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ABSTRACT

The purpose of this paper is to describe the development of children's measurement concepts and to outline implications of recent research on transitivity and counting. Discussion is confined to measurement of length and focuses on aspects of measurement outlined in the last two of Piaget's three measurement stages. It is argued that measurement involves interdependent application and knowledge processes (application processes include counting and division into units; knowledge about measurement involves understanding of transitivity and measurement units). Research described indicates (1) that young children apply the counting estimator in a measurement context with no regard for unit size and (2) that it is presently not known how the child switches in a measurement context from using the counting estimator to using the measurement estimator. At least three developmental sequences are proposed to account for this change; all assume that the child has attained the ability to conserve length. The first sequence suggests that motivation to change is based on cognitive conflict between the length estimator and the counting estimator. The second and third sequences suggest that motivation to change is based on knowledge about, respectively, the transitivity principle and direct instruction. Suggestions are offered for further research. (RH)

The Relationship of Counting and Transitivity to the Measurement of Length

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Measurement is a quantification skill which has been largely unexplored by those interested in cognitive development. However, by studying the application of counting and transitivity skills in a measurement context we can identify some of the boundaries of children's understanding of these skills and some of the relationships between them.

The purpose of this paper is to describe the development of measurement concepts and to outline the implications of recent research on transitivity and counting for our thinking about these abilities. The discussion will be confined to measurement of length since it is one of the first types of measurement to be acquired (Beilin & Franklin, 1962).

One major developmental trend identified in the Piagetian work on measurement (Piaget, Inhelder, & Szeminska, 1960) is that more global relative assessments precede quantitatively precise assessments. This trend is apparent in the obligatory three stages of measurement abilities outlined by Piaget. In the Piagetian tasks children were asked to measure the lengths of two objects. In stage 1 comparisons made between two objects were visual. In stage 2 children started to use an unmarked ruler to measure. However, they did not understand that objects used to measure two lengths must be a uniform size. Thus, when the ruler was shorter than the two lengths to be measured they often made errors. When children measure they have to move the smaller ruler several times across the longer length. For the measurement to be accurate the placement of the ruler must occupy the same amount of length each time. The common error is that the placements of the ruler do not occupy the same amount of length. In stage 3 children can apply the ruler consistently to make accurate measurements. I will be focusing on those aspects of measurement that are outlined in the last two stages.

The function of measurement is to assign numerosities to the properities of objects (Campbell, 1920). Procedures used to determine numerosity are called estimators (Gelman 1972) and common examples of estimators are counting and subitizing. Gelman and Gallistel (1978) have proposed five principles to describe the counting estimator. Three of the principles, labelled the how-to-count principles, deal with the application of counting skills. The two remaining principles, the abstraction and order-irrelevance principles, deal with knowledge about the properties of numbers. I propose that this same division can apply to measurement which can also be divided into the processes of application and knowledge. The application processes include counting and division into units. Knowledge about measurement involves an understanding of transitivity and of measurement units.

The discussion is based on the assumption that the knowledge processes and the application processes are interdependent. One feature underlying all of them is the necessity of understanding that units used to measre must be a uniform size. First the knowledge processes and then the application proccess will be discussed.

I propose that having an understanding of measurement units involves knowing that measurement units are individual elements which are members of the same class, and which are equal on some physical dimension. With this understanding children can divide continuous quantities into equal sized units and can equalize units of unequal sizes. The measurement literature supports the conclusion that first graders and younger children lack this understanding and rely on number information rather than coordinating number information with size information (Carpenter, 1975; Inhelder, Sinclair, & Bovet, 1974). Part of their difficulty may derive from a lack of understanding about transitivity.

The understanding of transitivity involved in the measurement process consists of combining two relationships to make inferences about a third (e.g. A > B, B > C, therefore A > C), and understanding that the judgements must have the same common term or reference point for example B in A > B > C. Some of the work on transitivity done by Trabasso (Bryant & Trabasso 1971; Trabasso

1975,1977) has examined whether children can combine two relationships to make inferences and suggests that young children can make transitive inferences. Therefore, young children should have the necessary transtive abilities to be able to measure accurately. However, recent work on transitivity suggests that the task used by Trabasso may be solved by a variety of noninferential processes (Breslow 1981; Kallio 1982; Blevins 1981) and that it is uncertain whether preschool children have the abilty to make inferences. Studies examining the understanding that a common reference point is essential for an inference have shown that children will make the inferences in the absence of this term (de Boysson-Bardies & O'Regan 1973; Blevins 1981). For example, if given the relationships of A > B and C > D they will infer A > D. It is not suprising then tha children make comparisons betwen two lengths that are based on measuring each length with a different size unit since they seem to be willing to make judgements based on categorical information about whether something is big or smal rather than relative information about whether something is bigger or smaller than another object. Understanding that a common linking term is essential for the inference should be related to knowing that measurement units have to be an equal size because it means that children know that each measurement has to relte to every other measurement.

Once a child possesses the transitivity principle the ease of applying it may depend on the measurer's familiarity with the measuring units involved. For example, it may be difficult for a child to measure a length in meters if the child does not know what a meter is. However, the principle that the units used to measure must be uniform remains the same across various measurement situations. This principle allows us to construct measures of our own when no standard measures are availible.

Applying the process of counting to measurement involves the assumption that it is possible to use numbers to quantify length. Within the area of number devlopment we see that children become increasingly precise when asked to make judgements about relative numerosity (Cooper, Starkey, Blevins, Goth, & Leitner 1978). They progress from knowing which of two arrays has more to knowing exactly how many more one array has than another. Part of this change is a result of the child's understanding of how and when to apply estimation skills. The same type of understanding can occur in measurement when children use numbers to quantify length. There is evidence suggesting that in measurement situations young children readily use numbers to quantify length (Carpenter 1975, Inhelder, Sinclair, & Bovet 1974). However, they focus only on the number of units present and not on both the size and number of the units. They do not seem to understand that there is an inverse relationship between number and size of units.

The difference between using number as an estimator and measurement as anestimator then is that numbers are used to answer two different questions about quantity, how many or how much. In the counting situation children are answering the how many question. This question can be answered by assigning one number name to one object. Gelman and Gallistel refer to this as the one-one principle. In the measurement situation children are answering the how much question. According to Fuson and Hall (1983) using counting skills in a measurement context involves attaching information about the type of unit counted to the count word, for example 7 in. vs 9 ft. There is only one number name per unit, and assigning information about the appropriate type of unit involves making a decision about the most appropriate level of measurement (in. vs ft.) and about the ratio of objects to measurement unit (12 in. corresponds to 1 ft.).

The second application process involved in measurement is division of a length into units. Part of the measuring process for length often involves moving a smaller length along a longer length and counting the number of smaller lengths composing the longer length. If the measurer is careful there will be no gaps between sequential placements of the smaller object, and the measurement will be accurate. Piaget and his colleagues have found that children often leave gaps between placements. Fuson and Hall report that children often use their fingers to mark the placements and fail to include the part of the length that their finger is covering in their measurement. Being able to accurately measure involves having the necessary motor coordination to move the smaller length precisely as well as an understanding of measurement units. Errors can be a result of a lack of motor coordination or a lack of understanding of measurment units.

The previous discussion indicates that a major difference between the measurement estimator and the counting estimator is that the units must be a uniform size for measurement. Previous work on measurement indicates that young children apply the counting estimator in a measurement context with no regard for unit size.

How then does the child go from using the counting estimator in a measurement context to using the measurement estimator? There are at least three developmental sequences that could account for this change, all of which presume that the child has conservation of length. The first sequence is based on cognitive conflict between the length estimator and the counting estimator providing the motivation for change. For example, some judgements about overall length can be made that are accurate in terms of whether one length is >, <, or = to another. If these judgements disagree with those obtained from counting, this conflict could focus the child on unit size as a way to coordinate the dimensions of length and number. In this case incorrect application of the counting process would lead to an understanding of transitivity and measurement unit which would then lead to correct application of the counting process.

The second sequence is based on knowledge about the transitivity principle providing the motivation for change. When children understand that a common linking term is necessary to make an inference, they have the cognitive rule necessary for discovering that units used to measure must be a uniform size. Piaget (Piaget et al. 1960) identifed two types of transitivity, qualitative and quantitative, which are relevant for understanding the relationship between transitivity and measurement. Qualitative transivity involves encoding relationships relatively, that is A > B> C. Quantitative transitivity involves encoding relationships in terms of units, that is A = 6in., B > A by 2 in., and C > B by 4 in. Piaget has found that qualitative transitivity precedes quantitative. Therefore, children may understand the necessity of a common linking term for qualitative transitivity and then use this rule as the basis for making the same discovery about quantitative transitivity. The realization that there should be a common linking term could focus the child on unit size. In this case transitivity triggers changes in the application process and in the knowledge about measurement units.

The third developmental sequence is based on direct instruction providing the motivation for change. Children are taught how to measure in school. It is possible that they memorize the rules for how to measure and come to understand them after obtaining feedback from correct and incorrect applications of the rule. In this case application processes trigger changes in the knowledge processes.

These possible developmental sequences and the preceding observations about measurement lead to the following suggestions for those pursuing research on measurement. First, it is important to make a distinction between measurement tasks assessing an understanding of unit size and those which do not. The importance of this distinction can be demonstrated by examining the claims made about a conservation training study done by Bearison (1969) which involved training children on measurement, then assessing the impact of this experience on the acquisition of conservation. The training used was with units of equal sizes and did not require the child to determine how to divide the quantity into appropriately sized units. Bearison concluded that conservation results in part from a conflict between logical operations and in part from the results of empirical discoveries based on measurement. The problem with this argument is that an understanding of conservation is a prerequisite for being able to divide a quantity into equal sized units so that it can be measured. Preconserving children believe that moving an object can change its length, so the entire measurement process would be based on units which are constantly changing in size. Training in an experimental situation might lead to conservation because the experimenter divides the quantity into units, but in real life the child is faced with this task.

The second suggestion is that studying the measurement estimator can give us information about how children define what to count. Gelman and Gallistel argue that young children will count heterogeneous groups of objects, and suggest that one issue to be pursued is the degree of heterogeniety the child will tolerate in counting. The issue of how the child divides the quantity to be counted is equally interesting because it provides us with additional information about what children believe the purpose of counting to be. Possibilities about their beliefs range from the belief that the prupose is to provide a number to the belief that it is to provide precise information about a specific quantity.

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