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ABSTRACT

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# Parametric ANCOVA vs. Rank Transform ANCOVA When Assumptions of Conditional Normality and Homoscedasticity are Violated



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Running head: Parametric ANCOVA vs. Rank Transform ANCOVA

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# Parametric ANCOVA vs. Rank Transform ANCOVA When Assumptions of Conditional Normality and Homoscedasticity are Violated

Data obtained from research studies based on the pretest-posttest randomized control group design (Campbell and Stanley, 1963) are frequently analyzed using analysis of covariance with the pretest as the covariate and the posttest as the dependent variable. This analysis strategy assumes that the data meet the following conditions:

1) The relationship between the pretest and the posttest is linear;
2) The relationship between the pretest and the dependent variable is the same for all groups (homogeneity of regression slopes);
3) The posttest scores are independent of each other both between and within groups;
4) Within each group the distribution of posttest scores for each value of the covariate is normal (conditional normality);
5) Within each group the variance of the dependent variable is the same for each level of the covariate and the conditional variances are equal for all groups (homoscedasticity).

The robustness of analysis of covariance to violations of these assumptions has not received the same level of attention as the robustness of the t-test and analysis of variance. Furthermore, statistical power, under variations of assumptions, has received little attention. Elashoff (1969) and Glass, Peckham and Sanders (1972) have reviewed the limited literature on the robustness of ANCOVA. The present paper focuses on the effects of violating the assumptions of conditional normality and homoscedasticity on a) the probability of Type I errors and b) statistical power.

Box and Anderson (1962) studied analytically the effect of conditional non-normality on the ANCOVA F-test and concluded that the robustness of ANCOVA to a violation of this assumption was dependent on the shape of the distribution of the covariate. With a normal covariate, violating the assumption has little effect on the F-test but when the covariate is non-normal violating the assumption may lead to a non-robust test. Atiquallah (1964) reached a similar conclusion using a different analytic approach. Since both Box and Anderson and Atiquallah used analytic techniques in studying the effect of non-normality on the ANCOVA F-test, their results do not provide an indication of the magnitude of the error in terms of the probability of a Type I error.

In studying the effect of violating the homoscedasticity assumption, Potthoff (1965) found that the robustness of ANCOVA depended on the sample sizes and the variance of the covariate in the comparison groups. When the sample sizes are equal and the variance of the covariate is the same across comparison groups ANCOVA is robust to the violation of this assumption. If the sample sizes are unequal and/or the variances are unequal then ANCOVA is sensitive to violations of the homoscedasticity assumption. The greater the discrepancies in sample sizes and/or covariate variances the more sensitive the F-test.

When distributional assumptions are violated nonparametric strategies are often suggested. This suggestion is made even when traditional parametric strategies are robust to the violations since the nonparametric procedures can be more powerful when assumptions are not met (Blair & Higgins, 1980). In the case of analysis of covariance several nonparametric approaches have been suggested (McSweeney & Porter, 1971; Puri and Sen, 1969; Quade, 1967). In the present paper the approach suggested by McSweeney and Porter is considered because of its computational simplicity and because previous investigations comparing this approach with Quade's approach have indicated that the two strategies provide similar results with respect to both the probability of a Type I error and power. (Conover & Iman, 1982; McSweeney & Porter, 1971).

McSweeney and Porter (1971) suggested that a nonparametric analog to the traditional parametric analysis of covariance could be achieved by separate transformations of the covariate and the dependent variable, substituting ranks across treatment groups for the original observations. The ranked data could then be analyzed using the same procedures as those used with parametric ANCOVA. The resulting test statistic has an F-distribution with the same degrees of freedom as those associated with the parametric ANCOVA test statistic. McSweeney and Porter compared the parametric ANCOVA with rank transformed ANCOVA and found that when the parametric assumptions were met the rank transform approach was only slightly less powerful than the parametric ANCOVA. The difference in power between the two approaches was greatest when a moderate to strong relationship existed between the covariate and the dependent variable. Although they did not consider comparison of the two approaches when the assumptions of conditional normality or homoscedasticity have been violated they suggested that the ranking procedure may be more powerful.

Conover and Iman (1982) conducted a small simulation study comparing parametric ANCOVA with the rank transform approach when the conditional distributions were: normal, lognormal, exponential, uniform and Cauchy. They found the rank transform approach to be robust to violations of conditional non-normality while the parametric ANCOVA strategy led to either an increased or reduced probability of Type I errors. In terms of power the results indicated that parametric ANCOVA lost power with the lognormal and Cauchy distributions but was more powerful when the conditional distributions were normal or uniform. Conover and Iman considered only situations involving 1) a covariate having a normal distribution, 2) sample size of 10 for each group of 4 groups, and 3) the effects of extreme conditional non-normality. The present study extended Conover and Iman's investigation to include both normally and non-normally distributed covariates and to include several degrees of heteroscedasticity. Furthermore, the present investigation considered less extreme violations of the normality assumption and also the combined effect of both heteroscedasticity and non-normality.

## Method

### Design

The simulation had 5 factors: 1) form the conditional distribution, 2) strength of the posttest-covariate relationship, 3) heteroscedasticity of the conditional distribution, 4) form of the covariate distribution, and 5) sample size. Details on the levels of these factors are given below. All combinations of all levels were not studied. Those that were investigated will be indicated after describing the factor levels more specifically.

Form of the conditional distributions. Six distributions were employed as conditional distributions, a normal distribution and 5 non-normal distributions. Table 1 presents descriptive statistics calculated on a sample of 100,000 scores from each of the 6 distributions as well as the proportion of observations found 1, 2, or 3 standard deviations from the mean. These results were obtained using the Statistical Analysis System (1979). Generation of the data is described in detail below.

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Insert Table 1 Here

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Strength of the posttest-covariate relationship. Two levels of strength of relationship were employed. For the homoscedastic condition $\rho$ the Pearson product moment correlation, was either .3 or .7. For the heteroscedastic condition $\eta$, the correlation ratio, was either .3 or .7.

Variance of the conditional distributions. In addition to the homoscedastic case, 3 levels of heteroscedasticity were employed. Data were generated so that the conditional variances increased as the value of the covariate increased. The increase in conditional variance was the same for both groups. To give an idea of the extent of heteroscedasticity considered in the simulation, Table 2 reports the conditional variances for 5 points on the covariate scale under the 3 heteroscedastic conditions. This table applies only to normally distributed covariates. For non-normal covariates different conditional variances were associated with the five scale points. Had the same conditional variances been associated with these scale points for non-normal covariates, it would not have been possible to compare different covariate distributions while maintaining a common correlation ratio. At the first level of heteroscedasticity and with $\eta$ = .7 the conditional variance one standard

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Insert Table 2 Here

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deviation above the mean (51) was 1.5 times as large as the conditional variance one standard deviation below the mean (49), [.612/.418 = 1.5].

For extreme heteroscedasticity (level III) the conditional variance one standard deviation above the mean was 7.5 times the conditional variance one standard deviation below the mean [1.590/.209 = 7.5]. The rate of change of the conditional variances was approximately the same when $\eta = .7$ or $\eta = .3$.

Distribution of the covariate. The six distributions employed for conditional distributions were also used as distributions for the covariate.

Sample size. All comparisons involved two groups. Both equal sample sizes and unequal sample sizes were studied. For equal sample sizes, data on either 5 or 15 subjects were simulated for each group. For unequal sample sizes, data were simulated for 5 subjects in one group and 10 in the second group.

Condition combinations for five subjects in each cell. Had all combinations of the remaining four factors been simulated the design would have been a 4 (degree of heteroscedasticity) x 6 (form of the conditional distributions) x 6 (form of the covariate distribution) x 2 (strength of relationship) completely crossed factorial. Of these 288 cells, the 120 cells involving combinations of conditional distributions A through E in Table 1 and conditions I and II in Table 2 were not simulated. Thus, we did not simulate data for the moderate degrees of heteroscedasticity in combination with the non-normal conditional distributions.

Conditions combinations for other sample sizes. Fewer combinations were simulated for sample sizes other than five in each group. The combinations simulated for the equal n conditions (15 in each group), and the unequal n condition (5 in one group and 10 in the other) were different. Table 3 summarizes the conditions which were considered. An asterisk (*) is used to indicate which conditions were studied when sample sizes equalled 15. For cases with sample sizes of $n_1 = 5$ and $n_2 = 10$ a number symbol (#) is used. It might also be noted that when sample sizes equalled 5, all cells in the table were studied.

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Insert Table 3 Here

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Generation of the data

The method for generating the posttest data differed slightly depending on whether the conditional distribution was homoscedastic or heteroscedastic. For the homoscedastic case the equation for generating the posttest data was

$$Y = c_j + \rho X + E\sqrt{1 - \rho^2} \quad (1)$$

For the null case $c_j$ was zero for both groups. For the non-null case $c_1$ was zero while $c_2$ was .5. For all cases X was distributed with a mean 50 and variance one while E was distributed with mean zero and variance one. The coefficient $\rho$ was either .3 or .7 depending on the strength of relationship being simulated. For a normally distributed covariate a standard normal variable, $z_1$, was generated using the normal function of the Statistical Analysis System (1979). X was then generated by adding 50 to $z_1$. For normal conditional distributions a standard normal variable, $z_2$, was generated and E was set equal to $z_2$. Non-normal variables were generated using a procedure developed by Fleishman (1978). This procedure transforms a standard normal variable to a variable with mean zero, variance one and known skewness and kurtosis. The skewness and kurtosis are controlled by choosing b, c, and d in the equation,

$$w = -c + bz + cz^2 + dz^3.$$

For the non-normal conditional distributions E was simply set equal to w. For the non-normal covariate X was generated by adding 50 to w.

For the heteroscedastic cases the posttest data was generated using the equation

$$Y = c_j + \rho X + E\sqrt{k(z_1 + 5)^r} \qquad (2)$$

The number k was chosen so that the average conditional variance would be $1 - \rho^2$. This permitted the correlation ratio to be either .3 or .7 depending on whether $\rho = .3$ or .7. The number r was chosen to control the rate of increase of the conditional variance. The variables $z_1$, X and E were generated as described in the previous paragraph. The use of equation (2) implies that regardless of whether $z_1$ is transformed to a normal or non-normal covariate, the same conditional variance is associated with the normal transform and the non-normal transform of a particular value of $z_1$. This in turn assures that the average conditional variance is $1 - \rho^2$ for both the normal and non-normal covariate.

## Results

The investigation generated data for several hundred situations in which the assumptions of conditional normality and homoscedasticity were violated to varying degrees. Since it is not practical to report all of these results, the findings are summarized in various forms. The adequacy of the observed proportion of Type I errors was judged as acceptable when any result was less than two standard errors above or below the theoretical probability of a Type I error. The standard error for a proportion is equal to $([p(1-p)]/N)^{\frac{1}{2}}$. For p = .05 the standard error for 1000 replications equals .007 while for p = .01 the standard error equals .003. Observed proportions outside the probability intervals of (.036, .064) at the .05 level and (.004, .016) at the .01 level were therefore considered as unacceptable.

The results are reported in two sections and each section is divided into two parts. The first part of the first section presents the effects of conditional non-normality on the proportion of observed Type I errors. The second part of the section compares the power of the non-parametric approach to ANCOVA. The second section presents the effects of heteroscedasticity and the combined effect of non-normality and heteroscedasticity. The first part of this section reports the effects on Type I errors and the second part compares the power of the two analysis strategies.

### Conditional non-normality

Type I errors. Considering the equal and unequal n conditions there were 21 conditions involving a homoscedastic, normally distributed error. Table 4 reports the estimates that were above the upper bound of the probability intervals for nominal α levels of .01 and .05. (No estimates were below the lower bound.) The results relevant to ANCOVA are a check on the simulation since all assumptions were met for these 21 conditions. The values above the upper

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Insert Table 4 Here

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bound for α levels of .01 and .05 are probably attributable to sampling error. For the R-ANCOVA there were 3 values above the upper bound when α was .01. The values .021 and .017 occurred with $n = 5$ in each group, the moderately leptokurtic (condition C in Table 1) covariate distribution and ρ equal to .3 and .7 respectively. Although not conclusive, the results suggests the rank transform approach may be slightly liberal for small sample sizes and a leptokurtically distributed covariate for nominal $\alpha = .01$.

There were 19 cases involving a non-normal homoscedastic conditional distribution and a normally distributed covariate. Out of these 19 cases the parametric ANCOVA and the rank transformed ANCOVA each had one estimated liberal and one estimated conservative α for the nominal α of .01. Neither had an estimated α outside the probability interval for nominal $\alpha = .05$. This result is consistent with Box and Anderson's (1962) and Atiquallah's (1964) conclusion that for a normally distributed covariate, the analysis of covariance is robust to violations of the assumption of conditional normality.

Table 5 presents stem and leaf diagrams of distributions of estimated α levels for conditions involving non-normal covariates and conditional distributions, nominal $\alpha = .01$, $n_1 = n_2 = 5$, and $\rho = .3$ and $.7$. The table indicates that rank ANCOVA is somewhat liberal for both $\rho = .3$ and $\rho = .7$, but is more liberal for the former situation.

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Insert Table 5 Here

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When the nominal $\alpha$ was .05 neither analysis showed a tendency to be liberal or conservative. (These results are not presented in a table.) With $\rho$ = .3 all 25 actual $\alpha$ levels fell in the probability interval for the parametric ANCOVA, while only one of 25 fell above the upper limit of the probability interval for the rank ANCOVA. For $\rho$ = .7 the corresponding frequencies are zero and three. For n = 15 in each group there were 6 combinations of non-normal covariates and non-normal conditional distributions for each of $\rho$ = .3 and $\rho$ = .7. None of these conditions resulted in estimated actual levels outside the bounds of the probability intervals for nominal $\alpha$ = .01 or $\alpha$ = .05. There were 6 combinations for the unequal n condition with $\rho$ = .7. None of these resulted in actual $\alpha$ levels outside the probability interval for nominal $\alpha$ = .01 or $\alpha$ = .05.

Power. Table 6 presents the first, second, and third quartiles of the power differences between the rank and parametric ANCOVA for combinations of $\rho$ and $\alpha$. It also presents all power differences that were greater than .05. As the results indicate, with a few exceptions minimal power differences occurred when the errors were homoscedastic. An exception to this generalization was observed however in cases involving the leptokurtic conditional distribution (E).

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Insert Table 6 Here

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Table 7 reports all power differences for $\rho$ = .3, and a conditional distribution following distribution E. With $n_1 = n_2 = 15$, all conditions resulted in at least 9% increase in power for R-ANCOVA over ANCOVA. In all six situations involving $n_1 = n_2 = 5$ power differences less than 5% were observed. These results suggest that with equal group frequencies of at least 15 and a skewed and leptokurtic distribution, a significant power advantage accrues to the rank ANCOVA. It also might be noted that the differences of .091 and .112 occurred with a normal covariate and so the results are not dependent on deviations from normality for the covariate.

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Insert Table 7 Here

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Conditional non-normality and heteroscedasticity

Type I errors. Table 8 presents the estimated actual α levels that were outside the probability intervals for nominal α's of .01 and .05. The table suggests that parametric ANCOVA was robust to the assumption of homoscedasticity when this assumption alone was violated. In situations involving the violations of conditional normality and homoscedasticity the parametric approach had a tendency to be conservative when the sample sizes were small ($n_1 = n_2 = 5$) and $\alpha = .05$. The rank ANCOVA strategy on the other hand had a tendency to be liberal in situations involving heteroscedastic conditional distributions and when the sample sizes were small, $\alpha = .01$ and $\eta = .3$. This was true for both normal and non-normal conditional distributions. In situations involving a normal covariate and the combined conditional non-normality with heteroscedasticity both parametric ANCOVA and R-ANCOVA provided appropriate actual probability of Type I errors.

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Insert Table 8 Here

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Power. Table 9 reports power differences for conditional distributions D and E with various combinations of group frequencies, η, α, and covariate distributions. Inspection of the Table 9 indicates that when there were 15 subjects in each group a practical power advantage accrued to the rank ANCOVA for $\rho = .3$ and .7, $\alpha = .01$ and .05, and both conditional distributions.

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Insert Table 9 Here

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The advantage appears to be greatest with conditional distribution E. Distribution D was a skewed distribution. Distribution E was a leptokurtic and skewed distribution. However the frequencies reported in Table 1 suggest that the predominant characteristic of a plot of distribution E was its skewness. This suggests that skewness combined with heteroscedasticity and an equal group frequencies of at least 15 will result in practically important power advantage for the rank ANCOVA. Table 10 reports the median, maximum, and minimum power difference for all levels of α and η and conditional distributions A, B, and C. Clearly the power differences are minimal for these conditional distributions.

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Insert Table 10 Here

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## Conclusions

The results of the analysis indicated that the parametric analysis of covariance was robust to violations of either the conditional normality or homoscedasticity assumption. In situations where both assumptions were violated however, and the covariate has a non-normal distribution, the parametric ANCOVA exhibited a slight tendency to lead a conservative test of the hypothesis when the sample size was small and the nominal level of significance was .05. These results are not consistent with those reported by Conover and Iman (1982). In that study the researchers found that the parametric ANCOVA led to a conservative hypothesis test when the conditional error distribution was lognormal or Cauchy. Conover and Iman did not consider violations of the homoscedasticity assumption. The discrepancy in findings might be explained by the difference in the degree to which the conditional normality assumption was violated. The Cauchy distribution considered by Conover and Iman had parameters of 0 and 1 for the median and scale respectively. With these parameters the distribution is the t-distribution with 1 degree of freedom. Conover and Iman therefore considered a far more leptokurtic distribution than the one considered here. The lognormal distribution studied by Conover and Iman had parameters of $e^2$ and $e^4(e^4 - 1)$ for the mean and variance respectively. With these parameters the distribution has skewness of 414.36 and a kurtosis of $2.64 \times 10^{10}$.

The present study did not consider as extreme violations of assumptions as Conover and Iman. The rationale behind the levels of skewness and kurtosis chosen in the present study was based on Fleishman's (1978) argument that simulation studies should reflect distributions commonly found with real data. Furthermore Fleishman points out that Pearson and Please (1975) found that most distributions they examined had skewness less than .8 and kurtosis between -.6 and +.6. The present study considered distributions similar to as well as distributions slightly more extreme than those considered by Pearson and Please. Since researchers generally do not report the skewness or kurtosis of their data it is difficult to determine how closely the distributions reflect actual data. However scores on the Metropolotian Achievement Test for the math and reading subtests were obtained for grades 1 through 10 on approximately 1500 students per grade. The skewness of these distributions ranged between -.55 and .05 and the kurtosis ranged between 1.37 and -.34. It was therefore concluded that the distributions considered in the present study were probably similar to those found by investigators in actual research studies.

The robustness of parametric ANCOVA found in the present study should not be interpreted to mean that ANCOVA is always robust to violations of assumptions. This study considered only moderate departures from the normality assumption; furthermore parametric ANCOVA may not be robust if the distributional assumptions are violated in different ways for the two groups. Havlicek and Peterson (1974) found this result in studying Student's t-test.

The rank transformation approach to ANCOVA was found to be robust when the covariate had a normal distribution and the errors were non-normal. These findings are consistent with Conover and Iman (1982). However when both the covariate and errors were non-normally distributed, the sample sizes were small ($n_1 = n_2 = 5$), $\alpha$ was .01, and the strength of the covariate-posttest relationship was weak ($\rho$ or $\eta = .3$), the rank ANCOVA tended to lead to a liberal test. Under all other conditions involving non-normal covariates and conditional distribution the rank ANCOVA was quite robust.

When the conditional distributions were homoscedastic and either normally or non-normally distributed the power differences, with one notable exception, were generally quite small. With a correlation of .7 and with n = 5 in each group the parametric ANCOVA was slightly more powerful. However under the other equal n conditions studied ($\rho = .7$, $n_1 = n_2 = 15$; $\rho = .3$, $n_1 = n_2 = 5$ $\rho = .3$, $n_1 = n_2 = 15$) the rank ANCOVA tended to be slightly more powerful. The only exception to the generally small power differences occurred when the conditional distribution appeared markedly skewed and $\rho$ was .3.

When the conditional distributions were heteroscedastic similar results occurred. Except when the conditional distribution appeared to be skewed, the power differences were small. However practically important power differences emerged with the skewed conditional distributions. The differences were larger with 15 subjects in each group than with 5. The differences also increased as the degree of skew increased, but were fairly similar for both $\eta = .3$ and $\eta = .7$. This finding is especially significant since the rank ANCOVA appeared to have an actual $\alpha$ level near to the nominal $\alpha$ level under the conditions described above so the power advantage is not an artifact of a non-robust procedure. The finding of greater advantage in power associated with the rank transformed ANCOVA when sample sizes were moderate ($n = 15$) is consistent with the results reported by Blair and Higgins (1980) in their comparison of the Wilcoxon t-test with Student's t-test. In that study they found very little difference in power between the two procedures when sample sizes were small but with moderate (9, 17; 18, 18) or large (27, 81; 54, 54) samples, greater power was associated with the nonparametric approach.

The power findings in the present study suggests it may be fruitful to conduct further simulation studies to determine the boundary conditions, on combinations of skewness, heteroscedasticity, strength of relationship and sample size that results in a practically important power advantage for the rank ANCOVA. Unequal group frequencies should be included since the failure for a power advantage to emerge in our study for unequal group frequencies may well be due to the fact that the total frequency was only 15. It may also be useful to include several degrees of group differences so that empirical power curves can be constructed.

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Table 1

Proportion of Random Variables Observed Within 1, 2 or 3 Standard Deviations of the Mean and Summary Characteristics of the Six Distributions Studied

| Standard Deviation from the Mean | Normal N | Platykurtic A | Slightly Leptokurtic B | Moderately Leptokurtic C | Skewed D | Skewed and Moderately Leptokurtic E |
|---|---|---|---|---|---|---|
| -∞, -3.0 | .08 | | .33 | .62 | | |
| -3.0, -2.0 | 2.17 | | 2.11 | 2.09 | | |
| -2.0, -1.0 | 13.95 | 19.62 | 11.89 | 9.71 | 18.02 | |
| -1.0, 0.0 | 33.90 | 30.22 | 35.50 | 37.67 | 37.81 | 63.35 |
| 0.0, 1.0 | 33.80 | 30.24 | 35.57 | 37.61 | 26.73 | 21.92 |
| 1.0, 2.0 | 13.87 | 14.83 | 12.13 | 9.91 | 13.16 | 9.36 |
| 2.0, 3.0 | 2.12 | | 2.20 | 2.02 | 3.91 | 3.66 |
| 3.0, ∞ | .11 | | .30 | .65 | .39 | 1.69 |
| Mean | .0018 | .0018 | .0054 | -.0054 | .0050 | -.0002 |
| Variance | 1.0036 | 1.0090 | .9887 | .9831 | 1.0043 | .9913 |
| Skewness | .0046 | -.0043 | -.0026 | .0277 | .7199 | 1.6928 |
| Kurtosis | -.1065 | -1.0160 | .7207 | 2.6584 | -.1031 | 3.3159 |

Table 2

Conditional Variances at Five Levels of X for Three Levels of Heteroscedasticity When $\eta = .7$ and $\eta = .3$

| | Levels of Heteroscedasticity | | | | | |
|---|---|---|---|---|---|---|
| | I | | II | | III | |
| X | $\eta = .7$ | $\eta = .3$ | $\eta = .7$ | $\eta = .3$ | $\eta = .7$ | $\eta = .3$ |
| 48 | .306 | .546 | .098 | .176 | .028 | .050 |
| 49 | .408 | .728 | .233 | .416 | .117 | .209 |
| 50 | .510 | .910 | .455 | .813 | .358 | .639 |
| 51 | .612 | 1.092 | .787 | 1.404 | .891 | 1.590 |
| 52 | .714 | 1.274 | 1.249 | 2.230 | 1.926 | 3.537 |

Table 3

Summary of Conditions Simulated for $n_1 = n_2 = 15$ and $n_1 = 5$, $n_2 = 10$[a,b]

| | | Covariate Distribution | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Error Distribution | | N | | A | | B | | C | | D | | E | |
| Heteroscedasticity | Shape | .7 | .3 | .7 | .3 | .7 | .3 | .7 | .3 | .7 | .3 | .7 | .3 |
| HOMOSCEDASTIC | N | # | * | | | | | # | * | | | # | * |
| | A | | | | | | | | | | | | |
| | B | | | | | | | | | | | | |
| | C | # | * | | | | | # | * | | | # | * |
| | D | # | * | | | | | # | * | | | # | * |
| | E | # | * | | | | | # | * | | | # | * |
| I | N | | | | | | | | | | | | |
| II | N | | | | | | | | | | | | |
| III | N | # | * | | | | | # | * | * | * | # | * |
| III | A | | | | | | | | | | | | |
| III | B | | | | | | | | | | | | |
| III | C | # | | | | | | # | | | | # | |
| III | D | # | * | * | * | * | * | # | * | * | * | # | * |
| III | E | # | * | * | * | * | * | # | * | * | * | # | * |

a. All combinations of factors were simulated when $n_1 = n_2 = 5$

b. A * indicates $n_1 = n_2 = 15$, a # indicates $n_1 = 5$, $n_2 = 10$

Table 4

Liberal Estimated α Values for Normal and Homoscedastic Conditional Distribution

| | Nominal Alpha | |
|---|---|---|
| Analysis | .01 | .05 |
| ANCOVA | .021 | .071 |
| R-ANCOVA | .017, .018, .021 | |

Note: For each combination of Analysis and Nominal Alpha, there were 21 conditions simulated.

Table 5

Distributions of Actual α Levels for Nominal $\alpha = .01$, $n_1 = n_2 = 5$, Non-normal Covariates and Conditional Distributions

| | Analysis | | | | | |
|---|---|---|---|---|---|---|
| | ANCOVA | | | Rank ANCOVA | | |
| ρ | Stem | Leaf | Count | Stem | Leaf | Count |
| .3 | 02 | | | 02 | 6 | 1 |
| | 02 | | | 02 | 01134 | 5 |
| | 01 | 899 | 3 | 01 | 777778 | 5 |
| | 01[a] | 0001333456 | 10 | 01 | 11333555666666 | 14 |
| | 00 | 555678889999 | 12 | 00 | | |
| | 00 | | | 00 | | |
| .7 | 02 | | | 02 | 6 | 1 |
| | 02 | | | 02 | | |
| | 01 | 7 | 1 | 01 | 7789 | 4 |
| | 01 | 011111222366 | 12 | 01 | 001124444556 | 12 |
| | 00 | 77778888999 | 11 | 00 | 77789999 | 8 |
| | 00 | 3 | 1 | 00 | | |

[a]The entries for the stems and leafs enclosed by the dotted lines are within plus or minus two standard errors of the nominal $\alpha = .01$. Stems should be multiplied by .01.

Table 6

Distribution of Power Differences for Homoscedastic Errors[a]

| α | ρ | $Q_1$ | $Q_2$ | $Q_3$ | Power Differences >= .05 |
|---|---|---|---|---|---|
| .01 | .3 | .008 | .013 | .021 | .091,.091,.106 |
| | .7 | -.007 | -.002 | .003 | |
| .05 | .3 | .000 | .005 | .015 | .067,.112,.108,.140 |
| | .7 | -.022 | -.013 | -.005 | -.059,-.067,-.068 |

[a]Power differences calculated by subtracting power for parametric from power for rank ANCOVA. Each distribution is based on 48 conditions.

Table 7

Power Differences with Conditional Distribution E and $\rho = .3$

| $\alpha$ | $n_1 = n_2 = 5$ | $n_1 = n_2 = 15$ |
| --- | --- | --- |
| .01 | .018,.021,.021,.029,.033,.044 | .091,.091,.106 |
| .05 | .028,.029,.031,.035,.039,.045 | .108,.112,.140 |

Table 8

Summary of Estimated α Levels for Heteroscedastic Condtional Distributions

| Condition Description | Number of Cases | Nominal α .01 ANCOVA | .01 R-ANCOVA | .05 ANCOVA | .05 R-ANCOVA |
|---|---|---|---|---|---|
| Normal and Non-Normal Covariate; Normal and Heteroscedastic Conditional Distribution | 47 | .003/ | /.017,.017,.019,.019 .021[a] | | |
| Normal Covariate Distribution; Non-Normal and Heteroscedastic Conditional Distribution | 17 | | /.018 | | |
| Non-Normal Covariate; Non-Normal and Heteroscedastic Conditional Distribution | | | | | |
| $\eta = .3$ $n_1 = 5$, $n_2 = 5$ | 25 | | /.017,.018,.018,.019 | .033,.033,.034/ | |
| $\eta = .7$ $n_1 = 5$, $n_2 = 5$ | 25 | | /.017 | .031,.032,.032,.033/ | /.065 |
| $\eta = .3$ $n_1 = 15$, $n_2 = 15$ | 10 | | /.017 | .035/ | |
| $\eta = .7$ $n_1 = 15$, $n_2 = 15$ | 10 | | | .028/ | |
| $\eta = .7$ $n_1 = 5$, $n_2 = 10$ | 6 | | | | |

a These five estimate liberal α values involved $\eta = .3$ and $n_1 = n_2 = 5$

Table 9

Power Differences For Conditional Heteroscedastic Distributions D and E[a]

| n | η | α | Conditional Distribution | Covariate Distribution A | B | C | D | E | N |
|---|---|---|---|---|---|---|---|---|---|
| 5,5 | .3 | .01 | D | .027 | .027 | .020 | .029 | .020 | .023 |
| | | .05 | | .042 | .045 | .030 | .023 | -.003 | .020 |
| | .7 | .01 | | .004 | .045 | .023 | .018 | .015 | .018 |
| | | .05 | | .003 | .016 | .001 | .053 | .010 | .013 |
| 15,15 | .3 | .01 | | .066 | .044 | .052 | .059 | .042 | .065 |
| | | .05 | | .100 | .084 | .087 | .067 | .067 | .120 |
| | .7 | .01 | | .104 | .074 | .067 | .136 | .136 | .100 |
| | | .05 | | .111 | .096 | .036 | .161 | .158 | .161 |
| 5,10 | .7 | .01 | | | | | -.008 | .008 | .003 |
| | | .05 | | | | | .007 | .019 | .27 |
| 5,5 | .3 | .01 | E | .009 | .025 | .033 | .013 | .027 | .057 |
| | | .05 | | .058 | .069 | .047 | .058 | .046 | .057 |
| | .7 | .01 | | .022 | .017 | .022 | .021 | .023 | .007 |
| | | .05 | | .015 | .029 | .031 | .043 | .023 | .007 |
| 15,15 | .3 | .01 | | .213 | .203 | .201 | .250 | .189 | .238 |
| | | .05 | | .264 | .283 | .272 | .244 | .274 | .282 |
| | .7 | .01 | | .186 | .188 | .223 | .210 | .209 | .161 |
| | | .05 | | .228 | .195 | .220 | .248 | .224 | .192 |
| 5,10 | .7 | .01 | | | | | .017 | .023 | .037 |
| | | .05 | | | | | .048 | .063 | .064 |

rences calculated by subtraction power for parametric ANCOVA from power for Rank ANCOVA.

Table 10

Distribution of Power Differences for Conditional Distributions A, B, and C[a]

| α | η | Number of Cases | Minimum | Median | Maximum |
|---|---|---|---|---|---|
| .01 | .3 | 24 | .007 | .023 | .035 |
| | .7 | 24 | -.013 | .003 | .021 |
| .05 | .3 | 8 | -.001 | .017 | .035 |
| | .7 | 8 | -.034 | -.019 | .007 |

[a]Differences calculated by subtracting power for parametric ANCOVA from power for Rank ANCOVA.