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Final Report

Application of Computer Graphics to Graphing in Algebra and Trigonometry

National Science Foundation
National Institute of Education

SED 80-12447

January 1, 1981 - December 31, 1982

J. Richard Morris
Virginia Commonwealth University

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This project was designed to improve the graphing competency of students enrolled in the elementary algebra, intermediate algebra and trigonometry courses at Virginia Commonwealth University. The immediate goal of the project was to design and implement computer graphics programs (called "graphing lessons") that would be available for use in the three courses mentioned above. The software package was to be interactive and would give the student control over the events occurring during a lesson. The grant provided funds for purchasing three Apple II Plus computers which were initially used to create the lessons and are now available to the students. Students enrolled in the three mathematics courses are assumed to have no special knowledge of or skills in using the computer. Indeed, the lessons are written so that the presence of the computer will not hinder the student's progress or cause the student additional anxiety. It is hoped, in fact, that use of these graphing lessons will decrease student anxiety and improve graphing skills. The lessons give positive reinforcement and get the student actively involved in the process.

The student interacts with the computer by typing in answers or otherwise replying to questions, plotting points and taking quizzes at appropriate places in the lesson. The lessons are written in Pascal and, again, they are designed so that no computer knowledge is required of the student; once a student is seated and the machine turned on by an attendant, he or she, with reasonable care, should be able to work through the lesson unaided. However, in the event of difficulty, an attendant will be on hand to assist.

The lessons are intended to be supplemental in nature and should be used by the student only after having been introduced to the topics in the classroom. By using the computer the student has the opportunity to see graphs drawn quickly and accurately - something that is generally difficult for an instructor to do with only chalk and blackboard. Thus, the graphing lessons reinforce the concepts covered in the classroom and, as a result, the student's graphing skills are expected to improve.

As previously mentioned, the lessons are intended to be supplementary. When an instructor completes a formal lecture on each topic, the student's attention is called to the fact that a graphing lesson is available for that topic. The instructor emphasizes that the lesson is instructive, easy to use, and will enhance the student's understanding of the lecture.

In keeping with the objective of providing a friendly, nonthreatening learning experience, the Apple computers are kept in a comfortable room with informal surroundings. An attendant is present to turn on the machines and generally help students begin each lesson. As noted, once the machine is turned on it should be possible for a student to work through each lesson unaided. However, an attendant is present if any difficulty - computer-related or mathematical - arises.

Each lesson is written so that the student has the option, at various times, of continuing the lesson, stopping altogether, or redoing a portion of the lesson. The lessons which are currently available for use are listed below.

List of Graphing Lessons

1. Introduction to Use of Keyboard
2. Linear Equations
   (a) equations of the form $Ax + By + C = 0$
   (b) equations of the form $y = mx + b$
3. What's My Line (Game)
4. Inverse Functions
5. Quadratic Equation $y = Ax^2 + Bx + C$
   (a) effect of A
   (b) effect of B
   (c) effect of C
   (d) experiment with choices of A, B, C.
6. The Parabola Game
7. Exponential Functions (Part 1)

In addition, specifications have been written for lessons on:

1. Exponential Functions (Part 2)
2. Logarithm Functions
3. The Trigonometric Functions (six lessons)
   (a) Sine Function
   (b) Cosine Function
   (c) Tangent Function
   (d) Secant Function
   (e) Cosecant Function
   (f) Cotangent Function

Writing computer code has proven to be a very time-consuming task and by far most of the time has been devoted to this phase of the project.

The completed lessons are currently being tested by students in our audio-tutorial mathematics laboratory. Students in the remedial algebra and college algebra courses are invited to use the lessons at anytime but preferably at the same time they study the corresponding material in the textbook. Data is being gathered to determine the frequency of use, the reaction towards the machine and the students' general reaction to the graphs lessons. A questionnaire is administered at the completion of the lesson.

Following the testing of the lessons the project members plan to write instructor and users guides and hope to be able to distribute the software package through some agency such as CONDUIT.

Description of the Graphing Lessons

1. Introduction to Use of Keyboard

The student learns how to enter numerical data and learns the proper keys to use to respond to the true-false and yes-no questions. The lesson contains enough examples and enough sample questions so that by the time the student has worked through it he or she should feel at ease using the machine.

2. Linear Equations

The student is introduced to equations of the form $Ax + By + C = 0$ and $y = mx + b$. The lesson emphasizes graphically the geometric meaning of slope as rise over run. The student actively participates in the lesson by using the paddles to plot points.

3. What's My Line

This is a game which tests the student's comprehension of Lesson 2. The computer draws a line and the student is asked to supply the values for m and b in the equation $y = mx + b$. A record is kept of the student's attempts when the student's answers are typed. At the end the student's score is printed. The level of difficulty gradually increases and the student may quit at any time.

4. Inverse Functions

The student is introduced to the concept of an inverse function via the geometric notion of reflection in the line $x = y$. The paddles are used to move points (x,y) to (y,x).

5. Quadratic Equations $y = Ax^2 + Bx + C$ (4 separate lessons)

(a) Effect of A The effect of the coefficient A is studied with B and C fixed.

(b) Effect of B The effect of B is studied by varying B while keeping A and C fixed.

(c) Effect of C The effect of C is examined by keeping A and B fixed.

(d) Experiment with Choices of A, B, C The student sees in this lesson the combined effect of all three coefficients.

6. The Parabola Game

The computer draws parabolas and the student is asked to supply the correct values of A, B, and C. A record is kept of a student's attempts and a score is printed when the correct answers are typed in. Students may continue playing as long as they wish or quit at any time.

7. The Exponentials Function (Part 1)

The graphs of $y = a^x$ are examined for various values of a greater than 1. The student actively participates in plotting points and when enough points have been plotted they are connected by the computer to draw a smooth curve.

Description of the Specifications for the Remaining Graphing Lessons

The specifications for the following graphing lessons have been written, but at this time the coding is incomplete.

8. The Exponential Function (Part 2)
9. The Logarithm Function
10. The Sine Function
11. The Cosine Function
12. The Tangent Function
13. The Cosecant Function
14. The Secant Function
15. The Cotangent Function

The specifications for Lesson 8 call for examination of the graphs of $y = a^x$ for values of a between 0 and 1. When completed, it should be very similar to Lesson 7. Lesson 9 specifications would produce a lesson which would introduce the student first to the graph of $y = \log_2 x$. This would be achieved by having the student find y values for given x values and then having the machine plot the points and draw the graph. Next, $y = \log_{10} x$ would be treated in a similar fashion, and finally the connection between the logarithm and exponential function would be explored using the inverse function concepts already discussed in Lesson 4.

The specifications for the six trigonometric functions are basically the same, so only those for the sine function will be described. The student is first given the equation $y = \sin(x)$ and a table of values for x. The x values given are $x = 0$, $\pi/2$, π, $3\pi/2$ and 2π. The student is asked to type in the correct values for y. It is then noted that additional points are needed for an accurate graph so the x values $\pi/12$, $\pi/8$, $\pi/6$, $\pi/5$, $\pi/4$, $\pi/3$ and asked to use a pocket calculator to find the y values. If the student does not have a pocket calculator, he or she is instructed to press return and the y values will be displayed. A similar process is used for the intervals $[\pi/2, \pi]$, $[\pi, 3\pi/2]$ and $[3\pi/2, 2\pi]$ so a complete graph of one full sine wave is obtained.

APPENDIX I

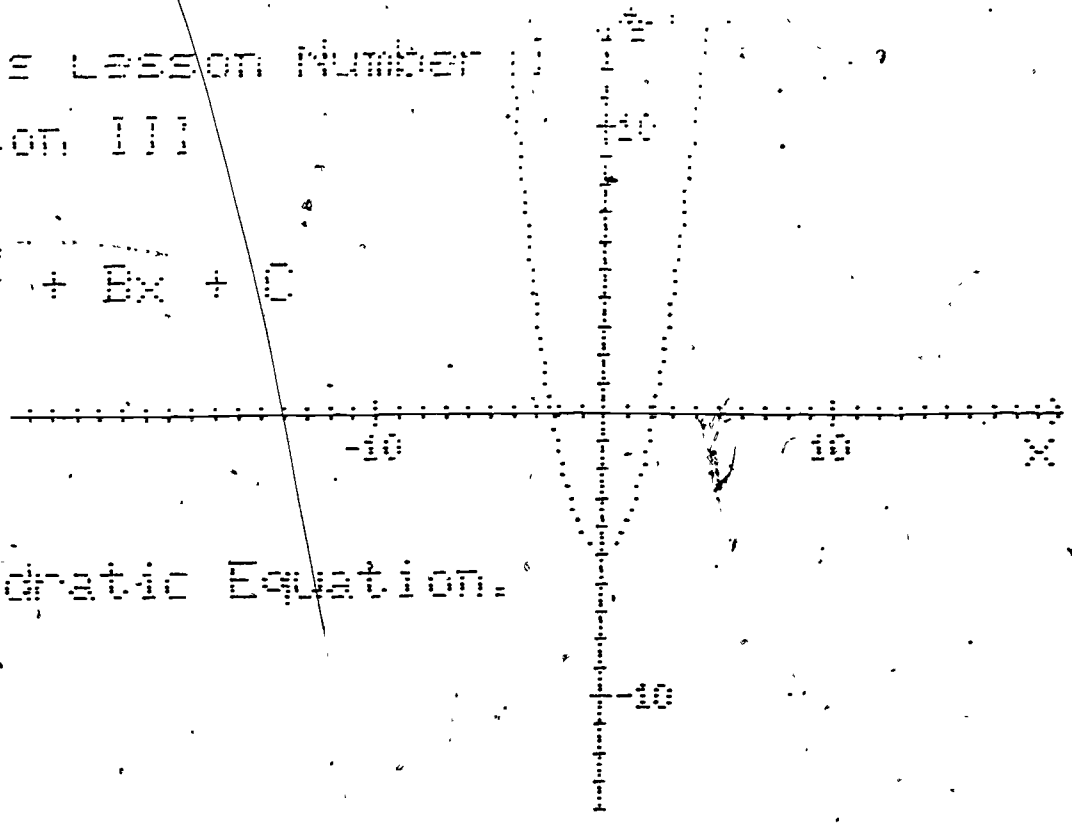
SAMPLES OF INPUT AND OUTPUT

The following pages represent a sample of
the interactions between the student and the computer.
The displays are taken from a sublesson, a quiz and a game.

Graphics Lesson Number 1

Sublesson III

$y = Ax^2 + Bx + C$



The Quadratic Equation.

To continue, press the RETURN key...

Let's see the effect of B.

To do this, let A = 1, B = 1, and C = 0

With these choices of A, B, and C, the equation

$$y = Ax^2 + Bx + C$$

becomes

$$y = x^2 + x$$

When you are ready press the RETURN key

$y = x^2 + x$

You will be given a series of numbers for which you will be expected to calculate answers using the equation which will appear at the top of the screen.

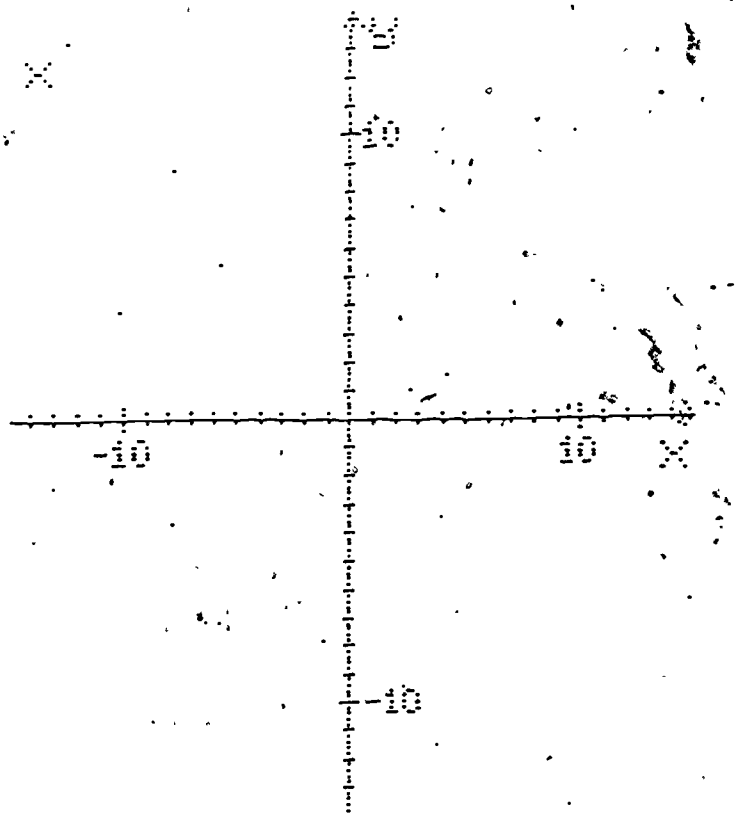
The numbers you will be given are the x-values of the equation. You are to calculate the y-values.

Press the RETURN key to continue...

EQUATION: $y = x^2 + x$

TABLE:

x	y
0	



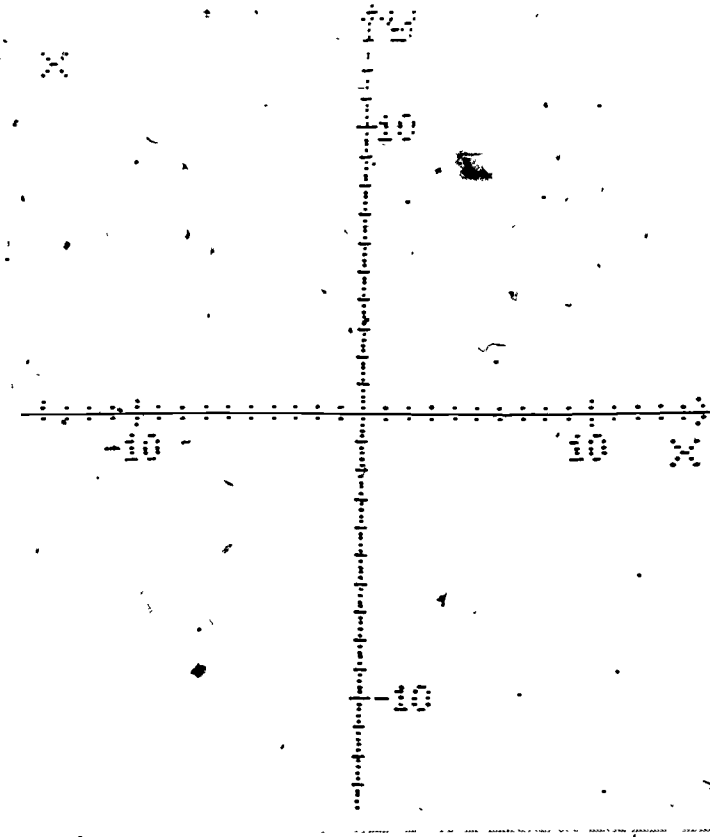
First we'll try x = 0

You calculate the y-value and type it in

EQUATION: $y = x^2 + x$

TABLE:

x	y
0	



To get y all you do is multiply x times x and add x.

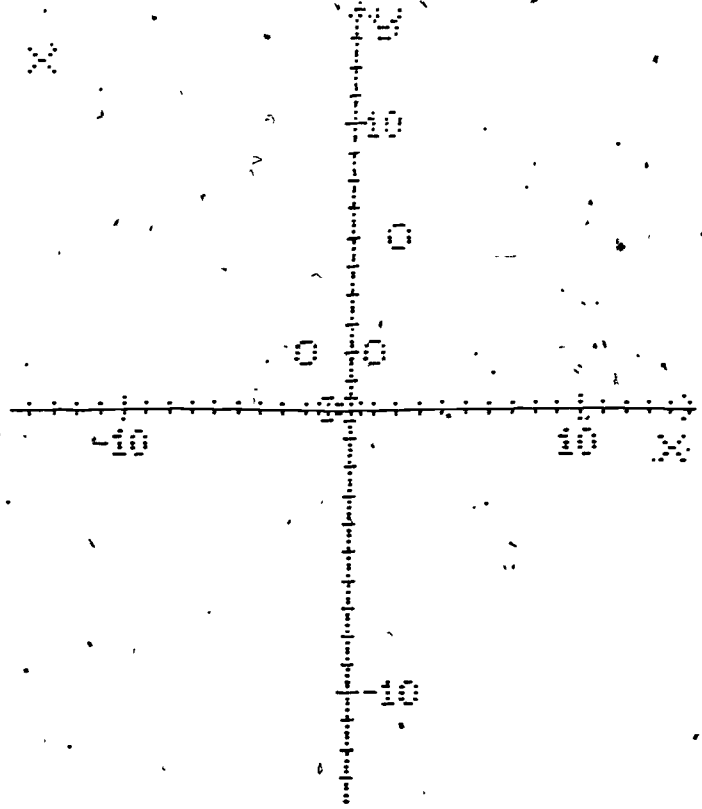
COMMENTARY:

The student has entered an incorrect number. The entry is not recorded in the table. The instructions above are given to aid the student in the calculation. The student is next asked to try again.

EQUATION: $y = x^2 + x$

TABLE:

x	y
0	0
1	2
-1	0
2	6
-2	2
3	



Next point...

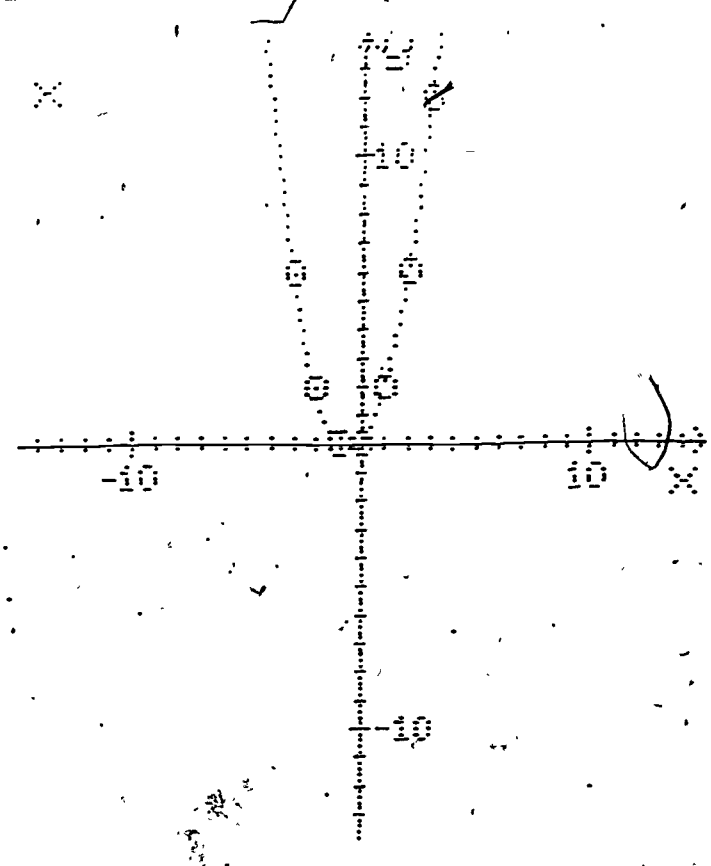
COMMENTARY:

As the correct numbers are entered, the corresponding point is plotted.

EQUATION: $y = x^2 + x$

TABLE:

x	y
0	0
1	2
-1	0
2	6
-2	2
3	12
-3	6

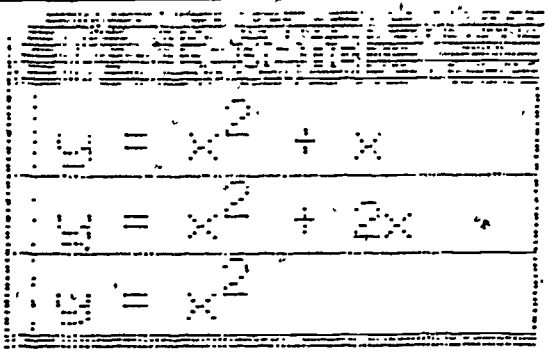


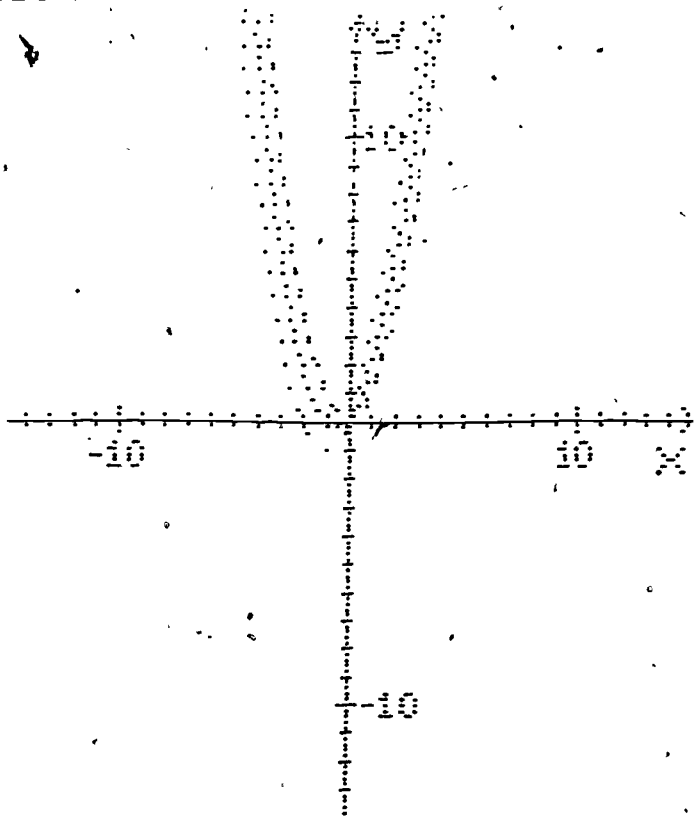
Now, try to draw the graph of $y = x^2 + 2x$

Press the RETURN key to continue...

COMMENTARY:

When the table is complete, the graph is drawn. The student next repeats the process for a choice of B = 2 to give $y = x^2 + 2x$.





Let's see the effect of negative B.

Now, try to draw the graph

of $y = x^2 - x$

Press the RETURN key when you are ready

COMMENTARY:

The curves above are displayed in the order in which they are listed in the legend. A delay occurs before the presentation of the next curve. The student is given ample time to compare the curves and observe the effect of B.

Negative values of B are studied in the second half of the sublesson.

If you think you can predict the effect of B on the graph and are ready for a quiz press Y, otherwise press N.

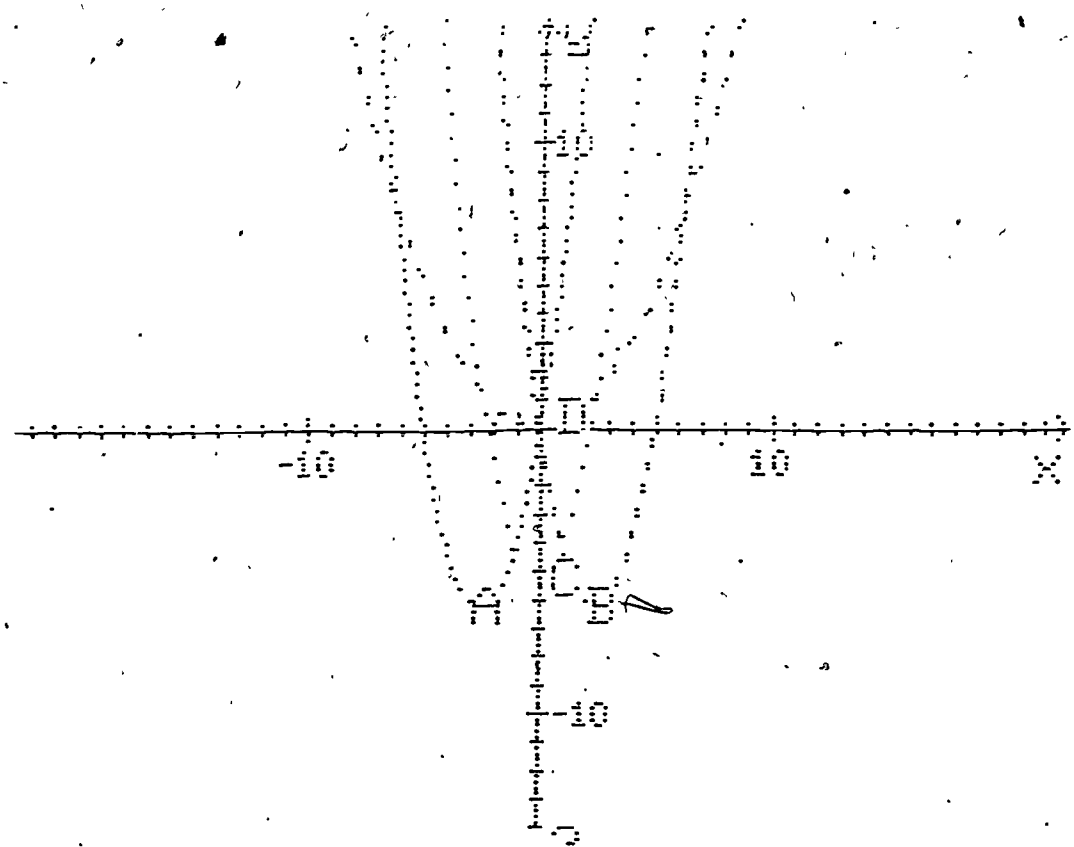
(Y/N) >

COMMENTARY:

A quiz on the effect of the coefficient B is given to complete the lesson. If N is pressed the student is branched back to the beginning of the sublesson.

You will be shown four curves on the same axes. They will be labelled A, B, C, & D. You are to choose which is correct for the equation $y = x^2 - 5x$

To continue, press the RETURN key...



Which is right for $y = x^2 - 5x$,
A, B, C, or D? >

COMMENTARY:

The curves above are displayed in the order of the letters A, B, C, D. A delay occurs before the presentation of each curve.

If an incorrect response is given the student is branched back to the sublesson.

A correct response will end the sublesson.

APPLE II PLUS
APPLE PASCAL version 1.1
Created June 25, 1981
APPLICATION OF COMPUTER GRAPHICS TO
GRAPHING IN ALGEBRA AND TRIGONOMETRY

NSF-NIE Program for the Improvement of
Mathematics Education
Using Information Technology
SED80-12447

January 1, 1981 - June 30, 1982

J. Richard Morris, James Wood,
and Steven Seidel
Mathematical Sciences Department
Virginia Commonwealth University
Richmond, Virginia 23284
(804) 257-1381

To continue, press the SPACE BAR.

THE PARABOLA GAME

If you would like to see instructions describing how to play this game type an I . If you already know how to play, press the SPACE BAR.

This exercise will give you a chance to study the affects the coefficients A, B, and C of the equation $y=Ax^2+Bx+C$ have on the graph of that equation.

The computer is going to pick A, B, and C and draw the graph of the equation, but A, B, and C will not be revealed to you. You are to study the computer's graph and determine the values of A, B, C it picked.

Press the SPACE BAR to continue_

You will be asked for A, B, and C. The computer will draw the graph of your equation so that you can compare it with the graph of the equation the computer picked. You will be allowed to alter your estimates of A, B, and C if your graph doesn't match the computer's graph

For fun, the computer will keep track of the number of attempts you make. The object is to match the computer's graph in the fewest number of attempts.

Press the SPACE BAR to continue

When you are asked to type in a number, type the same symbols you would write on paper and then press the key marked RETURN. If you make a mistake you can erase it by pressing the key with the left-pointing arrow on it.

The computer will accept numbers from -99.9 to 99.9 . For example:

7.6 12 -8.3 -6 are OK, but

137.6 -900 8.317 are not.

If you would like to read the instructions again, type an I . Otherwise, press the SPACE BAR to begin._

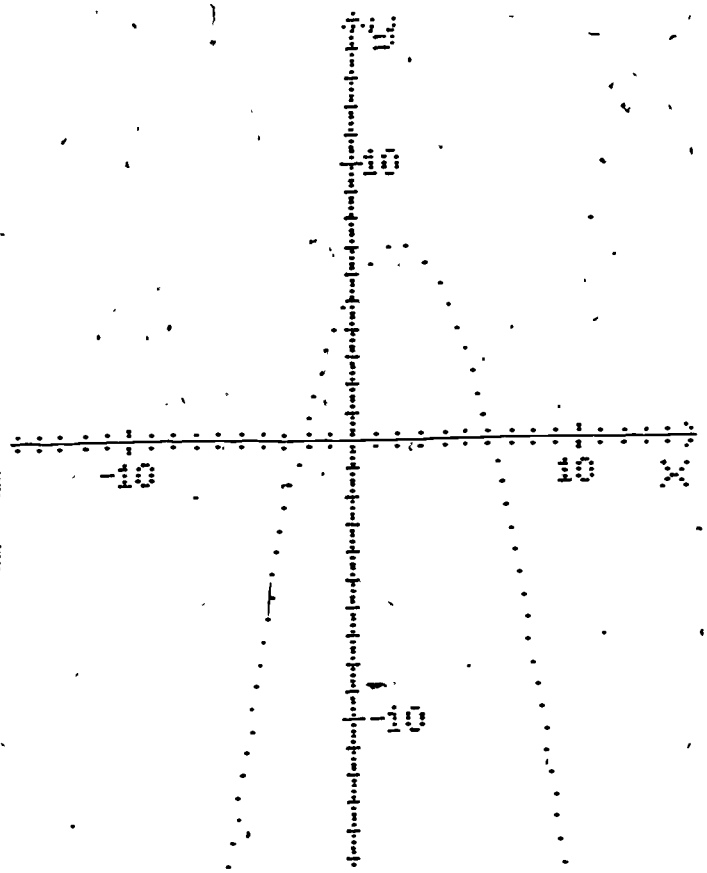
The computer's parabola is ---→
Its equation has the form:

$y = Ax^2 + Bx + C$

You may assume:

$-3.0 \leq A \leq 3.0$

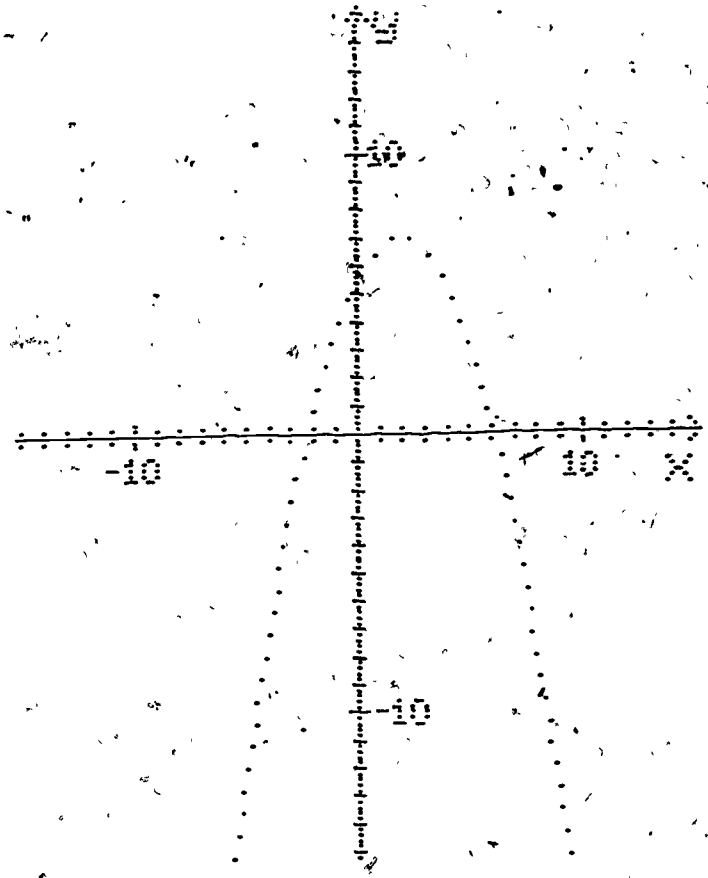
and $-3.0 \leq B \leq 3.0$



Enter your estimates for A, B, and C.
Press RETURN after each number you enter.

A = 0 B = 0 C = 54_

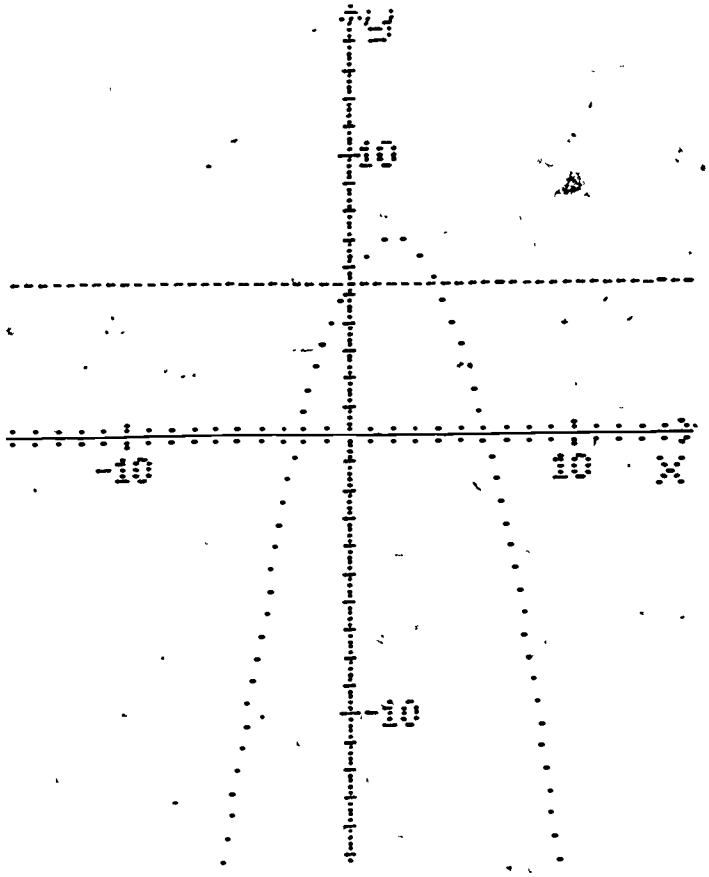
Equation:
y=54



The graph of your
equation is off
the screen. Sorry.
Try making C smaller. You could try C=0.
Press RETURN after each number you enter
C = 5.4

Old equation:
y=54

New equation:
y=5.4



Attempts: 1

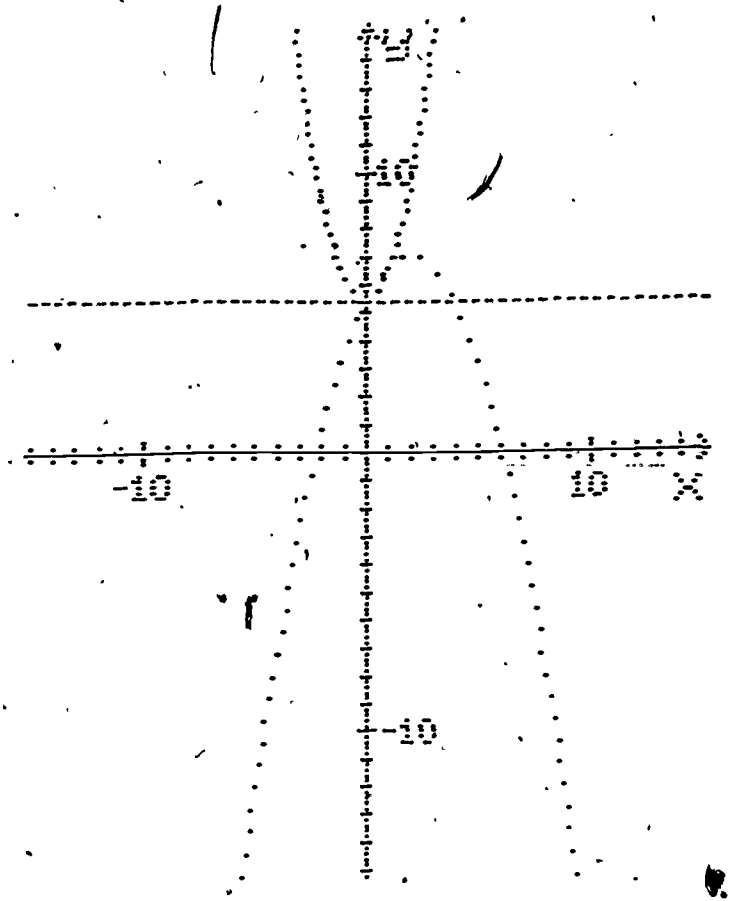
The graph of your equation is a straight line since A = 0.
Pick a value for A other than zero.

A = 1

Old equation:
y=5.4

New equation:
$y=x^2+5.4$

Attempts: 2

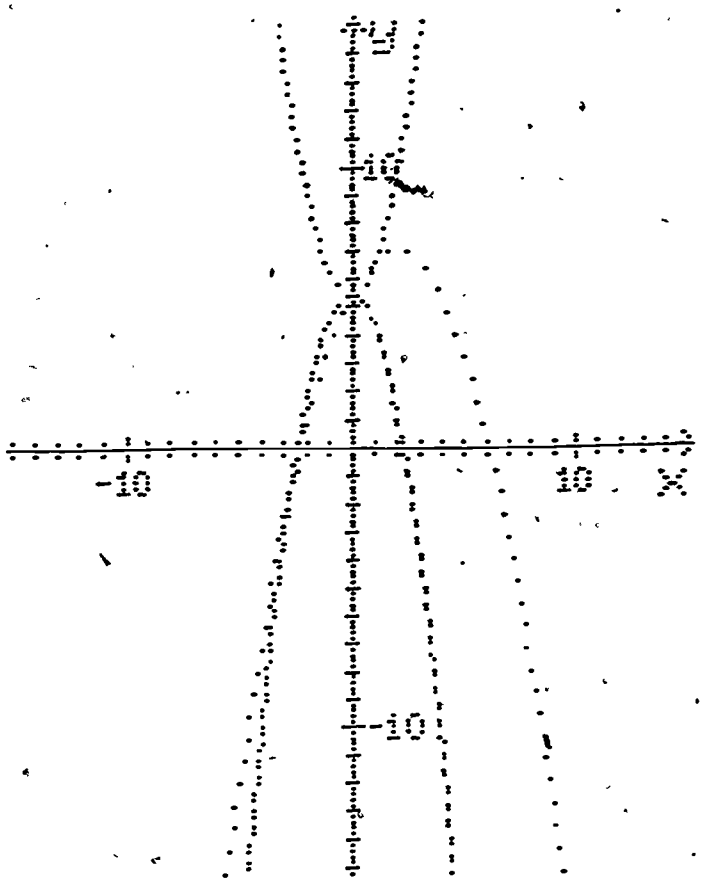


Change A, B, or C? (or Quit?)
(Type A, B, C or Q)
A = -1_

Old equation:
$y=x^2+5.4$

New equation:
$y=-x^2+5.4$

Attempts: 3



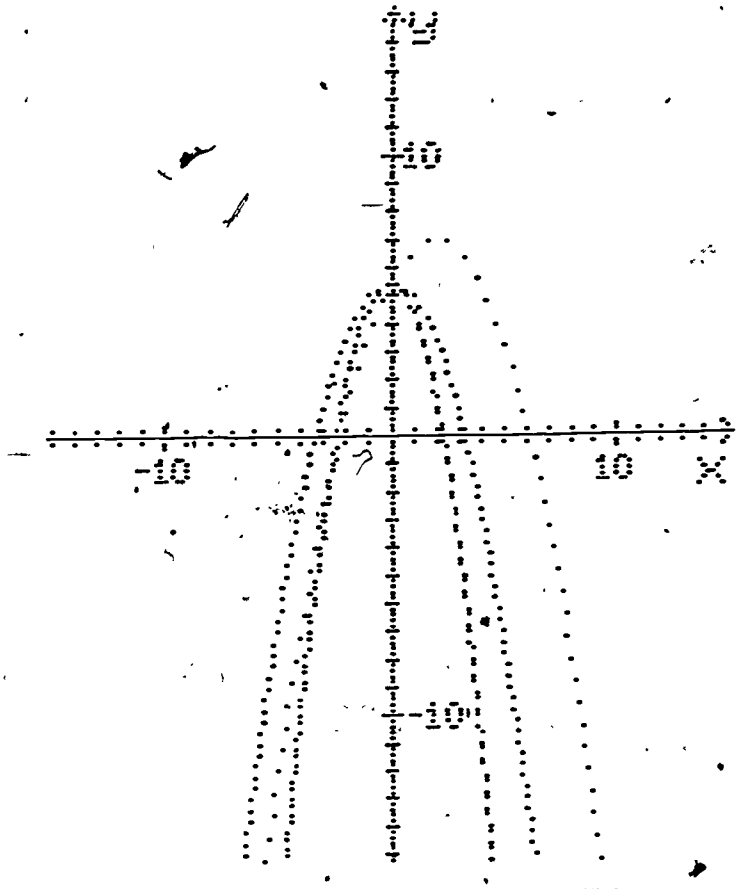
Change A, B, or C? (or Quit?)

A = -0.5_

Old equation:
$y=-x^2+5.4$

New equation:
$y=-.5x^2+5.4$

Attempts: 4



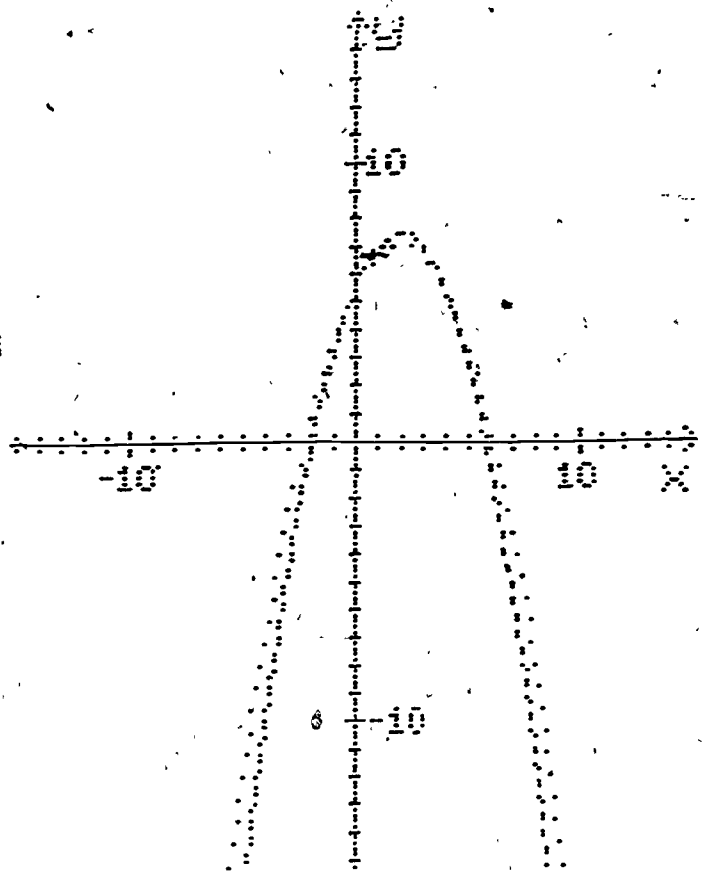
Change A , B , or C ? (or Quit?)

B = 2_

Your final
equation was:
$y=-.5x^2+2x+5.4$

The exact solution
was:
$y=-.4x^2+1.6x+5.4$

Final score: 5



To play again press the SPACE BAR.
If you are all done type an E :_

APPENDIX II

ABSTRACTS OF PAPERS PRESENTED

Scanning on the Fly:
An Approach to the User Interface*

Steven R. Seidel
Mathematical Sciences Department
Virginia Commonwealth University
Richmond, Virginia 23284

ABSTRACT

Educational software must provide a "friendly" user interface in order to prevent student frustration. A recurring source of such frustration is the task of responding to prompts issued by the computer. Scanning on the fly reduces the difficulty of this task. It prevents the possibility of errors of form in student responses by accepting only keystrokes that lead to correctly formatted responses. This approach has the advantage that all text displayed by the computer pertains to the content of the lesson and none to the technical use of the computer. Its implementation is based on the notion of a finite state automaton, yielding reliable and easily modifiable software. Scanning on the fly has been used in the development of microcomputer-based instructional software for college algebra and trigonometry. Applications to other CAI courseware are discussed, students' reactions are summarized, and a Pascal implementation is given.

To appear in Computers and Education

*This work partially supported by NSF-NIE Grant SED 80-12447.

Plotting Curves on Low Resolution Graphics Systems*

Steven R. Seidel
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Virginia Commonwealth University
Richmond, Virginia 23284

ABSTRACT

Standard techniques for plotting a curve on low resolution graphics systems represent the curve as a collection of straight line segments or unevenly spaced dots. Neither of these representations is appealing from a pedagogical standpoint. Both make it difficult for the student to relate his experience on the computer to the material presented in the classroom and in the textbook because neither bears much resemblance to the usual types of drawings made by the instructor and the illustrator. A technique for plotting low resolution curves is presented that overcomes this difficulty by taking advantage of the mind's propensity for filling in missing visual information. Curves are plotted as collections of evenly spaced points or short line segments. The spacing between successive points on each curve is kept constant, to within one pixel, even as the slope of the curve changes. By choosing a spacing between the points of as little as four or five pixels a "natural looking" curve results; the eye is encouraged to fill in the missing information and the underlying limitations of screen resolution are successfully hidden. This plotting technique has been used in the development of microcomputer-based instructional software for college algebra and trigonometry. Applications to CAI courseware are discussed, students' reactions are summarized, and a Pascal implementation is given.

Proceedings of the 21st Southeast Region ACM Conference, April 1983

*This work partially supported by NSF-NIE Grant SED 80-12447.