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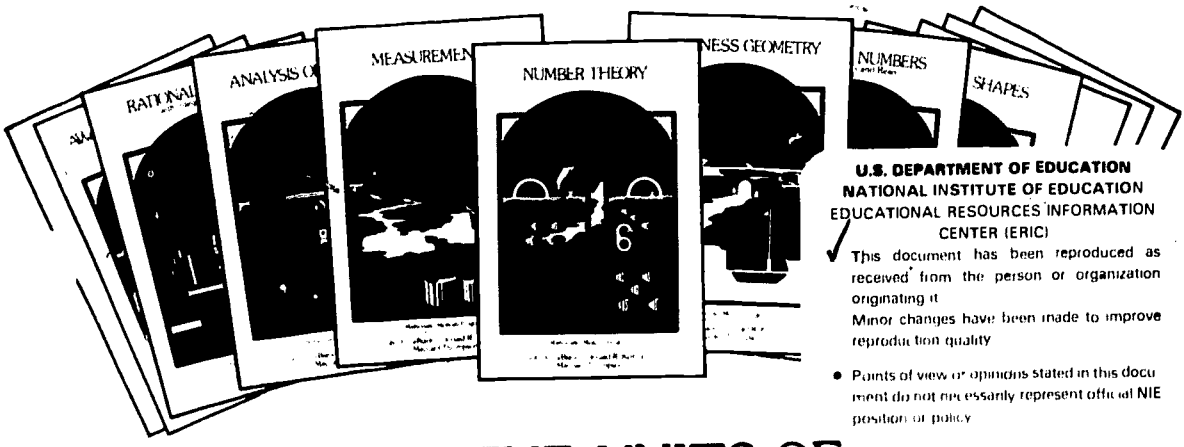
ABSTRACT

This guide is designed for the college instructor who plans to use some of the 12 units of the Mathematics-Methods Program. The program is based on three assumptions about the teaching and learning of mathematics: (1) Mathematics content and methods should be combined in the training of prospective elementary school teachers; (2) Mathematics should be learned in a laboratory setting; and (3) Teachers should be taught as they should teach. These units were written at the Indiana University Mathematics Education Development Center, and they combine the mathematics content and methods learning of college students who are training to be elementary teachers. The units are flexible and can be used in content courses, methods courses, or courses which combine both. In addition to the college classroom component, the program has a coordinated elementary school teaching experience component which is not developed through the units. This guide provides a detailed description of all units and some implementation suggestions. The teaching experience component as implemented at Indiana University is also outlined. (MP)

USER'S GUIDE



FOR THE UNITS OF THE

MATHEMATICS~ METHODS PROGRAM

John F. LeBlanc
Donald R. Kerr, Jr.
Maynard Thompson

MATHEMATICS EDUCATION DEVELOPMENT CENTER

INDIANA UNIVERSITY

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USER'S GUIDE

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MATHEMATICS-METHODS PROGRAM

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INTRODUCTION

Description

This Guide is for the college instructor who plans to use some of the 12 units of the Mathematics-Methods Program. These units were written at the Indiana University Mathematics Education Development Center, and they combine the mathematics content and methods learning of college students who are training to be elementary school teachers. The units are flexible and can be used in content courses, methods courses, or courses which combine content and methods. In addition to the college classroom component which is implemented through the units, the Mathematics-Methods Program has a coordinated elementary school teaching experience for college students. The teaching experience component is not developed in the units. This Guide will provide a detailed description of the units and some suggestions for implementing them. It will also outline the teaching experience component as it has been implemented at Indiana University.

Rationale

The Mathematics-Methods Program is based on three assumptions about the teaching and learning of mathematics.

- Mathematics content and methods should be combined in the training of prospective elementary school teachers. In developing mathematical concepts with children, teachers must interweave mathematical development and pedagogical considerations. Experience suggests that the ability of teachers to do this is not greatly enhanced by separate instruction in mathematics content and methods.
- Mathematics should be learned in a laboratory setting. A laboratory setting is one where mathematics is learned by doing and mathematics is developed from the standpoint of its relationship with the real world. It is only through actually doing mathematics that one gains the insight and confidence that are nec-

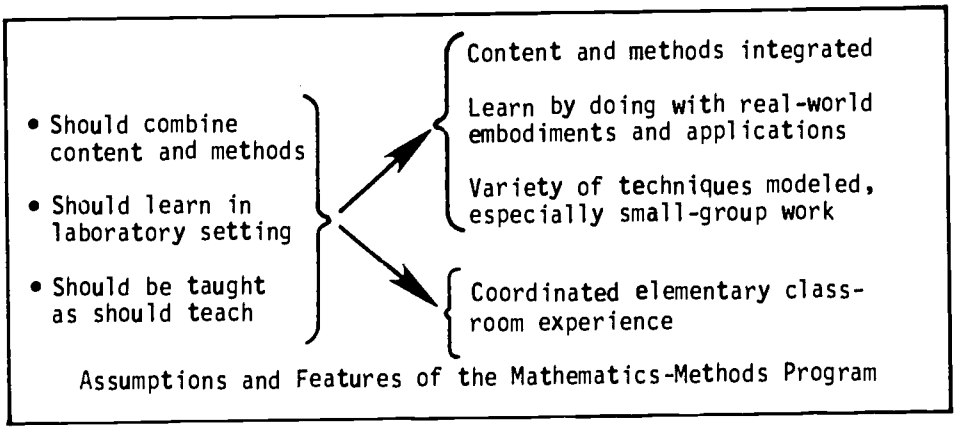
essary to make mathematics useful. While doing mathematics it is important to see the real-world applications of the mathematics being done as well as to experience that mathematics embodied in objects that are real. It should be noted here that for a prospective elementary school teacher the elementary classroom is an important part of the real world.

- Teachers should be taught as they should teach...and the last time most elementary school teachers are taught mathematics is in their undergraduate content and methods courses.

These three assumptions gave rise to the following four features of the Mathematics-Methods Program.

- Mathematics content and methods are integrated. Adult mathematics content is developed along with appropriate instances of that content in the elementary school and pedagogical considerations involved in teaching the content to children.
- The mathematics is developed in a laboratory setting. Mathematical and pedagogical problems are posed to the learner who then has the responsibility to work toward a solution. These problems require work with embodiments of mathematics in real-world objects and applications of mathematics to the real world. The activities frequently focus on the relationship between the mathematics for the adult learner and the mathematics for the child.
- A variety of instructional techniques are modeled for the preservice teachers. These include individual work, small-group work, whole-class seminars, peer instruction, work with learning games, work with concrete materials, outside reading, data gathering, etc. ... lectures are seldom called upon. The small-group instructional format is most frequently used. Experience suggests that this format provides a balance between individual responsibility for actually doing mathematics and the need of inexperienced individuals for help, encouragement and confidence.
- If possible, a coordinated elementary classroom teaching experience is supplied. This work with children emphasizes the

teacher's real world and can be the source of great motivation for the prospective teacher.

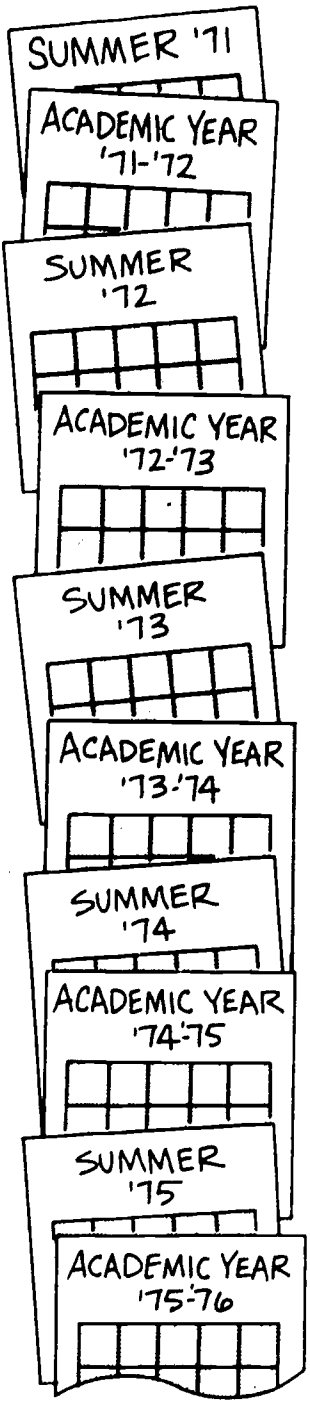


Assumptions and Features of the Mathematics-Methods Program

Development

The Mathematics-Methods Program was developed at the Indiana University Mathematics Education Development Center during the years 1971-76. The development was funded by the UPSTEP program of the National Science Foundation with the goal of producing an innovative program for the mathematics training of prospective elementary school teachers.

It was felt that the curriculum revolution of the 50's and 60's had greatly influenced the quality and quantity of the mathematics being studied by prospective elementary teachers. It was also felt, however, that a balance had not been reached between considerations of mathematics learning and considerations of professional teacher training. It was to this problem that a team of mathematics educators, mathematicians, and graduate students in mathematics and mathematics education addressed themselves in developing the Mathematics-Methods Program. There follows a brief historical outline of the development of the Program.



Conceptualization and exploratory writing.

Further exploration, mini-trials, writing of draft units, groundwork for school experience.

Writing and validation by consultant mathematics educators, writing by faculty-graduate student teams, plan for fall pilot classes.

Two pilot classes at Indiana University using draft units with school experience; many visitors, much unit writing and revision.

Summer workshop for users (pilot centers), continued writing and revision based on experience with pilot classes and suggestions from workshop participants.

Two pilot classes at Indiana University (some formal evaluation), 14 pilot centers, writing, revision based on feedback.

Summer workshop for users, more writing and revision.

Four Mathematics-Methods Program classes at Indiana University (some formal evaluation), 30 pilot centers, start on final revisions for commercial publication of units.

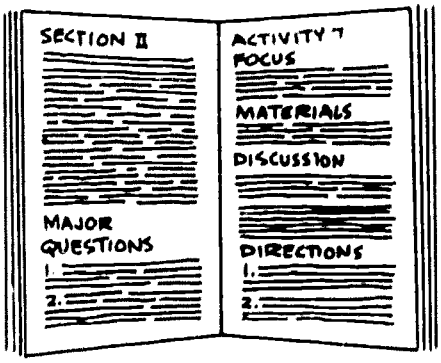
Summer workshop for users, preparation for final unit publication.

Four Mathematics-Methods Program classes at Indiana University, 42 centers using Program, final preparation of units for publication by Addison-Wesley Publishing Company.

COLLEGE CLASSROOM COMPONENT

Nature of the Units

The college classroom component of the Mathematics-Methods Program is implemented using the units. There are 12 units--each focusing on a mathematical topic and on how that topic relates to the elementary school curriculum. Each unit is organized into sections and each section consists of several activities. It is through working with these activities that the prospective elementary teacher learns mathematics content and confronts the problems of teaching that content to children.



Each unit has an introduction which gives a brief summary of the content of the unit and of its role in the elementary curriculum. Each section of a unit begins with a synopsis of the objectives and activities of that section. The activities in each section are also preceded by major questions which cover important concepts in that section. The purpose of each activity is explained in its focus which is followed by a list of materials needed to complete the activity. In addition to the directions in the activity which tell the student what to do, there is an occasional discussion of some topic or issue related to the activity.

The following should be noted about the units.

- Wherever possible they encourage the college student to learn by doing. This is often accomplished by putting the student in a problem-solving situation.
- The college student uses materials that children might use in an approach that is consistent with appropriate child instruction. Sometimes role-playing or peer-teaching is used.
- The activities in the units are sequenced to develop certain mathematical and pedagogical skills and concepts for the college student. This sequencing contrasts with the organization of those mathematics laboratory workbooks which are intended as resources of activities to be selected as needed.

In the development of the units of the Mathematics-Methods Program several techniques or features have evolved which are not typically found in a textbook or in a laboratory manual. Some of these deserve further discussion. Other unit-specific details are discussed in the Instructor's Manual which accompanies each unit.

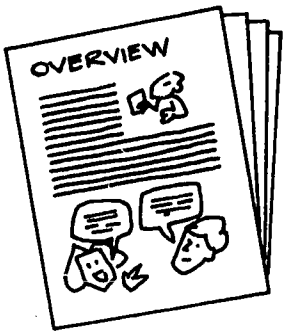
Special Features of the Units

Major Questions

Each section of each unit begins with major questions. These questions often ask the student to synthesize or extend concepts presented in the section.

- Major questions can be read before the activities of the section to serve as advance organizers for the section.
- They can be answered by the student as a written assignment when the section is completed.
- They can be used, possibly in a modified form, as essay questions on a test.
- They can be used as the basis of a summary, whole-class discussion of the section.

Overviews

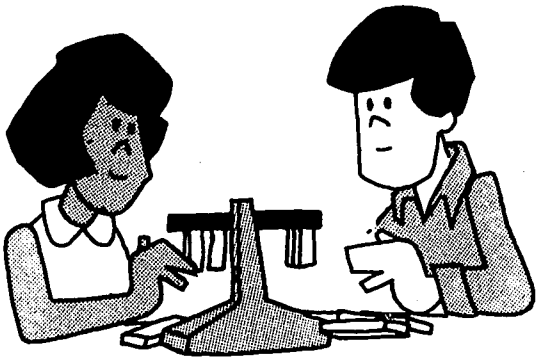


Each unit has an overview. Depending on the content of the unit, the overview

- surveys the occurrence of the content of the unit in the elementary curriculum, outlining the development of concepts throughout the child's elementary school experience,
- introduces the college student to the content of the unit and its role in history and today's society,
- develops in the college student a certain frame of mind toward the content of the unit.

Most overviews consist of an essay to be read (or slide-tape to be viewed if available) and some questions about the essay. One good strategy is to have the students read the questions...read the essay ... and then discuss the questions as a class. Some instructors may find it useful to return to an overview at the end of a unit or even at the end of the course.

Activities Involving Manipulatives



There are many activities which involve the prospective teacher with manipulative materials that may be available in the elementary classroom. For most of these activities there is a dual objective:

- to teach a certain mathematical concept to the college student,
- to give the college student experience, confidence, and insight with materials which can be helpful in teaching children.

The college instructor will need to make these dual objectives clear to the college student in order to avoid both the situation where the student who is familiar with a mathematical concept does not take the manipulative materials seriously and the situation where a student plays with the materials without adequate focus on the concepts which they embody.

Card-Sorting Tasks

One of the many important pedagogical concepts that a preservice teacher needs to confront is sequencing of child instruction. Card-sorting tasks have been developed in order to promote an awareness of the need for sequencing without setting forth a particular rigid sequence.

For a particular card-sorting task, each card represents a step in an instructional sequence for an elementary school mathematics topic. The college student is asked to put the cards in an order which makes sense. Then the students are asked to share, justify, and modify their sequences. The focus is on awareness of and experience with sequencing rather than on learning sequences.

Diagnosis and Remediation Activities

In order to prepare for diagnosing arithmetic learning problems of children and for prescribing remediation, the prospective teacher is presented with partially-completed child pages and is asked to

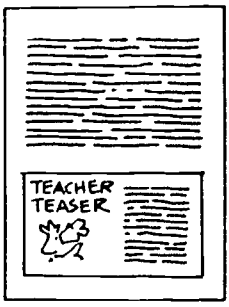
- identify the child's difficulty,
- complete the page as the child would have,
- describe possible causes of the child's difficulty,
- suggest possible procedures to be followed to remediate the difficulty.

Again, the idea is to place the college student in a situation of learning-by-doing through problem solving.

Seminars

In order to promote learning-by-doing, the activities are written in an open-ended, problem-solving format. Students do get involved and often raise interesting and important questions. There is a need for the instructor to bring the class together for the purpose of summary, clarification, and extension. Seminars are built into the units at key spots. Sample seminar questions are supplied as a resource for the instructor.

Teacher Teasers



Problem solving is an important part of learning and applying mathematics, and children can be motivated to solve problems--even (maybe especially) if they are challenging and unusual. In the spirit of teaching the college students as they should teach, problems called Teacher Teasers are interspersed throughout the units. Teacher Teasers are generally related to the content being studied. They can be assigned, encouraged or discussed. They can be for extra credit, for no credit, or they can be required. Most instructors will probably find that some in-class attention needs to be given to Teacher Teasers if many students are going to take them seriously.

Instructor's Manuals

Since the units of the Mathematics-Methods Program differ considerably from traditional college classroom materials, attention has been given to preparing a detailed Instructor's Manual for each unit. These manuals contain

- suggestions for possible routes through the unit, depending on the time available and the objectives of the course,
- suggestions for answers to the major questions for each section,
- suggestions for implementing each activity,
- answers or suggested answers to all questions,
- answers for all Teacher Teasers,
- general statement as to the purpose and spirit of the unit and of specific activities.

Many instructors will find these manuals helpful--especially on the first trip through a unit.

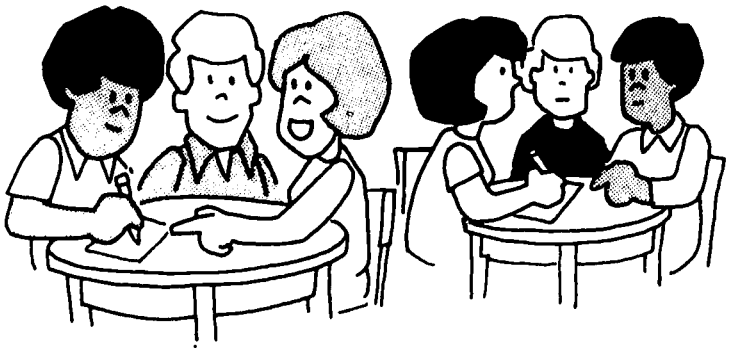
Suggestions for Using Units

While the units of the Mathematics-Methods Program can be implemented in a number of different ways, using different teaching styles and with different overall objectives, certain approaches have proved to be effective in fostering the basic goals of the Program. Some suggestions and tips follow which can be adapted to the particular needs of the instructor.

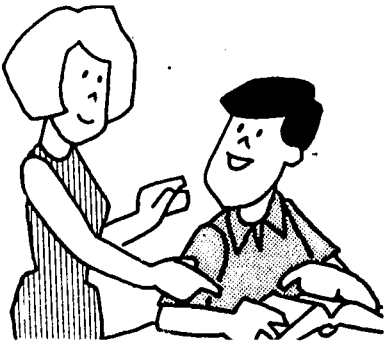
Classroom Organization

The nature of a topic, the style an instructor uses, and the constraints of time all bear on the decision of how to organize for instruction. For certain topics (maybe factual or complex) a well-constructed lecture may be the most effective means of instruction. This is particularly true under the pressure of time. For other topics--especially those where a process is involved--it is most important to foster actual involvement on the part of the learner. In all cases, it is important that the instructor become comfortable with the format employed.

Learning-by-doing is fostered by shifting responsibility (and, therefore, attention) from the instructor to the learner. However, if the learner lacks confidence and experience individual work can be unproductive. In order to foster learner responsibility without placing unreasonable demands on individuals, many activities in the units of the Mathematics-Methods Program have been written with small-group work in mind. Working in groups of three or four has the potential for reducing a student's apprehension and inertia--many prospective teachers fear mathematics and have trouble getting started on problems. Small-group work also has the potential for increasing communication skills and fostering peer instruction. It does raise the problem of identifying and assessing individual performance. (See the section entitled "Record Keeping and Grading.") Some further observations on small-group work follow.



- To promote learning-by-doing through small-group work the instructor must serve as a resource and facilitator--answering questions as needed and directing student work by asking timely and apt questions. This does not, however, preclude calling the entire class together from their groups in order to make a particular point, clarify instructions, or save time.



- Certain activities lend themselves to individual work. Besides, an instructor may want to include a certain amount of individual work in order to promote self-reliance in the college student. One effective way to combine individual, small-group, and whole-class work is to
  1. Assign a problem or investigation to individuals;
  2. Have small groups share and refine their results;
  3. Have the whole class share and discuss the small-group and individual findings.
- Many instructors find it profitable to end a small-group working session with a brief (five-minute) whole-class discussion, synthesizing and reviewing material covered and planning for future work.
- Most classes will require some time to get into the swing of small-group work; it requires interpersonal relationships and communication which are not practiced in many classes. The instructor may want to change group membership from time to time in order to promote more effective work (e.g., put a bright student in a group to help out, take a dominant student out of a group, or separate gossiping friends). If the instructor is likely to want to change group assignments, it is good to make assignments from the beginning (even randomly) so that later changes will not be interpreted as punishment.
- Efficiency in small-group work is enhanced if the classroom is set up with materials ready to go when the students arrive. This avoids wasted time at the beginning of a class period.

Record Keeping and Grading

Since much of the learning from the units is a result of doing rather than reading, it is particularly important for the student to have a record of what was done and what was learned in the various activities. This record, or journal, can be a resource to the prospective teacher for progress check and for grading, and, ultimately, it can be a resource to the inservice teacher for ideas for classroom instruction. The following are possible ingredients for the journal.

- unit pages--they are perforated and hole-punched,
- answers to questions and major questions,
- solutions to problems,
- notes from lectures, seminars and reading assignments,
- ideas, articles, references, games, etc., which might be of use for teaching children,
- a record of any elementary classroom teaching experiences.

Most instructors have found that the students require some guidance in preparing journals. Students need to learn what kind of record keeping is expected of them. It is good to collect student journals early in the grading period in order to provide feedback and direction.

Grading an activity-oriented course raises some particular problems. Possible resources for graders include:

- the journal--its thoroughness, accuracy and depth,
- quizzes--these are particularly useful when there is some specific content to be mastered,
- exams--it is important to make exams congruent with course objectives. Don't, for example, say that the course focuses on content and methods and just test for content, or don't say that the problem-solving process is important and then give quick-answer factual exams.
- attendance and class participation--this is particularly important in a course where class experiences are an object of the course in themselves. Many instructors find that a strong initial stand on attendance avoids subsequent problems.

How a Unit Might Go

The following is intended to provide some feeling for how a class might progress through a unit.

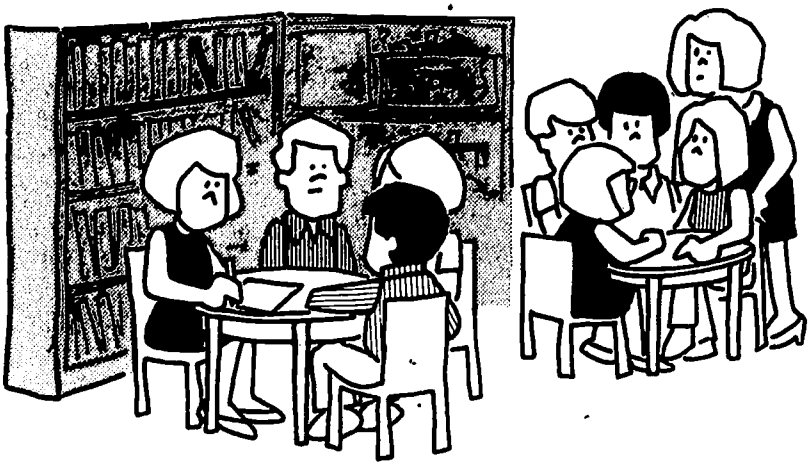
- The instructor should look through the unit and the Instructor's Manual to determine an appropriate selection of activities to do and to make plans for supplying any materials that might be necessary. Students should know in advance which activities are to be covered.

- At the end of the period before a new unit will start the instructor can assign the reading of the introduction to the unit and to the first section and the major questions for the section. The overview could also be assigned. Students should be reminded to bring the unit to the next class.
- After the overview of the unit has been read, ten minutes of the first class can be spent discussing the questions which accompany the overview.
- Following a brief introduction the students can start the first activity in small groups during the class period and finish it individually as homework. The instructor can move from group to group to see that everyone is getting into the activity.
- The class can progress through the unit, doing some activities in class, some at home, skipping some and doing some for extra credit. The instructor may find that a five-minute wrap-up discussion at the end of class is helpful at times.
- The students should be given ample advance notice of when their journal for the unit will be due and when their exam (if any) will be.
- If many students in a class lack a particular mathematical or pedagogical skill or concept, that skill or concept can be supplied by a lecture. If only a few students need help, tutoring by the instructor or by peers may be the best way to solve the problem.
- A good culminating activity is a seminar discussing the important issues and concepts encountered in the unit.

The Classroom

The classroom is an important part of the implementation of the Mathematics-Methods Program.

- Small-group work is enhanced by tables which will accommodate at least four students.
- Some activities call for the use of elementary mathematics texts. It is most helpful to have these available in the classroom.



- Considerable class time is saved if the materials needed for an activity are readily available. If all of these materials are stored in or near the classroom, the instructor's life is greatly simplified.

There is also the general point that in the spirit of "teaching as one should teach," a pleasant and rich mathematical environment will set the tone of the course. Posters, displays, materials, models and books all lend to such an environment.

THE CONTENT OF THE UNITS

There are twelve units in the Mathematics-Methods Program. The units are largely independent and can be used in any combination and in any order. The units can be classified into three categories as follows.

Basic Number Units:

- Numeration
- Addition and Subtraction
- Multiplication and Division
- Rational Numbers with Integers and Reals

Geometry Units:

- Awareness Geometry
- Analysis of Shapes
- Measurement
- Transformational Geometry

Mathematical Topics for Teachers Units:

- Experiences in Problem Solving
- Graphs: The Picturing of Information
- Number Theory
- Probability and Statistics

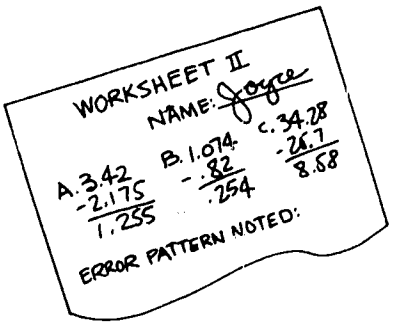
Each unit has its own unique characteristics, dictated by the particular content and objectives of that unit. In order to best communicate these characteristics and the content of the units, we will present

- a general synopsis of the units in each category,
- a description of the particular features of each unit,
- the table of contents of each unit.

Basic Number Units

The heart of the elementary school mathematics curriculum is arithmetic: numeration and the four operations on the systems of whole numbers, rational numbers, and integers. A great deal of experience and research has been accumulated on the teaching of arithmetic, and some very sophisticated instructional sequences have resulted. The basic number units of the Mathematics-Methods Program attempt to develop the concepts and methods of arithmetic in a way that adequately reflects the current state of the art and yet is not too sophisticated for the inexperienced learner. Among the features of these units are:

- Standard manipulative materials (commercial and homemade) are used and analyzed.
- Diagnosis and remediation are stressed.

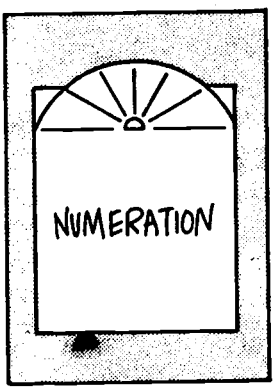


- The operations are carried through the concept, symbolization, basic fact, algorithm sequence.
- Some games, tricks, and puzzles are introduced.

The basic number units also share certain features with all of the units of the Mathematics-Methods Program. These include:

- The prospective teacher learns by doing in a problem-solving mode.
- Concepts are developed from concrete to abstract.
- Concepts are developed from and applied to the real world.
- Current elementary mathematics text series are drawn from and are analyzed.

Numeration



The concepts and attributes of numeration systems are introduced in the World Numeration Contest. The likely winner, the Hindu-Arabic system is the object of most of the unit's attention. The prospective teacher works with early number topics. The concepts of grouping, trading, and place value are developed, and different manipulative aids are analyzed to determine which of these concepts they most clearly embody. Expanded notation and scientific notation are introduced, and there is some work with "other bases." Various embodiments are used to introduce decimal numeration.

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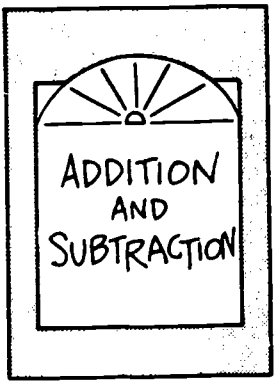
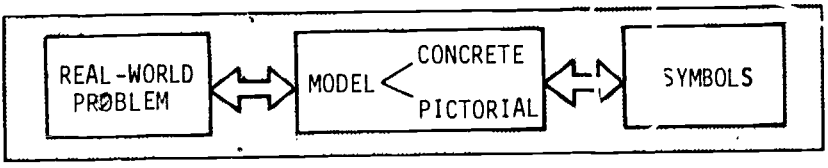


It is most important for a teacher to be aware when concepts are to be developed, when basic facts are to be learned, and when algorithms are the object of instruction. The Addition and Subtraction unit attends to these distinctions and provides the prospective teacher with ideas and experiences relevant to each of these stages. The interplay between a real-world problem, a concrete or pictorial model of that problem, and a symbolic representation of the problem is carefully developed in a problem-solving setting.



The properties and definitions related to the addition and subtraction of whole numbers are developed and applied to the problems of teaching basic facts to children. Standard algorithms are developed using concrete aids, and some nonstandard algorithms are studied. As in all of the basic number units, diagnosis and remediation problems are presented.

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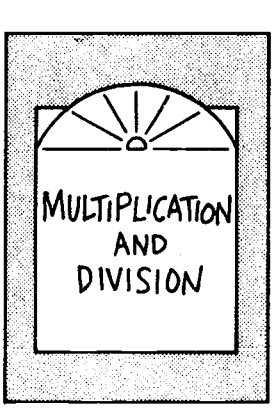
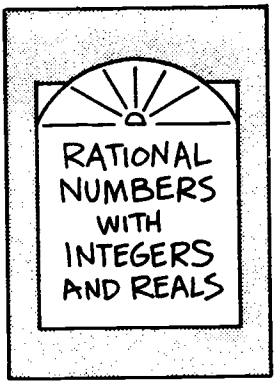
Multiplication and Division



The basic outline and objectives of the Multiplication and Division unit parallel those of the Addition and Subtraction unit. As can be seen from the table of contents, the section headings delineate the three stages in the development of multiplication and division with children. The development of concepts from real-world problems through a concrete or pictorial model to a symbolic representation is stressed. Throughout, the college student

works with concrete materials with an eye toward helping a child to work with these same materials.

Contents



The rational numbers and the operations on them present very important and difficult instructional problems. Embodiments become difficult and, for certain concepts, seem to become counterproductive. The operations depend on previous number work and the algorithms are more complex. The Rational Numbers unit deals with each of these problems. Some features of the unit are:

- Some concepts are developed from physical embodiments, some from pictorial representations, and some from mathematical relationships. The problems related to these alternatives are discussed.
- Each of the operations is related to the same operation on whole numbers--analogies with whole number work are stressed.
- The concepts of equivalent fractions and of equivalent decimal representation are carefully developed.

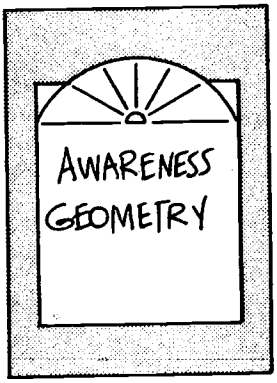
Many college students are not confident of the mathematics of the rational numbers. A self-test is provided as a guide to needed review and remedial study. The basic properties of rational numbers are reviewed. Their density on the number line and infinite decimal representations provide a natural bridge to a brief study of the real numbers, where representations, rational approximations, cardinality, and density are studied. The unit begins with a section on integers which stresses the embodiments of negative numbers and analogies with whole number work.

Contents

Geometry Units

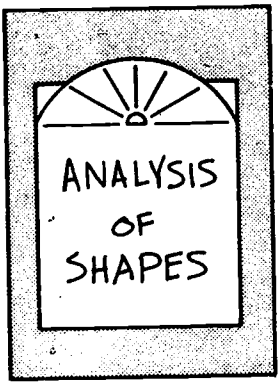
The geometry units of the Mathematics-Methods Program are based on the assumption that geometry will be taught if the teacher sees purpose and direction to the study of geometry. Geometry can be thought of as the study of space experiences. The main objects of study are shapes which are abstracted from the environment. In studying these shapes we can describe them, measure them, transform them, and we can apply what we learn to problems in the real world.

Awareness Geometry



The Awareness Geometry unit is designed to affect the college student's attitude toward geometry. The geometric potential of the environment is explored so that the college student will see ways of developing informal child geometry lessons based on the child's environment. The prospective teacher looks carefully at the immediate environment, experiments with shapes that are observed there and informally analyzes certain of those shapes. At the end of the unit there is an opportunity to plan geometry lessons for children.

Contents



Much of the geometric content in the elementary school curriculum can be classified as classical Euclidean geometry. However, the classical Euclidean approach to that geometry is not appropriate for young children. The Analysis of Shapes unit approaches this content from the point of view of the occurrences of the shape, the analysis of the shape, and the applications of that analysis to real-world problems. Straight lines, triangles, and circles are studied. The analysis of each is carried out in an informal, inductive manner. The last section of the unit deals with the issue of verification--attempting to put into perspective the informal geometry which is studied in the elementary school and the formal geometry of Euclid which is common in secondary schools.

Contents

Section III: Circles

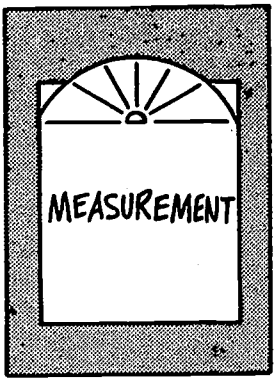
Activity 16 Circles: Their Role in the World
Activity 17 Circles and Points
Activity 18 Circles and Lines
Activity 19 Equation of a Circle
Activity 20 Area vs. Perimeter
Activity 21 Circles Around You
Activity 22 How to Teach Geometry

Section IV: Verification

Activity 23 How Do You Know It's True?
Activity 24 What Will It Take to Convince You?
Activity 25 Proof
Activity 26 Verification for Kids

Required Materials

Measurement



The current transition to the metric system underscores both the need for understanding the measurement process and the need for experience with metric units. The Measurement unit introduces the process of measuring in a way that is consistent with elementary school instruction by

- identifying and comparing attributes,
- comparing quantities of attributes with nonstandard unit quantities,
- using standard (metric) units.

This process is applied to some common attributes, and then attention is given to child instruction in measurement. The college student actually does measuring in activities that could be used with children. Metric units are used throughout.

Contents

Section I: The Measurement Process

Activity 1 Identifying and Comparing Attributes
Activity 2 Nonstandard Units
Activity 3 Metrics Are Coming

Section II: Certain Common Measurements

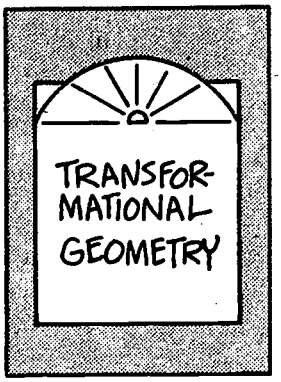
Activity 4 Area
Activity 5 Geoboards
Activity 6 Familiar Formulas for Area
Activity 7 π and Circles
Activity 8 Volume Measurements
Activity 9 More about Measurement
Activity 10 Measuring Minds
Activity 11 Measurement Versus the Real World

Section III: Child Learning of Measurement

Activity 12 Child Readiness for Measurement
Activity 13 Child Problems with Measurement
Activity 14 Doing and Analyzing Measurement Activities
Activity 15 Writing Measurement Activities
Activity 16 Building on Textbooks
Activity 17 Seminar on Child Measurement

Required Materials

Transformational Geometry



An important part of our space experiences involves changes in shapes. Rigid, projective, and topological transformations are studied in this unit--each being developed from real experiences. Rigid transformations are emphasized. It is seen that every rigid transformation can be decomposed into slides, flips, and turns or into flips alone. Rigid transformations are applied to symmetry and tessellations, and the prospective teacher is asked to analyze critically the current and potential study of rigid transformations in the elementary school.

Contents

Introduction to the Geometry Units of the Mathematics-Methods Program
Introduction to the Transformational Geometry Unit

A Working Overview of Transformational Geometry

Section I: Rigid Transformations

Activity 1 Slides, Flips, and Turns
Activity 2 Decomposition of Rigid Transformations into Slides, Flips, and Turns
Activity 3 Coordinate Analysis of Rigid Transformations
Activity 4 Symmetry
Activity 5 Using Symmetry to Analyze Shapes
Activity 6 Tessellations
Activity 7 Experiences with Geometry Materials
Activity 8 Rigid Transformations in the Elementary School

Section II: Projective Transformations

Activity 9 Casting Shadows
Activity 10 Invariants Under Projective Transformations
Activity 11 Similarity

Section III: Topological Transformations

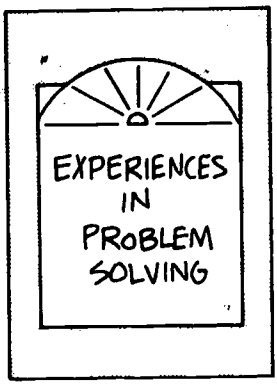
Activity 12 Exploring Topological Transformations
Activity 13 Certain Important Topological Invariants
Activity 14 Games, Graphs, and Euler
Activity 15 Seminar on Transformational Invariants

Required Materials

Mathematical Topics for Teachers Units

The topics of these units are chosen from those mathematical topics besides basic number work and geometry which have greatest relevance to the real world and the current elementary mathematics curriculum. These units are completely independent and can be selected from as time and the judgment of the instructor dictate. Most college students will find material in each unit which is new, interesting, challenging, yet do-able.

Experiences in Problem Solving

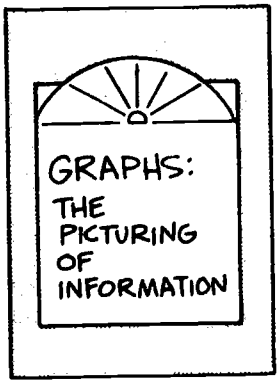


Many college students have little insight into the process of solving problems. It seems most important for a teacher to have some insight into this process in order to help children develop problem-solving skills. The Problem Solving unit introduces the framework for problem solving that is generally found in Polya's work:

- Understanding the problem
- Getting started with a plan
- Carrying out the plan
- Thinking back

Three activities introduce particular strategies (working a simpler problem, finding a pattern or formula, and working special cases). There are many problems to solve which are generally of a puzzle type. The problems require little specific mathematics background and are chosen to be interesting and motivating. The college student is asked to think back on her/his problem-solving experiences and to consider ways of helping children with their problem-solving experiences.

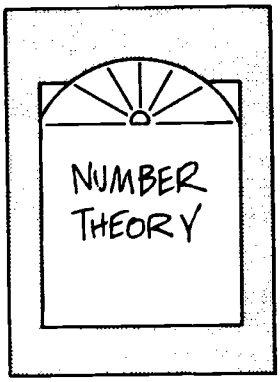
Contents



The unit is subtitled "The Picturing of Information," and this theme is carried throughout. Each section is concerned with picturing a different kind of information (see the section titles in the table of contents). Sections I and II are concerned with graphs of data and locations, which are a fairly standard part of the elementary curriculum. One of the messages of these sections is that there is considerably more potential for these kinds of graphs in the elementary school than is usually realized. Section III introduces the graphs of relations which are characteristic of the work of the Belgian mathematics educators, George and Frédérique Papy. Most college students will find the Papygrams to be new and interesting. In the fourth section graphs of functions are evolved from experiments which also give the college student experience with pattern finding. The unit has a diagnostic test and several class projects--including one where data is collected and represented. In another activity misrepresentations of data with graphs are explored.

Contents

Number Theory



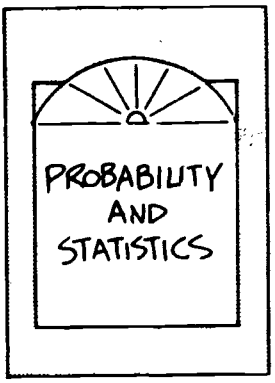
Number theory draws its importance for elementary teachers from four sources:

- Certain number theory is needed for basic number work (e.g., least common denominator);
- All of number theory reinforces basic number work;
- Number theory is rich in interesting, challenging, and solvable problems which are a source of problem-solving experiences for children;
- Number theory provides "concrete" examples of certain abstract structures which are important in the study of mathematics.

The Number Theory unit explores each of these roles. The first section deals with those number theory topics which have specific applications in the elementary school, including divisibility, prime factorization, and least common multiples. (The Euclidean algorithm and casting out nines are introduced later, in Section III). The second section focuses directly on the problem-solving process with illustrations chosen from the many challenging problems in number theory. Section III introduces modular arithmetic, which provides a nice em-

bodiment of the abstract concept of group. Throughout the unit concepts are illustrated and embodied using pages from elementary school texts as well as such common materials as Cuisenaire rods and the 100's chart.

Contents



Flipping coins, rolling dice, and spinning spinners are all part of the lives of many children. Moreover, every person, young or old, makes decisions based on estimates of likelihood. These estimates may be more or less accurate depending on the background and training of the individual. The Probability and Statistics unit begins with coin flipping, spinner spinning, and the like. It introduces such concepts as sample space, equal likelihood, and relative frequency which are needed to make reasonable likelihood estimates. It points out some of the prejudices which children may bring to a likelihood decision-making situation. Then the unit deals with data collection, certain basic statistics, relationship of statistics to probability and the use of statistics in decision making. The unit also contains some work on combinations, independence, and conditional probability which is cast at the level of prospective elementary teachers but which probably has only limited direct application in the elementary classroom.

Contents

CHOOSING UNITS TO USE

The time limitations of most courses make it necessary for the instructor to select from among the 12 units of the Mathematics-Methods Program. The following should be taken into consideration.

- The units are independent--however, some sequences of units make more sense than others.
- Done in their entirety in a laboratory setting most units require about one semester-hour of time (50-minute periods).
- Units can be speeded up by selecting activities, by assigning some work as homework, and by covering some material in lectures. The Instructor's Manual for each unit has suggestions for different paths through the unit.

The units of the Mathematics-Methods Program have been used by at least 43 different institutions in a wide variety of ways involving the following variables:

content emphasis/methods emphasis
school experience/no school experience
one instructor/team taught
total curriculum/part of the curriculum
preservice/inservice

Practically every meaningful combination of these variables has been tried successfully by at least one instructor at some institution.

There are several natural combinations and sequences of units for particular kinds of courses. Some examples follow.

- Combined Content and Methods (3 semester hours)

  Numeration
  Rational Numbers with Integers and Reals
  Experiences in Problem Solving

  It is worth noting that Numeration and Rational Numbers raise many of the important pedagogical issues of arithmetic. In particular, the physical embodiments and models for addition, subtraction, multiplication, and division are developed in Rational Numbers.

- Combined Content and Methods (6 semester hours)

  Numeration
  Number Theory or Graphs
  Experiences in Problem Solving
  Measurement
  Rational Numbers with Integers and Reals
  Analysis of Shapes
  Probability and Statistics or Transformational Geometry

- Combined Content and Methods (9 semester hours)

  Numeration
  Awareness Geometry
  Addition and Subtraction
  Measurement
  Multiplication and Division
  Experiences in Problem Solving
  Analysis of Shapes
  Number Theory or Graphs
  Rational Numbers with Integers and Reals
  Probability and Statistics or Transformational Geometry

- Methods Emphasis (3 semester hours)

  Numeration
  Awareness Geometry
  Measurement
  * { Addition and Subtraction
    { Multiplication and Division
    { Rational Numbers with Integers and Reals

  *These three units can be put together and selected from to effect the emphasis on the pedagogy of arithmetic which is desired by the instructor.

- Content Emphasis (3 semester hours)

  Number Theory

  Analysis of Shapes or Transformational Geometry

  Rational Numbers with Integers and Reals

  Probability and Statistics or Graphs

- Content Emphasis (6 semester hours)

  Number Theory

  Analysis of Shapes

  Rational Numbers with Integers and Reals

  Experiences in Problem Solving

  Probability and Statistics

  Graphs

  Transformational Geometry

- Individual Units such as

  Probability and Statistics

  Number Theory

  Experiences in Problem Solving

  Transformational Geometry

  can be inserted into a course which is using a conventional text. These units treat material which is not readily available for prospective elementary teachers.

Many combinations of units can make sense if they fit the objectives and procedures of the instructor.

ELEMENTARY SCHOOL TEACHING EXPERIENCE COMPONENT

Objectives and Features

An important goal of the Mathematics-Methods Program is to relate mathematics to the real world of the learner. Since the elementary teacher's real world includes the elementary school classroom, it is important to relate the mathematics training of a prospective elementary teacher to the classroom. The units attempt to do this in the college setting. The elementary classroom experience developed as a part of the Mathematics-Methods Program carries the relationship much further. The units can be implemented with or without a school experience. Those who have implemented both have found the school experience to be very effective.

Since the goals and the circumstances of each institution vary so widely, it is not possible to prescribe an elementary classroom experience. Instead we will present an outline of the procedures followed at Indiana University in implementing the school experience portion of the Mathematics-Methods Program. We hope that this outline will provide a source of ideas and procedures which can be adapted to the circumstances of others.

The school experience at Indiana University has the primary goal of providing the prospective teacher with insight into how children think about and learn mathematics...insight into both the problems children have and the freshness and enthusiasm that they can bring to the study of mathematics. There is no focus on the problems of classroom management. This goal has substantial implications for implementing the program. Other goals include the following:

- To initiate the development of skills in the teaching of mathematics to children;
- To lend relevance, context, and meaning to content and methods being studied in the college classroom;
- To start to build the preservice teachers' confidence in working with children;
- To provide some basis for career decisions--to teach or not--if

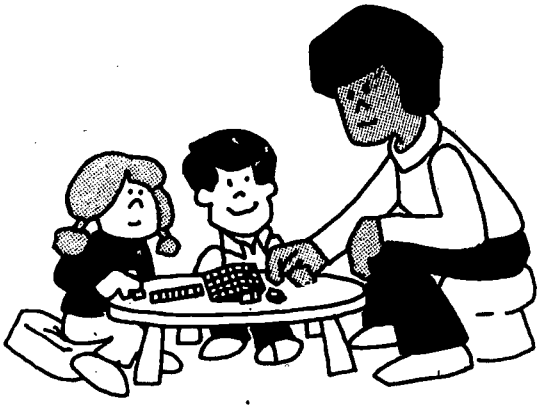
so, at what level?

- To do all of the above in a way that is manageable for the college instructor and the elementary school teacher and that is beneficial for the children.

To accomplish these objectives at Indiana University a pre-student teaching school experience has been implemented which has these features:

- Topics taught in the elementary school are coordinated with those in the college classroom.
- An entire elementary school class is taken over by an entire college class--reducing the planning problems for the classroom teacher and reducing the supervisory logistics for the college instructor. This step reduces those problems since the college instructor can contact a single elementary teacher to set up a classroom experience session. The college instructor can accompany the college class to the elementary classroom--eliminating the logistical problems which result from having different college students working in different classrooms. The classroom teacher does not have to worry about what to do with children who are not part of the school experience, and the effect of a college student's absence is reduced since the children can be divided among the several college students present.

- The college student works with individual children or small groups of children--reducing management problems and increasing attention on child thinking.



- The college student begins with class-tested lessons to teach; more responsibility for developing original lessons is given as experience and confidence grow.
- Lessons are designed to be introductory, enrichment, or extension in order to avoid interference with regular mathematics instruction.
- The college student works with children at several grade levels--from grade 1 to grade 6.

How to Proceed

Each college instructor has her/his own set of beliefs and circumstances which will dictate what should and can be done with respect to school experience. We describe here the school experience sequence which has been implemented at Indiana University.

Stage 1: Introductory visit of college class is made to the school to meet the principal and cooperating teachers and to chat informally with children.

Stage 2: Students in college class sit with children in elementary class during a lesson taught by a model teacher (e.g., the

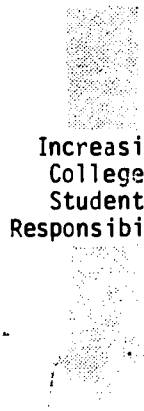
college instructor). College students help children on follow-up work sheet. In this way the college student has an opportunity to get some experience with children in a low-threat situation. Also the model teacher's lesson provides an ideal point of departure for a subsequent class discussion about the school experience.

Stage 3: College students are given the lesson plan of a class-tested lesson which has a high probability of success. They prepare the lesson, and the college class teaches the lesson to an elementary class on an individual or small-group basis. College students work with one or two students, depending on relative numbers of children and college students. Sometimes two college students pair up so that one can teach while the other observes.

Stage 4: College students teach a lesson which has been given to them, and then they extend the lesson during one or two subsequent visits.

Stage 5: College students write and teach a lesson based on a topic and objectives which are supplied by the instructor.

Stage 6: College students develop and teach their own lessons on a topic of their own choice.



1. Informal talk with children
2. Observe model lesson and follow up with one or two children
3. Teach provided lesson
4. Teach and extend provided lesson
5. Write and teach lesson based on provided topic and objectives
6. Write and teach lesson on own topic and objectives

Stages in Indiana University School Experience

At Indiana University we have experimented with different frequencies of visits. Some classes have had weekly school experiences while others have met at the university every other week to discuss the previous week's experience and to plan for the future. Whether or not this is feasible it has proved most important to have regular discussions of the school experience. If time can be scheduled immediately following the experience, that seems ideal. There follows a list of other observations which have proved to be helpful.

- Always plan ahead and let the classroom teacher know what to expect.
- Be sure that the college students are adequately supervised--preferably by the instructor.
- Encourage the college students to keep some sort of log of their experiences which includes anecdotes and insights gained into child thinking.

- If the early lessons which college students are to teach can be modeled for them--live or on videotape--it may greatly increase the probability of success.

It is most important to periodically review school experience objectives and the extent to which procedures are promoting these objectives. For example, if one were to have classroom management skills as an objective, one might not want to restrict school experiences to working with small groups of children.

Sample Lessons

The following is a list of lesson topics which have been used and found successful in the elementary school experience at Indiana University. The list is obviously not exhaustive of possible topics.

Topic	Grade Level
NUMERATION	
Early Grouping Experiences: 3's, 5's and 10's	1-2
Renaming Numbers: 1 Less Ten, 10 More Ones	3-4
Patterns on the Hundred's Chart	2-4
OPERATIONS WITH WHOLE NUMBERS	
Thinking Strategies with Addition Facts	2-3
Thinking Strategies with Multiplication Facts	3-6
Introduction to Multiplication	2-3
Introducing Division with Remainders	3-5
GEOMETRY	
Introduction to Line Symmetry	1-4
Graphing Ordered Pairs	4-6
MEASUREMENT	
Measurement: Introducing Area	3-5
Measurement: Area on the Geoboard	4-6
NUMBER THEORY	
Odd Numbers and Even Numbers	1-2
Prime Numbers and Composite Numbers	4-6
OTHER TOPICS	
Classification	K-2
Introduction to Negative Numbers	3-6
Introduction to Probability	4-6

Continued from inside front cover

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