Y)} \right] \tag{1}$$

Where p is the number of items, and

$$\text{VAR}(Y) = \sum_{i=1}^{p} \text{VAR}(X_i) + \sum_{i=1}^{p} \sum_{\substack{j=1 \\ i \neq j}}^{p} \text{COV}(X_i, X_j)$$

Equation (1) shows that alpha is mostly the ratio of the covariances between the items under the total scale variance.

Alpha obtained from equation (1) refers to the homogeneity of a scale whose total for each individual comes from the simple sum of p items; that is $Y_i = X_{i1} + X_{i2} + \ldots + X_{ip}$.

It is worth noting that in this situation items with largest variances will affect more the value of alpha than items with small variances. When the unit of measurement differs arbitrarily from an item to another, this way of computing alpha can be seriously misleading, if not totally inappropriate.

Generally, though it is not an absolute rule, it is more convenient to calculate alpha from the correlation matrix. In most situations items included in a scale are considered a priori equally important in the determination of the total score. For this reason it is more appropriate to standardize the items and consequently use correlations rather than covariances as measures of association between .e items.

However in practice the difference between alpha and alpha standardized is quite small when the items are all scored on the same arbitrary unit of measurement, as it is often the case when we examine scales of the Likert type.

In the standardized form, the total score (Y_i) for an individual is the sum of p standardized items.

$$Y_i = Z_{i1} + Z_{i2} + \ldots + Z_{ip}$$

The internal consistency of such a scale is given by equation (2)

$$\text{Alpha standardized} = \frac{p}{p-1}\left[\frac{\sum_{i=1}^{p}\sum_{\substack{j=1\\ i \neq j}}^{p} r_{ij}}{\text{VAR}(Y)}\right] \qquad (2)$$

$$\text{Where VAR}(Y) = \sum_{i=1}^{p} \text{VAR}(Z_i) + \sum_{i=1}^{p}\sum_{j=1}^{p} r_{ij}$$

Using formula (2) alpha becomes totally independant of the unit of measurement of the items, since all of them have the same mean and same variance.

Before arriving at the main objective of this paper which is the illustration of a method for maximizing alpha, we would like to point out a very common error made by many researchers who use alpha and alpha standardized without distinction. Imagine this fictitious example: possibly by using SPSS one finds for a particular scale that alpha equals .60 and alpha standardized equals .84; since .84 can be considered as an indication of a good internal consistency he concludes that his scale is rather valuable; however he uses the raw scores to compute the total score ($Y_i = X_{i\ 1} + X_{i\ 2} + \ldots + X_{i\ p}$) which will be the dependent or independant variable in his research. He is unaware that his measure has a rather poor internal consistency of .60. The fact is that in this example, alpha at .84 corresponds only to the scale whose total is obtained by adding the standardized scores, not the raw scores.

There are some simple avenues opened to those who wish to increase the internal consistency of their scale. A first approach is simply the extraction of the worst items. Generally the worst items are those whose variances are unrelated to the variances of the majority of the other items. This has proven to be a satisfying method in most cases where the number of items is large and the loss of some items doesn't really affect the quality of the scale. Adding new items is another way of improving the internal consistency. This method can be a very effective one. However if the new items are just repetitions or reformulations of items already in the scale alpha will certainly increase but not the quality of the scale. These two methods briefly described above are based on modifications of the original scale. Now we will present a method that maximized alpha for a given set of items.

Let Y_i be the total score on the scale obtained for an individual by adding his weighted p standard items.

$$Y_i = W_{i\ 1}\ Z_{i\ 1} + W_{i\ 2}\ Z_{i\ 2} + \ldots + W_{i\ p}\ Z_{i\ p} \qquad (3)$$

In the situation illustrated by equation (3), whatever the W_s alpha can be computed from the next equation

$$\text{Alpha} = \frac{p}{p-1}\left[\frac{\sum_{i=1}^{p}\sum_{\substack{j=1\\ i\neq j}}^{p} W_i\ W_j\ r_{i\,j}}{\text{VAR (Y)}}\right] \qquad (4)$$

$$\text{Where}\quad \text{VAR (Y)} = \sum_{i=1}^{p} W_i^{\,2} + \sum_{i=1}^{p}\sum_{\substack{j=1\\ i\neq j}}^{p} W_i\,W_j\,r_{i\,j} \qquad (5)$$

Rewriting equation (5) we obtain

$$\sum_{i=1}^{p}\sum_{\substack{j=1\\ i\neq j}}^{p} W_i\ W_j\ r_{i\,j} = \text{VAR (Y)} - \sum_{i=1}^{p} W_i^{\,2} \qquad (6)$$

Substituting the right hand side of equation (6) in equation (4)

$$\text{Alpha} = \frac{p}{p-1}\left[\frac{\text{VAR (Y)} - \sum_{i=1}^{p} W_i^{\,2}}{\text{VAR (Y)}}\right]$$

And thus

$$\text{Alpha} = \frac{p}{p-1}\left[1 - \frac{\sum_{i=1}^{p} W_i^{\,2}}{\text{VAR (Y)}}\right] \qquad (7)$$

From equation (7) it is clear that alpha will be at its maximum whenever

$$\frac{\sum_{i=1}^{p} W_i^2}{\text{VAR (Y)}}$$

will be the smallest possible, i.e. when VAR (Y) will be the largest possible. For those who are familiar with principal component analysis the solution is quite straightforward. Maximizing VAR (Y) is precisely what is done by a principal component analysis. In fact principal component analysis weighs the items in such a way that the variance of the linear combination of the variables is maximized; this solution is unique only if a constraint is imposed on $\sum_{i=1}^{p} W_i^2$; traditionaly we let $\sum_{i=1}^{p} W_i^2$ equal unity.

From this discussion it is obvious that we can express alpha max in terms of the first principal component. Rheaume (1980) has first shown the relation between alpha max and the first principal component.

$$\text{Alpha max.} = \frac{p}{p-1} \left[1 - \frac{1}{\lambda_1} \right] \qquad (8)$$

Where λ_1 is the eigenvalue or the variance associated with the first principal component which has the largest variance.

Consequently the W_s which maximize the variance of the linear combination given in equation (3) correspond to the respective coefficient of saturation of each item under the first principal component. Then alpha will be maximized when each standardized item will be weighted by its respective coefficient under the first principal component.

A numerical example should help to clarify the differences between alpha, alpha standardized and alpha maximized.

Considere Table I which gives a summary of a reliability analysis performed on an attitude scale comprising 7 items. These data were obtained from a SPSS run using subprogram reliability.

Table I

Reliability Analysis

	Variance	Corrected item-total correlations	Alpha if item deleted
Item 1	9.24	-0.008	0.61
Item 2	68.09	0.082	0.81
Item 3	7.61	0.757	0.40
Item 4	7.73	0.252	0.54
Item 5	9.36	0.686	0.40
Item 6	4.02	0.645	0.47
Item 7	4.97	0.623	0.46

Alpha = 0.566 Alpha standardized = 0.779

Table I shows that alpha calculated from the covariance matrix is rather low (.56) while alpha calculated from the correlation matrix reaches .779. The difference between the two coefficients is substantial. A look at the covariance and correlation matrices in Table II helps to explain such a gap. Item 2 appears to be the worst item in its correlation with the other items. When alpha is computed from the covariance matrix, item 2 receives greater importance because its variance is relatively large and consequently its covariance with other items is directly affected. What we have here is a poor item, with respect to its relation to other items, receiving the greatest importance due to its relatively greater variance attributable to its unit of measurement.

Normally item 2 should be deleted as suggested in Table I; then alpha would reach .81 but we don't know what alpha standardized would look like, although it is obvious that the difference between alpha and alpha standardized would be quite small.

Table II

Covariance matrix and correlation matrix

Covariances

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
Item 1	9.24						
Item 2	-11.77	68.09					
Item 3	1.33	11.47	7.61				
Item 4	2.16	-0.97	-0.23	7.73			
Item 5	2.91	3.11	6.13	4.56	9.36		
Item 6	3.78	1.28	2.72	3.29	4.33	4.02	
Item 7	1.21	4.69	4.68	0.84	5.05	1.85	4.97

Correlations

	Item 1	Item 2	Item 3	Item 4	Item 5	Item 6	Item 7
Item 1	1.00						
Item 2	-0.46	1.00					
Item 3	0.15	0.50	1.00				
Item 4	0.25	-0.04	-0.03	1.00			
Item 5	0.31	0.12	0.72	0.53	1.00		
Item 6	0.62	0.07	0.49	0.59	0.70	1.00	
Item 7	0.17	0.25	0.76	0.13	0.74	0.41	1.00

Now suppose we want to keep item 2 in the scale for some logical reasons. What is the maximum value that alpha can reach for our set of 7 items? To answer this question a principal component analysis must be done. Table III presents the main results of this analysis.

Table III

Principal component analysis

First eigenvalue: 3.35447

	Saturation coefficient under this first principal component
Item 1	.46
Item 2	.22
Item 3	.79
Item 4	.50
Item 5	.93
Item 6	.84
Item 7	.78

Using formula (8) we can calculate alpha max.

$$\text{Alpha max.} = \frac{p}{p-1} \left[1 - \frac{1}{\lambda_1} \right]$$

$$\text{Alpha max.} = \frac{7}{6} \left[1 - \frac{1}{3.3547} \right]$$

$$\text{Alpha max.} = \sim .82$$

Alpha max will be obtained if the total score for each individual is computed from the following linear combination of items.

$$Y = .46Z_1 + .22Z_2 + .79Z_3 + .50Z_4 + .93Z_5 + .84Z_6 + .78Z_6$$

Each standardized item is weighted by its respective saturation coefficient under the first principal component. We see that item 2 receives the smallest weigh, as expected from the previous analysis.

Conclusion

We have seen that it is possible to find alpha max. for a given set of data by weighting the standardized item with the aid of principal component analysis. Based on our experience, alpha max. do not differ substantially from alpha when there are many items in the scale (20 or more). However it may be worth calculating alpha max. when the number of items is less than 20.

Though the weighting of the items offers the possibility of increasing the internal consistency of a scale, it may be difficult to justify on rational bases the different weighs or importances given to each item. In most cases the statistical weighting of the items should not be too far from a rational weighting based on the nature of the items or other theoretical considerations.

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