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ABSTRACT

Several extended caution indices (ECIs) have been introduced earlier as a link between two distinctly different approaches: one based on standard statistics and the other, a model-based approach, utilizing item response theory (IRT). Expected values and variance of some ECIs are derived and their statistical properties are compared and discussed. Then, standardized ECIs are introduced and their distributions are investigated. It turns out that the standardized ECIs fit normal distributions well. A comparison of detection rates among appropriateness measures based on IRT theory is carried out with the signed-number data set. There is no noticeable difference in their detection rates using the 80 percent intervals. (Author)

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STANDARDIZED EXTENDED CAUTION INDICES AND COMPARISONS OF THEIR RULE DETECTION RATES

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Several extended caution indices (ECIs) have been introduced earlier as a link between two distinctly different approaches: One based on the standard statistics and the other, a model-based approach utilizing item response theory (IRT). Expected values and variances of some ECIs are derived and their statistical properties are compared

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Abstract

Several extended caution indices (ECIs) have been introduced earlier as a link between two distinctly different approaches: one based on standard statistics and the other, a model-based approach utilizing item response theory (IRT). Expected values and variances of some ECIs are derived and their statistical properties are compared and discussed. Then, standardized ECIs are introduced and their distributions are investigated. It turns out that the standardized ECIs fit normal distributions well. A comparison of detection rates among appropriateness measures based on IRT theory is carried out with the signed-number dataset. There is no noticeable difference in their detection rates using the 80% intervals.

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Introduction

An increasing number of researchers have begun to show interest in using response patterns of n items for analyzing performance on test scores. By so doing, more information is obtainable than by using only traditional total scores. Tatsuoka and her colleagues (Birenbaum & Tatsuoka, 1982a, b; Tatsuoka & Tatsuoka, 1982a) have demonstrated that some wrong rules of arithmetic computations (fractions and signed-numbers) can produce the right score of 1 on as much as 60% of the test items. If many students apply a variety of wrong rules consistently throughout the test, then these faulty rules cause a serious problem by violating the unidimensionality assumption of a dataset. After rescoring these correct responses obtained by faulty rules, the dataset became nearly unidimensional. They have developed several indices to detect aberrant response patterns resulting from consistent application of wrong rules (Tatsuoka & Tatsuoka, 1982b) and have shown one of them, the individual consistency index (ICI), to spot more than 90% of such aberrant response patterns (Tatsuoka & Tatsuoka, 1981).

Rudner (1982) investigated the detection rates of various personal indices (norm conformity index, caution index, personal biserial and appropriatness measures based on item response theory) and found that the indices based on IRT are more efficient for detecting anomalous response patterns than those based on observed item response and summary statistics. However, estimating parameters of IRT models requires a substantial number of subjects while it is often impossible to have such a large sample size in many classroom settings.

Sato (1975) developed the caution index in conjunction with S-P curve theory and succesfully used it for diagnosing students' performance and evaluating instructional materials in Japan. Harnisch and Linn (1981) demonstrated its usefulness by applying it to a NAEP dataset (National Assessment of Educational Progress). Although their analysis is based on a large dataset, their results show clearly that analysis of response patterns as a whole provides very useful information associated with individual differences, curriculum differences and school differences.

The concepts of S-P curve theory and caution index have been extended to the continuous domain of IRT models from the approach based on the discrete summary statistics by Tatsuoka and Linn (1982). They have developed five alternative indices and named them extended caution indices 1, 2, 3, 4 and 5. In this paper, further statistical properties of ECI1, 2, and 4 will be discussed and their detection rates will be compared.

Statistical Properties of Extended Caution Indices

Definition of the Extended Caution Indices

A group of extended caution indices (ECI) has been introduced as a link between two distinct approaches of detecting aberrant response patterns (Tatsuoka & Linn, 1981). One is based on the use of binary response patterns and their standard summary statistics (Sato, 1975; van der Flier, 1977; Tatsuoka & Tatsuoka, 1980, 1982a), while the other is a model-based approach. In the latter, the patterns of probabilities that are derived from item response theory are utilized in calculating appropriateness measures together with observed binary response patterns

(Wright, 1977; Drasgow, 1978; Levine & Rubin, 1979). ECIs are an extension of Sato's caution index to the approach using IRT. In this section, three of the five ECIs will be investigated in terms of their expected values, variances, and advantages and disadvantages.

Let y_{ij} $[i=1,\ldots,N;\ j=1,\ldots,n]$ be the binary score of subject i to item j, $y_{i.}$ be the ith row sum, and $y_{.j}$ the jth column sum of the data matrix (y_{ij}). Let P_{ij} be the probability of subject i answering item j correctly, which may be based on the one-, two- or three-parameter logistic model. That is,

$$P_{ij} = c_j + \frac{1 - c_j}{1 + \exp[-Da_j\,(\theta_i - b_j)]} .$$

where $c_j = 0$ and $a_j = 1$ for the one-parameter logistic model; $c_j = 0$ for the two-parameter logistic model. Thus, two data matrices -- one comprising observed binary scores of n items for N subjects (y_{ij}) and the other consisting of (P_{ij}) -- may be introduced. We refer to (y_{ij}) as the observed binary matrix and (P_{ij}) as the probability matrix.

Let G_j be the jth element of a vector approximating the group response curve (GRC) for item j, and T_i be that of the vector for the test response curve (TRC) for subject i. Then

$$G_j = \frac{1}{N} \sum_{i=1}^{N} P_{ij} \quad ,$$

$$T_i = \frac{1}{n} \sum_{j=1}^{n} P_{ij} \quad .$$

In other words, G_j for item j and T_i for subject i are the jth column sum and the ith row sum, respectively, of the probability matrix (P_{ij}).

Three of the five ECIs are defined as complements of the ratio of two covariances between various pairs of row vectors taken from the two matrices.

$$\mathrm{ECI1}_i = 1 - \frac{\mathrm{cov}(\underset{\sim}{y_i}\ ,\ \underset{\sim}{y.})}{\mathrm{cov}\ (\underset{\sim}{p_i}\ ,\ \underset{\sim}{y.})} \tag{1}$$

$$\mathrm{ECI2}_i = 1 - \frac{\mathrm{cov}(\underset{\sim}{y_i}\ ,\ \underset{\sim}{G})}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P_i})} \tag{2}$$

$$\mathrm{ECI4}_i = 1 - \frac{\mathrm{cov}(\underset{\sim}{y_i}\ ,\ \underset{\sim}{P_i})}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P_i})} \tag{3}$$

where $\underset{\sim}{y_i} = (y_{i1}, y_{i2}, \ldots, y_{in})$, the vector of binary scores for subject i or the ith row vector,

$\underset{\sim}{y.} = (y_{.1}, y_{.2}, \ldots, y_{.n})$, the column-sum vector in the observed binary matrix,

$\underset{\sim}{P_i} = (P_{i1}, P_{i2}, \ldots, P_{in})$, the probability vector from the ith row in the probability matrix, and

$\underset{\sim}{G} = (G_1, G_2 \ldots, G_n)$, the GRC vector which is the column-sum vector of (P_{ij}). Expression (1) is defined by forming the ratio of the following covariances: the numerator is the covariance of subject i's response pattern and the column-sum vector over n items in (y_{ij}), and the denominator is the covariance of the ith row probability vector derived from a logistic model and the column-sum vector in (y_{ij}). Expressions (2) and (3) have the same denominator, the covariance of the GRC vector and the ith probability vector, and the numerators are covariances of the response pattern vector with the GRC vector and the probability vector, respectively.

When $\underset{\sim}{y_i}$ consists of all 1s or 0s, the second terms of the ECIs become undetermined.

The expectations of ECI1, ECI2 and ECI4

In this section, the expectations and variances of the three ECIs given by Equations (1), (2) and (3) will be derived. The actual values of the ECIs for subject i can be calculated by replacing the item and person parameters with their estimated values $\hat{a}_j$, $\hat{b}_j$ and $\hat{\theta}_i$ based on the maximum likelihood method. It is known that the maximum likelihood estimates of item and person parameters satisfy the likelihood conditions (Lord and Novick, 1968) given in Equations (4).

$$\sum_{j=1}^{n} \hat{\theta}_i \hat{P}_{ij} = \sum_{j=1}^{n} \hat{\theta}_i y_{ij}$$

$$\sum_{j=1}^{n} \hat{P}_{ij} = \sum_{j=1}^{n} y_{ij}$$

$$\sum_{j=1}^{n} \hat{a}_j \hat{P}_{ij} = \sum_{j=1}^{n} \hat{a}_j y_{ij} \quad . \tag{4}$$

Since the ECIs are functions of the person parameter θ_i, the conditional expected values and variances of the ECIs for a fixed ability level will be introduced. Hereafter, the circumflex on $\hat{P}_{ij}$ (and its ith-row vector $\underset{\sim}{\hat{P}}_i$) will be omitted to simplify the notation.

ECI1

The conditional expectation of the first ECI defined in Equation (1) is given by the following:

$$E(\text{ECI1}|\theta_i) = 1 - E\left(\frac{\text{cov}(\underset{\sim}{y}_k \, , \underset{\sim}{y}_.)}{\text{cov}(\underset{\sim}{P}_i \, , \underset{\sim}{y}_.)} \middle| \, \theta_i\right)$$

$$= 1 - \frac{E[\text{cov}(\underset{\sim}{y}_k \, , \underset{\sim}{y}_. | \theta_i)]}{\text{cov}(\underset{\sim}{P}_i \, , \underset{\sim}{y}_.)} \quad . \tag{5}$$

The observed vector $\underset{\sim}{y}_k$ is a random vector at the level θ_i and the expectation is obtained over k. Now, we have to find the expectation in the numerator of the second fraction, $E[\mathrm{cov}(\underset{\sim}{y}_k , \underset{\sim}{y}_.)|\theta_i]$. First, the covariance of $\underset{\sim}{y}_k$ and $\underset{\sim}{y}_.$ is rewritten as the summation of the product of the deviations:

$$E[\mathrm{cov}(\underset{\sim}{y}_k , \underset{\sim}{y}_i)|\theta_i] = E[\sum_{j=1}^{n} (y_{kj} - p_{i.})(y_{.j} - p_{..})|\theta_i] / n$$

where $p_{i.}$ is the ith row mean of (y_{ij}) and $p_{..}$ is the mean of the row means or column means as follows,

$$p_{..} = \frac{1}{n} \sum_{j=1}^{n} p_{.j} = \frac{1}{N} \sum_{i=1}^{N} p_{i.} \quad .$$

By using the second members of Equations (4), this expectation reduces to the covariance of $\underset{\sim}{P}_i$ and $\underset{\sim}{y}_.$. Thus, the conditional expectation of ECI1 at the fixed level i becomes zero, as summarized in Equation (6).

$$E(ECI1|\theta_i) = 1 - \frac{\mathrm{cov}(\underset{\sim}{P}_i , \underset{\sim}{y}_.)}{\mathrm{cov}(\underset{\sim}{P}_i , \underset{\sim}{y}_.)} \equiv 0 \quad . \tag{6}$$

The conditional variance of ECI1 at the fixed level i is

$$\mathrm{Var}(ECI1|\theta_i) = E[ECI1 - E(ECI1|\theta_i)]^2 \quad . \tag{7}$$

By substituting the result from (6), the conditional variance (7) becomes $E(ECI1^2|\theta_i)$. That is:

$$\begin{aligned} E(ECI1^2|\theta_i) &= E([1 - \frac{\mathrm{cov}(\underset{\sim}{y}_k , \underset{\sim}{y}_.)}{\mathrm{cov}(\underset{\sim}{P}_i , \underset{\sim}{y}_.)}]^2|\theta_i) \\ &= -1 + \frac{E(\mathrm{cov}^2(\underset{\sim}{y}_k , \underset{\sim}{y}_.)|\theta_i)}{\mathrm{cov}^2(\underset{\sim}{P}_i , \underset{\sim}{y}_.)} \end{aligned} \tag{8}$$

where we have again used the fact that $E[\mathrm{cov}(\underset{\sim}{y}_k , \underset{\sim}{y}_.)] = \mathrm{cov}\ (\underset{\sim}{P}_i , \underset{\sim}{y}_.)$. The numerator of the last term of Equation (8), however, can be expanded to the sum of the diagonal and off-diagonal terms, and then by applying the conditions given in Equations (4), we obtain Equation (9).

$$\frac{1}{n^2} E([\sum_{j=1}^{n}(y_{kj} - p_{i.})(y_{.j} - p_{..})]^2|\theta_i)$$

$$= \frac{1}{n^2} E[\sum_{j=1}^{n}(y_{kj} - P_{i.})^2(y_{.j} - p_{..})^2|\theta_i]$$

$$+ \frac{1}{n^2} E(\sum_{j\neq h} (y_{kj} - p_{i.})(y_{kh} - p_{i.})(y_{.j} - p_{..})(y_{.h} - p_{..})|\theta_i)] \quad . \tag{9}$$

The first term, the diagonal part inside the parentheses of the above equation, is:

$$E[\sum_{j=1}^{n}(y_{kj} - p_{i.})^2(y_{.j} - p_{..})^2|\theta_i]$$

$$= \sum_{j=1}^{n}(y_{.j} - p_{..})^2 \, E[(y_{kj} - p_{i.})^2|\theta_i]$$

$$= \sum_{j=1}^{n}(y_{.j} - p_{..})^2[P_{ij}(1 - P_{ij}) + (P_{ij} - T_i)^2] \quad .$$

The second term inside the parenthesis is:

$$E(\sum_{j\neq h}(y_{kj} - P_{i.})(y_{kh} - P_{i.})(y_{.j} - P_{..})(y_{.h} - P_{..})|\theta_i)$$

$$= \sum_{j\neq h}(y_{.j} - P_{..})(y_{.h} - P_{..}) \, E[(y_{kj} - P_{i.})|\theta_i] \, E[(y_{kh} - P_{i.})|\theta_i]$$

$$= \sum_{j\neq h}(y_{.j} - p_{..})(y_{.h} - p_{..})(P_{ij} - T_i)(P_{ih} - T_i) \quad .$$

Adding the results of the two expectations gives Equation (10).

$$\frac{1}{n^2} E([\sum_{j=1}^{n}(y_{kj} - p_{i.})(y_{.j} - p_{..})]^2|\theta_i)$$

$$= \frac{1}{n^2} [\sum_{j=1}^{n}(y_{.j} - p_{..})(P_{ij} - T_i)]^2 + \frac{1}{n^2}[\sum_{j=1}^{n} (y_{.j} - p_{..})^2 P_{ij}(1 - P_{ij})]$$

$$= \text{cov}^2(\underset{\sim}{y_.}\ , \underset{\sim}{P_i}) + \frac{1}{n^2} \sum_{j=1}^{n}(y_{.j} - p_{..})^2\sigma_{ij}^{\ 2} \tag{10}$$

Substituting (10) in Equation (8), the variance of ECI1 becomes:

$$\mathrm{Var(ECI1)} = -1 + \frac{\mathrm{cov}^2(\underset{\sim}{y}_{.}\ ,\ \underset{\sim}{P}_i) + \sum_{j=1}^{n} \sigma_{ij}^2\, (y_{.j} - p_{..})^2/n^2}{\mathrm{cov}^2(\underset{\sim}{P}_i\ ,\ \underset{\sim}{y}_{.})}$$

$$= \frac{\sum_{j=1}^{n} \sigma_{ij}^2 (y_{.j} - p_{..})^2}{n^2 \mathrm{cov}^2(\underset{\sim}{P}_i\ ,\ \underset{\sim}{y}_{.})} \quad . \qquad (11)$$

ECI2

The conditional expectation of the second ECI is given by

$$E(\mathrm{ECI}_2|\theta_i) = 1 - E\left[\frac{\mathrm{cov}(\underset{\sim}{y}_k\ ,\ \underset{\sim}{G})}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)}\ \middle|\ \theta_i\right]$$

$$= 1 - \frac{E[\mathrm{cov}(\underset{\sim}{y}_k\ ,\ \underset{\sim}{G})|\theta_i]}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)} \quad . \qquad (12)$$

But

$$E[\mathrm{cov}(\underset{\sim}{y}_k\ ,\ \underset{\sim}{G})|\theta_i = \frac{1}{n} E[\sum_{j=1}^{n} (y_{kj} - p_{i.})(G_j - T)\ |\ \theta_i]$$

$$= \frac{1}{n} \sum_{j=1}^{n} E[(y_{kj} - p_{i.})(G_j - T)|\ \theta_i]$$

$$= \frac{1}{n} \sum_{j=1}^{n} (P_{ij} - T_i)(G_j - T) = \mathrm{cov}(\underset{\sim}{P}_i\ ,\ \underset{\sim}{G}) \quad ,$$

where $$T = \sum_{i=1}^{N} T_i/N = \sum_{j=1}^{n} G_j/n \quad .$$

By substituting this result in Equation (12), we get (13).

$$E(\mathrm{ECI2}|\theta_i) = 1 - \frac{\mathrm{cov}(\underset{\sim}{P}_i\ ,\ \underset{\sim}{G})}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)} = 0 \qquad (13)$$

The conditional variance of ECI2 is given by Equation (14),

$$\begin{aligned}\mathrm{Var}(\mathrm{ECI2}\,|\,\theta_i) &= E[(\mathrm{ECI2} - E(\mathrm{ECI2}))]^2\ |\theta_i) \\ &= E(\mathrm{ECI2}^2\ |\theta_i) \\ &= -1 + \frac{E[\mathrm{cov}^2(\underset{\sim}{y}_k\ ,\ \underset{\sim}{G})\,|\,\theta_i]}{\mathrm{cov}^2(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)}\quad. \end{aligned} \tag{14}$$

The expectation of the squared covariance of $\underset{\sim}{y}_k$ and $\underset{\sim}{G}$ can be simplified and given by Equation (15).

$$E[\mathrm{cov}^2(\underset{\sim}{y}_k\ ,\ \underset{\sim}{G})\,|\,\theta_i] = \mathrm{cov}^2(\underset{\sim}{P}_i\ ,\ \underset{\sim}{G}) + \frac{1}{n^2}\sum_{j=1}\sigma_{ij}^{\ 2}\ (G_j - T)^2\quad. \tag{15}$$

By substituting (15) in (14), we get (16).

$$\mathrm{Var}(\mathrm{ECI2}\,|\,\theta_i) = \frac{\sum_{j=1}^{n}(G - T)^2\sigma_{ij}^{\ 2}}{n^2\mathrm{cov}^2(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)} \tag{16}$$

ECI4

The conditional expectation of ECI4 is

$$E(\mathrm{ECI4}\,|\,\theta_i) = 1 - E[\frac{\mathrm{cov}(\underset{\sim}{y}_k\ ,\ \underset{\sim}{P}_i)\,|\,\theta_i}{\mathrm{cov}(\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)}] \tag{17}$$

where $\underset{\sim}{y}_k$ is a random variable from the distribution of binary responses to n items at the fixed ability level i. Since the denominator of the expected value, cov $(\underset{\sim}{G}\ ,\underset{\sim}{P}_i)$, is fixed at level i, the second term will be simply the expectation of the numerator divided by the covariance of $\underset{\sim}{G}$ and $\underset{\sim}{P}_i$, $E[\mathrm{cov}(\underset{\sim}{y}_k, \underset{\sim}{P}_i)\,|\,\theta_i]/\mathrm{cov}\ (\underset{\sim}{G}\ ,\ \underset{\sim}{P}_i)$.

$$\begin{aligned} & E[\mathrm{cov}(\underset{\sim}{y}_k\ ,\ \underset{\sim}{P}_i)\,|\,\theta_i] \\ &= \frac{1}{n}\,E[\sum_{j=1}^{n}(y_{kj} - P_{i.})(P_{ij} - T_i)\,|\,\theta_i] \\ &= \frac{1}{n}\sum_{j=1}^{n}(P_{ij} - T_i)\ E(y_{kj} - P_{i.}\,|\,\theta_i) \end{aligned}$$

But $E(y_{kj} - p_{i.} | \theta_i) = P_{ij} - T_i$ because of Equations (4)

Therefore,

$$E(ECI4 | \theta_i) = 1 - \frac{\operatorname{cov}(\underset{\sim}{P_i} , \underset{\sim}{P_i})}{\operatorname{cov}(\underset{\sim}{G} , \underset{\sim}{P_i})}$$

$$= 1 - \frac{\operatorname{Var}(\underset{\sim}{P_i})}{\operatorname{cov}(\underset{\sim}{G} , \underset{\sim}{P_i})} . \tag{18}$$

The conditional variance of ECI4 is given by Equations (19).

$$\operatorname{Var}(ECI4 | \theta_i) = E\left[[ECI4 - E(ECI4)]^2 | \theta_i\right] \tag{19}$$

Substituting the expectation of ECI4 from Equation (18), (19) becomes

$$\operatorname{Var}(ECI4 | \theta_i) = E\left[\left(\frac{\operatorname{cov}(\underset{\sim}{P_i} , \underset{\sim}{P_i})}{\operatorname{cov}(\underset{\sim}{G} , \underset{\sim}{P_i})} - \frac{\operatorname{cov}(\underset{\sim}{y_k} , \underset{\sim}{P_i})}{\operatorname{cov}(\underset{\sim}{G} , \underset{\sim}{P_i})}\right)^2 | \theta_i\right]$$

A straightforward expansion of the inside of the parentheses leads to Equation (20).

$$\operatorname{Var}(ECI4 | \theta_i) = \frac{E[\operatorname{cov}^2(\underset{\sim}{y_k} , \underset{\sim}{P_i}) | \theta_i]}{\operatorname{cov}^2(\underset{\sim}{G} , \underset{\sim}{P_i})} - \frac{\operatorname{cov}^2(\underset{\sim}{P_i} , \underset{\sim}{P_i})}{\operatorname{cov}^2(\underset{\sim}{G} , \underset{\sim}{P_i})} \tag{20}$$

The numerator of the first term, $E[\operatorname{cov}^2(\underset{\sim}{y_k} , \underset{\sim}{P_i}) | \theta_i]$, can be simplified in the same manner as in the case of ECI1.

$$E[\operatorname{cov}^2(\underset{\sim}{y_k} , \underset{\sim}{P_i}) | \theta_i]$$

$$= \frac{1}{n^2} E([\sum_{j=1}^{n} (y_{kj} - p_{i.})(P_{ij} - T_i)]^2 \mid \theta_i)$$

$$= \frac{1}{n^2} E[\sum_{j=1}^{n} (y_{kj} - p_{i.})^2 (P_{ij} - T_i)^2 \mid \theta_i)$$

$$+ \frac{1}{n^2} E(\sum_{j \neq h} (y_{kj} - p_{i.})(y_{kh} - p_{i.})(P_{ij} - T_i)(P_{ih} - T_i) | \theta_i) .$$

Because of local independence and Equation (4), we obtain the following two relations:

$$E[\sum_{j=1}^{n}(y_{kj} - p_{i.})^2(P_{ij} - T_i)^2|\theta_i)$$

$$= \sum_{j=1}^{n}[\sigma_{ij}^2 + (P_{ij} - T_i)^2](P_{ij} - T_i)^2$$

and

$$E\left[\sum_{j\neq h}(y_{kj} - p_{i.})(y_{kh} - p_{i.})(P_{ij} - T_i)(P_{ih} - T_i)|\theta_i\right]$$

$$= \sum_{j\neq 1}[(P_{ij} - T_i)^2(P_{ih} - T_i)^2|\ \theta_i] .$$

By adding the results, we obtain

$$E[\mathrm{cov}^2(\underset{\sim}{y}_k , \underset{\sim}{P}_i)|\theta_i)$$

$$= \frac{1}{n^2}\sum_{j=1}^{n}[(P_{ij} - T_i)^2]^2 + \frac{1}{n^2}\sum_{j=1}^{n}\sigma_{ij}^{\ 2}(P_{ij} - T_i)^2$$

$$= \mathrm{Var}^2(P_{ij}) + \frac{1}{n^2}\sum_{j=1}^{n}\sigma_{ij}^2\ (P_{ij} - T_i)^2 \qquad . \qquad (21)$$

By substituting (21) in (20), we get Equation (22), the variance of ECI4.

$$\mathrm{Var}\ (\mathrm{ECI4}|\theta_i) = \frac{\mathrm{cov}^2(\underset{\sim}{P}_i , \underset{\sim}{P}_i) + \frac{1}{n^2}\sum_{j=1}^{n}\sigma_{ij}^{\ 2}(P_{ij} - T_i)^2}{\mathrm{cov}^2(\underset{\sim}{G},\underset{\sim}{P}_i)} - \frac{\mathrm{cov}^2(\underset{\sim}{P}_i ,\underset{\sim}{P}_i)}{\mathrm{cov}^2(\underset{\sim}{G},\underset{\sim}{P}_i)}$$

$$= \frac{\Sigma\sigma_{ij}^{\ 2}(P_{ij} - T_i)^2}{n^2\mathrm{cov}^2(\underset{\sim}{G} ,\underset{\sim}{P}_i)} \qquad . \qquad (22)$$

Comparison of Some Statistical Properties of the Three Indices ECI1, ECI2, and ECI4

Comparison of the Standard Errors

The conditional expectations of the three indices are different in a manner that suggests that ECI1 and ECI2 are similar to each other, while ECI4 stands alone. ECI1 and ECI2 have the constant expectation zero, regardless of the level of person parameter θ_i. On the other hand, the expectation of ECI4 is a function of θ_i, as shown in Figure 1 for the dataset obtained from a 32-item signed-number subtraction test. The x-axis represents true scores and the y-axis the 127 students' expected ECI4 values. The curve in Figure 1 decreases monotonically as the true score decreases. The standard error of ECI4 is the square root of expression (22) and is also a function of θ. Figure 2 shows the relationship between the standard error and the true scores. (The estimated true score of IRT was used instead of θ_i so as to have a value between 0 and 1, which facilitates comparison across different tests.)

Insert Figure 1 about here

Insert Figure 2 about here

For students whose true scores are extremely high or low, the standard-error curve rises sharply, while for average scores, it becomes rather flat.

Figures 3 and 4 are plots of the standard errors [square roots of expression (11) and (16)] of ECI1 and ECI2 against true score as the x-axis. They are almost identical curves that are nearly horizontal for the average true scores but increase rather rapidly at both the high and low extremes of true scores.

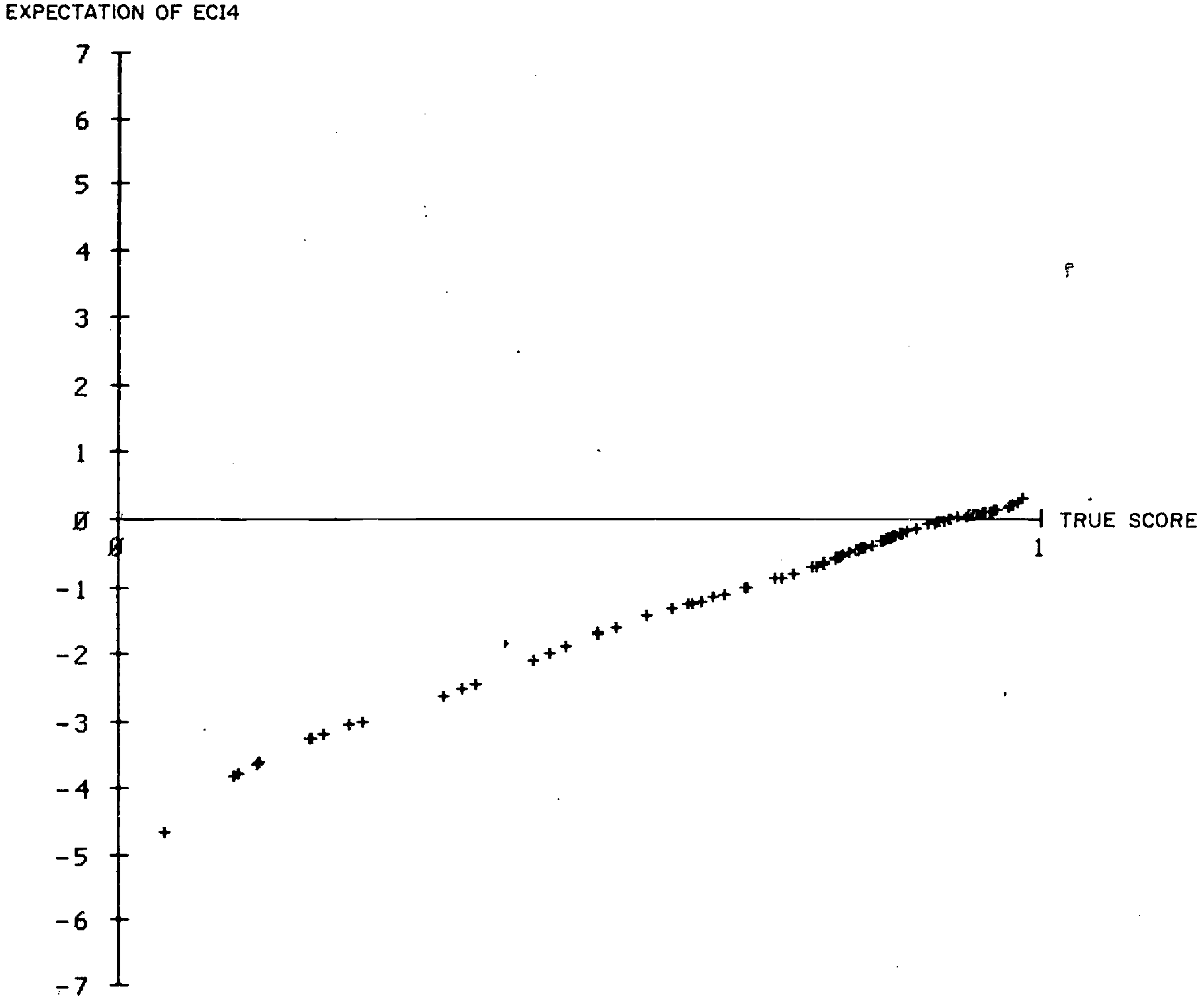

FIGURE 1: Expectation of ECI4 Plotted Against the True Score

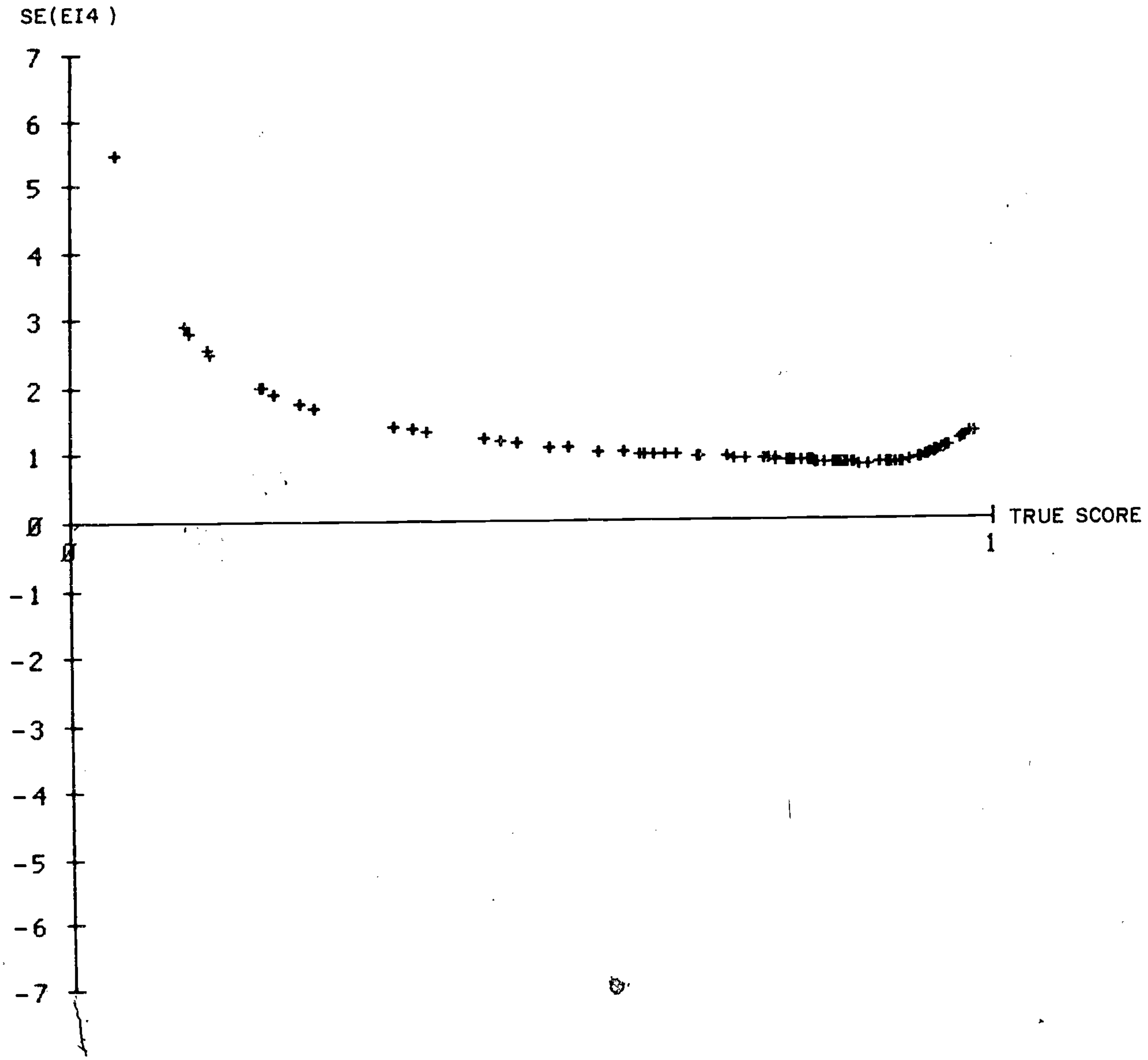

FIGURE 2: The Standard Error of ECI4 Plotted Against the True Score

Insert Figures 3 & 4 about here

ECI1 and ECI2 correlate highly ($r = .97$, see Appendix XI) and have the same constant expectation of zero. Moreover, their standard errors have almost identical curves when plotted against true scores, so we will drop ECI1 hereafter and make comparisons between ECI2 and ECI4. Since ECI2 is defined by using the elements in the probability matrix (P_{ij}), the investigation of ECI2 and ECI4 will be more interesting.

Standardized Extended Caution Indices, $ECI2_z$ and $ECI4_z$ and their Density Functions

ECIs can be standardized by subtracting their expected values and then dividing it by their standard errors. Equations (23) and (24) are the standardized extended caution indices ECI2 and ECI4.

$$ECI2_z = \frac{ECI2 - E(ECI2|\theta_i)}{SE(ECI2|\theta_i)} = \frac{n\,cov(\underset{\sim}{P}_i - \underset{\sim}{y}_i \, , \underset{\sim}{G})}{\left[\sum_{j=1}^{n} \sigma_{ij}^2 (P_{ij} - T)^2\right]^{1/2}}$$

$$ECI4_z = \frac{ECI4 - E(ECI4|\theta_i)}{SE(ECI4|\theta_i)} = \frac{n\,cov(\underset{\sim}{P}_i - \underset{\sim}{y}_i \, , \underset{\sim}{P}_i)}{\left[\sum_{j=1}^{n} \sigma_{ij}^2 (P_{ij} - T_i)^2\right]^{1/2}}$$

As can be seen in Equations (23) and (24), the second variables of the covariances in the numerators are $\underset{\sim}{G}$ and $\underset{\sim}{P}_i$, respectively. The denominator for $ECI2_z$ involves the group-oriented vector $\underset{\sim}{G} - T1$ while that for $ECI4_z$ involves the individual-oriented vector at the level i, $\underset{\sim}{P}_i - \underset{\sim}{T}_i 1$. Tatsuoka and Linn (1982) argue that ECI4 may correspond to the individual consistency index (ICI) introduced in Tatsuoka & Tatsuoka (1980, 1982a) while ECI2 may function similarly to the group dependent

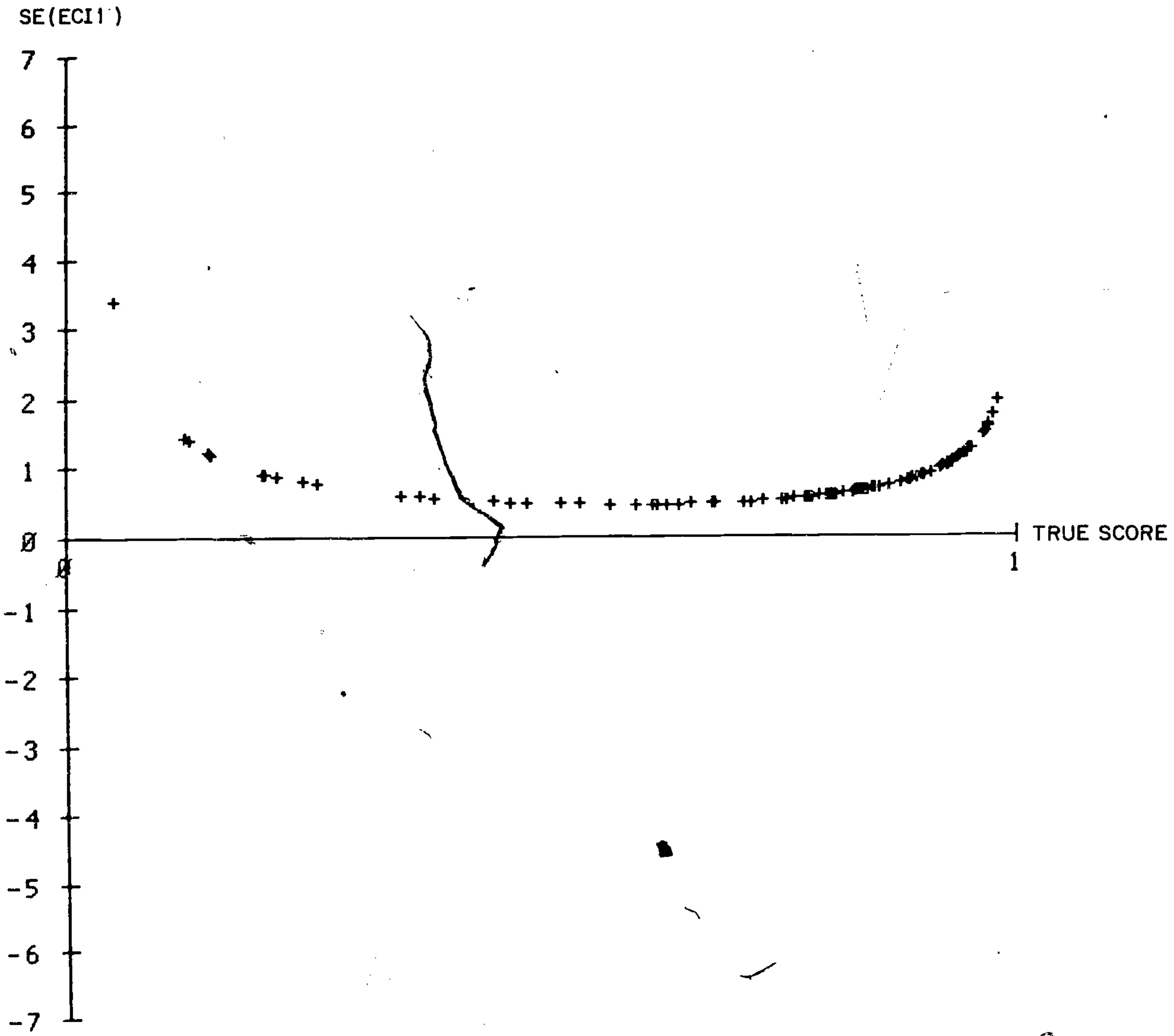

FIGURE 3: The Standard Error of ECI1 Plotted Against the True Score

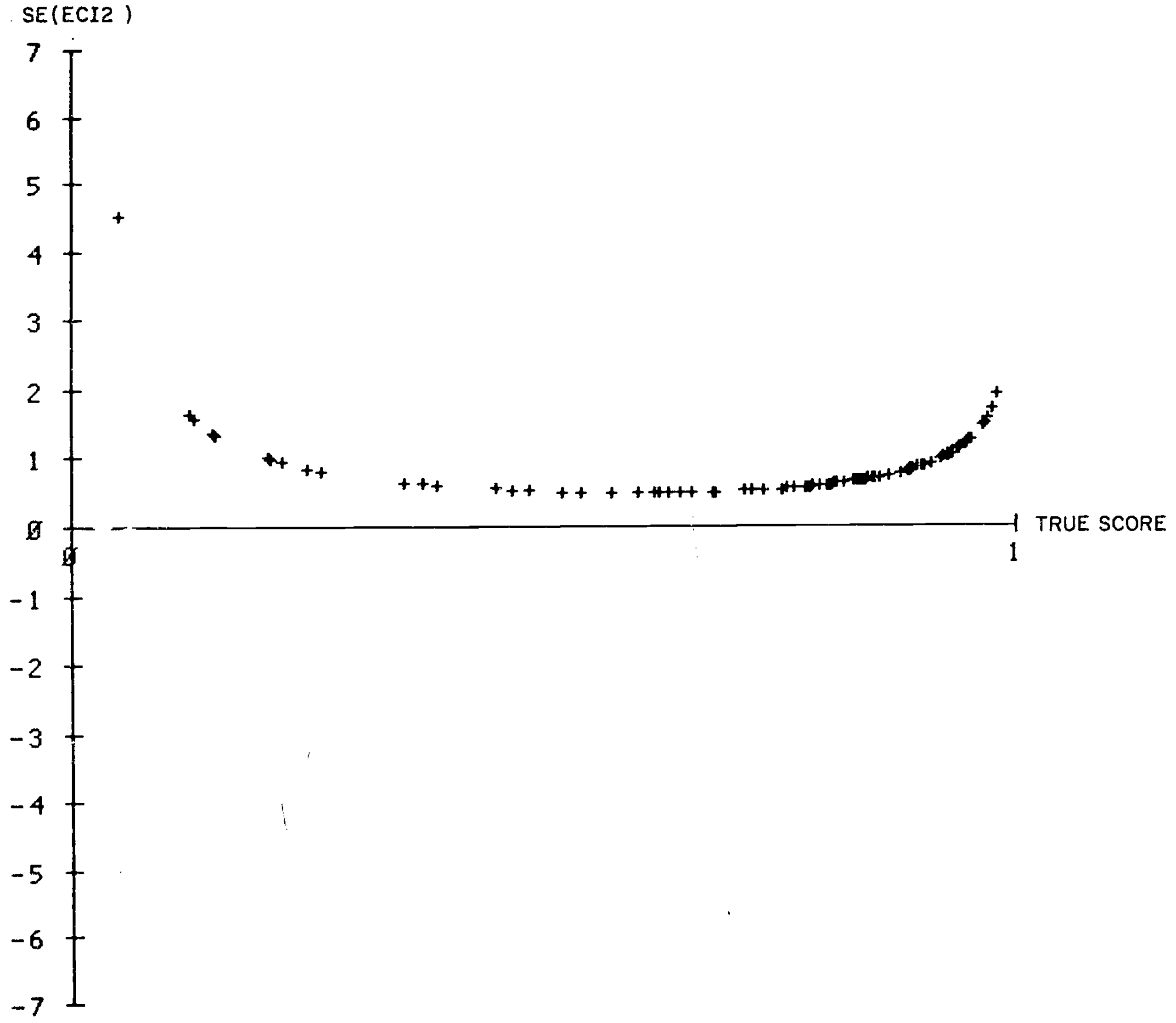

FIGURE 4: The Standard Error of ECI2 Plotted Against the True Score

indices, i.e., Sato's caution index (1975) or the norm conformity index (Tatsuoka & Tatsuoka, 1980, 1982a). The ICI has proven to be effective in spotting the aberrant response patterns resulting from consistent application of erroneous rules of operation (Tatsuoka & Tatsuoka, 1981). Our prediction with regard to detection rates of erroneous rules of operation is that ECI4 should be better than ECI2.

It should be noted that the scale of the original ECIs are functions of θ but those of the standardized ECI_zs no longer depend on θ. As a result, two $ECI4_z$ (or $ECI2_z$) values obtained from different θ levels are comparable in terms of the extent of anomaly they signify. However, the density functions of $ECI2_z$ and $ECI4_z$ have to be investigated in order to determine their differences statistically. Figures 5 and 6 show the goodness-of-fit test of the normal distribution for $ECI2_z$ and $ECI4_z$. Appendices I and II give the tests of the normal distribution for $ECI1_z$ and lz (Levine & Drasgow's standardized appropriateness measure, 1982), while Appendices III, IV and V give the goodness-of-fit tests of beta distributions for $ECI1_z$, $ECI2_z$, and $ECI4_z$. The data used in these figures are based on 2,400 students' scores obtained from a math test (National Assessment of Educational Progess series, mathematics for 13 year olds, Booklet 4). As can be seen in the figures, both the standardized ECIs fit normal distributions well. Similar results are obtained from the NAEP data, Booklet 5.

Insert Figures 5 & 6 about here

Appendices VII, VIII, IX and X give the standard errors of $ECI1_z$, $ECI2_z$, and $ECI4_z$ and the expectation of $ECI4_z$, obtained from the NAEP data. Although the NAEP data is used for testing "goodness of fit" of the ECIs with theoretical distributions, we will go back to the signed

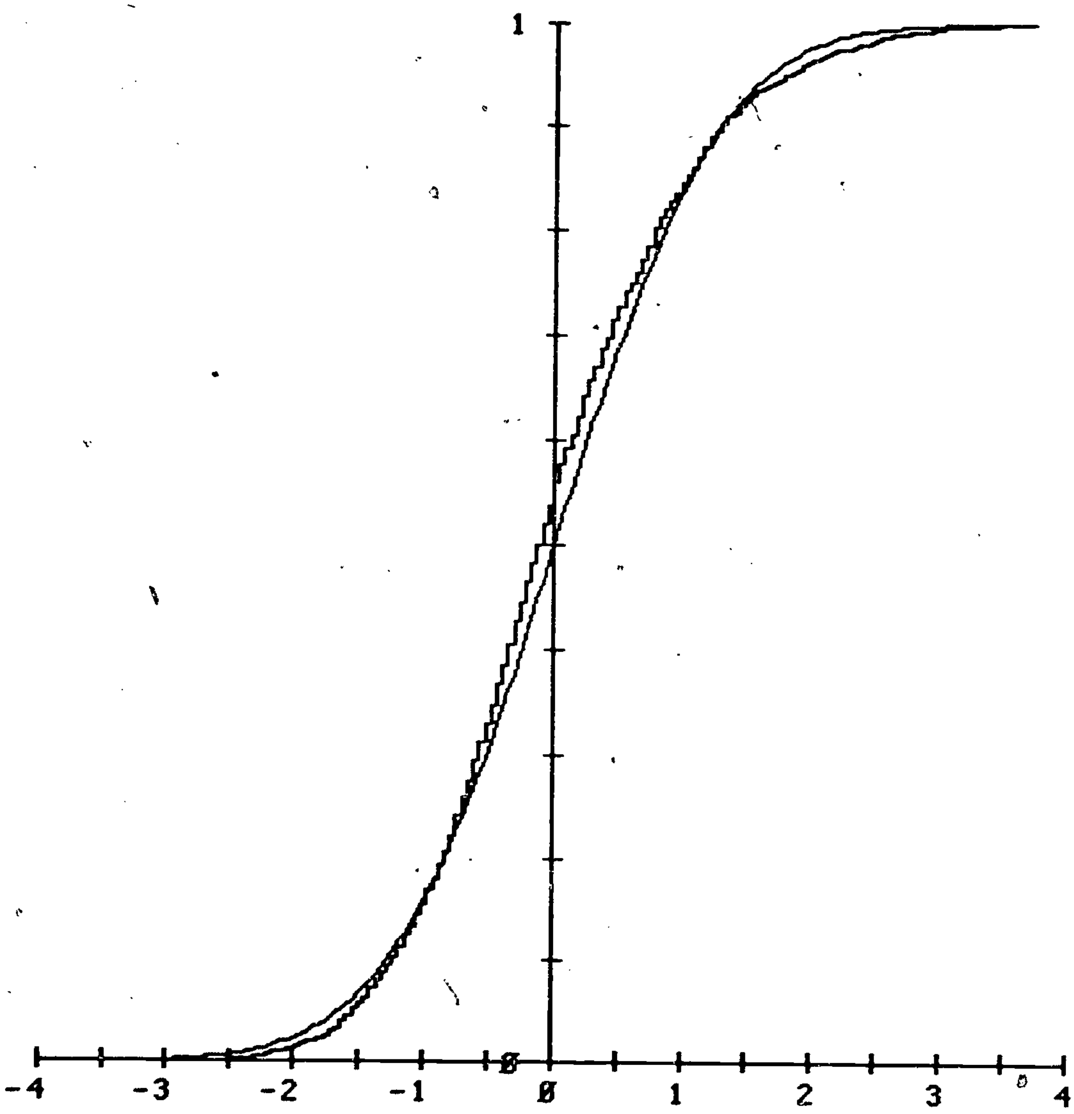

FIGURE 5: Goodness of Fit Test for the Normal Distribution:
The Stepfunction is a Cummulative Distribution of $ECI4_z$:
The Smooth Curve is a Theoretical Curve

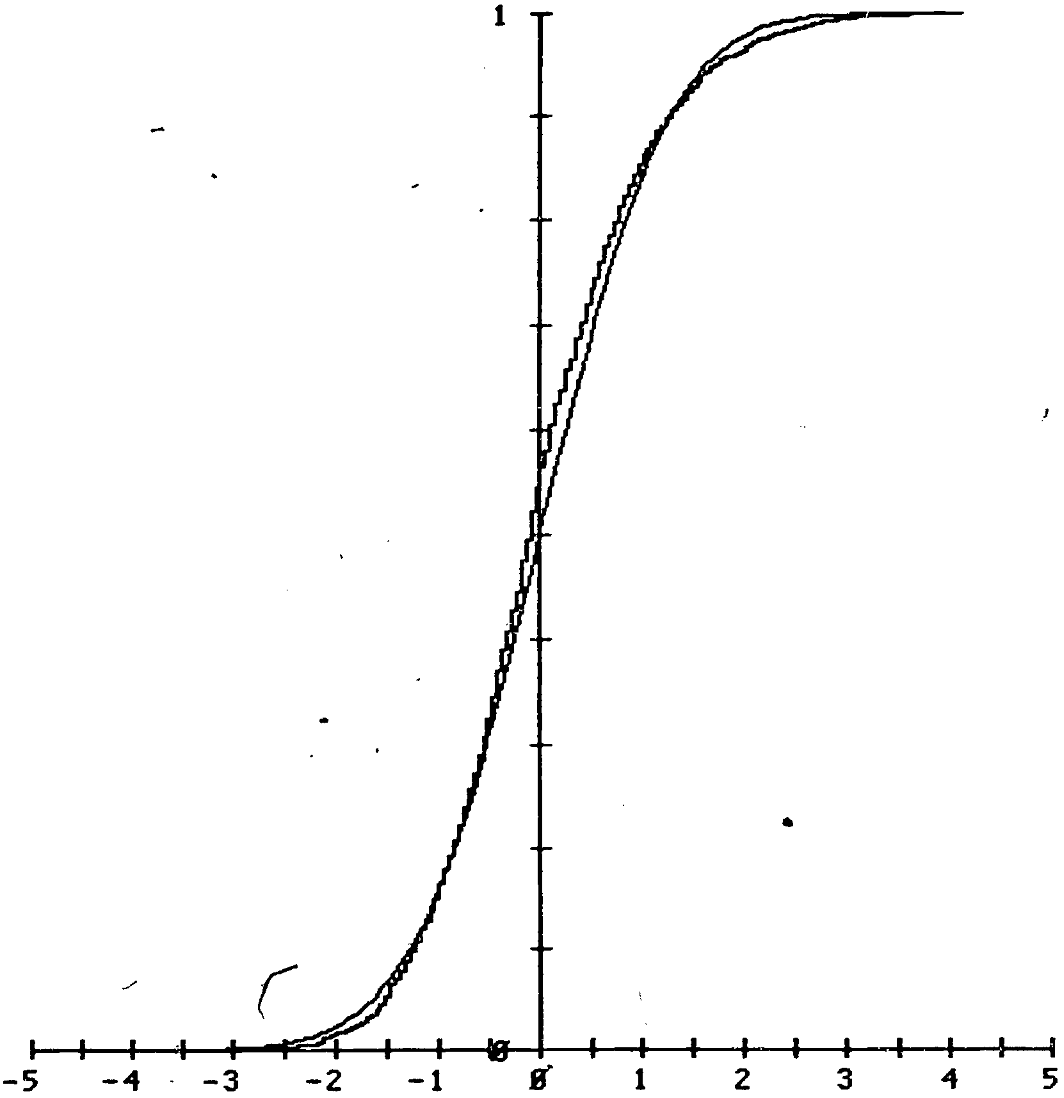

FIGURE 6: Goodness of Fit Test for the Normal Distribution:
The Stepfunction is the Cummulative Distribution of ECI2 z

number data in order to investigate the detection rate of aberrant response patterns by the standardized ECIs. In the next section, a brief description of the dataset and procedure for the comparions will be described.

A brief description of the dataset

Birenbaum and Tatsuoka (1982a) have demonstrated that the traditional zero-one scoring of incorrect and correct answers does not reflect a student's performance correctly because several erroneous rules frequently yield the right answer for some problems. By extensive error analysis performed on the original dataset (the 127 eighth graders test scores for signed-number subtraction problems) Birenbaum and Tatsuoka (1980) identified erroneous rules that were consistently applied by certain students. They rescored ones to zeros for items that students got right for the wrong reasons. The dataset used in Figures 1 through 4 are the modified dataset in which the scores of zero-one should reflect more accurately the student's performance than the original dataset of N = 127. The modified dataset was much more nearly unidimensional and had higher item-item and item-total correlations than the original, while the item-means and standard deviation remained almost the same (Birenbaum & Tatsuoka, 1982a). Fifteen erroneous rules were randomly selected from the 45 erroneous rules listed in Tatsuoka & Tatsuoka (1981) and responses based on these were added to the modified dataset. We refer to the new dataset of N = 142 as "Bugdata" hereafter.

Comparison of detection rates of $ECI2_z$ and $ECI4_z$ with respect to their 80% intervals

By using the item parameters estimated from the modified dataset, $ECI2_z$ and $ECI4_z$ for the 142 subjects in the bugdataset were calculated and plotted against the true scores. Figure 7 is the scatterplot of $ECI4_z$ against the true scores and Figure 8 is $ECI2_z$ against the same true scores. The 15 bugs are marked by a small circle "o" with the numbers and 89 real data points are marked by a plus sign "+" without being numbered.

Insert Figures 7 & 8 about here

The 80% intervals for both the ECIs and lz are constructed and listed in Table 1 along with the means and standard deviations of the indices. These are the intervals within which, theoretically, the values of the indices associated with 80% of the non-aberrant responses should fall. The intervals are marked by broken lines in Figures 7 and 8. We may choose, as a convenient decision rule, to classify response patterns with index values outside these intervals as "aberrant." The proportions of real response patterns classified as "aberrant" (which are essentially false alarm rates) by the four indices that are shown in Table 2 along with the proportions of the 15 bugs that are detected.

Insert Table 1 about here

Insert Table 2 about here

The unstandardized ECI4 seemed to have the best detection rates in comparison with the other four ECIs (Tatsuoka & Linn, 1982) but lost its high rate after it was standardized. Exactly the same dataset is used in both the cases, the standardized and unstandardized fourth extended caution index. In Table 2, the false alarm rates of the four indices

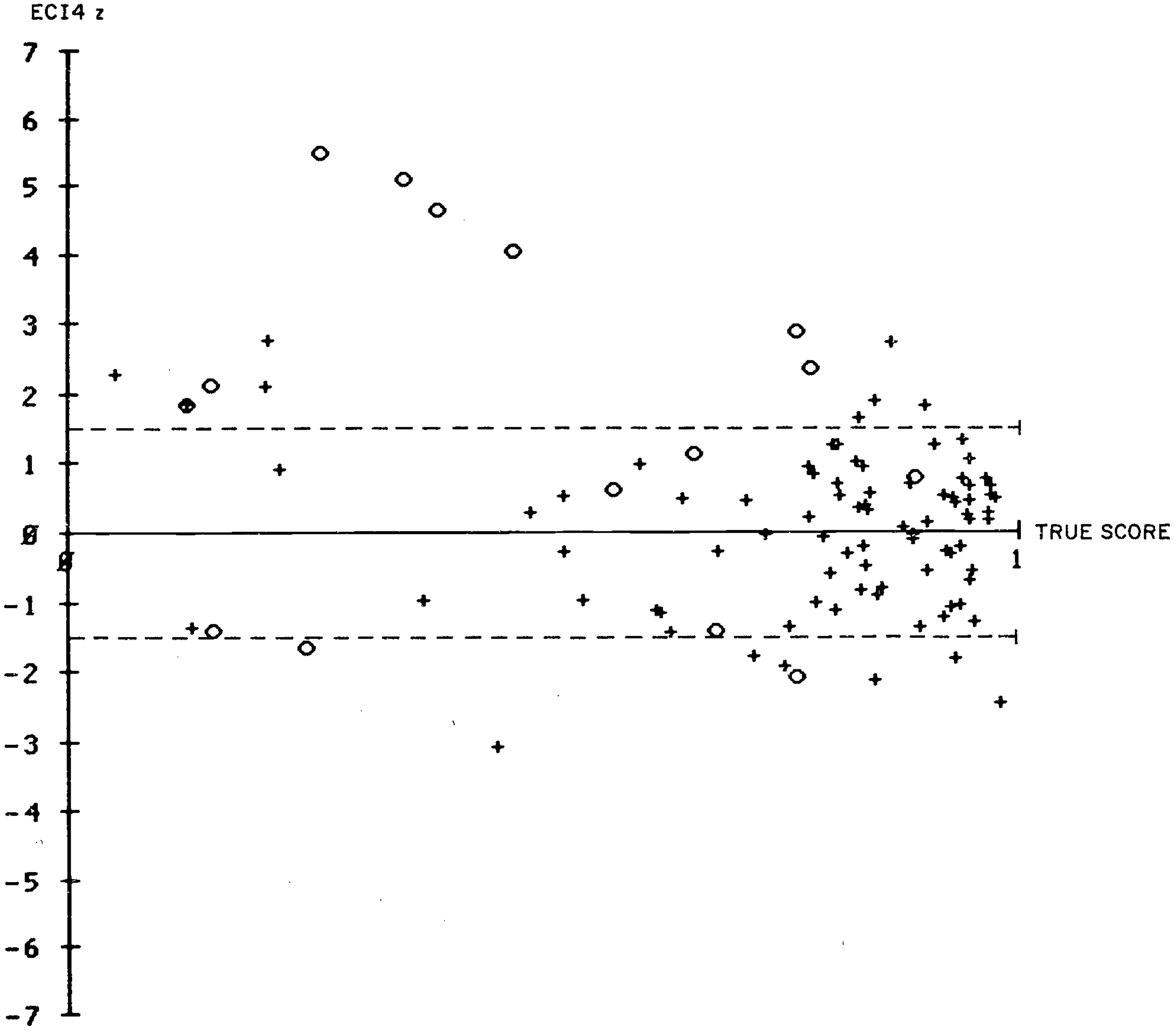

FIGURE 7: Plot of ECI4 z Against True Score for the Modified Dataset ("+") and Erroneous Rules ("O"), and 80% Probability Interval (-1.55,1.59).

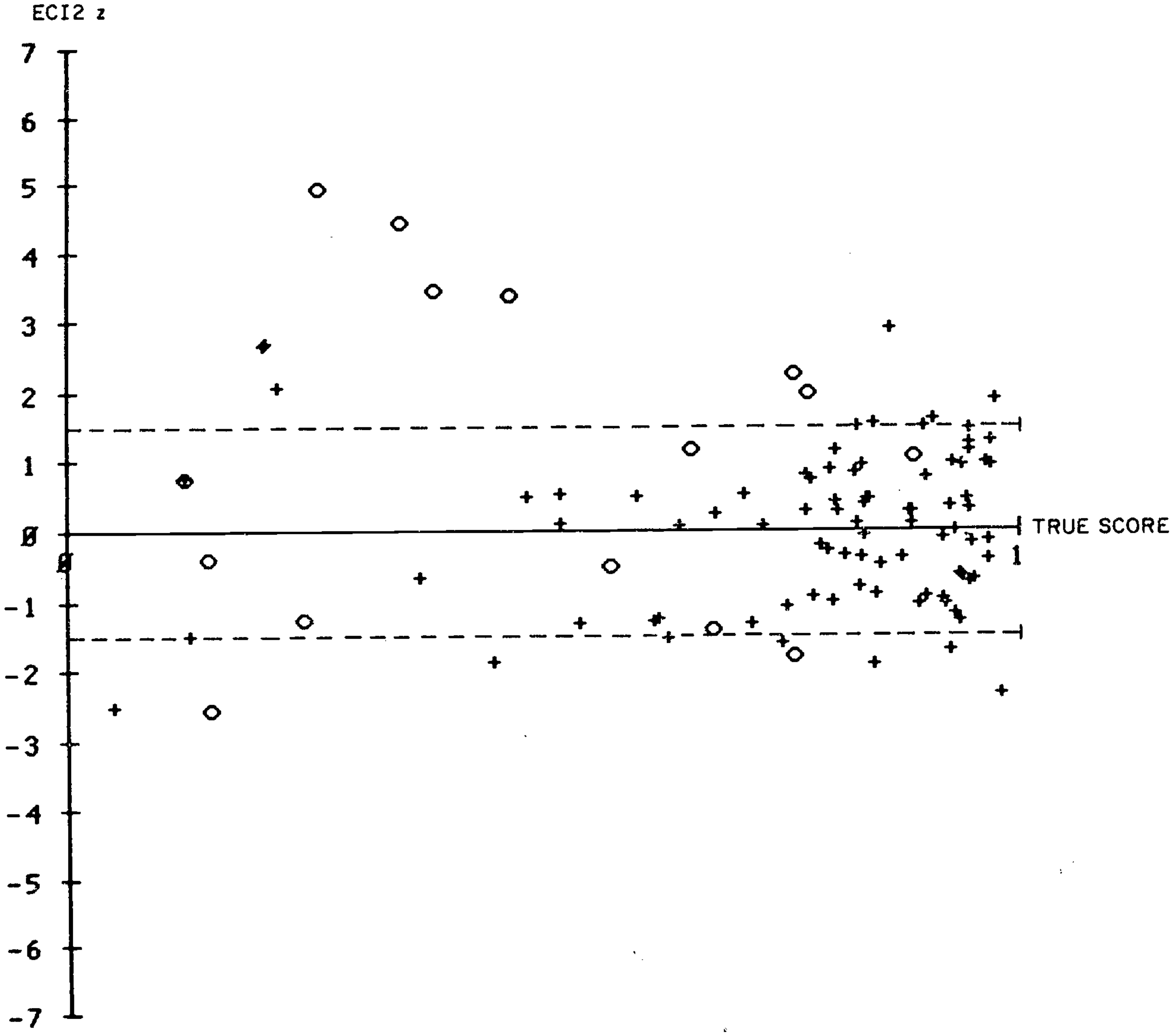

FIGURE 8: Plot of ECI2 z Against True Score for the Modified Dataset ("+") and Erroneous Rules ("O"), and 80% Probability Interval (-1.56,1.59).

Table 1

The 80% Intervals of $ECI1_z$, $ECI2_z$, $ECI4_z$ and lz.

Indices	Mean	S.D.	80% confidence interval
$ECI1_z$	.001	1.105	(-1.414, 1.416)
$ECI2_z$	.020	1.230	(-1.555, 1.594)
$ECI4_z$	.019	1.229	(-1.554, 1.593)
lz	.017	.619	(-.775, .809)

Table 2

Detection Rates of Erroneous Rules by Four Personal Indices Based on Item Response Theory with Bugdataset

	Real Students N = 89	Erroneous Rules N = 15
$ECI1_z$	.22	.60
$ECI2_z$	.15	.53
$ECI4_z$	.17	.67
lz	.18	.67

vary around 20% as they should, while the correct detection rate fluctuates around 60%. Considering the fact that the false alarm rate for the 89 students by using ICI with total scores (ICI $\geq$ .90 and scores lower than a certain criterion, Tatsuoka & Tatsuoka, 1981) was less than 5%, the results summarized in Table 2 are not as good as we had expected. One reason for the low detection rates may be the fact that the modification procedure of rescoring in the original dataset was carried out by an intuitive error analysis, and hence there are some responses affected by persistent misconceptions left in the modified dataset. Table 3 lists the percentage of "bugs" left in the modified dataset. The total number of bugs (including repetitions) has become 42. The mean absolute value of $ECI4_z$ in the two groups described in Table 3 are 3.141 for the bugs that were not found in the modified dataset, 1.353 for the bugs left in. However, the value of $ECI4_z$, 1.353, is still substantially high in comparison with the majority of real responses in the modified dataset.

Insert Table 3 about here

Summary and Discussion

The extended caution indices, ECI1, ECI2 and ECI4 are standardized by the usual transformation,

$$ECIm_z = \frac{ECIm - E(ECIm|\theta_i)}{SE(ECIm|\theta_i)} \quad \text{for } m=1,\ 2,\ \text{and } 4.$$

The conditional expectation of $ECI4_i$ is a function of the θ level, but those of the other two ECIs are identically zero. If we sample two students from different θ_i levels, then it is dangerous to compare their ECI4 values in order to determine which student's response patterns is more aberrant than the other. Moreover, the standard errors of all

Table 3

Percentage of Each Bug that was not Rescored and Remained in the November Modified Dataset (n = 8, N = 89) 356 Sets of Responses

	Bugs	%	Total Scores	* ECI4$_z$
Group 1	1	0	4	3.728
	3	0	3	4.309
	4	0	2	4.259
	8	0	6	3.059
	10	0	3	4.045
	12	0	2	-1.247
	13	0	1	1.338
Group 2	2	.006	6	2.554
	5	.011	5	-1.435
	6	.014	6	-2.197
	7	.003	4	.631
	9	.008	1	-.887
	11	.014	1	1.084
	14	.014	6	1.162
	15	.048	7	.876

*Mean of Group 1 = 3.141 S.D. = .503
Mean of Group 2 = 1.353 S.D. = .240

three ECIs are functions of θ_i and have U shaped trend curves. This explains the past findings that the correlation of personal indices, such as the caution index, NCI,or ICI, with total scores vary according to the shapes of the total-score distributions. The findings are that if the total-score distribution has a negative skewness, then the correlation is positive, if the distribution is positively skewed, then a negative correlation results (Harnisch & Linn, 1981; Tatsuoka & Tatsuoka, 1980). Since the ECIs are natural extentions of the caution index, we can safely impute some behaviors of ECIs to these discrete personal indices as well. ECIs provide inflated values at both the extremely high and low total scores. With the standardized ECIs, the bias of the values at the extreme scores is corrected, and moreover the responses from different levels of θ can be compared safely.

It would be ideal if the theoretical distribution of the standardized extended caution indices could be derived algebraically, but goodnes-of-fit tests of the ECI_zs with normal distributions provide satisfactory evidence that they may follow approximately normal distributions.

Regarding the detection rates of "bugs", they are unexpectedly low. We have tried to find the reason for this by investigating each response pattern in the modified dataset. The results indicate that if an otherwise normal dataset includes a considerable number of aberrant response patterns, then these patterns are no longer detectable with high probability by the ECI approach. A new method to detect such aberrant response patterns should be investigated in the future.

Rudner (1982) recently conducted a Monte Carlo study to compare the detection rates of various indices. He found that the indices based on item response theory performed consistently better with his data than the indices based on sample statistics alone. But IRT is not always applicable in practice. An advantage of ECIs in comparison with other appropriateness indices or Wright's index is that they can start from the caution index when a sample is small. Then it can be shifted to ECIs as the sample size becomes larger without loss of continuity because ECIs are natural extentions of the S-P curve theory. However, further investigation of the relationships between the original caution index and the ECIs will be needed.

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Appendices

Captions of Appendices

Appendix I: Goodness of Fit Test for the Normal Distribution: The Stepfunction is the Cummulative Distribution of ECI1$_z$

Appendix II: Goodness of Fit Test for the Normal Distribution: The Stepfunction is the Cummulative Distribution of lz

Appendix III: Goodness of Fit Test for the Beta Distribution: The Stepfunction is the Cummulative Distribution of ECI$_z$

Appendix IV: Goodness of Fit Test for the Beta Distribution: The Stepfunction is the Cummulative Distribution of ECI2$_z$

Appendix V: Goodness of Fit Test for the Beta Distribution: The Stepfunction is the Cummulative Distribution of ECI4$_z$

Appendix VI: Plot of lz Against True Score for the Modified Dataset ("+") and Erroneous Rules ("0"), and 80% Probability Interval (-.78, .81)

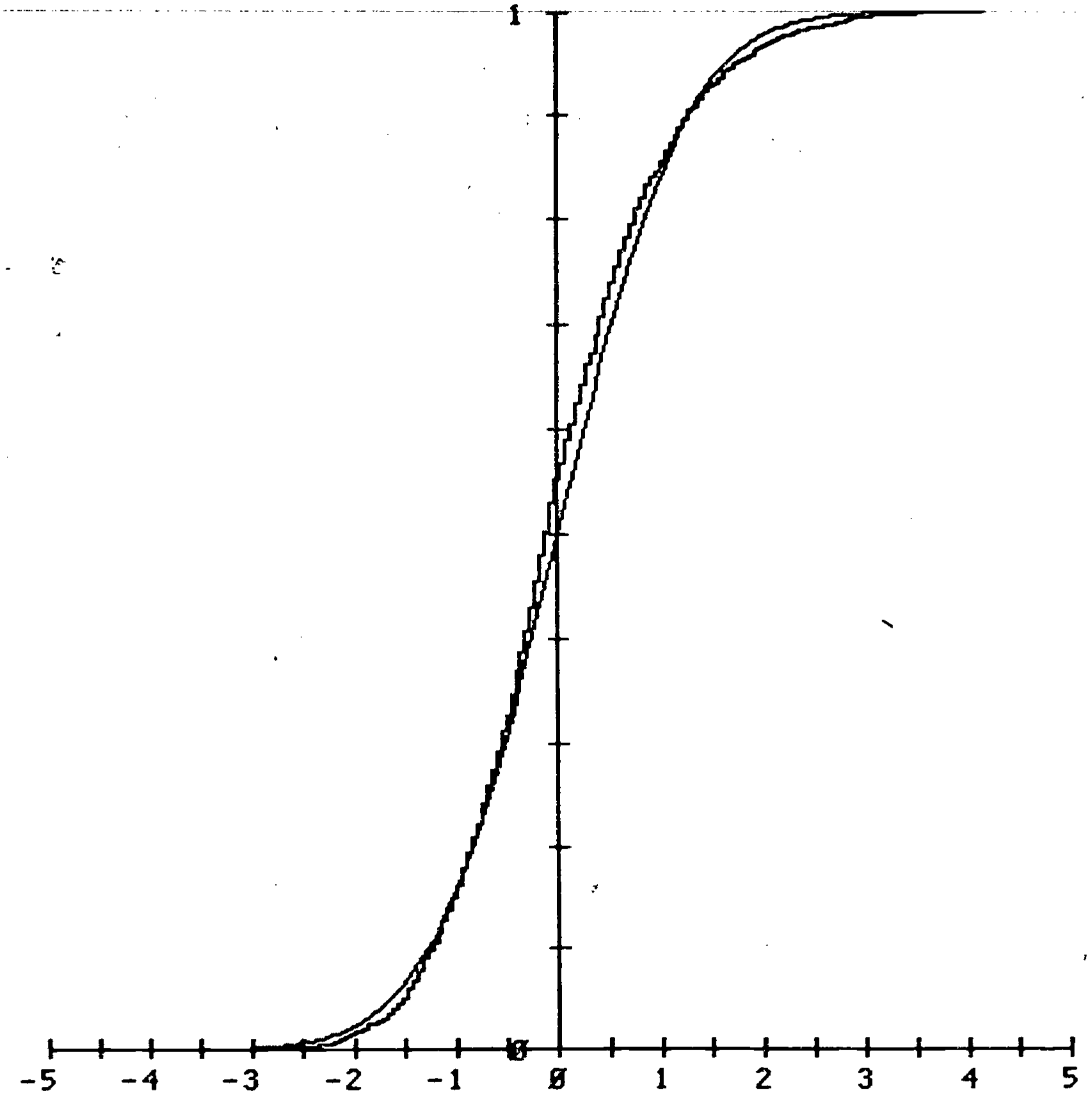
Appendix VII: Standard Error of ECI1

Appendix VIII: Standard Error of ECI2

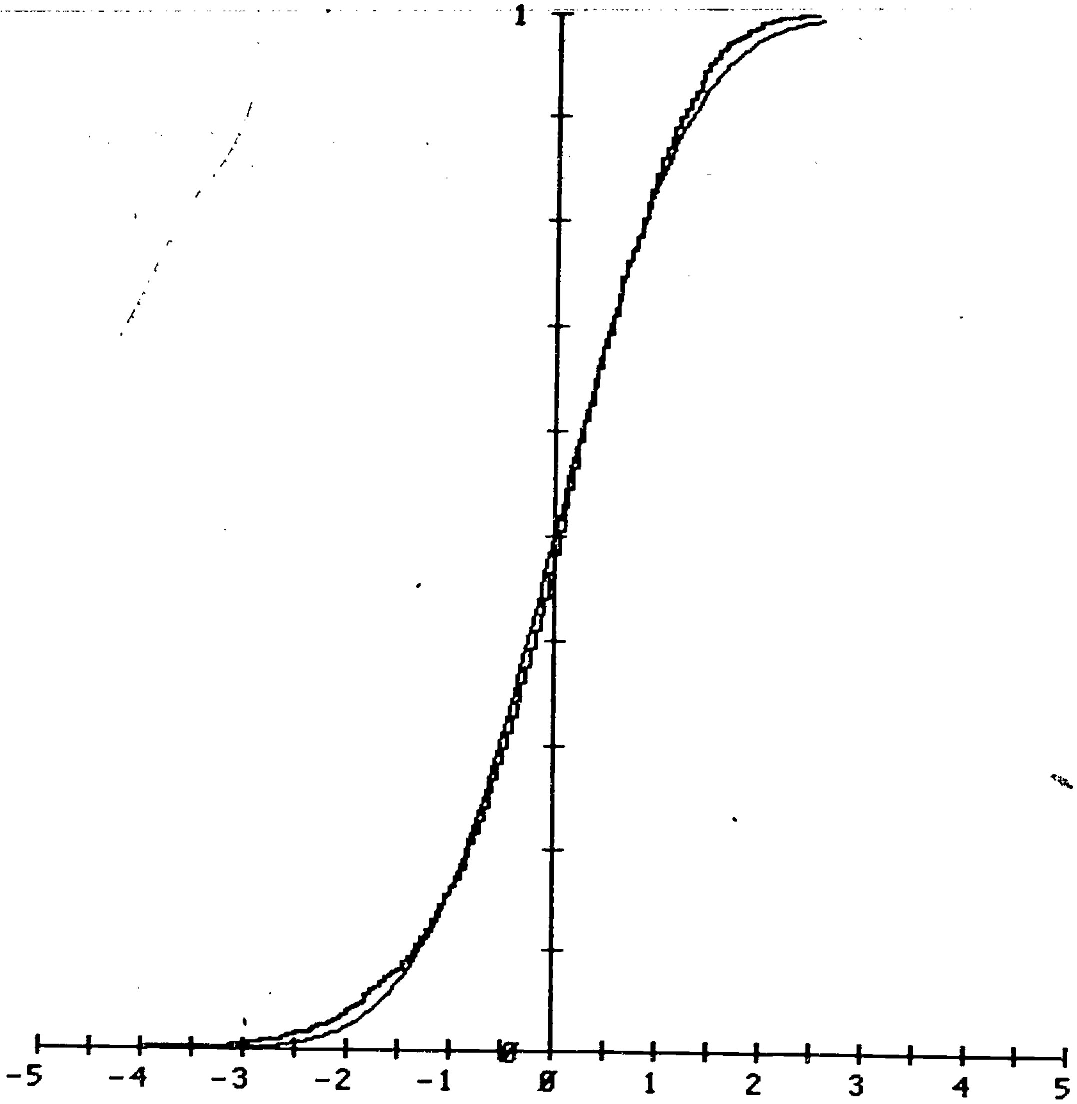
Appendix IX: Standard Error of ECI4

Appendix X: Plot of Expectation of ECI4 Against True Score

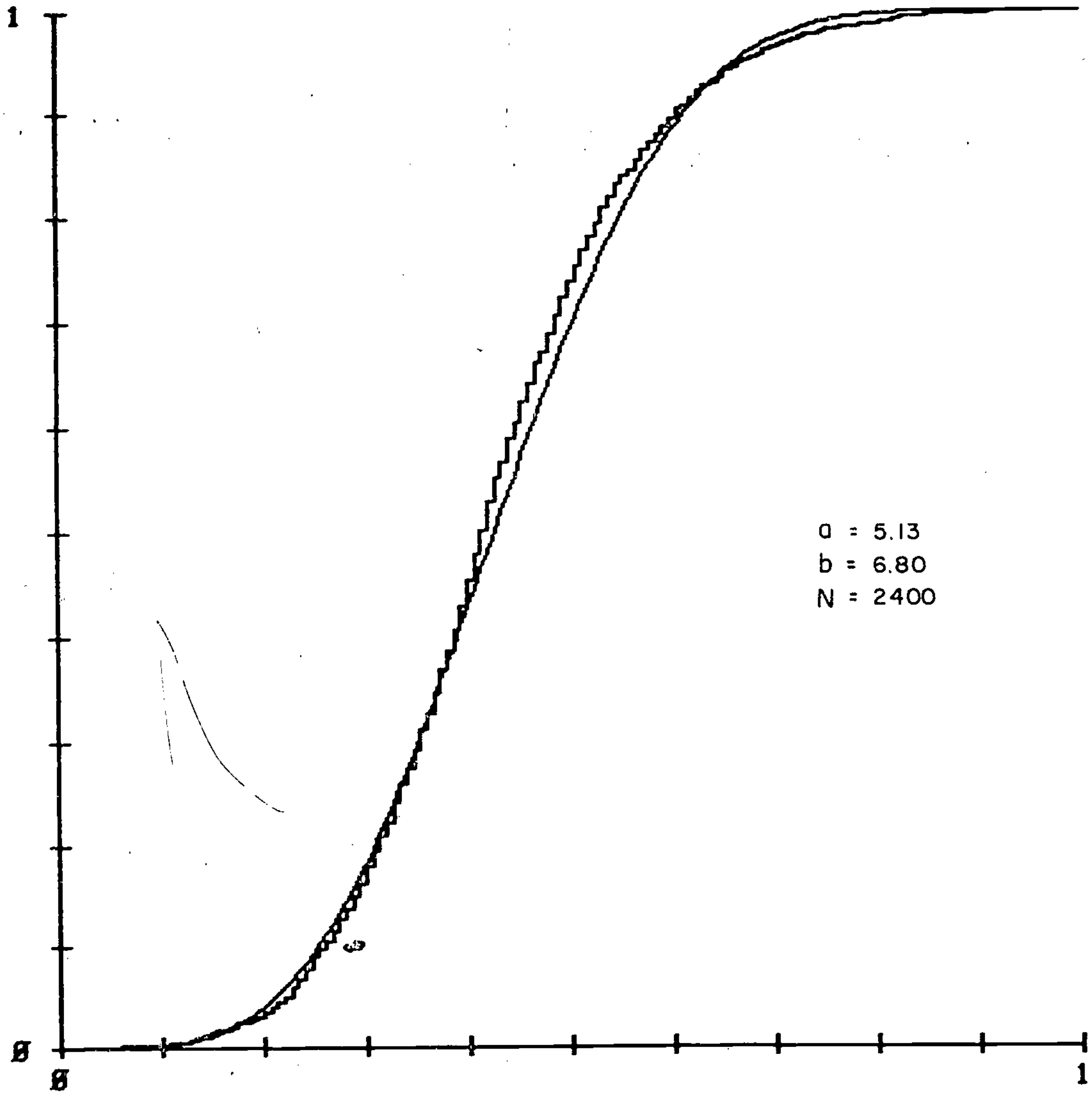
Appendix XI: Correlation Matrix of Standardized ECIs and lz with Bugdata



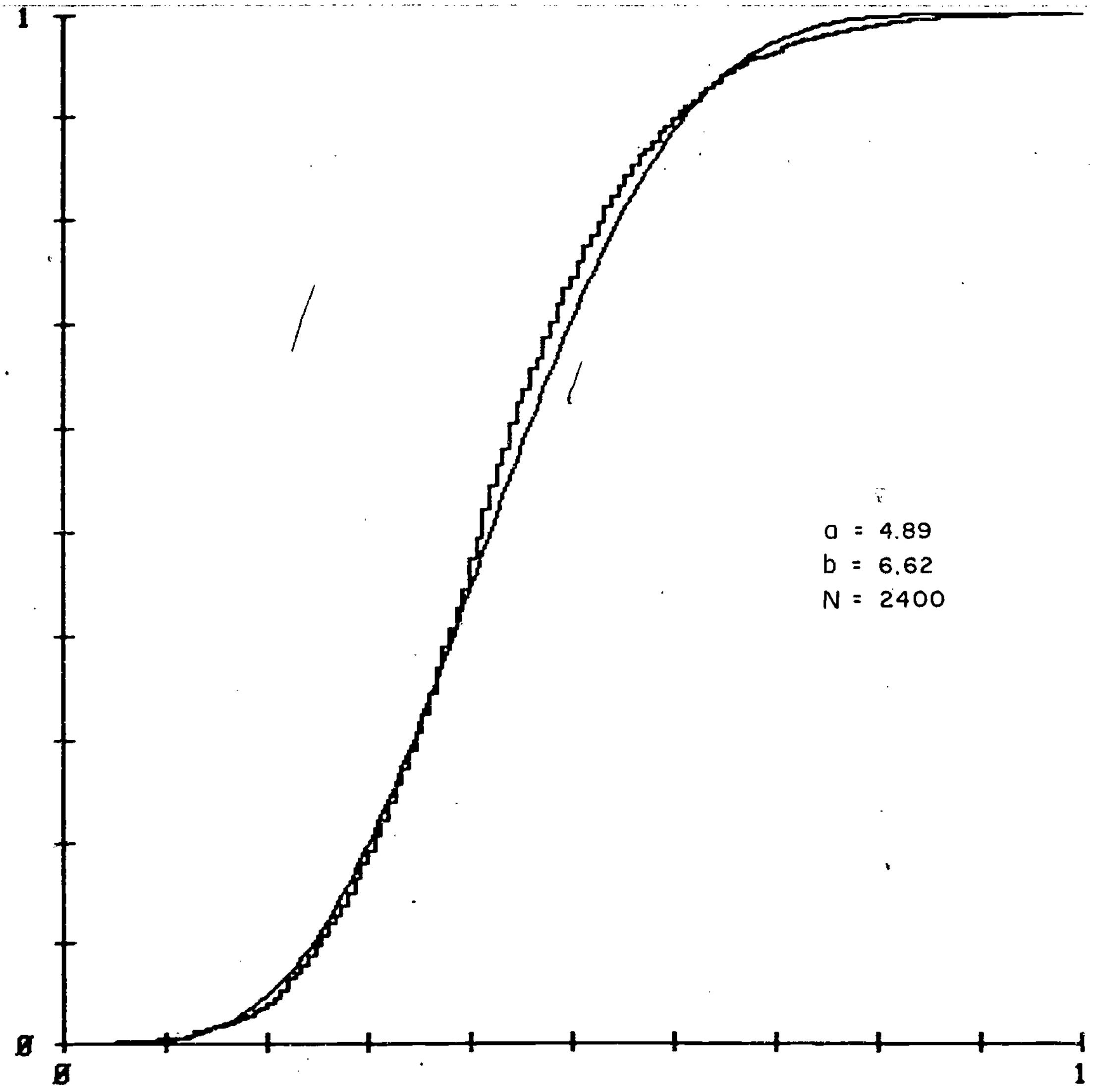
APPENDIX I : Goodness of Fit Test for the Normal Distribution :
The Stepfunction is the Cummulative Distribution of ECI1 z



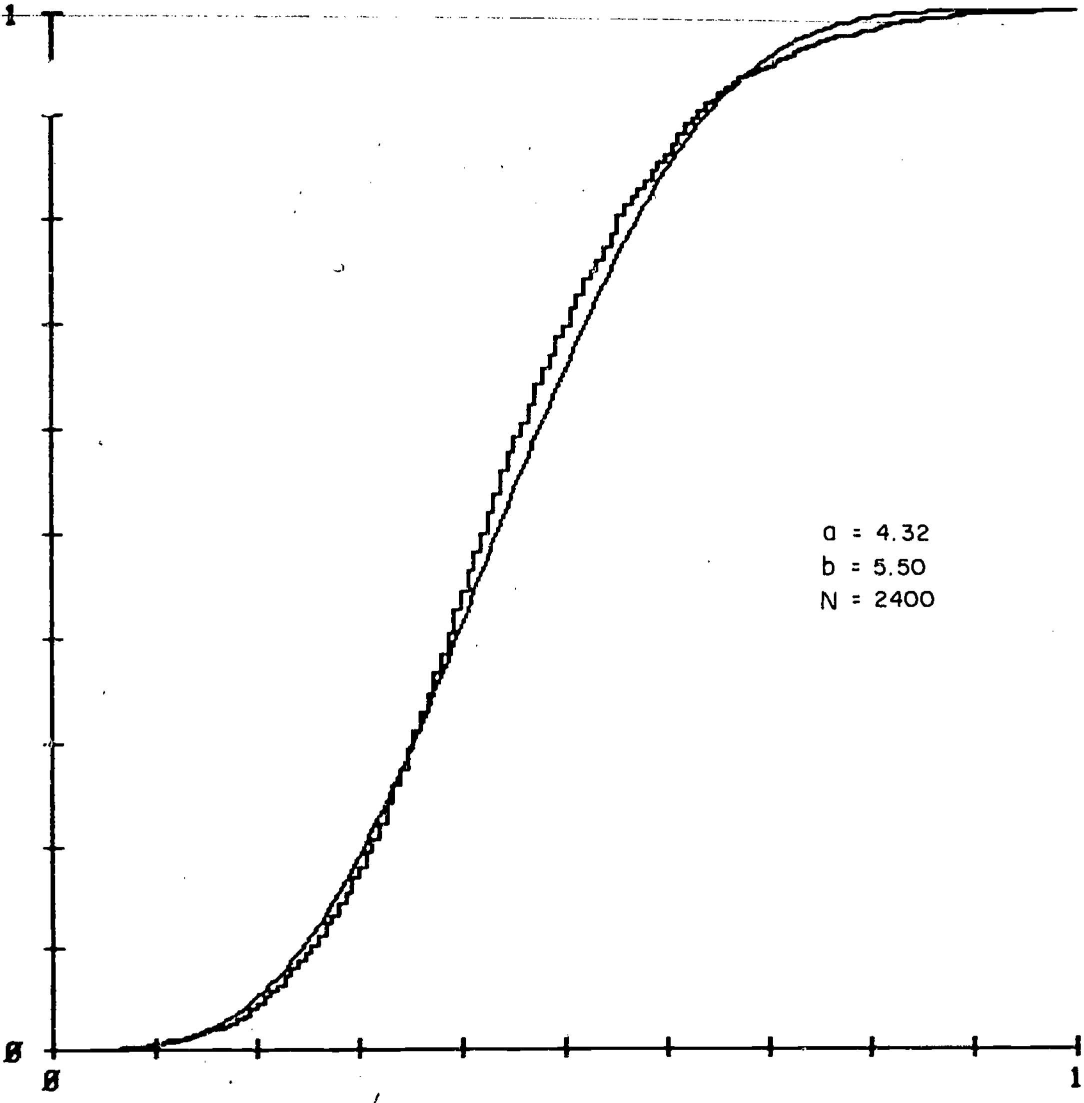
APPENDIX II : Goodness of Fit Test for the Normal Distribution :
The Stepfunction is the Cummulative Distribution of ℓz



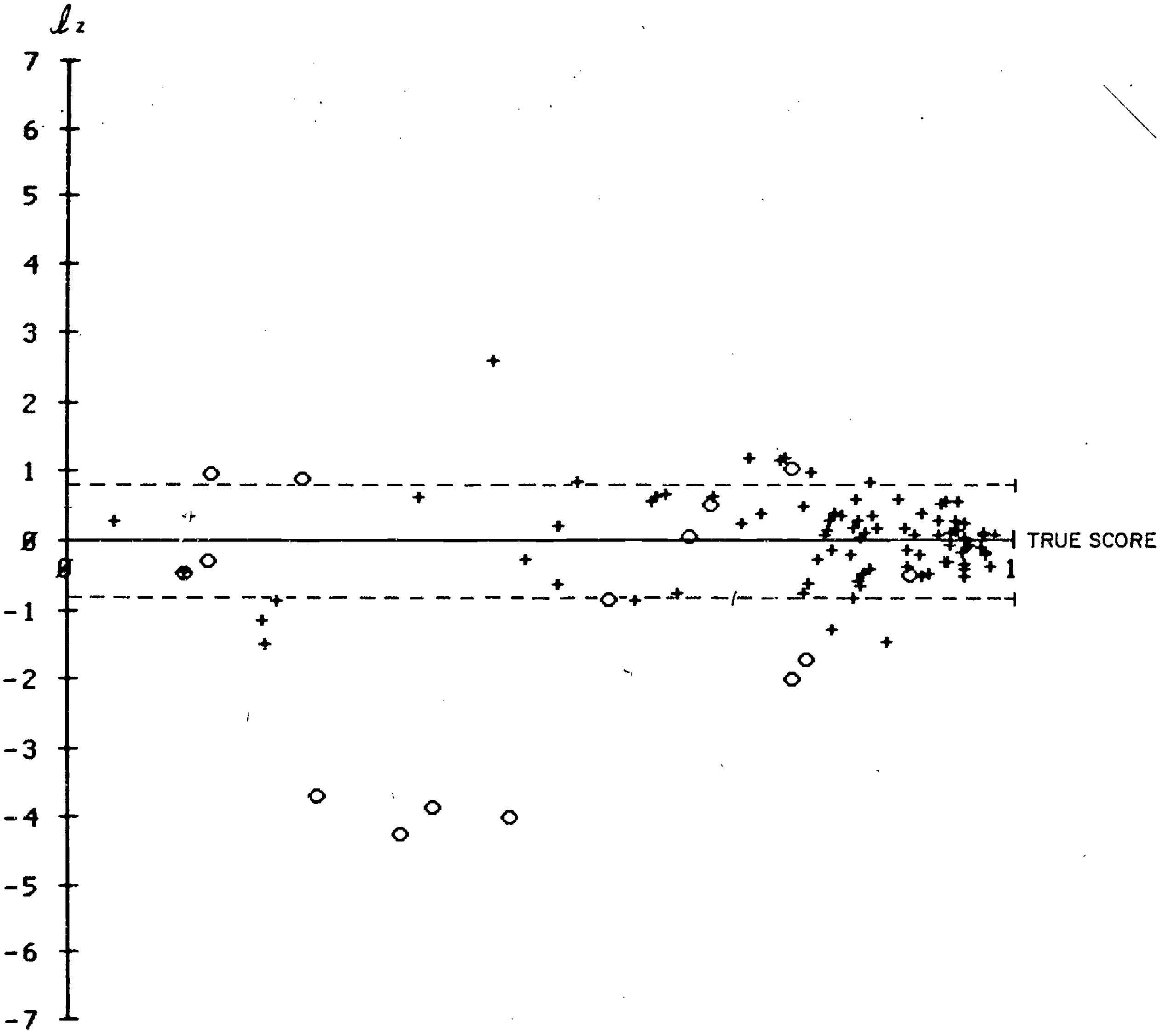
APPENDIX III: Goodness of Fit Test for the Beta Distribution:
The Stepfunction is the Cummulative Distribution of ECI z



APPENDIX IV : Goodness of Fit Test for the Beta Distribution:
The Stepfunction is the Cummulative Distribution of ECI2 z



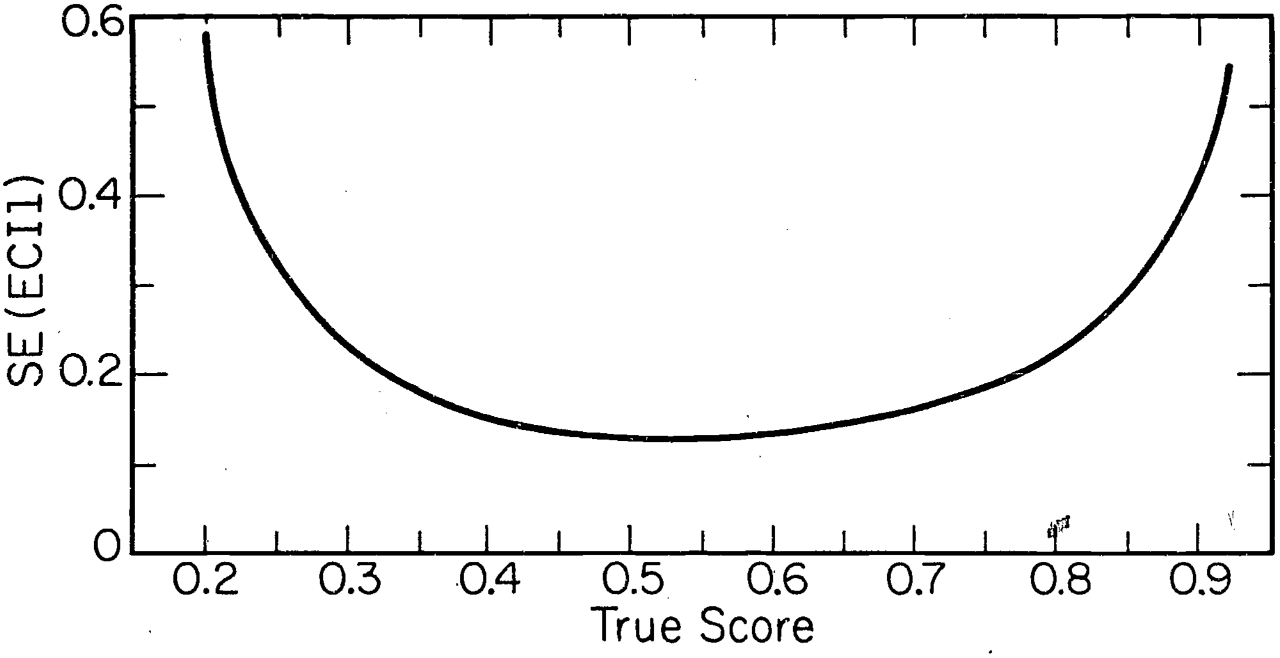
APPENDIX V: Goodness of Fit Test for the Beta Distribution:
The Stepfunction is the Cummulative Distribution of ECI4 z



APPENDIX VI: Plot of l_z Against True Score for the Modified Dataset ("+") and Erroness Rules ("O"), and 80% Probability Interval (-.78, .81).

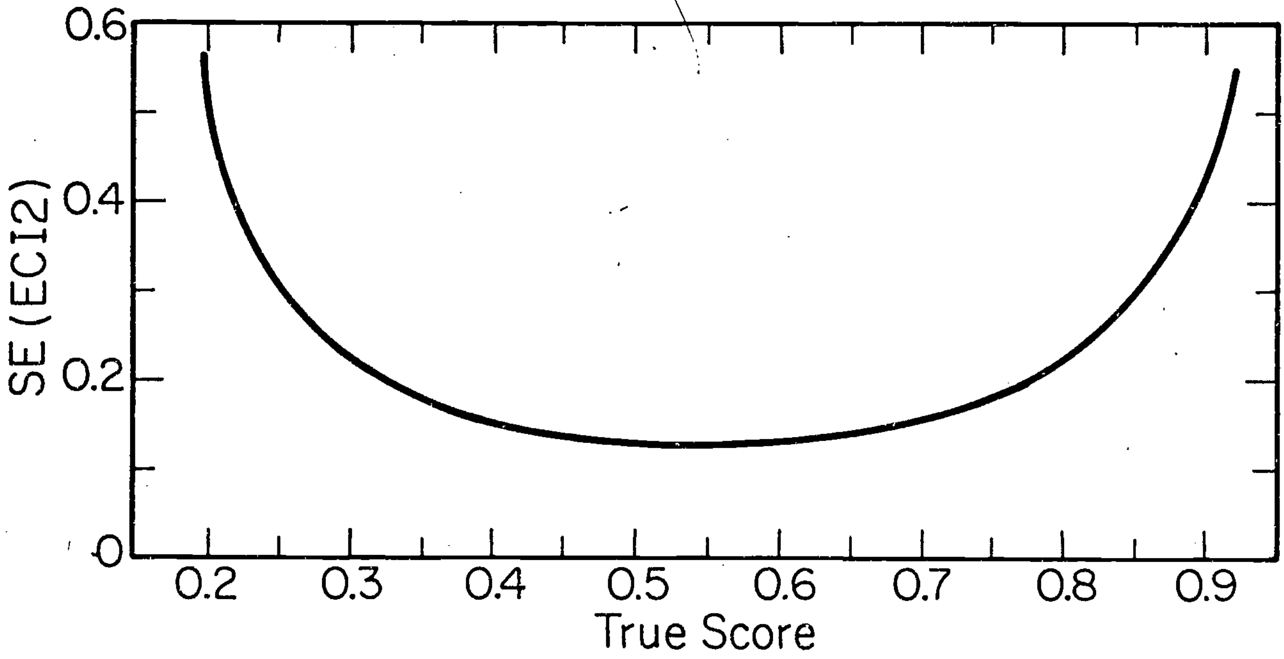
Appendix VII

Standard Error of ECI1



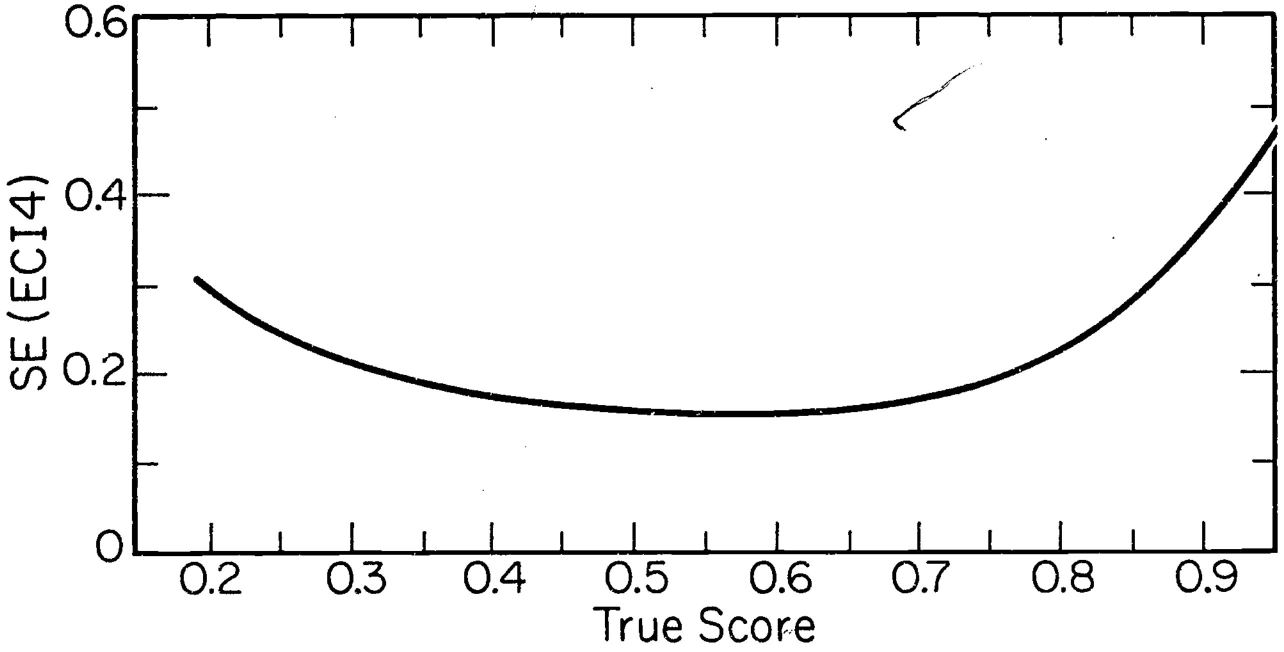
Appendix VIII

Standard Error of ECI2



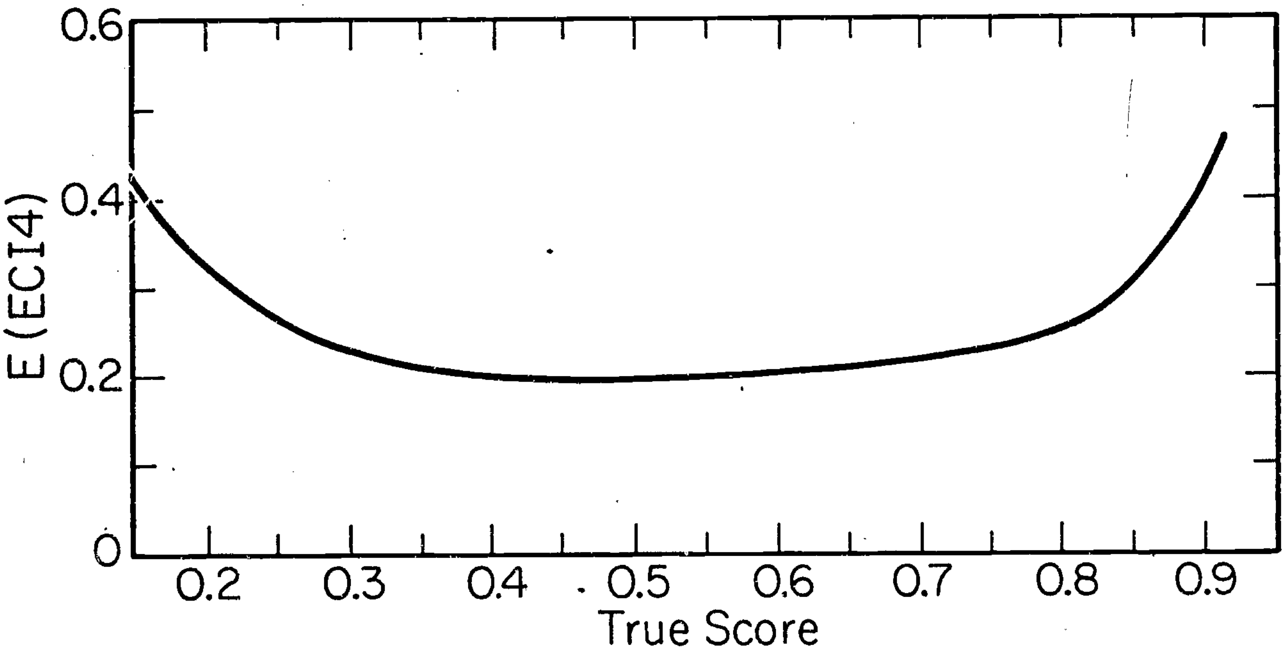
Appendix IX

Standard Error of ECI4



Appendix X

Plot of Expectation of ECI4 Against True Score



Appendix XI

Correlation Matrix of Standardized ECIs and lz With Bugdata

	ECI1_z	ECI2_z	ECI4_z	lz	Total Score	True Score
	1	2	3	4	5	6
1	1.00	.99	.92	-.88	-.11	-.14
2		1.00	.93	-.88	-.11	-.14
3			1.00	-.83	-.19	-.22
4				1.00	.22	.22
5					1.00	.99
6						1.00

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