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ABSTRACT

This simple method for simulating the Central Limit Theorem with students in a beginning nonmajor statistics class requires students to use dice to simulate drawing samples from a discrete uniform distribution. On a chalkboard, the distribution of sample means is superimposed on a graph of the discrete uniform distribution to provide visual evidence of the reasonableness of the theorem. The procedure requires less than an hour to provide a conceptual understanding of the theorem, which is necessary to learn large sample estimation and hypothesis testing procedures. The procedure can be reproduced using a microcomputer, which does not limit sample size. (CM)

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A CLASSROOM SIMULATION

OF THE

CENTRAL LIMIT THEOREM

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ABSTRACT

A simple method for simulating the Central Limit Theorem with students in a beginning nonmajor statistics class is presented. It requires less than an hour and leaves students with a conceptual understanding of the theorem as well as an acceptance of its plausibility.

KEY WORDS: Statistical Education; Central Limit Theorem; Simulation

A CLASSROOM SIMULATION OF THE CENTRAL LIMIT THEOREM

Most elementary courses in statistics for nonmajors feature the Central Limit Theorem as an introduction to statistical inference. The mathematical prerequisites associated with a proof of the Central Limit Theorem usually preclude its inclusion in such courses. Thus, students often do not understand fully the theorem or its implications. Those who do understand it often have a difficult time accepting its results.

This paper presents a simple method for simulating the Central Limit Theorem in the classroom. Although the simulation does not constitute a proof, it does demonstrate the reasonableness of the theorem and builds student confidence in its outcomes.

The Central Limit Theorem

The Central Limit Theorem is stated in most standard texts in elementary statistics. Although the statements vary, their thrust is that the sample mean, $\overline{X}$, is distributed asymptotically normal, with mean, μ, and standard deviation, $\sigma/\sqrt{n}$. A typical statement of the Central Limit Theorem is as follows:

> The Central Limit Theorem: If random samples of observations are drawn from a population with finite mean, μ, and standard deviation, σ, then, when n is

> large, the sample mean, $[\bar{X}]$, will be approximately normally distributed with mean equal to μ and standard deviation $\sigma/\sqrt{n}$. The approximation will become more and more accurate as n becomes large. (Mendenhall, 1976, p. 146)

Since texts at the level we are discussing do not include even a partial proof, the texts try to explain and/or demonstrate the Central Limit Theorem's plausibility. If a student does not understand and accept this cornerstone of inferential statistics, it is likely that the student will not understand the hypothesis testing and estimation procedures which would follow.

Classroom Simulation

The classroom simulation requires students to use dice to simulate drawing samples from a discrete uniform distribution. About 15 minutes are required for the simulation and 15 to 30 more minutes should be devoted to a discussion of the simulation. Details of the simulation follow.

A minimum of materials is required. Each student will need a standard die. These can be purchased quite reasonably at any department store. A handout (Figure 1) which includes the statement of the Central Limit Theorem, a frequency distribution table outline, and a histogram of the discrete uniform distribution, DU(1, 6), should also be provided.

The first step of the demonstration requires that each student toss his or her die five times and record the results. If there are less than 30 students in the class, each student

Statement of Central Limit Theorem

If random samples containing a fixed number $n (n \geq 30)$ of measurements are drawn repeatedly from a population with mean, μ, and standard deviation, σ, then the sample means ($\overline{X}$'s) will be distributed approximately normally with mean, μ, and standard deviation, $\sigma/\sqrt{n}$.

Demonstration of Central Limit Theorem

Summary of class samples:

$\overline{X}$	Tally	Frequency	Relative frequency
5.25-5.74			
4.75-5.24			
4.25-4.74			
3.75-4.24			
3.24-3.74			
2.75-3.24			
2.25-2.74			
1.75-2.24			
1.25-1.74			

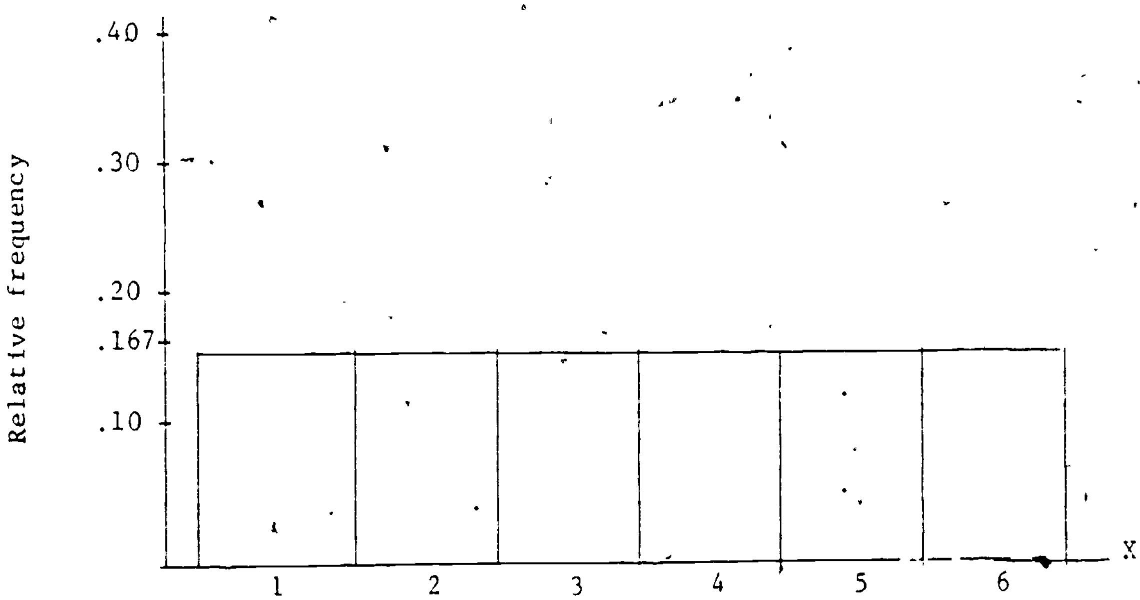

Figure 1. Student handout.

should perform the experiment twice. At this point in the simulation it should be emphasized that the theorem normally requires a "large" sample but we are using n = 5 for demonstration purposes only. In actual practice, "large" would likely mean a sample of at least 30.

Each student is instructed to compute the sample mean(s) ($\bar{X}$) for his or her sample(s). The instructor tallies the results on the board. I have found that a transparency of the handout (Figure 1) works very well for this purpose. The frequencies and relative frequencies can now be found and plotted over the discrete uniform distribution on the graph (Figure 1).

At this point several approaches are available to the instructor. Visual inspection of the distribution of $\bar{X}$ superimposed on the graph of X (the discrete uniform distribution) provides strong visual evidence of the reasonableness of the Central Limit Theorem even when an n of only 5 is used. It can be seen that the mean of the distributions of $\bar{X}$ and X are very similar. It can also be seen that the standard deviation of $\bar{X}$ is smaller than that of X, in fact, it is usually about one-half as large ($\sigma_{\bar{x}} = \sigma_x /\sqrt{n} = \sigma_x /\sqrt{5} = \sigma_x /2.24$).

A more rigorous approach would be to derive the population mean and standard deviation of a discrete uniform distribution with parameters 1 and 6 and compute the obtained mean and standard deviation of the simulated distribution of $\bar{X}$. In this way the actual μ and $\sigma/\sqrt{n}$ can be compared with the sample mean and standard deviation of the distribution of $\overline{X}$.

Illustration

The approach was used recently with a class of 30 students. Each student was asked to roll the die five times, record the results, and compute the mean. They were then asked to repeat the procedure. Thus, 60 sample means were available for plotting. Figure 2 illustrates the results.

It can be seen that the means of the distributions of the original discrete uniform (μ) and the graph of the sample means are almost identical. Further, the variability of the sample means is decidedly smaller than that of the discrete uniform distribution.

It is important to point out to the students that the simulation is intended only to demonstrate the reasonableness of the Central Limit Theorem. In actual practice most statisticians suggest a minimum sample of at least 30 is needed to apply it. While I have repeated the simulation more than a dozen times in class and have never had a situation where it produced unconvincing results, it should be pointed out that the results do improve as the n is increased.

A similar simulation using a microcomputer produced 60 sets of data for sample sizes of 15 and 30. Figure 3 illustrates the results of these simulations compared against the n = 5

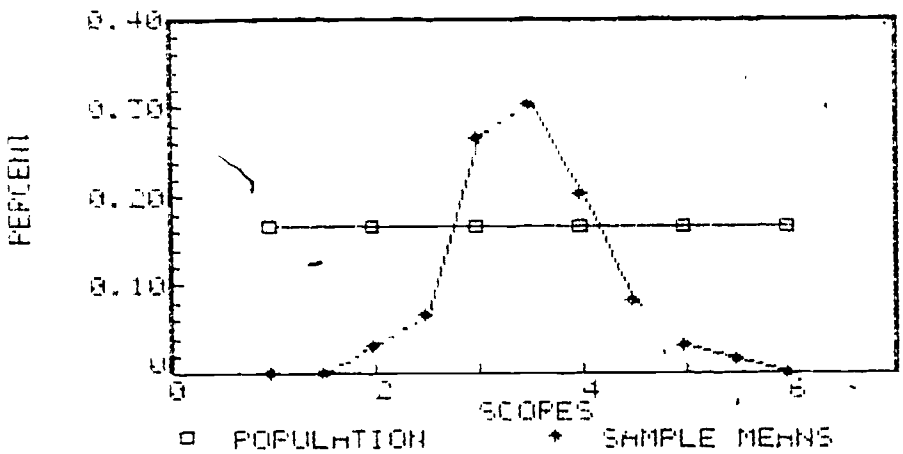

Figure 2. Graph of 60 sample means (n = 5) superimposed over discrete uniform distribution.

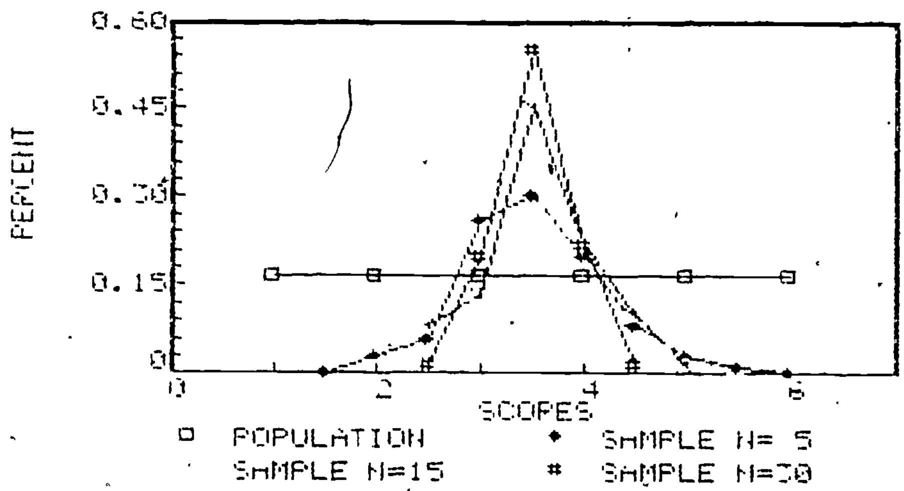

Figure 3. Graphs of sample means of varying n superimposed over discrete uniform distribution.

simulation. As the sample size increases, the distribution becomes more normal looking and the variance becomes smaller.

Discussion and Summary

Over the years the simulation described in this paper has been very successful for several of my colleagues and myself. It is particularly useful for demonstrating the plausibility of the Central Limit Theorem and promoting its understanding with nonstatistics majors. The teaching of large sample estimation and hypothesis testing procedures is a very logical followup to the simulation. I have found that an exercise such as the simulation proposed here puts the students more at ease with a concept that is often difficult for beginning statistics students and makes them place more confidence in its results.

It should also be noted that the procedure can easily be reproduced using a microcomputer. The advantage of this approach would be that the sample size would no longer be limited to five. In fact, students could try several sample sizes, plot them, and observe a result similar to that shown in Figure 3.

The procedure has been used very effectively to help students understand and accept the Central Limit Theorem. It provides a very effective lead in to a discussion of large sample estimation or hypothesis testing procedures. Students who have been through a similation seem to be more willing to approach an estimation or hypothesis testing situation by identifying a test statistic and trying to determine its sampling distribution. Such an approach demonstrates an understanding of the process rather than a rote approach.

Reference

MENDENHALL, WILLIAM.(1976), An Introduction to Probability and Statistics, Fourth Edition, North Scituate, Massachusetts: Duxbury Press.