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ABSTRACT

Addition strategies used by 36 kindergarten children were examined. Children were given written stimuli (such as "2+5" and "3+7") during two sessions taking place a week apart. Results indicated that once children came to rely on mental addition strategies, they often quickly invented more economical procedures to compute sums. Also confirmed was the hypothesis that a specific mental addition strategy recently discovered in a case study (counting-all starting with the larger addend) was not an uncommon labor-saving device among young children. On the other hand, the strategy of counting-on from the first addend was found to be relatively rare as a result of its being cognitively less economical than either counting-all starting with the larger addend or counting-on from the larger addend. A double count model of mental addition explained the above results. Finally, several devices that may help children make the transition from counting-all to counting-on were observed. (Author/MP)

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The Use of Economical Mental Addition Strategies by Young Children

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Abstract

This study examined the addition strategies of 36 kindergarten children. The children were given written stimuli such as 2 + 5 and 3 + 7, during two sessions that were one week apart. The results indicated that once children relied on mental addition strategies, they often quickly invented more economical procedures to compute sums. The study also confirmed that counting-all starting with the larger addend—a mental addition strategy recently discovered in a case study—was not an uncommon labor saving device among young children. On the other hand, counting-on from the first addend is relatively rare because it is a cognitively less economical strategy than either counting-all starting with the larger addend or counting-on from the larger addend. A double count model of mental addition accounts for the above results very nicely. Finally, several devices which may help children make the transition from counting-all to counting-on were observed.

The Use of Economical Mental

Addition Strategies by Young Children

Young children invent increasingly sophisticated and economical counting strategies to compute addition sum (e.g., Carpenter & Moser, 1982; Groen & Resnick, 1977; Ilg & Ames, 1951; Resnick & Ford, 1981). This study examined the addition strategies of kindergarteners in order to (1) gain insight into why and how children develop more sophisticated addition strategies and (2) confirm the existence of a newly discovered mental addition procedure.

The most basic addition strategy for children is counting-all with concrete supports (concrete counting-all). This involves counting out a number of objects or fingers for each addend (e.g., for 2 + 4: 1 2 and 1 2 3 4) and simply counting the total (1 2 3 4 5 6—6). This is a rather straightforward procedure and puts little demand on working memory. Counting-all done mentally, however, is a fairly sophisticated procedure and puts considerable demand on working memory. The child must (a) enumerate the first addend, and (b) continue the count sequence as the second addend is enumerated. The second step, then, requires two simultaneous counts—a double count. For 2 + 4, for example, the double count is four steps: "1, 2; 3(+1), 4(+2), 5(+3), 6(+4)—6." A somewhat more sophisticated strategy has occassionally been observed (Fuson, 1982; Resnick & Neches, in press). Counting-on from the first addend (COF) involves starting with the cardinal value of the first addend and continuing the count sequence as the second addend is enumerated (e.g., 2 + 4: "2; 3[+1], 4[+2], 5[+3], 6[+4]—6"). While the total count is reduced by this strategy, the double count is not (see Table 1). Eventually children invent the very economical counting-on from the larger addend (COL) strategy: Start the count with the cardinal value of the larger addend and continue the count sequence as the smaller addend is enumerated (e.g., 2 + 4: "4; 5[+1], 6[+2]—6"). This

COL strategy minimizes the total count and, more importantly, it minimizes the cognitively demanding double count (e.g., only two steps in the case of 2 + 4). Use of such a strategy depends on an ability to efficiently compare numbers and choose the larger. Development of a mental number line which can be used for such a purpose usually begins even before the child starts school (Resnick, 1983; Schaeffer, Eggleston, & Scott, 1974). Moreover, Fuson (Note 1, 1982) notes that, to abandon the counting-all for the COL (or COF) procedure, the child must realize that it is unnecessary to enumerate the larger (or first) addend to affirm its cardinal value. One focus of this study, then, was to examine the transition from concrete counting-all to mental addition and the development of more economical mental addition strategies.

Insert Table 1 about here

In a recent case study (Baroody, in press), a fourth mental addition strategy was discovered. Felicia, a pre-schooler, used a counting-all starting with the larger addend (CAL) strategy. She counted up to the larger addend first, and then continued the count with the smaller addend (e.g., 2 + 4: "1, 2, 3, 4; 5[+1], 6[+2]"). While Felicia's variation of the mental counting-all procedure does not reduce the total count (six steps), it does minimize the cognitively demanding double count. In the case of 2 + 4, the demanding double count is reduced to just two steps. Compare this to the four steps required by the counting-all starting with the first addend (CAF) procedure or, for that matter, by the COF procedure (see Table 1). Thus, Felicia's CAL approach is cognitively more economical than a CAF or even a COF strategy. The second focus of this study was to see if the CAL strategy was a unique invention by Felicia or a labor saving device commonly used by young children.

Method

Participants

A total of 36 children (15 boys and 21 girls) ranging in age from 5 years - 4 months to 6 - 9 (M = 5 - 11) participated in the study. The participants were drawn from three kindergarten classes in two middle- to upper-class surburban schools. All children participating in the study had parent/guardian permission.

Procedure

The addition ability of the children was evaluated as part of a larger study examining the developmental relationships between addition and commutativity.

During the familiarization session, both experimenters played math games with small groups of subjects. The addition task ("Car Race" game) was introduced at this time to ensure familiarity with the written addition format used in the study—including addition involving zero. If a child had no organized addition strategy, s/he was shown a concrete counting-all procedure (with blocks).

The experimental sessions consisted of a structured interview. The addition task took the form of a car race game. The subject was presented addition problems typed horizontally in large print on a 4 x 6 card. Problems were also read to the child. The sum indicated how many spaces the child or experimenter could advance his or her race car around the track. The child was instructed to solve the problem any way he/she wanted—using blocks, finger, or mental addition. Children who used mental addition strategies were encouraged to think out loud. When a child's strategy on a trial was not apparent, the experimenter asked the child how s/he figured out the problem. Testing was done in two sessions one week apart by different experimenters. The order in which children saw the experimenters was counterbalanced. In the first session, the following smaller addend first (SAF) addition were presented: 2 + 3, 2 + 5, 2 + 7, 3 + 4, 3 + 7, and 4 + 6. The following larger addend first (LAF) addition were presented: 4 +

2, 6 + 2, 8 + 2, 5 + 3, 6 + 3, and 5 + 4. In the second session SAF problems consisted of 2 + 4, 2 + 6, 2 + 8, 3 + 5, 3 + 6, and 4 + 5, and LAF problems were 3 + 2, 5 + 2, 7 + 2, 4 + 3, 7 + 3, and 6 + 4. For both sessions, the problems were introduced in random order. Scoring focused on the SAF problems, since these problems permit differentiation between strategies which start with the first addend and those that start with the larger addend. If a child had no organized strategy for adding, s/he was retaught concrete counting-all with blocks. Other strategies noted were spontaneous concrete counting-all, counting-all mentally starting with the first addend (CAF) (e.g., 2 + 3: "1, 2; 3[+1], 4[+2], 5[+3]—5"); counting-all mentally starting with the larger addend (CAL) (e.g., 2 + 3: "1, 2, 3; 4[+1], 5[+2]—5"); counting-on mentally from the first addend (COF) (e.g., 2 + 3: "2, 3[+1], 4[+2], 5[+3]—5"); and counting-on mentally from the larger addend (COL) (e.g., 2 + 3: "3; 4[+1], 5[+2]—5"). The predominant and most advanced strategy (when used more than once) for these problems were rated for each session (93% interrater agreement for 12 subjects) and across sessions.

A task adopted from the work of Schaeffer, Eggleston, and Scott (1974) was used to gauge the subjects' ability to mentally compare and choose the larger of two numbers (Baroody, 1979). The number comparison and addition tasks were presented in counterbalanced order. Half the trials involved N and N + 1 comparisons, half N + 1 and N comparisons. Nine trials (2 vs. 3, 4 vs. 5, 6 vs. 7, 8 vs. 9, 9 vs. 10, 2 vs. 1, 4 vs. 3, 6 vs. 5, and 8 vs. 7) were presented in random order during session 1; nine (1 vs. 2, 3 vs. 4, 5 vs. 6, 7 vs. 8, 3 vs. 2, 5 vs. 4, 7 vs. 6, 9 vs. 8, and 10 vs. 9) during session 2. The child was instructed: "We're going to play the 'Chase Game.' Do you want to be the cowboy or the indian? [The child then chose either a cowboy or indian toy figurine.] O.K., now the idea of this game is that the [experimenter's figurine] chases the [child's figurine]. I'll tell you two numbers. You tell me which is bigger—that way you won't get caught. Let's put the cowboy and indian here (at the starting line of a race track). Do you want your [figurine] to take 5 spaces or 1 space? Which is more, 5 or 1? So your [figurine] can move 5 spaces and my [figurine] can only move 1." After this practice trial (on which all subjects were correct, the experimental trials were presented. The child was scored on the total number of trials correct (0 to 18).

Results and Discussion

The subjects' addition strategies across sessions 1 and 2 are summarized in Table 2. Of the subjects who were retaught or generally relied on a concrete counting-all procedure during session 1, only a few (2 of 18 or 11%) adopted a more advanced procedure as their predominate strategy during session 2. On the other hand, of the subjects who initially relied on (CAF, COF, or CAL) mental strategies, nearly half (6 of 13 or 46%) adopted a more advanced strategy during session 2. (The difference between the groups was significant at the $p = .04$ level, Fisher Exact 2 x 2 Test.) Thus it appeared that once children made the relatively difficult transition to relying on mental addition, many quickly invented and adopted more economical procedures. That is, once children developed the relatively sophisticated ability to engage in a double count, they rather quickly found ways to minimize the memory demands of this cognitively demanding process.

Insert Table 2 about here

It appears that Felicia's CAL strategy (Baroody, in press) is not uncommon. It was or became the predominant addition strategy for six children (17% of the sample). Three children adopted CAL as their predominate addition procedure during the course of the study. For example, Andy (S# 23) appeared to invent the strategy during the first session. On trial 1 (2 + 3), Andy used a CAF approach. On trial 2 (3 + 7), he tried to employ this strategy again, but—after considerable difficulty—abandoned it. Note that Andy's CAF approach to 3 + 7 required a very taxing double count of seven steps. He then switched to a CAL strategy. Counting to himself, he reeled off "1, 2, 3, 4, 5, 6, 7" and then "8, 9, 10." Note that this strategy reduced the double count to a very manageable three steps. Thereafter, Andy always used the more economical CAL strategy.

Several other participants used Felicia's version of the mental count-all strategy occassionally. For example, Case used a CAF approach exclusively during session 1. He employed this strategy on only some problems (2 + 4, 3 + 5, and 4 + 5) during session 2. On the harder problems of this latter session (3 + 6, 2 + 6, 2 + 8), however, he used the labor saving CAL algorithm. Like Felicia (Baroody, in press), then, Case used this economical strategy selectively or strategically—when it was needed most. Eli also generally used a CAF strategy, but during session 2 switched to the more economical CAL strategy for 3 + 5, 2 + 6 and 2 + 8. Tami used a variety of strategies including Felicia's CAL approach. During the first session she used counting-all with concrete support (2 + 3, 4 + 6), CAL (3 + 4, 3 + 7) and COL (2 + 5, 2 + 7) approaches. During session 2, she used CAF (3 + 5, 3 + 6), CAL (4 + 5, 2 + 8), and COL (2 + 6) strategies (the strategy for trial 2 + 4 was unknown). In sum, a significant portion (about one fourth) of the sample either used CAL regularly or occassionally.

Discovery of the CAL algorithm raises the issue: What is the typical developmental order of mental addition strategies? It appears that, for some children at least, a neat, clear cut description of progress is not possible. Some children, such as Tami described above, use several different strategies at any one time. Nevertheless, some general trends do emerge. The data (see Table 2) suggest that a CAF algorithm is the first mental addition strategy for (nearly) all children. Some children (such as Andy described above), however, may need only a brief encounter with SAF problems with a relatively large double count to invent and adopt a CAL strategy. It seems unlikely that young children would reflect upon such problems before attempting to compute them and decide that starting with the larger rather than the first addend would be the wiser (easier) course of action. Indeed, because of their unary conception of addition (Weaver, 1982), children are likely to interpet even 1 + N problems as "one and N more" and not—at first—start with the larger addend. In any case, children who adopt CAL may never use a COF algorithm. After all, why would a child abandon a procedure which minimizes the cognitively demanding double count for a strategy that does not? While this study was not of sufficient duration to collect data on the matter, children who adopt a CAL algorithm would presumably next invent COL. This could help explain why a COF procedure has been observed so infrequently (e.g., Fuson, 1982). That is, a CAL strategy may be a more common transitional step from CAF to COL than a COF algorithm. A longitudinal study is needed to test this supposition.

Unfortunately, only one child (S #04), actually appeared to make the transition to COL during the course of the study. Meg used a CAF for a majority of the trials during session 1. On two trials she resorted to a COF procedure (once successfully and once unsuccessfully). On one trial she appeared to use COL. For the second session, she used the more advanced COL strategy exclusively. For 3 + 6, for instance, she responded, "Three, I mean 6 (pause); 7, 8, 9. In sum, it appeared that after a brief period of experimenting, she quickly dismissed a COF procedure in favor for the COL procedure (cf. Carpenter & Moser, 1982). That is, in order to minimize the demanding double count, she quickly abandoned COF and adopted COL. The great advantage in terms of cognitive economy of COL over COF may be another reason why COF is only occassionally observed.

We observed several mechanisms which may help children to make the transition from counting-all to counting-on—i.e., help them realize (at least implicitly) that counting out the first or larger addend is redundant to simply stating its cardinal value.

The first involved N + 1 and 1 + N types of problems. During the familiarization phase, the first author presented a girl with the problem 1 + 6. She looked perplexed and was unsure of what to do. Jenny, sitting in the next seat, whispered to her, "Oh, that's easy! Whenever you see 1, it's (the sum is) just the next number" (after the other addend in the count sequence—in this case 7). Jenny's N + 1 (1 + N) rule permitted her to enter the count sequence at N (the cardinal value of the larger addend) and count once to obtain the sum. This shortcut to the count-all process might then be extended to more difficult problems—yielding a general counting-on scheme (cf. Resnick, 1983). For example, a child might then reason that with N + 2 (2 + N) problems the answer is two after N in the count sequence. Therefore a problem such as 6 + 2 could be solved by counting "6; 7 (is one more), 8 (is <u>two</u> more)—so the answer is 8." To encourage the transition to counting-on, primary grade teachers might present children with N + 1 and 1 + N problems and encourage abstraction of the N + 1 (1 + N) rule. Once children can do N + 1 and 1 + N problems automatically, they can be encouraged to mentally compute N + 2 (2 + N) problems, and in turn, larger problems. Note that the transition may be difficult or impossible for children who have not yet developed the ability to use a double count. (N + 1 / 1 + N problems do not require a double count while larger problems do.)

A second possible transition mechanism involves concretely representing the first addend, labelling this perceptual unit (set of blocks) or kinesthetic unit (set of fingers) with the appropriate cardinal value, and continuing the count from this number <u>label</u>. We observed several variations of this theme. Dora (S #03) demonstrated the more basic form of this approach. For 6 + 2, she put out two piles of three blocks to represent six and one pile of two blocks to represent the second addend. She then <u>short-cut</u> the final count-all procedure by pointing to the concrete representation of the first addend, announcing its cardinal value ("six"), and continuing the count with two remaining blocks ("7, 8"). The same procedure was used to solve 6 + 3. This suggests that using dice games to practice addition might be helpful in encouraging the transition to counting-on. As children played such games, they would become familar with the various numerical patterns on a die (cf. Bley & Thornton, 1981). Eventually they would be able to subitize—that is, immediately "see"—the cardinal value of a die roll. They could then use this in the service of short-cutting the count-all procedure—much like Dora did. That is, they could subitize the value of one (the first) die and count-on from there—using the dots of the second die to help keep track of the double count.

Margie (S #13) and May (S #09) both demonstrated a more advanced form of Dora's technique. These children <u>automatically</u> represented the first addend with fingers and then counted-on from there, using additional fingers to keep track of the second addend (cf. Carpenter & Moser, 1982). For example, with 3 + 5, both children immediately put up the three fingers of one hand as they announced, "three," and then proceeded to put up, in turn, five fingers of the other hand as they announced, "4, 5, 6, 7, 8." Because the child concretely represented the first addend, some (e.g., Steffe, Thompson, & Richards, 1982) might argue that this procedure only has the appearance of counting-on. In fact, this procedure has two key features of a genuine COF strategy. First, it begins with a <u>cardinal</u> representation of the first addend (the immediate presentation of fingers to stand for the set and the announcement of the set's cardinal value), and second, it entails a double count.

Thus finger counting may facilitate the counting-all to counting-on transition for some children. Dantzig (1967) notes that fingers provide a device by which children can pass from ordinal representations of number to cardinal representations. Initially, children may be limited to only ordinal representation of 1 to 10: successively raising fingers as they count up to the desired number (see Figure 1). Later they develop the ability to make cardinal representations: automatically and simultaneously raising the required number of fingers. In the context of addition, some children may initially count out each addend on their fingers and then count the total number of fingers extended. This counting-all procedure entails using fingers to make ordinal representations of the addends (see Figure 1). Margie and May, described above, had arrived at the point where they could use their fingers to make cardinal representations of numbers. They used this ability, then, to short-cut the count-all procedure by starting with cardinal representation of the first addend. At this point, the concrete (finger) representation of the first addend is almost superfluous. With time, Margie and May may simply drop this unnecessary component and simply start with verbal designation of the first addend's cardinality. What developmental progression there actually is—if any—needs to be examined.

Insert Figure 1 about here

It appears that a number comparison facility is a necessary but not a sufficient condition for inventing and using addition strategies which involve starting with the larger addend. All the subjects were successful on the number comparison task at a statistically significant level. A total of 28 (78%) of 36 subjects were correct on all 18 comparisons ($p < .001$, Sign test). Seven more (19%) were incorrect on only one or two trials ($p < .01$, Sign test). Only one child (3%) exhibited some weaknesses on the task, missing four items ($p < .05$). Of the 11 children who (during session 2) used CAL or COL as their predominant strategy, all obtained a perfect score on the number comparison task. Of the 25 children who used other strategies, 17 achieved a perfect score (8 were successful but missed one to four items). Thus facility in comparing numbers develops prior to and is required for adopting a CAL or COL addition procedure but does guarantee the discovery of these more economical addition strategy. That is, in some children, number comparison and addition skills are initially isolated and only later are integrated in the service of cognitive economy.

In conclusion, the double count model is a useful heuristic for understanding the development of children's informal addition. First, the model explains why the transition from using concrete counting-all to using mental algorithms is so difficult and takes so long. With concrete counting-all, objects or fingers are used to directly model sets. With mental algorithms, objects or fingers are used in a more abstract role: to help keep track of the double count. Second, the model explains why more sophisticated strategies are invented so quickly over the transition to mental algorithms has been achieved. Mental algorithms require a double count. Double counts—especially large ones—tax working memory. Strategies such as CAL and COL that reduce the double count save mental effort and hence are favored (Baroody, in press). Lastly, the model explains why the COF procedure is used so rarely: It does not minimize the cognitively demanding double count. More economical alternatives (CAL or COL) are possible because kindergarteners already have an efficient number comparison scheme. In addition to an effort to reduce mental labor, recognition of dice patterns and automatic finger representation of numbers may be vehicles by which some children invent more efficient count strategies for addition. Thus, extensive use of dice games and finger counting should be encouraged early in school (cf. Baroody, Berent, & Packman, 1982).

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Table 1: A Comparison of the Cognitive Economy of Four Mental Addition Strategies in Solving "2 + 4".

Algorithm Name	Representation of the Algorithm	Total count (answer generating count)	Double count (count needed to enumerate/keep track of the second addend while simultaneously executing a portion of the answer generating count)
Counting-all starting with the first addend (CAF)	$1, \underline{2}; 3^{(1)}, 4^{(2)}, 5^{(3)}, 6^{(\underline{4})}$--6	6 steps	4 steps
Counting-on from the first addend (COF)	$\underline{2}; 3^{(1)}, 4^{(2)}, 5^{(3)}, 6^{(\underline{4})}$--6	5 steps	4 steps
Counting-on from the larger addend (COL)	$\underline{4}; 5^{(1)}, 6^{(\underline{2})}$--6	3 steps	2 steps
Counting-all starting with the larger addend (CAL)	$1, 2, 3, \underline{4}; 5^{(1)}, 6^{(\underline{2})}$--6	6 steps	2 steps

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Table 2: Addition Strategies Across Sessions 1 and 2

Addition Strategy	Session 1	Session 2
Retaught counting-all with blocks	9	3
Counted-all with blocks exclusively	7	9
On occassion used a mental strategy	3	5
CAF predominated	9	5
COF predominated	1	3
CAL predominated	3	6
COL predominated	4	5

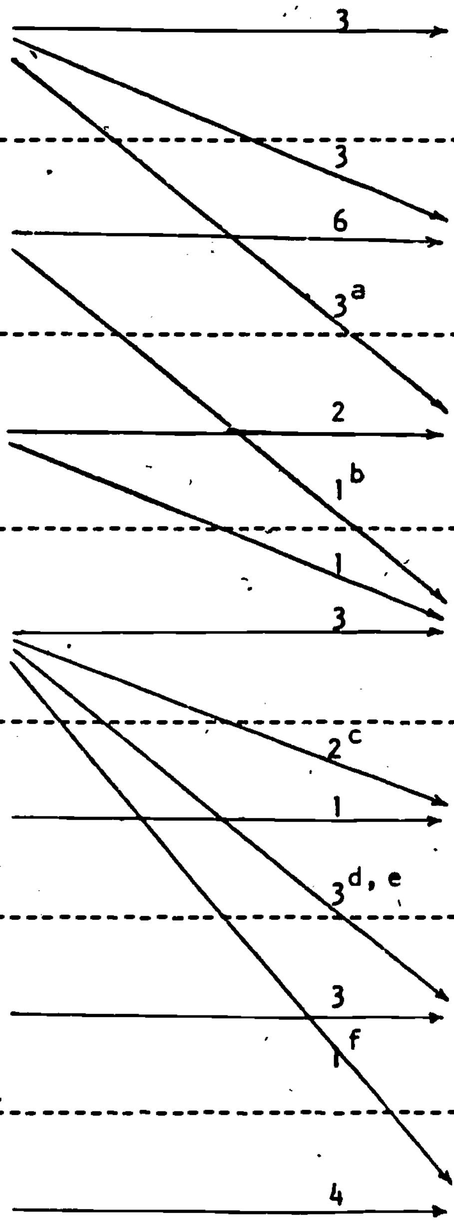


Table 2 continued

[a] All of these subjects used a CAF strategy on several occasions during session 1. In session 2, one of these subjects (S #26) resorted to using a CAL procedure twice, and another (S #08) appeared to use COF once.

[b] The subject (S #03) appeared to use CAF once during session 1 and counting-on or counting-on-like strategies twice during session 2.

[c] Both subjects (S #09 & S #13) used COF-like strategies during session 2--i.e., they immediately represented the cardinal value of the first addend with their fingers (as it was announced) and then used additional fingers to keep track of the second addend while they counted-on.

[d] S #23 actually switched to CAL as his predominate strategy during session 1 after using CAF first successfully and then unsuccessfully.

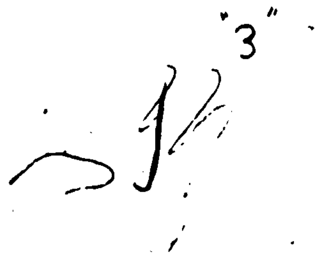
[e] S #18 actually used CAF and CAL equally during session 2.

[f] During session 1, S #04 used a COF strategy once successfully and once unsuccessfully. She also appeared to use a COL strategy once.

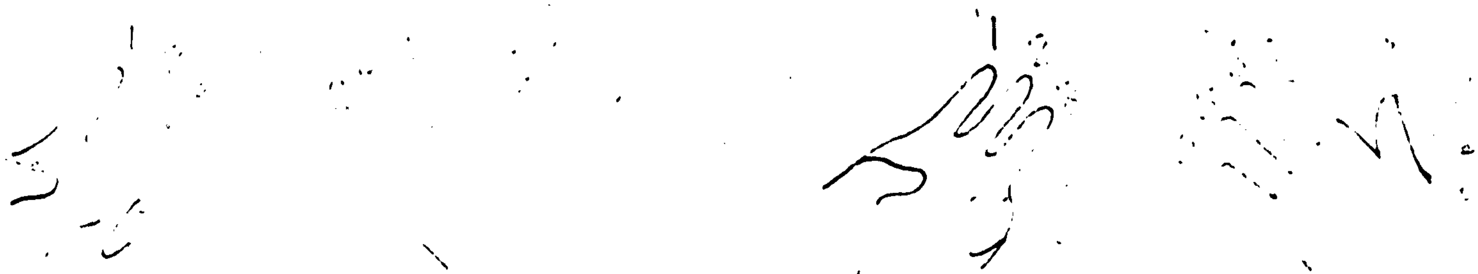
Figure 1: Ordinal and cardinal representations of numbers and addends

(a) Ordinal representation of three.

(b) Cardinal representation of three.



(c) Counting-all (using ordinal representations for the addends and sum).



(d) Using a cardinal representation of the first addend to short-cut counting-all.