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ABSTRACT

This report contains recommendations on needed changes (additions, deletions, and increased/decreased emphases) in elementary and middle school mathematics curricula and a statement of more general concerns about the secondary school mathematics curriculum. Separate sections are devoted to: (1) elementary and middle school mathematics; (2) traditional secondary school mathematics; (3) non-traditional secondary school mathematics; (4) role of technology; (5) relation of changes to other disciplines; and (6) teacher supply, education, and re-education. Recommendations in each section are based on issues and concerns addressed. Recommendations related to elementary/middle school mathematics include introduction of calculators/computers at earliest grade practicable, more emphasis on development of skills in mental arithmetic, estimation, and approximation (with less emphasis on paper/pencil operations), and direct experiences with collection and analysis of data. Recommendations related to secondary school curriculum include streamlining traditional curricula to make room for new topics. In addition, discrete mathematics, statistics, probability, and computer science should be regarded as fundamental and appropriate topics and techniques from these subjects should be introduced into the curriculum. Lists of participants and material related to the conference at which these recommendations were drafted are included in appendices. (Author/JN)

REPORT TO

NSB COMMISSION ON PRECOLLEGE EDUCATION

IN

MATHEMATICS, SCIENCE, AND TECHNOLOGY

"THE MATHEMATICAL SCIENCES CURRICULUM K-12: WHAT IS STILL FUNDAMENTAL AND WHAT IS NOT"

THE CONFERENCE BOARD OF THE MATHEMATICAL SCIENCES

DECEMBER 1, 1982

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*A limited number available upon request to Commission staff.

EXECUTIVE SUMMARY

Our charge from the NSB Commission was to identify what parts of mathematics must be considered fundamental for education in the primary and secondary schools. We concluded that the widespread availability of calculators and computers and the increasing reliance of our economy on information processing and transfer are significantly changing the ways in which mathematics is used in our society. To meet these changes we must alter the K-12 curriculum by increasing emphases on topics which are fundamental for these new modes of thought.

This report contains our recommendations on needed changes -- additions, deletions,and increased or decreased emphases -- in the elementary and middle school mathematics curricula and a statement of more general concerns about the secondary school mathematics curriulum.

With regard to elementary and middle school mathematics, in summary, we recommend:

- That calculators and computers be introduced into the mathematics classroom at the earliest grade practicable. Calculators and computers should be utilized to enhance the understanding of arithmetic and geometry as well as the learning of problem-solving.

- That substantially *more* emphasis be placed on the development of skills in mental arithmetic, estimation, and approximation and that substantially *less* be placed on paper and pencil execution of the arithmetic operations.

- That direct experience with the collection and analysis of data be provided for in the curriculum to insure that every student becomes familiar with these important processes.

We urge widespread public discussion of the implications of the changing roles of mathematics in society, support of efforts to develop new materials for students and teachers which reflect these changes, and continued and expanded experimentation within the schools.

With regard to the secondary school curriculum, in summary, we recommend:

- That the traditional component of the secondary school curriculum be streamlined to make room for important new topics. The content, emphases, and approaches of courses in algebra, geometry, precalculus, and trigonometry need to be re-examined in light of new computer technologies.

- That discrete mathematics, statistics and probability, and computer science now be regarded as "fundamental" and that appropriate topics and techniques from these subjects be introduced into the curriculum. Computer programming should be included at least for college-bound students.

Modern computer technology clearly has vast potential for enriching d enlivening the secondary school curriculum. However, we are not now in a position to make firm recommendations. There is need for research on the effects of incorporating technology into the traditional secondary school curriculum. We urge federal support for investigations into this question, including development of experimental materials and prototypes of actual school curricula.

Although we are generally optimistic about the future role of computers, we feel we must highlight one point that worries us even though it is not directly within our charge. The disparity of access between children who have a computer at home and children who do not threatens to widen the educational gap that already exists between different economic strata. It is urgent that programs be designed to address this problem.

We clearly recognize that the most immediate problem is not the mathematics *curriculum*, but the need for more, and better qualified, mathematics *teachers*. One section of this report is devoted to recommendations on attracting and training prospective teachers, better utilizing the talents of in-service teachers, and retraining teachers who are inadequately prepared for teaching mathematics. We feel that the coming changes in subject matter and emphasis not only will bring a new sense of vitality to K-12 mathematics, but also will encourage teachers actively to seek and participate in programs of professional development.

The Conference Board of the Mathematical Sciences stands ready to assist efforts to develop immediate strategies for addressing the teacher shortage and to develop long-term strategies for bringing about the curricular changes envisioned in this report.

I. The NSF/CBMS Meeting

In response to suggestions made at the July 9, 1982 meeting of the NSB Commission on Precollege Education in Mathematics, Science, and Technology and, specifically, to a request made by the Educators Panel of the Commission, The Conference Board of the Mathematical Sciences (CBMS) held a special meeting to address the topic THE MATHEMATICAL SCIENCES CURRICULUM K-12: WHAT IS STILL FUNDAMENTAL AND WHAT IS NOT. The meeting was held on September 25-26, 1982 at the headquarters of the Mathematical Association of America in Washington, D.C.

Participants in the meeting included the presidents of the American Mathematical Society, National Council of Teachers of Mathematics, Mathematical Association of America, American Mathematical Association of Two-Year Colleges, and Society for Industrial and Applied Mathematics. Two members of the Commission, Frederick Mosteller and Katherine Layton, and two members of the Commission staff, Ray Hannapel and Mary Kohlerman, also participated in the meeting. The other participants were representatives of the CBMS constituent organizations and the CBMS officers. (The list of meeting participants is in Appendix A.)

Six position papers on the fundamentals in the mathematics curriculum were written expressly for this conference. These position papers, along with various other background materials were distributed to all participants prior to the meeting. (Copies of the position papers are in Appendices D and E. A list of other materials is in Appendix E.)

The initial portion of the meeting was devoted to discussion of the position papers. Following this, participants joined working groups to address the question of what is still fundamental and what is not in K-8 and in secondary school mathematics. A general discussion of the written reports of the working groups was held during the last hour of the Saturday session.

On Sunday, new working group assignments were made to discuss the implications of changes in the K-12 mathematics curriculum. The reports from these groups were discussed in the closing session of the conference.

II. Recommendations to the Commission

INTRODUCTION

In the limited time available during the conference, it was not possible to establish full consensus on every detail of the working group reports. However, there clearly was broad consensus on the need to incorporate calculators and computers, as well as additional data analysis, into the K-12 curriculum and to make the necessary adjustments in the mathematical topics and modes of thought traditionally taught at these grade levels.

Some detailed recommendations on the fundamentals in the K-8 curriculum, what should be emphasized *more* and what should be emphasized *less*, are given in the working group report "Elementary and Middle School Mathematics." The corresponding adjustments needed in the secondary school curriculum, where the impact of technology is even greater, are described in more general terms in the two reports "Traditional Secondary School Mathematics" and "Non-traditional Secondary School Mathematics." In this area much more investigation and experimentation are required before a firm consensus can be reached.

Recommendations on dealing with the challenge of providing children with access to, and understanding of, computers and calculators pervade this entire report. They are dealt with specifically in the report "The Role of Technology." A statement of the relationship between the mathematics curriculum and what is, or can now be, taught in other disciplines is given in the report "Relations to Other Disciplines." The report entitled "Teacher Supply, Education, and Re-education" contains a variety of recommendations on attracting and retaining well-qualified mathematics teachers.

The question of the total time in the school curriculum which should be devoted to the study of the mathematical sciences was not addressed in any detail at the meeting. The general feeling was that, at the primary level, there appears to be an approximate balance between topics needing more emphasis and those needing less. At the secondary level, it is not yet clear how much time, in addition to the time that can be gained by streamlining the traditional mathematics curriculum, will be needed for discrete mathematics, probability and statistics, and computer science. This can only become clear after detailed examination of model mathematical sciences curricula and careful consideration of the many competing demands for time in the overall school curriculum.

There was general agreement at the conference that the most pressing immediate problem is the need for more, and better qualified, teachers in the classrooms. No curriculum, no matter how well-founded, can possibly succeed without dedicated and competent teachers to teach it. However, many participants felt that appropriate changes in the curriculum at this time, rather than detract from efforts to deal with the teacher shortage, could bring a new sense of vitality to K-12 mathematics and could serve to encourage teachers to actively seek and participate in programs of professional development.

Participants in the conference were also in agreement that their suggestions, even if influential in full, cannot be expected to constitute a "cure-all" for all the shortcomings of K-12 mathematics. In fact, a fundamental improvement in K-12 mathematics can be hoped for only within the framework of a general improvement of the total school environment. Remedies for the difficulties facing the teaching community (low teachers salaries, low prestige, lack of support by society, lack of discipline in the classroom, irregular attendance, etc.) are societal in nature and fall outside both the mandate and the competence of this group.

SOME ADDITIONAL RECOMMENDATIONS

In addition to the concerns and recommendations in the working group reports, a few points were emphasized in the general discussions which are of vital importance in the implementation of any curricular changes:

- **Textbooks**

Textbooks play a key role in the mathematical sciences curriculum at all levels. Any major changes in the curricula at the elementary, middle, or high school levels must be accompanied by corresponding changes in textbooks. For this to happen, the groups responsible for preparing textbook series and for adopting textbooks must be deeply involved in efforts to up-date these curricula.

- **Testing**

To a large extent the grade and high school teachers are under strong pressure to train their pupils so as to maximize their chances of doing well on standardized tests. As long as these tests stress computations, the pupils are bound to be drilled in computations, regardless of any other guidelines the teachers may have received, and even contrary to the sounder convictions the teachers themselves may have.

We call the attention of the Commission to the power and influence of standardized tests. Properly modified, these can have considerable effect in hastening the hoped-for improvements in the teaching of mathematics in grades K-12.

- **Articulation**

The entrance requirements and course prerequisites of the nation's colleges and universities are major factors in determining the topics in the secondary school curriculum as well as the amount of time devoted to them. Efforts to change the curriculum at the secondary level must be carried out in a cooperative effort with the colleges and universities.

● Equal Access

The disparity of access to computers between children who have a computer at home and children who do not threatens to widen the educational gap that already exists between different economic strata. It is urgent to design programs to address this problem.

● Women and Minorities

The conference noted with satisfaction the improvement during recent years in the participation of women in upper secondary mathematics. The many efforts that have led to this improvement must continue to be supported. We look forward to corresponding success with minority and handicapped students.

WORKING GROUP REPORT: ELEMENTARY AND MIDDLE SCHOOL MATHEMATICS

Arithmetic and, more generally, quantitative thought and understanding continue to become more important for more people, but the importance of various aspects of arithmetic has changed and will continue to change as computers and calculators become pervasive in society. The suggestions below are designed better to equip students for life and effective functioning in the developing age of technology. We believe implementation of these suggestions into the K-8 curriculum will make students more adaptive to future change, better equipped to use modern technology, better grounded in the mathematical bases for other sciences, and better grounded for further school mathematics.

A principal theme of K-8 mathematics should be the development of number sense, including the effective use and understanding of numbers in applications as well as in other mathematical contexts.

The changes we propose are fairly substantial, but are primarily in emphasis rather than in overall content. We believe they are consistent with, and are natural outgrowths of, recommendations relative to K-8 education of the earlier valuable documents, *Basic Mathematical Skills* by NCSM and *An Agenda for Action* by NCTM.

When implemented, the changes will be only modest at the K-3 level but more significant at the 4-6 and 7-8 grade levels. They essentially replace excess drill in formal paper-and-pencil computations with various procedures to develop better number sense on the part of the student.

Here is a list of special concerns:

1) Thorough understanding of and facility with one-digit number facts are as important as ever.

2) The selective use by students of calculators and computers should be encouraged, both to help develop concepts and to do many of the tedious computations that previously had to be done using paper and pencil.

3) Informal mental arithmetic should be emphasized at all levels, first aimed at exact answers and later at approximate ones. Such activity is necessary if students are to be able to decide whether computer or calculator printouts or displays are reasonable and/or make sense. Informal mental arithmetic involves finding easy, not formal algorithmic, ways of looking at number relationships.

4) There should be heavy and continuing emphasis on estimation and approximation, not only in formal round-off procedures, but in developing a feel for numbers. Students need experience in estimating real world quantities as well as in estimating numerical quantities which appear in complicated form. Methods requiring explicit (right or wrong) answers should be used where possible to help develop estimating procedures. For example, many exercises on comparing fractions with easy ones (eg., $\frac{12}{25}$ with $\frac{1}{2}$, and $\frac{103}{299}$ with $\frac{1}{3}$) can be used to get

students to think of complicated fractions as close to, but less than (or more than), easy fractions.

5) There should be a heavy and continuing emphasis on problem-solving, including the use of calculators or computers. Trial and error methods, guessing and guestimating in solving word problems should be actively encouraged at all levels to help students understand both the problems and the use of numbers. Naturally, examples and illustrations should be appropriate to the students' age, interest, and experience.

6) Elementary data analysis, statistics, and probability should be introduced, or expanded in use, including histograms, pie-charts, and scatter diagrams. The understanding and use of data analysis is becoming a vital component of modern life. The collection and analysis of data should include personal data of meaning to students, e.g., number of siblings, students' ages, heights and weights, data culled from newspapers, almanacs, and magazines, random data such as that produced by urn schemes and data from experiments in other school subjects.

7) Place value, decimals, percent, and scientific notation become more important. Intuitive understanding of the relative sizes of numbers that arise in the everyday world of applications becomes even more vital.

8) More emphasis on the relationship of numbers to geometry including, for example, number lines and plotting, should lead to better understanding of the concepts of arithmetic and of geometry.

9) Understanding of fractions as numbers, comparison of fractions, and conversions to decimals should have more emphasis while drill on addition, subtraction, and division of numerical fractions with large denominators should have less.

10) Drill on the arithmetic operations on three- (and larger) digit numbers should be de-emphasized. Such computations can and should be done by calculators and computers.

11) Intuitive geometric understanding and use of the mensuration formulas for standard two- and three-dimensional figures should be emphasized. More stress on why the formulas make sense is needed.

12) Function concepts including dynamic models of increasing or decreasing phenomena should be taught. (For more details, see 4) in "Traditional Secondary School Mathematics.")

13) The concepts of sets and some of the language of sets are naturally useful in various mathematical settings and should be used where appropriate. However, sets and set language are useful tools, not end goals, and it is inappropriate to start every year's program with a chapter on sets.

14) Based on motivation from arithmetic, algebraic symbolism and techniques should be encouraged, particularly in grades 7 and 8.

15) More extensive use of mathematics and computers in social science and science courses should be actively pursued. We encourage the consideration of this matter by experts in these fields and welcome opportunities to collaborate on further work in this area.

A discussion of possible computer programming or computer literacy courses is left to other groups for further study.

We call the Commission's attention to the fuller discussions and comments related to the K-8 curriculum in various position papers discussed at this conference including those of Anderson, Hilton, Pollak, and Willoughby.

Implementation Concerns

1) We hope the Commission will encourage widespread public discussion of the implications for K-8 mathematics of the changing roles of arithmetic in society. As an early step, we suggest discussions and conferences between teachers, supervisors, mathematics educators, mathematicians and editors of textbook series concerning this report and others on the same general topic. Such conferences could be quite inexpensive if most participants are local.

2) We hope the Commission will seek ways to encourage the development and use of textbooks for students and of teacher-training materials in the spirit of the suggestions made above.

3) We hope the Commission will seek ways to encourage changes in standardized tests toward number sense and problem-solving and away from single-operation computational skills.

4) We hope the Commission will encourage school systems to reassign interested teachers at the 4-6 grade level to become specialists at teaching mathematics or other disciplines. One mode might be a simple trade of classes between teachers with each teacher concentrating in areas of particular interest and competence. The needed changes in subject matter emphasis will be much easier to effect if those actually teaching any subject are selected for their special interests and aptitudes. Special inservice training programs should be developed for all such semi-specialized teachers, whatever their subject.

5) We hope the Commission will seek ways to improve the status of teachers and the conditions under which teachers attempt to do the important and difficult job of educating future citizens.

6) We believe that the needed changes can be brought about somewhat gradually and with general support of those concerned. There already is discussion in teacher and supervisor groups concerning many of the ideas put forth here.

The proposed changes generally involve modifications in the way mathematics is introduced and used in schools rather than adding new subject matter. The changes should permeate texts and not just be add-ons that can be ignored. There appears to be an approximate balance in time between topics needing more emphasis and those needing less. With the exception of computer use and the possible exception of parts of data analysis, the topics needing added emphasis have been taught and learned in American schools at various times and places in the past. The diminished role of paper and pencil computation is perhaps the topic which will provoke most concern and possible disagreement.

WORKING GROUP REPORT: TRADITIONAL SECONDARY SCHOOL MATHEMATICS

Current secondary school mathematics curricula are organized into separate year-long courses covering algebra, geometry, and precalculus topics. There are proposals that challenge this traditional division of school mathematics and the position of calculus as the primary goal for able college-bound students. Thus, the following analysis uses conventional course headings for discussion of proposed changes in traditional topics, not as endorsement of the status quo.

1) Overall Recommendation

The traditional component in the secondary curriculum can be streamlined, leaving room for important new topics. However, since breakthroughs in technology which allow this streamlining are so recent and the conceivable implications so revolutionary, it is not yet entirely clear what specific changes are appropriate.

2) Algebra

The basic thrust in Algebra I and II has been to give students moderate technical facility. When given a problem situation, they should recognize what basic algebraic forms they have and know how to transform them into other forms which might yield more information. In the future, students (and adults) may not have to do much algebraic manipulation -- software like mu-Math will do it for them -- but they will still need to recognize which forms they have and which they want. They will also need to understand something about why algebraic manipulation works, the logic behind it. In the past, such recognition skills and conceptual understanding have been learned as a by-product of manipulative drill, if learned at all. The challenge now is to teach these skills and understanding even better while using the power of machines to avoid large time allotments to tedious drill. Some blocks of traditional drill can surely be curtailed, e.g., numerical calculations using look-up and interpolation from logarithm and trigonometry tables.

3) Geometry

A primary goal of the traditional Euclidean geometry course is to develop logical thinking abilities. But not every fact need be given a rigorous proof to pursue this goal. Nor need this be the only goal of geometry, nor geometry the only means towards this goal.

We recommend that classes work through short sequences of rigorously-developed material, playing down column proofs, which mathematicians do not use. These proof sequences should be preceded by some study of logic itself. Important theorems not proved can still be explained and given plausibility arguments, and problems involving them can be assigned. The time which becomes available because proofs are de-emphasized can be devoted to study of algebraic methods in geometry, analytic geometry and vector algebra, especially in three dimensions. Work in three dimensions is essential if one is to develop any pictorial sense of relations between many variables, and handling many variables is essential if one is to model phenomena realistically.

There is much room for using computers in geometry. The power of graphics packages makes it much easier for students to get a visual sense of geometric concepts and transformations. The need to use algebraic descriptions of geometric objects when writing graphics programs reinforces analytic geometry. Finally, the algorithmic thinking needed to write programs bears much resemblance to the thinking required to devise proofs.

4) Precalculus

What often happens in this course is that students see the same topics yet another time, with more drill but with little new perspective. For better students there may not be a need for a precalculus course if drill is no longer so important and if algebra and geometry are done "right," with the concepts made clear. For instance, one justification for the precalculus course is the perceived need to develop the idea of functions; functions appear in Algebra I and earlier, but current teaching may give too static an understanding. With computers, the concept of function can be made central earlier and more clearly. The computer supports qualitative analysis of the graphs of functions, in a dynamic mode of display, and also allows detailed analysis of zeros, rates of change, maxima/minima, etc.

5) Algorithmics

Computers and programming have made the creative human talents and skills involved in developing and analyzing algorithms extremely important. These talents and skills, emphasized by the group on non-traditional topics, can be exercised quite naturally through traditional topics as well. Much of high school algebra consists of systematic methods for handling certain problems, e.g., factoring polynomials. Such methods are algorithms. Instead of making the student carry out such methods with paper and pencil a boring number of times, have the student do it just a few times and then program a computer to do it. The understanding gained should be at least as great.

6) The Average Student

For the many students in secondary school who are not specially talented in mathematics and not headed for careers in science or technology, current programs are a source of discouragement, anxiety and repetition in a dull "basic skills" program which serves them poorly. We cannot ignore the needs of this large and important group. Computers, as mathematical tools and media of instruction, offer a fresh window into mathematics for them.

7) Cautions

We have suggested that technology provides an opportunity to devote less time to traditional techniques while boosting understanding and allowing more time for more complex, realistic problem-solving. However, there are several cautions. First there are widespread and deep reservations about how much traditional goals should give way to technology. Second, there is little research data on the feasibility of such changes, and there are almost no prototype school curricula embodying the new priorities. Experimental programs, and research on the results, must be given major support. Third, changes in secondary programs must be carefully articulated with the expectations of colleges and employers, who often have conservative views about curricula. Finally, the syllabi of an extensive range of standardized tests play a very influential role in setting curricula and the actual classroom emphases of teachers. If curricula are to change, the tests must be changed. Clearly, strong national leadership and cooperation are necessary, from teachers, mathematicians and public policy-makers, to meet these challenges and implement significant change.

WORKING GROUP REPORT: NON-TRADITIONAL SECONDARY SCHOOL MATHEMATICS

On two basic principles the panel was unanimous:

- There is need for substantial change in both the subject matter of and the approach to teaching in secondary school mathematics.
- If changes are to be made in secondary school mathematics, we must make haste slowly, taking care at all times to insure full consultation with and support from the secondary school mathematics teaching community.

Our specific recommendations are grouped under five headings: Subject Matter, Approach to Teaching, The Use of New Technology, Teacher Training and Implementation.

1) Subject Matter

Careful study is needed of what is and what is not fundamental in the current curriculum. Our belief is that a number of topics should be introduced into the secondary school curriculum and that all of these are more important than, say, what is now taught in trigonometry beyond the definition of the trigonometric functions themselves. These topics include discrete mathematics (e.g., basic combinatorics, graph theory and discrete probability), elementary statistics (e.g., data analysis, interpretation of tables, graphs, surveys, sampling) and computer science (e.g., programming, introduction to algorithms, iteration).

2) Approach to Subject Matter

The development of computer science as well as computer technology suggests new approaches to the teaching of all mathematics in which emphasis should be on:

- algorithmic thinking as an essential part of problem-solving
- student data gathering and investigation of mathematical ideas in order to facilitate learning mathematics by discovery.

3) Technology

New computer technology allows not only the introduction of pertinent new material into the curriculum and new ways to teach traditional mathematics but it also casts doubt on the importance of some of the traditional curriculum, just as the hand calculator casts similar doubts about instruction in arithmetic. Particularly noteworthy in this context at the secondary level are:

- Symbolic manipulation systems which even now,but certainly far more in the near future,will allow students to do symbolic algebra (and calculus) at a far more sophisticated level than they can be expected to do with pencil and paper.
- Computer graphics and the coming videodisc systems which will enable the presentation and manipulation of geometric and numerical objects in ways which should be usable to enhance the presentation of much secondary school mathematical material.

One caveat which we would stress is that this technology and related software packages must be used not to enable students to avoid understanding of the essential mathematics but rather to enhance such understanding and to allow creative experimentation and discovery by students as well as to reduce the need for tedious computation and manipulation.

4) Teacher Training

There are two aspects of this:

a) Retraining of current teachers in the new topics, approaches and technology. One possible new approach to this might be the use of college students to aid and instruct secondary school personnel as part-time employees and perhaps through such incentives as forgiveness of student loans.

b) Education of new teachers

Crucial to long-term solution of the secondary school mathematics education problem is that the requirements for degrees in mathematics education be, as necessary, changed to incorporate modern content and approaches. In particular, we believe that all prospective teachers of secondary school mathematics should be required to take at least:

- one year of discrete mathematics in addition to traditional calculus requirements
- one semester or one year of statistics (with focus on statistical methods rather than mathematical statistics)
- one year of computer science.

5) Implementation

We recognize that the kinds of changes proposed here not only require much more study than has been possible by our panel but that also they will never be implemented unless there is dedicated cooperation among:

- secondary school teachers of mathematics and their professional organizations
- college curriculum people in schools of education and in mathematics departments and including their organizations
- state and local education authorities and their organizations.

A conference at an early date bringing together these groups to discuss the relevant problems and plan future action might be the most fruitful next step to provide some momentum for the changes we believe are necessary.

WORKING GROUP REPORT: THE ROLE OF TECHNOLOGY

Computers and related electronic technology are now fundamental features of all learning and working environments. Students should be exposed to and utilize this technology in all aspects of school experience where these devices can play a significant role.

We recommend:

1) The potential of technology for enhancing the teaching of mathematics and many other subjects is vast. Development of such resources should be supported at a national level. Specific examples include computer-generated graphics, simulations, and video-disc courseware materials. There should be efforts to create a network providing easy access to such banks of material.

2) While computing technology offers promise to enhance learning, differential access to the benefits of that technology could widen the gaps in educational opportunity which already separate groups in our society. It is imperative that every effort be made to provide access to computers and their educational potential for all sectors of society.

3) As a general principle, each mathematics classroom should have available computers and other related electronic technological devices to facilitate the computing and instruction required for mathematics learning and competency. Such availability of computers and other electronic technological devices in the mathematics classroom is as important as the availability of laboratory equiment for science instruction.

4) Hand calculators should be available in mathematics classrooms (both in elementary and secondary schools) for students on the same basis that textbooks are now provided.

5) Support should be given for broad developments in software that may be useful in the schools. School districts should encourage their teachers and students to engage in cooperative development activities and to find ways to recognize and disseminate the products of those efforts.

6) Computer literacy involves not only the use of computers to accomplish a great spectrum of tasks but also a general understanding of the capabilities and limitations of computers and their significance for the structure of our society. Development and implementation of appropriate programs to teach these more general concepts should be supported.

7) Possible curricular changes emanating from technological changes will require careful study and deliberation over a long period of time. This activity must be encouraged and supported from a national level. The exploratory projects should bring together teachers, curriculum developers, mathematicians, and affected interested parties from business and industry. The new programs developed should be tested extensively in a variety of settings to insure that they work with real students and schools before extensive implementation is attempted.

8) The interplay between word-processing, computers, data bases, and data analysis methods assist in breaking down barriers between disciplines thus offering an opportunity for schools to provide a range of holistic problem-solving experiences not typical in school today. Using the technology as an aid, students can plan and conduct data collection, analysis, and report writing that is realistic, attractive, and far beyond normal expectations in today's schools.

9) The need for well-trained, highly qualified teachers of mathematics is a must in a technological society. Support should be given to organizing programs for inservice training and retraining of current teachers of mathematics (elementary and secondary) who are inadequately prepared to teach a technologically-oriented curriculum, but have the capacity to profit from such programs to strengthen their mathematical preparation and teaching skills.

10) While technology provides opportunities, it also makes demands. The world becomes a more complex place in which to live. If we are to insure that a broad spectrum of society can function and participate actively in the business/industrial community and decision-making of the country, it is imperative that students become adept in the precise, systematic, logical thinking that mathematics requires.

WORKING GROUP REPORT: RELATIONS TO OTHER DISCIPLINES

Along with the effects of computational technology on the mathematics curriculum, it is also necessary to consider how this technology and the proposed curricular changes affect the relationships between the curricula in other disciplines and the curriculum in mathematics. We have interpreted the phrase "other disciplines" rather broadly.

First, using a narrow view, we must look at the effects these curricular changes will have on science education. There has always been a necessary interaction and coordination between the science and mathematics curricula, particularly with the physical sciences. At a minimum, this revised curriculum, which encourages student use of calculators and computers and emphasizes a good sense of estimation, provides an opportunity for elementary and high school education to be more realistic and eliminate the use of specialized problems with "easy numbers." If we raise our sights a bit, there is an opportunity for a better coordinated and integrated total science education. Furthermore, the introduction of statistical ideas, data handling procedures, and discrete mathematics provide an opportunity for a more mathematical discussion of social science problems at the elementary and high school levels. Similarly, changes in currently available tools will undoubtedly affect courses in business and commercial programs.

Related questions arise on the other side. What do the school programs and the college programs in natural sciences, social science, and business require in the mathematical preparation of entering students ? We believe the suggested curriculum can only be an improvement, but discussion with leaders of those disciplines are required.

Taking the broad view, we also believe that this modified curriculum, which provides students with the same (or greater) ability to use mathematics as well as an ability to use and appreciate the technology, will provide for a wiser citizenry. The graduates of such a program should be better equipped to deal with "poll results" and statistical data references to the economy and sociological problems.

We believe there is one serious area in which the nation needs more data for the development of an appropriate mathematics curriculum. Namely, what are the needs, in terms of mathematical skills, of the students who seek technical vocational employment without going on to further schooling ? Furthermore, what are the mathematical needs of students going on to technical or vocational schools ? Although we do not know the answers, we believe the new curriculum will do at least as good a job as the existing one. A conference or meeting to explore this area would be an excellent idea and would complement our work.

WORKING GROUP REPORT: TEACHER SUPPLY, EDUCATION, AND RE-EDUCATION

Efforts to improve and up-date the mathematics curriculum and to increase the mathematics, science, and technology literacy of all citizens require the support of qualified mathematics teachers at all levels. At present there is a serious and well-documented shortage of qualified teachers of mathematics at the elementary and secondary school levels in many areas of the country. Economic, employment, and social conditions forecast that the current short supply may indeed be a long-term problem. Furthermore, even in geographic locations where adequate supplies exist, the frequent turnover of mathematics teachers tends to impede learning.

The following recommendations address the need to increase the supply of mathematics teachers as well as to improve the quality of the teaching and thereby, the learning, of mathematics:

1) While state and local efforts by industry, business, and academe to deal with the teacher shortage are laudable and should continue, the magnitude of the problem is national in scope. An articulated national commitment with federal leadership and support is needed for its resolution. The public should be made aware of the problem through more effective publicity.

2) Incentives of all types need to be studied to *attract* and *retain* qualified teachers of mathematics. Financial incentives should be given special attention with priority assigned to those which do not create undue inequities and tensions among colleagues, in order to avoid being counterproductive.

Examples of possible incentives and support systems include the following:

a) Forgiveness of student loans and/or interest on loans for those who enter the teaching field.

b) High entry level salaries for special expertise (e.g., computer training).

c) Reduced teaching loads to allow teachers to pursue graduate study or other advanced training in the mathematical sciences and applied areas.

d) Financial support of graduate study or other advanced training in the mathematical sciences and applied areas.

e) Salary differentials by discipline.

f) Summer positions and other cooperative arrangements with business and industry to supplement a teacher's income (with the obvious caveat that the short supply of teachers is largely due to the fact that higher industrial salaries lure teachers away; industry would have to be discouraged from using this arrangement for recruitment purposes.)

3) In an era when content and technology are changing so rapidly, incentives are needed to keep qualified teachers in the field abreast of current trends in the mathematical sciences. Inservice workshops, NSF-type institutes, retraining courses, industrial experiences, and other forms of continuing education can serve to refresh the faculty and renew their commitment to teaching.

4) In some parts of the country, teachers from other disciplines are being assigned to teach mathematics classes. These teachers need special subject matter training and assistance in developing appropriate teaching strategies in order to succeed in their new assignments.

5) Encouraging colleges and universities to loan their faculty, and business and industry to loan their mathematically-oriented employees to teach courses in the secondary schools could be mutually beneficial. Qualified retirees or near-retirees might also be recruited to enter the teaching field. (Of course, the issues of appropriate teacher training and certification need to be addressed.)

6) In states where this is not the norm, it is recommended that teacher certification requirements be stated in terms of the specific topics to be covered in the subject area rather than in terms of just total number of credits.

7) Recommendations regarding the mathematical fundamentals to be covered in educating qualified teachers of mathematics include:

a) Elementary Level

It is strongly suggested that mathematics at the elementary school level be taught by teachers who specialize in mathematics. Whether the teacher specializing in mathematics should be assigned to all grades or just to grades 4-6 (or 4-8) requires further study. An alternative approach would be to identify those teachers in a given school who most enjoy teaching mathematics. Those teachers could be assigned to teach all mathematics courses across a grade level, while other teachers do similarly in reading and writing.

The following recommendations pertain to both the regular elementary school teacher and the teacher specializing in mathematics:

For entry into the mathematics education program for elementary school teachers, at least three years of college-track mathematics in high school is recommended. College mathematics courses should provide a sufficient background to understand the relationships between algebra and geometry, functions, elementary probability and statistics, instruction in the use of the hand-held calculator, and some exposure to computers. Creative approaches to problem-solving should also be included in the curriculum. Training should be at least one level above what is being taught. This background is particularly important in light of children's awareness of the world around them through television, other media, computers, and so on.

b) Secondary Level

Secondary school mathematics teachers should have course work in mathematics equivalent to a major in mathematics. Requirements for those who will teach mathematics should include the equivalent of a two-year calculus and linear algebra sequence, discrete mathematics, probability and statistics, and appropriate computer training. These courses should develop in the student a sense of "mathematical maturity" in the approach to problem-solving.

Note: College and university curricula for educating mathematics teachers should be re-examined and revised in accordance with the above guidelines and goals. Contingency plans should be developed in case separate departments of mathematics and computer science are established at the secondary level in the future.

Conclusion

The recommendations cited here require careful planning and implementation. With high technology a mainstay of our present and future society, it is imperative that we recognize and promote mathematics as a powerful, useful, and enjoyable component of our lives.

The reader is referred to the following documents for additional information:

Precollege Teacher Training and Retraining in Light of Expected Changes in School Mathematics. Richard D. Anderson, September 16, 1982.

Testimony Presented to the National Commission on Excellence in Education. Henry L. Alder, March 11, 1982.

Testimony Presented to the National Commission on Excellence in Education. Harold D. Taylor, March 11, 1982.

Appendix A

NSF/CBMS Meeting Participants
September 25-26, 1982

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Appendix B

Agenda
NSF/CBMS Meeting

THE MATHEMATICAL SCIENCES CURRICULUM K-12: WHAT IS STILL FUNDAMENTAL AND WHAT IS NOT?

September 25-26, 1982

Saturday, September 25

9:00 a.m. - 9:30 a.m.	Opening Remarks, Henry Pollak	Begle Room
	The NSB Commission, Katherine Layton and Frederick Mosteller	
9:30 a.m. - 10:30 a.m.	General Discussion Session: The Position Papers	Begle Room
10:30 a.m. - 10:45 a.m.	Break	
10:45 a.m. - 12:00 noon	General Discussion Session: The Position Papers (continued)	Begle Room
12:00 noon - 1:00 p.m.	Lunch	
1:00 p.m. - 2:00 p.m.	General Discussion Session The Position Papers (continued)	Begle Room
2:00 p.m. - 3:30 p.m.	Working Group Sessions	Conference rooms
3:30 p.m. - 3:45 p.m.	Break	
3:45 p.m. - 5:00 p.m.	General Discussion Session: The Working Group Papers	Begle Room

Sunday, September 26

8:30 a.m. - 9:00 a.m.	General Discussion Session: New Working Group Topics	Begle Room
9:00 a.m. - 10:30 a.m.	Working Group Sessions	Conference rooms
10:30 a.m. - 10:45 a.m.	Break	
10:45 a.m. - 12:30 p.m.	General Discussion Session: The Working Group Papers and The Report to NSF	Begle Room

SOME COMMENTS ON THE AGENDA

Henry Pollak

The proposal under which this meeting is being funded states the following. "The purpose of this meeting is to provide the NSB Commission with advice and guidance from leaders in the mathematical sciences community on the changes that are needed to bring the nation's mathematical sciences curriculum (K-12) into the age of technology. While CBMS cannot hope to produce a final definitive document from a single meeting, we do think it is possible to identify some general principles, to formulate theose questions most in need of further investigation, and to propose some mechanisms for conducting such investigations."

The word "curriculum" in this should not be narrowly construed. It includes questions of subject matter, of pedagogy, of teacher qualifications and teacher education, of relationships to other disciplines and of the relationship to technology. We want to look into all of these at this meeting and pull together our knowledge and our views. In some cases, we will be able to give some pretty definitive answers; in others we will formulate questions in urgent need of further investigation and propose mechanisms for undertaking such investigations.

In case you need convincing that there is enough work to keep us busy this weekend, here are some examples: Do we agree that estimation must have a much greater role in elementary mathematics than it has traditionally had? Do we agree that too much time is traditionally spent on arithmetic manipulations, and the hand-held calculator should have a major effect on the curriculum (in the broad sense that I have used the term)? What about the future of the material and of the point of view on symbolic algebraic manipulations in secondary school? What is the place of data analysis in the schools? What about decision making, statistics, algorithms, discrete mathematics, and probability? What about microprocessors and video discs and graphics terminals?

Why have we organized the Conference the way we have? We have position papers on many topics, and the next item of business will be to discuss them. After that, we need to break up into smaller groups, roll up our sleeves and get to work. It seemed to us that we should first break up by school level and then, using the results of the first discussions, break up again by some of the major issues on which our advice will be so important. We are going to ask each of you to give us your preferences within Working Groups A and Working Groups B. On each topic we may have one or more working groups, depending on how many of you sign up. We will respect your preference, with the boundary condition that each topic does need to be covered. Each working group will come up with opinions, statements, conclusions and recommendations for the benefit of the entire group.

There is an interesting and in my opinion constructive contradiction in what we are trying to do. We have here an excellent group of knowledgeable, innovative people who we hope will have lots of original ideas in the areas up for discussion. On the other hand, we want also to take up and reach consensus in areas that recently been much discussed, both by ourselves and others. The very important fact will be that this group representing the mathematical sciences community as no other can, has agreed. This says something different from an individual having come to a particular insight or conclusion. Both new ideas and affirmations of current ideas will be important to the NSB Commission.

There will be problems and issues on which our conclusion will be that further work is needed - and here is what form this might take. In some cases it will be further work which CBMS might undertake, in other cases it may be CBMS together with other organizations or individual organizations within CBMS. Such recommendations will also be part of our report to the NSB Commission. It is possible that additional efforts will indeed be undertaken as a result.

APPENDIX C

Position Papers by Meeting Participants

Richard, D. Anderson, *An Analysis of Science and Engineering Education: Data and Information*

Richard D. Anderson, *Precollege Teacher Training and Retraining in Light of Expected Changes in School Mathematics*

Richard D. Anderson, *Arithmetic in the Computer/Calculator Age*

James Baldwin, Position Paper

James M. Landwehr, *Memo on Activities of American Statistical Association*

Stephen Willoughby, Position Paper

APPENDIX D

Position Papers by Non-participants

Henry Alder, *List of Temptations to Resist*

Peter Hilton, *The Role and Nature of Mathematics: Implications for the Teaching of Mathematics Today*

Stephen White, *Notes on K-8 Math* .