DOCUMENT RESUME

ED 229 153 PS 013 510

AUTHOR Baroody, Arthur J.; And Others
TITLE The Development of Basic Formal Math Abilities.
PUB DATE Apr 83
NOTE 24p.; Paper presented at the Biennial Meeting of the Society for Research in Child Development (Detroit, MI, April 21-24, 1983).
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Basic Skills; Cognitive Development; *Elementary School Students; *Kindergarten Children; *Mathematics Achievement; *Mathematics Skills; Number Concepts; Primary Education
IDENTIFIERS *Formal Mathematics

ABSTRACT

Developmental patterns and interrelationships of various informal skills, reading and writing of numerals, and a range of base ten/place value concepts/skills were examined in a study involving 78 primary school children from four schools who were individually examined in a standardized interview. A total of 23 tasks were administered and scalogram, individual protocol, and error analyses were performed. Results indicate that the children learned to read and write numerals in a step-like fashion. For example, even though children wrote smaller terms correctly, they wrote larger, unfamiliar terms as they sound (e.g., 20090 for "two-hundred ninety"). Zeros caused many errors. While reading numerals preceded writing numerals for terms to 20, there was no consistent developmental relationship between these skills for larger terms. A "next-by-ten" elaboration of the mental number line appeared to underlie decimal ability. Base ten equivalents and place value appeared to be basic decimal knowledge, while operating with multiples of ten and an appreciation of the structure of the number system appeared to represent a deeper knowledge. Most second graders and even many third graders had only an imprecise appreciation of the repetitive pattern of the number system at the three digit level. (Author/RH)

The Development of Basic Formal Math Abilities

Arthur J. Baroody

Kathleen E. Gannon

Rusti Berent

and

Herbert P. Ginsburg

Graduate School of Education and Human Development

University of Rochester

Rochester, New York 14627

Paper presented at the biennial meeting of the Society for Research in Child Development, Detroit, April 1983.

Abstract

School introduces children to written numerical symbols and base ten/place value (decimal system) concepts—more powerful means of representing numbers than their informal (counting-based) representation. This study examined the developmental patterns and interrelationships of various informal skills, the reading and writing of numerals, and a range of base ten/place value concepts/skills. A total of 78 primary school (K-3) children from four schools were individually examined in a standardized interview. Scalogram, individual protocol, and error analyses were performed. Children learned to read and write numerals in a step-like fashion. For example, even though children wrote smaller terms correctly, they wrote larger, unfamiliar terms as they sound (e.g., 20090 for "two-hundred ninety"). Zeros caused many errors. While reading preceded writing numerals for terms to 20, there was no consistent developmental relationship between these skills for larger terms. A next-to-ten elaboration of the mental number line appeared to underlie decimal ability. Base ten equivalents and place value appeared to be basic decimal knowledge, while operating with multiples of ten and an appreciation of the structure of the number system appeared to represent a deeper knowledge. Most second graders and even many third graders had only an imprecise appreciation of the repetitive pattern of the number system at the three digit level.

The Development of Basic Formal Math Abilities

Even before school, children have informal (counting-based) means of representing number. They learn to count orally, and this provides the basis for a mental number line that allows them to mentally compare numbers and do basic addition and subtraction (e.g., Baroody, in press; Gelman & Gallistel, 1978; Ginsburg, 1982; Resnick, 1983; Starkey & Gelman, 1982). In school, these informal means of representation are supplemented with a formal and more powerful tool for representing numbers: a numeration system. School introduces children to reading and writing numerals and to computational algorithms. When larger values are involved, the use of written numerical symbols and computational procedures reduces memory demands and increases computing efficiency. School also introduces base ten/place value (decimal system) concepts, which provide the rationale for many formal procedures (e.g., writing numerals with more than one digit and renaming algorithms) (Resnick, 1983). Thinking in terms of ones, tens, hundreds, etc. also gives children flexibility in dealing with a wide range of mathematical tasks such as comparing and ordering larger numbers, addition and subtraction with larger values, and estimating (Payne & Rathmell, 1975).

Children learn formal skills and concepts in terms of their previous experience—in many cases in terms of their existing informal knowledge. Learning to read or write with the alphabet depends on the child's existing oral language. Similarly, learning to read and write numerals depends upon knowing the oral count sequence. Evidence from case studies (e.g., Ginsburg, 1982) indicates that primary age children initially read composite numerals such as 53 as separate numbers and write such numbers as they sound (e.g., 503 for "fifty-three"). Later they read and write numerals correctly but without appreciating their base ten/place value meanings. Only gradually do children appear to appreciate that position determines the value of a digit. Relying on the little research that exists (such as case study work), Resnick (1983) identifies three main developments in decimal knowledge. Initially, the child realizes that numerals can be partitioned into units and tens (e.g., 43 is 4 tens and 3 ones). The child then recognizes that multiple partitions are possible (e.g., 43 is also 3 tens and 13 ones) and finally that base ten partitioning processes underlie renaming algorithms.

In brief, there has been little systematic research on the development of basic formal abilities of children. Much of the work done on the topic has used the case study method and been exploratory in nature. This study, then, used a standardized interview format to examine the developmental precurors and patterns of basic formal math skills in primary school children. We were particularly interested in the relationship between informal and formal knowledge, the development of reading and writing numerals, and the development of base ten/place value concepts and related skills.

Method

Subjects

A total of 78 primary school children (25 kindergarteners from 4 - 5 to 6 - 4 years, 18 first graders from 6 - 6 to 8 - 3, 14 second graders from 7 - 5 to 8 - 11 and 21 third graders from 8 - 5 to 9 - 11) from four suburban schools participated in the study. Cooperating teachers were requested to choose children of high, average and low math ability so that their sample would represent the range of ability found in their classroom.

Procedure

The subjects were examined individually in a structured, standardized interview. The interviews were conducted in roughly equal numbers by the four authors.

As part of a larger test development project (Ginsburg & Baroody, 1983), which examined informal (I) and basic formal (F) abilities, the following 23 tasks were administered: two count by ten tasks (I), two mental number line tasks (I), two mental addition tasks (I), five reading numerals tasks (F), five writing numerals tasks (F), two place value tasks (F), two base ten equivalents tasks (F), a structure of the decimal system task (F), an adding ten/multiple of ten task (F), and subtracting by ten/multiples of ten task (F). The tasks were presented in the order delineated above. Within a task domain such as writing numerals, tasks were arranged in order of difficulty (as determined by rational analysis and pilot testing). An effort was made to find each subject's ceiling of competence within a domain and to proceed just beyond it. Testing was halted if the subject was unwilling to continue, if the child was obviously uncomfortable about continuing, or if previous responses indicated that there was clearly no chance of success.

Count by tens. Subjects' ability to count by tens (1) to one hundred and (2) from one hundred to two hundred was gauged. The subjects were instructed, "Now count by tens for me (starting with 100)." If necessary, the child was prompted with, for example, "Count by tens like this, 10, 20, 30...you keep going." If the child had to resort to counting the intervening numbers such as 10, 11, 12...19, 20, 21, 22, 23...29, 30, etc. or could not produce the count (in the correct sequence), he or she was scored as unsuccessful on the task.

Mental number line. Two tasks were used to evaluate a subject's ability to judge relative distances on a mental number line—one involving two digit terms and one involving three and four digit terms. The child was first presented a sample problem: "Which is closer to 6, 5 or 9?" The interviewer pointed, in turn, to each numeral (about 1 cm in height) printed on a 5 x 8 card. The numerals were arranged in the configuration of an isoceles triangle with the target (in the case of the sample: 6) at the apex, about 4 cm from each choice, which were about 5 cm apart. If the child responded correctly to the sample question, the interviewer commented, "That's right, 5 is closer; it's only one away from 6; 9 is 3 away from 6. If the child was wrong, the interviewer corrected with: "No, 5 is closer...." The interviewer then proceeded with the tasks: "Here is a X. Which is closer, Y or Z? The two digit task included the following comparisons: 32 and 24 vs. 62, 84 and 51 vs. 96, 48 and 23 vs. 54, 65 and 49 vs. 99, and 71 and 49 vs. 84. The larger mental number line task included the trials: 200 and 99 vs. 400, 5000 and 1000 vs. 8000, 700 and 300 vs. 900, 5000 and 2000 vs. 9000, and 3500 and 2000 vs. 7000. For each task, a subject was scored as successful if he or she got at least four of the five trials correct.

Mental Addition. Both mental addition tasks involved two digit terms; the first did not entail carrying; the second might have entailed carrying. The interviewer explained, "Now I am going to give you some adding problems to do in your head, like how much is 3 apples and 1 apple?" After the child computed the sample problem and feedback was given, the interviewer continued with: "Now try to get the right answer each time. You can figure it out any way you want to." For both tasks, the subject was asked, "How much are X apples and Y apples?" The trials for the no carrying task were 20 and 15, 14 and 13, 16 and 12. The trials for the carrying tasks were 29 and 12, 46 and 25, 28 and 17. A subject was scored as successful on each task if he or she was correct on at least two of the three trials.

Reading numerals. Five tasks were used to measure children's ability to read numerals: one digit numerals (2, 4, 7), teen numerals (10, 13, 16), two digit numerals (28, 47, 90), three digit numerals (105, 162, 280), and four digit numerals (1,002; 4,073; 2,301). Each numeral (1.4 cm high) was presented separately on a stimulus card. The experimenter said, "What number is this?" If necessary, the experimenter followed up with: "Read this number for me." Each trial was scored as either correct or incorrect. A subject was scored as successful on a task if he or she was correct on all three trials. Reading errors were also recorded.

Writing numerals. As with reading numerals, five tasks were used to evaluate children's ability to write numerals: one digit terms (two & six), teen terms (twelve & fifteen), two digit terms (twenty three & ninety seven), three digit terms (one hundred two & two hundred ninety), and four digit terms (one thousand ninety five & one thousand four hundred six). The experimenter instructed, "I'm going to tell you a number, and I'd like you to write it down on the answer sheet." The experimenter then read the numeral for trial 1, gave the child an opportunity to write his or her response on the answer sheet, correct the response if he or she wished to, and repeated the process with trial 2. The child was scored as successful if he or she wrote the numerals correctly on both trials. On the one digit and teen numerals tasks, a child was given credit even if the numerals were written in reverse (e.g., 1S for "twelve").

Place value tasks. The place value of digits in terms of (1) ones and tens and (2) hundreds and thousands was tested. In the first task, a child was shown a stimulus card on which was written: (a) 46, (b) 30, (c) 105. There were four trials, administered as described below.

(1.1) "Look at the number in example a. How many tens does that show?" (If necessary: "How many tens does that (pointing to the 4 in 46) stand for?")

(1.2) "Look at the number in example b. How many ones does that show?" (If necessary: "How many ones does that (pointing to the 0 in 30) stand for?")

(1.3) "Look at the number in example c. How many ones does that show?" (If necessary: "How many ones does that (pointing to the 5 in 105) stand for?")

(1.4) "Look at the number in example c. How many tens does that show?" (If necessary: "How many tens does that (pointing to the 0 in 105) stand for?")

In the second task, a child was shown a stimulus card on which was printed: (a) 873, (b) 2560, (c) 1049. A description of the four trials is delineated below.

(2.1) "Look at the number in example a. How many hundreds does that show?" (If necessary: "How many hundreds does that (pointing to the 8 in 873) stand for?")

(2.2) "Look at the number in example b. How many thousands does that show? (If necessary: How many thousands does that (pointing to the 2 in 2560) stand for?")

(2.3) "Look at the number in example c. How many hundreds does that show?" (If necessary: "How many hundreds does that (pointing to 0 in 1,049) stand for?")

(2.4) "Look at the number in example a. How many thousands does that show?" (If necessary: "How many thousands does that (pointing to the space before 873) stand for?")

On each task, a subject was scored as successful if he or she was correct on all four trials of the task.

Base ten equivalents. An appreciation of how many tens constituted a hundred and how many hundreds constituted a thousand were gauged in two separate tasks. In the first task, a child was shown a stimulus on which there was a play $10 bill and a play $100 bill. The experimenter commented: "In the picture here is a $100 bill (pointed to the bill). The $100 bill is worth how many $10 bills?" (If necessary: "If you traded the $100 bill in at a bank, how many $10 bills would you get?") In the second task, the stimulus contained a $100 bill and a $1000 bill, but the procedure paralleled that for task 1. For each task, if the subject knew the equivalence without computing, the child was scored as successful.

Structure of the decimal system. In this task children were asked to indicate what the smallest and largest one, two and three digit numbrs are. The experiment presented a stimulus containing the numerals 3, 24 and 578 and gave the following instructions. "Here are some written numbers. Three (pointed to the numeral 3) is a one digit number because when we write it, we need only one number. Written here is twenty four (points to the numeral 24). Twenty four is a two digit number because when we write it, we need two numbers: Written here is five hundred seventy eight (pointed to the numeral 578). Five hundred seventy eight is a three digit number, because we have to write three numbers for it. Write the answers to the following questions in the space provided on your answer sheet (pointed to the space in the lower right hand corner of the answer sheet). Now what is the smallest one digit number?" (If further clarification was needed, the experimenter explained, "Put here (pointed to the appropriate blank on the answer sheet) the smallest number that needs one number when we write it." When needed a parallel follow up question was used on the remaining trials.) After a child wrote down his or her response the following trials were given: "What is the largest two digit number?" "What is the smallest two digit number?" "What is the smallest three digit number?" "What is the largest three digit number?" For the first trial, an answer of one or zero was considered correct. A correct response to all trials constituted success on the task.

Adding ten and multiples of ten. The experimenter explained, "Here are some questions about adding money. We'll pretend that you have some money, and I'll pretend to give you some more. For example, if you start with ten dollars and I give you a ten dollar bill, how much do you have altogether?" After the sample trial, the child was given the experimental trials. In each case, the experimenter asked: "If you start with $_____ and add _____ ten dollar bill(s), what do you end up with?" The child was shown a stimulus card which specified the starting amount. The five trials are delineated below:

(a) $9 + one ten;

(b) $6 + three tens;

(c) $4 + three tens;

(d) $2 + ten tens; and

(e) $7 + zero tens.

A trial was scored as correct if the child responded (within about 5 seconds). A trial was scored as incorrect if the child had to count or go through laborious calculation. Success on the task was defined as 4 or 5 correct trials.

Subtracting ten and multiples of ten. The experimenter instructed, "Let's pretend that you are in a store. You have some money, you pay for something, and you want to find out how much you have left. For example, you have ten dollars, and you give the store clerk a ten dollar bill for a gift you buy. How much would you have left after paying the clerk?" After the example, the child was given the experiment trials. In each instance, the experimenter inquired, "If you start with $____ and take away ____ ten dollar bill(s), what do you end up with?" Accompanying the instructions for each trial was a stimulus card which specified the starting amount. The five trials were:

(a) $18 - one ten;

(b) $35 - two tens;

(c) $42 - one ten;

(d) $67 - six tens; and

(e) $113 - ten tens.

A trial was correct if the child responded quickly (within about 5 seconds). A trial was scored as incorrect if the subject had to count or go through laborious calculation. Success on the task was defined as 4 or 5 correct trials.

Results and Discussion

Two separate analyses were undertaken. One examined the development of reading and writing numerals; the second examined base ten/place value concepts and related skills.

Reading and Writing Numerals

As with other mathematical skills, (e.g., Gelman & Gallistel, 1978; Ginsburg, 1982), children learn to read and write numerals in a step-like fashion and interpret the unfamiliar in terms of their existing knowledge (see Table 1 and Figure 1). There were clear levels of ability by grade level. Kindergarteners could generally read one digit and teen numerals but not larger terms. First graders could read up to two digit perfectly but had difficulty with larger values. Second graders could generally manage to read three digit terms but often ran into difficulty with four digit terms. The third graders performed nearly perfectly on the reading numerals tasks.

Insert Table 1 and Figure 1 about here

Analyses of protocols revealed that, when children were presented with an unfamiliar numeral to read, random errors were infrequent. Error analysis revealed an age-related pattern in their systematic mistakes. Kindergarteners most often responded by substituting a term they knew (e.g., "seventeen" for 47) or made up novel number combinations (e.g., "ten-fifty" for 105 or "sixteen-twenty" for 162). Both errors are sensible in that the substituted or made up term in some way resembled the unfamiliar written numeral. First and second graders usually read terms as they were written (e.g., "one hundred two" for 1,002) or reduced large terms to smaller familiar terms (e.g., "four hundred seventy three" for 4073).

Reading terms containing zeros—especially "embedded zeros" such as 105 or 2,301 were particularly difficult for many subjects. Zero was often simply ignored (as in "four hundred seventy three" for 4073) or treated as an undifferentiated part of a larger unit (as in "ten-fifty" for 105). Such errors are often attributed to children's inadequate conception of zero as a placeholder in the base ten notation system (e.g., Kamii, 1981). Historically, zero may have been used as a placeholder (to represent an empty column in counting-boards) before it became a symbol for "nothing" or an "empty set" (Dantzig, 1967). However, children's initial and informal meaning of zero is "nothing." Sometimes children even interpret zero as meaning "nothing to notice"—as something that can be ignored (Ginsburg, 1982; Kamii, 1981). Moreover, children initially treat 10 and other decades (20, 30, etc.) as undifferentiated wholes rather than as composites of ten(s) and no (zero) ones (cf. Resnick, 1983). Only gradually does zero take on a second (formal) meaning as a place holder.

However, a formal understanding of zero as a placeholder does not assure that a child will correctly read large, unfamiliar terms containing zero. For instance, 12 second graders could correctly read numerals up to three digits including those with zeros, but six of these children could not correctly read all four digit terms. In such cases, zero errors (e.g., reading 1002, as "102," 4073 as 473) may simply be attempts simply to interpret the unfamiliar in terms of the familiar rather than the result of a misconception about the (formal) role of zero as a placeholder. For instance, one girl correctly read all terms but 2,301 to which she responded: "Two and thirty one—I don't know." While she read the composite numeral as separate numbers, she otherwise appeared to appreciate the role of zero as placeholder. Moreover, she was successful on basic base ten/place value concepts (base ten equivalents and place value for three and four digit terms). Her reading error then did not necessarily reflect a fundamental misconception about the role of zero as placeholder.

Apparently, learning to write numerals is a step-like process (see Table 1). Moreover, protocol analyses showed that while younger children (K-2) wrote smaller terms correctly, they often wrote larger, unfamiliar numbers as they are spoken. Kindergarteners were usually correct when writing one digit numerals but tended to write teen numerals in reverse (51 for "fifteen"). First graders were quite accurate with one and two digit numerals but wrote three digit numbers such as "one hundred two" and "two hundred ninety" with extra zeros (1002 and 20090, respectively). Second graders, for the most part, could accurately write numerals up to three digits, but nearly half had trouble writing four digit numerals. The most frequent error was again including extra zeros so that the number was written as it was heard. Basically, third graders were competent writing numerals including four digit terms.

Kamii (1981) argues that writing numbers as they are heard makes sense in light of the fact that young children are simultaneously mastering the written alphabetic system, which does tend to correspond to spoken language. In effect, young children may try to "spell out" "fifteen" as 51 and "one hundred two" as 1002, because they assume that the principles which underlie written numeration and alphabetic systems correspond. Kamii concludes that, in order to write numerals correctly, children must learn to recognize the similarities and differences in the written numeration and alphabetic systems as well as learn the correspondences and exceptions in spoken and written numbers (see Figure 2).

By contrast, the data of this study suggest that only learning the correspondences and exceptions in spoken and written numbers is necessary for improvements in numeral writing ability. It appears that, as they are required to write two, three, and then four digit terms, children must learn the spoken-written correspondence-exceptions for each level. That is, children generally do not seem to transfer the writing rules learned at one level to the next level. Left unresolved by this research is the question when or if children do make such transfer.

It is clear that, even after mastering writing numerals at lower levels, children tend to spell out unfamiliar, higher level terms. This evidence suggest that learning to write numerals does not depend upon children's ability to recognize the similarities and differences in the written numeration and alphabetic systems. Either children do learn to write numerals without differentiating between the two systems (as evidenced by spelling out errors) or differentiating between the two systems does not pose a problem to primary age children (numeral writing ability is a distinct domain). If the latter is true, then spelling out errors do not reflect young children's failure to differentiate written number and alphabetic systems but are simply attempts to deal with the unfamiliar in a sensible way.

Children who know, for example, how to write two digit terms but add extra zeros to three digit terms may do so because they fail to recognize the basic place value pattern of written numeral system and/or because they do not realize that "hundreds" represent the next higher level after two digit terms. Basically, children have to learn (a) that each successively larger level is indicated by writing <u>one</u> additional digit and (b) that the spoken unit terms ("one" to "nine"), two digit terms ("eleven" to "ninety nine"), hundred terms ("one hundred" to "nine hundred ninety nine"), thousand terms (one thousand to nine thousand nine hundred ninety nine"), etc. <u>each</u> represent successive levels. Without this prerequisite knowledge, children may resort to treating a term such as "one hundred two" in terms of two familiar entities (100 and 2) and thus yielding the common spell-as-heard error (1002).

Predictably, children learn to read one digit or teen numerals before they can write such numerals—even when writing reversals are scored as acceptable. With larger numerals, however, the developmental relationship between reading and writing is less clear. Initially, writing numerals is more difficult than reading numerals because a new set of rules must be learned. To read numerals, the child must appreciate their distinctive features and the part-whole relationships of the features (cf. Gibson & Levin, 1975). To write numerals, the child must <u>also</u> have a plan of execution—a set of rules for translating this (implicit) knowledge into motor actions (cf. Kirk, 1981). Once the child has learned a plan of action for writing each of the single digit numerals and the execution of these rules become fairly automatic, writing larger numerals is not necessarily more difficult than reading larger numerals. Indeed, different production strategies are required by the two tasks. Reading numerals requires recalling from long term memory the names of the decades. Production of the decades is a difficult task for many kindergarten children, (only 56% could count by tens). On the other hand, writing two digit numerals required recognizing of the spoken decade or a portion of the term (e.g., "nine" in "ninety-seven") and recalling the corresponding written symbol.

Base Ten/Place Value and Related Abilities

Consistent with LD case study work (Baroody, 1983), counting by tens (to 100 and over) appears to be an informal skill which develops prior to and may facilitate processing problems involving two digit terms or greater and appreciating the structure of the base ten/place value system (see Figure 3). As expected, a mental number line for two digit terms typically preceded base ten notions (cf. Resnick, 1983). A mental number line for three and four digit terms also appeared before most comparable base ten notions. This suggests that while base ten representation may be essential to estimation by rounding (to the nearest ten, hundred, etc. and then adding, subtracting, etc.), it may not be necessary for estimation via a front-end approach (i.e., using the left most digits to compute an approximate answer) (cf. Trafton, 1978). An appreciation of basic equivalents (e.g., the number of tens in 100) appears to be a basic base ten/place value notion. Apparently learning the names of the one's, ten's place, etc. and place value (e.g., that 42 represents four tens and two ones) represent relatively superficial knowledge (cf. Resnick, 1983). An appreciation of the effects of adding or subtracting ten and multiples of ten and the structure of the number system (e.g., knowing or deducing that the smallest and largest two- and three-digit numbers are 10, 99, 100, 999, respectively) seems to indicate a relatively deep understanding of the base ten/place value system. Only 21% of the second-graders and 76% of the third-graders were correct on the latter task. However, most of their errors involved questions at the three digit level and the "sensible" responses 199 or 900. Thus even these children had some—though an imprecise—appreciation of the repetitive pattern of the number system. Finally, mental addition accuracy involving two digit addends (e.g., 14 + 13, 29 + 12 or 46 + 25) typically lagged behind the various base ten/place value skills. This evidence is consistent with the views that the earliest stages of decimal knowledge are a "next-by-ten" elaboration of the basic number line representation (Resnick, 1983) and that mental addition depends on (at least implicit) base ten represenation (e.g., Ginsburg, Posner, & Russell, 1981).

Insert Figure 3 about here

References

Baroody, A.J. The case of Adam: A specific evaluation of a math learning disability. Paper presented at the annual meeting of the American Educational Research Association, Montreal, April 1983.

Baroody, A.J. The case of Felicia: A young child's strategies for reducing memory demands during mental addition. Cognition and Instruction, in press.

Dantzig, T. Number—The language of science. New York: Free Press, 1967.

Gelman, R., & Gallistel, C.R. The child's understanding of number. Cambridge, MA: Harvard University Press, 1978.

Gibson, E.J., & Levin, H. The psychology of reading. Cambridge, MA: The MIT Press, 1975.

Ginsburg, H.P. Children's arithmetic. Austin, TX: PRO-ED, 1982.

Ginsburg, H.P., & Baroody, A.J. TEMA— Test of early mathematics ability. Austin, TX: PRO-ED, 1983.

Ginsburg, H.P., Posner, J.K., & Russell, R.L. The development of mental addition as a function of schooling and culture. Journal of Cross-cultural Psychology, 1981, 12, 163-178.

Kamii, M. Children's ideas about written number. Topics in learning & learning disabilities, 1981, 1 (3), 47-59.

Kirk, U. Learning to copy letters: A cognitive rule-governed task. The Elementary School Journal, 1981, 81(1), 29-33.

Payne, J.N., & Rathmell, E.C. Number and numeration. In J.N. Payne (Ed.), Mathematics learning in early childhood (37th Yearbook of the National Council of Teachers of Mathematics). Reston, VA: NCTM, 1975.

Resnick, L.B. A developmental theory of number understanding. In H.P. Ginsburg (Ed.), The development of mathematical thinking. New York: Academic Press, 1983.

Starkey, P., & Gelman, R. The development of addition and subtraction abilities prior to formal schooling in arithmetic. In T.P. Carpenter, J.M. Moser, and T.A. Romberg (Eds.), Addition and subtraction: A cognitive perspective. Hillsdale, NJ: Lawrence Erlbaum Associates, 1982.

Trafton, P.R. Estimation and mental arithmetic: Important components of computation. In M.N. Suydam and R.E. Reys (Eds.), Developing computational skills. (40th Yearbook of the National Council of Teachers of Mathematics). Reston, VA: NCTM, 1978.

Table 1

Proportion of Subjects Successful on Each Task by Grade Level

	Grade Level			
	K	1	2	3
Task	(N = 25)	(N = 18)	(N = 14)	(N = 21)
Count by tens to 100	.56	.94	1.00	1.00
Count by tens to 200	.04	.56	.93	1.00
Mental number line--two digits	.12	.78	.93	1.00
Mental number line--three & four digits	.00	.33	.43	.86
Mental addition--no carrying	.00	.17	.43	.81
Mental addition--carrying	.00	.06	.07	.24
Reading one digit numerals	.92	1.00	1.00	1.00
Reading teen numerals	.72	1.00	1.00	1.00
Reading two digit numerals	.36	1.00	1.00	1.00
Reading three digit numerals	.04	.39	.86	1.00
Reading four digit numerals	.00	.11	.43	.95
Writing one digit numerals	.84	1.00	1.00	1.00
Writing teen numerals	.44	1.00	1.00	1.00
Writing two digit numerals	.52	1.00	1.00	1.00
Writing three digit numerals	.04	.28	.93	1.00
Writing four digit numerals	.00	.06	.57	1.00
Place value--ones & tens	.00	.06	.71	1.00
Place value--hundreds & thousands	.00	.00	.57	.95
Base ten equivalent: tens in 100	.00	.44	.79	.95
Base ten equivalent: hundres in 1000	.00	.33	.57	.81
Structure of decimal system	.00	.11	.21	.76
Adding ten & multiples of ten	.00	.17	.64	.76
Subtracting ten & multiples of ten	.00	.11	.57	.71

Figure Captions

Figure 1. Guttman scale results for reading and writing numerals.

Figure 2. Kamii's (1981) model of the relationships within and among written and spoken systems.

Figure 3. Guttman scale results for base ten/place value and related tasks.

Figure 1

Guttman Scale Results for Reading and Writing Numerals Tasks

Most difficult	Read 4 digits
	Write 4 digits
	Write 3 digits
	Read 3 digits
	Read 2 digits
	Write teens
	Write 2 digits
	Read teens
	Write 1 digit
Least difficult	Read 1 digit

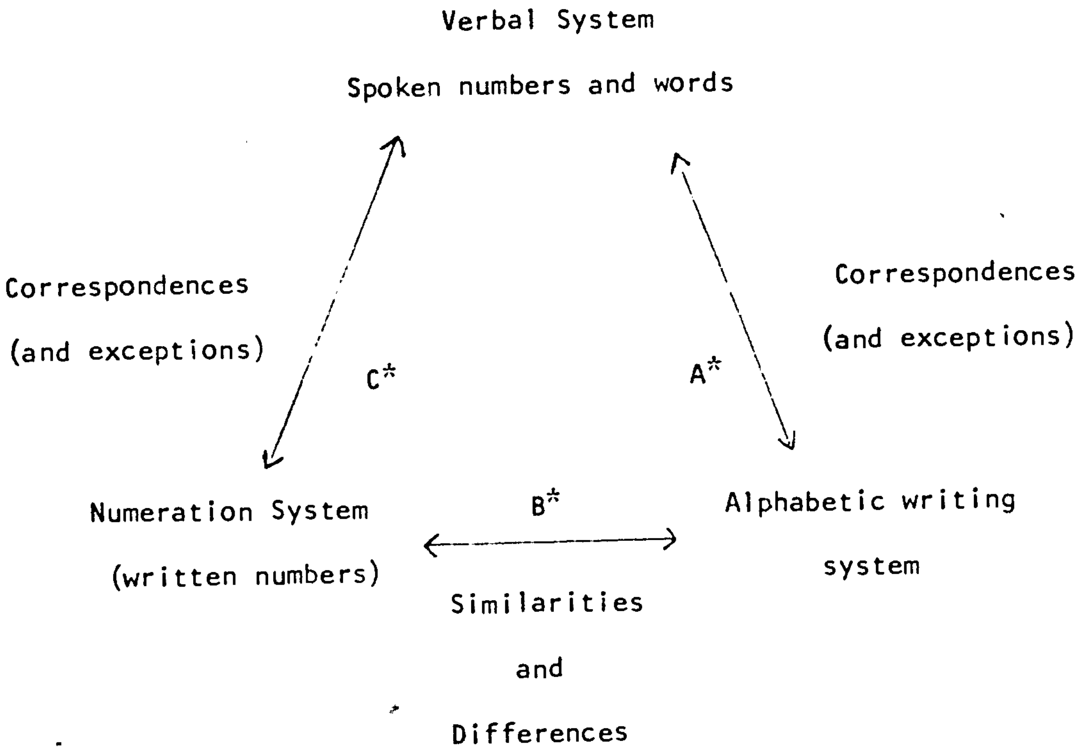
Coefficient of reproducibility = .97

Coefficient of scalability = .86

Figure 2

Kamii's Model of the Relationships

Within and Among Written and Spoken Systems



* What must be mastered in order to write numerals correctly

Figure 3

Guttman Scale Results for Base Ten and Related Tasks

Scale 1

Most Difficult	Mental addition of 2-digit terms with carrying
	Smallest and largest 1-, 2-, and 3-digit numbers (structure of base ten system)
	Mental addition of 2-digit terms with no carrying
	Adding ten and multiples of ten (decimal skill)
	Place value: one's and ten's place
	Tens in 100? (base ten equivalent)
Least Difficult	Count by tens to 100

Coefficient of reproducibility = .93

Coefficient of scalability = .78

Scale 2

Most Difficult	Smallest and largest 1-. 2-, 3-digit numbers (structure of base ten system)
	Subtracting by ten and multiples of ten (decimal skill)
	Place value: hundreds and thousands place
	Adding by ten and multiples of ten (decimal skill)
	Mental number line #2 (hundred and thousands)
	Hundreds in 1000? (base ten equivalent)
	Place value: one's and tens place
	Tens in 100? (base ten equivalent)
	Count by tens over 100
	Mental number line #1 (2-digit terms)
Least Difficult	Count by tens to 100

Coefficient of reproducibility = .91

Coefficient of scalability = .76