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ABSTRACT

This report presents findings from a study of the cognitive processes and learning difficulties of ninth-grade students in Algebra I. A clinical interview approach was used with 14 students. A group of six Hispanic students and a group of five Anglo students were enrolled in the same algebra class; three Hispanic students were enrolled in an "advanced" class, since they had started algebra one-half year before the others. Probed were students' appreciation of algebra as an abstract logical system, command of the formal operations of algebra, ability to use algebra, and ability to solve problems. Performance in English vs. Spanish was compared, and students' teaching styles were analyzed as they taught peers. It was found that students preferred not to use algebraic techniques in solving problems, were poor at verbalizing definitions and procedures and at translating problem statements into equations, did not use their textbooks very much except as a place to find assigned problems, and treated algebra as a rule-based discipline, not as a concept-based one. Findings are discussed in relation to the effectiveness of the textbook, to instructional approaches, and to the interplay of language in cognitive processes. (MNS)

The Learning of Algebra by 9th Graders: Research Findings Relevant to Teacher Training & Classroom Practice*

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Preface

This paper is a summary of a study entitled "A Study of the Cognitive Development of Hispanic Adolescents Learning Algebra Using Clinical Interview Techniques", funded by the National Institute of Education (NIE) under Contract Number 400-81-0027. The summary is taken from a detailed final report submitted to NIE in December of 1982 under the same title as above. What we have attempted to do here is present those salient findings and recommendations which would be particularly relevant to classroom practice and teacher training. Further details regarding the study can be obtained by requesting a final report from either the National Institute of Eduction, or the investigators, at the addresses given below.

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Introduction

There has been an increased concern in recent years over the poor mathematical preparation of the general populace. Of particular concern has been the decline of mathematical preparedness of entering college students. It is a well-documented fact that standard indicators of mathematical aptitude, such as the Scholastic Aptitude Test (Educational Testing Service, 1948-1982) have shown a monotonic decline over the last decade (Harris, 1976; Harneschfeger and Wiley, 1976; Braswell, 1978). Several explanations, both academic and sociological, have been offered for this decline (Edson, 1976). Perhaps the most cogent academic explanation for the lower mathematical preparation of college students is the decline in the overall enrollment in high school second-year algebra (National Science Foundation, 1980), coupled with the attenuation in high school mathematics enrollments as the level of the mathematics course becomes more advanced (National Assessment of Educational Progress, 1979). This situation has given rise to an anomaly which is rapidly gaining acceptance -- college level remedial mathematics programs (Whitesitt, 1982).

The purpose of the current research was to investigate the cognitive processes employed by ninth grade students enrolled in Algebra I. The choice of algebra for the subject of this study was not arbitrary. Algebra is traditionally the first mathematics course where students encounter concepts which are much more subtle and abstract than the simpler arithmetic manipulations of their prior math courses. Further, because algebra is the foundation upon which the majority of advanced mathematics is based, any insights gained from researching this area may eventually make the learning of algebra a more efficient process. Some national mathematical organizations have even gone on record to emphasize the importance of algebra for college level work over other mathematics courses taught in high school (The Mathematical Association of America and the National Council of Teachers of Mathematics, 1978).

A clinical interview approach was employed for data collection. The use of clinical interviews to study problem solving has met with some success. Unlike any paper-and-pencil assessment, the clinical interview approach is unique in permitting immediate feedback and interaction between the interviewer and subject. This allows the interviewer to probe for the cause of difficulties and misconceptions as they arise during the interview, something which is not possible with non-interactive data collection techniques.

The focus of this study was on the learning difficulties experienced by Hispanic students. There are several reasons for this focus. First, the number of investigations that have dealt with the learning and performance of Hispanics in mathematics is vanishingly small. Further, several statistics published in The Condition of Education for Hispanic Americans (National Center for Education Statistics, 1980) reveal that the educational, as well as socioeconomic, situation of Hispanics in the mainland is bleak. For example, even though in 1978, Hispanics in the mainland comprised 5.6% of the total population, the percentages of Hispanics enrolled in 4-year college programs, graduate programs, and professional programs were 2.8, 2.0, and 2.1, respectively, of the total student enrollment. In terms of college enrollment in fields where a strong mathematical preparation is indispensable, such as engineering and physical science, Hispanics are substantially underrepresented; the percentage of white-non-Hispanics enrolled in these fields out of the total white-non-Hispanic undergraduate enrollent is more than twice that of the percentage of Hispanics enrolled in these fields out of the total Hispanic undergraduate enrollment. There is, therefore, a need for more research efforts which may help in the development of strategies for increasing the number of Hispanics wishing to pursue math/science related professions.

Finally, we would like to comment that the results of this study are not idiosyncratic to Hispanic students. Even though this study focused on Hispanic students, the majority of the findings apply not only to the Hispanic participants, but also to a control group of Anglo students who participated in the study.

Methodology

The study was conducted in a junior high school (grades 7-9) in a small city in Western Massachusetts during the 1981-1982 academic year. There were a total of 14 students participating in the study. These 14 students were divided into three subgroups. The first group, which will be called "participating Hispanics" consisted of 6 Hispanic students (5 male and 1 female); these students were in the mainstream curriculum and not in any "special program". The second group, the "participating Anglos" consisted of 5 Anglo students (3 males and 2 females). The 11 students from these two groups were enrolled in the same Algebra I class taught by a teacher (henceforth, the "participating teacher") who was an integral part of the study. The last group consisted of three Hispanic students (one male and 2 females) enrolled in an advanced algebra class which was not taught by the participating teacher. These students will be called the "advanced Hispanics", where by "advanced" we mean that they had started studying Algebra one-half year before the participating teacher's Algebra I class. The participating Hispanics were interviewed a total of 8 times during the academic year while the other two groups were interviewed 4 times. The classroom style and textbooks used in both the Algebra I class and the advanced algebra class were "traditional".

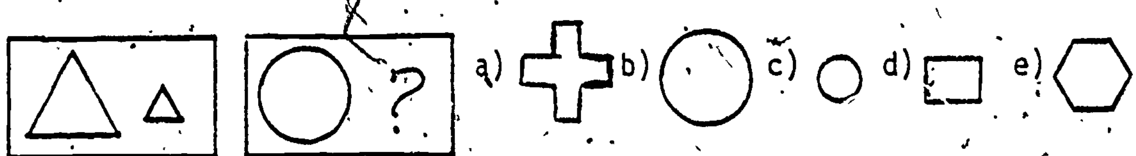
In the clinical interview approach used, an interviewer presented mathematics problems to a student and asked the student to "think aloud" while attempting to solve the problems. The interviewer was free to probe the student for clarifications and explanations of the mental processes he or she used to solve the problems. All interviews were audio-recorded. After the interview, the "protocol analysis" followed during which the interviewer reviewed the student's protocol (i.e., the events that transpired during the interview, including the student's worksheets and audio tapes) and made detailed comments on the student's performance, including strengths, weaknesses, difficulties, misconceptions, etc. Once all interviews were analyzed for a given session, a staff meeting followed during which the entire interview set was reviewed. The purpose of this meeting, attended by the researchers and participating teacher, was to propose and discuss explanatory hypotheses for the findings.

Student Profile

In April of 1981, all students in the school system in question were given the California Achievement Tests (CAT) (CBT/McGraw-Hill, 1977). This battery consists of 8 major divisions: 1 reading vocabulary, 2. reading comprehension, 3. spelling, 4. language mechanics, 5. language expression, 6. math computation, 7. math concepts and applications, and 8. reference skills. There were 3 sections which were made up of combinations of the 8 major sections above, namely, total reading was made up of the combined scores in reading vocabulary and reading comprehension; total math was made up of the combined scores from math computation and math concepts/applications; total battery was made up by combining the first seven categories listed above --that is, all major divisions except reference skills.

In addition, three other measures were administered to the participating teacher's whole Algebra I class, as well as to the 3 advanced Hispanics. The first of these is a Piagetian exam entitled "An Inventory of Piaget's Developmental Tasks" (Catholic University, 1970), designed to cover the following 18 areas: 1. quantity, 2. levels, 3. sequence, 4. weight, 5. matrix, 6. symbols, 7. perspective, 8. movement, 9. volume, 10. seriation, 11. rotation, 12. angles, 13. shadows, 14. classes, 15. distance, 16. inclusion, 17. inference, and 18. probability. There were 4 questions for each of these 18 areas making this battery a 72 question test.

The remaining two measures were the analogies and classifications subsections of the test of general ability level 5, form CE (Guidance Testing Associates, 1962). The analogies section contained 24 questions which were totally symbolic and asked the student to make one pair of drawings like a given pair, as shown in the following example:



The classification section was composed of 26 questions which again were totally symbolic and asked the student to identify the one drawing in a series of five which was "different". For example,

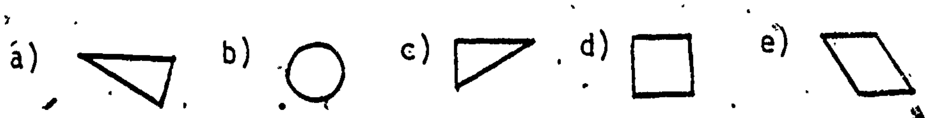


Table 1 gives the results of these tests for the various subgroups of students. The first two entries correspond to the mean and standard deviation. In the CAT scores, the third entry gives the national percentile ranking corresponding to that particular raw score. Also shown in Table 1 are the student's final grades in Algebra. The mean scores on all standardized measures are based on the total number correct.

Several observations can be made from Table 1. First, it is evident that the advanced algebra class is consistently "above average" on the CAT in comparison to the Algebra I class, as well as to national norms. Second, it is evident that the three advanced Hispanics are very comparable with their Anglo peers in the advanced algebra class in all the measures shown on Table I. Third, it is evident that the five participating Anglos are comparable to the 23 other Anglos in the Algebra I class, and that all of the Anglos in the Algebra I class are "average" students as determined by the CAT national percentile rankings. Finally, Table 1 shows that the 6 participating Hispanics are consistently "below average" in the CAT, both as compared to their Anglo classmates, and as compared by the national percentile rankings, except in the math computation section where this group scored "average".

In summary, it appears that the advanced Hispanics were extremely well prepared academically as measured by their performance in the CAT. The participating Anglos' performance in the CAT indicates an average preparation. The participating Hispanics appear to be academically underprepared, both in comparison to their Anglo classmates, and in comparison to national norms. In terms of the problem solving tasks given during the interviews, the 3 groups' performance was consistent with their performance on the standardized measures as shown on Table 1, namely, the advanced Hispanics performed extremely well in the interview problems, the participating Anglos showed an average performance, and the participating Hispanics showed a below average performance.

Interview Results

In this section, we will give a brief description of the kinds of questions investigated during the course of the study, and present a few of the interesting findings. The problems used in the interviews were designed to probe six relatively independent areas. In the discussion that follows, each of these six areas will be treated separately.

1. Appreciation of Algebra as an Abstract Logical System

The focus of this area was to investigate the degree to which the student creates and develops an abstract conceptual schema for mathematics that is independent of other logical systems, such as languages. Like language, mathematics has its own formal logical structure, largely composed of definitional constructs which form the "rules of the game". Bilinguals, who already have two languages and therefore two logical systems, may be able to develop a third logical system more readily than monolinguals. On the other hand, if the languages' logical structures are poorly developed, bilinguals may find it more difficult to develop new logical systems. Bilinguals therefore constitute a good "laboratory" in which to pursue these questions.

From a practical standpoint, we used the following general questions as a springboard for our investigations in this area:

- Does the student understand the role of definitions in mathematics? Does the student utilize definitions to resolve points of ambiguity?
- Does the student distinguish between mathematical statements (equalities and inequalities) and mathematical phrases (expressions and ratios)?
- Does the student recognize the difference between a variable and a label?

We will limit our discussion to answering the last question.

There have been several recent research studies which have revealed that many college students treat variables as labels (Mestre, Gerace, and Lochhead, 1982; Clement, 1982; Clement, Lochhead, and Monk, 1981; Rosnick, 1981). For example, the following problem,

> Write an equation using the variables S and P to represent the following statement: "There are 6 times as many students as professors at this university". Use S for the number of students and P for the number of professors.

the "variable-reversal error", where students would write $6S=P$, was committed consistently by approximately 35% of non-minority engineering undergraduate students. Using a population of Hispanic engineering students the frequency of the variable reversal error was 54%.

In clinical interviews of students solving the above problem, it was discovered that one of the major points of confusion regarding the variable-reversal error derived from treating S and P as labels for "students" and "professors", instead of treating them as variables to represent the number of students and the number of professors. It should be pointed out that the students interviewed displayed that they were aware that there were more students than professors in the problem statement. For these students the meaning of 6S=P was "6 students for every 1 professor". The mechanism leading students to write the variable-reversed equation appeared to stem from using a sequential left-to-right translation of the problem statement. That is, "six times as many students" becomes 6S, and since this is equal to the number of professors, students equate 6S to P.

To investigate how prone beginning algebra students are to committing this kind of error, we designed the three questions below:

1. Mr. Smith noted the number of cars, C, and the number of trucks, T, in a parking lot, and wrote the following equation to represent the situation: 8C=T. Are there more cars or more trucks in this parking lot and why?

Write an expression with variables for the following statements.

2. Six times the length of a stick is 24 feet.

3. If a certain chain were four times as long it would be 36 feet.

From the students' answers to these questions we found strong evidence that they had a proclivity to treat variables as labels. In problem 1, for example, 11 of the 14 students interviewed from the three groups said that there would be more cars in the parking lot as represented by the equation 8C=T due to the factor of 8 in front of the C. Of the remaining three students, one from the advanced Hispanic group said that there would be more trucks; her explanation displayed that she was using C and T correctly as variables for the number of cars and trucks. The last two students, both from the participating Hispanic group, gave rather unique answers; one said that you could not tell whether there were more cars or trucks because the values of C and T were not given; the other said that there would be an equal number of cars and trucks because of the "=" sign in the equation.

A comparison of the students' responses in problems 2 and 3 revealed an interesting phenomenon. All 14 students obtained the correct answer in both of these problems. In problem 2, eight of these students wrote a correct equation using the letter L for the variable, and the other six wrote a correct equation using some other letter for the variable, such as A or X; none of these six students, however, used the letter S for the variable; S would not be an unreasonable choice since the problem is about a stick. In contrast, the most popular variable name used in problem 3 was C and not L; seven students used C, two used L, and the remaining used some other variable name.

Given that both of these problems asked for a quantity involving length, and that many students were using the variable in problem 3 to represent "the chain", it is evident that the sentence construction of these two problems triggered, more often than not, specific variable names. That is, the manner in which problem 2 started, "six times the length..." makes it clear to the student that this is a problem about length, thereby triggering the use of the letter L for the variable. However, in problem 3, the first few words, "if a chain..." make it clear that this is a problem about a chain, thereby triggering the letter C to be used as the variable. Even though problem 3 is asking for a length just as problem 2 is, the student is distracted from this fact by having the references to length via the words "long" and "feet" appearing much later in problem 3. It therefore appears that the syntax structure of these problems is largely responsible for the triggering mechanism by which a variable name is chosen, with the first important noun in the problem statement serving as the trigger. This makes it somewhat of a random process whether the student will choose a variable name (i.e. a letter) as a label for a noun, or as a quantity to be represented by the variable name.

2. Command of the Formal Operations of Algebra

A complete logical system contains not only a set of definitions and relations, but also a set of rules for manipulating these relationships. Quite independent of the students' knowledge of the structure of algebra is their understanding of the dynamical laws, or permissible operations that may be used in the process of obtaining a solution. We are not concerned here with the students' ability to obtain answers, but rather with their knowledge and comprehension of legal algebraic manipulations. Thus, whereas in the last section we were interested in whether students knew "the rules of the game", in this section, we are interested in whether students "play by the rules of the game".

We were guided by the following general questions to probe this area:

- Does the student know what fundamental operations can be performed on an algebraic statement?
- Does the student realize that these operations do not change the solution to an equation?
- Does the student confuse what rules he/she may apply in a particular situation?

An example of some of the specific questions used during the interviews are the following:

4. Which of the following equations are true and why?

a) $a+b=b+a$ b) $a\div b=b\div a$ c) $a(7-b)=7a-ba$

5. Solve the following equations:

a) $2+X=3$ b) $-2X=12$ c) $4X+7=6X-5$

6. If $X=3$ and $Y=2$, what are the values of the following:

a) X^2Y^3 b) $(X+Y)^2$

7. Multiply out: $(X+2)(X-3)$

8. Factor the following: X^2+5X+6

9. a) What is -7 times 3?

b) What is -3 times 7?

Rather than giving a detailed breakdown of each group's performance we will only state that the performance on these types of questions was consistent with the groups' performance on the CAT. What we will do is discuss a couple of specific errors.

In an interview session near the beginning of the school year, the students were asked the following question: "Suppose $X=3$ and $Y=4$. What is $(X+Y)^2$?". At the time we asked students this question they had not covered polynomials, but had covered raising numbers to specific powers. Only five of the fourteen students were able to obtain a correct answer in this question. They did so by adding 3 and 4, and squaring 7 to get 49. At the time students were given problem 6 above, they had covered polynomials in class. Amusingly, all five students who obtained the correct answer to the question above, obtained the wrong answer to problem 6. The answer that all five of these students gave in problem 6 was 13, by taking $3^2 + 2^2$. This we think shows how students can become mind-locked when covering a new topic by attempting to solve all problems using the newly learned procedure rather than resorting to a previously learned, and perhaps easier procedure.

Finally, we would like to state that many of the students treated algebra as a rule-based discipline and not as a concept-based discipline. Because of this, students often confused which rules applied to which cases. For example, the participating teacher, in an attempt to help his students learn various algebraic rules, made a chart and posted it at the front of the class. The chart contained reminders of facts, such as how to determine the sign of the sum of a positive and a negative number. However, in explaining their procedures for solving some of the problems above, some of the participating Hispanics made it clear that they were applying some of the rules to the wrong cases. For example, some students stated that -3 times 7 was equal to 21 but that 3 times -7 was equal to -21; these students displayed that they were using the rule for determining the sign of a sum between a negative and a positive number to determine the sign of a product.

.3. Ability to Use Algebra

Before algebra can become a useful problem solving tool, the student must be able to formulate a problem in algebra and combine elementary operations into a strategy for solution. These abilities clearly require a deeper assimilation of the principles of mathematics than does learning to solve assigned problems. Knowledge of those factors which influence strategy formation, or of specific learning difficulties which inhibit strategy formation, will permit an optimization of the assimilation process. To extend the metaphor we have been using, this area attempted to ascertain how well the student can "manipulate the rules of the game to his/her own advantage while still remaining within the confines of legality".

We used the following general questions to guide us in this area of investigation:

- Does the student employ algebraic concepts to solve problems without being prompted, or does the student prefer to use other techniques such as trial and error or guessing?

- Does the student define variables?

- Does the student have a strategy for a solution, or resort to random manipulations in the hope of stumbling onto a solution?

- Is the student able to extend and apply his/her knowledge to a relatively novel problem?

We will only discuss the students' answers to one specific problem, and point out a general weakness we found among most of the students. This problem was given to all 14 students and goes as follows:

> Suppose I have a ball which I drop from a certain height. Every time the ball bounces, it only goes up to 1/2 of the highest point reached in the previous bounce. If I drop the ball from 16 feet how high does it go after the first bounce? How high does it go after the second bounce? Suppose I have another ball that goes up to 3/4 of the highest point reached in the previous bounce (repeat problem).

The results from this problem revealed that five students from the participating Hispanic group could work out the problem with the "bounce factor" of 1/2 without hesitation. However, three of these five could not generalize this procedure for a "bounce factor" of 3/4. The participating Anglos also suffered from this problem, albeit to a lesser extent; the advanced Hispanics were quite facile with either a 1/2 or 3/4 "bounce factor". In general, it appeared that students had an excellent intuitive grasp of "halving" but not of "three-quartering". It is our feeling that the difficulty with the 3/4 bounce factor was due to the students' not being able to verbalize or understand the procedure they used in the 1/2 bounce factor case. Had they realiazed that all they were doing was multiplying each maximum height times 1/2 to obtain the subsequent maximum height, they could have easily extended this procedure to the 3/4 case. Perhaps the difficulty was that in the 1/2 bounce factor case, students divided by 2, but in the 3/4 bounce factor case, they were not sure whether to divide by 3, 4, or some combination of 3 and 4. This inability to generalize a procedure from a simple to a complex case was observed in other contexts a well.

4. Ability to Solve Problems

This area of our investigations focused on the students' ability to solve word problems. Since algebra I is the first course where students encounter non-trivial word problems, and since word problems have always proved to be a nemesis of mathematics students at all levels, we will make our discussions in this section somewhat longer than those of the previous sections. The one area which will be stressed is the translation process from textual to symbolic representations, since it is this step which students found most difficult. The general questions which served to guide us were the following:

- Is the student able to translate word problems into algebraic equations and word phrases into symbolic expressions?

- Does the student misinterpret problems because of language difficulties such as poor vocabulary or reading comprehension?

- Does the student's success at solving problems depend upon the amount of linguistic processing that must take place?

- Does the student exhibit good problem solving procedures and habits?

The best way to start a discussion of word problem proficiency is to convey what students claimed they found hard about working out word problems. In one interview session, in which only the participating Hispanics took part, we asked the question, "What do you think is the hardest thing about solving word problems?" The following are the verbatim quotes given by the six students:

- "Getting the information you need to make the problem. Like if it says something about how many miles she walks, you gotta get how many miles--get it into numbers--take it apart."

- "Starting them. First it's hard for me to start a word problem. Like trying to find, you know, what's X."

- "Knowing what they want. Sometimes I read it wrong."

- "The whole thing. I hate 'em."

- "Finding the equation."

- "You don't know what to do--you don't know whether to add or divide--you don't know which number comes first."

These responses correspond very well with the difficulties which the students experienced in solving word problems. To paraphrase what the students found difficult about solving word problems, they realized that there was no set of rules or standard procedure which could be applied in a rote fashion to word problems in order to obtain an answer. Often, the approach taken by textbooks and teachers is to teach students specific techniques for solving specific types of problems. Students are then forced to remember tricks or rules which can be applied to "age problems", "per cent problems", etc. However, if there is a slight twist in a particular "age problem", then the tricks or rules they learned no longer apply and the student is left at a loss on what to try. A better approach would be to teach students sound, general procedures with which to approach any word problem, rather than specific procedures that apply to specific types of problems.

Something which is often overlooked when teaching students how to solve problems is how to deal with language-related subtleties, such as problem syntax, vocabulary, semantics, etc. As the student answers to some of the problems we are about to discuss will illustrate, the ability to understand a problem, and translate it into mathematical terminology is, we believe, the single most important step in the problem solving process.

Let us first consider the students' performance in the following four problems:

Write an expression using variables for the following statements:

10. A number added to 7 equals 18.
11. Six times a number is equal to a second number.
12. Nine times a number results in 36.
13. In seven years, John will be eighteen years old.

To aid in the discussion, we have coded the students' answers in Table 2. The letter "C" in Table 2 means that the student worked out the problem correctly. The entries "skipped" and "no idea" mean that the problem was skipped during the interview for that student, and that the student tried to solve the problem but had no idea what to do. Any other entry denotes the student's response in the problem.

Table 2 shows that students had little difficulty working out problems 10, 11, and 12, but that problem 13 caused inordinate difficulties. There is an obvious difference between problems 10, 11, and 12, and problem 13. In 10, 11, and 12, the problem structure is very clear -- the unknown to be represented by a variable always appears near the beginning as the noun "number". The remaining problem structure directs the student on what should be done with this unknown, for example, "a number added to 7...", or "six times a number...". On the other hand, the structure of problem 13 does not make obvious what the unknown is; the unknown, John's current age, must in fact be deduced. Evidently two students, one who wrote "7 + 11 = 18" and the other "7 + 11", understood the problem but were not able to write an equation using a variable. It appears that variations in the syntax of simple problems have an observable effect on performance. In particular, if the unknown or quantity being sought in the problem is not readily discernible, there is a higher likelihood for confusion than if the unknown appears clearly near the beginning of a problem statement.

Let us next consider the students' translational skills in slightly more difficult problems involving two variables:

Write an expression using variables for the following statements:

14. The number of nickels in my pocket is three times more than the number of dimes.

15. The number of math books on the shelf is equal to eight times the number of science books.

16. There are four times as many English teachers as there are math teachers at this school.

17. Last year, there were six times as many men cheating on their income tax as there were women.

The students' responses to these problems are shown in Table 3. The entry labeled "Reversal" implies that the student committed the variable-reversal error.

From looking at problems 14 and 15, it is clear that the syntax is such that a sequential left-to-right translation should yield the appropriate answer. For example,

"the number of nickels in my pocket"	"is (equal to)"	"three times more than the number of dimes"
N	=	3D

We therefore expect more correct answers on problems 14 and 15 than on 16 and 17, and further, we expect that those who translate problems using the sequential left-to-right method would likely get 14 and 15 correct, but 16 and 17 variable-reversed. This was borne out by the results; four students obtained both problems 14 and 15 correct as shown on Table 2, but of these four students, only one advanced Hispanic was able to get 16 and 17 correct. It should also be noted that the variable-reversal error constituted a significant percentage of the wrong responses.

Finally, we would like to discuss briefly some of the "unusual" answers in problems 11 and 14, as shown in Tables 2 and 3. Some of the student responses in these two problems are very suggestive of interpretational difficulties. For example, the participating Hispanic who wrote 6N=N in problem 11 explained that due to the phrase "is equal" in "six times a number is equal to a second number", both numbers must be the same, and therefore used N to represent both. The participating Hispanic who wrote 6X=2 explained that the "2" in the equation represented the "second number". The participating Hispanic who wrote 6X=12 explained that the "12" came from multiplying the "second number" times 6. As can be seen from Table 2, there were other answers very similar to these committed by students from all three groups. Although we have no first hand evidence to confirm it, we suspect that the three students who wrote variants of the answer "6N=2N" used the 6N to represent "six times a number" and 2N to represent the "second number".

In problem 14, we see from Table 3 that three students wrote answers with the inequality "greater than". There were also four answers which were not in the form of an equation, such as "$N^3 \cdot d$" and "N • D=". It is interesting to note that these types of answers did not occur with anywhere the same frequency in problem 15--a problem fairly equivalent in structure to problem 14. The biggest difference between these two problems is that in problem 14, the word "equal" was never explicitly used as was the case in problem 15; the equality in problem 14 had to be deduced from context. Those students who wrote an inequality seem to have interpreted (not unreasonably) the phrase "is three times more than" as a statement of inequality rather than as a statement of equality. Those who did not write a complete equation may have had problems figuring out where to put the equal sign.

5. English vs. Spanish Performance

Several word problems were constructed in Spanish to ascertain whether there existed observable differences in problem solving style, confidence, anxiety, etc. for the participating Hispanics when their English performance was compared to their Spanish performance. Since the algebra course was taught completely in English, all information, vocabulary, and experience in this subject will have been conveyed to the students in English. This may result in students being able to work out word problems "better" in English.

The overall success of the students in the Spanish word problems was not appreciably different than their success in word problems phrased in English. The styles used in solving the Spanish problems differed somewhat across students. Two students read the Spanish problems and proceeded to try and solve them directly. One student had not had any formal training in Spanish and "sounded out" the problems before attempting a solution; that is, the student read them aloud until the "gist" of the problem was understood. Two students translated the Spanish problems into English, and would not start on the solution until they understood the English translation. The remaining student stated that he could not understand the Spanish problems and did not attempt any solution.

The five students who attempted to solve the Spanish word problems encountered some Spanish vocabulary words they did not recognize and asked about their meaning. Once told, they had no further difficulty in attempting a solution. That is not to say that their attempts were successful. These differences in vocabulary did not, per se, create any additional difficulties in the ability of the students to solve the problems. We observed nothing in their overall performance to indicate that they could solve problems phrased in Spanish any better, or worse, than problems phrased in English. We should caution, however, that if these students had been solving the Spanish word problems totally on their own, it is very likely that their performance would have been worse than if solving English word problems due to difficulties with vocabulary, or inappropriate translations.

6. A Teaching Experiment

There was a teaching experiment tried with the six participating Hispanics during one of the interview sessions. The approach taken was to select six mathematical topics with which the students were unfamiliar and to give each student a 10 minute "mini-lesson" on one of the six topics. After the mini-lesson, the student was asked to teach the same lesson to one of his/her peers. Since the situation was contrived, there is no assurance that what we observed was what would normally take place if students were to tutor each other. Perhaps the experiment was only useful insofar as it let us evaluate what information the student deemed important enough to distill and teach to a peer.

Summarizing the results of this experiment, we can state that the teaching style used by the students did not differ appreciably. There were, however, distinct differences between the styles used by the students and the style used by the interviewer. The most obvious contrast between the two styles was that whereas the interviewer attempted to impart knowledge, the students attempted only to convey information. At no time did the students attempt to "motivate" the subject they were teaching to their peers. The students' presentations were aimed at answering the questions "what can you do with it?" and "in what does it result?", and never at answering "what good is it?"

For example, one presentation was on circles, and covered the topics of radius, diameter, circumference, and the mathematical relationships between them. Although the name "pi" was never mentioned, the interviewer attempted to convery that the ratio of the circumference to the diameter was a constant for any circle. Various wooden circular blocks were measured to illustrate the various relationships among the quantitites. In the student's presentation, he showed his classmate "what can you do with it?" by illustrating how to measure quantitites such as radius, diameter, and circumference; he answered "in what does it result?" by showing his peer how to plug some of the measured values into equations like 2R=D and C/D=3.1. He never conveyed two very important points, namely, that what was nice about the relationship C/D=3.1 was that it held for any circle, and that it allowed one to get C by knowing only D, or vice versa--points that were stressed in the interviewer's presentation.

In all cases we observed substantial attenuation of information between the original presentation by the interviewer, and the subsequent student presentation. There was a dramatic attenuation in the two-step process from the interviewer's presentation, to what the final student was able to glean from the presentation he/she heard from his/her peer.

Discussion

The fact that many of the errors we encountered have also been found in other investigations with non-minority students (Davis, Jockusch, and

McKnight, 1978; Matz, 1980) means that the students who participated in our study were not unique in what they found difficult about algebra. It is true, however, that the participating Hispanics were at a disadvantage in language proficiency as evidenced by their poor performance on the language portions of the California Achievement Tests. That this deficiency served as an impediment in the learning and problem solving processes was, we believe, apparent.

For the rest of this section we will focus our discussion on specific topics related to the effectiveness of the textbook, to instructional approaches, and to the interplay of language in cognitive processes. Before proceeding further, however, it would prove helpful to summarize the salient findings of this study:

- Students prefer not to use algebraic techniques in solving problems.
- Students are extremely poor at verbalizing definitions of mathematical terms, even when they possess a correct operational definition of the term.
- Students can often obtain a solution to a problem, but can seldom verbalize the procedure they used in obtaining the solution.
- Problem syntax is often the most important factor in determining problem difficulty.
- The step that students find most difficult in solving word problems is the translation of the problem statement into the appropriate mathematical equation(s).
- Students do not use their textbooks very much except as a place to find assigned problems.
- Students treat algebra as a rule-based discipline and not as a concept-based discipline.
- When applying algebraic rules, students do not apply them self-consistently.

1. The Textbook

Our discussion of the effectiveness of the textbook should not be construed as an indictment of the particular textbook used in the algebra I class. The Dolciani and Wooton textbook (1973) is not atypical of beginning algebra books, or of mathematics books in general. It has been, and still is, one of the most popular texts used in beginning algebra courses. As stated in the summary above, it became apparent soon after the beginning of the study, and was confirmed by the participating teacher, that the majority of the students in the class did not read the textbook as a means of supplemental instruction -- they merely used it as a place to find the problems that were assigned for homework.

The fact that students did not read the text is not something that can be easily blamed on the text. Perhaps the teacher should have made more of an effort to hold the students responsible for reading the book. What we can say is that the language and style of the textbook seems more appropriate for someone who already knows a little about algebra; than for 14 year old students with absolutely no prior training in algebra. The book does attempt to convey the precision necessary to "communicate" in mathematics. We believe, however, that the "incomplete" use of the text by the students may be responsible for some of the difficulties we uncovered, as we hope the following examples will illustrate.

In an interview session where we asked students to define several mathematical terms covered in the textbook, we found that the most misinterpreted term was "quotient". Popular interpretations given by the students to the term quotient were "answer" and "product". In the section of the textbook dealing with quotients (Chapter 4, section 4) we find the following instructions given in the quotient exercises:

"Read each quotient as a product. Then state the value of the quotient"

and

"State the value of each quotient".

For students reading these instructions, it is understandable why they might come away with the interpretation of "product" and "answer" for quotient -- the firt instruction above can be taken to imply that "product" and "quotient" are interchangeable; the second instruction makes perfect sense if one substitutes the word "answer" for "quotient".

Further, the textbook often attempted to draw upon "real life" situations for its word problems. This attempt to be relevant may be of questionable pedagogic value for students such as the participating Hispanics of this study. Even though we have no formal measure of socioeconomic status (SES) for the participating Hispanics, we were able to determine that they were "below average" in SES. The following two problems from the textbook (found on page 76) were used in an interview session to assess the understanding that the students had of the vocabulary, and will illustrate an important point.

> "A stock selling for $30 per share rose 2 dollars per share each of two days and then fell $1.75 per share for each of three days. What was the selling price per share of the stock after these events?"
>
> "On a revolving charge account, Mrs. Dallins purchased $27.50 worth of clothing, and $120.60 worth of furniture. She then made two monthly payments of $32.00 each. If the interest charges for the period of two months were $3.25, what did Mrs. Dallins then owe the account?"

Upon asking the six participating Hispanics to tell us what they thought terms like "stock", "share", "revolving charge account", "monthly payments", and "interest" meant, they displayed that they had little idea as to the meaning of these terms. Several students came close to being able to define "interest", and stated that it was something that banks and stores did to make more money.

What is of more questionable pedagogic value in using problems like the ones above is that students were being confused by the jargon, and not necessarily by the mathematics. Several students stated that even though they were not sure what some of the terms meant in the "revolving charge account" problem, they thought they could nevertheless solve the problem. Their attempts to solve this problem consisted of combining the four monetary quantities given in the problem in some fashion to obtain a final, albeit incorrect, answer.

2. Pedagogy

We would like to begin this section by briefly summarizing some characteristics displayed by the students which we believe are not conducive toward the learning of mathematics. The students' performance during the interviews revealed that they did not appreciate that when working in a subject like mathematics, a) the slightest degree of imprecision and sloppiness can lead to errors and b) there are logical and legitimate reasons for every stage in any series of algebraic manipulations. It is also evident from the students' unwillingness to use algebra that they had little "faith" in using algebraic procedures to obtain asnwers. In other words, the notion that algebra allows one to start with a word problem, and by applying certain procedures such as defining variables, translating the relationships among the variables in the problem into mathematical equations, and manipulating the equations, one is guaranteed of finding a solution even if one has absolutely no idea what the solution is anywhere along the way, is something students found quite incredible.

We also found that the students were neither very careful listeners, nor good at following instructions. One problem given in an interview session will illustrate this situation. The problem was the following:

3.7. For each of the following, underline all the operations that mean addition.

$3+7 =$

$4\div2 =$

$3+(4.8) =$

$9+(7-19) =$

Here we were not intereted in the students' answers to the problem inasmuch as we were interested in whether they followed our instructions. Our normal operating procedure for conducting interviews consisted of the interviewer presenting each problem to the student, as opposed to asking the students to read a problem silently before offering a solution. In the problem above, it was decided that upon reaching it, the interviewer would merely point to it and say to the student "do this problem". Admittedly, we were attempting to see how easy it was to trick the students, since the response we predicted was that they would solve the left hand side of the equality, and write an answer on the right hand side. Only two of the six participating Hispanics (only the participating Hispanics took part in the interview session in which the question above was given) attempted to read and follow the instructions of the problem; the other four did what we had predicted.

In order to address the difficulties that the students had with algebra, what is needed is a pedagogic approach which addresses all of the problems in a global fashion, as opposed to remedies which are applicable only in specific situations. To help evaluate the pedagogical suggestions we will make, it would help to have a viewpoint or ideal against which our suggestions can be judged. The viewpoint we will present will not only encompass the role of concepts and skills in learning algebra, but also include a perspective on the importance of communication skills in the educational process. In an oversimplified description of the situation, there appear to be two extremes in the approach used in teaching mathematics. There are those who feel that rigorous formality is indispensable to the learning of concepts and those who feel that the possession of manipulative basic skills is a precondition to learning concepts. We tend to agree and yet disagree with both of these views. If the formal aspects of mathematics are emphasized at the expense of training in basic skills, the student may learn the jargon of mathematics, yet remain quite incapable of solving problems. On the other hand, we will not be the first to point out that although working out lots of problems may be a necessary condition for problem solving proficiency, this does not mean it is a sufficient condition as well (Kilpatrick, 1978).

To perceive that this contention of views regarding formality versus basic skills is not easily reconciled, one need only observe that whereas basic skills are <u>taught</u>, concepts are <u>formed</u>. Concepts cannot be taught, although they are learned in some sense of the word. The actual formation of a concept, however, is a purely internal process on the part of the student. Although the possession of basic skills can certainly aid the student in this endeavor, no amount of honing of basic skills will necessarily force a student to conceptualize. Our resolution of this dilemma is based on our belief that the single most important ingredient in the educational process is communication. Our recommended <u>approach</u>, therefore, focuses on the use of the communicative process.

Communication between the teacher and the student can assist the student in the process of conceptualization. The teacher can not only suggest the existence of concepts and encourage students to grapple with them, but also, through interaction with the student, guide him/her toward the formation of correct concepts. It is imperative that first and foremost, the teacher convey to the student as early as possible the need to be precise when working in mathematics, whether it be in listening, following instructions, communicating, or manipulating mathematical expressions. The emphasis thereafter should be on helping students form generalizable concepts rather than on asking students to memorize algebraic rules and facts.* Further, we are in agreement with other researchers (Davis et al., 1978) in their advocacy of giving students the fullest possible appreciation of the importance of logically identifying and justifying algebraic procedures, and in their claim that "do it this way" approaches are not sufficient. If we were to choose one algebraic concept that is often taken for granted by experts, but which is a very difficult concept to grasp for neophytes, it is the notion that algebra is an artform in which unknown mathematical quantities can be manipulated via a set of rules in order to extract a known answer.

With the recent technological advances, one instructional approach that should be given serious consideration is the use or low-cost microcomputers for supplemental mathematics instruction. That microcomputers are gaining rapid acceptance in mathematics instruction, both as teaching and programming tools, is unarguable. For the types of students in this study, computer instructional modules which combine presentation of material with detailed step-by-step worked-out examples, and drill work would be particularly helpful. We would like to emphasize the phrase "presentation of material" above to distinguish this approach from the "drill" modules which have become the standard product of many software and publishing firms. There are several reasons why this approach may prove very effective for this age group. First, given that students are not inclined to read the textbook, having them read the material on a computer's screen may be a possible solution to this problem, particularly since the students may associate this with watching T.V. and not with reading a textbook. Second, the "mystique" surrounding computers can be exploited to have students spend more time working on mathematics. Finally, the fact that computers are so intolerant of sloppy communication would help in training these students to work and communicate in mathematics with more accuracy and precision.

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*We do not mean to imply that having a command of algebraic rules and facts is unnecessary. Although a command of algebraic rules and facts will aid in getting an answer, it does not help the student in designing a strategy for obtaining the answer.

3. Linguistics

Whenever a cognitive research study is conducted with a bilingual population, there is always one albatross with which to contend, namely, the question of how language proficiency affects the findings. The notion that language may have an effect on cognitive processes (not necessarily for bilingual populations) is not new. For example, in Vygotsky's (1962) view, many facets of intellectual functioning are intimately related to language acquisition. According to Vygotsky, the internalization of language introduces a restructuring of many mental processes. Of particular relevance to this study is Vygotsky's claim that problem solving strategies become more rational and sophisticated when they can be verbalized.

Another view, that of Whorf (1956), states that the language we speak can set certain limits or constraints on our perception. Perhaps the justification for this view derives more from cultural effects than from linguistic effects; that is, it may well be that cultural experiences are as important as linguistic experiences in forming our perceptions. The difficulty with the Whorfian hypothesis lies in how to distinguish between these two effects.

One hypothesis adduced by Cummins (1979) deserves particular attention due to its wide range of applicability to bilingual students. Cummins' linguistic threshold hypothesis posits that "there may be a threshold level of linguistic competence which bilingual children must attain both in order to avoid cognitive deficits and to allow the potentially beneficial aspects of becoming bilingual to influence their cognitive growth" (1979, p. 229). Cummins does not define the threshold level in absolute terms since it is likely to vary depending on the child's stage of cognitive development, and on the academic demands of the different stages of schooling.

Cummins does define three types of bilingualism. The first, "semilingualism", is characterized by a below-threshold level of linguistic competence in both languages. In semilingualism, both languages are sufficiently weak to impair the quality of interaction the student can have with his/her educational environment. The negative effects of semilingualism are no longer present in "dominant bilingualism", characterized by an above-threshold level of competence in one of the two languages. Dominant bilingualism is supposed to have neither a positive, nor a negative effect on cognitive development. The last category, "additive bilingualism", is one which has positive cognitive effects. Additive bilingualism is characterized by above-threshold competence in both languages.

In our investigations with college level Hispanic engineering students, we have found that, on standardized language proficiency measures, Hispanics score considerably below their Anglo peers (Mestre, 1981). This below-average performance holds across both English and Spanish. In terms of Cummins' definitions, these Hispanic engineering majors appear to be "semilingual". Even though we have not made an assessment of the Spanish proficiency of the participating Hispanics of this study, it appears from their performance on the language portions of the CAT that they are below-threshold, at least in English. Although it is extremely difficult to separate language effects from other effects, findings with college Hispanic students (Mestre, Gerace, and Lochhead, 1982; Mestre, 1982), as well as with the participating Hispanic group of this study indicate that this below-average language proficiency level has an adverse effect on mathematical performance. The facts that the advanced Hispanics of this study appear to be at least "dominant bilinguals" (and perhaps "additive bilinguals"), and that their performance in this study was extremely strong by any measure, lend support to Cummins' hypothesis.

Finally, it appears to us that any effort designed to increase the language proficiency level of bilinguals, at least up to the level of their monolingual peers, is most desirable. It does not appear to be too important which of the two languages is developed, as long as at least one of them is highly developed; however, for the obvious reason, the language used for instruction in the student's school may be the most appropriate to target for development. A word of warning is in order. Although there are strong indications that being highly proficient in language is a necessary condition for cognitive development, it certainly is not a sufficient condition as well.

Recommendations

Two points were made by the participating teacher which should be conveyed before moving to the specific recommendations proposed below. The first point made by the participating teacher was that since the "character" of mathematics classes varies from year to year, and from class to class, depending on the students comprising the class, it is unrealistic to expect that one pedagogic approach which proves very effective for one particular class would have the same result when used with another class. The second point concerned the realism of dealing with 14 year old adolescents. He claimed that, given this age group's maturity level, some pedagogic approaches were likely to prove more effective than others; in particular, approaches which demand the undivided attention of the whole class for a prolonged period of time would not prove fruitful.

In terms of the recommendations below, we believe that they would be of benefit to all students. However, we have designated several which we feel would be particularly beneficial for students such as the participating Hispanics of this study whose language proficiency level is below that of their Anglo peers. Our recommendations are the following:

1. Students should be asked to participate in the process of learning concepts. Although these students are not of an age where they are naturally introspective, encouraging and aiding them in forming general concepts are preferred over passively absorbing rules. Whenever possible, procedures which are generalizable to a wide range of problems should be emphasized over rule-oriented procedures which apply only to a narrow range of problems. Concept formation can be reinforced by presenting the students with both correct, and incorrect examples and asking them to recognize valid procedures as well as fallacious logic. It is often the case that the real hint of a concept lies in the path to the answer and not in the answer itself.

2. More important than the pedagogic style of the teacher or textbook is the harmony between the two. The teacher should use a textbook which is consonant with his/her teaching style, so that both text and the teacher emphasize a single approach. The teacher's primary objective should be to impart to the student his/her understanding of the material rather than some supposedly superior way of thinking about the subject -- one which he/she neither uses nor feels fully comfortable with.

3. Students should be held responsible for reading the textbook. The ability to learn from written material is an indispensable tool for self-learning and should therefore be incorporated as early as possible in the educational process.

4. Whenever a new definition or procedure is introduced, it should be compared and contrasted with previous ones. Equally important as telling students what something is, is telling them what something is not. The use of counter-examples or discussions of incorrect applications of rules/procedures would help students assimilate the correct rules/procedures more quickly.

5. Students should be made to realize that two very important ingredients in mathematical reasoning are precision and consistency. These same characteristics should be sought in the communication process itself. Due to the great redundancy present in oral communication, and aided by the context of the situation, most of us can tolerate large

doses of imprecision and inconsistency in oral communication. When attempting to teach or learn mathematics, however, imprecision and inconsistency can be very debilitating.

6. The use of microcomputers for supplemental instruction in mathematics should not be underestimated. Microcomputers may serve to motivate students to spend more time on mathematics, force them to communicate more precisely with a machine which is very intolerant of sloppy communication, and provide an opportunity to present material which students would not otherwise be inclined to read in the textbook.

The following three recommendations would be of particular benefit to students with language deficiencies.

7. Students should be asked to verbalize the rules, strategies, definitions, and procedures that they employ in solving problems. This would serve to a) monitor the precision with which the students communicate mathematical ideas, b) encourage students to always have a reason to justify what they are doing, and c) reveal any misconceptions the student has so that the teacher has an opportunity to address them.

8. In word problems, the emphasis should be on teaching students sound procedures in translating the problem statement(s) into mathematical notation. By defining variables and writing appropriate equations to represent the problem statement, students would begin to appreciate more quickly that they do not have to know the answer "all at once", but that the resulting equations are the means by which to obtain an answer.

9. A concerted effort should be made to increase the language proficiency level of "semilingual" students (in the Cummins sense) to at least the "dominant bilingual" level. Although the evidence is not conclusive, indications are that "semilingualism" may have an adverse effect on the communication process which we believe to be crucial in the educational process.

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TABLE 1

PERFORMANCE ON AVAILABLE MEASURES

	Read. Vocab.	Read. Comp.	Tot Read.	Spell.	Lang. Mech.	Lang. Expr.	Tot. Lang.	Math Cmpt.	Math Chcpts.	Tot. Math	Tot. Batt.	Ref. Skills	Anal.	Class.	Piaget	Alg. Grade
Algebra I Class N=34*	15.1	24.0	39.1	10.7	15.3	24.0	39.4	26.1	23.3	49.3	138.5	18.5	15.8	17.5	49.0	72.1
	5.7	5.7	10.5	2.9	4.1	5.3	8.5	5.5	6.3	9.9	26.1	3.2	3.6	4.8	7.9	15.3
	53	49	52	43	39	44	41	59	45	53	46	43	----	----	----	----
Nonparticipating Anglos N=23*	15.9	25.7	41.6	10.8	16.3	25.7	42.1	26.7	25.7	52.4	146.8	19.4	16.9	17.7	49.8	73.1
	5.6	5.3	9.6	3.2	2.8	4.7	6.3	5.4	5.9	10.0	23.5	2.5	3.1	4.6	8.8	16.0
	54	56	55	43	44	51	48	61	53	57	53	50	----	----	----	----
Participating Anglos N=5	17.4	23.8	41.2	10.8	15.0	20.2	32.5	26.0	19.8	45.8	133.0	18.0	15.2	18.2	49.4	78.8
	4.8	5.2	9.5	2.6	3.7	5.0	8.0	6.2	4.8	7.2	22.8	4.5	1.1	6.0	3.6	13.0
	59	48	54	43	38	29	31	59	34	46	43	39	----	----	----	----
Participating Hispanics N=6	10.5	18.3	28.8	10.3	12.2	21.2	33.3	23.8	17.8	41.7	114.2	16.0	12.2	16.3	45.8	62.3
	5.4	4.2	9.4	2.7	6.9	5.7	11.9	5.4	3.5	6.9	23.6	3.5	4.8	5.5	6.7	10.3
	30	29	29	39	24	32	27	52	27	38	30	30	----	----	----	----
Advanced Algebra Class N=29	23.7	31.3	55.0	15.4	20.7	32.3	53.0	34.0	35.4	69.5	192.9	22.1	----	----	----	81.4
	4.0	4.2	7.3	2.9	2.4	2.9	4.0	3.3	4.2	6.5	16.5	1.6	----	----	----	5.1
	79	79	79	83	78	85	84	81	81	83	86	78	----	----	----	----
Advanced Anglos N=26	23.7	31.6	55.3	15.3	20.7	32.7	53.4	34.0	35.5	69.5	193.5	22.3	----	----	----	81.9
	4.0	3.8	6.8	2.5	2.3	2.2	3.1	3.5	4.3	6.7	14.9	1.3	----	----	----	4.8
	79	81	80	83	78	87	85	81	81	83	86	80	----	----	----	----
Advanced Hispanics N=3	23.7	29.0	52.7	16.0	20.7	29.0	49.7	34.0	34.7	68.7	187.0	20.0	17.3	16.0	50.7	73.3
	4.0	8.2	12.2	6.1	3.8	6.2	9.0	1.7	3.8	5.5	31.2	2.6	2.5	1.0	4.5	7.1
	79	70	76	87	78	67	73	81	79	81	81	55	----	----	----	----

Note: The first twelve measures correspond to the California Achievement Tests. Analogies and Classification are from the Test of General Ability. The last two measures correspond to the Piagetian test and the students' final grade in algebra. The three entries correspond to the mean, standard deviation, and national percentile ranking.

* N for the Algebra I class is 32 for all CAT measures. N for the Algebra I class/nonparticipating Anglo subgroup is 21 for all CAT measures.

TABLE 2: STUDENT ANSWERS

		Problem #			
		10	11	12	13
	Correct Response	X+7=18	6A=8	9Y=36	J+7=18
Participating Hispanics	1	No Idea	6X=12 (12 was from 6 times second #)	C	Skipped
	2	7=18	6X=2 (second # was 2)	9·4=36	No Idea
	3	C	C	C	C
	4	C	6N=N	C	7+11=18
	5	C	6A=2A	C	7A·18
	6	B+7	6N·2N	9N+6=36	8+9=18
Participating Anglos	1	C	C	C	C
	2	C	C	C	7+A
	3	C	6A=2	C	7+11
	4	11N+7=18	6N=2N	9Ax4N=36	7=18
	5	C	C	C	X-7=18
Advanced Hispanics	1	C	C	C	C
	2	C	6X=X	C	y+7y=18
	3	C	C	C	C

TABLE 3: STUDENT ANSWERS

		Problem #			
		14	15	16	17
	Correct Response	N=3D	M=8S	E=4M	M=6W
Participating Hispanics	1	No Idea	Skipped	No Idea	No Idea
	2	No Idea †	8·6=48 *	4·4=16	2·6=12 (2=women, 6=men)
	3	N·D=	8xS=-8	4xS=-4	6·W=
	4	C	C	Reversal	Reversal *
	5	3N+D	Reversal	Reversal	Reversal
	6	$3d^2$ 4+N=	64M·8·8=	4MT=20ET	Reversal
Participating Anglos	1	3N+D=X *	M·8S=8 * (8 was tot. books)	4·E·M=T *	Reversal *
	2	a>3·B	Reversal *	Reversal *	Reversal *
	3	$N^3 \cdot D$	$b=S^8$	$E^4 \cdot M$	Reversal
	4	3N>10	N=8	Reversal	Reversal
	5	C	C	Reversal *	Reversal *
Advanced Hispanics	1	C	C	C	C
	2	C	C	Reversal	Reversal
	3	X^3 more than N	X^8 more than N	$X^4 \cdot N=X=1$ *	X^6 more than N

Note: Students appear in the same order as in Table 2.

†Upon prompting, student displayed an improper understanding of the relative sizes among the two quantities in the problem statement.

*Upon prompting, student displayed a proper understanding of the relative sizes among the two quantities in the problem statement.