DOCUMENT RESUME

ED 223 413 SE 039 476

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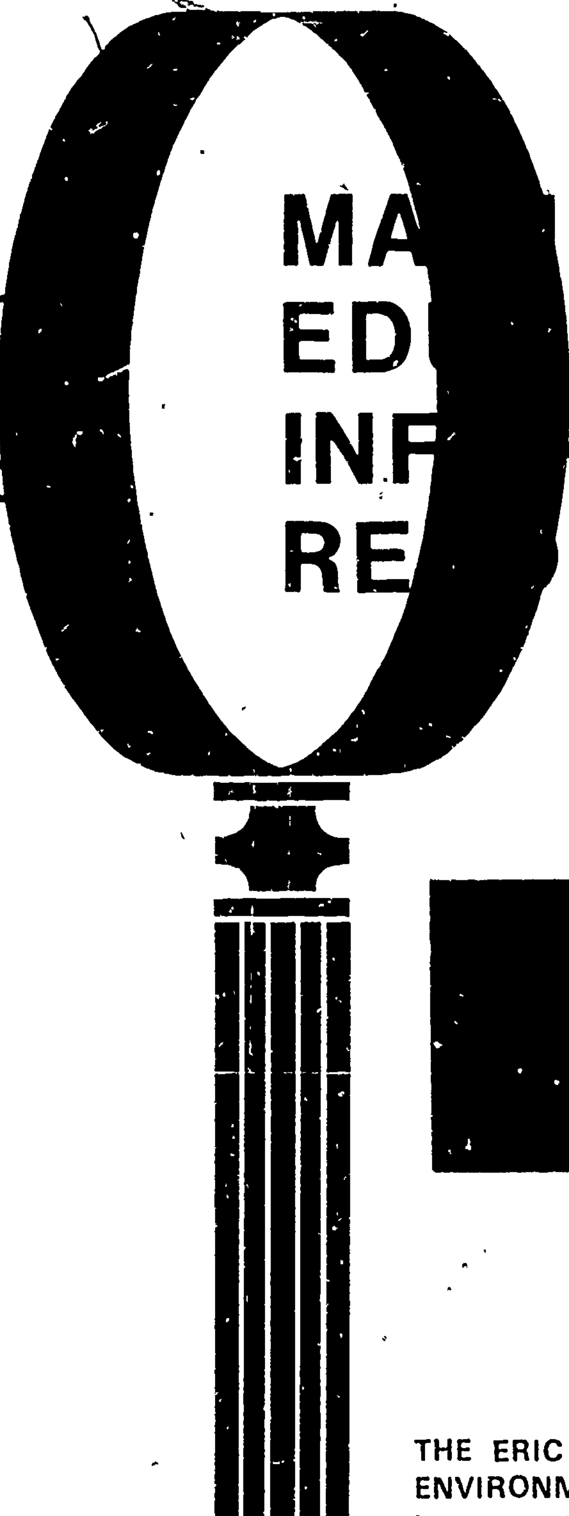
TITLE Mathematics Teachers: An Endangered Species.
INSTITUTION ERIC Clearinghouse for Science, Mathematics, and Environmental Education, Columbus, Ohio.; Ohio State Univ., Columbus. Center for Science and Mathematics Education.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE Dec 82
CONTRACT 400-78-0004
NOTE 117p.
AVAILABLE FROM Information Reference Center (ERIC/IRC), The Ohio State Univ., 1200 Chambers Rd., 3rd Floor, Columbus, OH 43212 ($4.50).
PUB TYPE Information Analyses - ERIC Information Analysis Products (071) -- Guides - Non-Classroom Use (055) -- Branch Around (888)

EDRS PRICE MF01/PC05 Plus Postage.
DESCRIPTORS *College Programs; Curriculum Development; Educational Change; Elementary Secondary Education; Higher Education; *Mathematics Education; Mathematics Instruction; *Mathematics Teachers; *Preservice Teacher Education; *Program Design; *Program Improvement; Undergraduate Study

ABSTRACT

The view discussed is that mathematics teachers are becoming a rare, if not endangered, species, and the public image of mathematics needs to be changed. The mathematics teacher is termed the crucial variable, and a need is seen for changes in mathematics teacher education. The approach described is based on the following assumptions: (1) mathematics teachers must know mathematics well beyond the level they may be expected to teach; (2) mathematics teachers need specialized knowledge; (3) teacher education is the key to educational reform; (4) teachers must be prepared for the realities and contingencies of teaching; and (5) teacher education is an on-going, developmental process. The material strives to: (1) present a broad overview of developments in mathematics teacher education as a perspective from which to view the current situation; (2) propose a taxonomy for teacher education; (3) illustrate the taxonomy and expand upon it through selected examples; (4) describe the operation of a teacher education program based on the taxonomy; (5) relate the taxonomy to the on-going, developmental nature of teacher education; (6) suggest content and experiences for inclusion in teacher education; (7) offer a model for program design; and (8) report preliminary experiences and evaluation. (MP)



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MATHEMATICS EDUCATION INFORMATION REPORT

THE ERIC SCIENCE, MATHEMATICS AND ENVIRONMENTAL EDUCATION CLEARINGHOUSE
in cooperation with
Center for Science and Mathematics Education
The Ohio State University

MATHEMATICS TEACHERS: AN ENDANGERED SPECIES

by

Peggy House

MATHEMATICS TEACHERS:

AN ENDANGERED SPECIES

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DECEMBER, 1982

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Foreword

In this book, Peggy House presents a taxonomy for a teacher education program from pre-service through in-service. But she does more than that. She develops the model for educating secondary school mathematics teachers, and proceeds to pack it full of specific ideas and suggestions that can be of use to all teachers educators, no matter what type of program they have. Many of these ideas can be transferred to the teacher education program for elementary school teachers. Thus, this book can serve as an invaluable resource for all teacher educators.

We are proud to make it available to you. We think it will help you in planning your program and your classes.

Marilyn N. Suydam

Associate Director
Mathematics Education

This publication was prepared with funding from the National Institute of Education, U.S. Department of Education under contract no. 400-78-0004. The opinions expressed in this report do not necessarily reflect the positions or policies of NIE or U.S. Department of Education.

Chapter One

Mathematics Teachers: An Endangered Species

Within the past week, local television and newspapers have called attention to the plight of timber wolves, snail darters, and trumpeter swans. It has become fashionable to show concern for endangered species. Unfortunately, some endangered groups never make it onto any of the official lists. Mathematics teachers are a case in point.

During the closing years of the 1970s there emerged a growing awareness that mathematics teachers were becoming a rare, if not an endangered, species. Newspapers across the United States reported shortages of mathematics teachers, especially at the secondary school level. Colleges and universities responded with reports of sharply declining enrollments in mathematics teacher education programs. Hypotheses were advanced about mathematically talented individuals being lured by high-paying jobs in engineering, technology, and the information sciences, especially the computer fields. The media contributed lengthy accounts of "teacher burnout" and teacher incompetence. Well-meaning parents, friends, relatives, even teachers and counselors, advised young people against careers in education.

We are faced with serious problems. Ours is a present and a future built on ever-increasing dependence upon information processing, technology, and applications of the mathematical sciences. More secondary school and college students are electing to study mathematics in preparation for careers in these fields. At the same time, sizeable numbers of mathematics teachers are themselves leaving their classrooms for new careers in those expanding areas. The demand for creative, effective mathematics teachers grows steadily.

The situation, however, is yet more complex. The past quarter-century has seen the passage of at least two major waves in mathematics education: the "new math" of the late 1950s and 1960s and the "back-to-the-basics" of the 1970s. As we enter the 1980s, we are challenged to create yet another new thrust, one which makes problem-solving the central focus of mathematics teaching. Backed by the recommendations of professional educators, especially the National Council of Teachers of Mathematics (NCTM), this focus on problem-solving recognizes the impossibility of teaching pupils all of the specifics of mathematics which they will need to know in the future.

Yet the public image of mathematics is, by and large, built on assumptions that mathematics consists of well-defined procedures and algorithms which produce precise "right" answers. Often mathematics is equated with arithmetic and algebraic manipulation plus, perhaps, the formal logic of deductive geometric proofs. To one with this view, the back-to-the-basics movement with its emphasis on rote learning and drill was a welcomed trend. Thus, we are faced immediately with the momentous task of changing public opinion on two key issues: (1) to gain acceptance of the fact that "basic skills" in mathematics include far more than computation, and (2) to gain recognition of the fact that "problem solving" encompasses more than the familiar "story problems" of school textbooks.

However we choose to address these challenges, though, we are forced to return to the mathematics teacher as the crucial variable. To say that the teacher is key is, perhaps, to recite a cliche. Yet it is a truism which cannot be ignored. The curriculum developers of the "new math" era were well aware of the need to teach classroom teachers the new content included in those programs. No less important today is the need to prepare mathematics teachers to adopt new teaching behaviors if they are to implement the recommendations proposed for mathematics teaching in the years ahead.

Significant among such recommendations is the Agenda for Action (1980), NCTM's response to present and future needs in mathematics education. It was conceived "to present its (NCTM's) responsible and knowledgeable viewpoint of the directions mathematics programs should be taking in the 1980s" (NCTM, 1980; p. i). The preface to the Agenda concludes with the challenge:

> These recommendations are not the end of our efforts but a beginning. They represent an agenda for a decade of action, and we call on all interested persons and groups to join us in a massive cooperative effort toward better mathematics education for all our youth.
>
> (NCTM, 1980; p. ii)

It is this writer's belief that an essential aspect of that massive effort lies in on-going education of all mathematics teachers. This book is one interested person's response to the challenge of the Agenda.

The book is addressed to persons responsible for the education and professional development of mathematics teachers through undergraduate (pre-service) or graduate levels of formal academic programs, as well as through the continuing in-service education of teachers offered either by school systems or by colleges and universities. While much of what is proposed could be generalized to other subject areas or to other levels of instruction, the discussion here is deliberately focused on the education of secondary school mathematics teachers. Discussions which are too broad or too general are frequently viewed as being vague or theoretical. They also are easier to ignore because they do not speak directly to a specific set of needs. To argue for the need to effect changes in the teaching of secondary school mathematics and in the education of secondary school mathematics teachers is not to negate equally serious needs in other areas. Nevertheless, to attempt to address too broad an area of need is probably to effect little change. Thus we will address specifically only the area of teacher education for secondary school mathematics in the hope that this one endangered species can benefit from these efforts.

Basic Assumptions

The approach to mathematics teacher education which is described in the pages to follow is based on several fundamental assumptions about the professional education of mathematics teachers. We state those assumptions here and refer to them throughout the remainder of the book. The assumptions are the following:

Mathematics teachers must know mathematics well beyond the level that they teach.

Let us acknowledge from the start that a firm foundation in mathematics continues to be an essential prerequisite for mathematics teachers. This assumption has long characterized the recommendations for the profession, as any reading of the history of groups setting guidelines for mathematics teacher education will verify. The "Guidelines for the Preparation of Teachers of Mathematics" published by NCTM in 1973 expressed the need thus:

> It is essential for teachers to know more than they are expected to teach and to be able to learn more than they already know, for without such knowledge, progress is essentially impossible.
>
> (NCTM, 1973; p. 5)

The 1979 revision of those guidelines repeated the sentiment in these words:

> Prospective teachers of mathematics at any level should know and understand mathematics substantially beyond that which they may be expected to teach.
>
> (NCTM, 1979; p 1)

We state this assumption explicitly and first to emphasize its importance, even though most of the discussion in this book will focus on other aspects of teacher education rather than on the formal study of mathematics. This is because, at least in recent years, secondary mathematics teachers have, in general, graduated from their teacher education programs with reasonably extensive training in mathematics, including work in calculus and several other areas such as analysis, differential equations, abstract algebra, linear algebra, probability, statistics, euclidean and non-euclidean geometries, and computer science. It is not uncommon, however, for the students or graduates of such programs to complain that they have no need of mathematics at that level since they will never teach it to pupils.

We reject that argument and continue to assume that teachers must study mathematics well beyond the level at which they teach. At the same time we must be careful not to assume that the mathematics component of the teacher education program is forever fixed. New discoveries in mathematics, new applications of mathematics, new emphases on various branches of the mathematical sciences all necessitate the periodic reevaluation of what mathematics we will require. For example, statistics, combinatorics, and numerical analysis are areas which are likely to assume greater importance in the future, yet their significance to the mathematics teacher will differ from their significance to the applied mathematician or to the researcher.

Mathematics teachers need specialized knowledge.

Knowledge of mathematics is only one aspect of the specialized knowledge that mathematics teachers must possess. In particular we stress that knowing mathematics is not the same as being able to deliver mathematics to others. Stated another way: being competent in mathematics does not guarantee that one will be a good mathematics teacher. Knowledge of mathematics is a necessary but not a sufficient condition for mathematics teaching.

In addition to knowledge of mathematics, teachers need knowledge of psychology and of pedagogy. They also need what Smith referred to as knowledge about the subject, in this case knowledge about mathematics. This, according to Smith (1969; p 125), is the knowledge used in thinking about the subject (mathematics) and the logical operations used in manipulating it. One can learn to perform mathematical operations without attaining knowledge about mathematics, and often it is the lack of a significant degree of knowledge about mathematics which most interferes with the delivery of mathematics to pupils. The programs we develop must give appropriate attention to all aspects of the specialized knowledge that mathematics teachers need.

Teacher education is the key to educational reform.

We noted earlier that the teacher is the critical factor in realizing changes in education practice. The NCTM Agenda for Action (1980), cited earlier, calls for mathematics teaching that requires new emphases within the curriculum, new uses of instructional materials, and new roles for teachers. Yet as recently as 1977, when the National Science Foundation (NSF) commissioned three large-scale projects to assess the status of mathematics and science teaching in the United States, the picture that emerged was far removed from what will be required if the Agenda is to be realized. The findings were summarized in one report of the NSF literature review (Suydam and Osborne, 1977) as follows:

> Classrooms have changed little over the past twenty years, despite the innovations advocated. Predominant patterns continue to be: instruction with total-class groups, tell-and-show followed by seatwork at the elementary school level, and homework-lecture-new homework at the secondary school level; and the use of a single textbook but few other materials. (ERIC, 1979, p. 2)

The other two NSF studies (Stake and Easley, 1978; Weiss, 1978) substantiate the conclusion noted above. One observer who compiled a case study of one school district as part of the NSF project recorded the following:

> In all math classes I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief

> explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (Stake and Easley, 1978: 5:6)

The observer went on to comment that "Although it seemed boring to me, students and teachers seemed comfortable with it."

While we recognize that there are and have been many outstanding mathematics teachers, the last observation above suggests the degree to which pupils and teachers have assumed that mathematics classes always follow the noted format. This conclusion was further verified in the second National Assessment of Educational Progress (NAEP) in mathematics (1977-78) which sampled pupil attitudes toward mathematics as well as their achievement in the subject. The NCTM committee that reported on the attitudes of secondary school students reflected in the NAEP results concluded:

> ...students perceived their role in the mathematics classroom to be primarily passive: they feel that they spend a lot of time listening to the teacher explain mathematics, a lot of time watching the teacher work problems, and a lot of time working problems from the textbook on an individual basis. (Carpenter _et al_., 1980b; p. 534)

If, as many studies have concluded, teachers teach as they were taught, than it is not difficult to see why the pattern of instruction described above generalizes so consistently across schools and classrooms. It also underscores the difficulties faced when one attempts to change the pattern, for teachers are not likely to change their classroom behavior unless alternative approaches are modeled in a meaningful way. Thus the assumption that teacher education is the key to educational reform carries with it the implication that teacher education programs should model desired teaching processes both in the college or university classroom and in the secondary schools. It also implies that teacher education must integrate formal instruction in the college classroom with meaningful practicum experiences in the secondary school.

<u>Teachers must be prepared for the realities and contingencies of teaching.</u>

While teachers may be the key elements in education, they are, none-the-less, restricted by the parameters of their school situations. Otte, summarizing conditions worldwide, concluded that teacher behavior is dependent on the "opportunities and limitations inherent in the context of their work." Among these opportunities and limitations he enumerated the decision-making and organizational structure of the school, the standards of colleagues, working conditions, working-time regulations, the curriculum, and available teaching aids. He further proposed:

> These elements need to be clarified so that the teacher can consciously consider them and improve the quality of his decision-making in his professional practice. Just as teacher education prepares teachers in the subject matter and methodology, so it ought to prepare them to cope with these demands. (Otte, 1979; p. 110)

What Otte described is another type of problem solving. Just as teachers cannot teach pupils all of the specifics of the mathematics they will need, so teacher educators cannot prepare teachers for each specific contingency that may arise. But also as the mathematics teacher hopes to teach pupils basic problem-solving skills, so, too, should teacher educators help teachers to develop the knowledge and understanding needed to make responsible professional decisions.

<u>Teacher education is an on-going, developmental process.</u>

We consider it essential to view teacher education as a developmental process beginning at the pre-service, undergraduate level and continuing throughout the teacher's professional life. We do not view teacher education as a collection of courses and workshops, and we do not dichotomize between undergraduate and graduate, pre-service and in-service education. Rather, we see teacher education as a unified whole and consider its various aspects (pre-service, induction, in-service, etc.) as points on a continuum. This facilitates a comparison of the similarities and differences among teachers at different points along the continuum, and it encourages a more realistic set of expectations of teachers at different stages of professional development. It further enables teacher education at any time to build on the past learning and demonstrated strengths of the teacher while focusing on the attainment of related objectives or new goals.

Dimensions of Concern

We have enumerated the assumptions that underlie our approach to mathematics teacher education. We must also acknowledge several areas of concern that impinge upon these assumptions, for unless teacher educators are attentive to these points, they may frustrate their own efforts to effect change.

A first concern arises from the recommendation of the <u>Agenda for Action</u> to make problem-solving the central focus of mathematics teaching. The concern is that "problem-solving" is a term often misunderstood because the speaker assumes that each listener understands it in the same way as the speaker intends. Too ofen it turns out that teachers assume "problem-solving" to signify textbook exercises stated in words ("story problems"), while mathematics educators generally use the term to signify, in particular, situations in which the solver lacks the prior knowledge and/or algorithms to guarantee a solution. Hence, teachers may believe they are emphasizing problem-solving when, in fact, they are falling short of that. Further, the situation is aggravated when teachers themselves do not engage in problem-solving; many mathematics teachers admit that they do not because they are unfamiliar or uncomfortable or threatened in a problem-solving situation. Teachers who are not problem solvers are not likely to teach problem-solving. The ramifications of this will be seen in a later concern.

The second concern arises from the picture of mathematics teaching drawn from the NSF studies and the NAEP response cited earlier. In particular, the infrequency of employing discovery learning, laboratory activities, manipulative materials, multimedia approaches, and other alternatives for mathematics teaching results in the fixed, colorless portrait of life in mathematics classes. Related to this there often is an assumption, either stated or implicit, that alternatives such as those listed above are appropriate for elementary school pupils but unnecessary in secondary school. Again we confront the dilemma that teachers who have not themselves learned through some of these alternatives probably will not employ them in their own classes.

Third, we must be careful in advocating goals like those of the Agenda for Action that we give deliberate attention to teacher education. A memorable passage from the report of the Cambridge Conference that convened in 1963 to propose goals for school mathematics was the admission that "Thus we ignored the whole problem of teacher training, and acted on the assumption that if a teachable program were developed, teachers would be trained to handle it" (Cambridge Conference, 1963; p. 3). We cannot assume a similar posture of ignoring teacher education relative to the Agenda. Indeed, that Agenda will never be translated into action except through the individual efforts of classroom teachers. We cannot take for granted the needs of the teachers upon whom we must rely for effecting change.

A fourth concern is for the prevailing attitudes toward mathematics exhibited not only by pupils but also by teachers and the public in general. A clue to those attitudes was dramatically illustrated in an item from the national assessment reported by Carpenter et al. (1980b; p. 538):

> "There is always a rule to follow in solving mathematics problems."

	Disagree	Undecided	Agree
Age 13	5 %	5 %	89 %
Age 17	8 %	4 %	88 %

Perhaps more than any other subject, mathematics is viewed in a very dualistic manner: there are "right answers" and "wrong answers," "right ways" and "wrong ways" for everything. This is closely allied to the view that mathematics is equivalent to arithmetic or to algorithmic manipulation and it is reinforced by the back-to-the-basics mentality which stressed drill and practice in computation.

We cannot suppose, however, that an overhwhelming majority of pupils arrived at this narrow view of mathematics entirely on their own. Otte placed the responsibility squarely with the mathematics teacher:

> To the greatest part of society the mathematics teacher is the primary source of people's views on and attitudes toward mathematics, and the question of knowledge about mathematics on the teacher's side is therefore not a merely academic question but of great social importance. (Otte, 1979; p. 124-125)

One explanation for the dualistic view of mathematics so common among pupils is that the mathematics they encounter in school tends to be mathematics in its finished form, not mathematics in the making. That is, they frequently are shown completed proofs, rules, formulas, or algorithms instead of experiencing the problem-solving process through which those conclusions were derived.

Here we return to the ramifications suggested earlier in our first concern. Because teachers have not made problem-solving an intrinsic part of their own behavior, they have not integrated it throughout their teaching. Pupils, in turn, have formed attitudes about mathematics that are inconsistent with true problem-solving as the central focus, the core of mathematics, and they have carried these attitudes into adulthood.

We must be careful here not to confuse responsibility and blame. Blame implies disapproval, reproach, censure, and there is nothing to be gained from blaming teachers for performing as they have been taught to perform. Responsibility, on the other hand, recognizes the vital role of the teacher as the principal agent or cause of pupils' attitudes about mathematics, and it should cause teachers to take seriously the significance of their impact on pupils and on the public. Responsibility also should cause teacher educators to take seriously their duty to help teachers to develop the kind of catholic view of mathematics that we seem to be expecting from them.

A final concern arises from the complex relationship that exists between developments in teacher education and research on teaching and teacher education. There exists a certain contemporary mentality of accountability that wants to demand proof or research based on an engineering model of the systematic manipulation of variables to arrive at conclusions characterized by an extreme degree of certainty. In the behavioral sciences, including education, this approach is not always applicable. However, influenced by the accountability mentality, we may be tempted either to demand firm evidence drawn from such research before we implement any change or to dismiss as inconsequential any attempts at change not based on such research.

A different position was taken by McKillip who, in making a case for competency-based teacher education (CBTE), asserted:

> It is not necessary, in advocating CBTE, to produce research evidence showing the CBTE is better than or even as good as existing programs. The non-CBTE programs in existence are not based on research and did not displace programs which preceded them because they were proved better. (McKillip, 1980; p. 46)

The above statement is not quoted as justification that "anything goes" or that we are free to be irresponsible in espousing any program at all. However, McKillip does remind us that often we have no more research-based evidence for retaining previous methods than we have for trying newer approaches.

Cooney gave further insight into our situation with this observation:

> Before much progress can be made with respect to research in mathematics teacher education, work needs to be done in delineating the nature of delivery systems, the nature of the content of mathematics teacher education programs, and the range of expected outcomes from the programs. Competency based teacher education (CBTE) programs have provided some impetus for laying out the various domains, although the task does not have to be couched in a CBTE context. (Cooney, 1980b; p. 469)

Elsewhere, reflecting on the type of research needed in mathematics teacher education, Cooney concluded:

> Not necessarily research in terms of highly sophisticated statistical techniques. But rather research in terms of a serious reflection about what we as teacher educators are all about, what we wish to accomplish, and how best we can accomplish whatever our primary objective is. (Cooney, 1980a; p. 12)

We raise this concern about research to acknowledge that the approach to teacher education which is discussed in these pages is not based on statistical analyses of experimental data. It is, however, conceived in the spirit of the serious reflection for which Cooney has called, and it is offered in response to the challenge, again from Cooney:

> If we wish to take seriously the education of mathematics teachers, we have a responsibility to seriously reflect upon what that special knowledge is which permits professional mathematics teachers (sic) and to consider what processes exist when teachers acquire that knowledge. The process of educating the professional mathematics teacher is too important to allow ourselves to be moved by whimsical forces. We should, nay, must reflect upon our work and continually pose questions for at least our own realization. That is what a profession is all about. (Cooney, 1980a; p. 15)

Goals of This Project

The major goal of this undertaking is to reflect upon mathematics teacher education in the manner described by Cooney. We will do this by suggesting a taxonomy of mathematics teaching as a framework for the development of an on-going teacher education program.

As noted, earlier, we will limit our application of this taxonomic scheme to the education of secondary school mathematics teachers for two principal reasons: first, because each subject field requires of the teacher certain unique types of knowledge and skills; and, second, because attempts to be too general usually result in no action. At the same time, we wish to avoid being so prescriptive that other educators cannot personalize and adapt these ideas to their own situations.

Specifically, then, we set forth the following goals for the remainder of this reflection:

1. To present a broad overview of developments in mathematics teacher education as a perspective from which to view our present position.

2. To propose a taxonomy for mathematics teacher education.

3. To illustrate and expand upon the taxonomy through selected examples.

4. To describe the operation of a teacher education program based upon the taxonomy.

5. To relate the taxonomy to the on-going, developmental nature of teacher education from preservice though induction to continuing professional education.

6. To suggest content and experiences for inclusion in a teacher education program.

7. To offer a model for designing a program based on the taxonomy.

8. To report preliminary experiences with and evaluations of the model.

In this way we hope to initiate the kind of reflection advocated earlier. We hope to call attention to many aspects of mathematics teacher education and to offer a framework upon which others can build. We wish only to describe, not to prescribe, and we invite others to find their own expressions of these ideas in each one's unique contribution to the continuing professional education of mathematics teachers.

Chapter Two

Overview of Developments in Mathematics Teacher Education

In mathematics, as in any field, one cannot dissociate activities in teacher education from trends and activities in the discipline as a whole. In order to consider the recent evolution of teacher education in mathematics, it is useful to view these developments in the context of activities in school mathematics overall.

Recent Developments in Mathematics Education

The 1950s and 1960s were decades of extensive curriculum reform in school mathematics -- the period which came to be known as the era of the "new math." Although there were numerous distinct and independent curriculum projects during that period, a common characteristic of the efforts was their emphasis on the content of school mathematics: on new topics, new organization, new emphases, new grade placement. As a result, there emerged a consequent urgent need for teacher training, especially for in-service education to prepare teachers for the new curricula. There also existed a prevailing climate of support and encouragement for those curricular efforts, and money for in-service teacher education usually became available. Hence, in-service summer or academic year institutes with financial support for participants grew in number. A later report estimated that twenty-five percent of junior high school and thirty-seven percent of senior high school mathematics teachers attended one or more NSF-sponsored institutes (Weiss, 1978; p. 69); another report placed the estimate at thirty-five percent of mathematics and science teachers (ERIC, 1979; p. 4).

Another phenomenon of the period was the establishment by professional groups of committees or commissions to study teacher education and to report their recommendations. Four such reports issued between 1959 and 1961 came from the Commission on Mathematics of the College Entrance Examination Board (CEEB), the Secondary School Curriculum Committee of NCTM, the Subcommittee on Teacher Certification of the Cooperative Committee on the Teaching of Science and Mathematics of the American Association for the Advancement of Science (AAAS), and the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematics Association of America (MAA). Their recommendations are reviewed in the Thirty-second Yearbook of the NCTM (1970), but in general the focus was on the college mathematics needed by secondary school teachers. The 1961 CUPM recommendations and their 1966 revision, in particular, have frequently been credited with having had an impact on mathematics education for elementary teachers, largely through a marked increase in the mathematics requirements of colleges and universities. Requirements for secondary school teachers continued to emphasize a major in mathematics with one or more courses in education, especially teaching methods.

In the decade that followed, the 1970s, the emphasis in school mathematics was on a "return to the basics," and less emphasis was given to teacher education. In 1975, the National Advisory Committee on Mathematical Education (NACOME) of the Conference Board of the Mathematical Sciences published its Overview and Analysis of School Mathematics, Grade K -12, an extensive report on objectives, current practices, and attainments in mathematics education. In attempting to survey developments in teacher education, NACOME was forced to conclude:

> The dominant feature of the mathematics teacher education picture is the absence of hard data concerning programs and practices, requirements, and characteristics of the products. Much of what is written, discussed in conferences, and used to justify recommended programs is based on sketchy impressionistic data, random cases of innovative activity and research, and opinion. It is impossible to even attempt a description of the "typical" graduate of a teacher education pre-service program, much less that same individual after possible exposure to a wide variety of in-service training experiences. (NACOME, 1975; p. 81)

Later, noting that a 1971 revision of the CUPM recommendations had failed to achieve the impact of the earlier statements but, instead, had been "largely ignored," NACOME suggested that:

> Attention to mathematical education in teacher training seems to be cyclical, ranging from positive emphasis to indifference. The climate of the 1970s has not been friendly to the concerns of mathematics teacher educators except as they fit the general trends and favored fashions in the field of professional education. (NACOME, 1975; p. 86)

The 1970s did, however, see a continuation of the trend in commission reports and recommendations. In 1971 the AAAS and the National Association of State Directors of Teacher Education and Certification (NASDTEC) published new guidelines and standards for the education of secondary school teachers of science and mathematics. The report posed twelve guidelines for planning, implementing, and evaluating programs and recommended content and experiences for secondary teachers. Of those twelve guidelines, three which spoke directly to the content of mathematics for prospective teachers were the following:

> Guideline VI: An undergraduate program for secondary school mathematics teachers should include a major in mathematics of sufficient depth to make possible further study of mathematics at the graduate level in areas appropriate for teachers.

> Guideline VII: An undergraduate program for secondary school mathematics teachers should include a substantial experience with the field of computing as it relates to mathematics and to the teaching of mathematics.

Guideline VIII: An undergraduate program for secondary school mathematics teachers should provide substantial experience with mathematical model building so that future teachers will be able to recognize and construct models illustrating applications of mathematics. (AAAS/NASDTEC, 1971)

Guidelines VII and VIII above reflected two emerging trends in mathematics teacher education. In addition to the traditional mathematics major based on calculus, algebra, geometry, and analysis, more attention began to be directed at computing and at the applications of mathematics.

Other guidelines in the AAAS/NASDTEC report called for:

-- experiences that foster continuous growth in those human qualities of the teacher that will enhance learning by his students. (I)

-- knowledge and experience to illustrate the cultural significance of science, to relate science and mathematics through technology to social conditions, and to apply the analytical methods of science in multidisciplinary approaches to studying and solving societal problems. (II)

-- opportunities for prospective teachers to gain insight into the intellectual and philosophical nature of science and mathematics. (III)

-- experiences which require the prospective teacher to seek out and study concepts which are new to him, and then to synthesize written and especially oral expressions of them designed for others for whom these ideas are also new. (IX)

-- experiences which will enable the prospective teacher to learn about the nature of learning, conditions that help young people learn, and how to maintain a proper learning environment. (X)

-- development of the ability of the future teacher to select, adapt, evaluate, and use strategies and materials for the teaching of science or mathematics so that teaching - learning situations for which he is responsible will be consistent with general knowledge about teaching and learning and will be appropriate both to the special needs of the learners and to the special characteristics of the science disciplines or the interdisciplinary problem. (XI)

-- development of the capacity and the disposition for continued learning in mathematics and science and the teaching of these subjects. (XII)

In addition to the twelve guidelines, the AAAS/NASDTEC recommendations also proposed four standards that outlined criteria for teacher training institutions. One of these, Standard 3, stated:

> Teacher education institutions should develop performance criteria as guides in planning teacher education experiences, in evaluating teacher education programs, and in assessing the ability of prospective teachers to contribute to effective learning.

This standard reflected another phenomenon of the 1970s, the emergence of competency-based teacher education (CBTE) or performance-based teacher education (PBTE) as a model for teacher training. Because the notion of competency-based education is related to the model of teacher education outlined in this book, we will discuss it at greater length in a later section.

In 1973, the NCTM issued its "Guidelines for the Preparation of Teachers of Mathematics" which proposed areas of academic and professonal knowledge, professional competencies and attitudes, and institutional responsibilities. Although stated as competencies, the guidelines did not constitute a framework of a CBTE program nor did they speak to CBTE *per se*. They did, however, suggest the scope of mathematical content, humanistic and behavioral studies, teaching and learning theory, laboratory and clinical experiences, and practicum and pre-certification teaching experiences recommended for mathematics teachers at various grade levels. In 1979 a revised set of guidelines was published that expanded on the competencies in the original list. These guidelines can serve as an excellent starting point for developing the kind of teacher education program that we will be describing in this book.

Another 1973 document was the "Report of the Conference on the K-12 Mathematics Curriculum" which summarized a conference held at Snowmass, Colorado. The Snowmass conferees also addressed the question of teacher education and called for:

-- the investigation and development of innovative approaches to better prepare teachers;

-- a focus on children and on the learning patterns and problems of pupils;

-- the combination and coordination of content, methods, and school experiences in teacher training;

-- the involvement of in-service teachers in pre-service teacher training;

-- needed research into methods of educating teachers to think mathematically; and

-- on-going support of change mechanisms for mathematics teaching in the schools.

The Snowmass Conference offers an interesting contrast to the Cambridge Conference held ten years earlier. The Cambridge Conference, which was attended only by mathematicians holding university positions or the equivalent, was cited earlier for its attention to curriculum without regard for teacher education. Elsewhere in the Cambridge Conference report was the acknowledgement that the conferees had made no attempt to take account of recent research in cognitive psychology such as the work of Piaget (Cambridge Conference, 1963; p. 3). These are some indicators of the shift that occurred in mathematics education, from a focus on curriculum to a focus on instruction and on the need for cooperation and interaction between colleges and universities on the one hand and teachers in the schools on the other.

The other major professional organization to propose guidelines for mathematics teacher education was the MASS. In 1975 the MAA guidelines were revised by a committee of both MAA and NCTM. The guidelines, however, focused on minimal course offerings expected of mathematics departments and on the quality of instructors for those courses, not on the requirements for the teacher education degree (NACOME, 1975; p. 90). In 1978 the MAA sponsored a conference, PRIME-80, to dicsuss prospects in mathematics education in the 1970s. Among its recommendations were a call for a redefinition of the mathematics skills essential for every citizen, a call for a reexamination of the content of pre-college and college mathematics, and recognition of the need to reexamine the mathematics needed by both pre-service and in-service teachers. The conference did not, however, elaborate at that time on the recommendations for teachers.

Two other events of the 1970s should be noted as significant bench marks in mathematics education: the three NSF studies of 1977 and the national assessment in mathematics conducted in 1972-73 and again in 1977-78. The NSF studies produced the portrayal of mathematics teaching described in Chapter One. They also provided data on some of the trends in mathematic enrollment, in curriculum, and in teacher characteristics and needs. Among the conclusions generalized from the three studies which have relevance for our discussion here are the following:

-- The terms "dull" and "boring" were used with great frequency when describing pupils' attitudes toward their mathematics classes.

-- Teachers reported one of their most difficult problems to be the motivation of pupils to learn mathematics.

-- Teachers at all levels identified learning new teaching methods and implementing discover/inquiry approaches as the aspects of their jobs with which they needed most help.

-- Teachers appeared to be very happy with the back-to-the-basics movement of the time and they applauded the emphasis on traditional content, instructional methods, and higher standards of student performance associated with the movement.

-- Many teachers thought too much emphasis had been placed on discovery learning, hands-on demonstrations, field study, and contemporary topics. Many also believed that mastery of certain skills is an essential prerequisite for concept learning and creativity.

-- The most common view of mathematics reported by elementary teachers was that mathematics is a collection of rules and procedures to be learned to a level of high proficiency. Secondary teachers also emphasized algorithmic performance and tended to see the major reason for studying mathematics to be preparation for more advanced mathematics.

-- Teachers were characterized as displaying lack of vitality, lack of interest, lack of creativity and curricular risk taking, lack of enthusiasm for teaching -- even sometimes a negativism toward children, colleagues, administrators, and university training programs. (Fey, 1979)

The above is a selective, perhaps overly simplified, list of conclusions from reports that fill many large volumes. Fey, in synthesizing these and other generalizations, added the following caveat:

> For many teachers, supervisors, and curriculum developers the picture of school mathematics assembled by the three status surveys will seem an exceedingly negative and inaccurate description of the teaching they see. It is tempting to dismiss the many critical quotations and interpretations of data as a product of biased investigators looking for a crisis that isn't really there. However, there is a consistency to the findings of all three NSF studies -- each by independent teams of investigators -- that makes the findings hard to ignore. (Fey, 1979; p. 503)

Fey went on to suggest that the most discouraging feature of the three NSF studies is the constant pattern of great difference between what appears to characterize mathematics education in most schools and what is recommended by prominent teachers, supervisors, and professional organizations.

The other studies that should be mentioned were the two national assessments in mathematics. The 1972-1973 assessment of the mathematical achievement of nine-, thirteen-, and seventeen-year-old pupils, plus a group of young adults, was the first nationwide testing of this type. Hence, the findings of that first NAEP gave only a picture of the mathematics achievement levels of pupils in 1972-73. No conclusions could be drawn about that achievement compared to past performances. Despite this limitation, NAEP results, together with test scores from local, district, or state testing programs, frequently were cited by the media, by the public, by school boards; and even by educators as evidence that mathematics achievement was dropping or as rationale for promoting the back-to-the-basics movement.

When results from the 1977-78 NAEP were released, beginning in 1979, the outcomes were widely publicized, probably because the results came as a surprise and/or a disappointment to some critics of education. In addition to official NAEP reports and analyses published in professional journals, hardly a newspaper or news magazine failed to report the bottom line: pupils at all age levels (nine, thirteen, and seventeen years) performed quite satisfactorily on items requiring computation and algorthmic manipulation; their performance on problem-solving items, however, either declined significantly in many areas or remained at unacceptably low levels.

The back-to-the-basics movement, advocated by its supporters as the remedy for declining mathematics achievement, had been a questionable success. It had apparently achieved that which it was designed to do: increase pupil performance in computational exercises. It had not, however, contributed to the development of higher level mathematical goals, especially problem-solving.

It was against the backdrop of this situation in mathematics education that NCTM issued its Agenda for Action, which set forth the following recommendations:

1. problem-solving be the focus of school mathematics in the 1980's;

2. basic skills in mathematics be defined to encompass more than computational facility;

3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;

4. stringent standards of both effectiveness and efficiency applied to the teaching of mathematics;

5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;

6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the divers needs of the student population;

7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;

8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society. (NCTM, 1980)

It is from this point, then, that we look to the question of the education of secondary mathematics teachers. First, however, it is appropriate that we examine more carefully the status of competency-based teacher education in mathematics.

CBTE in Mathematics Education

Competency-based teacher education came to the fore around the beginning of the 1970s. Briefly, CBTE designates a program in which performance goals are specified in advance of learning and in which the student is held accountable for attaining and demonstrating a designated level of competence in performing tasks considered to be essential to teaching. The emphasis is on output or performance; hence, achievement is held constant and the time for reaching that achievement level is allowed to vary. Thus CBTE is based on assumptions that differ significantly from the assumptions underlying course-based programs in which time is held constant and achievement is the variable. To accept the assumptions of CBTE is to undertake a significant reorientation in teacher education.

There was no widespread enthusiasm for CBTE within mathematics education. Perhaps this is due to timing, as the mounting momentum for CBTE largely coincided with a period of slow-down in mathematics teacher education which followed the period of fervor and generous funding of the earlier "new math" era.

In 1975 NACOME recognized CBTE as "likely to have the most profound consequences for teacher education in the immediate future" (NACOME, 1975, p. 97), but at the same time they leveled guarded criticism at NCTM for failing to take a stand on the issue:

> The NCTM Commission on the Education of Teachers of Mathematics has reported it will not take a stand until more evaluative evidence can be provided. This is no doubt an objective and scientifically sound position but has resulted in silence on the part of NCTM as an influence outside its own membership on a highly sensitive and political matter which has serious implications for mathematics education. (NACOME, 1975; p. 99)

NACOME further cited as politically more sensitive the action of the National Council of Teachers of English urging educators not to move exclusively into CBTE without more research and objective evaluation. They went on to advance their own very explicit position:

> The members of NACOME also take the position that neither teacher education nor certification procedures should be based solely on competency or performance-based criteria without a sound empirical rationale. Furthermore, we believe that the crusade-like zeal and bandwagon mentality with which these concepts are sometimes promoted and accepted is a real and present danger to mathematics education. We urge responsible authorities in both public and private sectors to insist upon sound and objective justification before embarking upon a course with such profound implications.
>
> There are undoubtedly positive results possible from a thorough, dispassionate, objective study of what competencies, abilities, skills, and knowledge relate directly to effective successful teaching. Clarification and agreement on broad goals for mathematical education are, however, a necessary condition and prerequisite to determination of such competencies. At present, we seem to be a long way from these ideas. Reasoned and responsible investigation and argument from both sides of the issue are sorely needed. (NACOME, 1975; p. 100)

In 1976, NCTM did take a stand consistent with the NACOME recommendations. The brief position paper read as follows:

> Recommendations
> On Competency-based
> Teacher Education
>
> The NCTM is convinced that there are good and bad competency-based teacher education (CBTE) programs just as there are good and bad non-CBTE programs. Any assessment of teacher performance must recognize that the teacher functions as an integrated whole, and the identification and assessment of competencies necessary for the successful teaching of mathematics require the skills of those working in the discipline. Some regions have mandated an approach to certification without specifying the need to include representatives from the fields of mathematics education. (The Council's document, "Guidelines for the Preparation of Teachers of Mathematics," is an effort to delineate better the competencies needed by the beginning mathematics teacher.) Therefore, to reassert the need to encourage a variety of creative approaches to the complex problem of teacher education, the Council makes these five recommendations:
>
> 1. That CBTE, however defined locally, not be used exclusively by certification bodies until more

research and evaluation of its outcomes are available.

2. That the competencies identified in the "Guidelines" be used as baseline competencies for purposes of teacher education and that efforts to identify and assess additional competencies, in particular those observable only in the classroom, be encouraged.

3. That evaluation in teacher education programs be characterized by systematic assessment of all competencies over a period of time to identify consistent and effective performance.

4. That the identification and assessment of performance related to mathematics teaching be chiefly the responsibility of professionals in the field of mathematics education: college professors of mathematics and of mathematics education, school mathematics teachers, and mathematics supervisors.

5. That representatives from the mathematics education community be involved in the development of competencies and assessment procedures relative to mathematics teaching and that if NCTM affiliates in these areas have prepared guidelines, those guidelines be used as a framework against which proposals can be judged, and if such guidelines are not available, the NCTM's "Guidelines" be used. (NCTM, 1976b)

Not so cautious, however, was an earlier recommendation by the AAAS/NASDTEC. That position was embodied in Standard 3 of the 1971 guidelines and standards cited earlier. It directed institutions as follows:

> Teacher education institutions should develop performance criteria as guides in planning teacher education experiences, in evaluating teacher education programs, and in assessing the ability of prospective teachers to contribute to effective learning.
>
> Some of the skills and competencies that an effective teacher has can be described in performance terms. Each faculty member who teaches teachers should describe in as much detail as possible what he expects his students to be able to do upon the completion of the part of the pre-service program for which he has a responsibility. The description will include the knowledge and skills that a beginning teacher would be expected to have or to demonstrate.

The development and general use of a checklist of performance criteria for the total program should become a joint effort of the teacher education faculty, students, and cooperating school personnel. Furthermore, it is important that any checklist be continually revised and updated to respond to new understandings about learning and to reflect changes in education in the schools.

A checklist of performance criteria, cooperatively developed by all those concerned with teacher education programs, can serve many useful purposes, such as:

1. planning modifications of programs and introducing innovations;

2. assessing the effectiveness of the teacher education programs of individuals and groups;

3. counseling students before and after admission to teacher education programs;

4. assisting future teachers in determining their own success and failures, in identifying competencies they need before entering teaching, and in assessing for themselves the likelihood that they will become happy and successful teachers;

5. recommending beginning teachers for certification on evidence of attainment of competencies described in the checklist; and

6. identifying problems for investigation and research.

Certification and accrediting agencies should urge teacher education institutions, where possible, to develop lists of performance criteria and to base their recommendations for certification on evidence of attainment of competencies described in these lists. Other information about future teachers should also be utilized in recommendations for certification, but evidence of acquisition of certain skills and competencies can be a source of important objective information (AAAS/NASDTEC, 1971)

A closer look at mathematics teacher education programs provides further insight into the extent to which CBTE trends and recommendations from the profession have been translated into practice. In describing the situation, NACOME reported that:

> The senior high school teacher's content preparation is little changed from the 1960s except that more recent graduates are more likely to have worked with computers, are more likely to have take courses

in probability and statistics and perhaps combinatorics, and may have been exposed to some serious work in applications or modeling. (NACOME, 1975; p. 85)

There is little evidence of recent widespread change or innovation in the methods component of the secondary teacher education program. Perhaps an exception is the growing use of video taping. In 1972 a survey of mathematics educators was carried out to determine recommendations for secondary teacher education... A large number of changes in practice were suggested. A near consensus was found on the need for earlier and more extensive observation and field experience prior to student teaching. (NACOME, 1975; pp. 87-88)

These NACOME generalizations are substantiated by other evidence. A set of papers entitled "Promising Practices in Mathematics Teacher Education" was compiled for a forum on mathematics teacher education at the NCTM annual meeting in 1972. Included were several papers describing secondary methods courses and/or practicum components based on competencies, behavioral objectives, or performances. The most common features in these seemed to be field experiences and microteaching. None reported full scale CBTE programs in secondary mathematics.

An examination of the programs of the next five annual meetings of NCTM (1973 through 1977, a period in which CBTE was receiving much of its attention) revealed seventy-three presentations classified as teacher education sections. One of these dealt with the theory and evaluation of CBTE in general; two secondary teacher education sections were of the "how to" variety (e.g., "how to write a module"); two sections described elementary CBTE programs for mathematics; and two others described programs in which the grade level was not clearly indicated. None of the descriptions gave an indication of a total CBTE degree program. It would appear that CBTE was not widely accepted by practitioners in mathematics education.

A similar survey of research reported in mathematics education supported the conclusion of lack of activity in the area of CBTE. Annual reports by Suydam and Weaver published in the Journal for Research in Mathematics Education reviewed the research, both journal articles and dissertations, published during those same years. There appeared a few assessments of "teacher competency" in which competency was determined by ascertaining the years of training and mathematical background of the teachers, but no evaluations of total CBTE programs for secondary mathematics teachers. In Roth's (1977) review of CBTE programs of institutions which could provide evaluation data, none reported were in mathematics. Presumably institutions with total CBTE programs also included mathematics teacher education, but, if so, they were not being reported in the professional literature in mathematics education.

Every alternative has strengths and limitations. CBTE is no exception. Elsewhere we have discussed some of the pros and cons of

CBTE (House, 1975), and the literature of education contains numerous other books and articles on both sides of CBTE. To recount those arguments is not to our purpose here. But, as many have recognized, the essential element of CBTE -- to identify and describe the desired outcomes of teacher education and to provide learning experiences that are designed to facilitate the student's development and demonstration of those objectives -- offers certain advantages. In particular, it can be the starting point for our reflection about what we wish to accomplish and how best to proceed. It is in this spirit that we recognize the potential in CBTE; the extent to which we espouse the approach will become clearer as the teacher education model develops.

Goals of Secondary School Mathematics

Since the goals of teacher education are derived from conceptions of the teacher's roles and responsibilities, let us consider next the goals of teaching secondary school mathematics and some of the forces that affect the teacher.

The mathematics courses commonly offered in the secondary school curriculum include one to three years of general mathematics (usually in the junior high school), at least two years of algebra, plane and solid geometry, trigonometry, introductory analysis, and, in some cases, consumer mathematics, introductory calculus, and computer science. In addition, short courses or units within one or more of the above courses are frequently available offering introductions to such topics as probability, statistics, transformational geometry, applied mathematics, linear algebra, set theory, group theory, logic, topology, number theory, or non-eculidean geometry.

On the surface, the basic courses enumerated above appear to be fairly constant and the list bears a strong similarity to the courses of studies listed by high schools in the past. In reality, mathematics is a discipline that has undergone marked change in the past quarter-century, and NACOME was forced to conclude that "today mathematics teaching is a troubled profession" (NACOME, 1975; p. ix).

Changing emphasis in mathematics

Perhaps mathematics teaching has, to some degree, always been a troubled profession. The Thirty-second Yearbook of NCTM recounts the history of mathematics education in the United States and points out the constant tension between the desire to approach mathematics as a formal system in its own right or as mental discipline, and the thrust to present mathematics as a tool useful to the learner. Over the years a number of committees and commissions have issued goals and recommendations for school mathematics (see NCTM, Thirty-second Yearbook, 1970). Underlying those reports are assumptions about the basic mathematical needs of pupils which Johnson and Rising summarized as follows:

1. Pupils need to know how mathematics contributes to an understanding of natural phenomena.

2. Pupils need to understand how to use mathematical methods to investigate, interpret and make decisions in human affairs.

3. Pupils need to understand how mathematics, as a science and as an art, contributes to the cultural heritage.

4. Pupils need to prepare for vocations in which one utilizes mathematics as a producer, and consumer of products, services, and art.

5. Pupils need to communicate mathematical ideas correctly and clearly to others.

(Johnson and Rising, 1972; pp. 44-45)

In order to meet such needs, mathematical goals must be broader and more inclusive than computational facility (arithmetic) alone. Both the Committee on Basic Mathematical Competencies and Skills of the NCTM (1972) and the National Assessment of Educational Progress (1970) have addressed the goals of school mathematics on three levels; i.e.,

1. as a tool for effective citizenship and personal living,

2. as a tool for functioning in a technological world, and

3. as a formal system in its own right.

Consideration of the three levels of mathematical goals suggested above helps bring into perspective the curricular changes of the last quarter-century. In surveying these changes, NACOME summarized the phenomena as follows:

> Mathematics program improvements of the "new math" 1960s were primarily motivated and designed to provide high quality mathematics for college capable students -- particularly those heading for technical or scientific careers. Guidance in the curriculum development came largely from university and industrial mathematicians, and the model for curriculum structure was the logical structure of mathematics. Today mathematics curriculum development focuses on issues largely ignored in the activity of 1955-1970. Responding to the concerns of classroom teachers, as well as educators and laymen interested in the basic goals of general education, attention has now shifted to programs for less able students, to minimal mathematical competence for effective citizenship, to the interaction of mathematics and its fields of application, and to the impact of new computing technology on traditional priorities and methods in mathematics. Furthermore, the dominant role of

> mathematical structure in organizing curricula has been challenged by many who advocate pedagogical or psychological priority in determining scope and sequence. (NACOME, 1975; p. 23)

The most obvious manifestation of the changes in mathematics education which dominated the 1970s was the movement popularly described as back-to-the-basics. To many the slogan suggested a return to drill and practice in computational skill, and it implied a swing of the educational pendulum to the content and methods of a previous era. We will elaborate on basic skills later, but back-to-the-basics also serves as a reminder of the danger of trying to oversimplify the complex task of teaching mathematics. In reality, many significant forces impinge on the mathematics curriculum, and these must be examined more closely if one is to derive an adequate conception of the role of the mathematics teacher.

Learning theories

One such force is the growing body of knowledge of how children learn mathematics. In particular, the work of Jean Piaget and the large number of studies influenced by Piaget's work indicate that the thinking of children is qualitatively different from that of adults and that, consequently, children cannot be expected to behave like little mathematicians. Further, the secondary school curriculum and secondary mathematics teachers must take account of the fact that many secondary school pupils are still concrete operational or, at best, in transition to the formal operational level, and this has implications for teaching and learning strategies. Unfortunately, the studies focusing on the formal operational behavior of pupils are considerably fewer in number than those dealing with earlier stages, and much remains to be learned. Nonetheless, the contributions of Piaget and others indicate the need for careful reexamination of the appropriateness of previous practices in school mathematics and for teacher competencies in designing instruction that meets the characteristics and needs of the pupils.

Instructional alternatives

Related to the growing awareness of how pupils learn mathematics is the increased attention to developing instructional alternatives. Notable in this regard is the role of manipulative devices and instructional aids as physical embodiments of mathematical concepts. Although most use of laboratory approaches to learning mathematics has been in the elementary school, secondary teachers need to develop competencies in teaching through laboratory lessons, and the mathematics laboratory should be given an expanded role in the secondary school.

Problem-solving

In a world of ever-accelerating change, it has become a truism that the school curriculum cannot provide pupils with the specific skills or algorithms that they will need for future life or work. This is particularly true of the technological applications of mathematics. Hence, algorithmic facility as a goal of school mathematics must be balanced against problem-solving in new or novel situations for which the pupil does not have a previously established algorithm. Teachers must facilitate intuitive and heuristic approaches on the part of the pupils in order to develop problem-solving abilities of their pupils, and teachers themselves must become problem-solvers.

Basic skills

Bolstered by widespread media publicity focused on declining test scores in a time of rising educational costs and in a general climate of increasing demands for accountability, mathematics and reading became targets of the movement back to the basics. On the one hand, the momentum of this movement gave rise to increased attention to drill and practice of computational skills, a proliferation of worksheets and textbooks promising to provide ample practice, and a demand for minimum competencies lists and/or minimum essentials tests. On the other hand, the movement gave mathematics educators occasion to reevaluate what constitutes "basic" mathematical skills. The position paper of the National Council of Supervisors of Mathematics (NCSM), issued in 1977, gained widespread publicity for its delineation of ten components of basic skills, and NCTM in the Agenda for Action reiterated that basic skills must be understood to encompass at least the ten NCSM skill areas. The ten areas of basic skills identified in the NSCM document were the following

Problem-solving

> Learning to solve problems is the principal reason for studying mathematics. Problem-solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem-solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions and they must be willing to subject these conclusions to scrutiny.

Applying mathematics to everyday situations

> The use of mathematics is interrelated with all computation activities. Students should be encouraged to take everyday situations, translate them into mathematical expressions, solve the mathematics, and interpret the results in the light of the initial situation.

Alertness to the reasonableness of results

Due to arithmetic errors or other mistakes, results of mathematical work are sometimes wrong. Students should learn to inspect all results and to check for reasonableness in terms of the original problem. With the increase in the use of calculating devices in society, this skill is essential.

Estimation and approximation

Students should be able to carry out rapid approximate calculations by first rounding off numbers. They should acquire some simple techniques for estimating quantity, length, distance, weight, etc. It is also necessary to decide when a particular result is precise enough for the purpose at hand.

Appropriate computational skills

Students should gain facility with addition, subtraction, multiplication, and division with whole numbers and decimals. Today it must be recognized that long, complicated computations will usually be done with a calculator. Knowledge of single-digit number facts is essential and mental arithmetic is a valuable skill. Moreover, there are everyday situations which demand recognition of, and simple computation with, common fractions.

Because consumers continually deal with many situations that involve percentage, the ability to recognize and use percents should be developed and maintained.

Geometry

Students should learn the geometric concepts they will need to function effectively in the three-dimensional world. They should have knowledge of concepts such as point, line, plane, parallel, and perpendicular. They should know basic properties of simple geometric figures, particularly those properties which relate to measurement and problem-solving skills. They also must be able to recognize similarities and differences among objects.

Measurement

As a minimum skill, students should be able to measure distance, weight, time, capacity, and temperature. Measurement of angles and calculations of simple areas and volumes are also essential. Students should be able to perform measurement in both metric and customary systems using the appropriate tools.

Reading, interpreting, and constructing tables, charts, and graphs

Students should know how to read and draw conclusions from simple tables, maps, charts, and graphs. They should be able to condense information into more manageable or meaningful terms by setting up simple tables, charts, and graphs.

Using mathematics to predict

Students should learn how elementary notions of probability are used to determine the likelihood of fugure events. They should learn to identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election forecasts.

Computer literacy

It is important for all citizens to understand what computers can and cannot do. Students should be aware of the many uses of computers in society, such as their use in teaching/ learning, financial transactions, and information storage and retrieval. The "mystique" surrounding computers is disturbing and can put persons with no understanding of computers at a disadvantage. The increasing use of computers by government, industry, and business demands an awareness of computer uses and limitations. (NCSM, 1977)

The posture that teachers assume with respect to basic skills and the degree to which teachers will be accountable for the learning of these basics by their pupils will have a significant impact on the teaching behaviors required.

Calculators

Few devices have burst upon society as rapidly as did the handheld calculator. Technology improved and prices dropped in inverse proportion, and within a few years of their appearance, calculators were priced well below textbooks. Further, calculators were readily available to everyone, and schools would have faced an impossible task had they attempted to ban or even ignore them. Quite to the contrary, NCTM (1976a) officially endorsed the creative use of calculators in school mathematics and reaffirmed its position in the Agenda for Action; and NACOME (1975; p. 138) went so far as to recommend that beginning no later than the end of the eighth grade, a calculator be available for each pupil during each mathematics class. The profession is only beginning to realize the potential impact of the calculator, but teachers must be prepared to examine the role of computation in school mathematics, the emphasis given to certain topics (for example, fractions), and the implications for instructional approaches in a calculator environment.

Computers and microcomputers

Although computers entered the school mathematics classroom before the handheld calculators, calculators soon outpaced the computer in availability, an understandable consequence of the low cost of calculators. However, the potential of the computer with its memory and programmable decision-making capabilities far outstrips the uses of calculators. Far from being antiquated by the advent of the calculator, the computer and, more recently, the microcomputer have become available for even more creative instructional uses, since many computational tasks can be accomplished without using computer time. Further, the computer and microcomputer, as the calculator before them, will soon be geared to widespread marketing for personal use. The implications of the yet unrealized and largely unimagined role of computers in mathematics classes will bring far-reaching demands for new teacher roles.

Individualization

Education in general has, in recent years, been increasingly sensitive to the goal of meeting the needs of individual pupils. At the same time, attention was being given to formulating goals and objectives for all pupils in terms of expected pupil behavior. Perhaps because mathematics is highly cognitively oriented and because much of mathematics (or, at least, much of arithmetic) deals with "right answers," mathematics seemed particularly suited for translation into behavioral objectives. This, coupled with the desire to individualize mathematics instruction and influenced by learning theories that favored hierarchical approaches to concept development, led many teachers, faculties, districts, and others to develop learning packages or modules. For the most part these were individualized only to the extent of being self-paced, but they nevertheless changed the role of the mathematics teacher to helper or resource person. To the extent that such programs are still operational, teachers will need certain relevant competencies; to the extent that these are replaced by other instructional alternatives, teachers will need other skills.

New mathematics

While certain topics of mathematics have thus far had only limited impact on the school curriculum, there is general agreement among mathematics educators that mathematics of the future will afford a larger role to areas like probability, statistics, and computer literacy. In addition, as practice of algorithmic calculation diminishes in the curriculum and as calculating and computing devices are more widely used, attention can be given to topics previously considered prohibitive because of the excessive or complex calculations involved. Examples are game theory and linear programming. The introduction of these or other topics into school mathematics presupposes the necessary competence on the part of the teacher.

Applications

Another area of increasing attention in mathematics is the application of mathematics to a wide variety of situations often referred to as "real-world" problems. Also included here is mathematical modeling. One implication of this attention to applications and modeling is an increasing concern for interdisciplinary approaches which, in turn, suggest new expectations for mathematics teachers both in terms of knowledge and in terms of instructional planning and implementation.

Affective concerns

Mathematics teachers cannot ignore the importance of the affective dimensions of learning. The role of the pupil's self concept of mathematical ability as a limiting factor in learning, the debilitating effects of "math anxiety" and "mathaphobia," the relationship of attitudes to learning, the importance of success, and the high attrition of female mathematics students are among the affective concerns of significance for teachers. Awareness of the nature and implications of these conditions, sensitivity to their existence in pupils, and effectiveness in reducing or eliminating negative outcomes must be counted among the necessary teaching competencies if teachers are to help pupils not only to do and to understand mathematics but also to appreciate, to enjoy, to choose, and to create mathematics.

Changing roles for teachers

The forces affecting the mathematics curriculum that are outlined above also affect the nature of mathematics teaching and the necessary skills and attitudes of the mathematics teacher. Some specific teacher competencies will be suggested in the model outlined in the remainder of this book, but several aspects of these competencies may be summarized as follows:

> Teachers must develop problem-solving skills, attitudes of thinking mathematically, and an awareness of mathematics as a human activity.

> Teachers must develop an attitude of openness to new learning and to new understandings of what is essential ("basic") in mathematics.
>
> Teachers need competencies and attitudes that are adaptive, that enable them to change as needed.
>
> Teachers must deepen their focus on children, on how they learn, and on the affective aspects of the learning process.
>
> Teachers must continue their own education with regard to new developments in mathematics and new applications of mathematics in all fields of human activity.

What, then, do we expect from a teacher education program? Peck and Tucker, reviewing the research on teacher education, suggested several generalizations based on their review. Among them were the following:

1. A "systems" approach to teacher education substantially improves its effectiveness. Such an approach consists of a series of steps which recur in cyclical fashion:

    a. precise specification of the behavior which is the objective of the learning experience;

    b. carefully planned training procedures aimed explicitly at those objectives;

    c. measurement of results in terms of the objectives;

    d. feedback to the learner and the instructor;

    e. reentry into the training procedure;

    f. measurement of results again following repeated training.

2. Teacher educators should practice what they preach. When teachers are treated as they are supposed to treat pupils, they are more likely to adapt the desired style.

3. Direct involvement in the role to be learned produces the desired teaching behavior more effectively than remote or abstract experiences such as lectures on instructional theory.

4. Using any or all of the techniques mentioned, it is possible to induce a more self-initiated,

self-directed, effective pattern of learning not only in teachers but, through them, in their pupils. (Peck and Tucker, 1973; p. 943)

These generalizations from the research on teacher education suggest some desired characteristics of teacher education programs. We believe, however, that each teacher education program is unique in certain ways and that it should remain so. We therefore do not presume to be able to specify all of the competencies that should be included in a teacher education program. Indeed, we reject the notion that anyone could prepare a definitive list of such competencies.

What we do hope to be able to do is to suggest a framework to assist teacher educators to accomplish the following:

-- to make the goals of mathematics teacher education more specific;

-- to enable a systems approach to mathematics teacher education;

-- to integrate formal classroom instruction with field experiences;

-- to conceive a teacher education program that is developmental in character and to differentiate between the competencies expected of teachers at various points in their careers;

-- to stress teacher education as an information processing system designed to help teachers analyze situations that they must confront in teaching and to make decisions based on a critical evaluation of available alternatives and their consequences; and

-- to facilitate continuing evaluation and updating of the teacher education system as new situations and needs arise.

We proceed with that task now by proposing a taxonomy of mathematics teacher education.

Chapter Three

A Taxonomy of Mathematics Teacher Education

In our approach to teacher education, whether pre-service or in-service, the starting point of program development will be the identification of expected competencies. Some competencies will be required of all teachers; others may be optional or recommended; all will be derived from conceptions of the role of the teacher in the education of pupils. Some approaches to the identification of teacher competencies are discussed later.

Operationally, to initiate a program for an individual learner, the starting point will be a needs assessment. The needs assessment is a measure of the student's present competency in order to determine the competencies still to be developed. From this assessment flow the identification of objectives to be achieved, the specification of enabling activities, the plan for evaluation, etc. Since the needs assessment is directly related to the prior specification of competencies, it cannot be discussed apart from the task of determining those competencies.

This model for a program for secondary school mathematics teachers approaches the task through a proposed taxonomy of mathematics teaching behaviors. Bloom (1956) indicated that a taxonomy is useful in helping educators gain a perspective on the emphasis given to certain behaviors by a particular set of educational plans. Further, a taxonomy should help curriculum builders to specify objectives, to plan learning experiences, and to prepare evaluation measures. McDonald (1972) pointed out the usefulness of a taxonomy for organizing descriptors of teaching and the related behavioral observations associated with the evaluation of teaching, for relating categories of behavior to one another, for discovering new relations between seemingly unrelated behaviors, and for placing the relations among categories of behaviors on a sounder empirical base.

It must be noted, however, that unlike a classification scheme which may contain arbitrary elements, a taxonomy must be constructed so that the order of the terms corresponds to some real order among the phenomena represented. A taxonomy must further be validated by demonstrating its consistency with theoretical views in research findings of the field it attempts to order (Bloom, 1956). For this reason, the "taxonomy" presented here is more properly an approximation to a taxonomy since it lacks verification. However, for convenience it shall be referred to as a taxonomy.

A Taxonomy of Mathematics Teaching

In this taxonomy of mathematics teaching, the ordering principle is teacher behavior. The behaviors described in the taxonomy are derived from the assumption that it is possible to specify in advance the desired outcomes of a teacher education program, and they include the behaviors expected of both pre-service ("beginner") and in-service ("expert") teachers. Hence, it is possible to delineate both entry-level competencies and competencies to be developed during the continuing professional development of the teacher.

The taxonomy will first be described in broad terms, presumably generalizable to other teaching fields; and in a later section, it will be illustrated with more specific performance objectives for secondary school mathematics. This taxonomy is deliberately developed to reflect Bloom's taxonomy and the category names are chosen for that reason, although more appropriate names may be found. The six levels of the taxonomy of mathematics teaching are the following:

Knowledge

The knowledge level concerns facts, processes, theories, techniques and methodology related to instruction. It also includes knowledge of mathematics and of the curriculum and materials of school mathematics. The knowledge level in the taxonomy of teacher competencies subsumes all of the levels of Bloom's cognitive taxonomy. This is the component of teacher education usually associated with the college classroom and usually measured by paper and pencil or other conventional classroom methods.

Comprehension

The comprehension level concerns performance of selected behaviors under controlled conditions such as peer teaching, microteaching, simulations, role playing, etc. It is a demonstration that the individual can do something, and the behavior to be demonstrated usually is called for in an explicit manner so that the individual is conscious of the goal of demonstrating the desired behavior.

Application

The application level refers to planning and administering learning activities and materials in a classroom setting. It is evidence not only that the individual can do but that he/she does do. Application involves the use of appropriate teaching skills at the proper time or with the desired frequency as a part of the normal teaching style.

Analysis

At the analysis level the teacher responds to pupil, teacher, subject matter, and environmental cues to select, organize, and administer effective programs and lessons. The teacher recognizes the constituent elements of the curriculum and the relationships among them and sees them as an organized whole. The teacher also responds spontaneously to students as individuals and his/her actions and decisions flow from a consistent and conscious rationale.

Synthesis

At the synthesis level the individual orchestrates his/her teaching behavior into a personalized whole, interiorizing and professionalizing the teaching skills and combining the underlying competencies into an effective style unique to the individual.

Evaluation

At the evaluation level the teacher judges the effectiveness of his/her teaching according to various internal and external criteria, including pupil progress toward desired goals, and he/she modifies the teaching in the direction of greater effectiveness.

Advantages of the Taxonomy

One anticipated advantage of adopting the proposed taxonomic scheme is to focus attention on aspects of teaching not readily described in specific behavioral objectives. More global, less atomistic competencies are suggested, and from these are derived more specific instructional goals and alternatives. This is in keeping with the definition of PBTE proposed by Gage and Winne (1975; pp. 146-147): "PBTE is teacher training in which the pre-service or in-service teacher acquires, to a pre-specified degree, performance tendencies and capabilities that promote student achievement of educational objectives."

In the above definition, "teacher performance" refers to observable behaviors--oral, written, or nonverbal. "Tendencies" are typical teacher behaviors in the average or normal teaching situation, while "capacities" are behaviors which the teacher is capable of when trying his/her best. All three of these--performances, tendencies, and capacities--are encompassed by the taxonomy.

Likewise, the taxonomy directs attention to the continuing growth and development of the teacher and related pre-service and in-service educational goals. For example, it allows program developers and evaluators to delineate those performances expected of entry-level teachers. (Many of these will be performances at the first three taxonomic levels, although some demonstrations of higher-order performance are definitely in order.) Combs et al. (1974) have insisted that the methods of experts are different from those of beginners, and that some of the methods of experts can be used only because the persons are experts. The taxonomy offers a framework for specifying higher-level performances to be expected of experts as opposed to those expected of beginners. This differentiation should result in more realistic expectations for beginning teachers which may alleviate some of the frustrations commonly experienced by them. It is also hoped that the taxonomy will help teachers to evaluate their own behavior and to plan for their own continuing professional growth

The taxonomy also encourages us to view the development of competencies from one level of the taxonomy to another. As an example, we might consider competencies related to the teacher's use of evaluation in teaching. At the knowledge level, such competencies include understandings about types of tests; reliability and validity characteristics of tests; uses of tests for formative, summative, or diagnostic purposes etc. Comprehension level competencies might include demonstrations that the teacher can construct tests that are appropriate for certain specified purposes. Application of these competencies indicates that the teacher actually does employ these various types of evaluation. At the higher levels of analysis, synthesis, and evaluation, it becomes apparent that the teacher's evaluation instruments and processes reflect the objectives of his/her instruciton, and that the teacher uses them to improve instruction and more effectively to help individual pupils.

Affective teacher characteristics also constitute a strand that may run through all levels of the taxonomy. For example, affective competencies at the knowledge level may include knowledge or recognition of the defense mechanisms or approach/avoidance techniques of pupils, or knowledge of the importance of teacher support, empathy, warmth, or positive regard with respect to pupil learning. At the comprehension level, the teacher may be expected to demonstrate learned behaviors for giving reinforcement or to identify clues to pupil or teacher attitudes in simulated situations. At the level of application and beyond, teacher competencies may include spontaneous use of reinforcements, empathy, etc. in teaching situations. Affective goals may also include positive changes in teacher attitudes toward various components, personnel, or systems in education, and a willingness to modify one's teaching behavior as one develops new competencies.

Finally, the taxonomy recognizes that teaching competency is a function of teaching style that is unique to each teacher, and it capitalizes on this by defining as necessary competencies to combine effectively the underlying competencies; to interact effectively with the pupils; the curriculum, and the learning environment, to develop a rationale for teaching; and to act purposively and consistently with the philosophy, rationale, and style.

Assessing Performances

Because the teaching taxonomy is structured on teacher behavior, a variety of assessment approaches is available. Performances at the knowledge level are generally assessed by conventional classroom methods including paper-and-pencil tests. Both pre-service and in-service teachers can be expected to demonstrate their knowledge about essential mathematics; about relevant theories, techniques, or processes of education; or about other content deemed appropriate. They will do this by written and/or oral responses including tests, papers, reports, discussions, and other forms of presentation.

Performances at the comprehension level may be assessed through controlled situations for a limited time, such as peer teaching, microteaching, simulations, or role playing. Observations by teacher training personnel or other "experts," rating scales, checklists, and video tapes are among the means of assessment. These apply at both the pre-service and the in-service levels.

At other levels, performance normally presumes demonstration in actual classroom settings over an extended period of time. These may more appropriately apply to in-service needs assessments and, to a more limited degree, to student teachers. The means of assessment include all those identified for comprehension, and they may also include assessment of pupil learning as one indicator of teacher effectiveness or need.

Whether performed in the college classroom, an assessment center, or the secondary school classroom, the needs assessment seeks to establish those teacher competencies already demonstrated by the teacher and to compare them to the expected or required teacher performances. The difference between these two sets of behaviors defines the objectives and the curriculum for the individual teacher. To illustrate elements of such a program for teachers of secondary school mathematics, the discussion below elaborates on representative competencies, objectives, learning activities, and evaluations corresponding to the six levels of the teaching taxonomy.

Representative Competencies

In designing a pre-service or in-service staff development program for secondary school mathematics, program developers will begin by identifying desired competencies corresponding to each level of the taxonomy. The more generic competencies will, in turn, give rise to more specific instructional objectives that the teacher is expected to accomplish and to demonstrate. The competencies and objectives below are meant to be representative of the various taxonomic levels. In no way should they be considered an exhaustive list. Neither should it be assumed that a given objective is necessarily the basic unit of instruction, as many of the listed objectives should themselves be further specified in terms of sub-objectives.

Subject: Mathematics
Level of taxonomy: Knowledge

Performance Goals	Instrucitonal Objectives
The student will:	The student will:
1. Recognize problem-solving as the central focus of school mathematics	1.1 Define the necessary conditions for the existence of a problem
	1.2 Differentiate between problems and exercises
	1.3 List various heuristics that are useful in solving problems
	1.4 Apply heuristics to the solution of problems

1.5 Discuss the importance of problem-solving as a goal of school mathematics

1.6 Suggest some strategies for introducing problem-solving in secondary school classes

2. Recognize the importance of diagnostic teaching in mathematics

2.1 For a given mathematical topic, identify the component concepts and prerequisite knowledge and skills

2.2 For a given mathematical topic, identify probable sources of pupil difficulty or error

2.3 Complete a task analysis for the given topic

2.4 Suggest appropriate instructional activities for developing the specified topic

2.5 Suggest appropriate remedial activities corresponding to anticipated or identified pupil errors

Level of taxonomy: Comprehension

Performance Goals	Instructional Objectives
The student will:	The student will:
3. Plan and teach a mathematics lesson	3.1 Select appropriate objectives and state them in terms of pupil behaviors
	3.2 Introduce the lesson in an effective, motivating manner
	3.3 Select and utilize appropriate intructional materials and teaching aids
	3.4 Present appropriate examples, nonexamples, problems, and applications
	3.5 Relate the topic to previous learning and pupil interest

3.6 Involve the pupils in practicing or applying the topic

3.7 Summarize effectively at appropriate points in the lesson

4. Demonstrate effective communication skills

4.1 Demonstrate the ability to implement a variety of communication patterns in the classroom (e.g., lecture, guided discovery, discussion)

4.2 Recognize defensive strategies and nonverbal behaviors of pupils

4.3 Use open-ended questions in instruction

4.4 Initiate and sustain discussion

4.5 Use pupil questions and comments in developing instruction

4.6 Give feedback and positive reinforcement to pupils

Level of taxonomy: Application

Performance Goals	Instructional Objectives
The student will:	The student will:
5. Plan and teach a unit of instruction.	5.1 Plan appropriate unit and lesson objectives
	5.2 Develop lessons related to the overall unit objectives
	5.3 Provide appropriate pacing, practice, application, and feedback during instruction,
	5.4 Vary the learning activities within and between lessons
	5.5 Manage the classroom in manner that promotes pupil learning
	5.6 Evaluate pupil progress and learning outcomes

6. Use a variety of learning experiences to develop mathematical content

6.1 Plan and present laboratory lessons in mathematics

6.2 Seek out, adapt, or create manipulative aids to enhance lessons

6.3 Make appropriate use of calculators and/or computers

6.4 Supplement textbook presentations with activity, laboratory, problem solving, and application lessons

6.5 Teach through problem solving as a regular part of the teaching style

6.6 Relate content to the applications of mathematical concepts in a variety of life situations

Level of taxonomy: Analysis

Performance Goals	Instructional Objectives
The student will:	The student will:
7. Analyze pupil, teacher, subject matter, and environmental cues to select, organize, and administer an effective mathematics program	7.1 Analyze the curriculum content into constituent elements, recognize the relationships among these, and see them as an organized whole
	7.2 Identify the key ideas and central goals of learning activities, and organize lessons to emphasize those key elements
	7.3 Identify aspects of lessons that could be enhanced by alternative learning activities instructional aids, etc.
	7.4 Provide a rationale for the selection and organization of content and materials in mathematics lessons, courses, and programs

7.5 Adjust classroom activity in accord with pupil needs

7.6 Plan remedial and enrichment activities appropriate for slow and fast learners

8. See pupils as individuals and respond spontaneously

8.1 Give individual recognition and acceptance to pupils

8.2 Listen to pupils

8.3 Expect success of every pupil and take measures to bring it about

8.4 Differentiate assignments according to pupil needs

8.5 Involve pupils in planning units and activities

8.6 Adjust instruction to meet individual pupil needs

8.7 Recognize variations in pupil learning styles and identify differences in ways in which individual pupils approach problems

Level of taxonomy: Synthesis

Performance Goals

The student will:

9. Combine the underlying competencies into an effective, personal teaching style

Instructional Objectives

The student will:

9.1 Model the mathematical behavior desired of learners

9.2 Display enthusiasm for mathematics, for teaching, for learning, and for pupils

9.3 Recognize the mathematical aspects of situations and integrate mathematics with other areas of learning

9.4 Regularly relate classroom learning to past learning, future topics, and pupil interests and experiences

9.5 Respond to classroom contingencies in an appropriate and consistent manner

9.6 Help pupils take ownership of their own learning tasks and assist them in bringing closure to those learning tasks

Level of taxonomy: Evaluation

Performance Goals	Learning Objectives
The student will:	The student will:
10. Judge the effectiveness of one's own teaching	10.1 Display confidence in one's own ability to teach mathematics
	10.2 Evaluate one's one teaching in terms of internal criteria (e.g., accuracy and logical consistency), external criteria (e.g., appropriateness for the given learners), and pupil progress toward goals
	10.3 Identify teacher behaviors that inhibit pupil learning and propose modifications of those behaviors
	10.4 Evaluate the curricula and materials for appropriateness in furthering one's instructional goals
	10.5 Plan and evaluate lessons in the light of current relevant research

Chapter Four

Operating a Program

We turn now to consideration of the operation of a program based on the taxonomy. For all performance goals and instructional objectives identified in the program, a variety of enabling activities also must be developed. Ordinarily it is expected that the student will be given a certain flexibility in selecting among these activities. In this, the assumption is made that the competency or performance objective represents a goal statement, and that ideally the student will have a choice among means for attaining the goal. In some cases the student also will have the choice of whether or not to develop a specific competency.

In planning and recommending enabling activities, program developers should be guided by certain principles of learning. In particular, the design of enabling activities should reflect the understanding that learning is promoted

- by the student's clear knowledge of the instructional goals;
- by the student's involvement in the educational planning;
- by the student's perception of the learning experiences as relevant to one's own life and goals;
- by reinforcement;
- by quick feedback;
- by opportunities to practice skills; and
- by provisions for individual differences, needs, and backgrounds.

In addition, it is further assumed that in practice the student will focus first on specific, individual skills and will later combine related competencies into skill clusters. Many of the instructional objectives suggested above actually represent such clusters which can be broken down still further. For example, Objective 3.1 includes specifying objectives in terms of pupil behavior. This might be further specified to include the following and more:

- Distinguishes between objectives that are behaviorally stated and those that are not.
- Rewrites nonbehavioral objectives in behavioral terms.
- Writes objectives at all levels of the cognitive domain.
- Writes objectives in the affective domain.

-- Explains the advantage to both teacher and pupils of stating objectives behaviorally.

Similarly, it is reasonable to plan activities to allow the beginning teacher to engage in instructional activities first with individual pupils (tutoring), later with small groups of pupils, and finally with entire classes.

Instructional Activities

The teacher education model proposed here also assumes that at least four types of instructional activities are integral to the program formal instruction, laboratory experiences, school-based practicum experiences, and individual study. Each of these warrants further discussion.

Formal instruction

Both pre-service and in-service teachers will need formal instruction from qualified personnel, usually college or university faculty. Most of the competencies at the knowledge level of the taxonomy are developed through such instruction. Formal instruction may be delivered thorough traditional quarter or semester courses, through short courses or workshops, through individually scheduled presentations, through readings, or through electronic media including audio and video tapes and computers.

A major portion of the formal instruction will surely be directed to the mathematics to be learned by the secondary school teacher. For beginning teachers this would be expected to include advanced study in calculus, analysis, algebra, and geometry, and at least an introductory study of probability, statistics, topology, and computer science. For in-service teachers, more advanced work in all the above is assumed. However, this paper will not attempt to prescribe specific mathematical knowledge to be demonstrated by secondary school teachers since such recommendations are readily available from the NCTM, from the MAA, and from numerous commissions as reported in the Thirty-second Yearbook of NCTM.

A second component of formal instruction is broadly described as "liberal education." Since this will vary widely between programs and individuals, no attempt is made to outline it here. However, every mathematics teacher should be expected to demonstrate understanding of the natural and social sciences and of the arts and humanities. They also may be expected to demonstrate a more extensive knowledge of at least one other area outside of mathematics.

Two types of professional educational knowledge also are delivered through formal instruction. The first, described here as "foundations," is common to education in general and includes theories of learning and cognition, human growth and development, general goals of education and of schools, principles and practices of school operation, principles and methods of testing and evaluation, classroom management, communication skills, human relations, legislation and its implications for teaching, and other topics of general applicability. In considering foundational topics, it is recommended that secondary school mathematics teachers be provided with opportunity to interact with teachers of other subjects and grade levels in discussions relating these topics to the broad spectrum of education.

The other dimension of formal professional instruction concerns the delivery of mathematics to pupils. This area, usually described as "methods," includes examination of the curriculum and materials of school mathematics, instruction in planning different types of lessons, consideration of the needs of various pupils, and applications of the foundations topics to the mathematics classroom.

It was noted earlier that a difference between the formal instruction component of a CBTE program and a traditional course-based program lies in the specification of certain kinds of levels of knowledge that the student must demonstrate to an acceptable degree. It is not possible, for example, for a high grade in algebra or analysis courses to compensate for ("average out") inadequate knowledge of geometry. Further, an individual who satisfactorily demonstrates the requisite knowledge is able to bypass some or all of the corresponding formal instruction.

Laboratory experiences

Laboratory experiences bridge the gap between formal instruction and school-based practicum experiences. Laboratory experiences usually take place on the college campus and include simulations, role playing, peer teaching, microteaching, hands-on experimentation with curricula and learning aids, viewing and critiquing video tapes, and more. Laboratory experiences most frequently represent enabling activities corresponding to competencies at the comprehension level. In general, they are of short duration under controlled conditions that afford the student the opportunity to practice specific behaviors explicitly identified, such as the use of questions to guide a discovery lesson or the use of physical objects to demonstrate or reinforce a concept. In general, laboratory experiences differ from formal instruction because they are designed to contribute to skill in interpretation and performance as opposed to contributing primarily to knowledge.

The case for laboratory activities and protocol materials has been made by several authors (e.g., Retzer, 1976; Gliessman, 1976; Smith, 1969; Gage, 1971). Gage argued forcibly for concreteness in research on teaching. His argument is no less valid for teacher education.

> First, the treatments should be embodied in materials and equipment . . . Textbooks, workbooks, instructional films, tests, audiotapes, videotapes, programmed textbooks, computer-assisted instructional materials, kits, manuals, models, games, simulations, and other devices for arranging instructional experiences in suitable sequences -- these are the vehicles through which good influences on what actually happens in schools can be most dependably exerted. Without such material embodiments, attempts to improve teaching and

> learning run into all the forces that keep people from acting on good educational advice. The advice tends to be too theoretical, too vague as to its meaning for practice, and insufficiently coercive, in the sense that it does not require the teacher to change his ways. Materials and equipment spell out the advice in practicable terms. If properly designed and accompanied with adequate instructions for use, they well-nigh force the teacher and student to do what is wanted of them by the experimenter. (Gage, 1971; p. 33)

Special attention is directed to the role of demonstration lessons presented on film or video tape. In developing the competency of both pre-service and in-service mathematics teachers, one must attend to several cognitive and affective variables. Among these, several critical concerns are the following:

- -- the intellectual development of the pupil and its relationship to the pupil's ability to function at an abstract, symbolic level;
- -- the learning styles of the pupils and the manner in which different individuals approach problems;
- -- the classroom behavior of pupils and its relation to learning;
- -- the pupil's attitudes toward mathematics and the relationships of attitudes and self concept to learning; and
- -- the development of alternatives for teaching mathematics, in particular through laboratory lessons and heuristic teaching.

Each of the above concerns is directly related to the pupil's classroom behavior, intellectual development, and affective characteristics. Hence, in order to consider these questions, one must be able to observe and evaluate the behavior of pupils in learning situations. This is done primarily through two means: direct personal involvement in mathematics classrooms and indirect classroom participation through television or films.

During the pre-service program, students will be expected to experience direct classroom observation and participation through an extended pre-student teaching practicum described below and through student teaching. In-service teachers draw upon the daily experiences in their own classrooms. While these school-based experiences are invaluable in many ways, they have three primary limitations:

- -- Classroom experiences are spontaneous and cannot be fully planned in advance. Hence, students may not have the opportunity to observe certain situations that are important in their professional development.

-- Classroom experiences are not preserved and cannot be replayed for systematic study.

-- No two students experience the same classroom happenings, so it is not possible draw upon a common learning experience for the development of new concepts or teaching strategies.

In order to supplement the school practicum component, the laboratory component includes video or films directed toward the following objectives:

1. To provide opportunities for both pre-service and in-service mathematics teachers to study common learning situations with an emphasis on the behaviors of secondary school pupils in these situations.

2. To show differences in the mathematical problem-solving behaviors of secondary school pupils.

3. To illustrate alternative strategies for teaching and learning mathematics.

4. To show pupils' classroom behaviors with regard to mathematics.

The tapes are intended to be used in an interactive fashion such as the following:

1. In seminars concurrent with the practicum, students can view selected portions of the tapes to observe pupil and teacher behaviors as these relate to classroom climate, classroom control, pupil attitudes, and other affective dimensions of mathematics classes, and they can relate these to their own classroom observations.

2. In practicum seminars and methods classes, students can observe pupils' behaviors in problem-solving or laboratory learning experiences to understand better the processes by which pupils learn mathematics and to identify more accurately evidence of a pupil's level of intellectual development. These, in turn, are used to motivate students in planning lessons or units appropriate for particular pupils.

3. Methods students can view tapes with a focus on the teacher's instructional objectives, learning set, teaching strategies, classroom interaction, questioning techniques, etc. These can become both a background for developing future content and a model for the development of similar lessons that the students are expected to plan, teach, and evaluate in peer teaching or microteaching sessions.

4. Students can view pupils of different age and grade levels engaged in a similar learning task in order to

consider the intellectual development and the mathematics learning of pupils during the secondary school years.

5. Students can see examples of the same content developed through a variety of instructional approaches and can discuss and evaluate the differential outcomes of each.

6. Student teachers can view and discuss the tapes as models of classroom situations that they may be trying to develop, both as models of other alternatives that a student mighy try and as examples of how a lesson might have been handled differently.

School-based practicum

As indicated earlier, it is considered essential for the training program to involve the students in classroom experiences. To this end, the pre-service program should include an extended pre-student teaching practicum during which the student participates in secondary school classroom experiences. The practicum also includes regular seminars involving students and staff development personnel. (Examples of possible practicum experiences and seminar content are suggested in the appendix.) The practicum, which probably will last at least one full academic year, would include, as a minimum, experience in both junior and senior high school classes. In addition, there should be opportunity for students to have practicum experiences in one or more alternative settings such as elementary schools, open or free schools, special education programs for slow learners or handicapped pupils, programs for the gifted, adult education programs, or other available experiences. In particular, secondary school mathematics teachers are especially encouraged to gain experience by participating in elementary and/or middle school classes.

One goal of the practicum is to help prospective teachers answer the questions, "What does it mean to be a high school mathematics teacher?" and "Do I really want to be one?" If an individual answers the latter question in the negative, this is regarded as a legitimate outcome, and it is hoped that such decisions can be made well before student teaching.

Another goal of the practicum is to enable students to relate the content of their formal instruction to actual classroom happenings. The seminars are largely designed to facilitate this by helping the students to focus their observations and experiences. In the case of the foundations component where formal instruction includes students from many disciplines, the seminar allows mathematics students to examine more carefully the foundations topics as they apply to mathematics instruction.

Finally, the practicum is designed to be a vehicle for the student's involvement in instructional activities. This begins with observation in mathematics classes, extends next to tutoring first individuals and then small groups of pupils, later expands to opportunities to present brief lessons or short units under the guidance of the classroom teacher, and eventually culminates in full-time student teaching.

This proposal considers some relevant practicum experience to be essential for all students. While it is possible for students to demonstrate knowledge without participating in formal instruction, the practicum component is the forum both for the development of certain competencies and for the assessment of others. Most of the competencies at the application, analysis, synthesis, and evaluation levels are demonstrated only in classroom settings. In the case of pre-service teachers seeking to develop and demonstrate entry-level performances, the practicum experience, including student teaching done under the supervision of a qualified mathematics teacher, is essential. It would be a contradiction in terms to suggest that a teacher could demonstrate required teaching competence without including demonstration in actual classroom situations over an extended time period.

A final word of caution is in order here: It is not sufficient merely to send students into schools. We cannot assume that they will see things we expect them to observe unless we deliberately call attention to those items. Many classroom realities are taken for granted both by pupils and by teachers, which may account for some of the unexpected surprises reported by beginning teachers. The suggestions for practicum experiences in the appendix are offered as examples of strategies for focusing the students' attention on some of these salient classroom realities.

Individual study

The fourth element of the program is the provision for individual and independent study. Activities in this domain may be focused on competencies at any level of the taxonomy. Alternatives include readings; video or audio tape presentations; computer-assisted intstruction; independent research, problem-solving, or laboratory investigation; interviews with pupils, teachers, or others; case studies; oral or written reports; peer group discussions; professional meetings; modules; and more. The nature and extent of the individual study is ordinarily determined jointly by the student and an advisor or supervisor, and this determination includes a specification both of the expected outcomes and of the criteria for evaluation.

It should be noted in considering the various types of enabling activities described above that if the student is to be given alternatives from which to select, it follows as a necessary consequence that he/she must also be given the opportunity not to select others. Program administrators must beware of the temptation to expect all students to complete all available learning activities. The one exception in this model, as noted earlier, is the insistence that the program include an appropriate practicum component for everyone.

Representative Enabling Activities

In a previous section, ten performance goals and some contributing instructional objectives were suggested. Each of these can be further illustrated by suggesting selected enabling activities as presented below. For convenience, possible means of assessment for each also are listed here. These are discussed more fully in a following section

Performance Goals	Enabling Activities	Means to Assess
1. The student will recognize problem-solving as the central focus of school mathematics.	1A. Individual problem solving	1a. Tests to assess knowledge about problem-solving
	1B. Discussion of problems and their solutions with peers and instructors	1b. Student solutions to selected problems
	1C. Live and/or taped demonstrations of problem solving lessons	1c. Observation and/or checklist of student use of heuristics
	1D. Readings (e.g., the writings of George Poyla)	1d. Evaluator-student discussions
	1E. Examination of mathematics texts to identify problems and exercises contained therein	
	1F. Student collection of good problems and identification of useful heuristics for each	
	1G. Observation of pupils engaged in solving problems	
2. The student will recognize the importance of diagnostic teaching in mathematics	2A. Formal instruction learning and developmental theories	2a. Tests to assess knowledge about learning, diagnostic teaching, and other concepts
	2B. Student completion of task analysis for selected topics	
	2C. Discussion of common sources of pupil error	2b. Oral and written reports from student diagnosing a particular pupil's difficulty
	2D. Analysis of pupil work to identify error patterns	

Performance Goals	Enabling Activities	Means to Assess
		2c. Critique of learning activities planned for remediation
	2E. Independent student search for resources and instructional alternatives appropriate for remediation of identified errors	2d Critique of outputs (task analysis, error patterns, test interpretation, etc.)
	2F. Case study and/or tutoring pupils with learning difficulty	
	2G. Examination of available diagnostic instruments	
	2H. Administration of diagnostic test to pupils and analysis of pupil performance	
	2I. Observation and tutoring in special education mathematics classes	
3. The student will plan and teach a mathematics lesson	3A. Formal instruction on the content of lessons writing ob-objectives, etc.	3a. Observation in laboratory or practicum
	3B. Writing objectives for a given topic; objectives critiqued by peers and evaluators; rewrite objectives	3b. Checklist of desired be-behaviors 3c. Peer evaluation of lessons
	3C. Planning instructional materials and activities	3d. Analysis of written lesson plan by instructor
	3D. Teaching lesson to peers; critique and discuss	3e. Self-evaluation of video taped lesson
	3E. Teaching of lesson to small group of pupils; supervisor observes, discusses	3f. Written test of ability to write objectives

Performance Goals	Enabling Activities	Means to Assess
	3F. Video taping lessons and performing self-evaluation	3g. Test of pupils to whom the lesson was presented
	3G. Viewing demonstration lesson; inferring teacher's objectives and plans	
4. The student will demonstrate effective communication skills.	4A. Discussion of the effects of various forms of communication in the classroom	4a. Written or oral discussion of the effect of communication in the classroom
	4B. Classroom observation and systematic recording of communication patterns	4b. Direct observation or video tape of lesson
	4C. Application of interaction analysis in demonstration lessons or in classroom observations or in video tape of student's own lesson	4b. Analysis of lesson plans 4d. Interaction analysis performed on students
	4D. Identification of levels of questions in demonstration lessons	
	4E. Planning of questioning sequence for selected lesson	
	4F. Comparison of demonstration lessons taught according to lecture, guided discovery, discussion, or other approach	
	4G. Observation of classroom situations for examples of pupil non-verbal behavior	

Performance Goals	Enabling Activities	Means to Assess
5. The student will plan and teach a unit of instruction	5A. Development of written unit and lesson plans	5a. Analysis of written plans by supervisor
	5B. Independent search of resources to identify useful lesson strategies and materials	5b. Observation by supervising teacher and evaluator
	5C. Teaching unit to secondary school pupils	5c. Classroom observation by peers
	5D. Development of pretests, posttests, and quizzes for unit	5d. Video tape of lessons for self-evaluation
	5E. Measurement of pupil learning during and after the unit	5e. Discussion of lessons with supervisor
	5F. Revision of lessons after instruction	5f. Measurement of pupil achievement of unit objectives
6. The student will use a variety of learning experiences to develop mathematical concepts	6A. Formal instruction to introduce student to new topics, applications, and resources	6a. Analysis of lesson plans by evaluators
	6B Participation in professional meetings	6b. Observation of lesson
	6C Interaction with resource persons such as industrial mathematicians, researchers, computer programmers, and statisticians	6c. Log of teaching experience to identify variety and frequency of learning activities
	6D. Student participation in mathematics laboratory lessons	6d. Pupil feedback on lessons in terms of interest, motivation and learning
	6E. Student involvement in computer-assisted instruction	6e. Evaluation by student, pupils, and/or supervisor of effectiveness of aids used in the development of concepts or generalizations
	6F. Discussion with peers and instructors of alternative presentations for given content	

Performance Goals	Enabling Activities	Means to Assess
	6G. Viewing demonstration lesson contrasting different instructional approaches to the same content	
	6H. Planning and teaching lessons of different types (laboratory, discussion, computer based, guided discovery, etc.)	
	6I. Making manipulative devices or audiovisual aids; creating orginal games	
	6J. Reviewing text materials and discussing ways in which learning might be enhanced by supplemental activities	
7. The student will analyze pupil, teacher, subject matter, and environmental cues to select, organize, and administer an effective mathematics program	7A. Reviewing a given course or school mathematics program to identify major topics in the curriculum	7a. Discussions with students 7b. Students' oral or written reports and recommendations
	7B. Comparing curriculum content with identified needs of pupils	7c. Long term record of lesson plans to assess degree to which the student alters his/her teaching behavior.
	7C. Reviewing texts for several grade levels to identify the spiral approach to selected topics	
	7D. Examining given texts to identify places where the textbook presentation might be enhanced or replaced by an alternative text or activity	7d. Feedback from pupils in terms of motivation, satisfaction, and learning
		7e. Observation of teaching to de-

Performance Goals	Enabling Activities	Means to Assess
	7E. Performing item analysis of pupils' test performance to locate learning difficulties and suggesting appropriate remediation	termine frequency with which teacher uses diagnostic tests, analyzes pupil errors, etc.
	7F. Consulting resources to develop enrichment activities that extend the basic mathematical content	
8. The student will see pupils as individuals and respond accordingly	8A. Formal instruction on relevant psychological theories; individual differences; needs and characteristics of slow learners, gifted handicapped, etc.	8a. Observation of student interacting with pupils 8b. Checklists to assess frequency of student's use of positive reinforcement, feedback to pupils, using pupil comments or questions in instruction, etc.
	8B. Practicum experiences in special education programs	
	8C. Practicum experiences tutoring gifted pupils and pupils with learning difficulties	8c. Examination of lesson plans and teaching logs to determine extent to which lessons and assignments are differentiated
	8D. Administration of diagnostic tests, Piagetian tasks, etc. to pupils	
	8E. Systematic observation of selected pupils in a variety of settings in and out of school	8d. Pupil feedback and evaluation of instruction
	8F. Case studies and discussions of these with experts and with other educators	
	8G. Video taping lessons for review and discussion by student, peers, and/or instructor	

Performance Goals	Enabling Activities	Means to Assess
9. The student will combine the underlying competencies into an effective, personal teaching style	9A. Reading of professional journals and participation in professional meetings 9B. Engaging in problem solving on a regular basis 9C. Building files of teaching resources, strategies, and materials 9D. Discussions with persons who regularly use mathematics in various contexts 9E. Interviewing or polling pupils to determine their attitudes and interests 9F. Discussions with teachers of other subjects on the relationships between mathematics and those disciplines	9a. Observations in classroom over time 9b. Comparisons of observations to assess consistency of behaviors 9c. Pupil feedback 9d. Pupil growth and learning 9e. Peer evaluation of the teacher's contribution in meetings, workshops, etc.
10. The student will judge the effectiveness of one's own teaching	10A. Formal instruction and independent study of relevant research in mathematics education 10B. Interacting with colleagues in professional meetings, workshops, and seminars 10C. Conducting formative and summative evaluations of pupil progress 10D. Observing colleagues and visiting other schools	10a. Observations by colleagues or outside evaluators 10b. Self-evaluation 10c. Pupil feedback 10d. Measures of pupil growth and learning 10e. Peer review of professional contributions

Performance Goals	Enabling Activities	Means to Assess
	10E. Preparing immediate and long-term proposals for one's own professional development	

Establishing Assessment Criteria

Establishing the performance criteria and assessing student attainment of them are undoubtedly the biggest obstacles to the adoption of any CBTE program. The lack of sound data on the relationship of teacher performance characteristics to pupil learning is much discussed in the literature (see, for example, Rosenshine and Furst, 1971). Yet CBTE operates on the assumption that teaching competencies hypothesized to effect pupil learning can be developed through appropriate kinds and numbers of enabling activities and that success in the attainment of the desired competencies can be established through the systematic assessment of teacher behavior. That assumption underlies our model here in the following manner: Generic competencies are identified which in turn give rise to specific instructional objectives. Attainment of the objectives is established when the student demonstrates an acceptable number and/or level and/or variety of specified behaviors, and from these is inferred the existence of the competency. Note that this approach is differenet from one that would equate competence with behavioral objectives that include specified mastery levels and conditions.

Clearly, the identification of behavioral criteria is always a judgment call. These criteria must flow from the specified objectives, and they should represent one's best professional judgment based on available, albeit fragmentary and tentative, knowledge. They are defensible not because they are based on certain knolwedge, but because they represent a systematic attempt to assure that nothing essential is overlooked and because they lead to assessment that is specific and descriptive and that therefore enables precise, corrective feedback. A further caution is imperative: The most important consideration is for the stability and growth of behaviors and behavior patterns over time, and one must always assume that the goal is growth in teaching competencies. Thus behavioral criteria must never be allowed to become maxima or ceilings in any program.

In establishing the behavioral criteria, it is common to consider six types of criteria; i.e.,

<u>knowledge</u> that the student is expected to acquire,

<u>outputs</u> (products, events) that the student is expected to produce,

<u>behaviors</u> that the student will demonstrate,

<u>attitudes</u> that the student will display,

<u>consequences</u> (usually pupil learning) that result from the student's intervention, and

<u>experiences</u> that the student will have.

This last category, experiences, acknowledges the fact that not all outcomes can be specified in advance. An example of an experience objective is that the student will have practicum experiences in a variety of grade levels, mathematics courses, ability levels, and school organizational patterns. The outcomes of these experiences are not predictable, but might include the student's recognition of the uniqueness of individuals and situations or a clearer insight into one's own strengths, weaknesses, attitudes, and preferences.

There are several approaches to setting behavioral criteria which include specifying the expected frequency of behaviors, identifying the expected degree of accuracy or adherence to some standard, establishing a rating system and an expected performance norm, or assessing the result of the students' performances in terms of their pupils' learning and growth. In any case, the criteria must take account of: How is the behavior to be demonstrated? When? How often? In what settings? In how many settings? Under what conditions? With what level of proficiency?

The variety of options in establishing behavioral criteria will be illustrated for a specific competency: The student will demonstrate the ability to teach a mathematical concept. (Cooney and others have identified a set of "moves" or patterns of behavior associated with teaching a concept that we shall assume in this illustration. See Cooney, Davis, and Henderson, 1975, for further discussion of the moves.) Sample criteria are suggested in each of the six categories listed earlier. No attempt is made, however, to set levels for these criteria. Such decisions are left to program developers, as they should be made in the context of total program goals. The sample criteria are as follows:

<u>Sample knowledge criteria</u>

The student will:

- accurately define a specified concept
- give examples and nonexamples of a specified concept
- give counterexamples for a false generalization about the concept
- state the necessary and sufficient conditions for the concept
- compare and contrast the specified concept with related concepts
- identify prerequisite concepts
- show the relationship of the specified concept to others in the curriculum
- differentiate between the concept and algorithms for the application of the concept

Sample output criteria

The student will:

develop a lesson plan for teaching the concept
design an activity that uses a physical model as an embodiment of the concept
plan a set of activities that contribute to the development of the concept and that provide both perceptual and mathematical variablity
plan an activity that provides an application of the concept

Sample behavior criteria

The student will:

teach a lesson using each of the moves for teaching concepts
teach a lesson using selected moves with a designated frequency
teach a guided-discovery laboratory lesson to develop a concept

Sample attitude criteria

The student will:

discuss the significance of the concept in relation to various goals of mathematics
discuss the importance of the concept in various applications

Sample consequence criteria

The student will:

teach pupils a concept they had not previously encountered
teach pupils a new application of a previous concept

(Attainment of consequence criteria is evidenced by resulting pupil knowledge and performance.)

Sample experience criteria

The student will:

teach the same concept to pupils of different ages
teach the same concept to pupils of different abilities
teach the same concept to individuals, to small groups, and to large groups

Assessing Student Competencies

Assessment of student competencies serves two major functions: a descriptive function that enables the recording and analyzing of behavior in order to provide corrective feedback, and a judgmental function that enables program personnel and students to make go/no go decisions. The problems associated with assessment are many and complex (see Merwin, 1973; Roth and Mahoney, 1975; Rosenshine and Furst, 1973; McNeil and Popham, 1973; Farrell, 1979), and they will not be enumerated here.

One consideration will be mentioned, however. In research, the traditional approach is to set high criteria for "certainty" (p .01, for example). The position is that it is better to reject something that is true than to accept something that is false. If the same is assumed as necessary for CBTE, then doubtless CBTE will fail. If, on the other hand, evaluation is approached as an attempt to establish descriptive data about what the student can or cannot do, then it is possible for CBTE programs to proceed. At the same time, research and development efforts should focus on establishing the reliability and predictability of various results based on the observed behaviors of teachers. From this knowledge, the expectations and means of CBTE can continually be refined and improved.

One thing that can be said is that the more explicit the behavioral criteria, the easier they are to assess and evaluate; also, the more specific and useful will be the feedback that flows from the evaluation. Ultimately, one is concerned with the stability and growth of competencies. Hence, a one-time demonstration that the student can do something is far less useful than an on-going evaluation of behavioral patterns and their use at appropriate times and with appropriate frequency. A constant concern is that the evaluation adequately sample the student's behavior in order to yield defensible inferences of competence.

There are, however, many means available for assessing student knowledge and performance. The varied means of assessment suggested earlier with the ten representative competencies and their related enabling activities illustrate the range of possibilities. The appropriateness of means is related to the type of behavioral criteria being assessed. In general, knowledge criteria are assessed by all conventional classroom means (quizzes, examinations, papers, problems solved, discussions, oral presentations, etc.). Outputs are examined and evaluated according to specified norms or characteristics. Behaviors can be counted and/or rated. Attitudes are elicited directly from the student's written or oral communication or inferred from his or her nonverbal communication. Consequences are evaluated by measuring pupil growth. Experiences are counted, logged, described, and/or submitted to self-evaluation by the student.

The earlier example of teaching a mathematical concept is used to illustrate some aspects that the assessment might consider. The student would be observed in the classroom and/or video taped, and the evaluator could note, count, rate, describe, and comment on the following:

Is the concept accurately and clearly delineated?
Is pupil interest motivated?
Is the importance of the concept established?
Are objectives and expectations communicated to pupils?
Is the concept related to: previous learning?
pupil interest?
pupil experience?
future learning?
Is the concept appropriate for: the age and intellectual development of the pupils?
the mathematical maturity of the pupils?
the course and unit objectives?
Is the presentation limited to the designated objectives?
How many different moves are used by the teacher?
With what frequency is each move employed?
Is there an adequate variety of moves?
Is there an adequate frequency of: examples?
nonexamples?
applications?
How appropriate for the pupils are the: examples?
nonexamples?
applications?
Are the necessary and sufficient conditions adequately developed?
Does the teacher ask for a definition but accept an example?
Do the applications and examples provide perceptual variability?
Do the applications and examples provide mathematical variability?
Are there adequate activities to practice or reinforce the concept?
Are there adequate opportunities to apply the concept to new situations?
Are learning aids and physical models used in the lesson?
Is there sufficient variety of learning activities?
Does the teacher elicit examples and applications from the pupils?
Does the teacher build on pupils' responses?
Do the teacher's questions depend on memorization or on understanding?
Does the teacher answer his/her own questions?
Does the teacher give all of the pupils sufficient opportunity to answer?
Does the teacher ask questions that help the pupils draw their own conclusions?
Does the teacher ask questions that lead to refinement and deeper understanding of the concept?
What is the proportion of guided discovery compared to the proportion of expository teaching?
Has the teacher identified the prerequisite concepts and skills?
Does the teacher build on the prerequiste concepts and skills?
Does the teacher differentiate between the concept and the algorithms for applying the concept?
Does the teacher elicit definitions or descriptions of the concept from the pupils?
Does the teacher ask the pupils to verbalize the concept too soon?
Does the teacher move too quickly to computation or algorithms?
Does the teacher use counterexamples to disprove false generalizations about the concept?
What is the proportion of teacher talk compared to pupil talk?

Does the teacher respond to individual pupil difficulties?
Is the teacher enthusiastic?
Does the teacher give feedback and reinforcement to pupils?
Does the teacher monitor and evaluate pupil learning?
Does the teacher adjust to unexpected happenings?
What is the degree of pupil understanding of the concept?

One advantage of an assessment such as that suggested above should be immediately apparent. Because the assessment is highly descriptive, the student can receive precise and specific feedback. Hence, one knows not only <u>how</u> one is evaluated but also <u>why</u> the evaluation is given. Thus, one also knows what one needs to attend to in further learning activities and one can mark progress over time.

Overall, the evaluation of students in the program will focus on the following broad aspects:

1. Growth and application of knowledge

   a. of mathematics
   b. of educational theory and research
   c. of human growth and development and of the intellectual and social needs of children
   d. of the goals and objectives of mathematics education
   e. of pedagogy and teaching methodology
   f. of general knowledge

2. Instructional planning, delivery, evaluation, and feedback

3. Oral and written communication

4. Classroom management

5. Problem-solving

6. Affective outcomes (motivation, enjoyment, self concept, concern for pupils, etc.)

7. Professional relations and interpersonal skills

Evaluation will take place in the college classroom, in assessment centers, in laboratories, and in secondary school classrooms. It will be conducted by college instructors, by cooperating classroom teachers, by students themselves, by peers, by trained observers, and by pupils. It will include paper-and-pencil assessments, discussions, interviews, direct and indirect observations, logs, outputs, self-evaluation, and more. Always it will need to be specific and descriptive, and always it must be communicated to the student and directed toward further professional growth.

Chapter Five

Teacher Education: A Developmental Enterprise

We began our presentation with an assumption that teacher education is an on-going, developmental effort, a unified whole with stages of teacher education viewed as different points along a continuum. We wish now to examine that assumption more carefully.

Pre-service Teacher Education

Before we consider specifics of the pre-service phase of teacher education, we should recognize the research of Perry (1970) and its relevance for teacher education. Perry studied the intellectual and ethical development of college students and conceived a scheme to represent the forms or stages of that development. Perry's nine positions described stages of development from an initial duality through multiplicity to relativism and commitment. He summarized those nine positions as follows:

Position 1: The student sees the world in polar terms of we-right-good vs. other-wrong-bad. Right Answers for everything exist in the Absolute, known to Authority whose role is to mediate (teach) them. Knowledge and goodness are perceived as quantitative accretions of discrete rightness to be collected by hard work and obedience (paradigm: a spelling test).

Position 2: The student perceives diversity of opinion, and uncertainty, and accounts for them as unwarranted confusion in poorly qualified Authorities or as mere exercises set by Authority "so we can learn to find The Answers for ourselves."

Position 3: The student accepts diversity and uncertainty as legitimate but still <u>temporary</u> in areas where Authority "hasn't found the answers yet." He supposes Authority grades him in these areas on "good expression" but remains puzzled as to standards.

Position 4: (a) The student perceives legitmate uncertainty (and therefore diversity of opinion) to be extensive and raises it to the status of an unstructured epistemological realm of its own in which "anyone has a right to his own opinion," a realm which he sets over against Authority's realm where right-wrong still prevails, or (b) the student discovers qualitative contextual relativistic reasoning as a special case of "what They want" within Authority's realm.

Position 5: The student perceives all knowledge and values (including authority's) as contextual and relativistic and subordinates dualistic right-worng functions to the status of a special case, in context.

Position 6: The student apprehends the necessity of orienting himself in a relativistic world through some form of personal Commitment (as distinct from unquestioned or inconsidered commitment to simple belief in certainty).

Position 7: The student makes an initial Commitment in some area.

Position 8: The student experiences the implications of Commitment, and explores the subjective and stylistic issues of responsibility.

Position 9: The student experiences the affirmation of identity among multiple responsibilities and realizes Commitment as an ongoing, unfolding activity through which he expresses his life style.

(Perry, 1970; pp 9-10)

Perry discussed the above scheme at length and illustrated the positions with longitudinal case studies of college students. While his research was not aimed at any one group of students or at any specific academic field, Perry's work suggests several considerations that may give insight into the education of mathematics teachers.

Undergraduate students frequently were found to be at the earlier positions of the scheme, quite frequently moving from dualism through multiplicity. Mathematics educators will recognize hints of this in the behavior of their own students. Consider, for example, the degree to which beginning education students seek answers on exactly how to do something: how to discipline, how to grade, how to teach fractions. These students look to their college professors and cooperating teachers for the answers, and it can be difficult to lead the students to formulate answers of their own

This difficulty manifests itself often in microteaching situations associated with methods classes An example of a typical assignment might be to present a short lesson to introduce a selected concept. The methods student prepares and presents the lesson to a small group of high school pupils or peers, and the lesson is video-taped for later playback and discussion. In that discussion, the professor may ask the student to explain why he/she chose to use a particular problem, example, application, or teaching aid in introducing the concept. Rather than receive an explanation that "I did it that way because . . .," it is not unusual to be met with the rustle of papers and notebook covers as the students prepare to write down "the right way" which, of course, they expect the professor is about to dictate

The tendency to cling to dualism may be more acute in mathematics than in other subject areas because, as noted in Chapter One, we so frequently encounter a dualistic view of mathematics itself. An example of this came not from an undergraduate, but from an older teacher with many years of teaching experience. During a graduate mathematics education course, the class was examining the procedure for finding square roots by the divide-average-divide again algorithm. Fascinated by the fact that this method, which was new to him, always worked, the teacher was at the same time deeply disturbed. At last he was compelled to demand "Ok! That's very nice. But when are you going to teach them the real way?"

Pre-service and beginning teachers are overwhelmed with concerns about their subject matter and about their own abilities. Will I know the mathematics? What if a pupil asks me a question and I don't know the answer? Will they obey me when I discipline? What should I do if they won't listen? They want answers.

Teacher educators, on the other hand, cannot give all the answers that the students would like. Rather, they should help the students to examine at least some of the alternatives available to them and to attempt to assess the probable consequences of each:

> How do you think pupils would solve that problem? What is likely to confuse them? What are some things they would have to know before they could work the problem? If a pupil doesn't understand X, what might you do to help him learn it? What are some questions you could ask your pupils to see if they understand the concept? What are some extensions or applications of that idea that you could present as a challenge to the pupils? How was this topic discovered? What was its significance to the history of mathematics? When will the pupils need this new concept again" What advantages are there in presenting the concept by method A rather than by method B? What are some good manipulatives to use in this lesson?
>
> What happened when the teacher disciplined the pupil? How did the teacher's action seem to affect the pupil who was punished? How did it affect the rest of the class? What other actions could the teacher have taken? What would you reasonably expect the outcome of those other actions to be? Which would you have chosen to use? Why?

Questions like these are intended to help students recognize that there are multiple approaches to teaching situations, that some may be preferred over others, and that different teachers may choose different responses. Rarely is there one approach, one "right way"; what is important is that the teacher learns to act from a rationale that expresses his/her personal beliefs and style.

What we are saying relative to Perry's work is that i is important to recognize the level of development of the future teacher and to help him/her move toward the higher positions. What we are saying relative to the taxonomy is that we want to lead the teacher to perform at the higher levels of analysis, synthesis, and evaluation. Earlier we suggested that many of the competencies associated with the pre-service phase of teacher education would be at the first three levels of the teaching taxonomy. Perry's work suggests that we must be realistic about our expectations. Many of the teaching competencies associated with the higher levels of the taxonomy presume attainment of the higher positions of Perry's scheme. For example, in order for a teacher to formulate and act upon a consistent philosophy and rationale for teaching, the individual must be able to accept the relativism inherent in the teaching situation and to make personal decisions and commitments.

This is not to suggest that the competencies expected from pre-service teachers be limited to the knowledge, comprehension, and application levels. The lists of questions suggested earlier, for example, are ultimately directed at the higher levels of the taxonomy. But the degree of attainment of higher-level competencies will differ from beginner to experienced teacher.

Smith proposed the following as a list of minimal abilities that a program of teacher education should develop:

> the ability to
>
> 1. perform simulant operations (questions, structure, probe);
> 2. manipulate the different kinds of knowledge;
> 3. perform reinforcement operations;
> 4. negotiate interpersonal relations;
> 5. diagnose student needs and learning difficulties;
> 6. communicate and empathize with students, parents, and others;
> 7. perform in and with small and large groups;
> 8. utilize technological equipment;
> 9. evaluate student achievement; and
> 10. judge the appropriateness of instructional materials.
>
> (Smith, 1969; p. 71)

These can be translated into competencies more specific to mathematics teaching, such as the following:

the ability to

differentiate among concepts, generalizations, skills, and problems;

describe and demonstrate appropriate strategies for teaching concepts, generalizations, skills, problem-solving;

use heuristic techniques and higher-order questions;

plan and teach guided-discovery lessons; and

locate, adapt, or design manipulative materials and use these in presenting mathematics lessons.

In generating these ideas about required and/or desired teaching competencies, it is useful to begin with the teaching process itself. Five major domains suggest themselves: knowledge, planning, instruction, evaluation, and development.

Knowledge

What will the mathematics teacher need to know? A good starting point is NCTM's list of "Guidelines for the Preparation of Teachers of Mathematics" (1979) which includes knowledge competencies in mathematics, in humanistic and behavioral sciences, and in teaching and learning theories. Important to include here is the ability to identify major unifying concepts of mathematics and to explain them in a way that is meaningful to a secondary school pupil. The ability to find the roots of an equation is quite different from the ability to explain to pupils the concept of a variable. The former is an example of knowledge of mathematics; the latter is an example of the knowledge about mathematics that we noted in Chapter One. Other examples of major unifying concepts are function, measurement, ratio, proportion, similarity, equivalence, and probability. Knowledge about these and other major concepts should be stressed in the pre-service program.

Planning

Examples of competencies related to planning for instruction include the following:

the ability to

identify key ideas and concepts in mathematics topics and activities;

specify long-range instructional goals and unit and lesson objectives;

plan a variety of learning activities designed to foster achievement of the specified objectives;

plan learning experiences that take into consideration individual pupil needs and backgrounds and make provision for individual differences;

plan lessons and units that reflect sound learning theory and/or research;

give a rationale for the selection of objectives and the proposed sequencing of learning experiences;

plan examples, non-examples, applications, problems, questioning, sequences, models, manipulatives, etc., for use in the lesson.

specify any concepts, principles, or skills that are prerequisite for the new lesson; and

anticipate aspects of the material that are likely to be confusing or difficult for the pupils and plan strategies to confront those difficulties.

Instruction

Competencies related to instruction might include these:

the ability to

motivate pupils to become involved in the lesson;

employ a variety of instructional strategies and materials in presenting lessons;

adjust teaching strategies, pace, difficulty, and content of the lesson to match the stated objectives and the needs and characteristics of the pupils;

bring the class to order, establish a relationship with the pupils, and structure the learning environment so pupils will be free to learn and to express their curiosity and creativity;

foster a wide variety of interaction styles between teacher and pupils, between pupils and pupils, and between pupils and the curriculum;

relate new mathematics topics to previous learning, to pupil interests and experiences, and to other content areas;

employ a variety of questioning techniques and problem-solving opportunities;

provide adquate feedback and reinforcement;

demonstrate effective communication skills and interpersonal relations; and demonstrate genuine enthusiasm for mathematics, for pupils, and for teaching mathematics to pupils.

Evaluation

Examples of competencies that derive from evaluation are:

the ability to

identify pupil errors and error patterns and diagnose pupil difficulties;

prescribe appropriate learning experiences based on diagnosis;

develop tests and other evaluation instruments based on the goals and objectives of the lesson, the unit, and/or the program;

evaluate pupil learning, teacher effectiveness, and program success using a wide range of techniques in addition to tests; and

analyze the results of evaluation procedures and use the findings (a) to provide feedback to pupils and their parents and (b) to modify and improve instruction.

Development

Competencies related to the professional development of the mathematics teacher may be represented by:

taking advantage of opportunities to improve teaching through;

reading the professional literature in mathematics education;

participating in professional meetings and activities at the local, state, and national level;

identifying and utilizing a wide range of instructional resources available both in and out of the school;

sharing promising ideas and effective techniques with colleagues;

planning a personal program for continued study of mathematics and mathematics education; and

developing expertise in curriculum planning, development, and evaluation.

Once again, the competencies above do not represent an exhaustive list of program goals. Rather, they are presented to call attention to the scope and complexity of mathematics teaching about which teacher educators must be concerned. The lists can and should be modified and extended. Also, the broad competencies stated here must be refined, clarified, and made specific. None will be fully achieved by the pre-service teacher, but their development is rooted in the pre-service phase of teacher education and progress toward them constitutes the focus of the program.

There are many approaches to the development of outcomes such as these in addition to lectures and discussions. We will suggest some here and illustrate them with examples.

Guided observations

Observation is a skill too often taken for granted. If students report they are bored with classroom observation, it may be because they don't know what things to look for. Perhaps never again will a teacher have as much opportunity to observe teaching as during the pre-service phase of teacher education. That opportunity should not be wasted.

Observation guides that focus student attention on specific things can be prepared and given to students before a classroom visit. Later the students discuss the observations in a small seminar and consider the implications of those observations for teaching mathematics. For example, an early observation might be planned to focus attention on the classroom itself. Some of its points might be these:

> Sketch the physical arrangement of the classroom, including all furniture and equipment.
>
> How does the classroom arrangement contribute to or distract from learning?
>
> Make a list of the learning materials and equipment in the classroom. Indicate with a (P) those items that are readily available for pupil use; indicate with a (T) those items that are for teacher use only; indicate with an (X) thoe items that are used by pupils only with the teacher's permission.
>
> Is the classroom noisy? Adequately ventilated? Adequately lighted? Too warm or too cold?
>
> Are the bulletin boards decorated? Sketch or describe them.
>
> Is student work displayed in the classroom? How? For what purpose?
>
> What elements of the room make it a pleasant (or an unpleasant) place to be?

Describe the conditions of the pupils' desks, the walls, floor, window coverings, chalkboards.

If you were unfamiliar with the school, what features of the room would tell you that you were in a mathematics classroom?

In the discussions that follow students can share and compare their observations and thereby be helped to consider how the physical conditions of the room contribute to the learning environment.

Because guided observations are an important part of the teacher education experience, additional suggestions for observation are given in the appendix.

Problem-solving

Students in the program should regularly engage in problem-solving themselves. A recommended strategy is first to present the students with a problem and allow time (hours, days, weeks, as needed) for their solutions. For example, present the game of Nim:

> Twenty-one markers are placed in a pile. Two players take turns removing one, two, or three markers from the pile. The player to take the last marker wins.

Have the students play the game and try to develop a strategy for winning. After they have solved the problem, discuss their solutions. How many different solutions were found? What heuristics helped in solving the problem? What heuristics did not lead to a solution? Is the problem similar to any other they have solved?

Discuss how the students think school pupils would do the problem. What difficulties would you expect pupils to have? Do you think the pupils will solve it? How?

View video tapes of pupils engaged in the same problem. Discuss the pupils' behavior, their degree of understanding of the problem, the difficulties they seemed to have, some reasons that might account for those difficulties. Evaluate the teacher's role in presenting the problem and/or have the students suggest how they would handle the lesson.

Have the students present the problem to friends or to high school pupils and report how those persons solved it.

Generalize the problem to new situations: What if there were more than twenty-one markers in the pile? What if you could take more than three on your turn? Suppose you start with N markers in the pile and that you can take any number of markers from a to b. How would you play then? Suppose your goal was to not take the last one (i.e., the person who takes last is the loser.) Now how should you play?

Not all problems need to be given to the students. Instead, the students should be encouraged to make files of interesting problems that they have found, to exchange problems with each other, and to create problems of their own.

Thinking about mathematics

Present the students with new questions about old topics so they will have to think about those topics more carefully. For example: If I give you a very large number, how would you determine if it is prime or not? What is the most efficient way to make the determination? If I gave you a very large number that is not prime, how could you be sure you had found all its divisors? How would you determine what is the smallest number with exactly twenty-four divisors? How would you prove to a high school pupil that there is no largest prime number?

Thinking about mathematics also involves thinking about the reasons behind well-known facts. For example: The product of two negative numbers is positive. Construct a developmental sequence of instances and/or examples that lead to this generalization inductively. Construct a developmental sequence of statements that lead to this generalization deductively. Write a deductive proof or verification of the generalization. Give a counter-example to the claim that the product of two negative numbers is negative.

Popping the question

There are many concepts and principles that should become second nature for a mathematics teacher but that sometimes become stumbling blocks because the students have "learned" or accepted them without really thinking about or understanding them. Asking students unexpectedly to explain a concept or mathematical procedure can encourage thinking about mathematics as well as provide opportunities to discuss and examine more closely some important ideas. For example:

What is π ? (Note that the most frequent response is numerical, 3.14 or the like. That is not the intent of the question, which seeks to elicit a response indicating that π is the ratio of the circumference to the diameter of any circle.)

What is area? or volume? (Again, it is common to receive an answer that indicates an algorithm: $A = l \times w$, for example. The question seeks to elicit a description or definition of the concept of area and/or volume.)

In algebra class you spend a lot of time teaching pupils to factor quadratic equations. Why is that important?

How many different ways can you think of to show that two lines are parallel?

Why do we say $x^0 = 1$?

Scavenger hunts

Unlike popping the question, where students are expected to produce a response without any special preparation, scavenger hunts pose questions for which the students must seek answers in various resources. Scavenger hunts may focus on different aspects of teaching such as content, methods, curriculum, or attitudes. For example:

On content:

When was the number zero invented and by whom?

Describe several non-euclidean geometries and compare and contrast them to each other and to euclidean geometry.

What are amicable numbers? Give some examples.

Find some examples of the fibonacci sequence in the physical world.

On method:

Find several different mathematical models and/or sets of materials that you could use in teaching multiplication or division to junior high school pupils who are having trouble with these operations.

Describe an activity, model, problem, or investigation for introducing the concept of a limit to senior high school pupils.

On curriculum:

Select a single textbook series and find where the following topics are introduced: negative integers; algebraic expressions, exponents, similar figures, trigonometric functions . . .

Find all of the places throughout the series where ratios are used. Outline the treatment of that topic each time it appears.

In the series, which is introduced first, fractions or decimals? Are pupils expected to know certain things about one before studying the other? If so, what?

On attitudes:

Take an informal poll of twenty of your friends who are not mathematics majors. Try to determine how they view mathemtaics and what they do or do not like about it. How did they develop their attitudes?

Interview a first-year mathematics teacher and try to find out what aspects of teaching were most difficult, most enjoyable, most frustrating, most surprising. For what was the teacher best prepared? least prepared? What advice does the teacher have for you?

Thinking on your feet

Role-playing typical classroom situations can be helpful to the prospective teacher. Here one student is selected to be the teacher; another is the pupil with the problem. The rest of the students ad lib the roles of the other pupils, something they generally prove to be very good at. The pupil with the problem (supplied by the professor) initiates the activity:

I don't understand why you marked these wrong on my paper:

$$\frac{2}{3} + \frac{4}{5} = \frac{6}{8}$$

$$\frac{19}{95} = \frac{1}{5}$$

How come you can't divide by zero? If you have nothing and you divide it by nothing, you still have nothing. So why don't we say 0   0 = 0?

Dilemmas

Students are asked to consider typical situations that they may face and to describe what they would do in each case and why. For example:

How would you react to the following reasons why a pupil doesn't have her homework?

I had too much homework in biology.
I had to work last night.
I forgot what the assignment was.
I lost it.
I did the wrong page so I threw it away.
I didn't do it.

> Anne always seems to learn new material easier and more quickly than anyone else. Whenever you try to develop a concept or present a discovery lesson, Anne "spoils it" by blurting out all the answers before the others have time to solve the problems. What can you do to keep Anne from spoiling it for the others without at the same time destroying Anne's enthusiasm and interest?

Activities like the examples above should begin well before student teaching, and they can be continued well past the pre-service preparation stage. Such activities facilitate the translation of knowledge into teaching behavior, and in them are the seeds for the evolution of a personal philosophy and style of teaching.

The Beginning Teacher

The first few years of teaching, especially the first year, are so critical in the life of a teacher that they warrant special attention.

Ryan's book, <u>Don't Smile Until Christmas</u> (1970), contains personal accounts of the learning, growth, joys, frustrations, successes, and failures of first-year teachers and is worthwhile reading for teacher educators, administrators, and supervisors. Elsewhere Ryan discussed the problems of beginning teachers identified in numerous studies, focusing on five areas:

> The ability to maintain discipline in the classroom;
> students' liking of them;
> their knowledge of subject matter;
> what to do in the case they make mistakes or run out of material;
> how to relate personally to other faculty members, the school system, and parents (Ryan, 1979)

Smith, too, noted the seriousness of the problems of beginning teachers and concluded:

> The bewildering network of activities into which the beginning teacher is drawn often confuses him and leads him to wonder what the role of the teacher actually is. In many instances he loses sight of his responsibility for educating his pupils as he attempts to meet and deal with all the influences that play upon him from one hour to the next. The natural tendency of an individual in such circumstances is to flee at the earliest opportunity. (Smith, 1969; p. 25)

What can we do about the problems of the beginning teacher? The answer to that question is suggested by identifying the two principal reasons why the problems exist to begin with: the failure to deal with these problems in the pre-service program and the lack of a support structure for the beginning teacher.

In the previous section we discussed at length approaches to pre-service teacher education that are designed to focus on the kinds of problems frequently encountered by teachers. Regular use of these strategies and emphasis on the teacher's responsibility to make choices and decisions, to formulate reasons for those decisions, and to anticipate the consequences of those choices will alleviate some of the unexpected realities of teaching.

We must be cautious, though, either of expecting too much or of being too critical of the undergraduate program. The pre-service teacher is essentially limited to simulated laboratory experiences and to student teaching -- hence, to behaviors at the second and third levels of the taxonomy. These are suggestive but not necessarily predictive of the behaviors they will exhibit in actual teaching, for while it is highly unlikely that one will do unless one can do, the fact that one can do something does not necessarily mean that one will do it. Further, the pre-service teacher often lacks the experience really to understand the implications of many of the aspects of the undergraduate program until he/she has more classroom experience.

The pre-service teacher has one definite advantage which the beginning teacher usually does not enjoy, however: a support structure of peers and mentors. The loss of these often is keenly felt by the beginning teacher who finds himself without a cooperating teacher, or methods instructor, or student teaching supervisor, or seminar group with whom to discuss ideas on problems. This is a need largely unfilled in teacher education.

We do not believe that beginning teachers should plunge immediately into formal post-baccalaureate educational programs, but we do advocate more informal in-service activities directed specifically at their needs. These can be offered by school districts or by the undergraduate or graduate institutions and they can include any of the activities suggested for the pre-service program. Indeed, we have found that beginning teachers not only do not object to repeating experiences from the pre-service program, but that they actually enjoy doing so because they bring to them an understanding and a felt need that they could not have had as students. Beginning teachers also have appreciated the opportunity to get together socially with other beginning teachers to share experiences and to trade ideas for classroom use.

Beginning teachers also need and appreciate supervision and help on the job, but that supervision must yield specific and constructive feedback. No teacher is helped by an occasional ten-minute visit from a supervisor or by an evaluation that simply states the class was adequate or the pupils were not learning. No one is more concerned about classroom problems than the teacher who must spend every day in that classroom, and most teachers sincerely want to deal with their problems and to continue to improve their teaching. They need specific feedback that recognizes their strengths and that offers ideas that can lead to action in building on those strengths while attending to their weaknesses. Again, all of the approaches described elsewhere in this book continue to be appropriate here.

Experienced teachers sometimes forget the special frustrations and pains of the early years of teaching. In particular, the transition in self-image which must occur from the pre-service teacher who views self primarily as student to the teacher who accepts self as teacher is a significant one. Teacher educators and teaching colleagues should not underestimate its importance.

In-service Teacher Education

We will take in-service teacher education to mean all those educational experiences of the teacher after the baccalaureate degree, including advanced degree programs; academic work outside of a degree program; and in-service education offered through school districts, professional organizations, colleges or universities, and state agencies.

The goals of in-service education were summarized by Downs to be the following:

1. To provide teachers the opportunity, the time, the means, and the materials for improving their professional competencies.
2. To assist teachers in applying to themselves new insights into the learning process.
3. To help teachers expand their perceptions of mathematics.
4. To assist teachers in developing creative instructional approaches (a) that are meaningful and mathematically correct and (b) that inculcate in students an enthusiasm and a satisfaction in learning and using mathematics.
5. To provide a means of maintaining quality in the existing curriculum.
6. To provide a means of assigning priorities to school problems and their need for solution.
7. To provide a mechanism for responding to problems of a curricular nature, to problems of an instructional nature, or to problems in human relations.
8. To facilitate a school's making full use of its resources.
9. To implement significant innovative curricular and instructional practices.

(Downs, 1977; pp 5-9)

All of these goals, especially those which refer to the teacher personally, reinforce the basic tenet that the goal of continuing professional education is to help teachers develop competencies at the higher levels of the taxonomy.

Yet in-service education has been widely criticized for failing to achieve its purposes. Otte, commenting on the situation worldwide, quoted a report of the Organization for Economic Co-operation and Development as follows: "There seems to be international unanimity on the crucial part that should be played by continuous in-service education for teachers, for teachers of teachers, and for others involved in administration in the education system" (Otte, 1979; p. 116). Yet, he went on to report that in most countries the situation in practice was badly wanting. Two contributing factors which Otte identified were "perceptible tensions between 'theory and practice', between institutions concerned with teacher education and the schools themselves" and "the disconnected variety of course offerings and the lack of continuity between courses" (pp. 116-117).

Otte's conclusions appear to be in agreement with the criticism of teacher education leveled by Smith a decade earlier. He charged that programs in universities and colleges were chaotic, with teachers often taking a potpourri of courses selected more for convenience than for usefulness to their jobs. Education beyond the pre-service level, Smith charged, was geared not to improving teacher performance but to increasing teacher salary. And he noted:

> The lack of connection between perennial education and improvement in performance can be attributed also to the failure of higher institutions to devise programs of advanced training. They provide courses in pedagogy and in the disciplines, but these are too often designed with little reference to the various roles that teachers play. Furthermore, these courses are not grouped and ordered so as to lead progressively to more effective performance in specified roles and positions. (Smith, 1969, pp. 153-154)

The approach to teacher education through the teaching taxonomy can help address the problem of tensions between universities and schools that Otte identified, the lack of programmatic integrity noted by both Otte and Smith, and the failure to construct programs derived from the roles and needs of teachers charged by Smith. Throughout this discussion we have attempted to show the relationship between the competencies needed by teachers and the educational experiences designed to bring about those competencies.

The continuing professional education of mathematics teachers can and should reflect the deepening, strengthening, modifying, and extending of the competencies developed earlier on the continuum, as well as the addition of new ones. Teachers engaged in in-service education enjoy the advantage of a classroom perspective on aspects of teaching that are real to them, and in-service education can bring to bear upon those situations any of the strategies described for the pre-service program

In-service teachers also should have a deepened understanding of the nature and relationships of mathematics to bring to their own continuing study of mathematics, and we wish to affirm the importance of mathematics teachers continuing to study mathematics themselves. Let there be no misunderstanding. We reject the argument that in-service education should focus only on methods, on the mathematics the teacher actually teaches, or on something for Monday morning. All of these are significant and worthy of consideration, but they cannot be allowed to displace mathematics entirely.

The in-service period also is a good time to reexamine old topics from a new vantage point. After some experience teaching about functions, limits, quadratic equations, trigonometric relations, the pythagorean theorem, etc., teachers benefit from a new encounter with some of the many ramifications of these ideas. Geometry teachers of considerable experience, for example, have been surprised to discover that the pythagorean theorem need not be restricted to squares constructed on the sides of the right triangle, but that it can be generalized to any regular polygons or even to any similar and similarly placed polygons on the three sides.

Finally, we should not lose sight of the importance of modeling teaching in the in-service program. Undergraduate students probably have more opportunities to observe teaching than do practicing teachers. Jackson observed that:

> . . . teaching is such an engrossing activity that many teachers do not have time to look at their own practice and that of their colleagues with a critical eye. Moreover, even when they are given the opportunity, many of them do not know what to look for. In other words, they lack a critical stance from which to examine the process of teaching in general and their own work in particular. (Jackson, 1971; 29)

Persons responsible for in-service education should take this as a serious challenge, for it is closely related to the goal of developing higher-level competencies.

Elsewhere we have suggested strategies for helping teachers focus their attention through guided observation in classrooms or via video tapes. Those same strategies can be used by practicing teachers who should be encouraged to video-tape and analyze their own classes as well as the demonstration lessons taught by others. Jackson's further comments support this position:

> Closely related to the problem of professional commitment is that of encouraging teachers to become critical of their own actions. Here the needs are both technical and conceptual. We must help teachers see themselves and others at work -- through videotapings, observational schedules, more frequent visits to neighboring classrooms -- but we must also help provide a critical perspective from which to examine the process of teaching. (Jackson, 1971; p. 30)

If we begin early in the teacher education program to develop this critical perspective, the task will not overwhelm the in-service teacher. And the teacher who develops the capacity and the disposition to honestly evaluate his/her own teaching is the teacher most likely to benefit his/her own pupils.

Stepping Back

We have contended that teachers should become evaluators of their own teaching. So, too, should teacher educators become evaluators of teacher education. In Chapter One we cautioned against assuming an engineering type research stance in evaluating teacher education programs. But we believe that the taxonomy can provide a framework to step back and to assess teacher education.

Many questions about teaching are unanswered. We suggest a few here:

> What competencies should mathematics teachers have at different stages of professonal development? Do teacher education students develop the knowledge and skills we desire? What behaviors do practicing mathematics teachers actually exhibit? Are they compatible with the set of desired competencies? How do they change over time?
>
> What are teacher education students' views of mathematics? of mathematics teaching? Do those views change as the students participate in various experiences? In what way? Do they change during the first year or two of teaching? In what way? Do the teachers' beliefs about mathematics affect their teaching behaviors?
>
> Do the teachers who complete the program increase their understanding of concepts? their understanding and skill at problem-solving? Do they use more and varied teaching strategies? more applications, problems, models, or materials? Do they vary their teaching strategies according to pupil characteristics or curricular content? Do their pupils improve their understanding of mathematics and their problem-solving skill?

Each question suggests many more!

Chapter Six

Designing a Program

It is one thing to recommend (or even to mandate) that one adopt a systematic approach to teacher education through pre-specified objectives, it is quite another thing to determine what those objectives should be. It is one thing to expect that enabling activities be related to objectives and that evaluation reflect demonstrated competence; it is quite another thing to establish how these are to be achieved. Obviously, the responsibilities of program planners in decision-making are enormous.

While programs may share many common characteristics, each, in the end, is unique. Needs, personnel, and situations differ, and it is not possible either to develop the definitive program or to transplant a program unchanged from one setting to another. What can be done, though, is to outline major steps to be taken, important questions to be asked and answered, significant decisions to be made. These, in turn, can be enhanced if program developers review the experiences of others undergoing similar development activities (see, for example, Giles and Foster, 1972). Below is an overview of the development process.

1. <u>Clarification of the task</u>

   Review and clarify the program goals and needs.

   Identify all who will be affected by the program changes and involve them from the beginning in planning and development.

   Establish a rationale to guide program development. For example, will the program be limited to what can presently be demonstrated and validated? Will competencies be derived from conceptions of what teachers should do or from what they actually are observed to do?

   Establish a time line for program development.

2. <u>Identification of the competencies</u>

   Determine the approach or combination or approaches to competency identification. Representative approaches include the following:

   a. Identify the desired pupil outcomes and from these infer requisite teacher knowledge, behavior, and attitudes.

   b. Examine existing courses in teacher education and translate their course content into competency statements.

c. Derive competencies from the consensus of practitioners, teacher educators, etc., and from the recommendations in the professional literature.

d. Observe teachers recognized as effective and record their behaviors.

e. Obtain competencies by theoretical generation derived from models of teaching such as:
-- teaching as interaction
-- teaching as behavior modification
-- teaching as information processing
-- teaching as concept formation
-- teaching as problem-solving

Generate competencies.

Review competencies to determine

a. if all levels of the taxonomy are appropriately represented;

b. if the competencies are consistent with known research, psychological theories, etc.;

c. if the competencies are appropriate
-- at various levels of professional development,
-- for desired pupil outcomes, or.
-- for preparing the teacher for various grade levels and school situations;

d. if the competencies are conistent with the recommendations of relevant professional gorups;

e. if individual competencies bear the desired relationship to the total curriculum;

f. if the set of competencies is internally consistent; and

g. if any essential competencies have been omitted.

Determine entry level competencies.

Specify competencies expected at various stages of professional development.

Delineate requried competencies, recommended competencies, optional competencies. Relate optional competencies to specific situations for which they are important.

Make the competencies public.

3. Establishment of assessment criteria

From the competencies, determine the expected level, variety, and frequency of criterion behaviors.

Where possible, determine the research base and the relationship of criteria to teacher effectiveness and pupil learning.

4. Development of enabling activities

Derive proposed activities from stated competencies and objectives.

Review proposed activities

a. to ensure activities in all four areas: formal instruction, laboratory, practicum, individual study;

b. to assess the adequacy of the variety of alternative activities in accommodating the different learning styles of the students; and

c. to ensure the existence of enabling activities for all identified competencies;

Determine materials, resources, and protocols to be located, obtained, or developed.

Determine the appropriate sequencing of the enabling activities.

Determine the cost of the instructional program including resources, space, and personnel needs.

5. Establishment of assessment procedures

Determine the assessment procedures appropriate for each assessment criterion.

Identify existing assessment instruments.

Develop needed assessment instruments.

Identify needed evaluation personnel and clarify the skills required for each.

Set criteria for the frequency, level, and variety of expected assessments.

Develop feedback mechanisms to communicate assessment results to students.

Set grading criteria and/or develop grading alternatives.

Review proposed assessment techniques for adequacy, appropriateness, consistency with objectives, effectiveness, usefulness for feedback, usefulness for decision-making.

Outline decisions to be made as a result of various assessments.

Determine the research base for proposed assessments.

Determine areas of the needed study, data collection, etc.

6. Determination of personnel needs

Identify needed personnel for all aspects of the program.

Clarify the roles of all personnel.

Determine staff development needs.

Plan and implement staff development program.

Identify resource persons outside of the program.

Hire additional personnel as needed.

7. Preparation for implementation and operation

Identify administrative concerns, including admission criteria, record keeping, space utilization, resource management, facilities renovation and maintenance, credit allocation and reporting, supportive services, funding sources.

Develop plans for each area of administrative concern.

Set timeline for program implementation, including subgoals and strategies for phasing in various components.

Determine special needs or difficulties.

Determine conditions for on-going operation and strategies to ensure continuation of the program.

8. Planning for program evaluation

Establish mechanism for on-going program evaluation that focuses on

a. the adequacy and appropriateness of the identified competencies and objectives, including review of their specificity, comprehensiveness, clarity, significance, relation to pupil learning, relation to assessment criteria, scope, level, research base;

b.. the effectiveness of the enabling activities, including their adequacy, variety, sequencing, relation to objectives, relation to assessment criteria, management, accommodation of varied learning styles, research base, cost effectiveness;

c. the personnel, including the extent to which they model the program objectives; their supportiveness, availability, flexibility, adaptability, unity, goal directedness; the adequacy of staff development activities;

d. program management, including its efficiency, effectiveness, goal directedness, cost effectiveness; and

e. student assessment procedures, including their relationship to objectives and criteria; research base; adequacy in type, frequency, scope, instrumentation; usefulness for corrective feedback; usefulness for decision-making.

Program developers must keep in mind that planning and implementation are incremental operations. All planning efforts have two simultaneous aspects: a vision of the desired outcomes and a set of enroute strategies that are to the program as the enabling activities are to the competencies. Developers must guard against adopting an all-or-nothing attitude that either the entire program must be implemented for all students at one time, or none of it can be operationalized. Often only certain aspects of the program can be initiated at a given time, or perhaps only certain students can be accepted into the program. This is an acceptable alternative as long as all concerned recognize the limited nature of the situation and do not delude themselves or others into believing that if they have implemented part of the program the goal has been achieved. Also essential to the success of the program is on-going communication with and involvement of all persons who will be involved in or affected by the program. Their "ownership" of its goals and processes is crucial to its survival.

Chapter Seven

Experiences and Critique

The model proposed here points to a thorough curriculum review and revision. It makes no pretense at being an educational panacea. It is admittedly speculative and idealistic, and it has not been fully implemented or tested under fire. It does, however, offer an exciting challenge, an opportunity for a clearer specification of goals, a call for a reexamination of curriculum practices and program components in the light of learning principles, an opportunity to refine learning experiences so that they may be made more effective in achieving goals, and a challenge to develop more precise assessment procedures both for evaluation and for diagnosis. All of these are exciting and challenging, not as exercises in their own right but because of their potential for developing more effective teachers of mathematics, both before and throughout their professional careers.

Experiences with the Model

Aspects of this teacher education model that have been implemented to date have been primarily focused on the pre-certification level. In particular, the undergraduate program for secondary school mathematics education majors at the University of Minnesota is built on this model. Consideration of that program and feedback from its graduates and cooperating teachers gives insight into certain strengths and weaknesses of the approach.

Program description

Students seeking to teach secondary (grades 7-12) mathematics enter the mathematics education program of the College of Education as juniors after completing two years in the College of Liberal Arts or the equivalent. Prior to admission to mathematics education, they are expected to complete 25 quarter credits of mathematics and must complete at least one full year of calculus. In addition to the liberal education credits required of all students, the mathematics education major includes the following components:

(a) A minimum of 48 quarter credits of mathematics at the level of calculus and above, including four quarters of analysis through multivariate calculus, linear algebra, and differential equations (the standard freshman/sophomore sequence for mathematics majors). Also required are three upper division courses in Fundamentals of Analysis/Algebra/Geometry, one course in probability or statistics, and one course in computer studies.

(b) General foundations courses in education, including general psychology, introduction to secondary school teaching, educational psychology, the school in society, human relations, teaching exceptional children, and drug education (total 28 credits).

(c) Mathematics education courses consisting of two quarters of methods, at least three quarters of pre-student teaching practicum, and student teaching. Three additional courses are recommended electives and may become requirements; these are teaching arithmetic/algebra/geometry and they are normally taken concurrently with the foundations of analysis/algebra/geometry courses of the mathematics major.

Mathematics education majors also complete a second major, a minor, or a supporting field in an area other than mathematics, and they may elect to take certain offerings from the mathematics education graduate program.

The program described above evolves from the belief that the role of the mathematics teacher is to provide pupils having different levels of interest and ability in mathematics with the opportunity

(a) to learn the basic concepts, principles and processes of mathematics necessary for effective participation as an adult in a technological society;

(b) to develop the problem-solving skills needed to solve new and unfamiliar problems;

(c) to foster positive attitudes and self-concepts about the importance of mathematics for society and for the individual and about each one's ability to learn mathematics; and

(d) to prepare adequately in high school so that pupils will not be eliminated from future educational and/or career opportunities because of inadequate background in mathematics.

Thus, the teacher education program seeks to prepare teachers to fulfill the above role by meeting the following broad goals:

1. To have knowledge of mathematics sufficient to enable the student to pursue advanced study in mathematics during the undergraduate program or during later continuing education.

2. To engage in advanced study of formal mathematics in the three basic areas of analysis, algebra, and geometry that emphasizes understanding, proving, and communicating mathematical ideas, including some responsibility for independent learning.

3. To develop fundamental ideas and skills in areas of special significance in the present and future, including probability, statistics, and computer literacy.

4. To have knowledge of several learning theories and their implications for teaching mathematics, and to be able to apply those theories in the planning, implementation, and evaluation of instruction.

5. To have knowledge of the organization and operation of secondary schools, with emphasis on facilitating a classroom environment that fosters learning, open communication, and acceptance of each pupil.

6. To be aware and accepting of different needs, goals, and experiences of individuals, and to be able to diagnose pupil needs and to plan instructional activities to meet those needs.

7. To have knowledge of the goals, content, instructional materials, and teaching methods of school mathematics, and to be able to plan, implement, and evaluate instruction and to make instructional decisions based on knowledge of the pupils, the subject matter, and the learning environment.

8. To develop effective interpersonal relations and communication skills, and to recognize the significance of affective goals in the mathematics classroom.

9. To have a basic liberal education, including courses in writing and in the arts, the humanities, and the social and natural sciences.

10. To pursue study in an area other than mathematics.

During the first year of the program, juniors in the introductory education and educational psychology courses, both of which enroll students from all teaching fields, also enroll for concurrent practica in mathematics education. For those two quarters they are assigned by the mathematics education advisor to one quarter each in selected junior and senior high schools which they attend at least once a week, and they meet with the mathematics education supervisor for a weekly seminar. In the seminars, mathematics students from across the sections of the general education courses translate the content of those courses into specific applications for the mathematics teacher, and they relate these topics to their field experiences and observations. Also, in the school practicum they have the opportunity to move from observation to limited performance for classroom routines to instructional experiences, first with one or a few pupils and later with whole classes for short periods of time. They also are helped to assess their personal choice of teaching as a career. In addition, juniors are ordinarily enrolled in the fundamentals of mathematics sequence and the corresponding teaching methods courses.

The senior-year sequence focuses on methodology and content. In the first methods course, emphasis is on designing, delivering, and evaluating instruction. The standard methods topics of writing objectives; teaching concepts, generalizations, problem solving, etc.; planning lessons; designing and evaluating tests; and more are integrated through course projects to develop units of instruction. Since the students will do student teaching later in the year, they are assigned to another quarter of practicum in the school where they will teach. Thus, for their methods assignments they select topics that they actually will teach, and they design their units of instruction for that purpose. This series of activities is an iterative and interactive process in which the students receive feedback on their efforts and redo them as needed until the final units are completed and acceptable. Units contain all lesson plans, pre- and posttests and quizzes, worksheets, overhead transparencies, and support materials. Also included in this course are biweekly peer-teaching sessions during which the students teach parts of their units, receive written and oral feedback from peers and instructor, and critique videotapes of their own performances. Throughout these sessions the emphasis is on why and how: Why did you use that approach? that example? that model? Why did you respond as you did to the pupil's question? How could you do it differently? What would you expect to be the result of an alternative approach? Later, during student teaching, the students are again observed while teaching parts of these units. They also make the units available as resources for their classmates.

A second methods course provides the opportunity to survey the curriculum with emphasis on major concepts and principles; models, methods and materials for teaching these; difficulties that pupils and teachers are likely to encounter; and ideas for remediation and expansion During this quarter the school practicum is optional, but in order to maintain contact with their student teaching sites, most of the students elect to continue the practicum. The program culminates in student teaching under one of two options: full days for one quarter or half days for two quarters. Student teaching also includes a weekly seminar with the university supervisor.

Program emphasis

In those program components directly under control of the mathematics education faculty (e.g., practica, methods courses, seminars, and student teaching), special emphasis is placed on the following five competency areas:

1. Knowledge about mathematics as described in Chapter One. Much attention is given to understanding the fundamental concepts and principles of school mathematics and to the ability to explain and/or demonstrate them in precise, concise, and varied ways, including the use of concrete manipulatives, models, and formal, symbolic representation.

2. Planning for instruction. Previous experience with student teachers indicated a tendency toward inadequate planning on the assumption that they could create examples and questions as they taught. To counter this, the program stresses and requires careful planning of lessons, units, and activities. Students are expected to include in their written plans the sequences of questions they will ask, examples they will give, and alternatives to consider in response to anticipated pupil errors or difficulties. The belief here is that for the beginning teacher, in particular, it is far better to over-plan than to under-plan.

3. Instructional decisions. Closely related to the concern for adequate planning is the focus on instructional decision-making. Throughout the program, students are expected to seek and try a variety of methods and instructional materials for presenting mathematical topics. They also are expected to weigh the expected outcomes of each alternative and to explain how and why they would respond to various instructional situations.

4. Classroom interaction and communication. Attention to inter-personal communication between teacher and pupils, pupil and pupil, teacher and parents, teacher and colleagues, etc., is considered an important part of the teacher preparation program. In particular, the teacher must take account of known learning theories, psychological principles, affective goals, and learner needs in order to establish an effective learning environment in the classroom.

5. Classroom management. Techniques for effective and efficient classroom management are given attention, and students are encouraged to experiment with various approaches during their student teaching.

The major vehicles for attending to the above are through formal instruction, seminar discussions, video tapes of secondary school pupils and teachers, peer teaching laboratories, and school practica and student teaching.

Evaluative Feedback

Fifteen students who were among the first to complete the program described above were assigned to student teach in seven senior high schools and three junior high schools, Each worked with one or more cooperating teachers who held a permanent licenses in secondary school mathematics and who had been identified as effective mathematics teachers. Almost all of the cooperating teachers had more than ten years of teaching experience, and most had previously worked with student teachers from a variety of colleges and universities.

The cooperating teachers were not informed of any changes in emphasis or content in the teacher education program. At the conclusion of the quarter each cooperating teacher was asked to write an open-ended narrative evaluation of his or her student teacher. While these narratives did not exhibit a common format, they were examined for common themes that pertained to the student teacher's teaching skill and classroom behavior. These are summarized below. Comments about personal traits, interests, or accomplishments of individuals are not included.

All fifteen student teachers were commended for their careful planning and for their use of a variety of instructional techniques. More than half were described as particularly skilled in the use of inquiry methods. Among the cooperating teachers' observations were the following: "I found his ideas on developing a lesson from the initial motivating ideas to the evaluation were thoughtful and creative." "Her planning was always thoughtful; she anticipated and prepared for many problems that students have in understanding complex concepts." "I have never seen a more efficient and effective planner and organizer for a new teacher." One experienced cooperating teacher commented, "I often observed her classes to obtain new ideas for me to use the next year"; and another observed that, "He is one of the first student teachers I've had who has been able to work through problem-solving techniques with a class."

Fourteen of the fifteen were described as having established outstanding rapport with their pupils and with their supervisors and fellow teachers. They also were commended for their effective classroom management skills, and thirteen were also cited for their flexibility and skill in adapting to unexpected situations. Cooperating teachers noted that, "During class presentations he changed his delivery, depending on class participation, to bring about the best learning," and "She has had excellent training . . . and was willing to share her own ideas with me."

Fourteen of the student teachers also were described as enthusiastic, energetic, and/or dedicated, and they were further commended for their self-confidence and poise. Typical comments were: "I found that they (pupils) would go to him many times before coming to me; they were that confident in what he could do for them." "After just a few days, the class functioned as if he had been there all year and the students enjoyed being in his class." "She was so poised that it was at times hard to remember that she was a student teacher rather than an experienced staff member."

Other frequently mentioned qualities were the student teachers' understanding of the subject matter and their ability to explain material to their pupils; the pupils' reported enjoyment of the student teachers' classes; the pupils' success as measured on achievement tests; the student teachers' abilities to communicate with parents, administrators, and other staff; and the student teachers' initiative in seeking evaluations and suggestions from the cooperating teachers.

While there are not similar data with which to compare students completing this program with those of previous years, the strong themes in the cooperating teachers' evaluations indicate that the major program objectives are being met to a degree that is observable in the daily classroom behavior of the student teachers. They suggest that pre-service teachers can develop competencies at the higher level of the teaching taxonomy. It remains to be determined the extent to which these competencies continue to characterize the teaching of these individuals as they enter the profession.

Strengths and Weaknesses

Graduates of the program attributed much of their success to the following factors, which they identified as the principal strengths of their preparatory experiences:

practicum seminars early in the program that facilitated a sense of support, cooperation, and personal friendship among the students;

a variety of field experiences in different types of schools that provided opportunities to observe and to try alternative teaching strategies and classroom management techniques;

regular seminars that helped students to interpret and to focus on the implications of those school practica;

integration of methods activities with the projected student teaching assignment in order both to make those learning tasks more relevant and to afford the chance to implement plans and ideas in their own classes;

laboratory experiences with peer teaching and with video tapes of experienced master teachers as an opportunity not only to observe teaching but also to develop a rationale for different instructional strategies;

emphasis on understanding mathematical concepts, principles, and problem-solving processes, and stress on how to present these meaningfully to secondary school pupils of various ages, interests, and abilities; and

emphasis on examining the rationale for various instructional and managerial decisions that teachers must make, and on reflecting on their own values, goals, and motives for teaching.

Principal concerns expressed by the graduates focused mainly on the perceived differences between their goals and beliefs and those of in-service teachers they observed. Frequently they encountered opposition or, at the least, indifference from classroom teachers who either did not support their attempts to implement various instructional strategies or who actively opposed their efforts. To counter the resulting frustrations demands time and active attention to providing adequate feedback and reinforcement. It also requires the careful selection of schools and cooperating teachers for field experiences, and it argues for the importance of extending the teacher education process to in-service as well as pre-service teachers.

Conclusion

Early experiences with the teacher education model outlined in this paper have been encouraging, and they lend preliminary support to the usefulness of the taxonomy in delineating specific teacher performance outcomes that include both knowledge and instructional behavior. Additional implementation and evaluation of the model will continue to contribute to the goal, stated at the outset, of delineating the nature of the content of teacher education programs, the nature of delivery systems, and the range of expected outcomes at various stages along the continuum of professional development.

APPENDIX

Focus on Observation

Because teaching is a complex activity, the observation of teaching is likewise complex. Classroom observation, at any stage in a teacher's development, is aided by observation guides that help the teacher focus on salient aspects of the teaching process, the learning environment, or the school organization. Whether observing another teacher or a tape of one's own teaching, the observer should have definite objectives for the observation.

The leading questions given in this appendix constitute a lengthy list of examples of foci for observations. Specific observational tasks can be formulated by selecting appropriate items from the list. As always, the items are presented here as examples to stimulate ideas on the scope of possible observational tasks. The lists are not comprehensive, and users are urged to modify, to substitute, and to add at will.

On the classroom environment:

Sketch the physical arrangement of the classroom. Include all furniture and equipment. How does the classroom arrangement contribute to or distract from learning?

Make a list of the learning materials and equipment in the classroom. Indicate with a (P) those items readily available for pupil use. Indicate with a (T) those items used only by the teacher. Indicate with an (X) those items used by pupils only with the teacher's permission. Which materials, if any, are kept locked?

Is the classroom comfortable? Adequately lighted? Adequately ventilated? Too hot or too cold?

Are the bulletin boards decorated? Sketch or describe them. Do the boards seem to be related to the mathematics that is taught in the room?

Is student work displayed in the classroom? How? For what purpose?

What features of the cla: oom make it a pleasant (or an unpleasant) place to be?

What is the condition of the pupils' desks? The wall, floor, window coverings, chalk boards?

If you were unfamiliar with the school, what features of the classroom would tell you that this is a mathematics room?

On the instructional process:

Describe the pupils' behavior as they enter the classroom. Note what they talk about. What indicators do you observe that give you some insight into how the students feel about being in math class?

Is there a bell or other official signal that class has begun? If not, how did the teacher signal that the class was beginning? How did the teacher begin the class?

How long did it take the pupils to settle down after the teachers called the class to order? What did they do in the meantime? How many times did the teacher have to call for attention or order before class could begin?

How did the teacher motivate the new lesson? Did the teacher appeal to any of the following: the significance of the topic for the pupils? the importance of the topic for future mathematics? applications of the topic? past experiences or learning of the pupils? the curiosity or interests of the pupils?

Does the teacher seem to be enthusiastic about the topic? How is the teacher's attitude manifested?

What pupil behaviors give you some hint about whether they really were motivated to learn the new material or not?

Drawing on your own knowledge of mathematics and on any resources you care to use, suggest at least three other ideas for motivating the same topic. Which of the options (including the teacher's) do you prefer for this topic? Why?

How did the teacher bring the lesson to an end? When did this happen? When did it appear the pupils had "tuned out" on the lesson? Had the lesson ended at that point?

Did the teacher have to take special measures to try to keep the pupils' interest and attention? If so, what?

Was there any specific aspect of the lesson that might account for the pupils' "tuning out" early?

How did the teacher make the homework assignment? When was the assignment given? What did the pupils do while the teacher explained the homework? Was the assignment clear? If not, what was unclear about it?

Outline the entire lesson as it develops during the class. After class write what you believe were the teacher's objectives, based on the lesson you observed. After you have written your list of objectives, ask the teacher to see his/her lesson plan. Do the objectives you wrote coincide with the teacher's? Did the lesson as you outlined it follow the lesson plan? Did anything happen during the class (for example, an unexpected pupil answer or question) that might account for the teacher's digression from the plan?

Discuss the lesson with the teacher. In particular, discuss any deviations from the lesson plan and find out why the teacher thought them appropriate.

Does the teacher think the objectives were accomplished? On what did he/she base that conclusion? If they were not accomplished, to what does the teacher attribute this?

Observe lessons in each of the following areas: concepts, generalizations, skills. In each case, list all the moves the teacher used. (See Cooney, Davis, and Henderson, 1975, for a discussion of moves.) How many times did the teacher use each move? Were there any moves not used at all in the lessons? Can you think of a reason why this might be?

Were there any times when the teachers asked for one thing but accepted something else (for instance, asked for a definition but accepted an example)? Do you think the teacher had a reason for this?

Who gave most of the examples, the teacher or the pupils?

If pupils made false generalizations, did the teacher respond with counter-examples? Give some instances.

How would you determine whether the pupils had understood the concept or generalization or had mastered the skill?

How much time was spent teaching the concept or generalization before the class moved to algorithms for applying the concepts?

Where in the lesson was the concept defined or the generalization asserted? Would you classify the lesson as discovery or expository?

How often does the teacher pose problems for the pupils? How can you be sure these really are problems and not exercises?

Describe a problem-solving lesson that you observed. State the problem. List the heuristics that the pupils used. Which ones did the pupils initiate? Which ones did the teacher suggest?

How does the teacher encourage problem-solving? Is the teacher's behavior different during problem-solving lessons compared to other lessons you have observed with the same teacher?

How many of the pupils were able to solve the problem? Did any solve it only partially? Did any go beyond the original problem? Did the teacher encourage multiple solutions? original approaches? How was that encouragement given?

Were the pupils' behaviors, responses, and attitudes noticeably different compared to other days in the same class? If so, in what way? To what do you attribute this?

What do you think were the teacher's objectives in presenting the problem? What does the teacher say were the objectives? Were those objectives met? How do you know? If they were not met, why not?

Over time, is there evidence that the pupils are becoming better problem-solvers?

What examples of "real-world" applications has the teacher used? How were these introduced? What purpose were they intended to serve?

Were the pupils more interested, attentive, or curious about the applications? On what behavioral evidence do you base your judgment?

How did the applications or examples make the lesson more understandable?

What learning aids are available to the mathematics teacher? List all the mathematics teaching aids. In particular, check for

calculators
computer or microcomputer

geoboards
scales
rulers
protractors
compasses
scissors
MIRAs
surveying tools
geometric models
special transparencies; e.g., for graphs
graph paper
blackboard coordinate grid
multibase materials
abacus
cards
dice
games
other (describe).

Indicate the following for each material available:

(a) there are sufficient for each pupil to have his/her own set of materials
(b) there are sufficient for pupils if they work in two's or three's
(c) there are only one or two available for demonstration
(d) the materials are kept in the classroom
(e) the materials are in a central resource center
(f) the materials are locked up or put away somewhere

In each case, what alternatives were available to the teacher? Give a probable reason for choosing the one the teacher chose. What was the consequence of that choice? What are the likely consequences of the other alternatives? Which alternative would you choose? Why?

Did the teacher "go off on a tangent" at any time? Describe what happened. What brought about the tangent? Was the tangent purposeful? Was it time well spent? Why do you think that it was/was not?

Were there any "suprises," such as an unanticipated question or a pupil's misunderstanding, that made it necessary to adjust the lesson? Describe what happened.

How many times during the class period did a student initiate communication to ask for directions or clarification of directions? to ask a question about content? to volunteer an idea, opinion, or answer? to answer a question put by the teacher either to the pupil directly or to the class as a whole? to direct a question or comment to another pupil?

How often do the students work in small groups (three or more)? with one other pupil? alone at their desks? at the chalkboard? at a lab station using materials or manipulatives? in a resource center or library? alone at a computer terminal?

During the class period, how many pupils participated four or more times? two or three times? only once? not at all?.

What proportion of the teacher's questions require a yes/no answer? a fact or other specific information already learned or memorized? a numerical value for a particular computation? an original expression of a concept? a new conclusion from the pupil?

How many questions are not really questions at all but statements followed by a question mark? Give some examples of these. What purpose do they serve?

How long, on the average, does the teacher wait for the pupil's answer? How many questions actually get answered by the pupils? How many does the teacher answer or rephrase or simplify? How many go unanswered?

Are any of the questions so "loaded" that it would be impossible to give the wrong answer? Give examples.

How does the teacher encourage the participation of all the pupils? Does that encouragement lead the pupils to initiate questions or comments or merely to respond to the teacher?

Does the teacher insist on any particular language or terminology? When does this happen?

What pupil behaviors do you observe when the pupils don't know or aren't sure of the answer to a question?

Does the teacher appear to direct the instruction to particular pupils or to certain parts of the room? Describe this. Is it a deliberate choice on the teacher's part? If so, for what reason? If it is not consciously intended, what is it that seems to draw the teacher's attention?

Which pupils do you notice most? Why is this? Which ones do you tend to ignore? Why? Which pupils seem to lead or to dominate in the class? In what way?

Do you observe differences in the pattern of teacher-pupil communication with certain pupils as compared to others? Describe.

What verbal or non-verbal means does the teacher use to control the class? Describe them.

How many minutes were spent in each of the following activities: correcting homework? reviewing previous materials? presenting new materials? supervising while the students work on assignments? giving directions or conducting management tasks? other (describe)?

How does the teacher manage the following classroom routines: taking attendance? collecting homework or tests? returning papers to students? other (describe)?

What procedures or rules are observed for: turning in homework or other papers? making up work missed due to absence? making up unexcused missing assignments? moving about the room to sharpen pencils, etc.?

Does the teacher assign pupils to their seats? How? Does the teacher assign pupils to small groups? How?

What are the rules about students talking to one another during class? Are students required to raise their hands for permission to speak?

What are the procedures when pupils forget to bring their books? paper? pencils?

Are students allowed to leave the room to go to their lockers, the bathroom, etc.?

When students are working on homework or tests or other activities, what does the teacher do?

How does the teacher handle the correction of homework? Do the pupils receive grades for homework?

What interruptions of the lesson occurred? Could any have been avoided? What was the effect of each interruption? How did the teacher handle these?

How does the teacher provide for periodic review and maintenance of skills? Is this accomplished through class activities? homework? quizzes or tests? other?

When a pupil gives an incorrect answer in class, how does the teacher respond? Does the teacher correct the pupil? help the pupil correct his own errors? embarrass the pupil?

Are the teacher's corrections directed at individual pupils or at the whole class? Is the effect on the class different in these two cases? How?

Correct a set of homework papers or tests. Analyze the kinds of errors the pupils make. Try to describe any patterns in these errors. Use those patterns to infer any possible misunderstandings on the pupil's part that would account for the errors. Suggest remedial activities to help correct the problems.

How does the teacher show positive reinforcement to the pupils? Is this given individually or to the whole class? Is positive reinforcement given only for good performance (a high test score, a good homework paper, etc.)? What other occasions resulted in positive reinformcement? What effect does the teacher's attention or approval seem to have?

What provisions are made in the school and in this class for slow learners? for gifted students? for handicapped stduents? Do these students receive special instruction? special schedules? special curricula? special learning materials? If so, describe. How does the classroom teacher have to modify his/her teaching to accommodate pupils with special needs?

On pupils:

Stand in the corridor between classes, in the cafeteria line, in the school bus line, student commons, or other place where students gather. What do they talk about? What things seem to annoy them? What things seem important to them? How much of the talk is about school? about particular subjects? about particular teachers?

Talk to students directly and try to determine what classes they like or dislike and why. What do they like or dislike about math class? Do they think mathematics is important, interesting, and/or "poor teacher"? What things about school do they like and want to keep? What things about school would they like to change and how? What goals do they have for themselves?

On teachers:

Talk to a beginning mathematics teacher. Try to determine what aspects of the first year were most surprising, most satisfying, most frustrating, most disappointing. For what was the teacher best prepared? least prepared? What advice does the teacher have for you?

Interview a student teacher just before student teaching. Determine what the student teacher's expectations are for student teaching. What are the apprehensions? Interview the same student teacher at the end of the student teaching period. Were those expectations and apprehensions fulfilled? In what ways? In what ways were they not met? What did the student teacher consider to be the most valuable aspect of the experience? What advice does the student teacher have for you?

Interview a teacher with several years experience. How have that teacher's attitudes and views changed over the years? Why did he/she become a teacher? Why is he/she still a teacher today? What are the most positive aspects of teaching? the most frustrating? What advice does the teacher have for you?

On the school:

What duties do teachers perform besides teaching, such as homeroom, cafeteria, study hall, etc.? Are these duties optional for teachers or are they assigned?

What is the purpose of homeroom? What activities take place there? How do the pupils behave during homeroom?

Attend a parents' meeting. Try to determine what parents want for their children in the mathematics program. What do parents like about the mathematics program?

Ask to read a copy of the teachers' contract for the district. What duties are specificed for teachers? What restrictions are specified, such as length of work day, sick leave, personal leave? What else does the contract specify?

If possible, attend a school board meeting when teachers' contracts are being negotiated. What items are included in the negotiations? Who represents the teachers? How are the teachers' representatives selected?

Is there a teachers' union in the district? Is membership required? What services does the organization offer its members?

What extracurricular activities are offered in the school? How many of these are athletic? academic (e.g., honor society)? creative (e.g., band, drama)? social (e.g., pep club)? governmental or service (e.g., student council)? Are the organizations open to all students? If not, who is eligible? How many students participate?

How are extracurriculars funded? Who supervises them? Are teachers required to sponsor an activity?

What are the goals of each organization? What activities do they sponsor? Try to participate in at least one group over time. Do the students relate to the teacher differently in extracurricular activities as compared to classes? In what ways?

List what you see as the advantages and disadvantages of sponsoring a student activity. Talk to teachers who are faculty sponsors. What do they see as the positive and negative aspects of that activity for them?

How are students scheduled for their classes? Which subjects are required and which are elective? Must students have their parents' approval before registering for classes? Can they choose their teachers?

Are students "tracked" in different curricula? Who determines into which track an individual is placed? Can students change from one track to another? How?

How are the teachers' schedules determined? Within the mathematics department, who decides which teacher will teach which classes?

Does the school have a testing program? What tests are given? To whom? What use does the school make of the test results? Are students required to pass certain tests for graduation?

Does the school have standardized grading policies? If so, what are they? How does the mathematics teacher determine student grades?

Are there "rewards" or "recognitions" for scholarship, such as honor rolls, honor society, college scholarships? How are they awarded?

What are the graduation requirements in the school?

Read the school's curriculum guide. What are the basic required courses? What is the range of electives? What mathematics courses are offered? Which are students required to take? How many students take mathematics?

Is there a student handbook? If so, read it and try to find what rules, if any, govern the following: excuses for absence from school and/or class; making up missed assignments; cars and parking; movement to and from lockers during the day; dress; smoking, drinking, drugs; movement in corridors during class periods; use of library, resource center, computer center, etc.; respect for teachers and other pupils; destruction of school property; textbook use and/or rental; field trips; conduct in cafeteria, assemblies, study hall, student commons; work release; participation in extracurriculars; holding school office; other.

Talk to the following persons and try to write a brief job description for each. What is their role in the school? What are their responsibilities? What aspects of their jobs directly affect teachers?

Principal	Computer center head
Assistant principals	School nurse
Guidance counselor	School board member
Secretary	PTA officer
Janitor	Mathematics department head
Librarian	Special education teacher
Audio visual director	

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