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ABSTRACT

Several special cases of the general Rasch model, varying in complexity, were investigated to determine whether they could successfully model realistic multidimensional item response data. Whether the parameters of the model could be readily interpreted was also investigated. The models investigated included: (1) the vector model; (2) the product term model; (3) the vector and product term model; (4) the reduced vector and product term model; and (5) the item cluster model. Of the models investigated, all but the reduced vector and product term model and the item cluster model were rejected as being incapable of modelling realistic multidimensional data. The item cluster model appears to be a useful model, but its applications may be limited in scope. Of the models studied, the reduced vector and product term model was found to be the most capable of modelling realistic multidimensional data. Although the estimation of the parameters of the reduced vectors and product term model may be more difficult than it would be for other models, this model appears to be the model that is most worth pursuing. (Author/PN)

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The Use of the General Rasch Model with Multidimensional Item Response Data

Robert L. McKinley
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Several special cases of the general Rasch model, varying in complexity, were investigated to determine whether they could successfully model realistic multidimensional item response data. Also investigated was whether the parameters of the model could be readily interpreted. Of the formulations of the model investigated, all but two were found to be incapable of modeling realistic multidimensional item response data. One of the remaining formulations of the model was found to have limited applications. The version of the model found to be most

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) useful is an extension of the two-parameter logistic model.

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CONTENTS

The Use of the General Rasch Model with Multidimensional Item Response Data

Latent trait theory has become an increasingly popular area for research and application in recent years. Areas of application of latent trait theory have included tailored testing (McKinley and Reckase, 1980), equating (Marco, 1977; Rentz and Bashaw, 1977), test scoring (Woodcock, 1974), and criterion-referenced measurement (Hambleton, Swaminathan, Cook, Eignor, and Gifford, 1978). While many of these applications have been successful, they are limited to areas in which the tests used measure predominantly one trait. This limitation is a result of the fact that most latent trait models that have been proposed assume unidimensionality. Because of this requirement, in some situations latent trait models have not been successfully applied. For example, in achievement testing the goal is not to measure a single trait, but to sample the content covered by instruction. Therefore, most latent trait models are inappropriate since tests designed for this purpose generally cannot be considered to be unidimensional. Even when the goal is to measure a single trait, if dichotomously scored items are used no generally accepted method exists for forming unidimensional item sets, for determining the dimensionality of existing item sets, or for testing the fit of the unidimensional model to the data.

An alternative to trying either to construct unidimensional item sets or to fit a unidimensional model to already existing item sets is to develop a multidimensional latent trait model. Several such models have been proposed (Bock and Aitkin, 1981; Mulaik, 1972; Rasch, 1961; Samejima, 1974; Sympson, 1978; Whitely, 1980), but little research has been done using these models. Some work has been completed on the estimation of the parameters of the Bock and Aitkin model (Bock and Aitkin, 1981), the multidimensional Rasch model (Reckase, 1972), and the Whitely model (Whitely, 1980), but no extensive research has been completed on the characteristics and properties of any of these models. The purpose of this paper is to present the results of research on the characteristics and properties of the multidimensional Rasch model. Before presenting these results, however, the multidimensional models that have been proposed will be briefly discussed, as will the research that has investigated the characteristics of these models.

Multidimensional Latent Trait Models

Three of the multidimensional latent trait models that have been proposed have been extensions to the multidimensional case of the unidimensional Rasch model (Rasch, 1960). The unidimensional Rasch model is given by

$$P(x_{ij} \mid \theta_j, \sigma_i) = \frac{\exp(x_{ij}(\theta_j + \sigma_i))}{1 + \exp(\theta_j + \sigma_i)} \qquad (1)$$

where θ_j is the ability parameter for Person j, σ_i is the item easiness parameter for Item i, and $P(x_{ij} \mid \theta_j, \sigma_i)$ is the probability of response x_{ij} (0 or 1) to Item i by Person j. The multidimensional model proposed by Rasch (1961) is given by

$$P(x_{ij} \mid \underline{\theta}_j, \underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j, \underline{\sigma}_i)} \exp[\underline{\phi}(x)'\underline{\theta}_j + \underline{\psi}(x)'\underline{\sigma}_i + \underline{\theta}_j'\underline{\chi}(x)\underline{\sigma}_i + \rho(x)] \qquad (2)$$

where $\underline{\theta}_j$, $\underline{\sigma}_i$, and $P(x_{ij} \mid \underline{\theta}_j,\underline{\sigma}_i)$ are as defined above; $\underline{\phi}$, $\underline{\psi}$, $\underline{\chi}$, and ρ are scoring functions which are functions of x only; and $\gamma(\underline{\theta}_j,\underline{\sigma}_i)$ is a normalizing factor necessary to make the probabilities of the response alternatives sum to 1.0. The scoring functions $\underline{\phi}$, $\underline{\psi}$, and $\underline{\chi}$ act as weights for the parameters, while the ρ term is used to adjust the scale for different scoring procedures. Both the scoring functions and the ρ term depend on the score obtained by a person on the item. In order to apply this model to multidimensional data, $\underline{\theta}_j$, $\underline{\sigma}_i$, $\underline{\phi}$, and ψ are defined as column vectors, $\underline{\phi}(x)'\underline{\theta}_j$ and $\underline{\psi}(x)'\underline{\sigma}_i$ are inner products of vectors, and $\underline{\chi}(x)$ is defined as a matrix. The terms ϕ and ψ now represent vectors of weights for the different elements in the σ- and θ-vectors. The $\underline{\chi}$ matrix is a matrix of weights. Rasch never attempted to apply this model.

Reckase (1972) tried to apply the generalized Rasch model to real and simulated item response data with limited success. In this study, the multidimensional model fit multidimensional data no better than did the unidimensional Rasch model. However, Reckase did not include the $\underline{\theta}_j'\underline{\chi}(x)\underline{\sigma}_i$ term in the model, which may have resulted in the poor fit of the model to multidimensional data. In addition, several methodological problems may have contributed to the poor results of the study. First, the sample size used to estimate the parameters of the model was relatively small for the number of parameters estimated. Second, in addition to estimating the parameters of the model, Reckase also estimated the values of the $\underline{\phi}$ and $\underline{\psi}$ scoring functions. Finally, in order to estimate the parameter vectors, the dichotomously scored items used in the study were combined into clusters to form nominal response patterns. The most appropriate way to form the clusters was not known, which may have caused problems in the estimation of the parameters. Despite these difficulties, a least squares estimation procedure was developed which did yield somewhat reasonable parameter estimates.

Mulaik (1972) also proposed a multidimensional model that is a generalization of the Rasch model. The model proposed by Mulaik is given by

$$P(x_{ij}|\underline{\theta}_j,\underline{\sigma}_i) = \frac{\sum_{k=1}^{m} \exp[(\theta_{jk} + \sigma_{ik})x_{ij}]}{1 + \sum_{k=1}^{m} \exp(\theta_{jk} + \sigma_{ik})} \tag{3}$$

where θ_{jk} is the ability parameter for Person j on Dimension k, and σ_{ik} is the difficulty parameter for Item i on Dimension k. Although Mulaik never applied this model, he did suggest procedures for estimating the parameters of the model for three separate cases: when item responses are normally distributed and have a common variance for all items and subjects; when item reponses follow a Poisson distribution; and when item responses are dichotomous.

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Samejima (1974) proposed a multidimensional latent trait model that is a generalization of a different unidimensional model. The model proposed by Samejima is based on the two-parameter normal ogive model. This model is given by

$$P(x_{ij}|\underline{\theta}_j,\underline{a}_i,\underline{b}_i) = \Phi[\underline{a}_i'(\underline{\theta}_j - \underline{b}_i)] \qquad (4)$$

where $\Phi(x)$ is the normal distribution function, $\underline{a}_i$ is a column vector of item discrimination parameters, $\underline{b}_i$ is a column vector of item difficulty parameters, and $\underline{\theta}_j$ is a column vector of ability parameters for Person j. Unfortunately, the basic derivation of this model used the continuous response version of the normal ogive model. Therefore, its use with dichotomous data requires that item scores be translated to the continuous scale. Since no procedure for translating item scores to the continuous scale is available, the model cannot at present be applied to dichotomous data. Like Rasch and Mulaik, Samejima never applied this model, but only suggested how the parameters might be estimated.

Sympson (1978) proposed a multidimensional model based on the three-parameter logistic model. The Sympson model postulates that the probability of a correct response is determined by the product of the conditional probabilities of a correct response on each of the dimensions being measured. The three-parameter logistic model is given by

$$P(x_{ij}=1\ |\theta_j,a_i,b_i,c_i) = c_i + (1 - c_i)\frac{\exp[Da_i(\theta_j - b_i)]}{1 + \exp[Da_i(\theta_j - b_i)]} \qquad (5)$$

where θ_j is the ability parameter for Person j, a_i is the item discrimination parameter for Item i, b_i is the item difficulty parameter for Item i, c_i is a pseudo-guessing parameter for Item i, and D = 1.7. The three-parameter logistic model is used to model the conditional probability for each dimension, although the c_i parameter does not have a separate value for each dimension, but rather is a scalar parameter related to the item as a whole. The multidimensional model is given by

$$P(x_{ij}|\underline{\theta}_j,\underline{a}_i,\underline{b}_i,c_i) = c_i+(1 - c_i)\prod_{k=1}^{m}\frac{\exp[a_{ik}(\theta_{jk} - b_{ik})x_{ij}]}{1 + \exp[a_{ik}(\theta_{jk} - b_{ik})]}\ ,\ x = 0,1 \qquad (6)$$

where the parameters are as defined above and m is the number of dimensions. Although Sympson has done some work on estimating the parameters of this model, no application of the model to multidimensional data has yet been attempted.

The model proposed by Whitely (1980) is somewhat similar to Sympson's model. This model, called the multicomponent latent trait model, defines the probability of a correct response to an item as the product of the probabilities of performing successfully on each cognitive component of the item. The Whitely model is given by

$$P(x_{ij}|\underline{\theta}_j,\underline{b}_i) = \prod_{k=1}^{m} \frac{\exp[(\theta_{jk} - b_{ik})x_{ij}]}{1 + \exp[\theta_{jk} - b_{ik}]}, \quad x = 0,1 \qquad (7)$$

where all the parameters are as previously defined. It can be seen that this model is essentially another extension of the Rasch model. The model focuses on the different cognitive skills required to perform on an item rather than the global dimensions hypothesized by Sympson. Estimation procedures have been developed for the model and some applications have been made to real data. However, because of the emphasis placed on identifying the different cognitive skills required by an item, the application of this model is limited to data collected under very restricted experimental conditions.

Bock and Aitkin (1981) have proposed a multidimensional two-parameter normal ogive model for use with dichotomously scored response data. This model is given by

$$P(x_{ij}=1 | \underline{\theta}_j,\gamma_i,\underline{\alpha}_i) = \Phi[(\gamma_i + \underline{\alpha}_i'\underline{\theta}_j) / \sigma_\varepsilon] \qquad (8)$$

where γ_i is the difficulty parameter for Item i, $\underline{\alpha}_i$ is a column vector of discrimination parameters for Item i, $\underline{\theta}_j$ is a column vector of ability parameters for Person j, $\Phi(x)$ is the normal distribution function, and σ_ε is given by

$$\sigma_\varepsilon = (1 - \sum_{k=1}^{m} \alpha_{ik}^2)^{\frac{1}{2}}. \qquad (9)$$

Bock and Aitkin described a method for estimating the parameters of this model, and presented the results of the application of the model to the data for the Law School Admissions Test (LSAT) presented in Bock and Lieberman (1970). The results of the application of the model to the LSAT indicated that a two-dimensional solution fit the data better than a one-dimensional solution. Fit was assessed using a likelihood ratio chi-square test.

Summary

Six different latent trait models have been proposed for use with multidimensional item response data. Of these six models, two are of little interest here. The Samejima model is not designed for use with dichotomously scored item response data, and the Whitely model is appropriate only for special experimental conditions. Of the remaining four models, only the Bock and Aitkin model and a special case of the Rasch model have been applied, and no attempt has been made to extensively investigate the characteristics and properties of any of the models. The purpose of this research is to extensively investigate the characteristics of one of those models, the generalized Rasch model.

Method

Design

The design of this research was to start with the most simple formulation of the multidimensional Rasch model, investigate its ability to describe multidimensional item response data, and if necessary to investigate increasingly more complex versions of the model until good model/data fit was obtained. At each level of complexity the properties of the model were investigated, and the reasonableness and usefulness of the model were explored. This was done by generating test data to fit the particular form of the model being investigated, and analyzing that data in an attempt to assess how well the characteristics of the data matched the characteristics of real data with multidimensional characteristics.

The most general formulation of the model investigated in this research is the model described by Rasch (1961), given by Equation 2. The simpler formulations of the model used in this research were obtained by eliminating different terms from the model statement by setting the appropriate scoring functions equal to zero for all item scores.

For each model statement that was obtained, simulated test data were generated to fit the model. Using the known parameters and model statement, predictions were made as to the dimensionality of the generated data and the characteristics of the hypothetical items. Analyses were then performed on the simulation data in order to test the predictions. If it were found that a model statement could not be used to generate realistic data, in terms of either dimensionality or item characteristics, then the model was rejected, and a different model statement was investigated. This involved altering the terms of the general model (Equation 2) that were included and those that were zeroed out. In some cases, all of the terms in a particular rejected model statement were retained, and one or more additional terms from the general model were added.

Analyses

The first analysis performed on the simulation data generated using the models was a factor analysis. Factor analysis, in this case, is not being used as a means of validating the models, but as a procedure for determining whether the data generated from the models have characteristics similar to those of real test data. All of the factor analyses performed in this research were performed using the principal components method on phi coefficients. When the obtained and expected factor structures of the data did not match, follow-up analyses were performed in an attempt to determine why the obtained factor structure was different from what was expected.

Follow-up analyses included plotting the true item parameters against the factor loadings and against traditional item statistics such as proportion-correct difficulty values and point biserial discrimination values. These analyses were performed using both the unrotated factor loading matrix and the factor loading matrix rotated to the varimax criterion. The purposes of these analyses were three-fold. One purpose was to determine whether the obtained factor structure was a result of the model statement, the values used for the model parameters, or both. The second purpose was to facilitate interpretation of the model parameters, and the third purpose was to determine whether the model yielded items with reasonable characteristics. In many cases it was necessary to generate additional data, using different values for the parameters of the model, in order to answer specific questions about a particular model statement.

Using the results of all of the analyses performed for a particular model, a decision was made as to whether the model adequately generated data similar to real test data. If a model statement were rejected, an attempt was made to determine from the results of the analyses what changes in the model would yield a more acceptable model.

Results

Vector Model

The first model that was investigated was a simple vector parameter model. The $\underline{\theta}_j{}'\underline{\chi}(x)\underline{\sigma}_i$ and $\rho(x)$ terms were eliminated, yielding the model given by

$$P(x_{ij}|\underline{\theta}_j,\underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j,\underline{\sigma}_i)} \exp(\underline{\phi}(x)'\underline{\theta}_j + \underline{\psi}(x)'\underline{\sigma}_i) \tag{10}$$

where all the terms are as defined for Equation 1, and $\underline{\theta}_j$, $\underline{\sigma}_i$, $\underline{\phi}$ and $\underline{\psi}$ are vectors. For this model the scoring functions all took on values of one for a correct response and zero for an incorrect response. This model was selected first because it appeared to be a straightforward extension of the unidimensional Rasch model (Rasch, 1960) to the multidimensional case. The expectation was that data generated according to this model would have a dimensionality that would vary with the number of elements in the parameter vectors. For instance, when data were generated using two elements in both the item and person parameter vectors, it was expected that the data would yield a two-factor solution when factor analyzed. This was not the case, however. Regardless of the number of elements in the parameter vectors, this model yielded one predominant factor. This was true regardless of the actual values of the parameters or the values that were used in the scoring functions.

Table 1 shows the first two eigenvalues from a typical principal component solution for the vector model. As can be seen, there is a dominant first factor, with one minor factor. Table 1 also shows the unrotated factor loading matrix obtained for these particular data, as well as the proportion-correct difficulty and the inner product of the item parameter vector and scoring function for each item (sum of the item parameters). As can be seen, there is little variation in the loadings on the first factor, while the minor factor is related to item difficulty. Factor II generally has positive loadings for easy items and negative loadings for hard items. Once it was ascertained that the vector model would not yield multi-factor data, it was not difficult to determine why. Equation 8 can be written as

$$P(x_{ij}|\underline{\theta}_j,\underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j,\underline{\sigma}_i)} \exp(\alpha_j + \delta_i) \tag{11}$$

TABLE 1

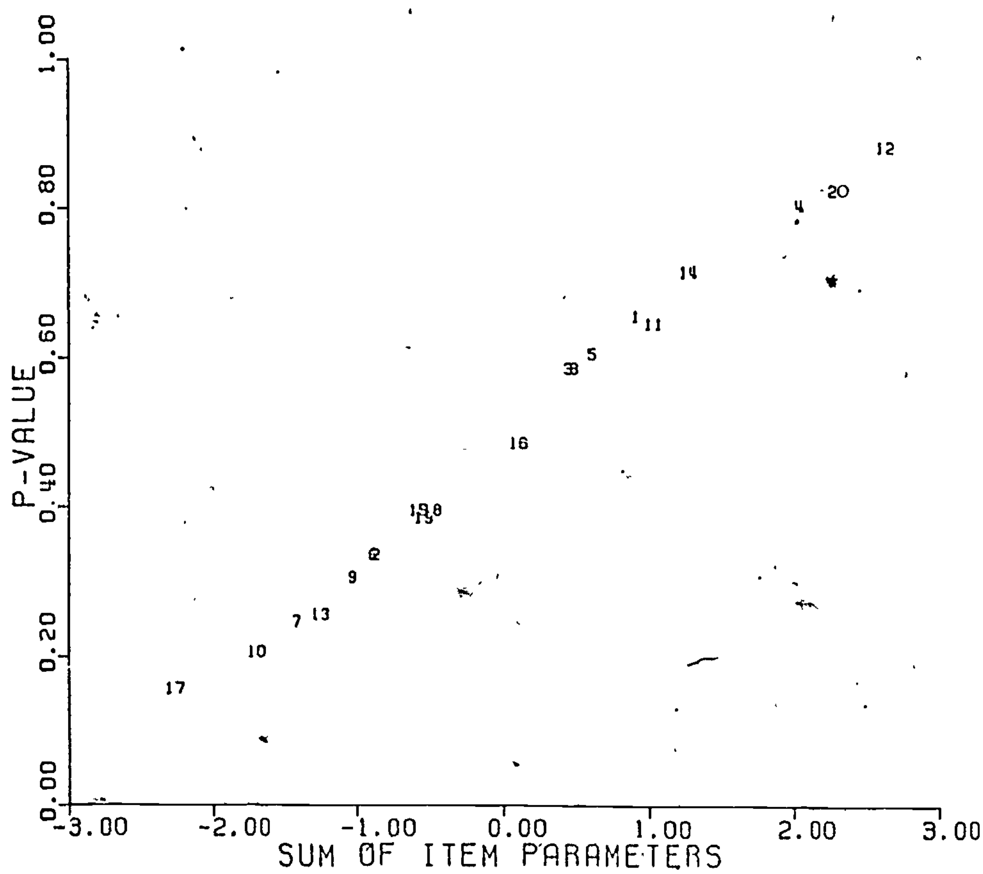
Principal Component Factor Loadings Based on Phi Coefficients with the Sums of the Item Parameters and Observed Proportion Correct for Two-Dimensional Vector Rasch Model

Item	Sum of Item Parameters	Observed Difficulty	Factor I	Factor II
1	.89	.65	.57	.28
2	-.89	.33	.52	-.24
3	.43	.58	.56	.04
4	2.02	.80	.47	.29
5	.59	.60	.59	.27
6	-.91	.33	.52	-.14
7	-1.44	.24	.51	-.27
8	.47	.58	.56	.13
9	-1.05	.30	.51	-.24
10	-1.76	.20	.48	-.19
11	.98	.64	.58	.07
12	2.58	.88	.43	.35
13	-1.31	.25	.50	-.34
14	1.22	.71	.53	.29
15	-.64	.39	.55	-.09
16	.05	.48	.56	.06
17	-2.33	.15	.46	-.36
18	-.54	.39	.54	-.05
19	-.60	.38	.53	-.26
20	2.26	.82	.43	.39
Eigenvalues			5.42	1.18

where $\alpha_j = \underline{\phi}(x)'\underline{\theta}_j$ and $\beta_i = \underline{\psi}(x)'\underline{\sigma}_i$. Equation 11 is the unidimensional Rasch model, with inner products of vectors as parameters. Therefore, regardless of the values of the model parameters, as long as the inner product remains the same, the probability of a response is the same. Therefore, the dimensionality of the vectors is unimportant, only the product is critical. The model is still a unidimensional model. The factor analysis results typified by the solution shown in Table 1 serve as an empirical demonstration that the vector model is a unidimensional model. It can also be empirically demonstrated that the inner products of the scoring function and parameter vectors serve as parameters for the model. Figure 1 shows a plot of proportion-correct difficulty by the inner product of the scoring function and item parameter vectors, which for this case is just the sum of the item paramaters. As can be seen, there is an almost perfect relationship between the inner products and the proportion-correct scores. When data were generated using the unidimensional Rasch model, with the inner products from the two-dimensional model as parameters, exactly the same plot was obtained.

Figure 1

Relationship Between the Proportion Correct and the Sum of the Item Parameters for the Two-Dimensional Vector Model



Note: The numbers in the plot are the item numbers.

Product Term Model

It was clear from the results just reported that using parameter vectors in an otherwise unidimensional model did not make it a multidimensional model. Therefore, the vector model was rejected as a multidimensional model. The next model that was investigated contained only the $\underline{\theta}_j{}' \underline{\chi}(x)\underline{\sigma}_i$ term. This was the next model investigated because it involved more than simple inner products of scoring and parameter vectors, but was simpler than using both inner products and the $\underline{\theta}_j{}' \underline{\chi}(x)\underline{\sigma}_i$ term.

When $\underline{\theta}_j$ and $\underline{\sigma}_i$ are vectors, $\underline{\chi}(x)$ must be a matrix. The product $\underline{\theta}_j\underline{\sigma}_i{}'$ represents a matrix of products of all possible pairs of the elements in the θ_j- and σ_i-vectors. For two-dimensional θ_j- and σ_i-vectors,

$$\underline{\theta}_j\underline{\sigma}_i{}' = \begin{bmatrix} \theta_1\sigma_1 & \theta_1\sigma_2 \\ \theta_2\sigma_1 & \theta_2\sigma_2 \end{bmatrix} \tag{12}$$

The $\underline{\chi}(x)$ matrix is a scoring matrix having an element for each element of the $\underline{\theta}_j\underline{\sigma}_i{}'$ matrix. If the $\underline{\chi}(x)$ matrix for the matrix in Equation 12 were

$$\underline{\chi}(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{13}$$

for a particular response x, then the numerator of the model statement for that response would be $\exp(\theta_1\sigma_1 + \theta_2\sigma_2)$. The nonzero elements of the $\underline{\chi}(x)$ matrix indicate which elements of the $\underline{\theta}_j\underline{\sigma}_i{}'$ matrix are included in the exponent. It is clear from this that by selectively using zeros in the $\underline{\chi}(x)$ matrix, various products of $\underline{\theta}_j$ and $\underline{\sigma}_i$ elements can be selected. Varying the values of the nonzero elements in $\underline{\chi}(x)$ assigns different weights to different combinations. Thus, the product term model, given by

$$P(x_{ij}|\underline{\theta}_j,\underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j,\underline{\sigma}_i)} \exp[\underline{\theta}_j{}' \underline{\chi}(x)\underline{\sigma}_i] \tag{14}$$

is a very rich model in terms of the number of alternative formulations of the exponent of the model that are available. Unfortunately, when data were simulated using some of these alternatives, it was discovered that this model had an inconvenient property. Regardless of which formulation of this model was used, and regardless of what values were taken on by the item parameters, the item proportion-correct difficulties were all approximately equal to each other. A closer examination of the product term model indicates why this occurred. Using the item parameters shown in Table 2, data were generated using

$$\underline{\chi}(x = 1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

for a correct response, and

$$\underline{\gamma}(x = 0) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{16}$$

for an incorrect response. This yields a model given by

$$P(x_{ij} = 1|\underline{\theta}_j, \underline{\sigma}_i) = \frac{\exp(\theta_{j1}\sigma_{i1} + \theta_{j2}\sigma_{i2})}{1 + \exp(\theta_{j1}\sigma_{i1} + \theta_{j2}\sigma_{i2})} \tag{17}$$

where the θ_{jk} and σ_{jk} terms are elements in the θ- and σ-vectors. From Equation 17 it can be seen that the item parameters are similar to the discrimination parameter in the unidimensional two-parameter logistic (2PL) model presented by Birnbaum (1968) since they are multipliers of the person parameters. In fact, if written as

$$P(x_{ij} = 1|\underline{\theta}_j, \underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j, \underline{\sigma}_i)} \exp[\sigma_{i1}(\theta_{j1} + 0) + \sigma_{i2}(\theta_{j2} + 0)] \tag{18}$$

the model is essentially a two-dimensional two-parameter logistic model with both of the difficulty parameters equal to zero for all items. Because the data used for Table 2 were generated using a bivariate N(0,1) with $\rho = 0$ distribution of ability, difficulty parameters of zero yielded a predicted proportion-correct difficulty of .5.

A principal components analysis of phi coefficients yielded evidence that the use of two item parameters resulted in a two-dimensional model. The first three eigenvalues obtained for the data generated using the item parameter values in Table 2 were 4.0, 2.4, and .9. The role of the item parameters as discrimination parameters in this model is indicated by comparing the item parameters shown in Table 2 with the rotated factor loading matrix, also shown in Table 2. The correlations between the item parameters and the factor loadings indicated that there was a strong linear relationship between the item parameters and factor loadings ($\underline{r}$ = .98 for σ_1 with Factor II, $\underline{r}$ = .99 for σ_2 with Factor I), supporting the conclusion that σ_1 and σ_2 are acting as discrimination parameters.

Vector and Product Term Model

The vector model that was investigated first was essentially a unidimensional model that contained a difficulty parameter (the inner product $\underline{\psi}(x) \cdot \underline{\sigma}_i$) as the only item parameter. The product term model is a multidimensional model that contains discrimination parameters as the only item parameters. In order to obtain a multidimensional model which contained a difficulty parameter, the vector and product term models were combined. A combination of these two models is given by

TABLE 2

Item Parameters, Proportion Correct Item Difficulty and Factor Loadings from a Varimax Rotated Principal Components Solution on Phi Coefficients for the Product Term Model

Item	σ_1	σ_2	p	Factor I	Factor II
1	0.150	1.150	.48	.52	.11
2	1.280	0.200	.50	.08	.56
3	0.260	1.350	.51	.57	.09
4	1.000	0.300	.52	.14	.52
5	0.250	1.050	.49	.47	.09
6	1.040	0.100	.51	.03	.57
7	0.110	1.150	.47	.52	-.01
8	0.200	1.200	.49	.57	.09
9	1.400	0.300	.50	.12	.58
10	0.300	1.200	.48	.54	.07
11	1.350	0.150	.51	.08	.59
12	0.400	1.200	.50	.53	.14
13	1.150	0.250	.50	.01	.52
14	0.150	1.300	.49	.61	.04
15	1.000	0.250	.51	.11	.49
16	0.100	1.400	.46	.61	.04
17	1.350	0.150	.50	-.01	.59
18	1.250	0.100	.52	-.03	.59
19	0.200	1.500	.48	.62	.03
20	1.150	0.500	.51	.25	.46

$$P(x_{ij} \mid \underline{\theta}_j, \underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j, \underline{\sigma}_i)} \exp[\underline{\psi}(x)'\underline{\sigma}_i + \underline{\theta}_j'\underline{\chi}(x)\underline{\sigma}_i]. \qquad (19)$$

As can be seen in Equation 19, the $\underline{\phi}(x)'\underline{\theta}_j$ term was eliminated when the two models were combined.

Table 3 shows the item parameters used to generate data to fit the vector and product term model. These data were generated using two-dimensional parameters. The scoring functions were also two-dimensional and were vectors of ones for a correct response and vectors of zeros for an incorrect response for all elements. Table 3 also shows the rotated factor loadings obtained for the first two factors from the principal components analysis of phi coefficients obtained for that data. The first three eigenvalues from the solution are 5.26, 2.28, and 1.07. Initial analyses indicated that this model could be used to model multidimensional data, and that item difficulties were not constant (see Table 3). However, these analyses also indicated that it was not realistic to use the same item parameters in both the parameter vectors and the product term. The problem is indicated by the magnitude of the correlation of the item proportion-correct difficulties with the item point biserials. Because of the double role played by the item parameters, the proportion-correct scores and point biserials had a correlation of $\underline{r}$ = .94. That is not a very realistic situation. Therefore, this model was also rejected as a reasonable method for describing multidimensional item response data.

Reduced Vector and Product Term Model

Since the analyses of the vector and product term model indicated that the same item parameters should not appear in both the parameter vectors and the product term, the item parameter vector and the scoring functions were altered so that parameters appeared in one or the other, but not both. In order to facilitate this, two additional elements were inserted into the item parameter vector. For a correct response the first two elements were zeroed out of the product term, while the last two were elements were zeroed out of the vector term. This procedure results in the first two item parameters acting as difficulty parameters and the last two parameters acting as discrimination parameters. Although four item parameters were used, only two dimensions were modelled in this case. For an incorrect response all of the parameters were zeroed out. All nonzero elements in the scoring functions were set equal to one. The resulting model is given by

$$P(x_{ij} = 1 \mid \underline{\theta}_j, \underline{\sigma}_i) = \frac{1}{\gamma(\underline{\theta}_j, \underline{\sigma}_i)} \exp[\sigma_{i1} + \sigma_{i2} + \sigma_{i3}\theta_{j1} + \sigma_{i4}\theta_{j4}] \qquad (20)$$

where the θ and σ terms are elements of the corresponding vectors.

The first three eigenvalues obtained from the principal components analysis for this model are 5.39, 1.30, and .99. Table 4 shows the item parameters that were used to generate the data, as well as the factor

TABLE 3

Item Parameters, Proportion Correct Item Difficulty, and Rotated Factor Loadings for the Vector and Product Term Model

Item	σ_1	σ_2	p	Factor I	Factor II
1	.230	.190	.56	.63	.12
2	.880	2.180	.77	.73	-.10
3	.900	-1.920	.35	.08	.71
4	-.900	-.110	.32	.39	-.22
5	-.640	.830	.52	.58	-.35
6	-.540	-1.040	.16	.10	.31
7	1.730	-1.350	.54	.34	.65
8	.940	.630	.69	.69	.21
9	.030	-.110	.46	.56	.11
10	-1.610	-.570	.13	.01	-.37
11	-1.170	1.260	.74	.75	.05
12	-.550	-1.070	.17	.04	.22
13	-.420	-.480	.31	.32	-.01
14	.220	-.070	.53	.60	.15
15	-.020	-1.670	.21	-.04	.55
16	2.420	.370	.78	.61	.40
17	1.230	.400	.69	.68	.28
18	.250	.410	.58	.65	.08
19	.140	.760	.64	.67	-.10
20	-1.770	.550	.30	.27	-.60

TABLE 4

Item Parameters and Rotated Factor Loadings
for the Reduced Vector and Product Model

Item	σ_1	σ_2	σ_3	σ_4	Factor I	Factor II
1	.206	-.503	.373	.997	.51	.01
2	-.164	.888	1.205	1.832	.60	.32
3	.448	.261	.766	.876	.34	.36
4	.814	-.008	1.321	1.714	.55	.34
5	.111	-.908	1.344	1.216	.41	.42
6	-.947	.044	1.758	1.694	.46	.51
7	-.490	.111	.687	.738	.40	.24
8	.553	-.502	.347	1.454	.61	.07
9	-.344	.639	1.307	.127	-.06	.64
10	-.257	.303	.851	.824	.26	.39
11	-.069	-.542	.472	.404	.22	.25
12	.779	.432	.392	.656	.30	.13
13	-.611	.571	.578	1.252	.59	.13
14	-.140	-1.032	.334	1.066	.60	-.04
15	-.705	.081	.821	.480	.07	.44
16	-.386	-.164	1.912	.244	.03	.71
17	-.154	.044	1.193	.537	.16	.56
18	.474	.249	1.385	1.287	.49	.44
19	.438	-.210	1.320	1.110	.42	.45
20	.294	.190	1.634	1.492	.45	.49

loadings obtained from a varimax rotation of the first two principal components. The item parameters that were used as multipliers (σ_3 and σ_4) were all positive in order to avoid having items with negative discriminations.

The results of the factor analysis of these data indicate that a dominant first factor is present. However, there was a second component present in the data which was strongly related to the item parameters ($\underline{r}$ = .87 for σ_3 and Factor II, $\underline{r}$ = .87 for σ_4 and Factor I). The item parameters in the product term were related to the factor loadings, while the sum of the item parameters in the vector term were found to be related to the proportion correct difficulty. The correlation between the sum of the parameters in the vector term and the proportion correct difficulty was $\underline{r}$ = .98, indicating that the sum of the vector parameters act as difficulty parameters. There was not a significant correlation between the item difficulty and point biserial values ($\underline{r}$ = .12). The sum of σ_3 and σ_4 had a correlation of $\underline{r}$ = .96 with the item point biserials.

The analyses of the model set out in Equation 20 indicate that it has many desired characteristics. The rotated factor loadings are highly related to the item parameters in the product term, the item difficulty is highly correlated with the sum of the item parameter vector elements, and there is no correlation between item difficulty and item discrimination.

One problem that does exist with the data that were generated is that the factor analysis results indicated that the data had only one predominant factor. One possible reason for this is that so many of the items had large values for both of the item parameters in the product term. In order to test this, data were generated for the set of item parameters shown in Table 5. The eigenvalues from the principal components analysis for these data are 2.49, 2.28, 1.05, and 1.03. As can be seen, when using the item parameters from Table 5 to generate data, there are two factors of approximately equal magnitude present in the data.

Item Cluster Model

Although the reduced vector and product term model appears to adequately model multidimensional data, the presence of the product term complicates parameter estimation, since the separation of the item and person parameters is not possible through techniques of conditional estimation. Because of this, one more model that does not have a product term was investigated. This model is the item cluster model.

One of the reasons the item vector model, given by Equation 10, does not adequately model multidimensional data is that no information about the different dimensions is preserved in the item score when the item is dichotomously scored. The elements for the different dimensions are summed, and the sums are treated as parameters. If it were possible to score the dimensions separately, then the vector model might be able to model multidimensional data. This requires, however, polychotomous item scoring.

TABLE 5

Item Parameters for the Reduced Vector and Product Model

Item	σ_1	σ_2	σ_3	σ_4
1	.977	.258	1.000	.000
2	.359	.728	.000	1.000
3	-.322	.377	1.000	.000
4	-1.289	1.128	.000	1.000
5	-.613	.219	1.000	.000
6	1.299	.797	.000	1.000
7	.029	-.213	1.000	.000
8	-.360	-.862	.000	1.000
9	.769	-.487	1.000	.000
10	-1.447	2.092	.000	1.000
11	-1.252	-.243	1.000	.000
12	-.778	-1.426	.000	1.000
13	.668	-1.860	1.000	.000
14	2.102	-.025	.000	1.000
15	-.724	.968	1.000	.000
16	1.230	-.535	.000	1.000
17	.260	-1.216	1.000	.000
18	-1.092	-.432	.000	1.000
19	-.994	1.479	1.000	.000
20	-.206	-.525	.000	1.000

Scoring an item on each dimension would require 2^n response categories, where n is the number of dimensions. Unfortunately, most test data are not scored polychotomously.

An alternative to having polychotomous item scoring is to consider more than one item at a time. If two dichotomously scored items are clustered together, and the cluster is treated as a single unit, then the cluster has 2^2 or 4 response categories - (0,0), (0,1), (1,0), and (1,1). The model given by Equation 10 can then be applied, with the exception that the σ-vector now represents a cluster rather than a single item, the scoring functions now take on values for 4 response categories instead of 2 and the response x is a vector with two elements. Further, the number of elements in the σ vector need not be the same as the number of items in the cluster, but rather should reflect the dimensionality of the cluster.

The procedure by which this model was investigated is as follows. For the two-dimensional case, item parameters were selected for 20 items. The items were paired so that Items 1 and 2 formed Cluster 1, Items 3 and 4 formed Cluster 2, and so on until 10 clusters were formed. For each cluster there were four response categories, which were scored as follows:

a) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]$ for $\underline{x}$ equal to both items incorrect;

b) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \left[\begin{smallmatrix}0\\1\end{smallmatrix}\right]$ for $\underline{x}$ equal to the first item incorrect, the second item correct;

c) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \left[\begin{smallmatrix}1\\0\end{smallmatrix}\right]$ for $\underline{x}$ equal to the first item correct, the second item incorrect;

and d) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \left[\begin{smallmatrix}1\\1\end{smallmatrix}\right]$ for $\underline{x}$ equal to both items correct.

For any one cluster two responses were generated, one for each dimension, using the parameters shown in Table 6. Table 6 also contains the unrotated factor loadings for the first two factors from a principal components analysis of phi coefficients obtained for these data. The first four eigenvalues were 3.61, 3.06, 1.33, and 1.21.

As can be seen, for the factor analysis the simulation data were treated as 20 items, rather than as 10 clusters. The eigenvalues listed above indicate that there were two roughly equal components in the data. Table 6 shows that the first component was defined by the items that were placed first in the cluster, and the second component was defined by the items that were in the second position in the cluster. Consistent with the scoring functions, there were two equal independent factors.

In order to demonstrate that the factors need not be independent, the same item parameters were used to generate data using the following scoring functions:

TABLE 6

Unrotated Factor Loadings on First Two Principal Components for the Two-Dimensional Item Cluster Model

Item	Item Parameters	Independent Model		Dependent Model	
		Factor I	Factor II	Factor I	Factor II
1	.893	.56	.00	.40	.21
2	-.850	.02	.65	.33	-.36
3	-.892	.66	-.02	.38	.24
4	.690	.01	.66	.33	-.33
5	.430	.64	.00	.41	.20
6	3.200	.02	.19	.25	-.24
7	2.016	.36	-.04	.36	.16
8	-3.310	.07	.22	.10	-.35
9	.594	.61	-.05	.40	.33
10	.470	.00	.69	.26	-.39
11	-.913	.66	.01	.47	.30
12	1.220	.06	.55	.32	-.38
13	-1.437	.58	-.01	.37	.20
14	-1.260	.00	.58	.32	-.43
15	.467	.65	-.07	.45	.28
16	.880	.04	.62	.28	-.36
17	-1.048	.66	-.04	.44	.28
18	-.970	-.02	.64	.28	-.37
19	-1.760	.56	.07	.43	.10
20	-2.140	.01	.42	.20	-.31

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a) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \begin{bmatrix}0\\0\end{bmatrix}$ for $\underline{x}$ equal to both items incorrect;

b) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \begin{bmatrix}.1\\.9\end{bmatrix}$ for $\underline{x}$ equal to the first item incorrect, the second item correct;

c) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \begin{bmatrix}.9\\.1\end{bmatrix}$ for $\underline{x}$ equal to the first item correct, the second item incorrect;

and d) $\underline{\phi}(\underline{x}) = \underline{\psi}(\underline{x}) = \begin{bmatrix}1\\1\end{bmatrix}$ for $\underline{x}$ equal to both items correct.

The principal components analysis of phi coefficients for the data generated according to this model yielded six factors with eigenvalues greater than one [2.46, 1.83, 1.09, 1.08, 1.01, 1.00]. Table 6 shows the unrotated factor loadings. As can be seen, there are still two factors present in the data. However, the factors are no longer defined only by the items in the corresponding position in the cluster. The first component is a general factor, while the second component indicates the position of the item in the cluster. Clearly these two sets of items are not independent in this case.

Discussion

The use of simulation data to study the characteristics of a model before selecting it for application is perhaps atypical of research on latent trait models. Usually a model is adopted, estimation procedures are derived, and the model is applied without ever going through the process this study has employed. In this study this approach has been taken for two main reasons. First, it was felt that when dealing with multidimensional latent trait models much of the acquired wisdom concerning latent trait models might no longer apply. It was felt that considerable research was necessary in order to gain an understanding of how these models work and what the model parameters represent before they could be applied. This belief has been borne out several times in this study by findings indicating that the models were not behaving in the anticipated manner.

A second reason for taking this approach was that it seemed impractical to attempt to develop estimation procedures for some of these models. Specifically, the general model set out by Rasch has a very large number of parameters. It seemed impractical to try to estimate all of them, and it was hoped that research on the model could help simplify the estimation process by eliminating some terms from the model and by discovering restrictions on the range of values for the parameters. With these considerations in mind, the results of this study will now be discussed.

Vector Model

The simplest formulation of the general model that was investigated was the vector model. This model is simply the unidimensional Rasch model, but with vectors for parameters instead of scalars. This model was found to be totally inadequate for modelling multidimensional data. When data were generated according to this model, the resulting data were unidimensional, with item characteristics determined by the inner product of the item parameter vectors and scoring functions. From this it follows that this model would fit multidimensional data no better than a unidimensional model having parameters equal to the inner products from the vector model.

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Product Term Model

Because of its slight similarity to Birnbaum's two-parameter logistic model, it was felt that the product term model would be better able to model multidimensional data. It was anticipated that the item parameters in the product term would behave as discrimination indices, and that is how they did behave. Unfortunately, without the vector terms in the model there were no terms playing the role of difficulty parameters. The data generated for this model had items of constant difficulty. From this it was concluded that this model would be useful only for modelling items of constant difficulty, and when items have varying difficulties this model is inappropriate.

Vector and Product Term Model

Based on the findings for the vector model and the product term model, it was hypothesized that a combination of the two models would be necessary to model items that were both multidimensional and of varying difficulty. Analyses of the vector and product term model indicated that it would model multidimensional data, and that it would model items of varying difficulty. However, it was also found that, as long as the item parameter vector elements appeared both in the vector terms and in the product term, the item difficulties and disciminations would be highly correlated. Since this is rarely the case in real test data, it was concluded that this model would be useful only in a very limited number of circumstances.

Reduced Vector and Product Term Model

In order to overcome the deficiencies of the vector and product term model, it was clear that a given item parameter vector element should appear only in the vector term or the product term, but not both. It was anticipated that similar problems might exist if the person parameter vector elements occurred in both the vector term and the product term, so the same restriction was placed on the person parameters as was placed on the item parameters.

The resulting model appears to be quite successful at modelling realistic multidimensional data. It is capable of modelling correlated as well as independent dimensions, and the item parameters are readily interpretable. The only real problem there seems to be with this model is with the estimation of the parameters. Although there are fewer parameters to estimate than is the case with the general model, there are still a fair number to estimate. Moreover, there are no observable sufficient statistics for the parameters in the product term. These problems do not make estimation of the model parameters impossible, and probably not even impractical. However, they do make estimation more difficult.

Item Cluster Model

The item cluster model was proposed as an alternative to the vector model. This model does not involve a product term, but it still can successfully model multidimensional data. However, it does involve clustering items, which gives rise to a number of new problems. For instance, as yet it is unclear what the effect is of forming different combinations of items, or whether all items should be clustered with the same item. Preliminary investigations seem to indicate that the optimal clustering procedure is to cluster all items on a subtest with one item taken from a different subtest. Another alternative, which has not been explored, is to apply the model only in situations where items are already clustered, such as is the case with passage units. Clearly more research is needed on this type of application of the item cluster model.

Summary and Conclusions

The purpose of this study was to investigate the application of the general Rasch model to multidimensional data. Several formulations of the model, varying in complexity, were investigated to determine whether they could successfully model realistic multidimensional data. Also investigated was whether the parameters of the models could be readily interpreted. The models investigated included: a) the vector model; b) the product term model; c) the vector and product term model; d) the reduced vector and product term model; and, e) the item cluster model.

Of the models investigated, all but the reduced vector and product term model and the item cluster model were rejected as being incapable of modelling realistic multidimensional data. The item cluster model appears to be a useful model, but its applications may be limited in scope. Of the models studied, the reduced vector and product term model was found to be the most capable of modelling realistic multidimensional data. Although the estimation of the parameters of the reduced vector and product term model may be more difficult than it would be for other models, this model appears to be the model that is most worth pursuing.

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