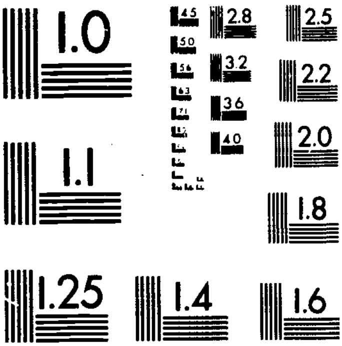


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ABSTRACT

Abstracts of 11 mathematics education research studies are provided. Each abstract is accompanied by the abstractor's analysis of or comments about the study. Studies reported include: "The Importance of Spatial Visualization and Cognitive Development for Geometry Learning in Preservice Elementary Teachers"; "Classroom Ratio of High and Low-Aptitude Students and the Effect on Achievement"; "Replacement and Component Rules in Hierarchically Ordered Mathematics Rule Learning Tasks"; "Intuitive Functional Concepts: A Baseline Study on Intuitions"; "Aspects of Proving: A Clinical Investigation of Process"; "Sex Differences in Teachers' Evaluative Feedback and Students' Expectancies for Success in Mathematics"; "The Position of the Unknown Set and Children's Solution of Verbal Arithmetic Problems"; "Use of Situations in Mathematics Education"; "Strategy Use and Estimation Ability of College Students"; "Story Problem Solving in Elementary School Mathematics: What Differences Do Calculators Make?"; and "Drawn versus Verbal Formats for Mathematical Story Problems." Lists of mathematics education research studies reported in CIJE and RIE from October through December 1982 are also provided. (JN)

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# MATHEMATICS EDUCATION INFORMATION REPORT



THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
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# INVESTIGATIONS IN MATHEMATICS EDUCATION



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An editorial comment . . .

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## Critical Problems in Mathematics Education: The Search for the Holy Grail

Thomas P. Carpenter
University of Wisconsin

At the turn of the century, Hilbert posed 23 problems whose solution would lead to fundamental advances in mathematics. It has been proposed that this exercise serve as model for identifying the critical problems in mathematics education. Richard Shumway asked the authors of chapters in Research in Mathematics Education (Shumway, 1980) to follow Hilbert's example and identify a small number of significant problems based on the research reviewed. Recently, David Wheeler invited a number of colleagues in mathematics education to go through the same exercise so that he might arrive at a synthesis of critical problems that would give direction to research in mathematics education much as Hilbert's 23 problems did for mathematics.

A case can be made that the failure of educational research to provide definitive answers to serious educational problems have not been clearly articulated. Platt (1964) has argued that the areas of science in which the most dramatic successes have occurred are those in which the practitioners have invested a substantial effort in identifying and analyzing the critical problems. It is not clear, however, that educational problems can be subjected to the same level of analysis or be as clearly solved as problems in mathematics, microbiology, or high energy physics. Cronbach (1975) argues that conclusions in social service are generally not absolute. He proposes that "we cannot store up generalizations and constructs for ultimate assembly into a network" (p. 123). In other words, even if fundamental problems in mathematics education could be identified, it is not apparent that they could be clearly solved.

In the last 10 to 15 years, a number of research areas promised to provide answers to fundamental questions in mathematics education, but what specific changes in mathematics instruction have been based on the extensive body of Piagetian research or research on discovery learning or aptitude-treatment interactions?

I believe that research is unlikely to provide definitive answers to broad fundamental educational questions. I think that the most progress will be made if we are more modest in our goals, our research is more clearly focused, and our conclusions are more carefully qualified. I am suggesting that research be directed at developing what Shulman (1974) calls middle-range theories. These theories fall between the task specific working hypotheses that are generated to explain individual behaviors and the comprehensive theories that attempt to encompass all of instruction in mathematics.

For the most part, I believe that attempts to draw all-encompassing conclusions from educational research at best have not been terribly productive and at worst have been misleading. For example, I think that the claims for academic learning time and direct instruction must be highly qualified if one acknowledges that the goals of instruction include understanding and problem solving. Broader conclusions based on research in this area could potentially lead to many inappropriate decisions about effective teaching. On the other hand, although I am generally sympathetic to the finding that teaching for understanding facilitates retention and transfer, I think that this conclusion is so broad that it has had relatively little impact on instruction in mathematics.

The kind of direction that I am suggesting is illustrated by a discussion at a recent conference on concept learning. In one of the working groups, the thesis was put forth that a certain sequence of positive and negative examples was most effective in teaching mathematics concepts. Alan Schoenfeld proceeded to identify a number of concepts that everyone present agreed would be most effectively taught using only positive examples. In fact, for every sequence of positive and/or negative examples the group could come up with, he was able to find a concept for which that sequence would be most effective. The point he was making is that conclusions about concept learning in general are not appropriate. The most effective way to teach a particular concept depends on the concept.

Research is beginning to provide a picture of how specific mathematics concepts are acquired and is beginning to provide an understanding of the instructional process in particular contexts (Romberg & Carpenter, in press). But much of this research is descriptive, and it is not clear that it can readily be captured in 23 critical problems. This does not mean that careful analysis is not necessary. A great deal of sloppy thinking is excused on the grounds that it is necessary to be flexible in clinical or ethnographic research. The clearest insights have come when the research was guided by some theory, and it was possible to put structure on the results. Thus, I do believe that it is necessary to identify the critical problems within a specific domain, but I am not sure that these critical problems will encompass all of mathematics education.

I would like to end this editorial with a disclaimer. There was nothing in either Shumway's or Wheeler's requests for critical problems to preclude the kinds of limits on problems that I have proposed. The straw person that I have attempted to knock down is my own creation, not theirs. Furthermore, I do not intend to disparage the search for larger questions. I do not believe that it is either naive or a waste of time. Like the knights of the round table searching for the holy grail, the search itself can prove instructive. The larger questions are worth asking; I'm just not sure we are going to find definitive answers that will significantly influence instruction. The questions themselves, however, may provide direction to the more clearly focused research, but we must be sure they do not limit it.

## References

Cronbach, L. J. Beyond two disciplines of scientific psychology. American Psychologist, 1975, 30, 116-127.

Platt, J. R. Strong inference. Science, 1974, 146, 347-353.

Romberg, T. A. & Carpenter, T. P. The teaching of mathematics. In M. C. Wittrock (Ed.), The third handbook of research on teaching. Chicago: Rank McNally, in press.

Shulman, L. J. The psychology of school subjects: A premature obituary? Journal of Research in Science Teaching, 1974, 4, 319-339.

Shumway, R. (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics, 1980.

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1983

Battista, Michael T.; Wheatley, Grayson H.; and Talsma, Gary. THE IMPORTANCE OF SPATIAL VISUALIZATION AND COGNITIVE DEVELOPMENT FOR GEOMETRY LEARNING IN PRESERVICE ELEMENTARY TEACHERS. Journal for Research in Mathematics Education 13: 332-340; November 1982.

Abstract and comments prepared for I.M.E. by GLENDA LAPPAN, Michigan State University.

1. Purpose

The primary focus of the study was to examine the effect of spatial ability and cognitive development on learning mathematics; in particular, on learning geometry. The effect of instruction in geometry on spatial ability was also studied.

2. Rationale

Correlational studies have long shown a positive relationship between spatial ability and achievement in mathematics. Although many studies in the last six to eight years have investigated this relationship in an attempt to understand the nature of the interaction of spatial ability and learning mathematics, the results leave much unexplained. There has also been considerable discussion of the importance of cognitive development. Since students in the concrete operational stage rely heavily on concrete and pictorial representations which have spatial components,

> there is reason to believe that investigating the interaction of spatial ability and cognitive development will shed some light on the roles that both factors play in mathematics learning. (p. 333)

3. Research Design and Procedures

The subjects of the study were 82 college students, mostly females, enrolled in four sections of a one-semester college course in geometry for preservice elementary teachers (PSET). The independent variables were spatial ability and cognitive development; the dependent variable was achievement in geometry. To measure spatial ability, the Purdue Spatial Visualization Test was given to the students at the beginning and at the end of the semester. The cognitive development of the PSET was measured at the end of the semester by a modified version of the Longeot Test of cognitive development. Thirty-one of the 82 subjects received a score of 12 or higher on the 15-item test and were classified as formal operational. Achievement in geometry was measured by the total of the students' scores on three tests given during the semester.

The geometry course was activity-oriented. The students were involved with many investigations and materials that had spatial components.

4. Findings

The students' pre and post spatial visualization scores and cognitive score were each significantly correlated with the course grade ($p < .001$). The spatial score correlations with the cognitive score were also significant (pre $p < .01$, post $p < .001$) as were the pre spatial scores with the post spatial scores ($p < .001$).

In the regression analysis of course grade, the primary predictor was the cognitive score ($p < .001$) with the pre spatial score, accounting for an additional 6% of the variance ($p < .01$).

The subjects with median scores on either the cognitive or pre spatial test were excluded and the remaining 59 subjects with high or low scores were anlyzed in a two-way analysis of variance. The main effect due to the spatial level and the interaction were not significant.

The posttest scores of spatial visualization were significantly higher than the pretest scores of spatial visualization ($p < .001$).

5. Interpretation

The authors state that their findings give a "strong indication that cognitive development is a better predictor of a geometry course grade than spatial ability" (p. 338), but both are important in learning geometry. The authors hypothesize that the analytic nature of many of the spatial items on course tests may have reduced the correlation between course grade and spatial scores.

Some Purdue Spatial Visualization test-retest data on 36 teachers not included in this study lend support to the claim that geometry activities such as those provided in the course contributed to the significant difference found in spatial visualization scores. The gain for the students in this study was significantly higher than the test-retest gain for the "control" teachers ($p < .05$).

The increase in spatial scores for students above and below the median was compared. The average gain for students below the median was 3.95; for those above the median, the average gain was .92. This suggests that further research is needed to clarify whether or not instruction such as the activity-based geometry instruction in this study helps improve spatial ability and if so, whether instruction benefits one group of students more than another.

## Abstractor's Comments

The authors are to be commended for adding an interesting twist to the question of the role of spatial ability in the learning of mathematics. They focused their attention on the learning of geometry, a critical but sadly neglected component of the K-12 curriculum, and raised the question of the relative importance of spatial ability and cognitive development in predicting achievement in geometry.

It is, of course, important to study preservice teachers, since they ultimately become critical factors in the education of children. However, we must be careful not to infer that this study tells us anything about children, or, for that matter, anything about college students (males or females) in general.

The authors give a great deal of information on both the Purdue Spatial Visualization Test and the Sheehan version of the Longeot Test of cognitive development to justify their choice of these two instruments. However, so little information is given about the three tests used as a course grade score, that one is not even sure that the same tests were given to all four sections of students. I am left wondering who designed the tests? What format was used for questions? If the items were not multiple choice, how was a protocol for scoring established? What was the reliability of each course exam? Since the mean for the group was over 80% on the course exams, were the exams discriminating enough to provide useful measures? Were each of the four sections taught by a different instructor? Was the weekly discussion of course content sufficient to assure that the treatment was fairly standard from class to class? Were there any differences among the four classes on any of the measures?

The spatial visualization test was given as both a pre- and post-test. However, the cognitive development test was given as a posttest. How can we be sure that the study of geometry itself did not affect the cognitive development of the students? Since the study was interested in how well cognitive development predicted success in geometry, would it not have been better to test the students' cognitive development prior to the treatment?

On the question of whether or not spatial ability can be improved by training, this study offers support for the effectiveness of instruction of an activity-based sort. This is an important result for mathematics education. We could place greater confidence in this result if a more careful look at the test-retest scores for other preservice elementary teachers without spatial training could be given. For example, Pre-Experimental design 3 from Campbell and Stanley (1963) would be appropriate:

$$\frac{X\text{------}\;O_1}{\phantom{X------}\;O_2}$$

Here X is the treatment of taking the Purdue Spatial Visualization test the first time; $O_1$, the scores on the same group the second time; $O_2$, the scores on the Purdue Spatial Visualization Test for an equivalent group of students. The authors acknowledge this problem and do report comparison data between gains for two groups, the students that received the instruction and another group that did not.

## Reference

Campbell, D. T., and Stanley, J. C. Experimental and Quasi-Experimental Designs for Research. Chicago: Rand McNally, 1963.

Beckerman, Terrill and Good, Thomas. THE CLASSROOM RATIO OF HIGH AND LOW-APTITUDE STUDENTS AND THE EFFECT ON ACHIEVEMENT. American Educational Research Journal 18: 317-327; Fall 1981.

Abstract and comments prepared for I.M.E. by RICHARD CROUSE, University of Delaware.

1. Purpose

The ratio of high-aptitude students to low-aptitude students in third- and fourth-grade classrooms would influence the mathematics achievement of these students.

2. Rationale

Recent research has demonstrated that the classroom process can be altered in ways that improve student achievement. However, in comparison to the growing literature on instructional process, there is very little information on how the types of students present in classrooms influence instructional process or outcomes. Also, much of the research in this area has used the school rather than the classroom as the unit of analysis for testing the student characteristic ratio/achievement hypothesis. This investigation was thus conducted using the classroom as the unit of analysis, since this analysis has more potential for explaining student progress than research analyzed at the school level if the variable of interest functions at the classroom level.

3. Research Design and Procedures

The sample for the investigation was 103 third- and fourth-grade classrooms drawn from a large metropolitan school district that basically served a middle-class population. Pre- and post-mathematics achievement data and aptitude scores were available for these students.

Within grade level, students were assigned to high, middle, or low aptitude groups on the basis of their scores on the Cognitive Ability Test. If classrooms did not have at least four students each of high, middle, and low aptitude, they were dropped from the analysis.

This criterion reduced the number of classrooms from 103 to 81.

Two types of classrooms were then operationally defined. More favorable classrooms were those in which low-aptitude students were less than a third of the classroom population and high-aptitude students were more than a third. Less favorable classrooms were defined as those in which low-aptitude students were more than a third of the class and high-aptitude students were less than a third. Fifty-five classrooms were classified as more or less favorable classrooms. This included 27 third-grade classrooms with 14 more favorable and 13 less favorable, and 28 fourth-grade classrooms with 17 more favorable and 11 less favorable.

The dependent measure used in the study was students' total mathematics scores on the Iowa Test of Basic Skills. Residual gain scores were computed for students by using each student's score on the pretest as a covariate. Before conducting an analysis of variance, it was ascertained that levels of student aptitude were comparable across classrooms.

4. Findings

Both low- and high-aptitude students in more favorable classrooms had higher achievement scores than comparable groups in less favorable classrooms ($p < .05$).

5. Iterpretations

The investigators concluded from their findings that:

(1) The possible effects due to the ratio of high-to-low aptitude students in a classroom are not accounted for by the usual statistical procedures used in process-product or teacher effectiveness research.

(2) Teacher effectiveness research may be confounded by the aptitude ratios in a classroom or by other uncontrolled classroom context effects.

(3) Classroom mean gain scores may be the result of the interaction of aptitude ratio and instructional variables.

(4) There are various explanations as to why a more favorable or less favorable environment influences student achievement. One explanation might be that the demands of the teacher might be different

depending upon the various aptitude ratios. Thus, the pace could be slower in less favorable classrooms, or perhaps more time is spent on management problems.

(5) "Having a greater ratio of high-to-low aptitude students in a class does not automatically create a more favorable environment for low achievers, but it may increase the chance for such students to receive appropriate instruction."

## Abstractor's Comments

This is an interesting study which attempts to attack an important problem in teacher effectiveness research. However, some information was not included which would have helped in the reading of this study.

Among the questions which arise in connection with the reporting of this study are:

(1) What was the duration of the experiment?

(2) Table III in the report gives means for low- and high-aptitude groups -- but means of what?

(3) Since the study was of intact classrooms, was randomness achieved?

In spite of these criticisms, this is an interesting study which has significance for the teaching of mathematics. As the investigators suggest, it would be important to further test the aptitude-ratio context effect to see if the findings are generalizable across grade levels and/or subject matter. Additional studies would also be needed to determine which ratio or ratios, if any, are most beneficial to students at various grade levels.

Bergan, John R.; Towstopiat, Olga; Cancelli, Anthony A.; and Karp, Cheryl. REPLACEMENT AND COMPONENT RULES IN HIERARCHICALLY ORDERED MATHEMATICS RULE LEARNING TASKS. Journal of Educational Psychology 74: 39-50; February 1980.

Abstract and comments prepared for I.M.E. by IPKE WACHSMUTH, Northern Illinois University.

1. Purpose

"The present study examined ordered and equivalence relations and performance errors in a set of hierarchically arranged fraction identification tasks to determine the extent to which observed relations and errors were congruent with the component-rule and rule-replacement hypotheses" (pp. 41-42).

2. Rationale

The investigation was conducted in the context of mathematical rule learning. For different rule-learning tasks, hierarchical orderings of tasks can be considered in which subordinate tasks are prerequisite to super-ordinate tasks. The authors contrast two hypotheses about the conditions under which two rule-learning tasks can be expected to be in an ordered relation:

(a) the component rule hypothesis suggested by Gagne, which states that two rule-learning tasks are ordered if one involves a rule that is a component of the second;

(b) the rule replacement-hypothesis suggested by the authors, which states that two rule-learning tasks are ordered if one involves a rule that has to be replaced by a more complex rule in order to apply also for the second.

For both cases, the authors cite studies in the realm of fractions to support the existence of rule-learning tasks that are ordered as indicated in the hypotheses.

The two hypotheses lead to different expectations about order and equivalence relations between tasks and about probable performance errors.

The authors believe that insights about the nature of the transition from nonmastery to mastery with respect to a particular rule-learning task can be obtained through establishing the validity of one of the hypotheses. These insights could have important implications for instruction.

3. Research Design and Procedures

The two hypotheses were investigated using a set of tasks in which a fractional part of a set of n objects was to be identified; for example, two-fifths of a set of five. A child's behavior emitted in mastering such a task could be conceptualized in two rules (not necessarily being verbalized by the child):

(i) the "denominator rule", which states that to identify a fraction with denominator r for a set of n objects, the set must be partitioned into r equivalent subsets;

(ii) the "numerator rule", which states that to identify a fraction with numerator s, one of r subsets established by the denominator rule must be taken s times.

Since the numerator rule refers to the r subsets established by the denominator rule, the denominator rule can be regarded as a component of the numerator rule. In this respect, identifying one-fifth of a set of five would be hypothesized to be a task subordinate to identifying two-fifths a set of five, because the counting involved in the numerator rule could be omitted in the first, while it could not be omitted without impairing identification in the second case.

In contrast, the authors illustrate rule replacement by an example involving another hypothetic rule:

(i)' the "one-element rule", which equates each of the r subsets mentioned in (i) with just one element in the set of n objects.

For instance, two-fifths of a set of five could be identified correctly in applying the numerator rule in connection with the one-element rule (in place of the denominator rule): one of the five elements (rather than subsets) established by the one-element rule is taken two times. Yet cases where the number of objects differs from the denominator could only be expected to be identified correctly if the one-element rule is replaced by the more complex denominator rule. In this respect, identifying two-fifths of a set of five would be hypothesized to be a task subordinate to identifying two-fifths of a set of ten.

The two hypotheses were systematically explored using the statistical technique of latent class models to assess equivalence and order relations among a set of eight fraction identification tasks. The tasks, presented twice in random order, required identification of a fractional subset of a given set of circles. Included were the identification of one-third of a set of three, two-thirds of a set of three, one-third of a set of six, two-thirds of a set of six, one-fifth of a set of five, two-fifths of a set of five, one-fifth of a set of 10, and two-fifths of a set of 10.

A total of 456 middle-class children (213 boys and 243 girls) of ages 7-12 from mixed ethnic groups were group-tested in public and parochial elementary schools. Sample responses to the questions given in testing booklets were demonstrated and understanding of all tasks and directions was ensured before and during the testing. Time was given as necessary to complete all problems. Each response was scored as passing or failing.

Four latent class models were tested:

Model $H_1$, representing equivalence of tasks, was composed of three latent classes: a nonmastery, a mastery, and a transition class. It was assumed that masters would pass all problems, nonmasters would fail all problems, and transitionals would have a passing probability equal across items.

Model $H_2$, representing equivalence of tasks, included all classes of $H_1$ plus four classes representing inconsistency of responses across different items. That is, pairs of tasks hypothesized to be equivalent were expected to be responded to inconsistently by transitional individuals.

Model $H_3$, representing ordering of tasks, included all classes of $H_2$ plus one class representing individuals who were masters of the subordinate task and nonmasters of the super-ordinate task with respect to two tasks hypothesized to be ordered.

Model $H_4$, representing ordering of tasks, involved an asymmetrical transition between nonmastery and mastery in that no latent classes were included to represent the case in which performance on a super-ordinate task was superior to performance on a subordinate task.

## 4. Findings

For each pair of tasks, it was determined which of the four models best fit the data. In no case was $H_1$ preferred. $H_2$ was preferred for all task sets involving denominator variations and for one task set that varied numerators. $H_3$ was preferred for two of the four task sets involving numerator variations. $H_4$ was preferred for one task set involving numerator variations and for all task sets involving variations in the number of subset elements. For all performance errors, 56% were consistent with the one-element rule.

Except for four cases, the preferred models were characterized by non-significant chi-square values. The four significant cases included the pairs one-fifth of ten vs. two-fifths of ten, two-thirds of six vs. two-fifths of ten, one-third of three vs. one-third of six, and two-thirds of three vs. two-thirds of six.

## 5. Interpretation

The findings for items involving denominator variations support the hypothesis that two tasks which involve a common rule will be equivalent with respect to mastery of that rule. A partial inconsistency in responding is part of the transition process from nonmastery to mastery.

The findings for items involving numerator variations, with one exception, support the component rule hypothesis. The exception suggests that in one case the acquisition of numerator rule and denominator (or one-element) rule occurred concurrently. In two cases of an established ordering of tasks, it occurred that nonmasters of a subordinate task were in transition with respect to the super-ordinate task.

The "most important finding in this study" (p. 49) was the preference for $H_4$ in all cases varying the number of subset elements. This finding is consistent with the rule-replacement hypothesis and supports the view that many children used two qualitatively distinct rules (one-element and denominator rules) in handling fraction denominators. The authors relate this finding to cognitive developmental theory and suggest that, as in the course of broad-scale development, qualitative changes may occur in children's thinking for specific learning tasks.

The following implications for instruction are formulated: The fact that advancement from nonmastery to mastery involves a transitional state suggests differential instruction with respect to the state of the learner. The fact that when varying numerators many nonmasters of a subordinate task were transitional with respect to a super-ordinate task suggests that both tasks may be learned at the same time. The rule-replacement hypothesis could be relevant for analyzing cognitive changes and diagnosing problems in children's learning through determination of the rules they use in task performance. The insights gained could provide a basis for instructional sequencing.

The authors raise the following research questions:

- to investigate the origin of rules used by students;
- to investigate factors that affect sequential and simultaneous rule acquisition;
- to investigate whether rule-replacement occurs in many areas of learning;
- to investigate to what an extent rule-replacement can be affected by instruction.

## Abstractor's Comments

Marking one-third of three circles should be as easy or difficult as marking one-fifth of five circles, marking two-fifths of five should be more difficult than marking one-fifth of five, and marking one-fifth of ten more difficult than marking one-fifth of five --

these were the key ideas in this very thoughtfully designed study. The aim was to obtain insights into the rules that govern children's thinking in performing such tasks, which could explain why one task would be as easy as, or more difficult than, another.

The two hypotheses formulated were hypotheses about rules that are hypothetic themselves. This makes the issue complicated since hypothetic rules are unobservable; they can only be observed implicitly in the results of their application. Latent class models were used to attack the problem of representing and testing hypotheses stated in terms of unobserved ("latent") variables as given with the rules.

The results, in their general trend, support both hypotheses. Of particular interest seem the findings formulated about transition from nonmastery to mastery of a rule-learning task. From the written-test data, insights could not be obtained that give an explanation for the observed partial inconsistencies. Clinical interviews with a smaller sample might provide additional information.

It should not be overlooked that the findings were not always as clearcut as one might have hoped with this promising approach. Four of the 12 task sets tested between model and data set. For one of these, one-third of three and one-third of six, this was due to the fact that the number of response patterns where both of the superordinate task items were passed and both of the subordinate items were failed was unexpectedly large under the preferred model, $H_4$. The preference for $H_4$ which modelled an asymmetrical transition not accounting for such response patterns was not rejected since the tested improvement of $H_3$ over $H_4$ was not significant.

In addition, I don't feel comfortable when a result is established through complicated modelling that, based on a p-value of .05, states the "occurrence of an equivalence relation" for the task set requiring identification of one-fifth and two-fifths of 10, which "simply suggests that the acquisition of the numerator rule and the acquisition of the denominator rule or one-element rule were concurrent for this task set," and then calls for research "to investigate factors affecting sequential and simultaneous rule acquisiton" (p. 48; the analysis of standardized residuals reveals that an unexpectedly large number of inconsistent responses to identical items was responsible for the observed discrepancy).

The impact of the approach taken in the study is not affected, however. With rule replacement, another important mechanism of learning has been identified that certainly can be found in many other areas of mathematics learning up to the calculus level (for example, compare the tasks of finding the derivative of sin x and sin $(x^2)$; non-masters of the second task frequently come up with cos $(x^2)$). The research questions raised in the context of rule replacement appear very worth considering. The origin of rules that have not been taught, as in the case of the one-element rule, presumably has to do with "minimal discrimination": for a restricted class of tasks this simpler way of attacking a problem may have proven successful and thus was adopted by the learner as a rule that is not abandoned as long as no failure is realized. Confronting the learner with instructional situations that require finer discrimination to be made might be a possible way of affecting rule replacement.

Dreyfus, Tommy and Eisenberg, Theodore. INTUITIVE FUNCTIONAL CONCEPTS: A BASELINE STUDY ON INTUITIONS. Journal for Research in Mathematics Education 13: 360-389; November 1982.

Abstract and comments prepared for I.M.E. by JOHN HUBER, Pan American University.

1. Purpose

The purpose of this study was to assess the intuitive background of junior high school students as they develop the concept of function. In particular, the following hypotheses were tested:

1. Intuitions on functional concepts grow with the pupils' progress through the grades.
2. Intuitions are independent of sex.
3. Intuitions of high-level students are more often correct than those of low-level students.
4. Intuitions are more often correct in concrete situations than in abstract ones.

2. Rationale

For the purpose of this study the term intuitions is taken to refer to "mental representations of facts that appear self-evident" (p. 360). The authors make the assumption that the intuitive meaning of a mathematical idea is essential in order to develop the mathematical reasoning process. In addition, they feel that (1) intuitions can be trained through appropriate activities, (2) a primary goal of education is to enlarge the base of intuitions, and (3) the teaching process should be based on the intuitive knowledge of the learner.

Based on its unifying nature and its frequent appearance throughout the mathematics curriculum, the function concept is one of the most central ideas in mathematics today. It is this high level of abstraction and generalizability that make the function concept quite complex. First, it is not a single concept. It has a number of subconcepts associated with it (e.g., domain, range, image of an element, etc.). These are called "functional concepts" (p. 361) in this study. Second, the same function may have various representations called "settings" (p. 361) (e.g., number of variables, finite domain, finite range, implicit definition, explicit definition, recursive definition, etc.).

Each of these aspects is a major contributor to the difficulties students encounter in learning the concept of function. Based on these three aspects of function, the authors present a three-dimensional "function block" (p. 364) structure in which the first dimension represents the various settings, the second dimension the various functional concepts, and the third dimension the levels of abstraction and generalization. Based on the first two dimensions of this structure, the four hypotheses were formulated.

3. Research Design and Procedures

The dependent variable, AVG, was the percent of correct items on a questionnaire designed by the authors. The items measured various concepts about abstract and concrete functions in three settings (diagrams, graphs, and tables.).

External validity for the questionnaire was provided by a panel of five high school and college mathematics teachers classifying a pool of items according to the concepts concerned. Reliability coefficients were estimated using the KR-20 formula for the full test and the concrete and abstract subtests. The reliability estimates were 0.91 (full test), 0.86 (concrete subtest), and 0.81 (abstract subtest).

The four independent variables were grade in school (Grade), ability-social-level (Absolv), Setting (diagram, graph, or table), and Sex. The contruct variable Absolv is a combination of ability level and the extent to which the learning environment was socially disadvantaged (pp. 367-369).

At the beginning of the school year before any classes had studied the concept of function, the questionnaires were administered to students in grades 6, 7, 8, and 9 in Israel. Only those students completing 90% of the questionnaire (a total of 443) were included in the sample.

4. Findings

Using a four-way analysis of variance, AVG by Grade X Absolv X Setting X Sex, 51% of the variance in total test scores was accounted for by the model. The independent variables Grade, Absolv, and Setting were found to be statistically significant ($\alpha = .05$). The significant interactions were Grade X Absolv, Absol X Sex, and Grade X Absolv X Sex.

5. Interpretations

An overall increase in performance through the grades was observed, with a significant decrease occurring from Grade 7 to 8. The main progress came in Grade 6 for high-Absolv and in Grade 8 for low-Absolv students. This supports the general cognitive performance theory of Lewy and Chen that socially disadvantaged students can learn the material, but it takes them longer to do so (p. 372).

Comparing the performance in the three settings, it was found that at all grade levels the diagram setting presented more difficulty than the other two. This was attributed to the complexity of the diagrams and poor reproduction of several questions. Comparing performance in the graph and table settings with respect to grade, no preference was found. However, in comparing the two settings with respect to Absolv, high-Absolv students preferred a graph setting while low-Absolv students preferred the table setting. This suggests that subconcepts should be introduced in a graph setting for high-ability students and in a table setting for low-ability students.

No differences in overall performance with respect to Sex were found. However, an interesting "switching" (p. 375) occurred in the Sex S Absolv and the Grade X Absolv S Sex interactions. Low-level males performed worse than low-level females, while high-level males performed better than high-level females. A similar switching occurred for high Absolv in the Grade X Absolv X Sex interaction. Two possible explanations were given, one based on differences in the rate of physical development of males and females and the other based on differences in the seriousness of male and female students at this age.

All factors contributing to the significant differences on the full test were significant on the concrete subtest. All factors except Setting and Grade X Absolv X Sex were significant on the abstract subtest. No additional factors appeared.

## Abstractor's Comments

The function concept is certainly one of the most fundamental and unifying concepts in mathematics. The authors are to be commended for undertaking a study in such an important area of mathematics. More studies in this area are needed.

Several important results need to be examined. The role of intuitions in the learning of mathematics needs to be pursued. Can intuitions be taught? Would a teaching process based on intuitions lead to more meaningful learning of mathematics? Answers to these questions are essential to the long-range implications of this study.

The "function block" paradigm not only provides a model for the study of functional concepts as they relate to the study of the concept of function, but will also provide a framework for the study of vertical and horizontal transfer. This model could also be applied to studies of the attainment of other mathematical concepts such as variable, set, and equation.

The conclusion that high-Absolv students prefer a graph setting while low-Absolv students prefer a table setting is very important. What type of learning results in each of these settings? Is one instrumental and the other relational? Are they different in level or type? These questions need answers.

The concept of variable is certainly essential to the understanding of the concept of function. No mention of this critical functional concept was explored in this study.

Several weaknesses in the statistical analysis need to be noted. Based on the "function block" paradigm, four hypotheses were formulated. However, in analyzing the results an ANOVA model with first- and second-order interactions was used. The authors should have either provided a theoretical base for hypothesizing such interactions or no such interactions should have been included in the ANOVA model. It is essential that the statistical model be the same as the model formulated by the hypotheses.

The first hypothesis formulated was, "Intuitions on functional concepts grow with pupils' progress through the grades" (p. 366). Again, the statistical technique was inappropriate. A significant main effect only implies that a trend (linear, quadratic, etc.) is present. A quadratic trend would not support the hypothesis the authors presented. A test of linear trend in grades should have been used.

In analyzing the four-way ANOVA, several interactions were significant. With significant interactions, interpretation of main effects is questionable.

Except for the few weaknesses noted above, this study should form an excellent foundation for future studies in the attainment of the concept of function. In addition, the function block paradigm should form an excellent theoretical framework for other studies.

Galbraith, P. L. ASPECTS OF PROVING: A CLINICAL INVESTIGATION OF PROCESS. Educational Studies in Mathematics 12: 1-28; February 1981.

Abstract and comments prepared for I.M.E. by LEWIS R. AIKEN, Peperdine University.

1. Purpose

The goal of this investigation was to determine the perceptions of secondary school pupils with respect to modes of argument having accepted status in mathematics. More specifically, an effort was made to attain insight into pupils' understandings of formal explanation and proof in mathematics and how objects are used in the mathematical world.

2. Rationale

The background research and theorizing concerned with the relationships of age and experience to mathematical reasoning are summarized briefly at the beginning of the paper. In particular, Bell's (1979) description of the meaning of proof in terms of verification or justification, illumination, and systematization is emphasized. Bell's research stemming from the proposition that pupils will not employ formal proofs until they understand the public status of knowledge and the value of public verification is reviewed. Van Dormolen's (1977) conception of different levels of functioning in logical thinking is also discussed. Van Dormolen's examples are related to Van Hiele's three levels of thinking: (1) a ground level, in which thinking is limited to a particular example; (2) Level 1, in which concepts are more abstract but still limited to the domain of discourse; (3) Level 2, in which local organization is understood and the person learns to reason about reasoning.

Relying on the research of Bell and Van Dormolen in particular, the present investigator was concerned with identifying discrepancies between the thought processes of pupils and accepted mathematical reasoning processes regarding specific problems. He was interested in determining the extent to which pupils are aware of the need for and

and actually use specific techniques and concepts in evaluating proofs and explanations.

3. Research Design and Procedures

The clinical interview technique was employed to study pupils' responses to three selected mathematical problem situations: Game of 25, Game of 7, and Quadrilaterals. Interviewing the pupil while he or she attempted to solve each problem consumed 30-40 minutes time per pupil, who was free to use paper and pencil and was asked a series of pre-planned questions while he or she attempted to solve the problem. The prompting questions continued until the problem was solved or the prompts were exhausted. Complete responses to each item were obtained from a minimum of 170 pupils aged 12-17 in the Brisbane (Queensland), Australia public schools. The interviews, which were tape-recorded for later analysis, were conducted by postgraduate students in education.

4. Findings

The 170 response protocols to each of the three items were analyzed in terms of the degree of completeness and methods of proof employed. Tabulations were made on such variables as complete checks, partial checks, etc., but no statistical analyses were reported. Empirical findings with respect to variety and completeness of checking, identification and use of principle, chaining of inferences, domain of validity of generalization, literal interpretation of statements and conditions, distinction between implication and equivalence, the meaning of definition, and proof structure were analyzed intuitively and considered at length in the paper.

5. Interpretations

The "clinical" interpretations of the findings of this investigation are in terms of eight components identified from clusters of responses given in the three problem situations: variety/completeness in checking; proof/explanation related to an external principle; linking of inferences; domain of validity of generalizations; literal interpretations of data; evaluating statements/distinguishing implication and equivalence; meaning of definitions; and proof structure. The investigator concluded that the response types identified in this investigation indicate that the majority of pupils do not have an objective, detached view of problems, but are rather restricted and occasionally even employ psycho-emotional approaches to problem solution. Correcting these limitations in pupil perceptions and thinking will require greater attention in schools to the development of high-level thinking across many contexts.

## Abstractor's Comments

This is an interesting, thought-provoking paper concerning the perceptions and logical thinking processes of secondary school pupils about mathematical problems. It is a heuristic paper in that, although it provides little concrete numerical data and few definitive findings, it should serve to generate numerous hypotheses for empirical investigation.

The clinical, or phenomenological-intuitive, approach used in this investigation has well-known limitations as a scientific method. The inherent subjectivity of the approach poses many questions concerning objectivity, reliability of testing and interviewing, generality of the findings, etc. In addition, insufficient data on the nature of the sample are included, and the lack of statistical tests of significance are noteworthy (at least to an American psychologist!). However, the clinical approach has many adherents and has generated some intriguing results. Furthermore, it is a necessary approach in studying such subjective phenomena as the thought processes or mental strategies employed by secondary school students in attempting to solve problems. Also, in defense of the investigator's methodology, it was not completely open-ended: specific, prearranged prompts were given by the interviewers. And although the reader is not told how many postgraduate students served as interviewers and how they were trained, the sample of respondents appears to have been sufficiently large.

## References

Bell, A. W. The Learning of Process Aspects of Mathematics. Educational Studies in Mathematics 10: 361-387; 1979

Van Dormolen, J. Learning to Understand What Giving a Proof Really Means. Educational Studies in Mathematics 8: 27-34; 1977.

Heller, Kirby A. and Parsons, Jacquelynne Eccles. SEX DIFFERENCES IN TEACHERS' EVALUATIVE FEEDBACK AND STUDENTS' EXPECTANCIES FOR SUCCESS IN MATHEMATICS. Child Development 52: 1013-1019; September 1981.

Abstract and comments prepared for I.M.E. by DALE DROST, Memorial University of Newfoundland.

1. Purpose

The purposes of this study were to investigate the existence of sex differences in teachers' use of evaluative feedback in junior high school mathematics classes and in students' expectancies for success in mathematics.

2. Rationale

Concern was expressed for the relative underparticipation of females in high school mathematics courses and the subsequent effects of this on future educational and career options. The researchers felt that the variables chosen for study might be related to participation in more advanced mathematics classes.

The portion of the study which dealt with the teachers' use of evaluative feedback was modeled on the work of Dweck et al. (1978). These researchers had identified differences in the type of praise and criticism received by males and females in fourth- and fifth-grade classrooms.

With respect to students' expectancies for success, research was cited supporting the position that performance is related to expectancy and that lower expectancies are often found with females than with males. The junior high school years were chosen for study based on the claim that these are the years when sex differences in attitudes towards and achievement in mathematics begin to emerge.

3. Research Design and Procedures

Data were collected by three methods: a classroom observational system, a student questionnaire, and a teacher questionnaire.

Five observers undertook approximately three weeks' training on a modified version of Brophy and Good's Teacher-Child Dyadic Interaction System (1970) and Dweck's observational procedures (Dweck et al., 1978).

Instances of teachers' use of praise and criticism were observed and recorded as they related to each student's quality of work, form of work, and conduct. The observers also coded teachers' explicit use of causal attributional statements into one of the four categories of task difficulty, effort, ability, or incorrect use of a mathematical operation. Finally, each explicit expectancy statement made by the teacher with respect to a child's performance was coded on a four-point scale ranging from most positive to least positive. The mean percentage of agreement for each observer with criterion coders was greater than 76% in all cases, with over 70% agreement being attained for individual categories.

The student questionnaire consisted of six questions about students' expectancies for success in mathematics, the questions being divided into those examining expectancy for success on a familiar task (i.e., current task), and those examining expectancy for success on a less-familiar task (i.e., future task). A seven-point rating scale was used, ranging from "not at all well" to "very well". Cronbach Alpha coefficients ranged from 0.77 to 0.85 and the correlation between the two scales was 0.62.

The teacher questionnaire contained two items. Teachers were asked to rank each student's position in class in terms of quintiles and also to indicate on a seven-point scale, ranging from "very poorly" to "very well", the expectancy for each student's performance in a future advanced mathematics course.

The observational system was used in eight seventh-grade and seven ninth-grade classrooms in middle to upper-middle class neighborhoods in a small northwestern city. Classes were volunteered by their teacher. The mathematics curriculum in each class was at grade level or slightly advanced. Observations were conducted for 13-15 hours in each classroom over a two-month period, with the last 10 hours of observations being recorded.

The student questionnaire was administered in 12 of the above classrooms to students who volunteered and also received parental permission. Fifty-nine percent of the total seventh-grade sample and 67% of the ninth-grade sample participated. Teacher questionnaires were apparently administered to all participating teachers. The questionnaires were administered after the completion of the observations.

Four sets of analyses were planned:

1. comparisons of the teachers' use of discriminant praise and criticism for boys versus girls;
2. comparisons of the teachers' use of praise and criticism for students having high versus low teacher expectancies;
3. comparisons of the teachers' causal attributions and expectancy statements for boys versus girls; and
4. comparisons of boys' and girls' expectations for their own mathematics performance.

4. Findings

1. Neither of the two main effects, sex or grade, nor the interaction between sex and grade was significant for any of the five variables: percentage of praise directed to the quality of work and the form of work; percentage of criticism directed to the quality of work, to the form of work, and to conduct. Praise directed to conduct was deleted from the analysis since it occurred very infrequently. The same conclusion was reached when the classroom, treated as a random factor nested within grade, was used as the unit of analysis, when the individual was used as the unit of analysis, and when sex and teacher were used as independent variables. In each case analysis of variance procedures were used.

2. Students were divided into high and low expectancy groups, based on the teacher's expectancies. With respect to praise there were no significant differences between grades or between expectancy groups, nor was there a significant sex-by-expectancy-group interaction. It was implied, although not stated, that there was no significant difference between sexes. For criticism, grade level was not significant nor was the sex-by-expectancy-group interaction. It was implied, though not stated, that there was no difference between expectancy groups. For criticism, boys received significantly more criticism in dyadic situations than did girls. The results for other interactions were not reported. For the above, the classroom was the unit of analysis and the

dependent variable was the mean score for each sex within each expectancy group and classroom.

3. Teachers made very few attributional or expectancy statements. Most attributional statements followed unsuccessful student outcomes and only these were coded. Chi square analyses revealed that teachers' use of attributional and expectancy statements did not vary as a function of either sex or teacher expectancy.

4. With respect to students' expectations for their own success the only significant difference reported was that girls had lower expectancies of success for future tasks than boys. Neither sex, grade level, nor the interaction between sex and grade level were significant for the complete expectancy of success scale or for the expectancy of success for the current performance subscale. ANOVAs were carried out using both the classroom and individual as the unit of analysis with similar results.

5. Interpretations

The investigators concluded that these findings did not support those found previously by Dweck et al. They suggest that possible reasons for this conclusion were that Dweck et al. had used only three teachers who might not have been representative. Also, the current study was conducted in junior high school mathematics classes whereas Dweck et al. observed fourth and fifth graders in a variety of subject areas. It was suggested that "teachers' feedback is in part determined by the age of the students" (p. 1019). It was also suggested that "sex differences in expectancies for mathematics do not emerge with any consistent regularity until late junior high school" and hence the findings in the present study were not surprising. Future research was recommended at a variety of grade levels to attempt to resolve reasons for the conflicting results.

The finding that girls had lower expectancies for future or unfamiliar tasks than did boys was considered to be in support of previous research. It was noted that a study examining the relationship between expectancies for success and participation in advanced mathematics was being undertaken.

## Abstractor's Comments

Research which helps educators better understand why fewer girls than boys choose to study mathematics in the high school can be valuable in alleviating this problem. Although the results of this study suggest that teachers' use of evaluative feedback in junior high school mathematics classes may not be a reason for underparticipation by females, as the investigators point out, this finding in itself is valuable.

Although the research was quite well designed and reported, several questions must be asked about the study:

1. One of the criteria for a class to be included in the sample was voluntary agreement by the teacher to participate. Are such classes representative of the larger population?

2. The student questionnaire was administered only to students in 12 of the 15 classes in the sample who volunteered and who had parental permission. Were these students representative of the larger population? From data reported in the study the participation rate of girls on the student questionnaire was 12% higher than for boys in grade 7 and 16% higher in grade 9. What are the implications of such a different rate of participation?

3. Details of the student questionnaire are scanty. Only six items were used in the current analysis, these consisting of two subscales. It is not stated how many items were on each subscale. One subscale is referred to as having novel or less familiar items at one time, and later as having items referring to success in later mathematics courses. These concepts need more explanation before valid judgments can be made about this instrument.

4. In the primary analysis the classroom was used as the unit of analysis. The researchers are to be commended for using this unit of analysis; however, they should have been content with this decision and not continue to do the analysis using students as the unit of analysis. What would have been their conclusions if the second analysis had indicated significant differences, when clearly the proper analysis revealed no such differences? The motives of the researchers become suspect when such inappropriate methods are used.

5. There was some indication in the rationale for the study that the investigators may have expected to find differences in teachers' use of evaluative feedback with girls and with boys. Yet when they found no such differences, they suggested in their discussions that this was not surprising since differences tend not to emerge until late junior high. One wonders why, if this was the case, they chose seventh graders for their sample rather than eighth or even tenth graders. Perhaps a similar study should be undertaken with students in these grade levels.

In spite of these criticisms, the study is a worthwhile contribution to the literature in this area. As the investigators indicate, more research is necessary before we can be sure if and where differences exist in teachers' use of evaluative feedback.

Hiebert, James. THE POSITION OF THE UNKNOWN SET AND CHILDREN'S SOLUTION OF VERBAL ARITHMETIC PROBLEMS. Journal for Research in Mathematics Education 13: 341-349; November 1982.

Abstract and comments prepared for I.M.E. by J. DALE BURNETT, Queen's University, Kingston, Ontario.

1. Purpose

Addition and subtraction problems were verbally presented to first-grade students in order to examine the relationships between "position of the unknown set" and the child's (1) method of representation and (2) strategy for obtaining a solution.

2. Rationale

The article quotes two studies from 1972 and 1973 which looked at students' solutions to number sentences and concluded that the position of the unknown in a number sentence affected the level of difficulty of the problem. One other study, a 1981 acticle which was co-authored by Professor Hiebert, is cited which indicates that, given the opportunity, many young students will represent such problems with small cubes and then manipulate the cubes to arrive at the correct answer.

3. Research Design and Procedures

Sample: 3 first-grade classrooms, n = 47. All students receiving parental permission were included.

Setting: March. The students had not received any previous formal instruction in solving verbal problems or in using concrete objects to represent problem situations.

Task: an interviewer read 6 problems to each student; 3 involving joining (addition) and 3 involving separating (subtraction). The order of presentation was randomized for each student. A set of small cubes was available. Factors such as syntactic complexity and number size were similar across all of the problems. Examples of the verbal problems and the associated number sentence are provided in the article. Thus, a joining problem where the position of the unknown set in the associated number sentence is that of the second addend is: "Bill had 3 marbles. Susan gave him some more marbles. Now he has 8 marbles altogether. How many marbles did Susan give to Bill?", which is paired, at least in the investigator's mind, with the number sentence $3 + \square = 8$.

Summary: A total of 47 first-grade students were each given 6 verbal arithmetic problems resulting in a total of 282 protocols.

Analysis: The analysis consisted of three phases. First, each protocol was examined and identified as exemplifying a particular strategy. From this stage a total of 9 "appropriate" strategies and 3 "inappropriate" strategies were identified.

These 12 strategies were then cross-classified with the 6 problem types and with whether or not the student used cubes to model the situation to provide a 6 x 2 x 12 table of student responses. Essentially the table consists of 282 classified protocols fitted into a structure with 144 cells.

Two additional columns were added to the table, one containing the simple sum of all of the "appropriate" strategies for that problem type and a second which indicated how many of these strategies resulted in the correct answer.

The final phase of the analysis consisted of examining this table and computing a few sub-totals and their corresponding percentages.

## 4. Findings

This study addressed two principal issues. With respect to the method of representation used to model the problem (i.e., whether or not the student used cubes), the author notes that 55% of the responses to problems of the form $a \pm b = \square$ involved cubes as part of the overall strategy, as compared with 40% for problems of the form $a \pm \square = c$ and only 18% for problems like $\square \pm b = c$.

The second issue, dealing with the cognitive strategies used by the students to solve the problems, noted dominant strategies for problems of the form $a \pm b = \square$ and $a - \square = c$, but a variety of approaches for $a + \square = c$ and $\square \pm = c$ types.

A review of the tabulations of the correct answers shows that most students (88% or 71%, depending on whether they used cubes or not) could solve problems of the form $a \pm b = \square$, whereas 50% or 33% could solve $\square + b = c$ and 22% or 37% could solve $\square - b = c$. The author also makes special note that only 39% and 28% could solve $a + \square = c$ whereas 80% and 37% could solve $a - \square = c$ types. There was a higher percentage of success in five of the six problem types for the group using cubes than for the group not using cubes.

The findings just noted are discussed on two levels. At a level closely related to the empirical context, the following conclusions are noted:

1. "....the position of the unknown had a substantial effect on children's modeling behavior" (p. 345) (i.e., cubes or no cubes). This is based on the differential percentages (53%, 40%, 18%) of students using cubes across the three main problem types.
2. "...the strategies used to solve the problems matched the action or relationships described in the problem" (p. 345). This is based on a careful comparison of strategy types for each problem type.
3. "....problems with the unknown in the first position not only are more difficult to model but also are more difficult to solve" (p. 345). This is based in part on the data used to support the first conclusion and in part on the lower level of success for the "first position" problems.
4. The major finding of the study is nicely summarized by the statement, "...the relative difficulty of a problem in this study seemed to depend on whether or not it was initially modeled with objects, which in turn depended on the position of the unknown set" (p. 347).

5. Interpretation

The second level of interpretation revolves around a brief discussion of two theoretical models, one by Skemp and one by Riley, Greeno, and Heller, which suggest "that arithmetic problems that are amenable to direct representation may be easier to solve than those that are not" (p. 348). "The results of this study indicate that the position of the unknown set in a verbal problem determines to a substantial degree whether or not the problem can be modeled successfully by first-grade children" (p. 348).

## Abstractor's Comments

I would like to organize my comments into three groups. First, I would like to summarize those aspects of the study that I liked. Second, I would like to play around with the data a little to show why I have some reservations about the conclusions. Finally, I would like to append a brief introspection of the review process itself.

There is much about this study that appeals to me. The main emphasis is one I endorse: it focuses on the actual processes used by children while attempting to solve a particular class of problems. Krutetskii (1976) has expressed his preference for this type of research strategy both forcefully and eloquently:

> It is hard to understand how theory or practice can be enriched by ... who computed, for 130 mathematically gifted adolescents, their scores on different kinds of tests and studied the correlation between them, finding that in some cases it was significant and in others not. The process of solution did not interest the investigator. But what rich material could be provided by a study of the process of mathematical thinking in 130 mathematically able adolescents! (p. 14).

Given the concern for process, the study is well-designed and carried out. I must also admit that I have a soft spot for researchers who do their best to let the data speak to them, and to help them improve their understanding, rather than simply to fit the data to confirm (or refute) a well-defined hypothesis. Perhaps my preference is based on my belief that the former group is faced with a more difficult task -- less structured and more open to methodological criticism -- but more interesting and, in the long run, I believe more fruitful. Parenthetically, I view both approaches as lying on a continuum, the exploratory mode eventually giving rise to the confirmatory mode. My suspicion is that we too often believe ourselves to be in a confirmatory situation when in fact an exploratory attitude would be more appropriate.

Having indicated my preference for process-oriented studies, this report is quite disappointing in one major sense. The report fails to give the reader any real understanding of the processes used by the students. All we are given is a generalized description of each strategy and a frequency table that indicates how many students used it in a particular situation. I suspect (perhaps wrongly) that there is a richness in the protocols that has not yet been captured. For example, three types of information are not reported: 1) the student's use of language; 2) timing information -- where are the pauses, the quick bursts of activity in the efforts to solve the problem --; and 3) the student's written work (if any). I suspect that the topic of protocol analysis itself is likely to undergo substantial development in the next decade. Studies like the present one give us an opportunity to begin this development. I would like to encourage the author to take one more step along this path.

Now for a comment on the table of data that is presented in the article. As noted in the abstract, the table is essentially a compilation of 282 events into a tabular structure with 144 cells. Because many strategies are used more than once in a particular context, the resulting table contains 62 non-empty cells and 82 empty cells. The two non-strategies (uncodable and indeterminate) account for 16 cells and 61 events (22% of the data). Simply stated, we do not have a lot of data here upon which to base conclusions. Rather than examine the complete table for noteworthy features, I created a number of sub-tables, a few of which prompt further comment.

For example, restricting attention for the moment to the strategy labeled "uncodable" (for me, an interesting situation deserving of further research), it is an easy matter to construct two 2 x 3 tables (addition or subtraction by position of unknown), one for students who used cubes and the other for those who did not.

| Used Cubes | Position of Unknown 1 | 2 | 3 | Did not Use Cubes | Position of Unknown 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| Addn. | 0 | 0 | 1 | Addn. | 6 | 4 | 1 |
| Subtr. | 0 | 0 | 1 | Subtr. | 2 | 4 | 2 |

Clearly, most uncodable strategies were used by those students who did not model the problem using cubes (and, reasonably, gave the researchers very little hint as to what they were thinking).

Errors are also interesting, and often informative, when investigating children's mathematical behavior. Three error strategies were identified in this study. Easily the most common error, and in fact the most common strategy found in the study, was that of saying that the answer is one of the two numbers given in the statement of the problem. Composing two tables as before for this strategy yields:

| Used Cubes | Position of Unknown 1 | 2 | 3 | Did not Use Cubes | Position of Unknown 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| Addn. | 1 | 6 | 0 | Addn. | 16 | 15 | 4 |
| Subtr. | 3 | 1 | 0 | Subtr. | 11 | 10 | 4 |

Three comments: 1) a total of 71 responses out of 282, or 25% of the sample of responses, were of this type; 2) the number of errors of this type increased as the position of the unknown moved from right to left; and 3) this type of error is much more prevalent among students who did not use the cubes than among those who did. A similar pattern, but with half the strength, is observed if you construct two tables for the "interminate" error strategy.

In summary, a review of the appropriate sub-tables for the various error and unknown strategies indicates 1) that students who use cubes to model the problem have fewer difficulties than those who fail to model the problems; and 2) the errors are much more frequent for problems of the type $\square \pm b = c$ and $a \pm \square = c$ than for $a \pm b = \square$.

The latter conclusion prompts two additional comments: 1) I somehow doubt that any classroom teacher would be surprised by this finding; and 2) the ordering of difficulty of the problems also corresponds to the linguistic complexity of the problems. Thus, problems of the form $a \pm b = \square$ require two factual statements followed by the question; forms $a \pm \square = c$ and $\square \pm b = c$ all require three factual statements for the student to keep track of before the question is presented.

Three of the successful strategies are strongly identified with one specific problem type and with the fact that they all used cubes to model the situation. Thus, "counting all" is used almost always for problems of the type $a + b = \square$; "separate" ["the larger quantity is represented, and the smaller quantity is removed from it. The remaining objects are counted to find the answer" (p. 344)] and "separate to" ["the larger quantity is represented, and objects are removed until the smaller quantity remains. The removed objects are counted to find the answer" (p. 344)]. The other common successful strategy was that "based on recall of that particular number fact" which was used to those students who did not use cubes. And now a concern emerges. Most of the strategies used in this study essentially require (or fail to require) that cubes be used (or not) as an integral part of their very definition (i.e., if you use the "counting all" strategy you are virtually required to use cubes. Similarly, if you know the required number facts, why would you use the cubes?) The remaining components of the master table are sufficiently sparsely populated as to question the value of including them in the overall analysis.

It is now instructive to compare the conclusions emanating from the above treatment with those of the author. It is worth emphasizing that the data base is the same -- frequencies of strategies under particular conditions; the difference is in the selective focus on specific subsets of the data.

The author claims that "... the relative difficulty of a problem seemed to depend on whether or not it was initially modelled with objects, which in turn depended on the position of the unknown set" (p. 347). The data support this conclusion. They also support other interpretations. For example, whether or not a problem was initially modelled with objects seems to depend on whether or not the student knew the relative number facts, or whether or not the problem is actually understood (i.e., would a student who really understood the problem ever use the strategy of "repeating a given number"?). The relative difficulty of the problem is also dependent on the position of the unknown set. Although the author did not make this mistake, it still deserves emphasizing that the nature of the data is descriptive and correlative -- it is possible to make statements of a relational nature (e.g., as the position of the unknown moves from right to left, the problems become more difficult), but it is not appropriate to make statements of a causative nature. We still do not know why this particular effect is observed.

One final thought: Is the idea of "position of unknown" an adult surface structure feature masking a deeper semantic level structure for the child? It is easy enough to compare the two algebraic representations $a + b = \square$ and $a + \square = c$ and see them as simply differing in the location of the unknown. However, these representations are not available to first-grade students. Their task is to make sense of a string of short verbally presented sentences requiring them to use both memory and logic (who has what and what is unknown). These problems are verbally, semantically, and logically more complex when the unknown is on the left side of the algebraic equation.

As an aside, I would like to thank Professor Hiebert for conducting the original study and Marilyn Suydam, the editor of IME, for inviting me to prepare this abstract and comment. The exercise has been a personal joy. I rarely read articles with the precision that this task required (a sad admission). Writing a succinct abstract of a research study is useful in any context. I must do more of it (independent of IME!). Most of the comments were a spontaneous outgrowth of preparing the abstract --

a dominoe effect. How much more rewarding this has been than my more typical skim reading which would simply note that some kids use cubes and some do not when solving problems of this type and that the position of the unknown affects the level of difficulty of the problem.

## Reference

Krutetskii, V.A. The Psychology of Mathematical Abilities in Schoolchildren. Chicago: University of Chicago Press, 1976.

Janvier, Claude. USE OF SITUATIONS IN MATHEMATICS EDUCATION. Educational Studies in Mathematics 12: 113-122; February 1981.

Abstract and comments prepared for I.M.E. by WILLIAM E. GEESLIN, University of New Hampshire.

1. Purpose

The author examines the effect of a situation on children's cognitive behavior. Situation refers to the physical and verbal environment (context) in which a question or problem is posed.

2. Rationale

Educators encourage relevant or meaningful problem situations. Likewise, many psychologists feel the context in which instruction or problems are presented is a determinant of children's mental structures. Consequently, a wide range of manipulatives, physical materials, and "realistic" problems have been developed for classroom use. Interest in this area has come from the writings of Piaget, Bruner, Dienes, Fischbein, and others.

3. Research Design and Procedures

The sample apparently consisted of 40 first-year pupils in an English secondary-level school. However, an additional 20 pupils from the first- (n = 7), second- (n = 7), and fourth-year (n = 6) secondary level were interviewed also. Subjects were presented with a graph which gave the speed of a race car during the second lap around a race track. Students were asked about the number of bends in the race track and later asked to select the corresponding track from seven alternatives. No information was given concerning selection of the sample. The author stated that the 40 first-year students received the task in written form, but all discussion seems to refer to interview data. No statistical analysis of data was presented.

4. Findings

The main difficulty students had in determining the number of bends in the track was confusing the speed graph with the track. In selecting the track that matched the speed graph, many students were unable to get relative perceptions of the track bends. Sex differences were noticable on several aspects of the task.

5. Interpretations

Familiarity with racing cars helped some boys to complete the first part of the task. Girls, on the other hand, could count on little "situational" support. However, in selecting the correct track, familiarity with the situation appeared to inhibit the necessary abstraction process. Mental images conflicted with the basic abstract aspects of the problem. Too much information, i.e., knowledge of racing cars, made the task more difficult.

Wide individual differences were noted. Situations should be used primarily to assist students in developing their ability to abstract. The author recommended the use of large-scale situations (i.e., involvement over a long time period) that stress the child's point of view rather than mathematical structure. Students need "verbal tags" if they are to deal successfully with abstract concepts. Most importantly, the use of situations does not necessarily make learning easier and is not the panacea for transforming abstract ideas into "concrete" representations.

## Abstractor's Comments

The major contribution of this article is the idea that educators may make mathematics more difficult in their efforts to help children learn. Introduction of physical materials, manipulatives, motivating situations, and realistic problems may confuse the student rather than clarify the mathematics. Research that attempts to investigate accepted "truths" is important and often quite revealing.

Unfortunately, this study is reported poorly and thus does little to answer the questions raised. The results are confounded with sex, spatial ability, knowledge of graphs, and type of problem presentation. The author chooses to disregard some results (e.g., sex differences) while emphasizing others (e.g., errors on the task) and does not explain his choices. In fact, little data are presented, leaving the reader unable to judge the validity of the many inferences made. It was stated that the task was administered in written form to first-year students, yet the discussion of methodology and results indicated an interview technique was used. Discussion includes results from non-first-year students even though the author earlier stated that only results of the written (and thus first-year students') task presentation would be discussed. Therefore neither sample, data, nor procedures are presented clearly. Editors and referees should not allow this much confusion in reporting and should assist the author in locating points of confusion.

Note that the above criticisms are not criticisms of interview methodology (or case studies), but rather are criticisms of the reporting. Interviews are a valuable methodological tool. However, they require controls, planning, and detailed reporting just as the traditional large-scale statistical studies do.

It is my hope that the ideas of Janvier will not be disregarded. The reader will not find much solace in the article as is. However, many interesting questions arise: Was the task used by Janvier a "mathematical" one? Is spatial ability related to abstracting ideas from a graph? Does the use of even a "good" manipulative or problem context confuse the learner? or change the task for some individuals? How does one select appropriate mental processes/retrieve appropriate information when faced with a task? Does the use of "large situations" (such as USMES problems) promote the learning of either problem solving or mathematical concepts? Some of these questions clearly require numerous investigations if we are to obtain answers to them. Janvier's article provides us with some hints as to how we might pursue these questions.

Levine, Deborah R. STRATEGY USE AND ESTIMATION ABILITY OF COLLEGE STUDENTS. Journal for Research in Mathematics Education 13: 350-359; November 1982.

Abstract and comments prepared for I.M.E. by OTTO BASSLER, George Peabody College for Teachers of Vanderbilt University.

1. Purpose

The study investigated the question, "Among college students, what are the relations among computational estimation ability, the number and types of estimation strategies used, and quantitative ability?" (p. 351).

2. Rationale

Estimation skills are useful in daily living and have been recommended as necessary basic skills by the National Council of Teachers of Mathematics and the National Council of Supervisors of Mathematics. Little research has focused on adult estimation skills or the strategies that adults use to estimate.

3. Research Design and Procedures

The sample consisted of 89 college students who volunteered to participate. Descriptive information about the subjects indicated sex (34 men, 55 women) and previous mathematics courses (a mean of 3.0 years of high school mathematics; 53% had not completed a college-credit mathematics course; no mathematics majors were included).

Instruments used in the study were (a) Test of Estimation Ability (TEA) and (b) School and College Ability Test (SCAT) quantitative subtest. The TEA is an investigator-constructed test consisting of ten multiplication and ten division exercises. Items contained two whole numbers, a whole number and a decimal fraction, or two decimal fractions. Directions to subjects were to "think aloud" to obtain an oral estimate to the solution of the exercise. This provides a score on each item determined by the accuracy of the estimate as well as a determination of the strategy used to obtain the estimate. Reliability of scores was .80

and percent of agreement by two raters on strategies was 90.4. The SCAT quantitative subtest, used to measure ability, had a reported reliability of .89.

Eight strategy classifications were developed based upon pilot data, estimation literature, and a logical analysis of the test exercises. Classifications were:

1. Fractions (F) -- use of common fractional relationships.
2. Exponents (Exp) -- a form of scientific notation.
3. Rounding Both Numbers (R2) -- both numbers estimated by a multiple of a power of 10.
4. Rounding One Number (R1) -- only one number is rounded.
5. Powers of Ten (Pow) -- rounding to a power of 10.
6. Known Numbers (K) -- rounding to numbers having a known product or quotient.
7. Incomplete Partial Products (Quotients) (IP).
8. Proceeding Algorithmically (Alg).

Examples for each classification were provided.

Testing time for each subject was approximately one hour. First the TEA was administered by presenting the items in random order. Subjects thought aloud as they obtained estimates without using pencil or paper. To clarify the strategy that was used, the investigator asked probing questions. This portion of the testing session was tape-recorded for later scoring. Next, subjects were asked to provide a brief educational background by completing a short questionnaire. The SCAT was completed last and took 20 minutes.

4. Findings

1) The correlation coefficient between scores on the SCAT (quantitative ability) and TEA (estimation ability) was .74.

2) Analysis of variance followed by the Scheffe procedure was conducted to test for differences in the frequencies of use of the eight strategy types. The results indicated Exp, IP, Pow, and K were used least frequently; F and R1 were used more frequently; and R2 and Alg were used most frequently.

3) Analysis of covariance, where the covariate was quantitative ability, indicated no systematic relationship between accuracy of estimate and strategy used when scores on individual test items were analyzed.

4) A one-way ANOVA was used to compare quantitative ability of students using different strategies for each item. Results were significant for 12 of the 20 items and indicated consistently that students using Alg were of lower ability than students using one of the types R1, F, K, or Pow.

5) The correlation coefficient between number of strategies used and scores on the SCAT was .55.

6) No significant relationship was found between number of estimation strategies and score on the TEA when quantitative ability was partialled out.

7) The mean score on the TEA, which has a maximum score of 60, was 25.9.

5. Interpretations

1) Scores on the TEA were generally low, which suggests that estimation is difficult for college students.

2) Estimation ability is closely related to quantitative ability. In general, high-ability students are better estimators and use more strategies than low-ability students. Also, lower-ability students tend to use the strategy Alg.

3) The most frequently used strategy types were R2 and Alg. Use of Alg may stem from a dependence on exact paper-and-pencil calculations, whereas R2 may be the technique most often taught as a method of estimation.

4) Student estimates when quantitative ability was statistically controlled seemed to have similar accuracy when different strategies were used.

## Abstractor's Comments

This study describes and analyzes the estimation strategies of a particular group of college students. Whether these results can be generalized to a broader population is debatable -- especially since all students were volunteers from a single New York City college and no descriptive statistics pertaining to ability as measured by the SCAT were provided. Another factor which might influence the results is the low level of achievement on the Test of Estimation Ability.

One surprising finding was that, when ability was controlled, there was no relation between estimation strategy used and accuracy of estimate. This may be due to the particular items on the test, or perhaps to poor application of the strategy. In any case, the strategies do produce different accuracies. For example, for the test item 824 x 26, using F (824 x 1/4 x 100) yields an estimate 4% too small, and a test score of 3; whereas, using R2 (800 x 30) yields an estimate 12% too small and a test score of 2. Other methods of estimation may yield more diverse results. This finding needs further investigation.

The study did provide a useful way of assessing estimation abilities and strategies. It provides support for the view that college students are poor estimators. Why would we expect different results when "they (the students) reported having been taught little if anything about estimating?" (p. 357). It is interesting to note, however, that despite this lack of instruction, individual students used an average of over four estimation strategies in completing the TEA. It seems to me that this study should provide the impetus for additional research and emphasis on teaching estimation strategies.

Szetela, Walter. STORY PROBLEM SOLVING IN ELEMENTARY SCHOOL MATHEMATICS: WHAT DIFFERENCES DO CALCULATORS MAKE? Journal for Research in Mathematics Education 13: 381-389; November 1982.

Abstract and comments prepared for I.M.E. by STANLEY H. ERLWANGER, Concordia University, Montreal, Quebec.

1. Purpose

Main Study: To determine if students who use calculators in story problems tend to try more problems, to use more correct operations, and to obtain more correct answers than students who use paper and pencil only. Supplementary Study: To compare the use of calculators on a posttest of problem-solving with the use of paper and pencil only, after all groups had used calculators during 8 weeks of instruction.

2. Rationale

First, in many studies, students who used calculators during instruction used only paper and pencil in achievement posttests. Roberts (1980) has criticized this tendency and suggests calculators should be used instead. Second, Szetela (1980a, 1980b, 1981) has shown that the calculator is a critical factor in solving story problems. In all three studies, students who used calculators performed better than those who did not. There were no significant differences when paper and pencil was used. Third, Wheatley and his colleagues have suggested that when calculators are used students can "focus on choosing the correct operations, determining the reasonableness of their answers, and further, a broader range of strategies is possible" (p. 21). Fourth, although there are hypotheses concerning superior problem-solving performance when students use calculators, there is little evidence to support them.

3. Research Design and Procedures

The investigation consisted of a main study and a supplementary study conducted simultaneously.

Main Study

Subjects: Two classes in each of grade 3 (n = 50), grade 5 (n = 36), grade 7 (n = 49), and grade 8 (n = 52) participated. Each grade was from a different school. Students in grades 3, 5, and 7 were randomly assigned to the Calculator group (C) or the Non-Calculator group (N). Grade 8 was partially randomized because of scheduling problems. One teacher in each grade taught both the C group and the N group.

Instruments: One pretest and two posttests were given in each grade.

Pretest: A 40-item pretest on numerical skills developed by Robitaille and colleagues (1979) was used for grades 3, 7, and 8. A similar test was constructed for grade 5.

Posttests: In the first posttest, consisting of 16 items on computational skills and 10 problems, only paper and pencil was allowed. In the second posttest, consisting of 20 problems, the C group used calculators.

Procedures: After the pretest, regular instructional activities were followed by the N groups in grades 3, 5, and 7 for 8 weeks, and those in grade 8 for 12 weeks, since they started 4 weeks earlier. The topics for each grade were:

Grade 3: Whole number operations in multiplication, basic division facts, and problem-solving applications.

Grade 5: Introduction to decimals, operations with decimals, and problem-solving applications.

Grade 7, 8: Decimals, ratios, percents, and problem-solving.

The C groups followed similar instruction at the same time, except that they used calculators and materials designed for calculators. One calculator was provided for every two students. In grade 3 the calculator was used mainly for problem solving, while in grades 5, 7, and 8 it was used for other activities as well as problem solving.

Data Analysis. Posttest data were analysed using analysis of covariance with pretest scores as a covariate. Three measures were analyzed for the problem-solving tests on the number of problems (i) attempted, (ii) with all operations correct, and (iii) with correct answers.

## Supplementary Study

Subjects: Seventy-six grade 7 students from three classes (two with the same teacher), 23 grade 6 students in one class, and 25 students in a split grade 5/6 class from two schools.

Procedures: Procedures were as in the main study, but calculators were were provided in each class, with one calculator for every two students.

Posttesting Treatment: Students were posttested on the 20 problems in a Calculator Testing Mode (CTM) and in a Paper-and-Pencil Mode (PPM) during one 50-minute session. For the first section of 10 problems, half of the students in each class were randomly assigned to CTM and the other half to PPM. For the second section of 10 problems, the groups reversed testing modes.

Data Analysis: Data were analyzed by analysis of covariance with pretest scores as a covariate. Measures were taken for the number of problems (i) attempted, (ii) with all correct operations shown, and (iii) with correct answers.

4. Results

Main Study:

(a) The pretest results, with Hoyt estimates of reliability ranging from 0.81 to 0.90, show that for all the comparison groups no pretest means were significantly different.

(b) First posttest (paper and pencil only - C and N groups)

The means, standard deviations, and F Ratios were calculated for the four measures: Skills (160), Problem attempts (10), Correct operation (10), and Correct answers (10). The Hoyt estimates of reliability for the computation test ranged from 0.63 to 0.87. Only the grade 3 results were significant, in favor of the C group on three measures - Skills, Correct operation, and Correct answer. All other differences were nonsignificant. The author states: "Overall, it is evident that the use of calculators over periods from 8 to 12 weeks did not diminish skill in paper-and-pencil computation and problem-solving ability ..." (p. 384).

(c) Second Posttest (C group with calculators)

The means, standard deviations and F Ratios were calculated for three measures on the 20 problems: Problem attempts, Correct operation, and Correct answer.

The results show significant differences in favor of the C groups as follows:

Grade 3 (Correct answer); grade 7 (Problem attempts, Correct operation, Correct answer); grade 8 (Correct answer). All other differences were nonsignificant.

Supplementary Study:

(a) The results of the pretests, with Hoyt estimates of reliability ranging from 0.58 to 0.91, show that the pretest means for comparison groups were not significantly different.

(b) The posttest results show that the groups using calculators performed significantly better than the paper-and-pencil groups on four out of 8 comparisons for Correct answers. The other four differences were nonsignificant, except on Correct operations in grade 6.

5. Interpretations

The author points out three findings from the two studies:

- The considerable degree of consistency of results in problem-solving performance with and without calculators that was found in the two studies. Also, the differences found were in favor of students using calculators.
- The results are consistent with other research that indicates that no loss of paper-and-pencil skills occur after using calculators during instruction.
- The results indicate the advantage that calculators provide in obtaining correct answers, but not in the number of problems attempted or in the number of problems in which students choose the correct operations.

The author observes further that, according to the evidence presented by Zweng (1979) from the National Assessment of Educational Progress, it appears "that it is the inability of students to choose the correct operations rather than computational weaknesses that contributes most to their inability to solve problems" (p. 387). The study indicates that calculators helped students to compute correctly, but not to attempt more problems or to choose correct operations any better than students without calculators.

The author concludes that although the study shows that the benefit of using calculators with story problems is limited to avoiding computational errors, this is an advantage - especially since calculators are accessible and inexpensive.

Abstractor's Comments

The results from the study have implications for the teaching of mathematics. It is disappointing to find out that the main role of a calculator in solving problems is that of a computational aid. The study should be replicated.

The research itself was well planned. The main study and supplementary study complement one another. The design and method of analysis were adequate aside from three limitations: one calculator for two students, the random assignment of grade 8 students, and two teachers for three grade 7 classes. My main concern is that the style of reporting is too concise and factual, so that it is difficult to see what assumptions were made. The questions and comments below illustrate some of those aspects I feel would have enhanced the value of this research.

1. The study attempted to find out whether the use of calculators makes a difference in solving story problems on three measures: number of problems attempted, correct operations, and correct answers. The report should thus provide enough details about how these aspects were taken into account.
2. Instruction: The instruction of the calculator and non-calculator groups in each grade is an important aspect of the study. The report says both groups received similar instruction which appears to consist of "problem-solving activities" and "other activities". How were these two aspects organized and controlled to ensure that both groups received the same treatment? Did students know how to use calculators prior to the study? Moreover, since the study was concerned primarily with problem-solving, would it not be better to confine the use of calculators to this aspect of the instruction only?
3. Problem-Solving Activities: This is a critical aspect of the whole study, but we know very little about it. The report does not

describe the approaches used by the teachers to teach problem-solving and we do not know whether or not the problem-solving activities were related to the purpose of the study. A more serious issue is: What were the criteria for "use of paper and pencil" and "use of calculator" in solving story problems and how is this related to the purpose of the study? In what way was the calculator intended to make a difference? Did the use of calculators influence what was done? It is not clear why one calculator was provided for two students. Was this intentional or what?

4. The Posttests: The report gives a sample problem for each grade. This is useful. However, the posttests were used to compare the performance of the comparision groups on three measures. One would therefore want to know the criteria used for selecting the problems, constructing the tests, and determining each measure. Was the selection of problems based on number of operations, complexity of computation, problem structure, etc.? Was time a factor in the test? What instructions were given to students?

I do appreciate the author's difficulty in that one is expected to produce a fairly short report for publication. However, a factual report seems to be inappropriate for a study which deals with aspects of problem-solving behavior. In my view, the value of this study would be greatly enhanced by detailed descriptions and explanations of assumptions and procedures, which include a combination of quantitative and qualitative data. For example, a description of how teachers usually taught problem solving with and without calculators would have been useful. Similarly, a description of observations of how individual pupils in the comparison groups solved problems would help the reader to evaluate the results.

Threadgill-Sowder, Judith and Sowder, Larry. DRAWN VERSUS VERBAL FORMATS FOR MATHEMATICAL STORY PROBLEMS. Journal for Research in Mathematics Education 13: 324-331; November 1982.

Abstract and comments prepared for I.M.E. by MICHAEL T. BATTISTA, Kent State University.

1. Purpose

This study investigated the effect on problem-solving performance of two formats for presenting routine mathematical story problems: verbal (as typically encountered in textbooks) and drawn (line drawings with minimal verbiage depicting a problem situation).

2. Rationale

Several studies have indicated that when students' performance on story problems is compared, the picture or diagram format is superior to the verbal format. But these studies have been restricted to test situations only. The authors suggest that the results may have been due to the fact that the picture/diagram format was novel to the students. Thus, the present study included practice with both verbal and drawn format problems before testing.

The study also investigated the effect of field independence and spatial visualization on performance in solving verbal and drawn format problems. In the case of field independence, the authors hypothesized that "The prominence of the essential information allowed by a drawing of a problem could negate any handicap a field dependent student might have with the problem in verbal format" (p. 325). Spatial visualization was included as a variable because it has been hypothesized in the literature to be related to visual encoding and flexibility in transforming data.

3. Research Design and Procedures

The subjects of the study were 262 students from ten participating fifth-grade classes in the Calgary, Alberta public school system.

Field independence was measured by the Hidden Figures Test, spatial visualization by the Punched Holes Test, and general reasoning by the Arithmetic Reasoning Test. All three measures were NLSMA adaptations of the French Kit of Cognitive Factors. A set of problems appropriate for fifth-grade students and requiring all four arithmetic operations was written by the authors in the usual verbal format. Each of these problems was also constructed in the drawn format.

There were two equivalent forms of the 16-item posttest. Each form presented the problems in the same order, alternating between verbal and drawn formats. Problems in the verbal format on the first form appeared in drawn format on the second form, and vice versa. The computataional difficulty of the 8 verbal versus the 8 drawn problems on each test was controlled by requiring that each problem in the verbal format be paired with a problem in drawn format that required similar computational skill to solve. Each item on the posttest was scored for correct arithmetic operation and correct solution. Each student was assigned a Drawing Score, Verbal Score, and Total Score for the posttest.

After the three aptitude measures were given to the students, five classes were randomly assigned to both the verbal and drawing treatment groups. During the five-to-six-week treatment period, students in the verbal group were given four sets of 8 verbal problems for practice, and students in the drawing group were given four sets of 8 drawn problems. The two forms of the posttest were then randomly administered to the students.

4. Findings

The Drawing Scores ($\bar{X}$ = 11.87, s.d. = 3.67) were significantly, but not substantially, higher than the Verbal Scores ($\bar{X}$ = 11.21, s.d. = 3.79). In order to test for a difference in problem-solving performance between the verbal and drawing treatment groups, ANOVAs were run, first using classes as the unit of analysis (no significant differences), then using students as the unit. In the latter case, the verbal group scored significantly higher ($p < .05$) on the Total and Verbal Scores. It was noted, however, that the verbal group also scored slightly higher on the Arithmetic Reasoning Test, so the two treatment groups could not be assumed to be equivalent. Informal interviews of students indicated that most students preferred problems in the drawn format.

Each of the three posttest scores was regressed on the aptitude measures to test for aptitude-treatment interactions. (All three aptitude measures were positively correlated with the posttest scores.) The only indications of ATIs were disordinal interactions between treatments and the Hidden Figures Test for both the Total Score and the Drawing Score ($p < .10$). For the Drawing Score, the slope of the regression line for the verbal treatment group was slightly greater than that for the drawing group.

5. Interpretations

The authors state that "Presenting problems by way of drawings was clearly more effective than the standard words-only presentation for these students. Students interviewed about preferences indicated that the drawings helped clarify problems" (p. 329). Furthermore, the authors suggest that the interaction between treatments and field independence on the Drawing Score and Total Score indicates that practice on drawn format problems may be more helpful than practice on verbal format problems for field dependent students, with the reverse true for field independent students. They state, "Perhaps practice on drawn problems serves to distract the field independent students by providing them with unnecessary mediators" (p. 329).

## Abstractor's Comments

When I first examined the pair of example problems provided in the article, one in verbal format, the other in drawn, I thought the drawn problem would be more difficult for students. It seemed that students would have to analyze the drawing more carefully than the verbal problem in order to decide what the problem asked and what was given.

The results of the study indicated that, at least for fifth-grade students, performance on drawn format problems was somewhat higher than on verbal format problems. However, if in fact the drawn format problems require more analysis than verbal format problems, we should expect that there would be a greater difference in performance between field independent and field dependent students on drawn format problems than on verbal format problems because the field independent students are more likely to utilize analysis (Witkin et al., 1977). Testing this hypothesis would require comparing the regression line for Drawing Scores on Hidden Figures Test scores to the line for Verbal Scores on Hidden Figures Test scores. This was not done in the present study.

Instead, the authors chose to focus on the observed (though not significant) ATI that suggested that the difference in Drawing Score performance between field independent and field dependent students was greater for the verbal treatment group than the drawing treatment group. The authors hypothesized that this result could have been caused by the fact that practice on drawn problems was distracting for field independent students (and not for field dependent students.) An alternate explanation is that when the verbal group was tested with drawn format problems, the novelty of the drawn format required the students to do more analysis than they would have done with a familiar format. Thus, since field independent students are more likely to use analysis, the observed ATI is consistent this hypothesis.

In addition to field independence, the other variable investigated by this study was spatial visualization. Although spatial visualization was more highly correlated with the problem-solving scores than field independence, not much mention was made of its effect on problem-solving performance. Apparently, there was no ATI between spatial visualization and treatment on any of the posttest measures. But it would have been interesting to see some of the relevant data. For instance, it was hypothesisized that one reason spatial visualization was important to consider as a factor in solving drawn problems is the likelihood that visual encoding involves some use of spatial relationships. This would seem to imply that spatial visualization would be more important for success in solving and practicing drawn format problems than verbal. However, with some verbal problems, a key element in solving the problem might be to visualize or imagine a situation. In this case, spatial visualization would seem to be very important in solving the verbal format version of the problem, but not so important in solving the drawn format version, since in the drawn format the visualizing is already done for the student.

There are several other questions that should be considered when interpreting the results of the study: Is there an interaction between treatments or testing format and reading ability? For instance, are students with low reading ability better able to solve problems in drawn format, or is the drawn format treatment more effective for them? What is it about the drawn format that makes the problems easier to solve? Are drawings simply more interesting to students? The example problem given in the article had drawings of human-like characters. Did all of the problems used in the study have such characters? Maybe students are more attentive to such problems -- especially field-dependent students (Witkin et al., 1977).

All in all, I found this to be an interesting study. It raised many theoretical questions on which I would like to see further research. As for instructional implications, the authors state, "The confirmation that with fifth graders a drawn format can give a problem-solving performance superior to that of verbal format has clear implications for textbook publishers and teachers" (p. 329). Since the difference in performance on drawn and verbal presentation format was moderate, and since we don't know if the results hold true at other grade levels or what the long-range effects would be if too great an emphasis were placed on drawn format problems, I would hope that textbook publishers and teachers move cautiously in utilizing the present results. For now, it seems prudent to say only that practice on drawn format problems is an alternate instructional strategy that can be used to help improve students' problem-solving performance.

## Reference

Witkin, H. H., Moore, C. A., Goodenough, D. R. and Cox, P. W. Field-dependent and field-independent cognitive styles and their educational implications. Review of Educational Research, 1977, 47: 1-64.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
October - December 1982

EJ 264 419 Benbow, Camilla P.; Stanley, Julian C. Intellectually Talented Boys and Girls: Educational Profiles. Gifted Child Quarterly, v26 n2, 82-88, Spring 1982.

EJ 264 930 Silverman, Irwin W.; Rose, Arthur P. Compensation and Conservation. Psychological Bulletin, v91 n1, 80-101, January 1982.

EJ 264 939 Peterson, Penelope L.; Swing, Susan R. Beyond Time on Task: Students' Reports of Their Thought Processes during Classroom Instruction. Elementary School Journal, v82 n5, 481-91, May 1982.

EJ 264 940 Winne, Philip H.; Marx, Ronald W. Students' and Teachers' Views of Thinking Processes for Classroom Learning. Elementary School Journal, v82 n5, 493-518, May 1982.

EJ 264 942 Smitsman, A. W. Perception of Number. International Journal of Behavioral Development, v5 n1, 1-31, March 1982.

EJ 265 341 Stigler, James W.; And Others. Curriculum and Achievement in Mathematics: A Study of Elementary School Children in Japan, Taiwan, and the United States. Journal of Educational Psychology, v74 n3, 315-22, June 1982.

EJ 265 345 Holzman, Thomas G.; And Others. Cognitive Dimensions of Numerical Rule Induction. Journal of Educational Psychology, v74 n3, 360-73, June 1982.

EJ 265 533 Bryant, Peter, Ed. A Special Issue on Piaget: Issues and Experiments. British Journal of Psychology, v73 n2, 157-311, May 1982.

EJ 265 641 Tishler, Anne G. Attitude-Achievement Interaction in Mathematics with Preservice Elementary Teachers. Capstone Journal of Education, v2 n2, 40-44, Winter 1981-82.

EJ 265 964 Ross, Steven M.; Rakow, Ernest A. Adaptive Instructional Strategies for Teaching Rules in Mathematics. Educational Communication and Technology: A Journal of Theory, Research, and Development, v30 n2, 67-74, Summer 1982.

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