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AUTHOR Kerr, Donald R., Jr.; And Others
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ABSTRACT

This unit is 1 of 12 developed for the university classroom portion of the Mathematics-Methods Program (MMP), created by the Indiana University Mathematics Education Development Center (MEDC) as an innovative program for the mathematics training of prospective elementary school teachers (PSTs). Each unit is written in an activity format that involves the PST in doing mathematics with an eye toward application of that mathematics in the elementary school. This document is one of four units that are devoted to the geometry units of MMP and to the unit itself, the text has six activity sections titled: Geometry Around You; A Sagging Door--Stability of Shapes; Constructing Solid Shapes from Plane Shapes; Cross Sections; Vertices, Edges, Surfaces, and Euler; and Implications for Teaching Geometry. (MP)

MATHEMATICS EDUCATION DEVELOPMENT CENTER

INDIANA UNIVERSITY

John F. LeBlanc, *Director*
Donald R. Kerr, Jr., *Assistant Director*

The following is a list of faculty and project associates who have contributed to the development of the Mathematics-Methods Program.

Mathematics Education Faculty

Frank K. Lester, Jr.
Sally H. Thomas
Paul R. Trafton
Ronald C. Welch

Project Associates — Mathematics Education

Gertrude R. Croke
Carol A. Dodd-Thornton
Nancy C. Fisher
Fadia F. Harik
Kathleen M. Hart
Tom S. Hudson
Calvin J. Irons
Graham A. Jones
Charles E. Lamb
Richard A. Lesh
Barbara E. Moses
Geraldine N. Rossi
Thomas L. Schroeder
Carol L. Wadsworth
Barbara E. Weller
Larry E. Wheeler

Mathematics Faculty

George Springer, *Co-principal Investigator*
Billy E. Rhoades
Maynard Thompson

Project Associates — Mathematics

Glenn M. Carver
A. Carroll Delaney
Alfred L. LaTendresse
Bernice K. O'Brien
Robert F. Olin
Susan M. Sanders
Barbara D. Sehr
Karen R. Thelen
Richard D. Troxel
Karen S. Wade
Carter S. Warfield
Lynnette O. Womble

Resource Teacher

Marilyn Hall Jacobson

Continued on inside back cover

AWARENESS GEOMETRY

DONALD R. KERR, JR.

Substantial contributors

JOHN F. LEBLANC

Carol A. Dodd-Thornton
Fadia F. Harik
Graham A. Jones

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PREFACE

The Mathematics-Methods Program (MMP) has been developed by the Indiana University Mathematics Education Development Center (MEDC) during the years 1971-75. The development of the MMP was funded by the UPSTEP program of the National Science Foundation, with the goal of producing an innovative program for the mathematics training of prospective elementary school teachers (PSTs).

The primary features of the MMP are:

- It combines the mathematics training and the methods training of PSTs.
- It promotes a hands-on, laboratory approach to teaching in which PSTs learn mathematics and methods by doing rather than by listening, taking notes or memorizing.
- It involves the PST in using techniques and materials that are appropriate for use with children.
- It focuses on the real-world mathematical concerns of children and the real-world mathematical and pedagogical concerns of PSTs.

The MMP, as developed at the MEDC, involves a university classroom component and a related public school teaching component. The university classroom component combines the mathematics content courses and methods courses normally taken by PSTs, while the public school teaching component provides the PST with a chance to gain experience with children and insight into their mathematical thinking.

A model has been developed for the implementation of the public school teaching component of the MMP. Materials have been developed for the university classroom portion of the MMP. These include 12 instructional units with the following titles:

Numeration
Addition and Subtraction
Multiplication and Division
Rational Numbers with Integers and Reals
Awareness Geometry
Transformational Geometry
Analysis of Shapes
Measurement
Number Theory
Probability and Statistics
Graphs: the Picturing of Information
Experiences in Problem Solving

These units are written in an activity format that involves the PST in doing mathematics with an eye toward the application of that mathematics in the elementary school. The units are almost entirely independent of one another, and any selection of them can be done, in any order. It is worth noting that the first four units listed pertain to the basic number work in the elementary school; the second four to the geometry of the elementary school; and the final four to mathematical topics for the elementary teacher.

For purposes of formative evaluation and dissemination, the MMP has been field-tested at over 40 colleges and universities. The field implementation formats have varied widely. They include the following:

- Use in mathematics department as the mathematics content program, or as a portion of that program;
- Use in the education school as the methods program, or as a portion of that program,
- Combined mathematics content and methods program taught in

either the mathematics department, or the education school, or jointly;

- Any of the above, with or without the public school teaching experience.

Common to most of the field implementations was a small-group format for the university classroom experience and an emphasis on the use of concrete materials. The various centers that have implemented all or part of the MMP have made a number of suggestions for change, many of which are reflected in the final form of the program. It is fair to say that there has been a general feeling of satisfaction with, and enthusiasm for, MMP from those who have been involved in field-testing.

A list of the field-test centers of the MMP is as follows:

ALVIN JUNIOR COLLEGE
Alvin, Texas

BLUE MOUNTAIN COMMUNITY COLLEGE
Pendleton, Oregon

BOISE STATE UNIVERSITY
Boise, Idaho

BRIDGEWATER COLLEGE
Bridgewater, Virginia

CALIFORNIA STATE UNIVERSITY, CHICO

CALIFORNIA STATE UNIVERSITY, NORTHRIDGE

CLARKE COLLEGE
Dubuque, Iowa

UNIVERSITY OF COLORADO
Boulder, Colorado

UNIVERSITY OF COLORADO AT DENVER

CONCORDIA TEACHERS COLLEGE
River Forest, Illinois

GRAMBLING STATE UNIVERSITY
Grambling, Louisiana

ILLINOIS STATE UNIVERSITY
Normal, Illinois

INDIANA STATE UNIVERSITY EVANSVILLE

INDIANA STATE UNIVERSITY
Terre Haute, Indiana

INDIANA UNIVERSITY
Bloomington, Indiana

INDIANA UNIVERSITY NORTHWEST
Gary, Indiana

MACALESTER COLLEGE
St. Paul, Minnesota

UNIVERSITY OF MAINE AT FARMINGTON

UNIVERSITY OF MAINE AT PORTLAND-GORHAM

THE UNIVERSITY OF MANITOBA
Winnipeg, Manitoba, CANADA

MICHIGAN STATE UNIVERSITY
East Lansing, Michigan

UNIVERSITY OF NORTHERN IOWA
Cedar Falls, Iowa

NORTHERN MICHIGAN UNIVERSITY
Marquette, Michigan

NORTHWEST MISSOURI STATE UNIVERSITY
Maryville, Missouri

NORTHWESTERN UNIVERSITY
Evanston, Illinois

OAKLAND CITY COLLEGE
Oakland City, Indiana

UNIVERSITY OF OREGON
Eugene, Oregon

RHODE ISLAND COLLEGE
Providence, Rhode Island

SAINT XAVIER COLLEGE
Chicago, Illinois

SAN DIEGO STATE UNIVERSITY
San Diego, California

SAN FRANCISCO STATE UNIVERSITY
San Francisco, California

SHELBY STATE COMMUNITY COLLEGE
Memphis, Tennessee

UNIVERSITY OF SOUTHERN MISSISSIPPI
Hattiesburg, Mississippi

SYRACUSE UNIVERSITY
Syracuse, New York

TEXAS SOUTHERN UNIVERSITY
Houston, Texas

WALTERS STATE COMMUNITY COLLEGE
Morristown, Tennessee

WARTBURG COLLEGE
Waverly, Iowa

WESTERN MICHIGAN UNIVERSITY
Kalamazoo, Michigan

WHITTIER COLLEGE
Whittier, California

UNIVERSITY OF WISCONSIN--RIVER FALLS

UNIVERSITY OF WISCONSIN/STEVENS POINT

THE UNIVERSITY OF WYOMING
Laramie, Wyoming

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INTRODUCTION TO THE GEOMETRY UNITS OF THE MATHEMATICS-METHODS PROGRAM

Geometry to most people just means proving theorems about angles opposite equal sides, squares of hypotenuses, and such. This is natural since most people have their only exposure to geometry in high school where the traditional course has been built around such proofs. Geometry has been gradually working its way into the elementary school. Today's new textbooks contain a considerable amount of geometry.* Much of this material is being ignored or badly taught since many teachers see little relevance of this geometry to their own lives, to other aspects of the elementary school curriculum, or to the lives of their pupils. Moreover, some of the topics that are currently contained in textbooks were not taught when the teacher went to school and, therefore, are not fully understood by the teacher.

The geometry units of the Mathematics-Methods Program attempt to present geometry from a point of view that will bring out the potential for geometry with children. Geometry is presented as the study of space experiences. This point of view is not only consistent with the historical development of geometry, but it also keeps the focus on the relationship between geometry and the objects and shapes in our environment.

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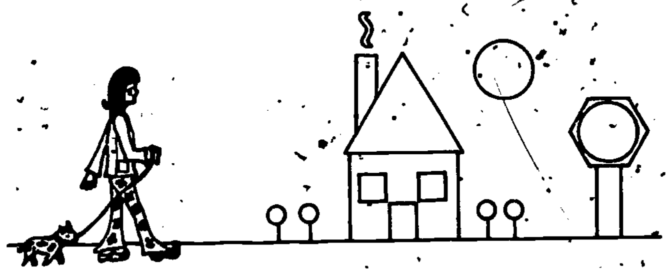
*Paul R. Trafton and John F. LeBlanc, "Informal Geometry in Grades K-6," in The 36th Yearbook of the National Council of Teachers of Mathematics, 1973: Geometry in the Mathematics Curriculum.

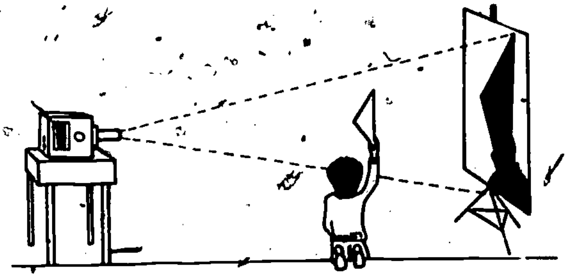
The study of space experiences addresses itself mainly to shapes. Shapes are abstractions from the environment. They can be informally investigated and analyzed. One can also study the changes (or transformations) that shapes undergo.

To effect this study of space experiences, four units have been developed.

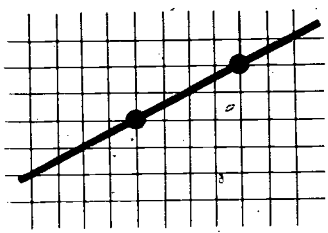
- The Awareness Geometry unit is designed to orient the prospective teacher to the informal study of geometry. In this unit one looks carefully at the environment, experiments with shapes that are observed there, and informally analyzes certain shapes. At the end of the unit, one is given experience with planning for geometry lessons with children.



- The Transformational Geometry unit studies changes that shapes can undergo. The unit is organized into the study of rigid transformations, projective transformations, and topological transformations. The presentation is informal and the focus is on concrete real-world examples of the concepts.

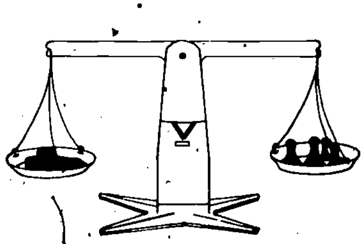


- The <u>Analysis of Shapes</u> unit studies straight lines, triangles and circles. The real-world occurrences and importance of each shape are investigated; each shape is informally analyzed to determine some of its important properties; and then the fruits of these analyses are applied to real-world problems. Many of the traditional topics of Euclidean geometry, including coordinate geometry, are considered here from a nontraditional point of view. There is also a section which deals with problems of verification and places into perspective the informal methods of elementary school geometry and the formal approach to high school geometry.



$$\frac{\text{up}}{\text{out}} = \frac{2}{4} = \frac{1}{2}$$

- The <u>Measurement</u> unit provides experiences with identifying attributes, choosing unit quantities of attributes, and determining numbers through comparisons. The emphasis is on informal, concrete, conceptual activities. There is a separate section which is devoted to child readiness and the planning of measurement activities for children. Metric units are used throughout. While measurement could have been included in the <u>Analysis of Shapes</u> unit, it has been placed in a separate unit because of its importance in the elementary school curriculum and in order to provide flexibility in the use of the units.



The four geometry units of the Mathematics-Methods Program are independent of one another. Any number of them can be used in any order. They can be used in a separate geometry course; they can be interspersed among other units of the Mathematics-Methods Program; or they can be used in conjunction with other materials.

These geometry units, like the other units of the Mathematics-Methods Program, involve one as an adult learner in activities which have implications for teaching children. One works with concepts that children might learn, with materials that children might use, and on activities that might be modified for use with children. The objective is to provide growth in understanding and enjoyment of geometry along with increased ability and desire to teach geometry to children.

INTRODUCTION TO THE AWARENESS GEOMETRY UNIT

Long before the development (some 2000 years ago) of the formal geometry which most people study in high school, human beings were interacting informally with the shapes in their environment. Certain shapes were observed to occur frequently. They became familiar and were, perhaps, even given names. The experience gained from centuries of trial and error established for these shapes special uses, roles, and properties. The most interesting shapes were subjected to informal analysis, which provided further information for extension and refinement of their use. It was only after millenia of such informal experience and investigation that it was possible to organize what was known about geometry into a formal, axiomatic system in which each geometric fact could be logically deduced from a few assumptions. Since Euclid's formalization of geometry (around 400 B.C.), the development of geometry has continued through a combination of informal and formal investigation.

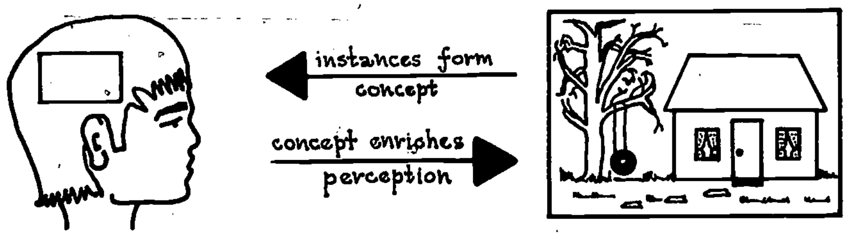
Interestingly, it seems appropriate for young children to learn geometry in much the same way that it has evolved historically:

- Young children respond well to opportunities to <u>look</u> around them, <u>observe</u> their environment, and <u>describe</u> what they see.
- <u>Experimentation</u> with shapes can provide insights into their properties and uses.
- Informal analysis of shapes can be done by children in order to make <u>inferences</u> which refine and extend their knowledge. Such

analysis can involve the child in dissecting as well as constructing objects and in recognizing patterns.

Only after such a sequence of experiences which develop the child's spatial insights and intuition does it seem appropriate to consider a formal study of geometry.

You (most adults for that matter) have probably gained most of your geometric knowledge and experience through a high school geometry course in which the primary focus was on deductive proofs. There are, in fact, many other aspects of geometry which you may never have studied. One object of this unit is to make you more aware of the geometry which is in your environment. Hopefully, this greater awareness will enrich your interaction with the world around you and will broaden your perception of geometry. Another subject of the Awareness Geometry unit is to better equip you to take advantage of the relationship between the real world and geometry when you are teaching children. Some psychologists feel that a child can best learn a concept by experiencing physical examples of it. Whether this is true or not, a child's grasp of a geometric concept will certainly be strengthened by real-world instances which help to form and refine the concept. On the other hand, the child who has grasped a geometric concept and who is sensitive to the geometric content of the environment can have a much richer appreciation for and understanding of the physical world.



Geometric concept of rectangle

Real-world instances of rectangle

In order to increase your geometric awareness and to broaden your view of appropriate geometric activities for children, the Awareness Geometry unit will follow three of the steps in the historical development of geometry:

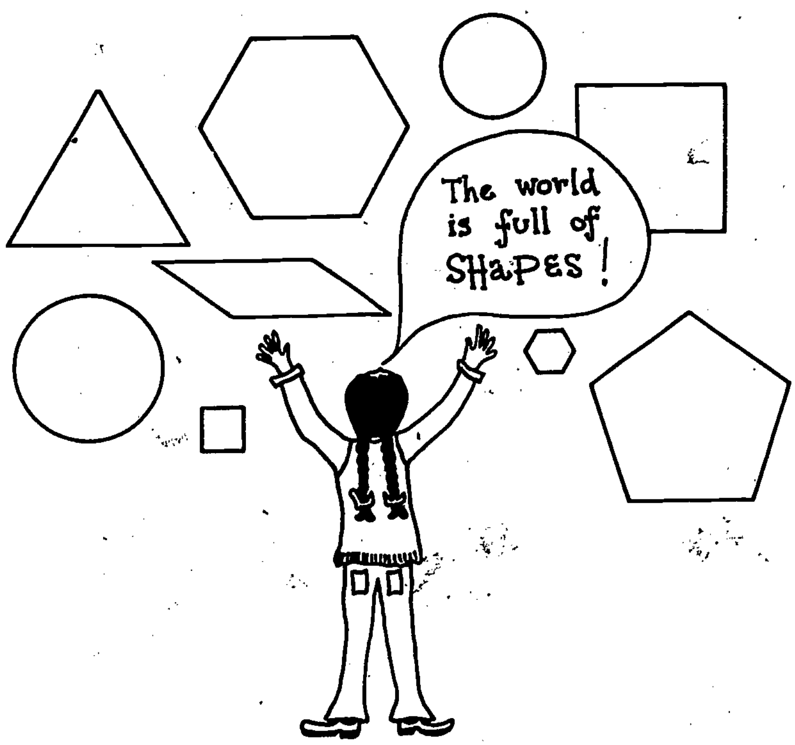
- You will be asked to look around, observe your environment, and describe what you see.
- You will experiment with shapes in order to gain insights into their properties and uses.
- You will informally analyze shapes in order to make inferences which refine and extend your knowledge. In making these analyses you will be involved in dissecting, constructing, and in recognizing patterns.

Finally, you will be asked to apply the experience of this unit to plan for geometric experiences with children.

In Activity 1 you are encouraged to look around you, to observe many different shapes, to look at a few of these more carefully, and to describe them. Activity 2 provides you with an opportunity to experiment with a shape in order to determine an important property that it possesses. Activities 3, 4 and 5 involve the analyses of various shapes. In Activity 3 you will dissect and construct three-dimensional (solid) shapes in order to determine how they are constructed from two-dimensional (plane) shapes. In Activity 4 solid shapes are analyzed to see what plane shapes they consist of. You will have some numerical pattern-recognition problem-solving experiences in Activity 5. It is important to note that Activities 2, 3, 4, and 5 have been selected from a myriad of such activities which can be done with children. One criterion for selection was that the activities present different techniques for teaching geometry to children. Finally, in Activity 6 the planning of geometry lessons for children is broached. You will take certain elementary textbook pages and use these as a basis for planning geometry lessons for children. Also in Activity 6 optional readings are assigned which concern geometry instruction for children. As a result of your experiences with this unit, you should be able to respond to the following questions.

MAJOR QUESTIONS

1. Describe the geometry of some interesting object in your environment (e.g., a piece of furniture, a football field).
2. Describe a learning situation that you might create to focus a child's attention on the geometry in his or her environment. Indicate the kinds of questions you might ask the child.
3. Choose a geometry lesson from an elementary mathematics textbook and indicate how you would motivate, implement, and extend the lesson in order to provide the greatest possible impact on children.



ACTIVITY 1

GEOMETRY AROUND YOU

FOCUS:

In this activity you will be engaging in the most primitive geometric endeavor both from an historic point of view and from the standpoint of developing geometric concepts in children. You will look at your environment in order to discover its geometric potential. You will observe shapes and hopefully become sensitized to similarities and differences among them.

MATERIALS:

The Mathematics-Methods Program slide-tape overview entitled "Awareness Geometry." (This is required for 1 below. If it is not available, choose 2 or 3.)

DIRECTIONS:

Do one of 1, 2, and 3 below and then proceed to do 4, 5, and 6.

1. View the slide-tape overview entitled "Awareness Geometry." Then, working with a group of students, spend five to ten minutes compiling a list of objects in your environment which have interesting shapes. Underline those objects whose shapes would be of greatest interest to children.

2. Go on a "shape scavenger hunt." That is, carefully investigate your daily environment (where you live, eat, work, play) for shapes of objects. Record as many shapes as you can. In particular, make a list of the five most common shapes that you find and of the five most unusual shapes.

3. Go on a "shape walk" with your class. As a class, investigate your classroom and portions of your campus and town to observe shapes of objects. Compile a list of objects with common and uncommon shapes.

4. Choose three objects from your list that are distinctive. For each object list as many characteristics or properties as possible.

5. Fill in the answers to the following questions for one of your three objects.

a) Name of object is ____________________.

b) Name(s) of the shape or shapes of the object is/are ____________________.

c) Is the object a surface or does it have a surface? ________

d) Does the surface of the object have flat regions? ________
If so, how many? ____________________

e) Does the surface of the object have curved regions? ________
If so, how many? ____________________

f) Does the object have straight edges? ____________________
If so, how many? ____________________

g) Does the object have curved edges? ____________________
If so, how many? ____________________

h) Does the object have corners (vertices)? ____________________
If so, how many? ____________________

i) Is the object symmetrical? ____________________

j) Does the object have congruent parts? ____________________

k) Would the object roll on the floor? ____________________
Why? ____________________

ℓ) Would the object slide on the floor? ____________________
Why? ____________________

m) The object is made of ____________________.

n) Write the name of an object that has the same shape as your object but which is made of different material. ____________

o) Are the two objects the same color? ____________________

p) Are the two objects the same size? ______________________

q) If you ask yourself questions b through ℓ about the new object, would you give the same answers as for the first object? ______________________

r) In what ways are the two objects geometrically different?

____

6. Answer the following questions concerning shapes in your environment.

a) A flying jet sometimes causes a vapor trail. What shape does the trail take?

b) A child drops a pebble in a pond. What do you see?

c) A child drops a pebble from a high building. What kind of path does the pebble follow? Where the pebble hits the street, what angle does the pebble's path make with the street?

d) What shape are stop signs?

e) Have you ever heard of a cubic planet?

f) Why are things the shape that they are? (Pick an interesting object and analyze it.)

7. OPTIONAL: *Start a shape scrapbook in which you place pictures, drawings, and descriptions of interesting shapes. Such a scrapbook will prove to be a nice resource for geometry work with children.*

TEACHER TEASER

Why are manhole covers round instead of square?

ACTIVITY 2

A SAGGING DOOR--STABILITY OF SHAPES

FOCUS:

What makes a shape stable? How can one stabilize a structure economically? These questions have certainly faced human beings since prehistoric times. A solution to the problem was probably arrived at through experimentation and trial and error long before any formal geometric analysis provided an answer. In this activity you will be given a brief chance to experiment with a stability problem. This activity is one example of the many such investigations that one could engage in.

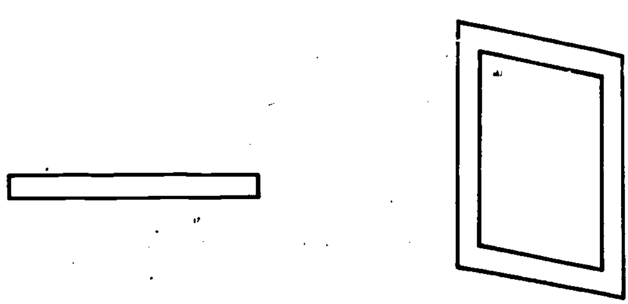
MATERIALS:

Paper strips (at least eight 3-cm x 20-cm strips per student), paper hole punches (one per group), brass brads (at least eight per student), scissors (one pair per group).

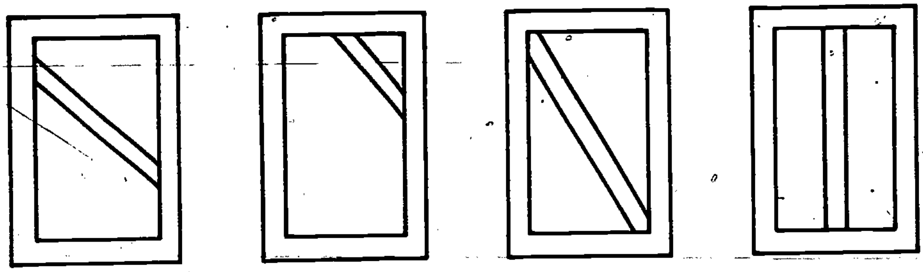
DIRECTIONS:

PROBLEM

Suppose that you move into an old house, and the screen door is sagging; i.e., its corners are loose, and it is no longer rectangular. How can the door be stabilized using a portion of the lumber below?



1. Before trying to solve the problem you should experiment with your strips and brads. Investigate the configurations below to see which ones are stable and which are not.



2. Draw a picture showing your solution to the sagging door problem.

3. Using your strips and brads determine which of the following shapes are stable: triangle, rectangle, pentagon, and hexagon. Stabilize each of the unstable shapes with the fewest strips possible.

4. Discuss with your group as many instances as you can of real-world structures where the stability principle investigated here is used.

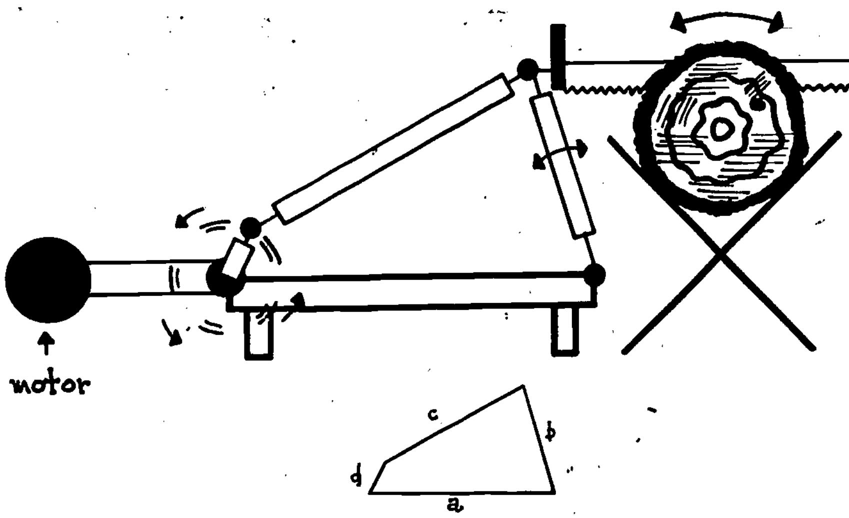
5. OPTIONAL: Construct a cube using milk straws as edges (the straws can be joined by threading the thread through them and then tying). Is the cube stable? If not, figure out a way of stabilizing it.

6. OPTIONAL: The fact that a quadrilateral is not rigid is of great technical importance. Nonrigid quadrilaterals have many different applications in the form of machines for doing and making things. One example of such a quadrilateral is the crank-and-rocker mechanism shown below. The longest side a is fixed, with the result that when the shortest side d makes a complete revolution, the rocker b traces out a certain path. By experimenting with your strips and brads, determine:

   i. the relationship that must exist between a, b, c, and d for d to be able to make a complete revolution,

   ii. the path traced out by the rocker b when condition i holds,

   iii. a practical application of the crank-and-rocker mechanism.



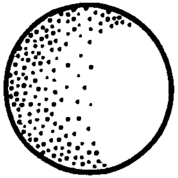
Crank-and-rocker mechanism

SOLID SHAPE CARDS

cone

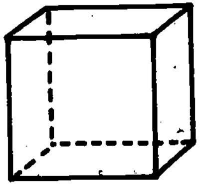
cone

sphere



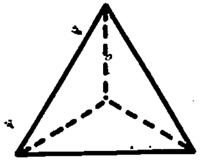
sphere

cube



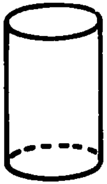
cube

tetrahedron



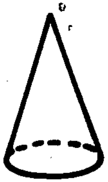
tetrahedron

cylinder



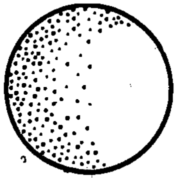
cylinder

cone



cone

sphere



sphere

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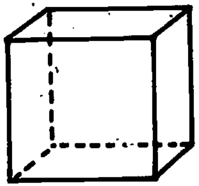
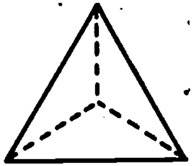


cube

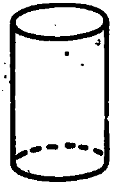
SOLID SHAPE CARDS

tetrahedron



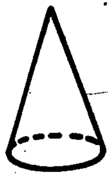
tetrahedron

cylinder



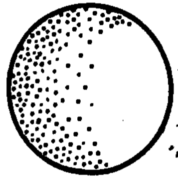
cylinder

cone



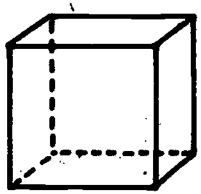
cone

sphere



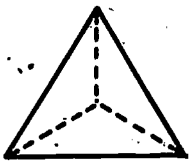
sphere

cube



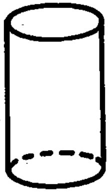
cube

tetrahedron



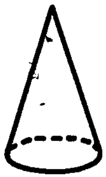
tetrahedron

cylinder



cylinder

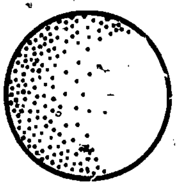
cone



cone

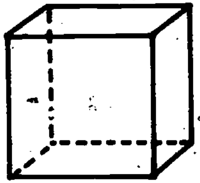
SOLID SHAPE CARDS

sphere



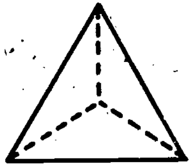
sphere

cube



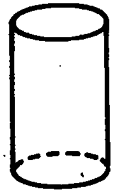
cube

tetrahedron



tetrahedron

cylinder

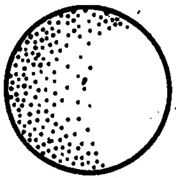


cylinder

cone

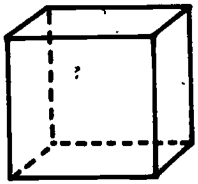
cone

sphere



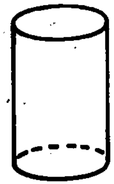
sphere

cube



cube

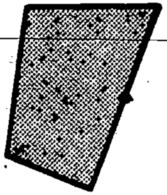
cylinder



cylinder

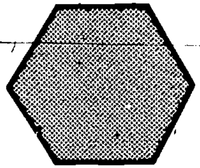
CROSS SECTION CARDS

quadrilateral region



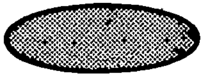
quadrilateral region

hexagonal region



hexagonal region

elliptic region

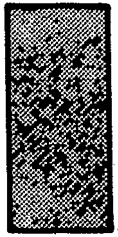


elliptic region

rectangular region

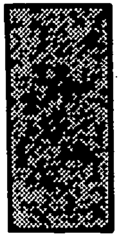
rectangular region

rectangular region



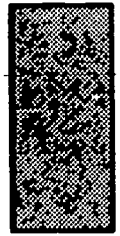
rectangular region

rectangular region



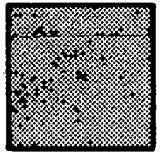
rectangular region

rectangular region



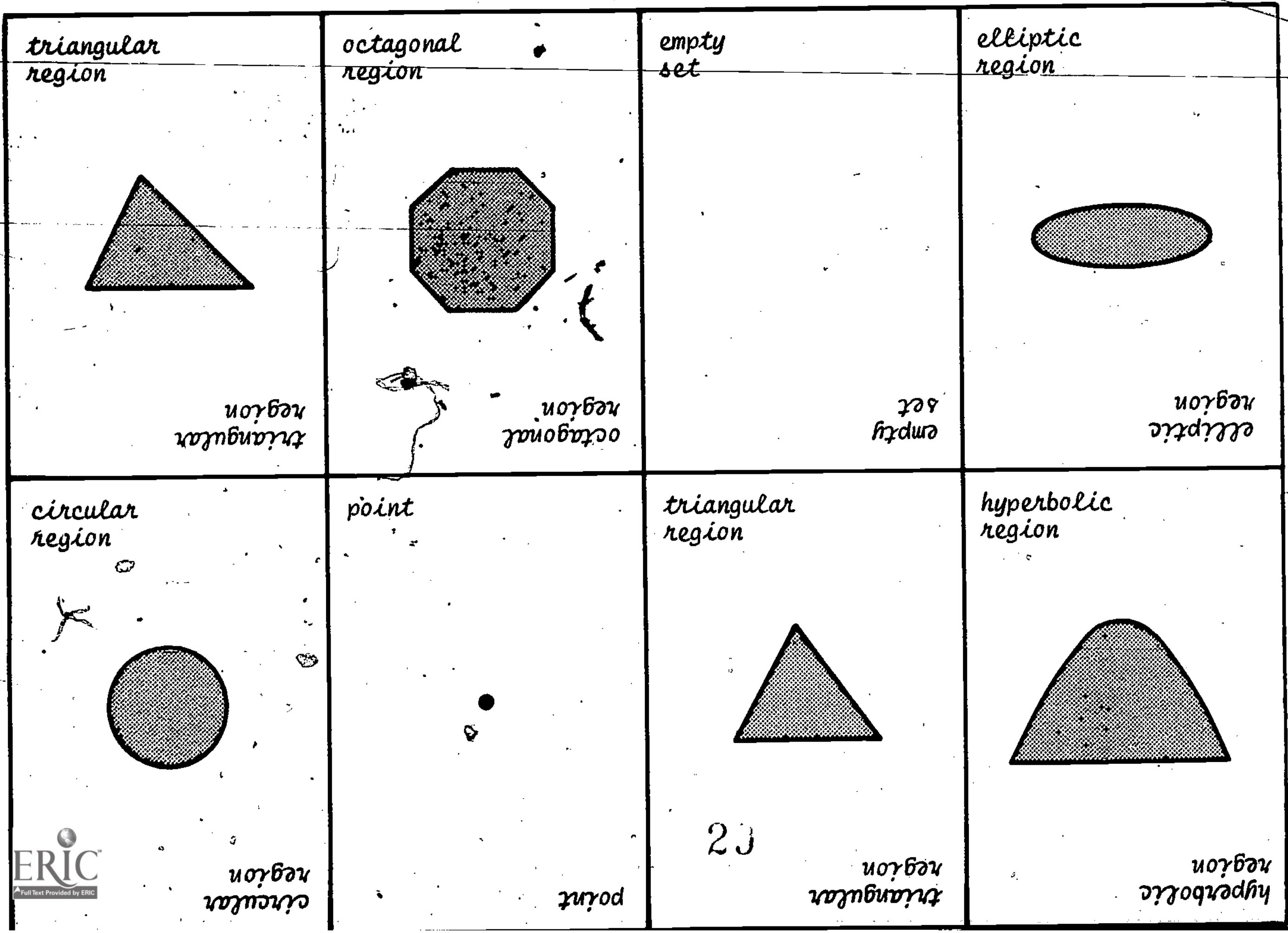
rectangular region

square region



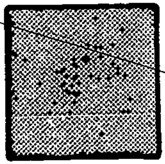
square region

CROSS SECTION CARDS



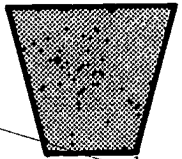
CROSS SECTION CARDS

square region



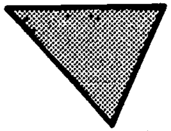
square region

trapezoidal region



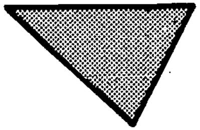
trapezoidal region

triangular region



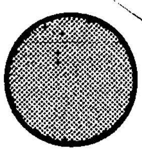
triangular region

triangular region



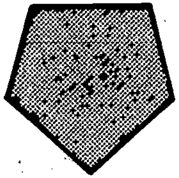
triangular region

circular region



circular region

pentagonal region

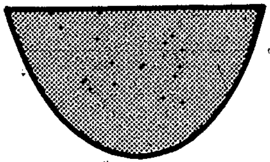


pentagonal region

point

point

parabolic region



parabolic region

ACTIVITY 3

CONSTRUCTING SOLID SHAPES FROM PLANE SHAPES

FOCUS:

In Activity 2 you briefly used a trial-and-error method to discover certain facts about the rigidity of shapes. In this activity and the next you will take another step in the historical development of geometry--that of analyzing shapes. Here you will analyze three-dimensional (solid) shapes by dissecting them to see what two-dimensional (plane) shapes they can be constructed from. It is important for you to realize that this is just one of many possible shape-analyzing activities that one might do with children.

MATERIALS:

Construction paper, ruler, scissors, paste or glue, tape, and containers (solid shapes) such as cereal boxes that are constructed by folding and fastening plane shapes.

DIRECTIONS:

1. Containers or solid shapes* are probably among the earliest shapes actually constructed by humans. It seems likely that humans have always wanted to carry more than they could conveniently hold in their hands.

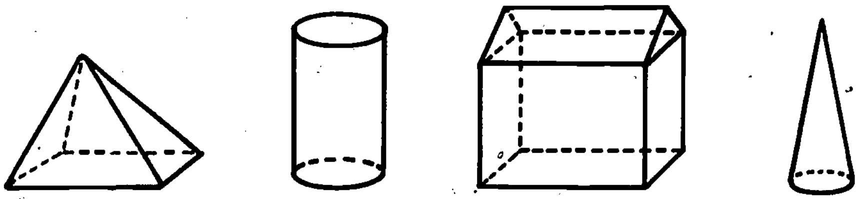
   a) Look over the solid shape supplied to you and roughly sketch the plane shape from which you think its surface was constructed.

   b) Dismantle (dissect) the shape to see if your sketch was correct.

*Actually containers are not solid shapes, but closed containers can be thought of as models for solid shapes.

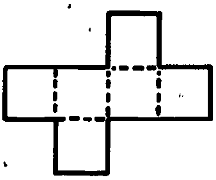
c) Carefully draw a different plane shape that can be folded and fastened to form the same solid shape. Cut it out, fold it, and fasten it to check yourself.

2. Choose from the solid shapes pictured below one that you have not yet worked with in this activity. Draw a plane shape that you feel can be folded and fastened to construct its surface (dot in the fold lines). Cut, fold, and fasten your plane shape to check yourself. Revise your plane shape if necessary.



Comment: You are seeing here that many solid shapes* can be analyzed in terms of plane shapes. This fact provides one justification for the focus on plane geometry in much of the school curriculum. There are, however, many who feel that more emphasis should be put on solid shapes, especially with young children. It may also be true that the connection between solid and plane shapes should be brought out more clearly to geometry students at all levels.

3. The shape below can be folded (along the dotted lines) to make a cube.



It is not the only shape that can. Construct several different

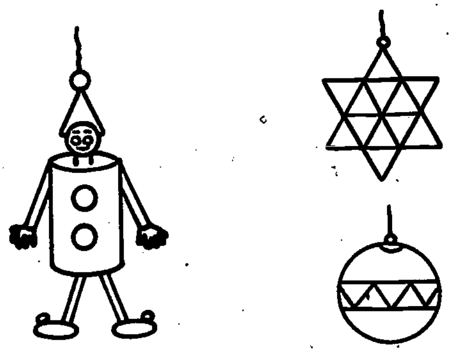
---

*Solid shapes are referred to as three-dimensional. Any surface, either flat or curved, is two-dimensional. Any curve is one-dimensional. Points are called zero-dimensional.

shapes that can be folded to make a cube. Draw each one that you find (be sure to draw in the dotted lines).

4. Children love to make things. Many of the things that they make are shapes that can have some potential for geometric analysis.

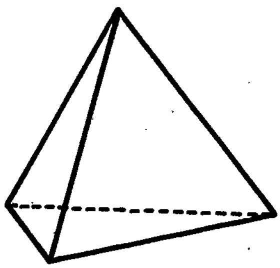
   a) Design and make a Christmas tree ornament out of standard geometric shapes.



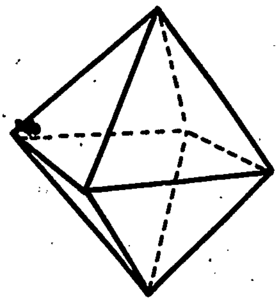
   b) Describe briefly how you, as an elementary teacher, might plan a sequence of experiences near Christmas, Thanksgiving, or Easter that would combine geometry and art objectives.

5. One must assume that manufacturers give some thought to the construction of containers. Considerations such as the economic use of materials, ease of construction, appearance, and the container's ability to be stacked or packed may be considered. Analyze several containers to see if you can determine the reasons for the manufacturer's choice of design and construction.

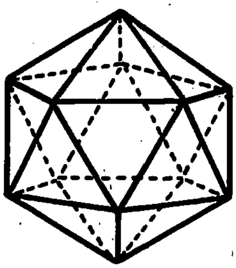
6. *OPTIONAL: Regular polyhedrons are solid shapes like those below whose faces are all the same polygon of the same size. One of the amazing mathematical facts is that there are only five of these beautiful shapes (sometimes referred to as the "Platonic Solids"). Read about them; e.g., look at* <u>*Giant Golden Book of Mathematics*</u> *by Irving Adler (New York: Western Publishing Co., 1960), or* <u>*Polyhedral Models in the Classroom*</u> *by Magnus J. Wenninger (Reston, Va.: NCTM, 1966). Construct each of the Platonic solids by folding and fastening an appropriate plane shape.*



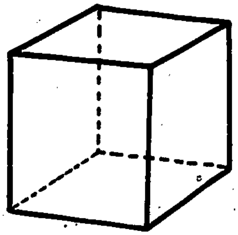
Tetrahedron - 4 equilateral triangle faces



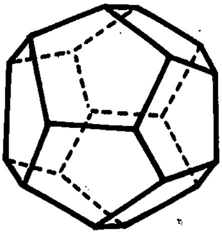
Octahedron - 8 equilateral triangle faces



Icosahedron - 20 regular triangle faces



Cube - 6 square faces



Dodecahedron - 12 regular pentagon faces

ACTIVITY 4
CROSS SECTIONS

FOCUS:

In Activity 3 you analyzed how to construct solid shapes out of plane shapes. In this activity you will analyze what plane shape you can make out of certain solid shapes by making plane slices or cross sections. A card game has been chosen as a teaching aid to help broaden your instructional repertoire.

MATERIALS:

Modeling clay and a piece of wire with washers on ends or potato and knife.

DISCUSSION:

Cross sections provide a mechanism for dissecting solid shapes. A cross section of a solid shape can be described as <u>a plane shape which results from intersecting the solid shape with a plane</u>. Another way of thinking of it is as the result of cutting straight across the solid shape with a knife. For example, consider some cross sections of the prism shape illustrated below.

This cross section is a triangular region.

This cross section is a rectangular region.

This cross section is a trapezoidal region.

DIRECTIONS:

1. Form solid shapes out of modeling clay and use a taut piece of wire to cut the shapes in order to observe various cross sections, or cut the solid shapes out of potatoes and then observe the cross sections which can be cut.

2. Draw a cube and then draw in

   a) a triangular cross section

   b) a rectangular (nonsquare) cross section

   c) a hexagonal cross section.

3. Cut out the cards which are in the center of the unit and play the game of Cross Section Rummy with two or three classmates.

Rules for Cross Section Rummy

a) The object is to run out of cards first.

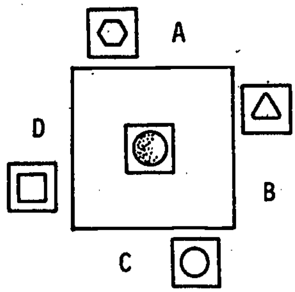
b) There are "solid shape" cards and "cross section" cards. Separate them and shuffle the two decks separately. Put the solid shape cards face down on the table. Deal out the cross section cards as far as you can and still have each player have the same number of cards.

c) The first player turns over a solid shape card. If the first player has a plane shape card which is a cross section of the turned over solid shape card, he or she can lay the plane shape card down. If the first player does not have a cross section card for the solid shape card, the first person on the left of the first player who does, lays it down.

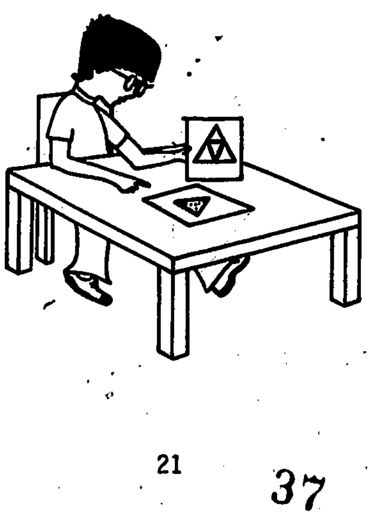
d) The turn then passes to the player on the left of the first player. This new player turns over a new solid shape card and play resumes as in b.

e) If there is doubt about the appropriateness of a play, the issue should be decided among the players.

For example, suppose player A is first and turns over a sphere from the solid shape pile. If neither player A nor player B has a card with empty set, point, or circle on it in his or her hand, and if player C has a card with a circle, then player C lays down the circle card. The turn passes to player B who turns over a new solid card, and play continues.



4. *OPTIONAL: Card games can be fun and instructional for children and are not too difficult to make up. Make up another card game which is designed to alert children to some aspect of geometry. For example, a player could remove a two-dimensional shape from his hand if it could be used to construct the particular three-dimensional shape which had been turned over. For younger children, a player could play a card which pictures a real-world instance of a shape that is turned over.*



ACTIVITY 5

VERTICES, EDGES, SURFACES AND EULER*

FOCUS:

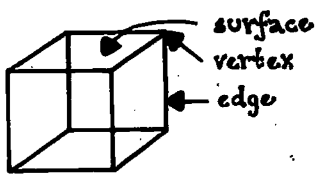
So far your informal analysis of shapes has been strictly qualitative and descriptive. In this activity you will use numbers as tools in a quantitative analysis of certain solid (three-dimensional) shapes in terms of their (two-dimensional) surfaces, (one-dimensional) edges, and (zero-dimensional) vertices. You will try to find the classical relationship (Euler formula) for polyhedrons by first choosing the polyhedrons from among a larger group of solid shapes and then engaging in the technique of pattern finding. There are many such activities that one can do which provide children with a problem-solving experience in geometry.

MATERIALS:

An assortment of solid shapes which includes a variety of polyhedral shapes (enough for each group to have several).

DISCUSSION:

A polyhedron is a solid shape which consists only of flat (not curved) surfaces which intersect in straight edges which in turn intersect in vertices. For example, the cube below has six surfaces, each of which is a square; it has 12 edges which are equal straight lines, and it has eight vertices.



*Leonhard Euler, pronounced "oiler," was an eighteenth-century mathematician. He produced more mathematics than anyone in history even though he worked for 13 years in total blindness.

DIRECTIONS:

1. Select four different solid shapes from those provided by your instructor and answer the following questions with respect to each.

   a) The shape has how many surfaces, how many edges, and how many vertices?

   b) The shape has how many flat surfaces and how many curved surfaces?

   c) The shape has how many straight edges and how many curved edges?

   d) Of the flat surfaces, how many are triangular and how many are rectangular (or square)?

   e) Of the flat surfaces, how many are neither triangular nor rectangular (nor square)? What shape are they?

2. Among the various solids in your classroom are some polyhedrons. Fill in the following table for as many different polyhedrons as possible. Look for any patterns* in the numbers as you are doing so.

Name of Polyhedron	Number of Surfaces	Number of Vertices	Number of Surfaces + Number of Vertices	Number of Edges

*Pattern finding is one of the most important techniques for finding mathematical relationships. Children should be given experience with using this technique in both arithmetic and geometry.

3. Do you see any patterns in your table? Let s represent the number of surfaces, e the number of edges, and v the number of vertices. Write below the relationship* that you see between s, e, and v.

4. Draw or name solids with each of the following combinations of surfaces, edges, and vertices. Indicate if it is impossible to do so.

s	e	v	Name or Sketch a Shape
6	12	8	
8	10	6	
4	6	4	
3	6	5	

5. Does the Euler relationship hold for solids that are not polyhedrons? Explain your answer.

*One such relationship is called the Euler formula.

6. Can you find a relationship between the vertices and edges of polygons? A polygon is a plane shape that is closed and is made up of straight edges; e.g., triangles, rectangles, and pentagons are all polygons.

ACTIVITY 6

IMPLICATIONS FOR TEACHING GEOMETRY

FOCUS:

The Awareness Geometry unit has emphasized the geometric richness of the real world. In this activity you will be asked to apply any knowledge and insights that you have gained in your work with this unit to the planning of geometry lessons for children.

MATERIALS:

One or two elementary mathematics textbook series (teacher's edition).

DISCUSSION:

Almost every elementary school teacher uses a mathematics textbook. While the texts may be carefully written and provide content and direction for mathematics instruction, a teacher should not be limited by a text. The teacher will frequently want to motivate, enrich, and extend a lesson. The teacher should look for applications of concepts and skills contained in a lesson and for other ways of relating the lesson to a child's life. It may be that some lessons should be modified so that they are consistent with the teacher's point of view or approach.

In this unit you have had to:

- look around you
- observe your environment
- describe what you see
- experiment with shapes
- dissect and construct objects
- recognize patterns
- make inferences

In this activity you will be asked to choose a geometry lesson from an elementary text and determine what its objectives are. Then you will be asked to indicate how you would motivate, enrich, and extend the lesson in order to get children to look, observe, describe, experiment, analyze, dissect, construct, and infer. In short, you will be asked to take advantage of the message of this unit to make those textbook lessons more effective and meaningful for children.

DIRECTIONS:

1. Choose a geometry lesson from a teacher's edition of an elementary mathematics text. Read the lesson to determine its objectives and to determine how the textbook authors intend that the lesson be implemented.

2. Decide how you would motivate, enrich and extend the lesson. In particular, decide

   a) how you would get children to relate the concepts of the lesson to their own lives,

   b) what questions you might ask the children to get them to probe the concepts of the lesson more deeply,

   c) what additional related activities you might have some or all of a class do.

3. *OPTIONAL: Teach the lesson to a child or to a small group of children.*

4. *OPTIONAL: The following are references which take an interesting point of view toward elementary school geometry.*

   - *John Egsgard, "Geometry All Around Us--K-12," The Arithmetic Teacher, (October 1969), pp. 437-445.*
   - *Environmental Geometry, (Nuffield Mathematics Project), New York: John Wiley & Sons, Inc., 1969.*

   Read one or both of the references keeping the following questions in mind as you read.

   a) General Questions:

   1) What is the theme or main point of the reading?

   2) Does the author's perception of geometry differ from your own? If so, in what way?

   3) What is the author's view of the value of geometry for children?

   4) What is the author's view of the role of exploration and experimentation in geometry instruction?

b) Questions concerning the Egsgard article:

1) In the author's view which should come first: the study of three-dimensional shapes or the study of two-dimensional shapes? Why?

2) Does Egsgard advocate looking to the real world for examples of a geometry concept that has been introduced, or does he suggest abstracting geometry from the real world?

3) For whom, and at what level do axiomatics and deductive proof enter into Egsgard's model for instruction in geometry?

c) Questions concerning the Nuffield article:

1) What is the main resource for geometric ideas which is emphasized in the Nuffield reading?

2) Do children see the world in the same way that we do? If not, why not?

3) What effect can the setting of an object have on one's perception of the object?

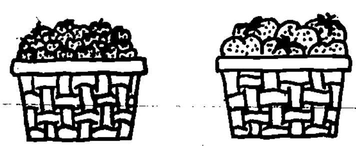
5. Choose one or two of the elementary textbook series which are in current use. For each series chosen,

a) List the major geometrical topics considered.

b) Is the spirit of the presentation consistent with the theme of the Awareness Geometry unit? Justify your comments with illustrations from the text series.

c) Describe any activities in the text series which you thought were particularly interesting.

TEACHER TEASER



Would you rather have a box of small strawberries or a box of large strawberries (assuming that the boxes are the same size and that the strawberries taste the same)?

REQUIRED MATERIALS

ACTIVITY	AUDIO-VISUAL RESOURCES	SUPPLIES	READINGS
1	Slide-tape: "Awareness Geometry," cassette recorder and projector. (Optional)		
2		Paper strips, paper hole punches, brass brads, scissors. (Optional: milk straws, elastic thread.)	
3		Construction paper, ruler, scissors, paste or glue, tape, containers such as cereal boxes.	Adler, Irving. Giant Golden Book of Mathematics. New York: Western Publishing Co., 1960. (Optional) Wenninger, Magnus J. Polyhedral Models in the Classroom. Reston, Va.: NCTM, 1966. (Optional)
4		Modeling clay, a piece of wire with washers on ends, or potato and knife.	

ACTIVITY	AUDIO-VISUAL RESOURCES	SUPPLIES	READINGS
5		An assortment of polyhedrons and other solids.	
6			One or two elementary mathematics textbook series (teacher's ed.). Egsgard, John. "Geometry All Around Us--K-12," Arithmetic Teacher, (October 1969), 437-445. (Optional) Nuffield Mathematics Project. Environmental Geometry. New York: John Wiley & Sons, Inc., 1969. (Optional)

Continued from inside front cover

Project Associates — Audio-Visual

Stephen B. Walter, *Design Coordinator*
Wallace S. Goya
John W. Hiner
Deane W. Hutton
Jean A. Lescohier
Carrie Hendron Lester
James L. McKittrick
Alexander C. Millar
Kyu-Sun Rhee
Vicki J. Semler
Donna J. Toler
Anne S. Walker

Staff

Susan E. Coté, *Administrative Assistant*
Susan J. Bakshi
Joyce A. Ford
Jane Kaho
Linda A. Svrcek

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This unit integrates the content and methods components of the mathematical training of prospective elementary school teachers. It focuses on an area of mathematics content and on the methods of teaching that content to children. The format of the unit promotes a small-group, activity approach to learning. The titles of other units are *Numeration, Addition and Subtraction, Multiplication and Division, Rational Numbers with Integers and Reals, Transformational Geometry, Analysis of Shapes, Measurement, Graphs: The Picturing of Information, Number Theory, Probability and Statistics,* and *Experiences in Problem Solving.*

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