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ABSTRACT

Details about an investigation into the nature of the mathematics curriculum at the elementary level and suggestions about ways to improve existing programs are the focus of this material. It is noted that there appeared to be a great gulf between the type of instruction typically provided and the type viewed as the best possible to provide, with much that was available not used by most teachers. It is felt that ways were found in which teachers and others could work together in developing activities and materials, and that better diagnostic tools were found as well. Further, ways that modest resources could be expended to effect noticeable improvements seemed to have been found. This document is presented with the intent of sharing experiences with others who might be interested in the elementary school mathematics experience. The bulk of the material consists of appendices which individually detail aspects of the exploration. (MP)

# EXPLORATIONS INTO WAYS OF IMPROVING THE ELEMENTARY MATHEMATICS LEARNING EXPERIENCE*

Max S. Bell
The University of Chicago

February, 1976

*Final report on a project supported by the National Science Foundation, grant number PES 74-18938.

# Explorations into Ways of Improving the
# Elementary Mathematics Learning Experience

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## Preface

The explorations discussed in this report were proposed in the early spring of 1974 and supported in May of 1974 with a relatively small grant from the National Science Foundation. By then my own concerns had shifted from the secondary school to the elementary school mathematics experience for several reasons that bear on these explorations: First, serious deficiencies in school mathematics education were apparent from test evidence and from the many adults all too willing to proclaim their inability to use mathematics, even in simple everyday affairs. Second, in working with secondary schools it became apparent that large numbers of students entering high school had negligible understanding of mathematics, or arithmetic even, which clearly indicated flaws in elementary school mathematics. (For example, 80 to 90 percent of ninth graders in some high schools I worked with were assigned to remedial mathematics classes and the average nationwide is perhaps 40 to 50 percent.) Third, most of the otherwise very capable elementary school teachers and prospective teachers I had begun to work with were almost completely lacking in confidence with respect to mathematics and its teaching. Fourth, observation of "mathematics" lessons in elementary school classrooms indicated that in most of them only calculation was taught, usually by rote from textbooks and workbooks. That nearly always meant a narrow and symbol oriented mathematics experience (even in the primary grades) and one with few links to children's lives. This seemed contrary to the best thinking about what would be appropriate for early learning of mathematics.

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*Much of the work reported on here was supported by grant number PES 74-18938 from the National Science Foundation. The present report expands on a preliminary report submitted to NSF in November 1975.

A survey of the output from various innovative mathematics and science curriculum efforts (Nuffield, Dienes, USMES, etc.) turned up a variety of nice materials for this or that specific purpose. But it appeared to me that even the most interesting of these alternate materials might be difficult to use in ordinary classroooms, since they often cover only fragments of a normal year of work, or require special knowledge few teachers have, or special equipment or arrangements that few classrooms have, or special planning and creation of new materials for which few teachers have time. Published opinion frequently cited Piagetian ideas and advocated more variety in the mathematics taught, more activities, and better ties to children's experience. But too little of this sort of discussion seemed informed by first hand knowledge about the realities of the school situations in which most teachers work. In short, the gulf between what was available and what was used seemed quite wide, and the gulf between what was said to be the best sort of early mathematics learning experience and the actuality seemed equally wide.

Given such observations, it seemed important to try to gain a better understanding of the usual elementary school mathematics experience. It seemed that a good start might be to focus on that experience in the public and private elementary schools of my own compact and well defined Chicago neighborhood. In addition to better understanding, I hoped that such an inquiry might locate some threshold not impossibly above present practice where the addition of relatively modest resources could achieve substantial improvements. I also wanted to see if school people themselves could be involved in fruitful development efforts. Such notions were the basis for the explorations that have led to this report.

At this writing, some two years after proposing these explorations, it seems to me that they did succeed in some of their more modest aspirations and that a report on them might clarify some issues and raise some questions for further investigation. We did find ways in which teachers and others can work together to generate excellent activities and materials and, in the process, most participants gained a broader perspective with respect to mathematics and its teaching. We did find efficient diagnostic procedures and preliminary investigations using them have indicated where some of the major learning difficulties lie. I believe we went a long way toward clarification of many of the issues that surround possible classroom use of small calculators. I believe we found ways that modest resources can be expended in a local school system with high likelihood of effecting noticeable improvements. (But zero addition of resources won't do, and patience and persistence over at least several years would be required.)

This report is intended to share our tentative findings with others who may be interested in the elementary school mathematics experience. The report elaborates upon our explorations in four progressively more detailed levels. Section I gives some background for our efforts and a list of the explorations undertaken. Section II attempts an assessment of results, proposes some additional explorations and lines of development, and comments briefly on the place of such work as undertaken here in research efforts in mathematics education. Section III discusses each investigation in some detail. Finally, a series of appendices provides documentation or more extended commentary with respect to some of the explorations.*

My debt to local teachers and principals will be obvious to any reader of this report and is gratefully acknowledged. Similiarly, the considerable contribution of such co-workers as Pamela Ames, Warren Crown, Mary Page, Marilyn Thompson, and Dale Underwood is acknowledged. University of Chicago people helped in various ways and as usual the help of a fine secretary, Dana Gregorich, eased many difficulties.

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*Not all who read this report will have the appendices. Within limits of availablility I will respond to requests for any of them.

## I. Introduction to the Project

### 1.1 Some background remarks

As I began in 1974 to sort out possible ways to inquire into the early mathematics learning experience, such paradoxes and problems as these seemed amenable to preliminary exploration with only modest resources:

(1) Teachers and principals are closest to school problems, yet they are very infrequently consulted about what might help in coping with those problems. I could imagine ways to achieve such consultation and get useful information from it.

(2) In spite of considerable group achievement testing in schools (usually of calculation skills), we know little about what actual mathematical concepts result from the school experience. We even have little detail about patterns of development of specific calculation skills. (Most testing gives only percentile or "grade level" scores but identical scores can come from quite a variety of hits and misses within a test.) Individual clinical testing seemed a way to get more detailed information.

(3) As I began to consider this project it was already obvious that hand-held electronic calculators would become very widespread in the society outside of schools, perhaps even to the extent that most people in their common lives would have little use for the work on calculation algorithms that so dominate the elementary school mathematics experience. Issues arising from that, plus possibilities for using calculators in teaching seemed amenable to preliminary exploration.

(4) Although teachers are the key to doing anything in school, the fact that so many of them have a quite uncertain grasp of mathematics and its possibilities makes genuine change in elementary school mathematics difficult. Although this is an enormous problem, it seemed essential to seek simple ways to make some headway toward its solution.

(5) The very limited range of mathematics concepts dealt with in actual elementary school classrooms plus the remoteness of that learning experience from the common life experience of children made me want to explore with teachers possible ways of broadening the experience along the lines suggested in "What Does 'Everyman' Really Need from School Mathematics?" (<u>The Mathematics Teacher</u>, March 1974).

In such areas as these I found few materials well enough developed to justify controlled experiments or strenuous efforts on behalf of any particular solution. Hence the word "explorations" in the project title.

## 1.2 A listing of the explorations

In retrospect, that initial list of things in need of exploration seems to have been a pretty good one, though other issues intruded and though it led to somewhat scattered efforts. I and five part-time consultants (Pamela Ames, Warren Crown, Mary Page, Marilyn Thompson, and Dale Underwood) found ways to engage each of the issues at least in preliminary ways. This resulted in a number of small scale investigations, as follows:

1. We engaged in a number of useful conversations with school administrators and with teachers.
2. We obtained detailed written responses to a questionnaire about the elementary mathematics experience from some 39 Hyde Park teachers.
3. We sponsored a number of classroom trials of calculators, with teachers supplying impressionistic and anecdotal reports on student responses to the calculators.
4. We tried in several ways to bring about more effective cooperation between the local Teacher Center and the University in increasing services to local teachers.
5. We completed development and pilot trials of two clinical instruments to assess children's grasp of basic mathematical concepts and started a diagnostic and remedial pilot project at one public school which will continue in this school year.
6. We made it easy for local teachers to audit our regular courses at the University of Chicago on behalf of activity-oriented elementary school mathematics teaching, and some took advantage of this.
7. We conducted trials of several models for providing expert consultant help to teachers. One of them has resulted in a close working relationship with a local public school which will continue over this next year.
8. We conducted a series of quite productive working groups that included teachers (with released time and modest stipends) along with myself, the part-time consultants, and others from the University. These working groups specified a large number of non-bookish activities

and experiences to help children acquire a good intuitive feeling about some basic mathematical ideas. (The sessions specified activities for young children with respect to measure, probability, relations, graphing, estimation, variables, logic, and non-computational uses of numbers.)

9. We took a fairly close look at typical textbook materials that tend to dominate the school experience.

As noted earlier, these inquiries are far from constituting precise experiments. Knowledgeable people who are also good observers have tried a variety of things and looked critically at the results, which seems an essential beginning in understanding the early mathematics learning experience. The explorations have led us to a much clearer view of the actual situation in elementary schooling (at least locally) and a correspondingly clearer view of the problems that need solving.

The year-long encounter with schools has also tempered my initial optimism considerably, especially with respect to the prospects for rapid change. Yet I believe that flexible, pragmatic, and patient work over several years could continue to illuminate the problems and, perhaps, produce an existence proof that modest resources carefully expended can lead schools to a very considerable improvement in the early school mathematics learning experience.

## II. Overall Assessment and Some Possible Next Steps

### 2.1 An assessment of the project

The NSF funded part of the project ran for about sixteen months. (It had been projected for six months but this proved quite unrealistic.) In that time we undertook a broad range of initiatives, as listed above. A description of each of these is in Part III, with more detail on some of them in the appendices. Briefly, I can identify at least these tangible products from the explorations:

(1) Some data about how elementary school teachers view the teaching of mathematics, some data about the teachers themselves, and a variety of suggestions about what would help them in their teaching of mathematics. (See Appendix D)

(2) Quite a few non-computational classroom activities specified by a working group of teachers and project people under the title "Everyman Mathematics Development Group." I believe these give us an excellent start in some important but neglected areas. (See Appendix F)

(3) Some clinical instruments for investigating children's knowledge of numeration and place value and the role of these in computation (See Appendix C) We have also attempted a novel use of a commercial written diagnostic test. A simple instrument for assessing children's awareness of some non-computational mathematics was also devised and tested. Again, these seem to me promising beginnings in neglected areas.

(4) Increased knowledge about some of the effects of the usual school experience.*

(5) We have feedback from many informal trials of calculators in the classroom that can now form the basis for more pointed studies and curriculum development initiatives. (See Appendices A and B)

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*For example, cooperation with one local school in its efforts to improve mathematics in the upper grades has confirmed anew that many older students have rather astonishing and pervasive deficits in their understanding of basic concepts. As one instance, many students who are able to compute the answers to the following problems cannot predict in advance of calculation which will give the largest and which the smallest answers: 345 - 34.5, 3.45 + 345, 34 + .0345, .345 - .0345.

(6) A certain number of local teachers are better trained in mathematics and its teaching from attending our courses or from participation in the "everyman" working groups.

It also seems fair to identify at least these more or less intangible results:

(1) I and my co-workers have considerably more familiarity with the elementary school mathematics experience as experienced by most youngsters.

(2) We have increased appreciation of the difficulties under which elementary school teachers work along with the conviction that teachers are given far too little practical help or guidance.

(3) We have enhanced links to the localteacher and administrator corps which should stand us in good stead in any continuation of such inquiries as these.

(4) I have identified a number of very capable people willing to work on such problems as undertaken in this project. The fact that some of the best of these can make a contribution on a consultant or part-time basis helps solve certain problems in such a University as this in working on practical school problems.

Some project activities are continuing by exploiting materials provided by the grant, momentum from grant supported activities, and volunteer efforts. These continuing efforts include the following:

(1) Consultation and help for one local public school in its efforts to improve the fourth through eighth grade mathematics experience by departmentalizing it and setting up specially equipped rooms.

(2) A pilot diagnostic and remedial project in the same school which has parents working with their own children with concrete materials and a "script" that we provide.

(3) Some revision of the output from the everyman activities working group.

(4) Some additional trials with calculators in classrooms.

The initial proposal for support of this project spoke of a search for "leverage points" where relatively modest inputs might bring about substantial improvements. It now seems to me that the most likely such leverage points are these:

(1) Work directed at materials that will help teachers make the early school experience richer in mathematical possibilities, more concrete,

and more "playful."

(2) Work on more accurate assessment of the actual school experience and its results, probably through clinical work with children in school settings.

(3) Providing more practical help for teachers in the form of powerful in-service training opportunities and help in implementing those materials just spoken of to make the mathematics experience richer, more concrete, and more playful.

Developing a variety of ways to provide leverage at such points seems to me the best way to exploit what has been learned from our explorations so far. Some possible next steps are outlined in the section that follows.

## 2.2 A tentative proposal for some further explorations

I believe that we have achieved some interesting and non-trivial results. But promising insights and leads from such informal work often remain undeveloped. Given our progress so far, I believe work of this sort could profitably continue for at least several more years. That is, I would like to see through to completion some sequences that begin with informal exploratory work, develop the results into school usable procedures, test and further develop those through carefully monitered work in classrooms, and, for some of these fully developed procedures, end with some well designed treatment and effects experiments. If such further work could be supported, I would propose about a three year extension of present efforts, perhaps with roughly the following features:

1. We would attempt to map more precisely the actual results of the present early mathematics experience. Since doing that involves a variety of diagnostic efforts, we would also experiment with various ways of intervening with remedial procedures. For example:
   a. A number of teachers would be asked to do some individual clinical testing of their own students, using instruments and procedures already developed in this project. Training would be offered and substitutes provided while teachers worked with their pupils. There would be certain controls for uniformity and quality. By pooling and analyzing the results we should get a pretty good picture of the actual results of schooling over several grade levels. Results could also be used by the teachers themselves and we would devise means of assessing to what extent this

serves to sensitize teachers to problem areas and to what extent they make use of the information thus gained.

b. Given promising results from the pilot project now in progress that helps parents help their youngsters in specific ways, a carefully controlled test of this method of intervention would be conducted.

c. A small working group would explore the potential of using calculators in various ways for diagnosis of difficulties with number and computation concepts.

2. Working groups would be formed to develop materials and activities in support of a richer and more concrete early mathematics experience.

   a. Several of us (probably Ames, Bell, Crown) would revise some of the materials developed by the "Everyman Mathematics Development Group." We would put them in a couple of formats intended for easy use by teachers with a minimum of explanation or training required. We would then arrange for classroom trial and feedback.

   b. Working groups would develop everman activities similiar to those at hand but in additional areas.

   c. In perhaps the most ambitious curriculum development effort of an extended project, a carefully chosen working group would attempt a thorough overhaul of the primary grades calculation curiculum. This would be an elaboration of the alternative approach suggested at the end of Appendix B and it would greatly enrich the numerical and computational content of grades 1-3, with considerable concrete work and using calculators as teaching aids in developing concepts. I would help teach the first drafts of such materials and several teachers would with a lag of a few weeks for revision then try out a first revision of them. These first efforts would aim to establish existence and practicality of alternative approaches; further development would naturally depend on whether or not that were successful. There would be careful evaluation, both to protect the youngsters involved and as a check on feasibility.

3. We would attempt to find practical ways to increase the mathematical expertness of average teachers.

   a. In cooperation with local schools and with the Hyde Park Teacher Center, several careful trials would be conducted of "Edith Biggs-type" workshops. (E. Biggs, <u>Freedom to Learn</u>, Ch. 6) A typical workshop would begin Friday noon (in the in-service time already provided periodically for local public schools). It would continue into that evening and for most of the next day with intensive work on single themes. Specific materials and suggestions for classroom work would be offered participating teachers. Over about a two month period there would be various sorts of consultation and help offered these teachers. A second Friday noon through Saturday workshop would then be held. It would include some activities desighed to assess the effectiveness of this form of in-service activity. (If such efforts proved to be effective, then they could be duplicated by many people in many places.)

b. As we identify especially competent people through our work with teachers in a continued project, we would try in a variety of ways to help them develop further. We would do this both to make them more effective in the project itself and also to experiment with ways of increasing over the long run the presently very meager supply of mathematical expertness resident in school staffs. Tuition scholarships for work in our courses and in mathematics courses, grants for special projects, pairing with especially able staff members for certain tasks are some of the leadership development things that might be provided. As such in-school leadership emerges and is further cultivated a small working group including some local administrators would work out ways to best exploit this talent.

These probable directions of a continued project do not exhaust the possibilities but they are illustrative of the things we think might now be accomplished. Such explorations would continue in much the same way as those now completed, except that I would free myself to spend much more time in them than has been possible up to now. I would continue to rely heavily on gifted part-time consultants. It would remain a small scale and low key effort, with a yearly budget of under $50,000. A small local advisory committee would be appointed, as well as a neutral outside observor/evaluator.

Some might think that the project's base in a small local system makes it marginal for national funding but I would hope that such a misconception would be easily dispelled. The nature of the inquiries makes and actual school system laboratory essential, but the results should have wide applicability. (In fact, the school development base should assure increased relevance to other settings rather than the reverse.) This is certainly true of specific materials and procedures; for example, the clinical work and the everyman activities. I would also be true of most of the things that might be developed in a continued project; for example, alternatives that may be developed for the primary school calculation curriculum. Furthermore, I believe that any models we develop for fruitful school-university interaction in school improvement should also have wider significance than to merely the local situation. Certainly there is far too little such interaction at present.

Some might object to the lack of "precision" and controls in some of the explorations reported here, but relatively loose mehodology is an appropriate consequence of the uncertain state of knowledge at present with respect to early mathematical learning. Some comments on how to proceed in the face of our relative ignorance are in the section that follows.

## 2.3 Some remarks on "exploration" and "research" in mathematics education

It may be helpful to attempt comment here on how explorations such as those discussed herein may fit into other scholarly efforts to better understand schooling. It must first be noted that at present most "research" about mathematics learning is characterized by "no significant difference" results from tests of this as against that curriculum segment or teaching method. Many of these tests are limited studies for Ph.D. theses and many are poorly set up with respect to such basic matters as sampling and controls. There exist a few careful studies that can serve as models for some true experiments in mathematics education, but it seems to me that there are at least two fundamental barriers to conducting relatively precise treatment-and-effects research. The most obvious barrier is the scarcity of attractive treatments; that is, there are few curriculum and pedagogical alternatives to normal practice both well enough developed and promising enough to justify precise testing. A second and more fundamental barrier is that our understanding of mathematics learning is so incomplete that we hardly know what to treat. Some elementary school teachers are very effective in teaching mathematics but too many are not. Some children learn mathematics very well from a given set of experiences, but many of their classmates with the same classroom exposure do not. Whether for teachers or for youngsters, our knowledge about why some succeed and some do not is very far from precise, and relatively little effort is spent on increasing that particular knowledge, compared, say, to the considerable effort expended on multiplying printed treatments of surface symptoms. To risk an analogy, it is as if severe abdominal pains were epidemic in a certain population and we didn't know either for the group or for individuals whether to treat for indigestion, food poisoning, ulcers, or appendicitis.*

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*To extend this analogy, at even more risk, a typical mathematics education sort of response to the abdominal pains might be for one doctor to be sold on treatment with Valium coupled with 100 pushups per day and another to adopt Miltown with 75 situps per day. Each would point to the survivors as evidence of efficacy of the treatment, and someone would get a doctorate with a "no significant difference" finding.

In this situation it has seemed to me that there should be more stress on close observation, shrewd trial and error, and development activities by mathematicians and mathematics educators working in school settings with school people. This is not common in the United States but some quite intriguing results have come from Soviet efforts of that sort. (For example, see Kilpatrick, Wirszup, et. al., editors, Soviet Studies in the Psychology of Learning and Teaching of Mathematics , SMSG.) For such work to be most effective, schools are needed both as laboratories and for the insights from first hand experience that many teachers can provide. Knowledgeable and committed university people are needed for their expertness in subject matter, for their disposition to survey possibly relevant literature and consult experts in various disciplines, and, not least, for the fact that they have time and resources usually not available to school people.

I do not, of course, believe that such explorations are the only things appropriately to be done on behalf of improving the early school mathematics experience. I believe that in a few areas we can already move from trial and error to carefully observed (but small scale) clinical trials of specific things (for example, our diagnostic-remedial efforts with repsect to place value now in progress in a local school). Closely observed trials of excellent curriculum materials seem important. Where these clinical efforts and small scale trials are promising, true experiments on a larger scale might follow.

## III. Reports on the Explorations

The paragraphs that follow correspond to the small scale investigations that were listed in Part I of this report. More detail about some of these projects is to be found in the appendices.

### 3.1 Conversations with teachers and principals

First it must be said that it is enormously helpful in generating useful conversations to have even a few hundred dollars of funding that can be flexibly used. For one thing the responsibility for using such funds has caused myself and those who worked with me to think about what might be done and to take the time to ask a number of people for advice. For another, having something tangible to offer, no matter how modest, is a novelty when university people talk to school people. (Usually we are somewhat exploitive--wanting a locus for research, help with teacher training, tuition for our courses, or whatever.) Third, being able to back up the trial of good ideas lets a conversation proceed fairly freely without constantly coming up against money barriers.

Local administrators and teachers are more than willing to give time and thought to considering what might be done to improve the school experience of youngsters. Furthermore, far from the stereotype sometimes presented of bureaucratic and uncaring principals and harassed and neglectful teachers the level of concern and willingness to consider new alternatives is remarkably high. Some school principals and teachers are very astute and insightful observers. The fact that (especially in the public schools) both principals and teachers feel frustrated by countless things beyond their control does not lessen the level of concern nor their willingness to try to overcome the handicaps that they work under.

The yield in information and ideas from these conversations, which I regard as considerable, can at most be implicit in the summaries that follow. In addition to that, much good will and many useful contacts have been generated by these conversations that will be very helpful as we follow up on these exploratory efforts. Of the 6 public elementary schools in Hyde Park an excellent day-by-day working relationship has been established with one principal and many of his staff; an excellent foundation for further work has been established with two other principals; we have friendly but not close relationships with two additional public school principals; we have not worked with one principal. Of the private schools, one teacher who worked with us closely and effectively in the winter quarter is the new elementary grades principal at a local private school; a Laboratory School assistant principal that we have worked with for some time is now director of another local private school; the local Catholic parochial school principal has been more than cooperative and helpful and has given released time to a number of her staff people for work here. We also have links to all Hyde Park schools except one through teachers we have worked with during the year.

In sum, the informal conversation and consultation phase of the project has been productive and provides a sound base for future work.

3.2 A questionaire about the school mathematics experience

We learned (by administering it) that our questionnaire was more detailed and time consuming than need be. Also we asked for responses late in the school year when teachers have much else on their minds. Nevertheless, we got detailed and very helpful responses from thirty teachers from four local public schools and nine teachers from the Laboratory Schools.

The complete record of responses to the questionnaire is included in Appendix D to this report. Here are some of the results:

1. 87% of the teachers responding used a regular, standard textbook series in their classrooms and more than half of them were satisfied or enthusiastic about it.
2. The best known "alternative" math programs were Madison Project (with 18 of the 39 respondents expressing familiarity with it), Nuffield Foundation (17), SMSG (15), and Distar (14). Even the most familiar programs, however, were only being actually <u>used</u> by 6 teachers.
3. The best known manipulative teaching aids were Cuisenaire Rods (35 of the 40 teachers expressed familiarity with the rods.), number lines (also 35), geoboards (34), graph paper (34). However, access to these materials seemed to be minimal with fewer than one-third of the teachers who expressed familiarity with a material actually having access to it.

4. Teachers were asked to rate the helpfulness of eight sorts of added resources. All were rated high with little distance between them to get a clear-cut consensus on preference. The three highest rated were help in diagnosing learning difficulties, help with youngsters who are discipline problems or need extensive individual attention, and help with learning more about manipulative and laboratory materials.
5. This exploratory questionnaire included several questions about how teachers feel about teaching various subject matters (reading, language arts, mathematics, social studies, and science), how confident teachers are of their own ability to teach those subjects, and how important each is. Reading consistently ranked first by a wide margin with respect to enjoyment, self-perceived competence in teaching, and importance; language arts was consistently second. Mathematics consistently ranked third in enjoyment, preference, and self-perceived competence, often close to language arts, and in the case of "most important" even ahead of it. Science was in last place in all respects, and by a wide margin. I find that disturbing in spite of the higher ranking of mathematics because I believe that most teachers responded with respect to mathematics on the basis of the sort of symbol-oriented computational exercises that they are accustomed to teaching. I believe that the activity orientation that may be associated with the teaching of science should characterize the early mathematics learning experience. Hence the low estate of science bodes ill for improvement of mathematics teaching in the direction that I believe would be most fruitful.
6. In three places in the questionnaire we asked teachers to write down their own ideas about what might be done to help improve the mathematics learning experience and to put a price tag on their ideas. The responses we got were interesting; they are included in the full report on the questionnaire in Appendix D.
7. We asked if responding teachers would like to participate in certain of our regular summer workshops (audit or for credit).

Sixteen said yes and 10 maybe but that summer only 3 teachers actually participated. (The questionnaire was given very late in the school year after summer plans have largely been made.) Similarly, there were 14 yes and 14 maybe responses to the possibility of participating in our regular Autumn quarter course on activities oriented teaching of elementary school mathematics; 5 teachers did participate to some degree in that course. For a similar inquiry about the possibility of joining an Autumn quarter discussion group aimed at improvement of the early mathematics learning experience, 23 teachers said yes and 3 maybe. This appears to have been a genuine commitment, for when the opportunity was actually offerred (in the winter quarter) a large number of teachers responded.

8. For these teachers, a mean of something over 40 minutes every school day is spent teaching mathematics.
9. Teachers responding had experience ranging of from 1 to 25 years with a mean and median experience of about 11 years. This confirms our impressions that we have a stable and experienced teacher corps in this community.
10. The questionnaire results very much confirmed that these teachers have little training for the <u>mathematics</u> teaching task. Of the 39 teachers 10 have <u>not</u> had a mathematics "methods" course in college, 21 have had just one such course. With respect to college mathematics courses, 17 have had <u>no</u> such courses, 10 have had one, 7 have had 2. It is safe to assume that even those who have had one or two courses have not gotten much beyond basic school mathematics.

I have from the first felt that this lack of training in mathematics is perhaps our most serious handicap. Most elementary teachers simply do not have a basis in mathematical knowledge by which to judge what an improved mathematics experience might consist of. Furthermore, they do not have the basic concepts and language that allows informed conversation with expert consultants about the possibilities. I continue to believe that we must, in the words of Marshall Stone, "stop prating about this obstacle and take counsel as to how to remove it!" (Our experience in courses we have developed here indicates that considerable headway on this can be made in a single course of a certain kind, and much more in two such courses.)

## 3.3 Classroom use of small electronic calculators

When this project was proposed in early 1974, a request for $1500 was included with the expectation that it would buy about 15 calculators to lend for classroom trial. By the time we bought the first calculators they were $35 instead of $100 (later they were $25, now they are $10), and we also obtained calculators from other sources. This meant that we were able to obtain and lend about 45 calculators and this enabled quite a number and quite a variety of classroom trials, ranging from an exploration with first grade students to one with Kenwood High School remedial classes. Most of the trials were quite informal and although a couple attempted pre- and post- testing none was done with anything like "scientific" controls, randomization, and the like. This category of exploration was one of two things that elicited the greatest "grass roots" response both in number of interested teachers and volume of feedback. Some reports on these informal trials are in Appendix A.

Here are some impressions from those efforts:

1. Youngsters of all ages and of all ability levels respond immediately and enthusiastically to the opportunity to work with calculators.
2. Generally speaking it is unnecessary to provide instruction in using them. That is, given a half-hour of exploration no one failed to figure out how to use their calculator and in some cases students had by then already figured out the individual quirks of various brands of machines. Students were respectful and careful of the calculators and none were either damaged or lost in actual classroom use.
3. The general pattern is that students at first do everything in sight on the calculator, but quickly settle into selective use of the machines for those problems where they are really most helpful.
4. At first the novelty of punching buttons to get results was sufficient motivation, and indeed the punching of buttons and

watching the numbers appear remained a continuing fascination, but fairly quickly students tired of doing only that. They did not then become bored with the machines but demanded to be given something to do with them. At that point many teachers were stuck because the materials they use in class simply do not include the stimulating projects and opportunities for fruitful use of such tools that the youngsters were demanding. In most cases, then, the machines became merely a way for students to check their written work and an occasional resource when some genuinely interesting problem with computational difficulties came up. When these machines catch on in schools they will make newly apparent what we must have known all along; namely that there is in school work very little that is interesting to do with calculation! We must do something about that.

5. While there was considerable demand for the calculators among teachers there was also some apprehension from most teachers and downright resistance from others. The apprehension and resistance seemed to be based on several factors. First was the feeling that the calculators would rot the mind by leading either to a decline in calculation skills for students with those skills or a failure to learn such skills. Akin to this there appears to be an underlying feeling that it borders on the immoral to use such a machine to do what can be done by hand. Thus in commenting on the use of calculators many teachers felt it might be OK to have them available for certain selected problems after about 5th grade, when calculation skills were presumably already in hand, but not before then and never for continuous use.

   Another source of unease in teachers relates to possible difficulties in keeping track of the machines--one teacher flatly refused to even consider having them in his classrooms because "Those machines have legs!" Thus the first batch of machines that I lent to teachers quickly ended up in the school vault for fear that they would catch hell if any were lost or broken. The calculators only came out of the vault when I explained that part of the experiment was to find out whether they would be stolen or broken. As already observed no machines were broken or lost in classroom use but the fears of teachers were justified by the fact that a total of 5 machines in several places were stolen by breaking into cabinets or pilferage from teacher's desks, apparently not by the users but by people outside the class who knew of their existence.

Another difficulty was the fact that all these machines are battery operated and teachers by and large simply couldn't go to the trouble or the expense of obtaining replacement batteries. Often when I checked back I would learn that students and teachers liked the calculators very much but they were in storage because of batteries that had run out. This may seem a trivial barrier to using such a tool but it confirms the experience that many people have had in advocating various kinds of "hands on" materials. Added to other duties it is often just too much trouble for teachers to keep such things operating and in proper repair. In addition, the non-trivial expense of batteries is very likely to be troublesome since the cost shouldn't come out of the hide of the teacher and schools are generally not set up to provide for such things. Rechargeable machines might be an answer but that also requires extra work on the part of the teacher and having machines plugged in makes them less secure than in a cabinet. It is just such apparently trivial difficulties that have swamped the use of promising educational aids in the past.

6. As noted, many teachers say essentially that calculators are probably fine as computational aids but only for someone who has already learned computation. But both Mary Page working with first grade youngsters and Sheryl Garmony working with third graders in a remedial summer school situation found that the machines have high potential for helping kids in explorations of numbers. For example, young children seem to get a lot of fun out of displaying a number on the machine and asking a friend if he can read it, with some competition developing in that respect. Such explorations could very well inductively teach many things about numbers and computation. That is, carefully considered and staged developmental work with the calculators as one tool might very well

pay dividends. (See Appendix B.)

7. As noted before, children very quickly learn from their own efforts the peculiarities of each machine and are able to switch from one machine to another. But some intriquing inquiries can follow from various machine types. For example, Mary Page working with first graders found that the type of display of nearly all machines (feeding in from the right) appeared to lead to more number reversal errors (e.g., reading 86 for 68) than did the Hewlett-Packard type of display that records from the left of the machine much in the way that numbers are written. Also, we had only simple machines, but we can imagine that some good pedagogical sequences could begin by questions <u>from youngsters</u> about the extra buttons on fancier machines: tan $x$, $10^x$, $\sqrt{x}$, and so on. We intend to explore that possibility.
8. Even though these machines are now very cheap (which in itself will alleviate some of the problems that come from fear of theft and breakage) no local school is considerring the purchase of them as a routine item although several teachers who borrowed our machines last year persuaded their schools to buy calculators for their use this year. These machines are sure to become very widespread in the world outside school--one prediction is that there may be 70-million calculators in use in the U.S. by 1980. Hence, schools probably should take account of them, but up to now they seem to intend to ignore them.

In sum, our preliminary explorations with small calculators indicate several things: They generate considerable and durable enthusiasm on the part of youngsters--and on the part of those teachers who are willing to try them at all. There are too few meaningful problems in school books to exploit the calculators and so they usually end up as electronic answer books for conventional computation work. Problems of keeping machines in batteries makes them troublesome to teachers. Security and theft is a very real problem, apparently not so much from users as from outsiders. Youngsters appear to develop good judgment about when it is and is not appropriate to use the machine, relying on doing simple things in their head, if they are able to do so.

Some intriguing research questions are opened up by the removal of computational barriers and by the potential of the calculator as producer of patterns and sequences. We believe some careful clinical trials of these possibilities are now in order.

### 3.4 Collaboration with the Hyde Park Teacher Curriculum Work Center

A modest amount of money was included in the initial NSF grant to be used by the Teacher Center in ways consistent with the grant. Their report is included as Appendix E.

In our opinion the special efforts of the teacher center in providing some special workshops and an on-going seminar were well conceived and well meant but not very successful in increasing the extent to which local teachers use the Center. Use of the Center by local teachers is not as heavy as we believe its high quality would justify, except among that minority of teachers in local schools that also live in this community. That is, use of the Center appears to be more a matter of residential proximity than teaching in the community. We should continue to work on this, for the Center has enormous potential.

### 3.5 Development of instruments for clinical work with children

A graduate student and experienced teacher of teachers, Dale Underwood, worked with modest support from the grant through the spring and summer of 1974 on the problem of assessing children's grasp of place-value, base ten numeration, and links of those to computation. Underwood's study of the problem was inventive and thorough. He identified some key issues and then embodied those in an instrument that takes about a half hour to administer in a one-to-one interview with a child. Underwood and another graduate student, Susan Beal, then did 30 interviews at second, third, and fifth grade levels in the Kozminski summer school to validate the usability of the instrument and establish some preliminary norms. The instrument is included in Appendix C. We believe it is a useful beginning in a key area. As long as we concentrate as heavily as we do in American schools on computation, there is little doubt that children's ability to understand and flexibly use place value numeration is a very important matter.

A second interview instrument that takes only 10 to 15 minutes and that can be used quite informally was developed by Warren Crown, formerly an elementary school teacher in Philadelphia, now a graduate student here at the university as well as a teacher of teachers at Governors' State University. That instrument focuses on non-computational skills and perceptions of youngsters, is quick and easy to administer, and we believe it should prove useful in getting a different sort of information than is usually sought in schools. That instrument was validated by having a number of Crown's pre-service and in-service teachers report on their administration of the instrument to a few children each. The instrument and some preliminary results are included in Appendix C.

A third clinical project using grant funds has just begun at Kozminski school. This pilot project involves an initial screening with respect to computation skills using the Stanford Diagnostic Arithmetic Test (published by Harcourt Brace) followed in the pilot project by clinical interviews to validate and refine the findings from the written instrument. This in turn will be followed by an attempt to specify remedial sequences using concrete materials to be administered by the parents of the youngsters involved. If the results from the pilot project are promising we hope to set up a larger scale experiment later in this school year with some attempt to randomize selection of experimental and control groups.

Though limited only to validation, our clinical investigations give disturbing indications that many children have little or no understanding of counting, base, or place value as these operate in numeration and calculation. This is true even of many who can usually get correct answers to calculation examples typical of achievement tests, provided they are told what operation to perform. Though similarly limited, our interviews directed at non-computational mathematics give discouraging indications with respect to such pervasive matters as measure, order, approximation, and reasonable cost or amount in various everyday situations. These matters certainly merit further investigation.

## 3.6 Including teachers in workshops and courses

As noted in the description of responses to our questionnaire, many local teachers expressed an interest in auditing courses or attending workshops here at the university. All teachers who responded to the questionnaire were then sent an invitation to audit our regular summer and autumn courses in mathematics education. Through summer 1975, ten local teachers have opted for such participation and have followed through with varying degrees of regularity. This has given us excellent contacts with some local teachers and some new links with certain schools. Working with a teacher in such a course gives an excellent basis for further communication; it is easy then to talk about adaptation to classroom use of whatever the content of the course might be.

Based on this experience I believe it is feasible and appropriate for this University to offer such low-cost opportunities for teachers in Hyde Park schools. The quality of local schools is certainly important to the quality of life in a university community, hence a non-trivial matter for the university as a whole. Demand is not so heavy as to be burdensome, though some courses must be scheduled at after-school hours and this is sometimes a troublesome matter. To make this routine rather experimental will, of course, require explicit permission and encouragement from the University.

We responded to a request from Laboratory School teachers for a series of five workshops on things specified by their steering committee. These were successful and established a good basis for cooperation with the 15 or so teachers who participated. But as a general rule responding to such individual requests is too demanding of regular university resources--to respond separately to each of the schools in Hyde Park would be an impossible addition to the load of regular staff here. It could however, be accomplished (with appropriate funding) using those consultants that were so effective in working with me on this project. A more basic problem is to follow-up such workshops to make them more operative in actual classrooms than is usually the case.

With respect to workshops, we don't believe that isolated workshops are very productive except perhaps in very special areas. We would like to test here the sort of in-service work described by Edith Biggs in

Freedom to Learn. She holds intensive workshops for a few days with a fairly large number of teachers. Participants then follow-up on those workshops in quite specific ways in their own classrooms for several months and then participate in another intensive workshop. Such a scheme seems to us both more manageable and more effective than responding to a number of separate requests. It could be done cooperatively with the Teacher Center, which would also have a good effect on University, Teacher Center, and local school links.

3.7 Providing consultants to schools

In elementary schools generally there are very few teachers expert in mathematics and our questionnaire results indicate that this is true in local schools as well. One obvious way to deal with this lack of inside expertness is to provide expert consultation from the outside, but being helpful in this way is not easily accomplished and effects often seem not to outlast the consultants tenure. A few years ago, we worked out one spectacularly successful instance of an outside consultant making a durable difference when Pamela Ames (as part of the Ford Training and Placement Program here) made herself available on a low-key, everyday basis to the mathematics department of DuSable High School. By the end of that year the staff had made giant strides in establishing close internal working relationships and in curriculum innovation suited to that school, and the effects have persisted now for over five years. But in that case we were working mostly with people well trained in mathematics and with mathematics teaching as their main job--a situation very different from most elementary schools. Hence as part of the present explorations we sought to get some experience with consulting relationships in local elementary schools as a basis for trying to find ways to make a similarly durable difference by this route.

1. Pamela Ames spent the Spring 1974 quarter in a local school three or four mornings each week. We asked the school to specify how she could best help and the principal asked that she work primarily with a single teacher with a special interest in teaching mathematics. Mrs. Ames' efforts to expand the role to include

consultation with other teachers were politely turned aside by the principal and the teacher she was working with. Mrs. Ames did learn a great deal about practical difficulties in implementing some of our pet suggestions for improving mathematics learning. (For example, the difficulties of setting up cooperative group activity with youngsters not used to such cooperation, or non-bookish activities with those used to relying heavily on books.) The one teacher who monopolized Mrs. Ames benefited; she later sent us a letter describing with some enthusiasm a mathematics project exploiting paper airplanes that she was working through with her classes. Mrs. Ames created some nice activities for building computational skills, which appears to be the overiding preoccupation of elementary schools. That is, from our standpoint, it was a useful experience, but we don't think we found a way to "cost effective" consulting with durable effects.

2. Marilyn Thompson worked with us part-time during Autumn and Winter quarters and several schools were invited to use her as consultant, again according to their own specifications. The schools involved seemed eager to have such help, but were puzzled about how to use it. Mrs. Thompson worked individually with a half-dozen or so teachers and helped at least one interest group of teachers in a school get started on discussing their mathematics program. She made some useful contacts and fostered considerable good will but we did not find a way to truly effective consulting relationships.
3. Mary Page worked part-time through the entire 1974-75 school year in a somewhat different way with Nancy Hanvey, a gifted first grade teacher at Kozminski School. Several mornings each week Mrs. Page worked in the hall with 3 small groups of youngsters. This suited our wish to explore young children's responses to calculators and to use of concrete materials and was Mrs. Hanvey's preferred way of receiving help. (We have often found that teachers offered help think first of just such an

arrangement: "Please do something with this group of kids that I haven't time to help in the way they need.") Again, we gained valuable experience with younger children, and helped a single teacher, but we don't claim to have set up a good model for use of consultants. Some of Mrs. Page's interesting notes on the experience are included in Appendix A.

4. As noted earlier, during the course of the year I had conversations with many teachers and most of the principals in Hyde Park, but I spent more time in Kozminski school than any other place and I believe I became accepted there as a helpful consultant. This also served my purposes of becoming more intimately acquainted with what goes on in elementary schools so it was a good bargain all around. By year's end the very able principal of Kozminski, Alan Travis, had persuaded himself of the usefulness of adding more of an emphasis on mathematics learning to his existing emphasis on learning reading and had taken several steps in that direction for the 1975-76 school year. These included designating the teacher most able in mathematics as specialist teacher in the upper grades. He took the further step of departmentalizing for 1975-76 the 4th, 5th, 6th grade learning experience and providing for specialized mathematics teaching there. Neither "specialist" teacher is especially well-trained in mathematics, but they like mathematics and should be effective, especially if we can give them some help and advice. In both cases the principal offered financial support for setting up their classrooms as mathematics laboratories. In the equipping of these rooms and the selection of special materials our services as consultant were used. Small amounts of money remaining from the NSF grant were used to continue a special consultant arrangement at Kozminski in the person of Pamela Ames as school began this autumn. I will also continue to involve myself there. It seems a good place to develop further some of the things we think we have learned by these explorations.

It is clear from our experience that the mere existence of excellent and expert consultants does not assure their effective use in schools. For one thing, many teachers are ambivalent about consultants--they ask for such help, but then often hold the consultants at arms length. I believe the main factor in this is, again, poor training and basic insecurity with respect to mathematics and its teaching on the part of most elementary school teachers. One teacher I worked with made this explicit as he tried to find time to take one of my courses: "I don't want to talk with you about my teaching of mathematics until I know what I'm talking about." Local teachers know that we feel calculation skill is often too exclusively emphasized and that we'd like more non-bookish inquiry and activity. They may well agree with us in theory, but their present way of making it through the year at least works (for most of them) and they feel just too swamped to remake themselves in this respect. That is, they need more than good advice.

Another factor in teachers' ambivalence about consultants may be the fact that in most teachers' experience such people are here today and gone tomorrow, or on hand too infrequently for appropriate follow-up. Certainly it is true that we are most accepted as consultants in those two schools (Kozminski and the Laboratory Schools) where we have appeared regularly and consistently over one or more years and have gotten to know some teachers fairly well so that a call for advice on a personal basis is possible. That sort of relationship is too expensive to be provided as a general thing even to a small subset of schools by regular University faculty or by a city school system. If we knew better how to make consulting effective in helping teachers that are far from expert in mathematics, it would not be expensive to provide part-time consultants such as the five people that worked with us so helpfully over the past year (graduate students and women professionals now raising families). But before proposing that we need to further explore ways to make consulting roles more productive. We intend to do that this coming year as Mrs. Ames and myself continue to work closely with Kozminski school (and less closely with some other Hyde Park schools). It seems likely to me that building teacher knowledge (and confidence) through participation in working groups, workshops and courses plus giving them ready access to expert advice may work well but any such factor in isolation is relatively ineffectual.

### 3.8 "Everyman Mathematics Development Group"--teacher working groups to create activities for non-computational mathematics

Many teachers in their questionnaire responses indicated their interest in working groups. We tried one such session in the autumn quarter by getting together a number of teachers and asking them to specify what they wanted to work on. Such open-endedness was clearly not the way to go about things and so we abandoned that attempt. From that experience we concluded that it would work best to provide considerable structure for the working sessions and to make them very task-oriented. We therefore planned sessions for the Winter quarter around a list of concepts and skills from my article "What Does 'Everyman' Really Need from School Mathematics" (The Mathematics Teacher, March, 1974). We then offered released time to teachers and modest stipends ($15 per two hour session) for a session to be held every Tuesday afternoon starting during school hours. At one time or another about 25 people participated in these sessions. (More expressed interest but we had difficulty in getting substitutes in order to provide released time. We know how to handle that now and another time it would probably not be a barrier.)

For each session we selected one of the 'Everyman' topics for attention. For the first part of each session there was a presentation and a discussion about that topic which was continued until it appeared that those present understood both the topic and the sense in which it could be useful to most people. Each participant would then write out activities relevant to that topic keyed to certain grade levels (usually the grade levels at which they were experienced teachers), and add them to the pile accumulating in the center of the table. Two staff people working with us in the project (Mary Page and Marilyn Thompson) would take the pile of contributions thus gathered, sort them into categories and edit them. Those raw materials were typed and then duplicated and returned to participants during the next session.

Some of the results of working in that way for eight topics in nine sessions are included in Appendix F. (Activities were listed for measure, probability, relations, graphing, estimation, variables, logic, and non-computational uses of numbers.) We were impressed by the output. We were also impressed by the number of times teachers would take an idea for an activity from a session, try it with children, and report results in the next session.

We believe several things were important to the success of these sessions and nearly all of them have something to do with the signals that we gave teachers that their contribution was needed, respected, and valued. Providing released time means that we got the best of their effort and not just what was left over after a tiring day. The stipends, though modest, were a tangible signal that what they had to offer was valued by us. We presented the topics in the spirit of "Here's what we think ought to happen but we think you know best how to make it happen" and this elicited warm and cooperative responses. The preliminary discussion at each session was probably essential because relatively few teachers would have a good idea without such discussion about what might be meant by such words as "relations," "coordinates," "probability," or "approximation." Even if they know the words they might lack a sense of how each denotes skills or gut-level intuitions for everyman rather than something taken from pages in a book.

Three of us who participated in most of those sessions (Bell, Ames and Crown) are now working through that raw material again. We are attempting to add written prefaces which approximate the exploratory discussion that began each session and that defined what is meant by each topic. We will then rework the raw material into a form that might be useful to teachers outside the group that produced them.

In sum, we found these to be productive sessions in the actual materials produced, in the fine working relationships that developed between us and a number of Hyde Park elementary school teachers and in the amount of self-education and increase in confidence that was evident. Hence, even if the written production were not to achieve the high quality that we believe was achieved in this case, the process and the dynamics of the experience itself make this sort of effort very worthwhile. I believe this sort of effort is among the most promising of the things we did this past year and among those things most deserving of follow-up and further work of the same kind.

3.9 Summary

In the original proposal these inquiries were to have been completed within about six months but that proved unrealistic and instead the NSF funded part of the project ran for about sixteen months. In that time we chose to try a broad range of things even at the cost of some superficiality. Among more or less intangible results for those of us who have worked directly in the project I would list considerably more familiarity with the actual early school experience of youngsters;increased appreciation of the difficulties under which elementary school teachers work; increased awareness that teachers are given far too little guidance or practical help in arranging the mathematics learning experience; and considerably enhanced links to the local teacher and administrator corps. As to tangible results, it seems to me that the activities produced by the "Everyman Mathematics Development Group" have high potential; the clinical instruments for investigation of place value and numeration concepts, and of children's awareness of non-computational mathematics are promising beginnings in neglected fields; and the feedback from many informal trials of calculators in classrooms forms a solid basis for more pointed studies and curriculum development. We have found teachers and administrators very willing to respond to initiatives on behalf of an improved mathematics experience but, with few exceptions, unlikely to take the initiative themselves. We have found that many teachers are aware that help is needed and we have found several promising ways of helping them. In at least one local public school a serious effort has been undertaken to improve the mathematics experience and this school may be a nice laboratory in which to test out various routes to such improvement.

Our most discouraging finding is the gulf between prevailing school practice and a widespread consensus outside of schools that the early school mathematics experience should be solidly anchored in activities, direct experience, playful manipulations, and so on, with mathematical ideas built into these experiences. One need not be allied to Dewey, Piaget, Bruner, Dienes, etc., to recognize that such early experience is at least useful and is perhaps crucial, yet most classrooms have none of it at all. (Most first grade books have <u>pictures</u> of objects and activities, but that is surely no adequate substitute for direct experience.) This violation in early school practice of what is probably the most fruitful means to mathematical competence may go a long way toward explaining the failure of the school mathematics experience for so many people.

The initial proposal for these investigations spoke of a search for "leverage points" and work directed at the early school experience seems to me the most crucial such leverage point. That might take the form of more accurate assessment of results of the typical early experience, better in-service training for teachers, and further development of alternate materials and activities that can make the early experience richer and more concrete and that are *easily* usable on a day to day basis in classrooms. Any continuation of the present project would focus on those things plus some interesting possibilities raised by widespread availability of inexpensive electronic calculators.

# APPENDICES

# Appendix A: Commentaries Based on Calculator Trials in Schools*

## Introduction

At the time I started these explorations, little had been done with hand-held calculators in schools. My approach was simply to lend calculators to teachers, have them make whatever use they thought fruitful of them, and ask them to comment on their uses and the children's reactions. Obviously, more systematic work than this now needs to be done, but these simple trials with feedback did give quite a lot of preliminary information, answered some questions, at least tentatively, and suggested some problems that should be investigated further.

This appendix begins with tentative comments and conclusions with respect to a number of questions that often come up in considering school use of calculators. Following that some evidence for those comments and conclusions is given, first by reports from several teachers (first, third, 5th-6th, 8th, and 9th grades), and second by a potpourri of brief remarks that are representative of many teacher comments.

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*This is an appendix to M.S. Bell, Explorations into Ways of Improving the Elementary Mathematics Learning Experience, a report on a project supported by NSF grant PES 74-18938.

## A.1 Some frequently asked questions with some tentative answers from classroom experience with calculators

It cannot be too strongly emphasized that these comments are based on quite informal trials and feedback from only about twenty teachers. Consideration of any particular question may depend on feedback from only one or two teachers. That understood, it still seems useful to try for first approximation comments on a variety of questions. Roughly speaking, questions about learning possibilities are at the head of the list, with questions that touch on administrative matters at the end of the list.

### 1. Is explicit instruction in using the calculator necessary?

Based on our trials we believe that at least from fifth grade on children learn to use a calculator very quickly (usually within an hour) with at most a worksheet that makes them confront various possibilities. They tend to learn to do the things they know something about; e.g., if they are unfamiliar with division they ignore that key. They learn both from the machine and each other; discoveries about shortcuts and particular quirks are quickly shared. (Despite this, there are already advertisements--e.g., in Learning magazine--for filmstrips on how to use a calculator!)

Certain concealed features, such as automatic constants or repeating operation keys sometimes need to be pointed out but correct use of them is quickly mastered once attention is drawn to them.

Some teachers who used the calculators in grades 1-4 taught several students explicitly and these students taught others. But self-teaching was effective in the Page first grade and Garmony third grade groups.*

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*The references herein to specific teachers or grade levels relate to anecdotal reports on classroom trials of calculators that are included in the next part of this appendix.

2. Do children "naturally" detect errors; that is, do they reject clearly unreasonable answers?

On this question we have mixed reports; some explicit research would be profitable. Tentatively we feel that older children who have good "number sense" can carry this over readily to calculator work but do not do so automatically (see Schafer report). Similarly, for younger students, some but not all of Page's first graders were uncomfortable about wrong answers and asked the teacher to look at them. Some upper grade students doing worksheets with division by .1, .01, etc., claimed the calculator was broken because it gave bigger answers--that is, they noticed what seemed to them to be a wrong result. But many youngsters in all grades seem accustomed to accept whatever "answer" a calculation leads them to and this carries over to calculators. Thus, for example, many children unfamiliar with decimals ignore the decimal point altogether in writing answers, no matter how ludicrous the result. As to how many significant figures to keep, nearly all children write whatever the calculator says--and often wish aloud there were more than the usual eight digits.

It seems to us that school neglect in teaching of approximation skills and of good sense about significant figures is plainly revealed when kids use calculators. Such things simply must be more emphasized if calculators are used in schools, but they should be more emphasized in any case.

3. Has the calculator any potential for diagnosing gaps in understanding of content?

We think there is considerable potential for this though none of our trials were directed to that end. For example, when using calculators with eighth graders we learned very quickly who does and who does not understand what decimals mean, even for youngsters who could do certain calculations using decimal numbers. As another example, in a certain sixth grade class, the problem 38 ÷ 144 was answered (on a six-digit machine) as 3.78947 by six students, 378947 by two, 26388 by four, and .26388 (the correct answer) by only two students. Those various responses give very clear signals for follow-up. Using the same problem with about 75 eighth graders gave similarly clear clues to difficulties (but far more students did it correctly, as one would expect). Also, every problem-solving use reveals only a minority that understand about significant figures in measure and calculation.

We believe the diagnostic possibilities are considerable and further work on them would be warranted.

4. Do children become curious about functions on the machine that are unfamiliar to them?

As far as our trials go, this is an open question. Third graders asked for more information about multiplication, but they already knew about multiplication in simple cases. These same youngsters asked about the meaning of decimals and settled (for the time being) for a simple "whole number plus a little more" sort of answer. They did not ask about division and ignored the division key. Similar results were found with first graders. All the sixth graders discussed in #3 above did the division problem demanded of them (they might have ignored division otherwise), but most did it wrong and no one, apparently,was moved to ask about it. Tentatively, the obvious answer seems probable: valid discoveries from completely unguided exploration of unfamiliar keys is unlikely but there may be considerable potential for guided exploration of partly familiar things. The unknown keys seem not to be distracting, so it at least may do no harm to keep further mathematical possibilities in the environment (by way of extra machine keys) and for certain youngsters and certain teachers some nice explorations might result. That possibility needs to be balanced against extra cost. I would like it if at least $\sqrt{\ }$ were on nearly all machines, perhaps instead of the usual % key.

5. Are children interested in the calculators and if so, does the interest last over an extended period?

There is invariably very high initial interest. From our trials, we find that high interest persists over a long time period provided students are given interesting things to do with the machines; indeed they demand to have things to do with them. Nearly every teacher in these trials commented on how motivating the calculators appear to be to kids and not a few teachers have said that "discipline problems" virtually disappear when the machines are used, even in quite difficult situations. The main problem, of course, is that few school mathematics materials have really interesting problems in them that exploit the power given youngsters by the calculators--a situation that most certainly should be remedied, and the sooner the better!

It could happen, of course, that interest in the calculators will wane as they become a very familiar feature of our society. But we have seen no evidence that "familiarity breeds contempt" either in these trials or for individual youngsters of our acquaintance who have now had calculators for several years.

6. Do children become dependent on the calculators? Does it matter?

Without discounting the possibility of overdependence as a long range effect or a result of unwise pedagogy it is clear from our trials to date that this is not a significant problem. Children at first do everything in sight on the machine but in all our trials they rather quickly gained good judgment about doing easy things in their head (whenever they could) and using the machine at most for things they would otherwise have done by pencil and paper algorithms. It is clear that many critics of the use of calculators in schools would regard any use in place of pencil and paper as leading to overdependence but there appear to be easy safeguards. For example, most teachers in these trials periodically demanded paper-pencil computation even with calculators present and the students seemed to go along with this without resentment.

It is unclear to us how much to worry about possible increased dependence on calculators. "The batteries may run down" as the main argument for no de-emphasis at all of hand calculation seems somewhat silly. More troublesome is the fact that we know very little about how children learn mathematical concepts, or even <u>what</u> they learn from the usual sequence of experiences. That being so, it would be unwise to discount the possibility that algorithmic manipulation of numbers <u>as such</u> contributes to the learning of important mathematical concepts. It is easy to imagine that the very intricacy of the manipulations plus the patterns and rules that make them work may sometimes result in important, even if unspecified, learnings. The existence of calculators suggests ways of inquiring into the issues here and this is another area where research should be fruitful (with, of course, appropriate safeguards). In the meantime, a conservative (but not immovable) posture toward the learning of calculation in schools seems warranted.

There can be no doubt of one thing: good "reflexes" with respect to basic multiplication and addition results ("learning and tables") remain very essential. With or without a calculator it is crippling not to have such reflexes. It is also crippling not to have a sure feeling for effects of multiplication and division by powers of ten--a feeling that far too many people now fail to acquire.

## 7. Is choice of machine configuration an important issue?

The first thing to be said is that in our trials children adapted to a variety of machines without difficulty and seemed to be able to switch from one machine to another without confusion. Thus it seems safe to say that at least for older children, any machine that gives reliable results could be used. But there still remain intriguing unanswered questions that merit investigation, and this is especially true for use of calculators with children in the primary grades.

In addition to power source (rechargeable battery, ordinary battery, AC adaptor) three aspects of configuration need consideration:

a) display: left to right entry or right to left entry; scientific notation or not; number of digits and possible choice of that; size and color of display; etc.

b) keyboard: which functions and number of functions; change sign keys; memories; constant--automatic or keyed; repeating keys; multipurpose keys; etc.

c) type of logic: algebraic (equation) logic; arithmetic logic as on some inexpensive machines; reverse Polish notation (RPN), with a "stack" that sets it markedly apart from the similar arithmetic logic.

These various aspects are combined in a bewildering array of choices in the calculator marketplace. But by now an inexpensive (under $10) consumer oriented machine has emerged that is virtually identical among major distributors: non-rechargeable battery with adaptor available; eight digit display with entry from the left; the four standard operations and a fairly versatile percent key; automatic constant which makes powers, reciprocals, repeated products, sums, and differences available by repeated punching of operation keys; no multiple use keys, except possibly C/CE. A few more dollars buys very similar models with memories, and a few more dollars buys similar models with square roots, reciprocals, and, sometimes, scientific notation. Beyond that considerable variety remains the rule.

A number of people have by now delivered themselves in print of opinions about what sort of machine should be purchased for school use. These opinions are remarkably similar to each other: algebraic logic, keyed or automatic constants, rechargeable batteries, and no multiple use buttons. The trouble is that this standard advice is not based on any evidence about what may work best pedagogically. Over the long run such issues deserve investigation. Findings from investigations can influence design since the schools market will presumably be important enough that calculators to given specifications can be made available.

The possible pedagogical ramifications of various combinations of display, keyboard, and machine logic are far from clear at present but such issues certainly exist and they may be non-trivial, especially for calculators used with younger children. Here are a few such issues and questions suggested by our eighteen months of informal work with calculators in schools:

a) Work with first graders suggests the possibility that the standard display that feeds in digits from the right may lead to more number reversal errors in writing or reading numerals than displays (e.g., Hewlett Packard--HP) that feed in from the left. (That is, for 78, first 00000007., then 00000078. on most calculators, versus 7.0000000, then 78.000000 on some calculators.)

b) All displays obviously have limits on the number of digits displayable, and children somehow find that dismaying. That is, even though with eight places larger numbers can be displayed than children normally see or use, they want more. But there are obvious pedagogical possibilities in the lessons to be learned from such limitations, and in ways of coping with them (for example, with scientific notation--see below)

c) Children are dismayed by the penchant of calculators to give decimal answers where there "should" be whole numbers (e.g., (1/3) X 3 = 0.9999999). Some calculators (HP again) cleverly avoid such "difficulties" and presumably it would be possible to program a calculator to perform integer arithmetic correctly, or even to switch between integerand floating point modes. But it may be that the lessons of round-off error are too valuable in themselves to try to overcome the "wrongness" of the standard machine. How careful to be in this respect may be a function of the age of the child, but then again it may not be.

d) Again with respect to integer versus decimal arithmetic, most divisions give decimal answers, but there are many real life situations leading to division where quotient and remainder is the appropriate response, or where a remainder expressed as a fraction is easier to interpret than a decimal. (E.g., if I divide these 13 cookies among three children...). Remainders can be retrieved from the decimal answer and doing so may be a valuable lesson at some point, but perhaps not for children first learning about division. Calculators with a "Q" or"(Q,R)" option switch might be useful for early grades, first perhaps set by the teacher, later a choice to be made by the child according to the sort of problem at hand.

e) All the printed advice we've seen on buying school machines specifies algebraic logic, but that may be very wrongminded advice indeed. First of all, there is no indication that the equation format (e.g., 7 + 8 = ☐ ) for calculation is the natural way for children to think as opposed to the column format suggested by RPN logic (enter the numbers, then operate). Indeed some of our work with first graders suggests the contrary. Second, with equation format either parentheses are needed (more expensive machines increasingly have them on the keyboard) or intermediate results must be recorded (or stored in memory) and re-entered, or calculations must be rearranged in fairly ornate ways. There are useful learnings in doing these things, of course, but they must be balanced against the simplicity and lack of fuss of an RPN logic with automatic stacking and recall of intermediate results. Keeping track of what is in the stack (plus the capacity to verify that) also has valuable learning potential. It seems possible that RPN logic may be more easily transferable to thinking about computer programming. In other words, the choice is not nearly so clear-cut as has been suggested and there seems to us to be a rich field for pedagogical inquiry here. (The fact that essentially all inexpensive machines have algebraic logic may decide the issue without investigation, which would be unfortunate.)

f) We have already suggested that interesting pedagogical inquiries could follow from having machines available with more functions than are immediately familiar to a child. Might a third grader ask about square root? (If so, it could be easily explained, especially with a calculator at hand.) What about base-10 logarithms, say at a time when integer powers of ten and their link to our numerations system is already well in hand? (With a calculator and hence no nonsense about extrapolation from tables, etc., the discussion might be manageable much earlier than is now the case.) Would just the fact of seeing function names daily, even if never used or explained, make children more receptive later on to work with, for example, trigonometric functions? And so on.

g) In the typical calculator, all input and intermediate data is lost unless recorded by hand. Printing calculators keep track of all that, so are preferred by many for school use. But keeping a written record may be valuable in itself. It may also decrease the remoteness and mysteriousness of the answer machine. (We have long felt that keeping a written record provides non-trivial links between concrete experience and abstract number processing and something like that may carry over to calculators.) There is no doubt that children would have to be carefully <u>taught</u> to keep such records; every teacher is familiar with children who regularly erase all the work they do (or do it on a separate sheet to be thrown away), recording only the final answer. (Indeed, we find this in the majority of children, indicating that "neatness" etc. is <u>taught</u> as a virtue that takes precedence over clarity and demonstration of process.)

h) When decimal answers are involved, the standard eight-digit display often gives much more "accuracy' than is warranted. For younger children so many decimal places is often an overload of information. One can imagine pedagogical advantages to the ability to choose how many decimal digits will be displayed. If one can keep shifting the display (with automatic roundoff) for a given result (as with several of the HP calculators), then one can imagine a variety of beneficial learning exercises. Just the necessity to choose in advance how much information to keep would have benefits.

i) Scientific notation as available on many calculators might seem at first glance to be too complicated a code for use with young children. (This code, in effect, changes every number to a pair of numbers with the first giving significant digits and the second a power of ten.) But our first reaction may be faulty and we should at least check on it by investigating when and how children can learn to use such a code with understanding. It may be that even primary children can use that code to some extent, and it seems very likely that many children could use it before it is taught in the present curriculum. Whenever it is teachable, it obviously opens up many new possibilities for using numbers in applications.

### 8. Are calculators durable enough for classroom use?

The history of classroom aids shows pretty clearly that things that get broken are simply no longer used. Teachers haven't the time or energy to get things repaired, even assuming there is money for repairs and they know where to send them.

With calculators, there are usually some initial failures with new machines--in one batch we bought, five of 24 machines had to be exchanged almost immediately but the failure rate in other batches was much lower. It seems to be the case that if they operate o.k. for a few hours, they are good for the long run. Even so, in handling about 50 calculators we have had to send four for repairs within the first year, even after initial exchanges. Dropping and other rough handling is infrequent--children seem to be pretty careful--and in any case we can trace no failures directly to such treatment. For inexpensive machines, repair costs after the usual one year warranty may exceed the price of a new machine.

From our experience it seems that if a school is to furnish machines, it should make sure of steady use for the first few hours to weed out and exchange initial failures; monitor carefully during the warranty period. (usually one year), and then plan for about a 20% per year replacement (or repair) rate after the warranty period. We haven't had calculators long enough to know how long they will last before wearing out altogether.

### 9. Are losses from theft frequent, unmanageable?

This question has represented one of the major barriers to thinking about school use. One teacher flatly refused to try out our machines with the comment "Those things have legs!" The first batch of machines lent to several teachers quickly ended up in the school vault and only came out on our assurance that part of the trials were aimed at finding out if thefts would be unmanageable. Even so, on three occasions when a calculator from a batch was lost (by theft from desks or cabinets, not in use) teachers insisted on taking up a collection to reimburse us, and written feedback from one such class made it very clear that students caught hell from a teacher who donated to the collection.

Our experience in lending about fifty calculators in a wide variety of situations over about a one year period is that as far as we can tell none were stolen by the children actually using them. Five have been lost in schools from after-hours thefts from desk drawers or cabinets. Another five have been lost from loans to students and teachers from our somewhat loosely controlled mathematics/science curriculum laboratory at the University--which has the usual library problems of keeping things in circulation versus the risk of loss. In other words, with few special precautions our loss rate from school use and lab use has been about 10% for each. Lately we have etched numbers on calculators and made up boxes with numbered storage cells (so it is obvious when one is missing) and we have tightened up security arrangements; it is too early to tell the effect of this.

Our machines were lent when they were relatively expensive and relatively novel. With the same machines now less than $10 and with more and more in circulation, temptations may diminish. But our experience makes us cautious with respect to distribution of expensive special purpose machines. Also, we will now routinely require that students in our teacher training classes have their own calculators rather than lending calculators from the laboratory. The problems (to the students themselves) of loss, breakage, or extortion have made us think it unwise to issue calculators directly to children for out of school use, especially young children. Even with cheaper machines direct ownership (or rental) seems preferable to schools supplying calculators gratis.

## 10. How should calculators be powered?

Most of the printed advice to school buyers that we have seen advocates rechargeable calculators, but we aren't so sure. The extra cost is substantial. For a batch of machines kept in the classroom it is a major nuisance to keep them on chargers. Some rechargeable batteries are said to acquire a "memory" for undercharging--that is, if used before fully charged, they may not then take a full charge unless allowed to run completely down. If batteries do run down, a calculator can't be used until recharged, unless extra (and expensive) battery packs are available.

Although short life of (non-rechargeable) batteries was a major nuisance (and expense) with an early batch of machines we acquired, those we've gotten lately with alkaline 9v transistor batteries last for about 20 hours of actual use, which is good for several months for an individual who is careful about turning off the machine when not in use. On the other hand, a classroom set of calculators in continuous use during several class periods per day would use up the 20 hours in just a few days. A-C adaptors (not re-chargers) are available for battery operated calculators but few classrooms are set up with enough electrical plugs to make that practical.

Our experience is that problems in this area are a major deterrent to routine classroom use of calculators. On checking back with teachers to whom we loaned calculators we were frequently told that they and the kids loved them, but they were out of service from worn out batteries. (They were put back in service again only when I supplied the batteries.) Few classrooms have enough outlets to keep machines plugged in, and in any case that severely restricts portability. Batteries are expensive; teachers shouldn't buy them and schools may be unwilling to. The problem is unsolved and the most likely solution may be, again, individual student ownership of their own calculators, and hence responsibility to keep them in service. For school owned sets, and perhaps even for school use of student owned or rented calculators one solution might be <u>rechargeable</u> 9v transistor batteries (if these exist at modest cost), with straight exchange of run down for charged batteries, and special rechargers that handle many batteries at once. A teacher would then simply keep a supply of charged batteries, and turn in run down batteries for recharging in some central place.

## A.2 Teacher Reports

These brief reports are annotated from written feedback provided by teachers who used calculators in various ways. In each case, the context is outlined, followed by comments from the teacher.

These reports are included:

| Page | Teacher | Context | Grade or Age |
|---|---|---|---|
| A-17 | Mary Page | Public school | 1st grade |
| A-21 | Sherye Garmony | Public summer school | ages 7-10 |
| A-23 | Pauline Schafer | Private school | 5th grade |
| A-24 | Childrens Comments | Public school | 5th-6th grades |
| A-25 | Kaye Letaw | Public; "remedial" | 5th-8th grades |
| A-26 | Sherye Garmony | Public; "remedial" | 8th grade |
| A-28 | Blythe Olshan | Public "alternative" school | 10th-12th grades |
| A-30 | Mary Ann Chory | Public high school | 9th grade |
| A-31 | Katy Blackburn | Public high school | 9th grade |

Context

Mary Page; Working November 1974 through March 1975 with small groups of 4 to 6 children from Nancy Hanvey's first grade at Kozminski school. (Mrs. Page was paid a consultant fee through U. of Chicago from NSF project funds.) This was an effort to explore young children's reactions to certain sorts of concrete materials on the one hand and calculators on the other hand. Only the calculator explorations are reported here. Three types of machines were used: Texas Instruments Exactra 20 (6-digit, 4 function, algebraic logic); Rockwell 10R (8-digit, 4 function, algebraic logic, some multiple use keys, including "+ =," instead of separate "="); Hewlett Packard 35 (reverse Polish notation).

Summary Remarks at end of the year:

1. Someone used a calculator almost every day. Self-generated activities ranged from doing problems with large numbers or checking problems they had done, to displaying telephone numbers or seeing what happened when the calculator couldn't handle a large number. Usually kids started out by doing addition combinations which they already knew. Those who were really interested (in this case well over half) would proceed to number combinations that they didn't know--many times trying to guess the answer to a problem they'd created and then seeing what the calculator said. Those who weren't very interested would end up aimlessly pushing keys. Even this activity wasn't totally without redeeming consequences, however, for in most cases kids would ask me to read to them whatever number was ultimately displayed.

2. Very few kids did subtraction problems, and no one showed much interest in the multiply or divide keys.

3. No one was able to do a $m+\square=p$ problem on the calculator, although I'm sure several could have if they'd had the calculators all the time. They were working on it when our sessions ended.

4. The calculator was an excellent motivational and learning tool. One problem that I had as a teacher was being "fair" about who got available calculators for the day. I was, of course, much more inclined to give them to those kids who I knew were interested in discovering something with the calculator instead of those who I was pretty sure just wanted to have it as a prestige symbol. It's difficult to be the judge of what constitutes useful experimentation for a given child.

5. The primary mood of the kids in my groups seemed to be one of competition. All of them showed a real desire to succeed, but emotions generated by feelings of competition often hindered success. This was especially true among the boys. For example, for data gathering preparatory to a graphing exercise we decided to count certain things in the hall (school rules forbade going outside). Each boy chose something to count--lights, doors, students, etc. Once in the hall, however, the desire to have the most quite overcame any compunctions about accuracy. When the totals were announced, the boy who had most accurately counted had the lowest number and was ridiculed. When I made up any kind of paper work, I found it worked best to give a different set of problems to each student in the group. This way they only had to worry about who finished first. This type of individualized instruction (I made problems to fit the abilities of the student) not only reduced competitiveness and copying, but in several cases led to real cooperation between kids when one couldn't figure out an answer. The calculator was a good non-competitive influence once we got over the hurdle of who was going to get it first.

6. Although we were all very careful of the calculators, they did get dropped every now and then; two were not functioning by the end of the spring. Since the battery pocket was the only part of the internal machine they could examine, these were frequently opened.

7. I'd urge HP to make a cheap durable version of their calculator for the following reasons:
   a: It offers, of course, a much broader range of mathematical activities.
   b: There's no left-right confusion in order of display which (I believe) can lead to, or at least perpetuate, number reversals.
   c: The reverse polish notation method of using the machine not only makes understanding the process which goes on inside the calculator easier, but also de-emphasizes the standard m'n p equation format. The latter is especially advantageous for kids who have (in my opinion) been introduced to equations before they were ready--in other words, most of them.

Notes from diary during year:

1. November 18: Today we just had one calculator, so in each group each person got about 10 minutes with the machine while the others played with the rods. There was great interest in the calculator--especially among the boys. When asked the question "What can you do with a calculator?" answers ranged from "Do your pluses" to "Write your telephone number." Most had no idea. No one tried to write an equation.

2. November 20: For the first 10 minutes or so the children paired up and used the calculator for writing answers to such questions as "What is the smallest number you can write on the calculator?" The largest?" etc. One child would display the numeral, the other write it down. As it turned out, this exercise was beyond the competence of most. For the smallest number, nearly everyone wrote "1". Answers for the largest ranged from 5 to 1009. In one group there was heated argument over which was larger, 51 or 74. I think they need more work on the concepts of more and less. For numbers of 10 and under, I'd suggest using concrete objects--for larger numbers, a number line displayed prominently around the room, along with posters showing random displays of say 25 objects, 52 objects, 5000 objects, etc.

3. November 26: We had two calculators. I let them play around for awhile, then it appeared that several children were trying to work out addition equations and were becoming frustrated by the fact that the entire equation was not displayed. I helped a few of the children and the rest soon caught on from them. Using simple sums to ten, they soon found out that a) You couldn't press two keys simultaneously, b) you could only press a certain key once if you wanted it to work right. As they began to calculate sums such as 3/2 and 5+1 I had them stop and check with their fingers to see if the calculator was actually displaying the correct answer. In some cases, because keys were pressed twice or the wrong sign was pressed, the answers were incorrect. Most children expressed disbelief and brought the answer to me. Many times they were not immediately sure of the exact sum, but knew that the answer they got could not be right. This type of experience lays the groundwork for healthy doubt and early work in estimation. Children, (and adults) often find it difficult to question a solution they have worked out themselves. In this case, of course, it is not the calculator that is at fault, but the operator; but the children don't see it this way yet. (The children still have difficulty distinguishing "2" and "5" on the machine. They also continue to reverse digits in two digit numerals.)

4. November 26: After we'd done some verbal equations, I gave each of them a list of about 5 simple sums to work out by themselves or on the calculator. (Everyone chose to use the calculator, but many checked with their fingers before or after.) When they were finished, they were encouraged to work out some of their own. Of the 12 children, 3 could not write their own equations, 3 proceeded to do sums with 2-digit numerals, (usually 10n or 10n + m) and one boy didn't do any equations I had written, nor wrote any of his own, but did many 10n + 10m problems on the calculator.

5. December 3: Everyone was excited about writing equations with the calculator. Often two people came up with different answers and would, after much argument, redo the problem to see who was right.

6. December 4: Some children regularly do problems in the hundreds now, and are able to read numbers in even hundreds. No one has tried to do a subtraction problem yet.

7. December 9: The children in the first group used the calculators for about 10 minutes--doing equations and writing large numbers. For the second day in a row one girl repeated a series of n + n problems. When I quizzed her afterwards, she didn't know any past 6+6, but counted out blocks to get the answers to 7+7 and 8+8. For 9+9 she used the calculator.

8. December 10: Today the girl in the second group who'd been doing 6+6, 7+7 etc. took a sheet of paper and showed me that she could do those same problems without the calculator. Two children tried subtraction problems today. One girl announced that 7 take away 8 was 1. I had her take some blocks, count out seven, and then asked her to take 8 away. We then talked about the difference between 1 and -1 on the machine. I just said that they weren't the same number. When trying to read large numbers (3 or more digits) on the calculator, the majority of the kids start reading from the right. I really think this has something to do with the order of the display.

9. Janaury 7: Mostly getting back into the swing of things. The calculator still intrigues the boys. However, when given a story problem, all but one found it difficult to apply it to methods with the calculator, and tried to do it on their fingers. Two boys who did zero problems correctly (6+0, etc.) were unable to generalize n+0=n. Not surprising. After playing with the calculator for a while, the girl who was doing n+n problems before Christmas began writing them down, trying to see if she could remember the solutions. She got as far as 8+8 and then had to count on her fingers.

10. January 7: The group of 4 girls got very enthusiastic about giving number names to the rods. They would each choose a rod, figure out its number name (using the white rods) and then whisper the name to me. After we'd done all the rods, they lined up 3 oranges and 6 whites--counted by tens to get 30, then were stymied by the 6 whites. Suddenly one girl asked, "Where's the calculator?" and proceeded to add 30 + 6 on it. Unfortunately, time ran out at this point.

11. February 21: We used the bean boards to do problems of the 10+n, 10x+n variety. They caught on very quickly, and several counted by tens, and then counted on to find the answer. The biggest mistakes were in counting units--usually the answer would be off by one less. They used the calculators to check the answers.

12. February 21: Showed them the HP 35 and gave very brief instructions for using it: "You want to put the first number into the machine so it can save it, so push the Enter key. OK. The machine has that number. What other number do you want it to work with? Now what do you want the machine to do with those numbers?" After the first couple of tries, no one

had any trouble. Three girls came down during recess to finish checking their problems, and one girl wanted a whole new set of problems to do.

13. February 21: Had three children in the last group. Gave one boy--always interested in the rods--the task of assigning number names to them, while the other two girls worked with the calculators. First, I used the HP then the Exactra to display two digit numbers, some in the teens, and some with 1 as the second digit, and asked them to say the numeral as I displayed it. Both girls made no mistakes when they read numerals from the HP but one made one reversal with the other machine, and the second made 5 reversals. I used these children for this because they haven't had as much exposure to the calculator as the kids in the other groups. It might be a good idea to do some serious testing in a couple of first and second grade rooms to see if order of display really makes such a difference. The problem of reversals is so prevalent among younger children, and causes so much confusion when they get to place value and two-digit addition, that it would be well to avoid any device that might cause further uncertainty.

Context:

Sherye Garmony; remedial 1974 summer school for youngsters, aged 7-10, identified by teachers in home school as in need of remedial reading instruction, but with 3.5 hours per day available, part of the time was given to mathematics. Ms. G. had six machines with about twelve youngsters per day; hence she divided the group so each child had a calculator to himself every other day.

Teacher comments: (abstracted from a longer report)

I was interested in seeing what would happen with respect to motivation to find out things about numbers--I just gave them out without instruction to see what they would do with them:

1. They first perceived from looking at the keys that they could use the machine to add, subtract and multiply. They weren't too sure about that other symbol (÷) but no one asked and I decided just to wait for questions.

2. First they began to test out problems they knew; e.g., 2 + 2 = 4--they feed it in and the calculator says 4. So they began to trust the calculator--they knew some things and the calculator knew the same things.

3. We had a store set up in the classroom, with empty containers the kids brought from home. The "cashier" would have a calculator (set for 2-place decimal, as was possible with these machines) and would total up purchases. They asked themselves about how accurately they would do this when their turn came. They began to check not only the calculator, but themselves and each other. They had recognized that they could make mistakes, and so could the machine. For this it wasn't necessary for every child to have a calculator and I concluded that generally speaking one calculator per child would not be necessary.

4. Multiplication and what it meant became an issue when they tried some and got big numbers, ("Well how does a calculator come up with a number like this?") because their own understanding of multiplication hadn't reached the level of that of the calculator. This gave me the chance to show how a multiplication problem could be broken into smaller parts using the distributive property. They were prompted to be curious by the calculator results so they were very interested when it came to me lecturing and them listening because it was going to help them understand something the calculator knew and they didn't.

5. Some of the children were more advanced than others and they explored division by pushing the division key. But on many problems a decimal point popped up and they asked "What does this mean in terms of the answer?" I felt it wasn't needed to go into a full explanation--their concern was just "How do you read a number like this?" or "What does it mean?" So I told them that the decimal located the whole number to the left, and the number they had was bigger than that but smaller than the next higher number. If they had pursued it further, I would have too, but they were satisfied with that answer. So, for example, if they got an answer like 1.5 and I asked for explanation they would say that it was bigger than one, but smaller than two--which was sufficient for what they were trying to find out at the point.

6. The presence of the calculator made the idea of mathematics instruction more appealing to the kids what with hot days, no airconditioning, their friends out of school doing interesting things. I believe the calculators are one reason attendance stayed high.

7. Word got around and children would come from other classes if their teachers would let them. My students would share their knowledge. They became more assured about themselves, not simply as mathematics students but as people and in their relationships with other children in the school. It changed the attitude in and out of the class from remedial/losers to a group with something special.

8. Since they didn't have a calculator for each child every day, they continued with regular math activities, then used their time with the calculators to check things they had done without them. Sometimes they would pair up and check each other. Hence the calculators became not a separate thing from the regular instruction but a part of it, which proved to be a very positive experience.

9. The end of term evaluation suggested that they had in fact become more proficient in terms of their understanding of basic principles of elementary school mathematics.

Context:

Pauline Schafer working Spring 1974 at U.C. Laboratory School with five 5th grade classes. (These students tend to do very well on standardized tests.) The experimental group (3 classes, 75 kids) explored calculators for 50 minutes per day on each of two days and then used them on a standardized test. (STEP Form B; STEP Form A taken two months earlier served as pre-test). The control group (2 classes, 50 kids) had no such work and took the test without calculators. Analysis was for whole test, and for "calculator" and "non-calculator" subsets of items; i.e., items agreed on by three referees where the calculator would be helpful (e.g., 744 + 578) or relatively useless (e.g., 90 minutes = ? hours).

Comments:

1. Results of testing:
   a) no significant differences on pre-test.

   b) There was a significant post-test difference favoring experimental group on calculator problems. There was slight but not significant difference on non-calculator problems favoring the control group. No significant difference on entire test; i.e., the usual reported score.

   c) That is, for these relatively able and test-wise students the use of calculators made no difference in the achievement test scores that would be reported from such testing.

2. Other results reported from observing students in their exploration of the calculators:
   a) All initially had trouble with problems where parentheses are needed ($8 \times 8 - 8 \times 8 \neq 0$) but eventually resolved the difficulties themselves.

   b) Using the calculators led to good questions about remainder in division and changing fractions to decimals.

   c) Few students detected wrong answers; few estimated answers.

   d) There was very high curiosity and interest.

   e) Students quickly mastered quirks of machine without explicit instruction.

   f) Student concensus: "Calculators useful not to replace our brains but to check on them."

Context

Five 5th, 6th, 5th/6th grade classes at Lewis Champlin Public School; Black, inner city, low income: one week (5 40-minute periods each); each child had a calculator

Comments

1. Reported results: Uses varied. Drill, practice, "games" from G. Immerzeel's Using the Hand Calculator.
2. In transfer between rooms one calculator was taken. Teachers insisted on taking up a collection among themselves (over strong MSB protests) to pay for it and nine student letters mentioned the loss, and the cost to teachers.
3. Feedback was via letters from kids, usually 2-3 sentences, 90 letters in all. See tally below.

Comments from 90 children's letters about their use for one week of Rockwell 12R calculators. (Four function plus % and square root; no separate "=" key, "+/=" is marked on one key, but punching any operation key gives "the answer" up to that point; automatic constant, so repetitive operations possible by multiple pushing of operation keys.)

a. I like it because it is fun; I like to push the buttons; I like to see the numbers light up; they are pretty; etc. (35 responses).

b. I like it because we can use it for math; good for work without pencil and paper; easy to work with (7); faster so you don't have to spend so much time on a problem (16); easier (7); you can do math problems and number problems too; etc. (35 responses)

c. Push a button and it gives you the answer (i.e., reliable) (10); if you don't know the times-tables, push the button; it helped me (6); helps me save the time of counting; etc. (21 responses).

d. They are smart (or smarter than me); very mindful-- think like a brain; it seems like it is human; it is a robot and we are not; etc. (8 responses)

e. I can add (or subtract or multiply or divide) with it. (Most of one class, plus four others.)

f. I don't like it because: the +/= change the numbers; my teacher didn't let me use it one day (but when I'm alone I can use it by myself); when I push one number and then another and then times it goes out; someone took one and teachers had to pay for it (6 responses)--so now we can't use it anymore (4 more responses); sometimes it came out wrong; sometimes I/it messes up; etc. (14 responses).

g. Other comments:
--Our class should keep them since we have more interest
--I have learned from it (4)
--I like to add up long hard numbers (2)
--If you had the wrong answer you could see it from others' answers
- You learn a lot of numbers
--I want one myself (2)
--I hope/wish we could keep them (9)
--Hard at first, easy after a week (2)
--I use it to check my work (4); with cross word (sic) puzzles (3); to learn how to do two number division and do a lot of this (2); help me think; improve my skill.
--It helps us know the way to do arithmetic
--It helps us with things we wouldn't have thought of with our own brain

## Context

Kaye Letaw; March-June (3 months); remedial math lab at Lewis-Champlin P.S. (Black, low income); students in grades 5-8; n = 33. Pre/post adapted from NLSMA tests--attitude toward math, solving word problems, computation (4 ops. with whole numbers, fractions, decimals)

## Teacher Comments

1. Calculator uses
   a) Exploration and drill for 3 weeks (6 lab periods) until each could use the calculator accurately
   b) Word problems, with money problems frequent
   c) Students write their own word problems, solve them, share with classmates (5 to 7 each).

2. Results of testing
   a. Hypothesis A: Students will increase skill in solving word problems.
   Result: One-point increase in mean; not significant. Reject hypothesis.

   b. Hypothesis B: Students will show positive change of attitude toward math as whole
   Result: Slight but not significant decrease in mean. (Maybe from end-of-school blahs?) Reject hypothesis as far as tested attitudes are concerned (but see #3,4 below).

   c. Hypothesis C: Use of calculators will not adversely affect computational skills.
   Result: .6 point increase in mean; not significant. Accept hypothesis.

3. Other Results Reported
   a. In initial exploration of calculators, "students became inquisitive, self-sufficient, and cooperative.. Questions were..directed to other students...For the first time students felt...one of their peers could have an answer to a mathematical question."
   b. The calculators...became another object to be used...rather than an expensive item to be stolen.
   c. When discrepancies were found...loud arguments...then "I'll do mine over again!" This became standard and made the students more self-sufficient.
   d. When calculators removed, "withdrawal symptoms."
   e. Most students could recognize activities that are easier and more accurately done with calculator

4. Overall summary by the teacher:

   The calculator experience was truly worthwhile. Students could work together without teacher direction. They relied on and trusted other students for answers to difficult problems. It helped students make judgements about what it is really necessary for them to learn.

Context

Sherye Garmony; Twenty eighth-grade remedial students, Wirth Middle School, 1974-75 school year. Eight calculators. (Four function, 6 place, floating decimal or fixed at two places)

Teacher Comments

1. Some youngsters in remedial classes have good skills but choose not to show them and instead exhibit behavior leading to the "remedial" label. This is true of some in this group. Others can perform well, but at a slower rate than "average" students.

2. The initial introduction to the calculators was a day in which they were passed out with the invitation to explore and see what they would do. This went fine; no instruction in use of the machines was needed.

3. The first sort of activity was merely in checking their own work for accuracy. For an average class one might think of checking several problems in a set just to make sure, but for this group it might be necessary to check after every single problem. If in a class of 20 a student can take a calculator and check himself, there is a more immediate response, which is very important to kids with these skill needs--they can't wait too long to find out if they are doing o.k. Waiting for a teacher they lose interest or their enthusiasm winds down.

4. Second, the calculators helped with word problems, which for these kids are not exciting adventures. The words get them hung up. Once I can help them understand what the words are asking them, then the calculator becomes an important tool. The question for them is still, however, what operations are needed to get the answer, and these questions I want them to answer for themselves, because that is what I'm basically concerned about. The accuracy through drill and practice we can cultivate as need be, but we can't get them down to the business of being accurate until they are sure they know what to do.

In this area the calculators have been very beneficial because their whole approach to word problems is different now. Those problems were always the ones left out, don't understand, can't do, but now they are done just as regularly as numeral problems. I think this is very important for these particular students.

5. Initially when we worked with decimals all the problems had a constant number of decimal places, so they could be added or subtracted just as with whole numbers and then insert the decimal where it belongs. To see how well they understood I gave some problems with different numbers of decimal places to see if they could use the calculator to do these problems. Eleven of 15 I checked could, because they had already seen the floating decimal point and how it worked. I assume that those who couldn't do the problem haven't grasped the idea of floating decimla point and what that means in calculation. I'm going to give them time to try to discover this for themselves and only step in if I think it is getting in the way and discouraging them. [This was written about the third month of the year.]

6. We have begun to look at integers from an apparent fluke in calculation--when you subtract something like 24 from 13, something happens in the calculator. We haven't talked much about it yet (third month of school)--it is just an answer that has come up. I ask all of the young people to keep records of what they do and the kinds of things they get on the calculators and then when I have time to work with them individually I try to find out what they know about what is happening and help them understand more.

7. I think an obvious advantage of calculators is that they give students the opportunity to check up on what they are doing. This is fundamental. But they also motivate a kind of curiosity that I as a teacher am not always able to come up with. I have seen an actual change in the performance level of these students. I have seen an enthusiasm and a desire to really become involved in mathematics and what it means. The calculator is a teaching aid that is very important to me now.

8. I have now begun to use the calculator in my other (non-remedial) classes. My advanced classes use it as any other instrument, just to check up on what they already know. It is fun and they enjoy it, but they can solve problems without it, while for remedial kids this is part of the problem--"I know what to do but I can't come up with the right answer." For my regular classes it is just another activity, like working with geometric designs, or whatever. It becomes very handy when we are doing problems with exponential notations and that sort of thing because here they are interested in getting an answer quickly so they can get on with something else. That is, there are probably skills to be taught, skills to be improved on, skills to be introduced for any student at whatever level.

## Context

Spring 1975 for about six weeks; Blythe Olshan at 'Metro" High School--an officially sponsored Chicago city experimental "school without walls." (Students and teachers plan courses together; many experiences have individual or groups working in a variety of places in the city, etc.) Ms. Olshan set this up as an experimental elective in calculation and problem solving with or without calculators.

## Teacher Comments

1. Setting up the experiment:

a) Students in this course ranged in background from an inability to do multiplication to the ability to do calculus.

b) The first day of class all students were allowed to play with the calculators and they each had to fill out an index card with information about their mathematical background. The students were told that half of them would be working with calculators and the other half would not.

c) The index cards were used to divide the students in groups. First we divided the cards according to levels: Level A - no algebra experience, $B_1$- less than 1/2 year of algebra, $B_2$ - presently enrolled in last quarter of algebra, C - geometry experience, D - advanced algebra and trigonometry experience, E - was for anyone at a higher level than trigonometry. These cards, from each group, were then divided in half. One half to use the calculators and the other half wouldn't.

d) The next time the students came to class they were given the results as to whether or not they would be using calculators. There was a great deal of disappointment on the part of non-calculator students. But after several discussions most students were ready to prove that they could do just as well and even better than those with calculators. Others immediately dropped the class.

e) The instructions for the class were: Attendance is mandatory. When you come to class you pick up your workbook and begin working. Each page was to be timed.

2. Results for students with calculators:

a) The first several weeks were very exciting for everyone. The students with calculators at first used the calculators for everything and within a week most of them realized that some calculations were better to be done in your head and the calculator could be used as a tool only when necessary.

b) The "Basic"students with calculators seemed to be the most enthusiastic. They would shout when they got correct answers and would say "This is great, how come we can't do math like this all the time?" "This is the first time I like math class!" "Hey, this is real easy." These comments were very frequent among several students that have felt like they've never succeeded in mathematics. They have had no confidence in themselves until now.

c) The 'poor students" just spoken of didn't want to leave the class when the hour was up. They would make up games and race to see who would finish first. One game was to keep adding by 2's and to see who could get the farthest in one minute. Soon they learned how to cheat and add more than two but they knew it had to be even. They also would do similar games with the other operations and learned a great deal from the results. After a week they became "too smart" and didn't trust each other anymore.

d) The more advanced students found the calculator as a welcome relief to many tedious computations. But they soon became bored with the idea of it as a game and needed interesting problems to keep them going. Most of the problems were at a very low level but if you had the knowledge of basic algebraic properties the work went very quickly.

3. Results for students without calculators:

a) The non-calculator students had different reactions. Those that stuck through the course did it for one of the following reasons: they needed the credit, they had nothing better to do, they liked doing these types of problems.

b) Again the basic students were the most enthusiastic. They liked the idea of racing against a calculator and being able to do as well. This again was one of the first times they felt real success and pleasure in a mathematics class.

c) The higher level studetns got bored very quickly and the number of these students in the class kept decreasing.

4. The results as to which groups did better, worked faster, etc. are still being calculated. The grading procedures were a great deal more detailed than anticipated in the designing of the course. In conclusion (for now) I would really like to see calculators available to all of my students!

Context:

Mary Ann Chory; practice teaching in an algebra class at Kenwood High; 27 calculators for one week; 32 people, mostly freshmen; roughly half do very poorly on tests.

Teacher Comments:

1. All of them had seen a calculator before. Roughly half had used one before. About 7 said that someone in their family owned one.

2. I gave out 4 worksheets from "Experiences with Mini-Calculators" (G. Imerzeel). They really liked having worksheets and were glad that they were allowed to fill them in, etc. Here are some observations from the different pages:

page 1 (count by n): Some people didn't know what counting by ones, etc., on the calculator, meant. Others were doing an extra step by adding the number everytime; when they discovered that they could do things faster by just pressing the + button again and again they almost became excited. Sometime during the first 10 minutes, a student found that the calculator had a limit (only 8 places). I then asked what was the largest number the calculator could exhibit? People were able to answer that, although some got mixed up in how to say it.

page 37 (many products, with concealed patterns): An interesting thing that occurred was the following: D. 76000 x 8  E. 760 x 800
One girl (above average for class) got the same answer for both and it freaked her out. She asked the girl (a poor student) sitting next to her what she got. I tried to lead them into why "this mysterious occurence" happened but it was too close to the end of the period.

page 4 and page 6 (check calculations): They liked these, especially where the answers were wrong. They worked together extremely well in figuring out which were wrong.

3. I told them to figure out what 3, $3^3$, $3^3$, $3^4$, $3^5$, ... was by using their calculator. Some didn't realize that you could do repeated multiplication the same as with addition. Joel (a poor student) pointed out this fact to the class. That psyched them out.

Some problems we did were: How many times do you push the x button (after you enter the number 3) before the number gets too big for the calculator? How many times do you think we will have to push the x button before the powers of 2 get too big for the calculator (more or less than for powers of 3)?

Then we found the powers of .5. This freaked them out. I asked them why did the product reach 0? Some kids said because "numbers got so small that the calculator couldn't read them anymore.' Then a group of kids started experimenting with other numbers between 0 and 1 to see if the same thing occurred.

4. The class atmosphere was great. People were really turned on. Some kids were so intense on what they were doing, they actually hated to see the period end--a very rare occurrence.

Context:

Katy Blackburn; Kenwood High; freshmen; age 15-17; low achievers; 70-109 IQ; Math stanine 1-2; Mostly low income; 22 students; 15 calculators; January-June (5 months)

Teacher Comments

1. With one exception, students enthusiastic and remained so.
2. "Greatest advantage...was that discipline problems were reduced to a minimum...when the calculators were present it was difficult to believe it was the same class." *
3. "Students begged for problems with multiple operations,"(even though they must write down intermediate results.
4. "Often they would compete...on their own initiative and not suggested by the teacher."
5. Based on (a) two general attitude scales, (b) computation tests (+, -, x, ÷ whole numbers; decimals; fractions; percents; a, $a^n$, integers, equations, area), (c) survey of attitude with respect to calculators: ..."Using the calculator in a class of basic math students may have contributed to a slight increase in computational skills, had little effect on students' tested attitudes toward math, were significant in motivation, and were enjoyed immensely by the students. Descipline problems were reduced to a minimum."
6. Problems:
   a) None with breakage, theft (but teachers were very careful)
   b) Resented having to share: "I want to use one by meself"
   c) Battery low but functioning gives strange results
7. Uses
   a) Counting 1's, 2's, 10's, 100's, 1000's
   b) place value
   c) +, -, x, ÷
   d) reading large numbers
   e) secret messages
   f) estimating
   g) perimeters, volumes, areas
   h) lengths of paths
   i) rounding off numbers
   j) placement of decimal points
   k) totals for grocery lists
   l) multiplication by 10's, 100's, 1000's
   m) patterns
   n) exponents
   o) decimal equivalents of fractions
   p) "Magic problems" (e.g., after a number, do this and that, get it back again)

*This persisted over the 5-month trial.

## A.3 A potpourri of brief comments on classroom use of calculators

A. My first grade children loved it. I taught one child how to use the calculator and he taught the others. When a child would do something wrong, others would correct him. At one point, two children were actually fighting over the machine. Both girls and boys were very interested, while girls of this age are usually less interested in mathematics. I'm not afraid it will prevent arithmetic learning for first graders; indeed I would like to try the experiment of having them available for a full year in class, then compare scores and attitudes with comparable classes. (Hanvey)

B. I left it on the desk and my second and third graders discovered it and began to use it for checking work, giving each other numbers to read, and so on. Children need to learn the logic of problem solving and, especially with young children, the calculator should help this by taking away the fuss about mechanics of calculation. For example, even with such a problem as "I have 5 pencils and someone gives me two more..." some of the children don't know that addition is the appropriate thing to do. (Longo)

C. With various groups of 6th-8th grade students, I have used different materials. These are some of the activities:

1. For checking answers: "Do it another way" becomes a real possibility, instead of teacher-imposed drudgery. Instead of having students use a "key" for checking their computation, it would be ideal to have a calculator available in the classroom for students to check their own work. If the answers differ, they would probably want to find out why.
2. Cryptograms: Using the relative frequency of letters in a sample, compared with the relative frequency of letters in the English language. Calculators are useful for the many computations involved.
3. Circles: Finding $\pi$ from measuring the circumferences and diameter of a large number of small and large circles, using calculators for the computation.
4. Astronomy: How many miles are in a "light year?" How could you find out? How far is it, in miles, to the galaxy that is 500 light years away? How long would it take to get there in a rocket? What is an astronomical unit? How many miles is it? How far is it, in miles, to a place 600 astronomical units from the earth?
5. Probability: There are many possible experiments to perform, to assist in understanding about probability. Calculators are useful in doing some of the computation.
6. Compound Interest: How long does it take a sum of money to double at 5% annual interest, compounded annually? How much money would you have now if your great-great-grandfather had put $2500 in the bank 100 years ago?

7. "Broken calculator" problems: Representing a sequence of numbers using just one digit and certain agreed-upon operations. (Once the idea is understood this can be done without calculators and then checked with a calculator. This activity helps in understanding the importance of parenthesis, and the order of operations.) (Macfarlane)

D. 1. Instant reinforcement when working with sixth graders on placement of decimal points: Do a series of problems with multiplying and dividing decimals in which the students first decide where the decimal point will be in the answer (i.e., how many digits to the right of the point). Then do the problem on a calculator. They will see instantly whether they placed the point correctly or not.

2. An aid in 6th graders learning prime factoring: Young students who are slow at dividing sometimes get "lost" before they can finish. Also, some have trouble finding prime factors above five since there are no easy divisability tests. I would have the class construct Prime Sieves and I would put one up to 100 on display in my room. When a student is prime factoring a number, he could use his calculator to do the necessary divisions and to test prime factors which he finds by looking at the Sieve. (Muelder)

E. An example of a spontaneous conversation kept going by easy calculating power that would very likely have petered out earlier without such power:

Conversation with Colleen, a 6th Grader, about time:

C. When Kevin (her 19-year-old brother) was twice as old as me,wow!--but now he is only 7 years older.

A. Yes, but when you were a year old he was 7 times as old; when a month old, 84 times as old, more or less

C. I don't believe it...

A. He'd lived 7 years, 84 months, when you had lived only 1 month.

C. MMM, and when I was younger, say only a week...

[For this a calculator came out. C then figured her present age in years (11 yr., 10 mo. = 11.83...), days, hours, minutes (over 6 million), seconds (off the 8 digit scale) "Well I'm glad I finally went off the scale. ..."]

C. Hey, you told me that if I tried to count to a million it would take me forty years, but I've lived over 6 million minutes???

A. MM... Did I? I sure was wrong!! (Bell)

F. (Notes on MSB discussion with a Principal (A. Travis) after several of his teachers had used some calculators in their classrooms. June 1974.)

We talked mostly about the practical problems of getting such machines into classroom use. He was perfectly willing to have them a feature of the classroom. The Board of Education does approve purchase of such devices (though this particular one hasn't been requested as far as he knew) but pretty much only at the regular cycle of concentration on mathematics materials--which would be in 1976. He felt that they could be bought from textbook funds at that regular adoption cycle; if not from such funds, forget it, he said, for few other funds are available. He suggested discussing the matter with certain Board of Education people downtown. (Bell)

G. In using mini-calculators with 6th, 7th, and 8th graders one of the first things one notices is the great intrinsic interest calculators seem to have, for both kids and adults. Perhaps this is because they are a new toy; in any case, mathematics steps out of the realm of the dull and routine when kids have a chance to use calculators in their math. Another thing which rather surprised me is the ease with which the students I worked with were able to go from one make and model of calculator to another. We were using various kinds of calculators, all at the same time, and after just a few trials the students could all make the calculator they were using "work." (Macfarlane)

H. Students who have not mastered the basic facts and computational skills by seventh grade have, in my opinion, only a slight chance of ever doing so. These students then need to learn to use an aid, such as a calculator, to supplement their poor computational background. A calculator will allow these students to perhaps develop some measure of problem-solving skills related to real life situations in spite of the aforementioned handicap. (Similar to comment by Charles Nelson, teaching 7th grade in Chicago public school.) (Smith)

I. Calculators are useful in mathematical reasoning. The logic of word problems is made easier without the burden of computation. I feel it is better for a child to gain an understanding of math principles, even if he is not adept at computation, than to lose out in both areas due to his lack of skill in computation. In the future, the availability of calculators may compensate for computational skills, but nothing can compensate for the lack of understanding of mathematical logic. ... However, I would restrict the use of calculators in the classroom. I would try to teach computation as in the past, although I have no rational reason for doing so. (Latin, for Latin's sake?) Calculators would be used to supplement computational skills. (Longo)

J. I love "my" (borrowed) calculator--both for myself and to correct kids' classwork. Calculators for kids? I'm not sure. Minus: I couldn't give them busy work. Plus: I could spend lots of time teaching them <u>when</u> to divide, multiply, etc. (Hennebach)

K. Even though I think actual use of calculators should wait till high school, the <u>effect</u> of the calculator should definitely be felt in the grade school curriculum. Fractions need to be de-emphasized and decimals and percents should be examined more closely. Also, with the advent of the metric system decimals take on new importance. (Gentile)

L. Calculators may be used by a child to check his own computation. They provide immediate and independent reinforcement, and are also an incentive to working the problem. They are fun to use, and the child can "compete" in private, without losing face if he's wrong. (Longo)

M. Calculators have encouraged me to attempt math problems I never would have touched before. They allow for much more rapid production, so that most of the tedium of long problem-solving is gone. They are also fun to use and make math exciting. I don't feel they should replace the learning of multiplication tables, etc. This feeling is partly intuitive, and partly based on the need for such knowledge in situations where it would be impractical to use a calculator. I see the calculator as opening doors to more creative problem solving using data from the real world--mass data. (Weiss)

N. It is very exciting to get answers just like that without going through the hard labor of calculation. But one must have a good sense of estimation and prediction how small or large the answer you expect. Errors can only be detected by ones' intuition or his sense of expectation. If a child doesn't realize this he would not necessarily end up with all right answers. Also the machine doesn't mean anything unless you know what procedures or operations are involved in getting the answers. ... Sometimes I feel insecure of not knowing what takes place inside the machine. You know the elements you feed into the machine but it only shows you the end product and not the process with which you got it. (Woo)

O. In the intermediate grades, I feel that calculators should, at this point, be used with caution. There is a danger in undermining instruction in number theory with too extensive use of calculators. Calculators do not clearly show intermediate steps in calculations; for example, how digits that are "carried," or how 1/8 gets to be equivalent to .125. How to continue to develop intuition about the correctness of calculator answers (in terms of magnitude, etc.) could become a problem. Possilby this aspect of calculator usage may also spur an increase in the use of manipulatives as intuition-building aids. This, to me, is one of the main questions, as calculators become more readily available. ... The dependency upon the user for the order and accuracy of the input data (just as with a computer programmer) should be stressed. The calculator could be used here not only as a motivational tool, because of its novelty, but again in terms of more realistic applications. In terms of finding patterns a calculator could allow for more instances of the phenomena to be observed than would be feasible to do with calculation by hand. (Smith)

Short Description of Teachers and Context in Which Calculators Were Used

Nancy Hanvey: Teacher at Kozminski Public School (elementary, urban); about 30 youngsters in a grade 1-2 classroom; one calculator for about a week in June 1974.

Patricia Longo: Teacher at Kozminski Public School (elementary, urban); self-contained classroom of about 30 students in 2nd-3rd grade; one calculator for about a month at end of 1974 school year.

Eleanor Macfarlane: Then teacher aide in suburban "learning center", now a teacher in a suburban elementary school; various 6th-8th grade students.

Richard Muelder: Mathematics teacher at University of Chicago Laboratory School; 6th graders.

Joyce Smith: Chicago Public School teacher; manages Math Lab for 4th, 5th, 6th grades.

Mark Hennebach: Regular substitute in Chicago schools.

Kathy Gentile: High school teacher.

Marna Weiss: Graduate student in Master of Science in Teaching Program at the University of Chicago; training to teach elementary school.

Matsuko Woo: Graduate Student in Master of Science in Teaching Program at the University of Chicago; training to teach elementary school.

# Appendix B: Some Remarks on the Calculation Curriculum

## Introduction

This is an appendix to M.S. Bell, Explorations into Ways of Improving the Elementary Mathematics Learning Experience, a report on a project supported by NSF grant PES 74-18938. Part B.1 was originally written for Z. Usiskin and M. Bell, Calculators and School Arithmetic: Some Perspectives, which in turn was prepared for the NSF supported "Electronic Hand Held Calculator Project," directed by Maqrilyn Suydam and Richard Shumway, Ohio State University. A somewhat different version of part B.2 also appears in that report. Zalman Usiskin's help in revising and improving both B.1 and B.2 is gratefully acknowledged.

## B.1 The canonical calculation curriculum: numbers and their processing in grades K-6

Since the calculator principally calculates, any assessment of its role in the curriculum is bound to focus on how it might interact with that part of the curriculum that deals with numbers and their processing. Hence it seems useful to attempt an assessment of typical patterns of schooling in those areas.

It is the impression of many of us that "the calculation curriculum" and "the mathematics curriculum" for grades K-6 are essentially two names for the same thing--that is, concern with calculation dominates those years to a very large extent. Even topics like measure, geometry, and probability frequently appear to be in books mainly to provide another context for calculation. However that may be, calculation concerns are at least the largest single component of the curriculum in grades K-6. To examine that curriculum, we looked at widely used textbook series from major publishers, because in the great majority of classrooms such textbooks determine the pace, structure, and actual learning materials for the students. For that purpose we went through two textbook series year by year and page by page and tallied the main content of each page into various categories. Here we include comments mainly on the calculation content, which does indeed dominate these books. The first such book is by now an old standby and has been very widely used: Investigating School Mathematics, Addison Wesley Publishing Company, 1973 edition--referred to hereafter as AW. The second is a newer series that seems to be attracting a considerable following, Mathematics Around Us, Scott Foresman and Co., 1975 edition, hereafter referred to as SF.

Before dealing with the calculation content of these textbooks (summarized on page 15) a comment on the ways in which such textbooks contribute to methods of teaching in the classrooms they dominate may be in order. First, in spite of Piaget, and Dewey before him, there is little acknowledgment in the textbooks themselves that the early (say grades 1-3) number experience of children might profitably be based on work with concrete materials such as counters or colored rods. To be sure, the books for these early years are lavishly illustrated with <u>pictures</u> of many objects and diagrams of concrete situations (e.g., for addition: pictures of animals in a pasture with others coming to join them) but it seems pretty clear that looking at pictures can't qualify as a concrete operation. Teacher manuals for both books (but especially for SF) suggest activities with concrete materials but our overwhelming impression from observing classrooms is that these suggestions are rarely followed. This overweaning dependence on symbol manipulation of one sort or another as the nearly exclusive diet for early work with numbers contrasts with what nearly all of the best informed people in our field say would be more appropriate. It is easy to imagine that it accounts for the poor intuition for and understanding of number work on the part of many people that has characterized the results of school mathematics education for a very long time.

To turn now to calculation as taught in these books some general impressions from our tally of calculation content in SF and AW can be summed up as follows:

(1) It is quite uncommon in these books for significant preparatory work for a given topic to appear at grade levels before the main introduction of the topic into the sequence. For example, there are essentially no fractions treated before grade 4, and no operations even with simple fractions before grade 5. This lack of gradual buildup may be another result of the dependence of books on symbols, since the appropriate preparatory experiences should very likely be quite concrete in nature.

(2) The general pattern of developments in both books as revealed by the page count is very similar although the books themselves seem quite dissimilar. An exception is that the SF book stresses decimal work on more pages (by a factor of 3 or so) and usually a grade level earlier. (The SF also has a more "applied" flavor throughout as context for the computational work.)

(3) The general pattern is introduction of number facts or new notation at one grade level with no prior work, then development of algorithms and the like with increasingly complex examples at the next two grade levels, with maintenance thereafter by review, use in story problems, etc.

As to the actual calculation content, Table 2 and Table 3 summarize the tally of pages in two ways. The relative emphasis in each book varies somewhat, but the mean captures very well a general impression of the sequence and emphasis in these books. Table 2 gives those means. Table 3 is a very much simplified version of Table 2; a check (√) indicates a beginning on a topic, often near the end of the textbook for that year; a plus sign (+) indicates main emphasis; a zero (0) indicates little or no attention to the topic at that grade level; the letter (m) indicates at most skill maintenance exercises.

Table 2 Number of Pages per Calculation Topic Averaged from Two Textbook Series

| | | Grade Level. | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Whole Number Addition | Addition Facts | 104 | 80 | 26 | 10 | 10 | 8 |
| | No "carry" | 10 | 15 | 10 | 0 | 0 | 0 |
| | Isolated "carry" | 0 | 28 | 26 | 12 | 0 | 0 |
| | Arbitrary sums | 0 | 0 | 10 | 15 | 20 | 17 |
| Whole Number Subtraction | Subtraction Facts | 88 | 58 | 28 | 9 | 7 | 7 |
| | Single regrouping | 0 | 27 | 24 | 7 | 0 | 0 |
| | Arbitrary a-b (b $\leq$ a) | 0 | 0 | 14 | 18 | 22 | 17 |
| Whole Number Multiplication | Multiplication Facts | 0 | 17 | 35 | 27 | 9 | 9 |
| | x 10, 100, 1000 | 0 | 0 | 9 | 15 | 9 | 7 |
| | Single digit 2nd factor | 0 | 0 | 18 | 16 | 6 | 2 |
| | Arbitrary $a \cdot b$ | 0 | 0 | 0 | 17 | 25 | 20 |
| Whole Number Division | Division Facts | 0 | 1 | 28 | 19 | 13 | 7 |
| | Single digit divisor | 0 | 0 | 6 | 16 | 15 | 0 |
| | Arbitrary $a \div b$ | 0 | 0 | 4 | 20 | 28 | 34 |
| (Rational) Fractions | Meaning; equivalence | 0 | 0 | 0 | 40 | 45 | 21 |
| | $\frac{a}{b} + \frac{c}{d}$ | 0 | 0 | 0 | 0 | 28 | 23 |
| | $\frac{a}{b} - \frac{c}{d}$ | 0 | 0 | 0 | 0 | 27 | 18 |
| | $\frac{a}{b} \cdot \frac{c}{d}$ | 0 | 0 | 0 | 0 | 13 | 22 |
| | $\frac{a}{b} \div \frac{c}{d}$ | 0 | 0 | 0 | 0 | 2 | 12 |
| (Finite) Decimals | Decimals as | | | | | | |
| | Money: add, subtract | 0 | 0 | 5 | 6 | 0 | 0 |
| | Meaning of decimals | 0 | 0 | 0 | 0 | 9 | 4 |
| | Add or Subtract | 0 | 0 | 0 | 4 | 17 | 18 |
| | Multiplication | 0 | 0 | 0 | 0 | 11 | 20 |
| | Division | 0 | 0 | 0 | 0 | 0 | 16 |

## Table 3 Grade Level of Introduction and Emphasis of Computation Topics in Two Textbook Series

| | | Grade Level | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| Whole Number +, - | Facts | + | + | m | m | m | m |
| | Easy | 0 | + | + | m* | m | m |
| | Mature | 0 | 0 | √ | + | m | m |
| Whole Number x | Facts | 0 | √ | + | + | m | m |
| | Mature | 0 | 0 | 0 | √ | + | m |
| Whole Number ÷ | Facts | 0 | 0 | + | + | m | m |
| | Long Division | 0 | 0 | 0 | √ | + | + |
| $\frac{a}{b}$ | Meaning | 0 | 0 | 0 | + | + | m |
| | +, - | 0 | 0 | 0 | 0 | + | + |
| | x | 0 | 0 | 0 | 0 | √ | + |
| | ÷ | 0 | 0 | 0 | 0 | 0 | √ |
| Finite Decimals | Money: +, - | 0 | 0 | √ | √ | m | m |
| | Meaning | 0 | 0 | 0 | 0 | √ | m |
| | +, - | 0 | 0 | 0 | √ | + | + |
| | x | 0 | 0 | 0 | 0 | √ | + |
| | ÷ | 0 | 0 | 0 | 0 | 0 | √ |

*The "m" designation, meaning "maintenance" of skills, can mean work in problem sets or implicit in other operations. In that case, pages devoted to the subject don't appear, and hence are tallied as "0" in Table 2.

The main thing that strikes us in this tally is the extent to which new symbolic work of a fairly complicated sort is piled on in fifth and sixth grades. The first four years consists of fairly leisurely work with counting and whole numbers: the meanings of the whole number operations, extensive algorithmic work with addition and subtraction, and some with multiplication. The final two years of the K-6 school sequence have a heavy load of precisely those topics well known to be troublesome to students: long division of whole numbers; common denominators; division of fractions; and virtually all work with decimals beyond addition and subtraction. Both the amount and complexity of symbolic calculation is very substantially escalated in these two years and it is not surprising that this proves to be very difficult for very many students.*

This curriculum sequence may help to explain the gut reaction of so many teachers to the use of calculators in schools: "Fine, but not until after at least sixth grade." More is at stake in that than merely the feeling that one should learn to compute "by hand" before getting a machine to do it. Even moderate use of calculators before about fifth grade would upset this standard sequence radically by bringing a number of things into the school situation before their existence is acknowledged by the canonical curriculum. For example, calculators introduce multiplication of anything by anything; division of anything by anything; division that usually leads to decimals; decimals more generally, and not merely decimals for money. That is, if one accepts that the present sequence of

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*According to followers of Piaget, many children in fifth and sixth grades may still be working at least in part in a concrete operations stage. But most of the work just referred to is heavily symbolic, and "understanding" what is going on requires fairly ornate formal arguments. This is especially so in the likely absence of work with concrete embodiments of the operations.

calculation development in schools is necessary and logical, then one must view the calculator as disruptive.

We believe that better sequences could easily be devised and that the calculator could play a fruitful role in these sequences. At a minimum, it would not be difficult to argue that the canonical curriculum has very serious weaknesses and is overdue for reconsideration.

For example, the canonical curriculum of the primary school years is surely quite weak. Too little is known about how young children learn best, but even what is known is often ignored. One signal of this is the almost universal failure to include even the simplest of concrete manipulations in early number work. First grade workbooks usually consist of page after page of equations with single digit addition and subtraction accompanied at most by pictures of corresponding groups of objects and most children apparently do spend their mathematics learning time filling in the blanks in those workbooks. Suitable concrete work should enable earlier introduction of non-negative decimals (as an extension of place value) and, if so, addition and subtraction of simple decimals could probably be included not much later than the addition and subtraction of whole numbers.

One example of curriculum materials that attempt (only) the rudimentary reform of including considerable concrete work without changing the content and grade level sequence of the canonical curriculum is Developing Mathematical Processes. At present it has not gained wide acceptance. It may be that it attempts so much by way of variety of concrete work (admirable to be sure) and demands such special sets of equipment that it is seen as impractical for school use.

A more promising approach to the early school calculation curriculum is the fairly radical departure from it implicit in the Comprehensive School Mathematics Program (CSMP). The program includes considerable concrete work. But more than that, decimals and fractions as well as operations using them are included from first grade on. Furthermore, the operations in the primary grades are not restricted to addition and subtraction, contrary to the practice in both the canonical curriculum and the DMP variant. This is accomplished with concrete devices that allow for fairly ornate algorithmic processing. (The "Papy minicomputer" is the principal one used in CSMP, but other possibilities also exist.) These concrete calculators do more than accomplish the calculation work. They also embody important features of standard algorithms--for example, "carrying" and "borrowing" in addition and subtraction of whole numbers; power of ten shift rules for multiplication and division. To some, this might suggest that the variants suggested by CSMP are ideally suited to a calculator based curriculum, with the more efficient electronic calculator replacing the likes of the concrete minicomputer. But we think this suggestion would be unwise--unlike the concrete calculators an electronic calculator obscures the process by which an answer is arrived at. But it would be an interesting exercise to consider the calculator as a teaching aid in the primary grades of such a curriculum, as an occasional alternate to the concrete algorithmic processing.

## B.2 Notes for an alternative calculation curriculum

### B.2.1 General remarks

In the usual elementary school curriculum, arithmetic skills have been taught as an end in themselves. Lip service is always given to the ability to apply these skills but most school materials do not reflect that goal and the recent NAEP tests suggest that airthmetic skills are learned <u>much</u> better than the applications of those skills. The canonical curriculum contains almomost no work in estimation or approximiation, only rudimentary applications, and little in the way of handling data. In classroom practice, there is virtually no concrete work, even in the primary grades. The numbers and operations at a particular level tend to be strictly controlled, parcelled out slowly, and with little grounding at one level for what comes next.

Suppose now that calculators are put into this canonical curriculum without any changes in it. Exactly those problems for which students spend hours and months learning how to compute answers are the ones for which the calculator gets answers very quickly. In a curriculum that already has virtually no manipulation of concrete objects, the calculator removes most of the manipulation even of numerals. Furthermore, what the calculator does well is not reflected in the canonical curriculum. The calculator allows for the handling of more complicated data. It makes trial and error methods feasible in many contexts. It displays real numbers not usually encountered in the textbooks for the early grades. (Indeed, it is not until grade 7 at the earliest that the canonical curriculum includes all the numbers and operations with them that can be handled with the simplest calculators.) Also, exploiting the calculator requires estimation skills unlikely to be taught in the canonical curriculum.

Hence, it seems to me that if calculators are to be used in the elementary school grades, alternative curricula must be designed with them in mind. I believe that it is possible to devise such alternate curricula that not only make room for fruitful use of calculators but also alleviate some of the weaknesses in the canonical curriculum. As a partial existence proof for this possibility, the pages that follow include pieces of a rough outline for one such curriculum alternative for the elementary school mathematics curriculum, or rather for the calculation part of that curriculum. The main features of this alternative are greatly enriched content in the early grades (as in CSMP), considerable work with concrete materials, and, of course, work with calculators from the beginning of the school experience

In considering the material that follows the reader should keep in mind the fragmentary and tentative nature of the suggestions made. With respect to content, the outline deals only with number and calculation and even in that is not exhaustive. With respect to sequence, the outline is at most suggestive. With respect to concrete materials, only a small sample of the possibilities is listed. With respect to calculators, the suggestions made by no means exhaust the possibilities. (Most of the calculator suggestions come from teachers in the trial uses of calculators described in Appendix A.) The outline is uneven, with much more detail for early work than for later work. This last is mainly in response to the apparently widespread belief that calculators have potential at most for the upper elementary grades and beyond. That is, the early grades are outlined in more (but far from sufficient) detail because that is where there appears to be the most scepticism both with respect to richer content and with respect to fruitful teaching uses of calculators.

Finally, the reader should know that what I regard as possibly the most promising teaching use of calculators is virtually absent from the fragmentary outline that follows; namely, calculators used to help youngsters make sense out of numerical data. At almost any level there are situations interesting to students for which both the data and answers from processing the data are understandable, but for which the actual computations or other processing are beyond their pencil and-paper calculating power. The calculator can give them the needed processing power and thus open up many more reasons to be interested in numbers and calculation. In any alternative proposed to the canonical curriculum such possibilities should be fully developed. Examples could come from everyday experience; from USMES-type challenges; from many nice science materials; from Nuffield-type activities; from class projects--including some in social studies, language arts, etc; and so on.

To sum up, the pages that follow include some first draft fragments for what might profitably be developed into a full scale alternative to present practice. It clearly is not yet a fully developed alternative but it may suggest that serious efforts to find a useful place for calculators in teaching elementary school mathematics (including the primary grades) could be rewarding.

### B.2.2 Some possible activities for a first phase of a calculation curriculum: introducing the child to arithmetic

In a first phase, each concept is introduced through manipulation of concrete objects. Unlike the canonical curriculum, from the start attention is given to fairly large whole numbers, to fractions, to decimals, to all four operations. Inputs and results are recorded to provide links among the verbal, the concrete, and the symbolic; it is important that this be done both for concrete and for calculator work. Recording of inputs and results is also important just to have a record of the processing.

Please keep in mind the cautions outlined above:

(a) Only calculation content is dealt with here, but I believe that any reasonable elementary school curriculum should include much which is not calculation

(b) The outline is not intended to be exhaustive in any respect: useful content may have been neglected; many other useful concrete activities exist; there are many other ways to exploit calculators for increased understanding of arithmetic.

(c) Using calculators to make sense of interesting numerical data can reinforce arithmetic learning in many ways, but few of these possibilities appear in the material that follows.

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (a) Whole Numbers as Counts | | |
| *Count by 1's, say to 126<br>*Count by 10's, say to 500<br>*Start at n, count on; count back<br>*Recording of results of a counting process, say up to 126<br>*Read a two-digit numeral<br>*Count out a set of n objects, n reasonable | *Attach verbal counts to many sets of objects<br>*Count out sticks, bundle by 10's Count by 10's with 10-bundles<br>*Embody two-digit numbers with sticks and bundles; count on (continue bundling) and back (unbundle)<br>*Form n; predict and act out (((n + 10)+ 10)+...); ((n - 10)- ...)<br>*With reversible grocery counter, count on and back by 1's; by 10's | *Set constant so that repeated button pushing counts by 1; child counts verbally in pace with display<br>*Similarly, count by 10's<br>*Display number, then use constant to count on and back by 1's<br>*Display n; use constant to count on and back by 10's; ask for prediction before pressing button<br>*One child displays number he can read; challenges partner to read it<br>*Children challenged to display biggest, smallest 1- or 2 digit numbers: disputes settled with concrete embodiments<br>*One child says a number; partner displays it |
| (b) Whole Numbers as Measures | | |
| | *Usual play and preliminary work with colored rods (e.g., Cuisenaire rods) augmented with clear acetate 1 x 1 cm squares to represent 0<br>*Measure lengths with rods; express results with 10-rods and at most one other<br>*Use meter stick to measure; record to nearest cm | |

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (b) Whole Numbers as Measures (continued) | | |
| | *Children weigh themselves on bathroom scales (kg); weigh other large objects (kg) and small objects (g): record results<br>*Water and sand table play with volumes--how many of these in this? | |
| (c) Meanings of addition of whole numbers | | |
| *Counting--i.e., combining<br>*Measuring--i.e., joining | *Combining sets of objects<br>*Usual work with "trains" of rods; some of this with rods on number line<br>*Add by counting on with grocery counter<br>*Simple two-digit addition with sticks and bundles<br>*"Add" using volumes, weights, etc. | *Get a + b results from games, data, a classroom store, other problems of interest to students, etc; occasionally use data beyond that which is easily acted out concretely<br>*Explore patterns such as "adding 10," "adding 2," "adding 9," to arbitrary x<br>*Fill in table of basic results with combined concrete and calculator work |
| (d) Meanings of subtraction | | |
| *Counts or measures:<br>take-away<br>comparison<br>*allow negative answers closely tied to real situations | *Use counters and rods in both take-away and comparison situations<br>*Count back with grocery counter<br>*Unbundle sticks for borrowing<br>*Act out some money problems; classroom store is possible; also integers via losses, lending, etc.<br>*Act out subtraction with various measures<br>*With rods marked with arrow, add and subtract on the number line | *Explore patterns like (a+b)-b, (a+b)-a, a - a, etc.; explain the answers in terms of real situations<br>*Explore a - b and b - a; compare a - b with b - a<br>*Estimate answers to large problems; verify with calculator |

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (e) Meanings of multiplication | | |
| *Arrays<br>*Repeated addition using counts and measures<br>*Combinatorial problems<br>*Count by n | *Count m groups of n counters<br>*Count m X n array by n<br>*Train of m n-rods; convert to 10a + b<br>*Measure array of n-rods across by an m-rod<br>*Act out combinations<br>*Money for 10 X 1, 10 X 10, 10 X 100, etc. | *With a constant, count by n's; explore patterns<br>*Calculate plausible problems with areas and arrays (e.g., how many tiles on the floor or ceiling)<br>*Units in units: seconds per hour; minutes per day; meters in n kilometers, etc. |
| (f) Meanings of division | | |
| *a things or something of size a split evenly b ways<br>*How many b's in a<br>*One-bth of a vs a÷b<br>*Correspond with multiplication<br>*Rate | *Share things evenly (e.g., cookies)<br>*Forming groups for how many b's in a<br>*a counters with b rows, "one for you and one for me," sometimes remainders<br>*One-bth of a in money, sticks and bundles<br>*Cut up ribbon of length a into b equal pieces; into pieces of length b | *Illustrative concrete problems; observe non-whole number quotients correspond to remainders<br>*Numeral n.d is between n and n+1; begin rounding and also lead-in to decimals<br>*"Best buy" pricing |
| (g) Introduction to decimals | | |
| *$n \leq n.d < n + 1$<br>*Decimal system and money; X 10 and $X \frac{1}{10}$ or ÷ 10<br>*Record metric length measures with decimal notation; notion of "more accurate" | *Decimals from money transactions and from linear measurements<br>*Calipers | *Grocery store problems; calculator as check-out machine<br>*Explore accuracy needed in real situations |

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (h) Meaning for fraction notation; equivalence of fractions | | |
| *Parts of whole<br>*Ratios<br>* $\frac{a}{b} = a \div b$ | * $\frac{1}{b}$ , $\frac{a}{b}$ of a set of objects<br>*Mark 24-cm "unit" on number line with rod trains ($\frac{1}{2}$'s, $\frac{1}{3}$'s, $\frac{1}{4}$'s, $\frac{1}{6}$'s, $\frac{1}{8}$'s, $\frac{1}{12}$'s, $\frac{1}{24}$'s); note many names for some points<br>*Reprise concrete work for $a \div b$<br>*Paperfolding of unit strips $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{2}$ of $\frac{1}{2}$, etc. | *Check equivalence of fractions by division<br>*Order fractions by division |

### B.2.3 Some possible activities for a second phase of the calculation curriculum: building up skill and concepts

Skill-building begins before Phase 1 ends and includes the basic addition, subtraction, and multiplication reflexes. During Phase 2 the algorithms for the fundamental operations are established and some paper-and-pencil competence and accuracy is expected.

The "concrete" work in this phase becomes more coded. For example, dowels of the same approximate size can replace 10-bundles (with only a few actual bundles left for regrouping). Work with counters can be done on a place-value sheet. This allows for work with decimals without needing physically smaller pieces for tenths, hundredths, etc.

The work of this phase is specified below in less detail than before, with the examples simply meant to suggest types of uses of concrete materials and of calculators. The main objectives here are to demonstrate the existence of fruitful work with these aids and to stimulate thought in these directions.

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (i) Addition, multiplication reflexes | | |
| | *Check doubtful combinations with concrete work, but goal is reflex responses | *"Beat the calculator" to encourage mental processing<br>*"Broken calculator" problems as practice--i.e., postulate only certain buttons work, ask for display of 1, 2, 3, ..., n |
| (j) Addition and subtraction skills | | |
| *$a + b$ ($a$, $b$ any integers, easy decimals)<br>*$a - b$ ($a$, $b$ any non-negative integers, easy decimals)<br>*$a - b = c \Leftrightarrow c + b = a$<br>*$a + b + c + d$ for reasonable data | *"War" with black and red cards, the latter negative; with pairs: $a + b$, $a - b$<br>*Chip computers to keep track of operations with multi-digit numbers<br>*Meter sticks, tapes, trundle wheels for data gathering or picturing | *Break down calculations by parts--e.g., use calculator for one's column-record, then for 10's column, record, etc. (in subtraction, the negatives suggest borrowing)<br>*Estimation of answers by using only first significant digit, or two significant digits |
| (k) Multiplication and division skills | | |
| *$a \times b$ for up to 2-digit whole numbers on paper; arbitrary on calculators<br>*$a \div b$ where $b$ has up to 2-digits on paper, arbitrary on calculators<br>*$a \div b = c \Leftrightarrow cb = a$ | *Repeated addition (for single-digit $b$ in $a \times b$) using dial-a-matic, grocery counter, chip computer, 10 blocks, or sticks and bundles<br>*Exploration of 10-shifts with matereals, e.g., each X 10 shift is an exchange for a next higher value<br>*Repeated subtraction (for single-digit $b$ in $a \div b$) using above materials | *Exploration of X 10, X 100, ..., shifts both for whole numbers and decimals<br>*Exploration of ÷ 10, ÷ 100, ..., shifts... Compare with X .1, X .01, ..., shifts<br>*How do you multiply with multiplication key inoperative?--begin with repeated addition, move to combination of repeated addition and X10, X100, ... shifts to get to the usual algorithm |

| Some Skills or Concepts | Some Sample Concrete Embodiments | Some Sample Calculator Uses |
|---|---|---|
| (k) Multiplication and division skills (continued) | | |
| | *Use dot arrays to exhibit 2-digit factors; group results by powers of 10 to suggest an algorithm. (This might well come earlier, as a concrete way to get answers for situations that involve products of two digit factors) | *Obtain partial products, then add<br>*How do you divide with division key inoperative? (1) begin with repeated subtraction, move to combination of repeated subtraction and X10, X100, ..., shifts to show possible algorithms; (2) revert to multiplication and careful trial and error |

### B.2.4 Some possible activities for a third phase of a calculation curriculum: applying and enriching arithmetic processes

This phase begins before the second phase has ended, perhaps in part as early as grade 4, and continues through the remainder of a student's arithmetic experience into high school and beyond. Here one uses the skills and concepts built up in the first two phases in a variety of ways to gain sophistication and power in processing and interpreting numerical information. For example, one uses the skills to estimate answers; the concepts to develop shortcuts and efficient application in arithmetic; uses both to arrange data in various ways to better understand it.

In this phase there is an explicit focus on algorithms as such (and on flow charts and other programming tools). for paper-pencil calculation, for organizing calculator work, and, possibly, for using computers. New content may include such things as percentages; square roots (probably trial and error squares using calculator); scientific notation (which may have been anticipated earlier); various rounding off processes; linear, quadratic, cubic relations (e.g., in connection with scaling effects involving linear, area, volume measures); exponential versus linear growth and $a^b$ for _a_ arbitrary, b an integer (various compound interest-type applications); attention to formulas and functions; and other content.

In this phase one relies on number patterns more than concrete embodiments. There would be some reliance on concrete work, especially for new concepts, but it would diminish and the embodiments themselves might be more abstract. To compensate for less concrete work, there should be even increased work with collecting and making sense of actual data, always with attention to questions of "reasonableness" of answers.

No outline is included for the work of this phase.

# Appendix C: Assessing Numeration and Place Value Concepts*

The interview instrument that resulted from several stages of development and validation is reproduced here. See 3.5 in the main report for more details.

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# Notes for a Clinical Interview on Number and Numeration Concepts

Draft
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To be used only with permission

Dale Underwood
The University of Chicago
April, 1974

*This is an appendix to M. S. Bell, Explorations into Ways of Improving the Elementary Mathematics Learning Experience, a report on a project supported by NSF grant PES 74-18938. As indicated by the cover page reproduced above, it is the work of Dale Underwood, now at Florida State University, Talahassee, Florida.

# THE TASKS IN BRIEF

Writing Numerals. The interviewer dictates a list of numbers which the child writes.

Reading Numerals. The child reads a set of numerals. If there are examples written incorrectly by the child in the preceding task at least two of these are included.

Magnitude. a) In one task the child is given various sets of small cards on which single digits are written and is asked to make different numbers and compare their magnitudes, to construct numbers bigger and smaller than given numbers, and to construct with a given set of digits the biggest and smallest numbers possible.

b) In a second task he is asked to determine for two numbers how much bigger one is than the other.

Counting. a) By ones. The child is asked to count (by rote) to 40. He is then asked to continue counting from various starting points, these starts being determined by his prior performance.

b) By tens. In a similar fashion the child is asked to count by tens to 250 and then to continue by tens from various starting points.

Algorithms. The child is asked to work some (written) addition and subtraction exercises. He is encouraged to "think aloud" and/or to explain or rationalize his procedure after the fact.

Tens and Ones.

# WRITING NUMERALS

Materials. The record sheet has the list of numbers you will dictate.
The child needs a pencil and the sheet to write numbers on which follows the record sheet.

Procedures.

1. Introduce the task by saying something like, "I'm going to call out some numbers and I'd like you to write them down on this sheet." Give the child his sheet. "The first number is 68. Just write it here." Indicate the first space on his sheet.

2. Continue through the list of numbers.
   (a) Read numbers without saying "and", e.g., "five hundred four" rather than "five hundred and four."
   (b) The numerals in your list are printed small, but try to keep them discretely from the child's view anyway.

3. Repeat a number as many times as necessary.
   (a) Don't say a (large) number piecemeal until you have said it twice without pauses. (The child who intends to write 400090098 for 4,998 will signal you to say "four thousand--[pause]--nine hundred--[pause]--ninety-eight." By all means do so, but only after you have said it twice without pauses.)

4. Make up Read cards from at least the first two errors the child makes in writing numerals (in any). Put these cards at the end of the Reading Numerals deck.
   (a) Write these numerals just as the child did, preserving his digit order and use (or lack) of commas. However, form all digits just as you did in the rest of the Read deck, e.g., don't write "ɥ" for "4" even though the child did.
   (b) Wait until the child is writing another numeral to make up one of these cards.
   (c) You may want to do more than two of these cards if the child's errors seem to be of different types.

Comments.

2(a) This suggestion is not motivated by a puritanical notion of correctness, but by the fact that we simply don't know what effect the two different ways of reading a numeral might have on the way a child writes it. Until we do know whether there is such an effect, a (perhaps arbitrary) consistency will enable us to learn at least how children respond to this way of reading numerals.

4(b) This procedure does save time, but primarily it minimizes the probability of signaling the child that he has made an error.

4(c) The object of making up Read cards from the child's errors is to find out which types of these errors are decoded (read) consistently with the way they were encoded (written). Consistency in encoding and decoding indicates a more conscious, deliberate, or rational pattern of thought than does inconsistency.

WRITE NUMERALS
(Record after interview)

68 ____________________

100 ____________________

4,998 ____________________

504 ____________________

1,000 ____________________

348 ____________________

5,004 ____________________

112 ____________________

872,000 ____________________

6 million ____________

12,000 ____________________

Don't say "and".

Make Read cards from errors.

READ NUMERALS
(Record after interview)

68 ____________________

100 ____________________

4,998 ____________________

504 ____________________

1,000 ____________________

348 ____________________

5,004 ____________________

112 ____________________

12,000 ____________________

from Write errors

__________ ____________________

__________ ____________________

__________ ____________________

__________ ____________________

Repeat after child if necessary.

# READING NUMERALS

Materials. The deck of cards (used in common with all children) on which you have written the numerals from the Read list, together with any cards made up for the current child in Writing Numerals.

Procedures.

1. Introduce the task simply, saying something like, "Now I'd like you to say (read) some numbers for me." Show the child the deck with the first card on top. If he doesn't start, ask him, "What is this number?"

2. Hold the deck so the child is speaking as directly as possible into the recorder. Put each card to the back of the deck after he reads it.

3. If you have any doubt about the child's responses being recoverable from the tape, repeat each of his responses after him. When you do this you must repeat them all--not just those that are wrong. (A simple request for the child to speak louder may be a sufficient alternative.)

4. Remove all Write error cards made up for current child before you put the deck back. Put the error cards with the rest of the child's materials to be recorded later on the Read record sheet.

Comments.

2 It is important that the child see only one numeral at a time since there are certain similarities. Some children will read "5,004" as "five hundred and four," and thus should not see this numeral together with "504."

4 You might want to mark the backs of the cards made up from the child's Write errors so they can be easily and accurately separated from the deck. (Using his initials or code number will also enable you to return them to the right folder if they should drop out.)

# MAGNITUDE

Materials. From the set of common materials, you will use the 4 sets of digit cards and the 3 sheets on which digit cards have been glued.
From the individual materials you will use the sheets on which 57 and 47 (and other pairs) are written and the Record sheet which you will fill out during the interview.

Task I. Discovering the child's word for "greater" when comparing numbers. Telling "how much more."

Procedures.

1. Lay before the child the sheet on which 57 and 47 are written. Point to 57 and ask, "How does this number compare to that one?"
   (a) You will frequently have to probe to get a magnitude comparison. Try "How is this number [57] different from that one?" or other non-suggestive questions initially. Before the situation becomes a "guess what's on my mind" hassle, however, just ask the child directly 'Would you say this number [57] is 'greater' or 'bigger' or 'higher' or what?"

2. When you get the child's word for "greater" record it and use it (and it's equivalent for "lesser") during the remainder of the task.

3. Ask "How much (bigger)* is this number?", pointing to 57. Record the response.
   (a) Probe a bit if the child just indicates, for example, that 57 has a 5 and 47 has a 4. You might use something like, "If I had this many pennies [47] and you had this many [57], how much more would you have than I do?"

4. Continue with the other sheets having pairs of numbers written on them. Ask the child to draw a ring around the number which is (bigger) and the to tell how much (bigger) it is. Record "how much bigger" on the sheet with the pairs of numbers.

Comment.

1 The principle of discovering and using the child's own language for concepts has an innediate intuitive appeal for one trying to assess the thought processes of the child. There is at least one instance in the literature of an investigator being mislead about a child's level of maturity by the incongruence of the child's language and that of the investigator. In the present context, some children will be lead by an interviewer's word choice to compare the physical sizes of two numerals rather than the magnitude of the numbers they represent.

---

*Substitute the child's word for "bigger" here and wherever this expression appears subsequently.

57 47

673 573

832 842

Task II. Digit cards.

Procedures.

1. Introduce these as "cards to make numbers with."

2. Present the first set (7 and 8) and ask, "What numbers can you make with these?" Record his constructions.

3. As the child constructs a numeral, ask "What is that number?" Record his reading.
   (a) Prompt the child as necessary in constructing and reading. The object of this phase of the task is to make sure the child is seeing a construction as a single numeral rather than as only discrete digits.

4. After he has constructed both 87 and 78, ask the child "Which of those two numbers that you made is (bigger)?"
   (a) If he responds "8", reform 87 and 78 and repeat the question, emphasizing the "two numbers that you made, 87 and 78."

5. Present the next 3 sets of digit cards in turn and in each case ask the child first to "make the very (biggest) number you can with all these cards." Then ask him how he would say that number. Next ask him to "make the very (smallest) number you can using all these cards."
   (a) Present the cards by laying them out before the child in an unarranged group rather than in a row. (There is no need to touch them between the "biggest" and "smallest" tasks.)
   (b) If he only uses some of the cards for a task, ask him again to use them all.
   (c) In each case record at least the child's final construction. (Record as much as you can of the succession of partial constructions leading to the final response.)
   (d) Record his reading of the "biggest" construction in parentheses beside the record of his construction.
   (e) On the third set of cards some children will (cleverly) construct "00257" for the "smallest" number. Record and accept this, but ask the child then to make the smallest number using all the cards but so that the "zeros are not in front."

6. Present the sheet on which 746 is glued and explain, "I've got a number glued down here: 746. What I want you to do is take these cards and make a number up here [indicating above 746] which is (bigger) than 746." Give the child the digit cards tucked under 746. Record his construction.
   (a) In this and the two subsequent tasks, read the glued numbers in full, as "seven hundred forty-six," etc.

7. Present the sheet on which 476 is glued, and explain similarly, "Now I've got another number glued down: 476. This time make a number down here [indicating below 476] which is smaller than 476." Give the child the digits tucked under 476. Record his construction.

8. Present the sheet on which 814 and 736 are glued, and explain, "This sheet has two numbers on it: 736 and 814. This time I want

you to make a number that is between these: above this [indicating above 736] so it's (bigger) than 736, and below this [indicating below 814] so it's (smaller) than 814." Give the child the cards tucked under 736.

(a) Repeat and amplify as necessary. In particular you may need to emphasize that a single number is to do both jobs at the same time.

(b) Record the construction.

Comments.

3 This task will provide more information about the child's numeral reading (a tendency to reverse digits, in particular), but is primarily a final opportunity before the main digit cards tasks to ensure that the child can treat a construction with digits as a single numeral.

5(c) It is not difficult, and often informative, to record the succession of partial constructions by recording from left to right each digit as the child places it. When a child deviates from a left-to-right construction (by inserting a digit between two already placed, changing the order of placed digits, etc.), just put a dash after the preceding record and then record the full digit sequence resulting from the deviation. For example, the record "27-25-2457" indicates the child was firm in his initial choice of "2" to begin the "smallest" number with these digits, but that he had to try several alternatives before he was satisfied with the order of the remaining digits. It also indicates that his final response was constructed deliberately and rationally rather than randomly or unthinkingly.

5(d) Reading responses are of two types (sometimes intermixed). For the type in which the child uses the words "hundred," "thousand," "million," etc., use abbreviations "H", "Th," "M," etc. Thus "seventy-five thousand and twenty-four" as a reading of "7524" may be recorded "75 Th 24." For the type in which the child reads off single digits and/or2-digit numerals, use dashes. Thus "seven, fifty-two, four" as a reading of "7524" may be recorded "7-52-4." The child's reading of his construction may provide a clue to his thinking. If he reads his "biggest" construction, 7254, as "72-54" one can see that even though he is not coordinating all four digits properly, he is thinking adequately in terms of digit pairs. Thus his thinking is at some intermediate stage of maturity. Note that you would have no right to conclude from just the construction itself that the child was thinking of digit pairs: thus the usefulness of his reading. Another instance that arises is the reading of "70052" as "7H52." This response is indeed the "biggest" numeral that can be constructed with these digits when numerals for "hundreds" are thought of in this fashion. (It is of interest to compare such an instance with the child's responses to the Writing Numerals task.)

6a

## MAGNITUDE (Record during interview)

| | |
|---|---|
| | Child's word for "bigger" ______ |
| 57, 47 | How much (bigger)? ______ |
| 832 v. 842 | Circle (bigger). How much bigger? ______ |
| 573 v. 673 | Circle (bigger). How much bigger? ______ |

"Cards to make numbers with"

| | |
|---|---|
| 7, 8 | What numbers can you make? ______ ______ [Prompt as needed] |
| | What is that number? ______ ______ |
| | Which of these is (bigger)? ______ ______ |

| | Biggest (Read each one) | Smallest |
|---|---|---|
| 2,4,5,7 | ______ | ______ |
| 1,4,4,5,7,8 | ______ | ______ |
| 0,0,2,5,7 | ______ | ______ |

| | | |
|---|---|---|
| 3,5,7 | ☐<br>746 | Make a number up here which is bigger than 746. |
| 4,5,8 | 476<br>☐ | Make a number down here which is smaller than 476. |
| 2,5,7 | 814<br>☐<br>736 | Make a number that is between these: bigger than this and smaller than this. |

# COUNTING

Materials. You will use the flowcharts "Ones" and "Tens"to determine successive starting points from which the child continues to count after his initial count.
You will probably need blank sheets of paper and a pencil.

Task I. Counting by ones.

Procedures.

1. Ask the child to count to 40.
   (a) If he hesitates, start him off with a remark, "You know: one, two, three..."
   (b) With younger children you may want to ask initially how far they can count. If they give a number less than 40, ask them to count up to that number. If they give a number greater than 40, ask them to show you by counting just to 40.

2. Using as starting points numbers determined by the flowchart "Ones" (as described below), say, for example, "Suppose you kept on counting and you got to 68. What would be the next number after 68?"
   (a) For starting numbers which end in 8 or 9, continue asking "And the next number after 69?", "And the next after 70?", for example, until either the child has reached the next number that ends in 1 or given an incorrect response. For starting numbers which end in 0 or 1, continue asking for the next number until either the child has reached the next number that ends in 3 or given an incorrect response.
   (b) When either the child hesitates or stumbles over the number because it is just too much verbiage to keep in mind or when he makes an error, write the starting number down for him. Say something to the effect, "That's a long one. Here, let me write it down. Maybe you'd rather just write the next number." Once you write a number, write the starting numbers for the rest of the task.

3. Following the flow chart. There are 3 basic rules:
   (i) If the child gives all correct responses following a starting point, follow the "down" arrow for the next start.
   (ii) As soon as the child makes an error (after also having seen the number written), follow the "right" arrow for the next start.
   (iii) The task ends when you reach "END".

Comments.

2(a) Asking the child successively to give the next number rather than merely continuing to count avoids the problem of having to stop the child. You always want to avoid interrupting a child and possibly disconcerting him.

2(b) Since the task is not intended as a test of short term memory, but of number sequencing ability, you need not hesitate to give the child the advantage of seeing the numeral as well as hearing the number word. One merely saves time and simplifies the procedure by not writing numerals until necessary.

3 By examining a child's path through the flow chart you can determine at least two things with a fair degree of confidence: that the child has mastered (forward) sequencing of numbers of up to N digits (N 5), and that he can sequence numbers of up to M digits within a decade. Not surprisingly, M is usually greater than N.

Task II. Counting by tens.

Procedures.

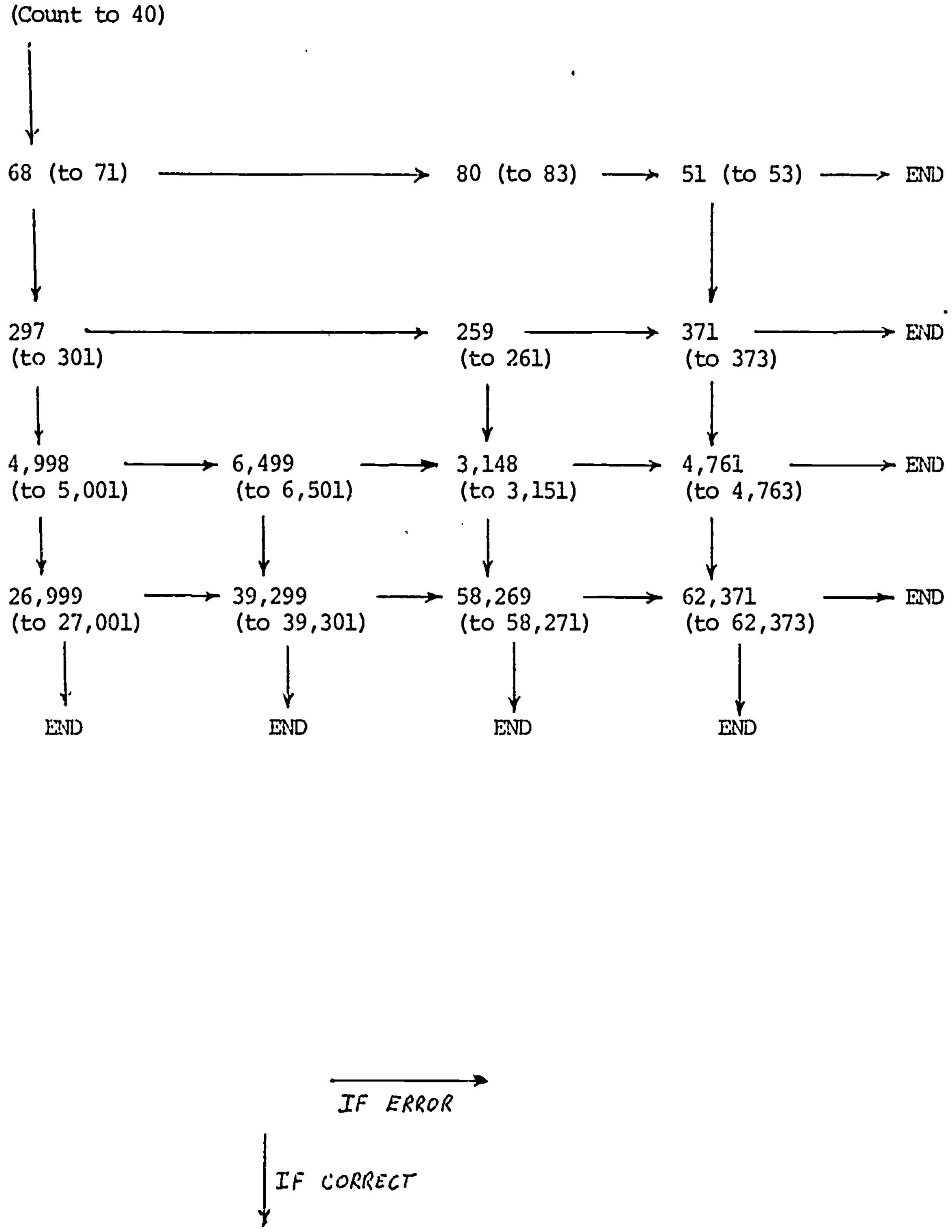
The procedures are similar to those for counting by Ones.

1. The initial task is to count by tens to 250.
   (a) As with Ones, start the child with "ten, twenty, thirty..." if he hesitates.

2. Ask the child to give the next numbers from starting points determined by the flowchart "Tens' in the same fashion as with "Ones."
   (a) Write the starting number as necessary and give the child the option of writing his response.
   (b) Continue after each starting point either until the child makes an error (after also having seen the number written) or reaches a number ending in 10(when the starting point ends in 80 or 90) or reaches a number ending in 30 (when the starting point ends in 00 or 10).
   (c) Be sure to repeat "the next number counting by tens" for each new starting point.

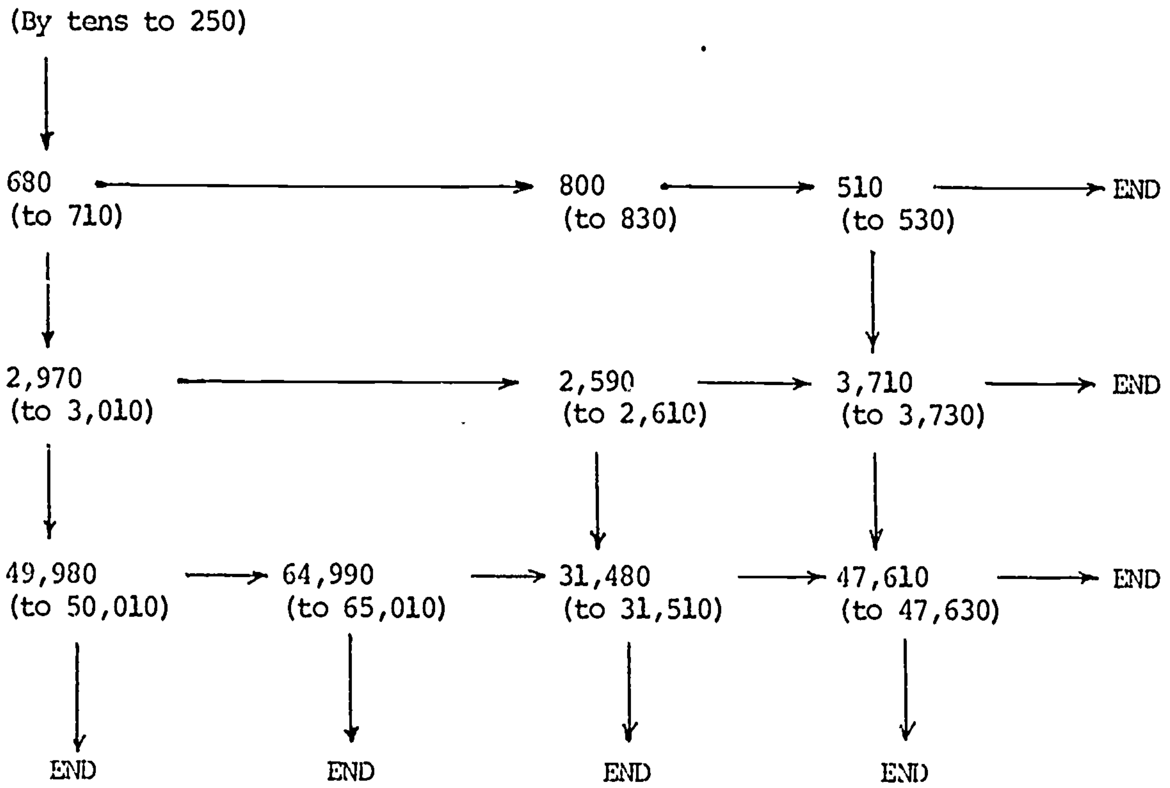
# COUNTING (ONES)

(RECORD AFTER INTERVIEW)

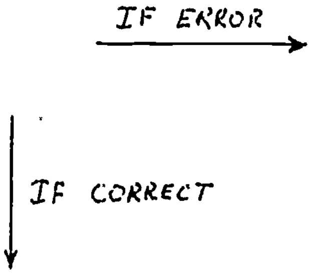


# COUNTING (TENS)

(RECORD AFTER INTERVIEW)



Remember "next number counting by tens" with each new start.



# ALGORITHMS

Materials. You will use the record sheets on which the exercises are written down the side.
The child will work on the (unlabeled) sheets over which the exercises are spread. He will need a pencil, and poker chips should be within his reach.

Procedures.

1. Explain, "I'd like you to show me how you add numbers. I'll give you some problems and I want you to tell me how you figure them out." Present the addition problems. Indicating the first one say, "Tell me what that problem says." Then, "O.K. Work it out and tell me how you do it."
   (a) Many children will not "think aloud." Once the child starts to work silently don't interrupt him. When he has completed an exercise ask him how he did it. Sometimes a child will interpret this request as a signal that he has made a mistake, and he will start to erase his answer. Whether he has made an error or not reassure him that he can change his answer if he wants to but that you were not telling him he was wrong, and then reiterate, "I just want you to explain how you did it."

2. While the child is working record as much as possible of his relevant non-verbal behavior. (Do this to the side of each exercise on the record sheet.)
   (a) Draw an arrow beneath the exercise on the record sheet to indicate whether he worked (wrote his answer) from right to left or from left to right.
   (b) Record any overt counting behavior (use of fingers, tally marks, chips, etc.) and the nature of his counting (counting from 1 vs. counting-on or counting-back). Even when you show by your attitude that you don't consider fingers a no-no some children will use their fingers in a surreptitious manner. This makes determining the nature of their counting very difficult, and you just try to draw them out with an accepting sttitude.

3. Probe the child's verbal responses and behavior to the extent that you will be able to assess his thought on the following dimensions (described below).
   (i) Number processing.
   (ii) Numeral processing.
   (iii) Verbalization in terms of place value (as appropriate).
   (a) Confident assessments on these dimensions is an ideal seldom attained. However, if you enter the situation having in mind what you are looking for you are more apt to find it than if you ask random questions and try to draw conclusions from the answers afterwards. The probing must be adapted to the child and what he has done. There is no way to anticipate all the contingencies.
   (b) Verbalization in terms of place value is not applicable if the child does not deal with digits as constituents of a numeral.

4. Follow the same procedures with the subtraction problems.

Comments.

1 A child's reconstruction of what he did is not reliable. This is the reason for trying to get him to "think aloud" as he works. However, this reconstruction may confirm observations of non-verbal or subvocal behavior made while the child was working. Sometimes the explanations after the fact are the only way you can tell what the child was doing. (In one case the interviewer was baffled by a third grade child's solution which proceeded:

$$\begin{array}{r} 48 \\ \underline{+76} \end{array} \dashrightarrow \begin{array}{r} {}^{2}48 \\ \underline{+76} \end{array} \dashrightarrow \begin{array}{r} {}^{2}48 \\ \underline{+76} \\ 0 \end{array} \dashrightarrow \begin{array}{r} {}^{2}48 \\ \underline{+76} \\ 30 \end{array}$$

Then the child explained that she "borrowed from the 6," indicating the 6 and 4, "had 2 left so I put that up there - then I counted." That is, she subtracted 4 from 6 to get 2, then counted to add 6+4, getting 10. She then wrote 0 and carried 1, adding it to 2 to get 3. This is not a typical procedure, but neither is it unique in its bizarre quality.)

2 This recording is not cut and dried either. You can usually do the things suggested, but you will have to adapt to the unexpected.

3 Although you want to get as much understanding of the child's thought as possible, it is good to keep in mind the principle that if a child does something deliberately and rationally once he will most likely do it again. Thus there is no need to push a child up against the wall to make sure you know what he is thinking on any single example. After a certain point let it pass and see if you can pin him down better on the next exercise.

## DESCRIPTION OF THE DIMENSIONS ON WHICH THE CHILD'S THOUGHT IS ASSESSED

Number Processing. In solving addition and subtraction exercises a child's treatment of numbers and physical embodiments of numbers can generally be placed on one of the levels of the Number Processing hierarchy which follows (adapted from W.A. Brownell). Note that placement on this hierarchy is independent of the child's accuracy in counting or in recall of basic conbenations. Accuracy should be note quite separately from thought process. The levels in the hierarchy follow.

(i) Simple counting. In adding the child displays or counts out both addends and finds the sum by recounting from 1. In subtracting the child displays or counts out the minuend, "takes away" the subtrabend, and then counts the difference from 1.

(ii) Counting-on and counting back. In adding the child starts counting from one addend, displaying or counting out only the other addend to determine when he should stop counting. In subtracting the child counts back from the minuend to the difference, not having to recount the difference.

(iii) Solution. The child solves a given combination from another which is recalled (even if incorrectly so). For example, "8+8 is 16, so 8+7 is 15."

(iv) Recall. The child simply "knows" a combination without calculation. (Brownell was concerned with the important distinction between meaningful recall and mere rote recall. This distinction and its detection are not considered here, but is partly covered by Numeral Processing.)

Numeral Processing. This dimension concentrates on the child's treatment of numerals in working algorithms, though of course it is impossible completely to separate consideration of numerals from the numbers they name. The categories in the hierarchy are constructed from examination of children's actual responses. However, the ordering of the levels is logical and is not meant to imply a progression of mental development of an individual child. As in classifying number processing, this classification is made according to the process irregardless of accuracy. (See next page for hierarchy.)

Verbalization. In the case of a child who utilizes digits in working algorithms you can note whether he spontaneously uses the words "tens," "hundreds," etc. (or "twenty," "six hundred," etc.). If he does not will he do so under probing? (If he writes $\begin{array}{r} {}^{1}24 \\ +37 \\ \hline 1 \end{array}$ and says "carry the one," how does he respond to "Do these two ones mean the same thing?" or "Why did you write one here and one here?"

| HIERARCHY OF NUMERICAL PROCESSING | TYPICAL EXAMPLES | |
|---|---|---|
| | Addition | Subtraction |
| (i) Numerals as indivisible wholes. The fact that numerals are composed of digits is ignored. Numbers are typically treated as endpoints of a count.* | $\begin{array}{r}24\\+17\\\hline 41\end{array}$ (CALCULATED BY COUNTING ON PAST 24) | $\begin{array}{r}73\\-48\\\hline 25\end{array}$ (CALCULATED BY MAKING 73 TALLIES AND CROSSING OUT 48 OF THEM) |
| (ii) Digits in unlike places not distinguished. The child utilizes the digits of which numerals are composed, but does not distinguish digit values determined by the places occupied by the digits. | $\begin{array}{r}426\\+10\\\hline 536\end{array}\quad\begin{array}{r}24\\+17\\\hline 14\end{array}$ (2+4+1+7) | $\begin{array}{r}50000\\-10\\\hline 40000\end{array}\quad\begin{array}{r}10000\\-1\\\hline 00000\end{array}$ |
| (iii) Digits in like places combined in isolation from other places. In adding the child either fails to "carry" or writes a 2-digit sum in one "place." In subtracting the child fails to "borrow" and either subtracts the lesser digit (kn the minuend) from the greater digit (in the subtrahend) or writes 0 as the difference. | $\begin{array}{r}24\\+17\\\hline 31\end{array}\quad\begin{array}{r}49999\\+10\\\hline 499109\end{array}$ | $\begin{array}{r}313\\-226\\\hline 113\end{array}\quad\begin{array}{r}313\\-226\\\hline 100\end{array}$ |
| (iv) Incorrect interaction between places. In adding the child "carries" from left to right. In subtracting the child "borrows" from a non-adjacent place. | $\begin{array}{r}2\overset{1}{8}\\+76\\\hline 1{,}5\\\longrightarrow\end{array}\quad\begin{array}{r}4999\\+1\\\hline 5990\end{array}$ | $\begin{array}{r}\overset{0}{\cancel{1}}000\overset{1}{0}\\-1\\\hline 00009\end{array}\quad\begin{array}{r}\overset{2}{\cancel{3}}1{}^{1}3\\-226\\\hline 0\ 7\end{array}$ |
| (v) Correct interaction between places. | | |

---

*In special cases this process may be the most mature: to add 299+1, for example, place by place with "carries" is less mature than using the fact that "add 1" is synonymous with "next number in the counting sequence."

# ALGORITHMS (ADDITION)

(RECORD DURING" INTERVIEW)

24
+17

48
+76

4999
+1

1468
+805

4999
+10

$$\begin{array}{r} 24 \\ +\ 17 \\ \hline \end{array} \qquad \begin{array}{r} 48 \\ +\ 76 \\ \hline \end{array} \qquad \begin{array}{r} 4999 \\ +\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 1468 \\ +\ 805 \\ \hline \end{array} \qquad \begin{array}{r} 4999 \\ +\ 10 \\ \hline \end{array}$$

# ALGORITHMS (SUBTRACTION)

(ACCOR[illegible] [illegible] [illegible]
INTERVIEW)

31
- 7

73
-48

300
- 1

313
-226

3200
- 436

$$\begin{array}{r} 31 \\ -\ \ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 73 \\ -\ 48 \\ \hline \end{array}$$

$$\begin{array}{r} 300 \\ -\ \ \ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 313 \\ -\ 226 \\ \hline \end{array}$$

$$\begin{array}{r} 3200 \\ -\ 436 \\ \hline \end{array}$$

General Remarks.

1. Always phrase questions "How would you...?" or "How con you figure out...?" or "What is...?" rather than "Can you...?" or "Do you know...?" One of the main objectives is to learn how a child thinks, how he relates or applies the numeration system in various contexts. Thus you will not be interested in whether a child thinks he can or can't do something, but in how he does it. Thus if you ask a question to which he can respond "yes" or "no", you still have to ask a "How," "What," or "Why" question. This is not meant to suggest that it is inappropriate to insert such questions as "OK?" or "Would you do that for me?" which not only make the interview more amiable, but also give opportunities for the child to signal when he is too tired to go on.

2. Don't be pushy or rush the child. After you ask a question or pose a task give him some thinking space. For example when dictating numbers for him to write, don't repeat the number unless he gives you a clear signal that he wants you to. Your judgment about when to repeat, change or drop a question will probably be right if you are aware simply of the tendency of most people not to tolerate silence.

3. Do not give the child approval or disapproval during a task, but do be overtly accepting of all he does. ("OK.") That is, do respond, but without value judgments. After the task thank and/or praise the child for doing the task.

4. You should not give the child feedback on his responses even when he asks for it. ("Is that right?" "Should that have one or two zeros?") Pass such requests off as gently as possible. ("I want to see how you would do it." "I just want you to do the best you can" or "...the way you think it ought to be.") At the conclusion of the interview you can go back to any such questions. This teaching is, of course, the most enjoyable part of the task and you will probably find that restricting it to the end goes very much against your teacher instincts. Of course the ultimate aim of setting such tasks for children is to have immediate information on which to base teaching. However, if you teach during the interview you will be unable to distinguish stable responses from transitory responses resulting from your help.

5. Don't interrupt the child more than necessary. The necessity for interruption can usually be avoided by setting or implying limits to responses as in the Counting task in which you ask the child to give only the "next" number rather than simply to continue counting.

6. Don't explicitly make the number-numeral distinction for the child. In fact, there should be no need for you to use the word "numeral."

7. Have the child sit on your right-hand side. Since most children are right-handed you will then be able to see more easily what he is writing.

8. Have several pencils and a stack of blank paper available.

9. Before you begin interviewing you need to write the numbers in the Read list on, say, 3x5 cards. These should be in your hand so the Read cards you make up during the interview from the child's Write errors will look like the numerals in the rest of the Read deck.

10. A final word: As formidable as this interview procedure appears from this description, you will be able to relax with it after you've done it a couple of times. Kids do enjoy it--to the extent that there is no dearth of volunteers by the second day you go in. Furthermore, teachers usually feel the observations made from the interviews are helpful and more relevant than usual diagnostic work.

## "EVERYMAN" QUESTIONS

How tall are you?

How high is this room?

How high with kids? (how many kids your size standing on each others shoulders-- to get to ceiling)

Do you know someone in another city? How far? How do you go there? How long does trip take?

How many beans in bag?

How much does the bag of beans cost?

How much does a new car cost?

How much do you weigh?

How much does this table weigh?

How much does a car weigh?

Draw inch, foot, centimeter.

Show yard.

## Appendix D: Summary of Teacher Questionaire Results*

### Introduction

The questionaire is reproduced below just as it was administered, with summaries of responses filled in as appropriate. The questionaire was administered in late May and Early June of 1974. Thirty-nine teachers from local elementary schools (public and private) responded. See 3.2 of the main report for additional details.

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## Survey of Hyde Park Teachers About the K-6th Grade Mathematics Program

(Max Bell, U. of Chicago, May-June 1974)

Note to teachers:

Many (perhaps most) adults admit that they don't know much mathematics and are not able to use math very well, which perhaps indicates that the school experience in mathematics could be improved. Many people have given prescriptions about what "ought" to be done about this but their notions may have little relationship to the situation as seen by working teachers. This survey seeks to consult teachers directly about what things might be done in this local school system (public and private) to help youngsters learn mathematics better. I can make no particular promises, but I believe that if specific problems can be identified, ways to work on them can be found.

Responses to this survey will, of course, be treated confidentially and data from individual teachers or schools will never be released. An overall summary of responses will be shared with participating teachers.

We recognize that some of the questions may be difficult to respond to in the form given but please give us as good an answer as you can to every item. We are more interested in the general picture than precise details. If you need more space to comment on any item, use the back of the sheet.

As a token of appreciation for your help, we want to send you one of the books listed at the end of this survey form. Please record your summer address on the last page and indicate which book you prefer.

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*This is an appendix to M. S. Bell, <u>Explorations into Ways of Improving the Elementary Mathematics Learning Experience</u>, a report on a project supported by NSF grant PES 74-18[illegible]38.

School____________________ Grade level you now teach________

39 respondents

I. Resources available to you:

A. Which mathematics textbook series do you use in your classroom?

How do you like it?

What are its advantages and disadvantages?

| (Response) | - | 0 | + |
|---|---|---|---|
| None 6 | | | |
| SMSG | 1 | 0 | 5 |
| AW | 0 | 2 | 2 |
| H-M | 0 | 6 | 3 |
| GCMP | 1 | 0 | 0 |
| Laidlaw | | 1 | |
| S Burd. | 3 | 0 | 0 |
| Other | 0 | 2 | 2 |
| Blank 3 | | | |

S.R.Bur

B. Put an "F" next to any of the following "alternative" mathematics programs that you are familiar with. Put an "A" by those you have access to, and a "U" by those that you actually use in your classroom. (Any or all of these symbols may appear for any particular materials.)

**Familiar With, Use**

Nuffield Mathematics Program 17,5

Laura Rasmussen's Pupil 11,2

Other Drill and Practice Workbooks or worksheets (specify) 15,12

Dienes Mat'ls 9,3

Madison Math Project 18,3

SMSG 15,6

Distar 14,0

Math Workshop (Wirtz, Botel, Sawyer) 8,4

Stretchers and Shrinkers (UICSM) 4,1

Other printed materials (specify) 10,10

C. What time is allocated to mathematics in your classroom (e.g., "30 minutes per day, "40 minutes on three days," etc.) *

| Blank | 0-10 | 11-20 | 21-30 | 31-40 | 41-50 | 51-60 |
|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 14 | 10 | 5 |

Mean ≈ 43 minutes

* Converted to time each day

D. Put an "F" beside the following manipulative materials with which you are familiar, an "A" beside those to which you have access, and a "U" beside those which you use in your classroom.

Familiar, Use

Cuisenaire Rods 35,9

Geoboards 34,13

Number lines 35,17

Multibase Blocks 19,1

Attribute Blocks 19,3

Graph Paper 34,15

Calculators (what kind?)
Electronic 1,1 Dial-a-Matic 3,3
Unspecified 9,2

Slide Rules (what kind?)
Home-made 1,1 Variety 2,1
Unspecified 7,0

Tangrams 22,4

Balances 24,11

100 Number Boards 26,10

Building Blocks 23,5

Games (specify) 22,7

Other (specify) 17,7

Counters (e.g., poker chips, counting blocks, beads, etc.)
(specify) 27,17

List in-service workshops in mathematics you have participated in since last June, with the subject and the organization that offered it (Bd. of Ed., Tchr. Center, etc.). "Grade" each from "A" (excellent) to "F" (flunk).

E. Are you aware of the services offered by the Teacher Center located in the YMCA building on 53rd Street? Yes-26; No-6; Blank-7
If yes, how do you make use of those services?

| None | Rarely | Make Materials | Math Workshops | General Workshops |
|---|---|---|---|---|
| 15 | 2 | 1 | 1 | 6 |

F. Are you aware of the mathematics/science laboratory in Judd Hall at the University of Chicago? Yes-7; No-25; Blank-7
(Most teachers are not; we may need to do something about that.)

IIA--Are you happy with your math program? Explain.

---

6th: When they come to me, kids have only mastered addition and (not always) subtraction. I spend most of the year teaching multiplication and division.

3rd: I don't feel I ever enjoyed math but am beginning to enjoy teaching it at this level. However, I don't feel very secure. I want to master early math so that I can get my children to enjoy it as well as just pick up a skill!

7-8: Entirely too formalistic and/or traditional as now taught. Programs should be divided (a) conceptual, (b) computational, (c) correlation of both.

5th: I have not had enough time or help to meet individual needs. Because of large numbers of slow learners or behavioral problems individualization has been difficult--sometimes impossible.

5th: I would like to be more prepared for new methods.

4th: OK, but the majority of the students have not learned the basic facts of addition, multiplication, etc. and much time must be devoted to this.

4th: OK, but there should be more opportunities for the students to explore on their own.

7th: My class understands new concepts but has difficulty retaining ideas. (I teach some 7th grade students who are performing at 2nd to 4th grade level.)

7th: There should be more materials and time to meet a wider range of abilities than we now do.

8th: I think math teachers need to be more expert in their field. The problem in our school is that we have no math teacher *per se*--each of us has a specialty (social studies, science, language arts) and we each teach a reading and a math class.

8th: Children no longer have the skills in fractions, decimals, and percents necessary for advanced work.

3rd: (1) Better integration with other grades needed. (2) Need more manipulative materials for concept development. (3) Pupil-teacher ratio of 1/25 is not reasonable given the ability range.

5th: I'd like more manipulative materials; budget is a real problem.

5th: Need more variety

K: I feel I need to understand Piaget better. My personal goal involves more study of the five year old mind in order to provide better math experiences of all kinds. I do feel one cannot separate math in kindergarten--the subject is thinking, not math, science, etc.

IIA continued

2nd: I have just finished writing and rewriting supplementary materials for a multi-age or open classroom. Now it's all printed and available and I am ready to start over again. I miss messing around with homemade junk. There are however, nice aspects about it.

3rd: Yes! I sometimes (it varies with year, month, week and day) feel that I haven't done enough! But I thoroughly enjoy doing it always! I am always finding something curious, intriguing exciting; really.

4th: Not really. Too much of the year is taken up by what I see as grade level expectations (becoming proficient in understanding and use of operations). Not enough time to do other things as work in geometry, rationals, sets, logic, etc.

Lrng Lab: Not really. I would like to have greater insight into math and how to teach it.

2nd: No. SMSG is going out of print, and there needs to be a central area in our school for stocking a great variety of manipulatives.

1-2: Not entirely. Children understand basic concepts but I can't see 3rd graders still counting on their fingers. Once a solid understanding is established children can learn number facts by heart. I've approached this as a game in the last few weeks and the children enjoy it.

7-8: No. There are too many "gaps" in students' backgrounds and classes are too large for individual tutoring.

Special Ed: Not especially. Would like better materials. Want to be given opportunity to become familiar by having publishers reps come to school with goods as we did this year with reading program. Dislike new math, perhaps because of own poor background.

II. Your opinions:

A. Are you happy with your math program the way it now stands? Explain.

See summary of comments, above

B. Rate each of the following with respect to its potential for helping youngsters learn mathematics better. First consider what might help in your own classroom, then what might help in the generality of classrooms in this community. Use a number as indicated by the following scale:

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| Unnecessary or wouldn't help | | Would help but is not especially important | | Would help <u>very much</u> |

| | Your classroom | | | | | | Many other classrooms | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Distribution: | B* | 1 | 2 | 3 | 4 | 5 | B* | 1 | 2 | 3 | 4 | 5 |
| 1. Help the teacher learn more about mathematics itself or help make the teacher more confident of his or her own competence in mathematics.<br>Averages: 3.94, 4.58 | 4 | 2 | 4 | 6 | 5 | 18 | 6 | 1 | 0 | 3 | 4 | 25 |
| 2. Help the teacher learn more about how to use manipulative and laboratory materials in the classroom.<br>Averages: 4.03, 4.42 | 4 | 3 | 2 | 4 | 8 | 18 | 6 | 0 | 0 | 7 | 5 | 21 |
| 3. Make available more or better printed materials such as workbooks, work cards, drill sheets, and so on.<br>Averages: 3.91, 4.25 | 4 | 4 | 2 | 5 | 6 | 18 | 7 | 1 | 1 | 5 | 7 | 18 |
| 4. Make available more or better manipulative materials.<br>Averages: 3.97, 4.25 | 5 | 4 | 1 | 4 | 8 | 17 | 7 | 2 | 1 | 2 | 9 | 18 |
| 5. Provide help in diagnosing learning difficulties in mathematics and prescribing additional work for those youngsters with poor understanding or skills.<br>Averages: 4.29, 4.65 | 4 | 4 | 0 | 2 | 5 | 24 | 8 | 1 | 0 | 1 | 5 | 24 |
| 6. Find better ways of handling those few youngsters whose behavior patterns make it difficult to work with the class; i.e., help with youngsters who are "discipline problems" or who simply demand more individual attention than most teachers have time to give.<br>Averages: 4.09, 4.81 | 4 | 1 | 4 | 7 | 2 | 21 | 8 | 0 | 0 | 0 | 6 | 25 |
| 7. Mathematics specialists or consultants expert in mathematics and the teaching of mathematics readily available to classroom teachers.<br>Averages: 3.43, 3.97 | 4 | 5 | 3 | 9 | 8 | 10 | 7 | 1 | 3 | 7 | 6 | 15 |

*Blank

| | Your classroom | Many other classrooms |
|---|---|---|
| 8. More assistance in the classroom by use of parent volunteers or teacher aides or university students in training to teach or older children, etc. (Assume that such extra hands will have had some training in helping out with the mathematics learning experience.)<br>Averages: 3.72, 4.13 | 3 3 3 10 5 15<br>_____ | 9 1 1 8 3 17<br>_____ |

- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -

[Add to this list your own ideas about what might help a lot in improving the elementary school mathematics experience.]

See compilation of responses, on following three pages.

9. (Your idea) _____ _____

10. (Your idea) _____ _____

11. (Your idea) _____ _____

C. Of all the things listed above, including those things you may have added to the list, which one would help you personally the most? _____
Which one do you believe would be the most helpful to many other teachers?

_____

| | You | Others |
|---|---|---|
| Blank | 7 | 12 |
| 1. Math. Course | 6 | 5 |
| 2. Manip. Course | 3 | 3 |
| 3. Written Materials | 1 | 2 |
| 4. Manip. Materials | 1 | 1 |
| 5. Help with Diagnoses/remediation | 3 | 3 |
| 6. Help with Discipline | 1 | 1 |
| 7. Consultants | 2 | 4 |
| 8. Aides/Volunteers | 6 | 1 |
| 9. Other | 9 | 7 |

II B 9: "Rate each of the following with respect to its potential for helping youngsters learn mathematics better. ... Add to this list your own ideas about what might help a lot in improving the elementary school mathematics experience." Here are the additions:

3rd: There are many teachers like myself.who do not see the whole picture of math; that is, where does it go from year to year and what end results are we striving for. If more of us did, the primary teachers would probably'do a better job. I also personally feel that the biggest burden of this problem of non-achievement and negative attitudes of math rests with us. I don't mean that every child must be on grade level but we have got to be doing something wrong when the average child has problems adding and subtracting in 4th and 5th grade. I find that most other teachers don't understand mathematics any better or as well as I do. As a matter of fact many of my colleagues don't even teach addition and subtraction as reversal operations, and I am angry when I get a second or third grade class where there has been no system to their earlier teaching. I find in third grade I can't cover the curriculum because they don't understand tens and ones, commutative principles, etc.

5-6: Parent education in this area may be of great value. Many parents today seem to feel that the math being taught is so completely different from the math they learned that they are most hesitant to give help to the child at home. I feel that very often the message conveyed to the child is that "this is such a difficult subject that even the parent cannot master it so how can the child?" I also feel that terminology has confused teachers as well as children in understanding and explaining certain very basic concepts.

5th: (1) More individualization of program - making each child have success at his own level. (2) Using more teacher-made materials better suited to each child - games, puzzles, etc. (3) Would like to have more peer-tutoring and also older-child/younger-child tutoring.

EMH: In service during school year in Math. (Every 2nd and 4th Tuesday).

5-6: (1) A self-teaching text -- not discovery alone. Some children never arrive at the idea the author has by himself. (2) Mini-units independent of the text for reteaching areas that a child may not have absorbed at an earlier level. Good for review; brush up on skills. (3) Loose leaf or duplicating master text: Hand back with examples and explanations only. Initial presentation of whole idea. Many pages for step by step development and comprehensive end of chapter reviews. Makes self pacing possible. Also extension materials for able students.

5th: More parent involvement

8th: Real work through in-service on metric system

8th: Return to skill building--fractions in 4th, decimals in 5th, percent in 6th, pre-algebra in 8th.

II B 9 continued:

4th: A qualified teacher to give extra help for the children who find math difficult. A parent could come under this category, perhaps one who was a teacher but not teaching now.

2nd: (1) Not seeing math as workbook but experiencing a lot of variety, not just computation. (2) Using manipulacive materials to prove a point (3) It helps to have correcting books be a shared activity with children so they can work (4) Teachers need to know that children who are scared can't see relationships! Feel stupid when don't understand. Teachers need to stop saying "but don't you see..." and figure out a way to make it clear.

2nd: A math lab (in our school preferably) where books, manipulative materials, etc., are available to examine.

3rd: Math should be taught at the start of the day. By the last period the children in grade 3 are pooped (so is teacher).

3rd: (1) No set criteria be imposed by the next higher grade level producing pressures -- more cross-grouping (age-wise) of children according to abilities (2) More tutorial help of older children to younger ones - great for the "discipline problem" older kids. (Example: first and sixth graders) We have a regular reading program of this sort. (3) Main thought: math should be an integral part of the day and not a "set aside" period.

4th: (1) Give teachers courses in math (not method teaching) so that they can feel the fun of doing math. (2) Give teachers workshops in making their own manipulative materials. (3) Decrease class load.

5th: Since math is not my main interest, I'd like a consultant to discuss side trips that can be made from the text used.

K: Beyond the pre-school age, my ideas are not important. I lack the experience and confidence. But I do have this feeling: most of the N-2 teachers would be more interested in a workshop dealing with children ages 4-7. This age group is always passed over quickly so as to get to the multiplication tables. In other words, K-8 covers too much territory. We would like to be able to apply what Piaget tells us about the 4-7 year old.

2nd: (1) A place to go to talk about and make manipulatives, like your office, not the teachers center which isn't "open" enough. (2) Continuing workshops to discuss problems related to math learning in the classroom. (3) Ideas for relating math experiences to other areas of the classroom. I.e., art, science, reading. ("Ideas" = dialogue about.)

3rd: (1) My Airplane Flying Activity for graphing, computing averages, developing fine skills. (Inexpensive -- used yearly -- successful.) (2) Graphing and mapping school rooms (or home) and adjacent areas Geometry scale drawing, etc. (3) measuring, gross to fine. Use of two disparate measuring devices -- metric balance and 3 meter rule for instance. Everything is measured, from classmates to egg shell thickness (varies throughout egg!).

II B 9 continued:

4th: (1) Less fixed curricular expectations so that a child (or group) who suddenly becomes interested in some concept can pursue it fully. (2) Time (and perhaps help) to work out curriculum ideas -- perhaps to write a unit on history of numbers, or on permutations, combinations. (3) Somebody qualified (not necessarily a consultant -- we've had that and it has not been helpful) with whom to discuss various approaches to use with kids who have trouble or with kids who really fly with math.

4-5: (1) Weed out pupils who have little or no interest in math; put them in a math manipulatives class. (2) Have math program set up for the year for a specific classroom of children rather than follow a textbook.

(1) Learning to relate math to everyday living experiences. (2) Emphasis on speed and mental arithmetic. (3) A good crash course taught by an expert in math who could provide a good overview of the teaching of elementary school math.

D. Please enter a number in the blank following each statement according to the following scale:

1. not at all
2. a little
3. a middling amount
4. very much

| | B* | 1 | 2 | 3 | 4 | Mean |
|---|---|---|---|---|---|---|
| I enjoy teaching reading ______ | 0 | 0 | 0 | 6 | 33 | 3.85 |
| I enjoy teaching other language arts ______ | 2 | 0 | 0 | 4 | 33 | 3.89 |
| I enjoy teaching mathematics ______ | 0 | 0 | 2 | 12 | 25 | 3.59 |
| I enjoy teaching social studies ______ | 5 | 1 | 5 | 6 | 22 | 3.44 |
| I enjoy teaching science ______ | 4 | 7 | 4 | 15 | 9 | 2.74 |
| I feel competent to teach reading ______ | 0 | 0 | 2 | 8 | 29 | 3.69 |
| I feel competent to teach other language arts ______ | 0 | 0 | 1 | 7 | 31 | 3.77 |
| I feel competent to teach mathematics ______ | 0 | 1 | 1 | 15 | 22 | 3.49 |
| I feel competent to teach social studies ______ | 1 | 2 | 1 | 12 | 23 | 3.47 |
| I feel competent to teach science ______ | 2 | 5 | 5 | 17 | 10 | 2.87 |
| I think it is necessary for most students at my grade level to work on: | | | | | | |
| reading ______ | 1 | 2 | 0 | 4 | 32 | 3.74 |
| other language arts ______ | 5 | 0 | 0 | 6 | 28 | 3.82 |
| mathematics ______ | 2 | 0 | 3 | 2 | 32 | 3.78 |
| social studies ______ | 9 | 2 | 1 | 13 | 14 | 3.30 |
| science ______ | 11 | 1 | 2 | 12 | 13 | 3.32 |

*Blank

E. Please rank the following subjects--reading, other language arts, mathematics, social studies, and science--on the scale below. In each blank to the right, fill in the name of one subject area. Use each name only once.

a) Probably is most enjoyable for me to teach:
b)
c)
d)
e) Probably is least enjoyable for me to teach

| Rank | B | 1 | 2 | 3 | 4 | 5 | Avg |
|---|---|---|---|---|---|---|---|
| Rdng | 3 | 20 | 6 | 5 | 2 | 3 | 1.94 |
| LA | 6 | 7 | 11 | 7 | 6 | 2 | 2.55 |
| Math | 5 | 7 | 8 | 10 | 8 | 1 | 2.65 |
| SS | 10 | 1 | 7 | 6 | 8 | 7 | 3.45 |
| Sci | 4 | 2 | 2 | 2 | 7 | 22 | 4.29 |

Use the same ranking system and the same subject areas for the scale below.

a) I probably am most competent to teach:
b)
c)
d)
e) I probably am least competent to teach:

| | B | | | | | | |
|---|---|---|---|---|---|---|---|
| Rdng | 4 | 18 | 8 | 6 | 1 | 2 | 1.89 |
| LA | 6 | 7 | 16 | 6 | 3 | 1 | 2.24 |
| Math | 5 | 8 | 5 | 8 | 10 | 3 | 2.85 |
| SS | 11 | 1 | 5 | 9 | 7 | 6 | 3.43 |
| Sci | 4 | 2 | 2 | 1 | 9 | 21 | 4.29 |

Use the same ranking system and the same subject areas for the scale below.

a) The most important subject for children at my grade level to master is:
b)
c)
d)
e) The least important is:

| | B | | | | | | |
|---|---|---|---|---|---|---|---|
| Rdng | 5 | 32 | 1 | 0 | 0 | 1 | 1.15 |
| LA | 12 | 2 | 6 | 13 | 4 | 2 | 2.93 |
| Math | 10 | 0 | 22 | 6 | 1 | 0 | 2.28 |
| SS | 11 | 0 | 1 | 5 | 13 | 9 | 4.07 |
| Sci | 8 | 0 | 0 | 2 | 9 | 20 | 4.58 |

III. How can we help?

A. Is there anything you would like to do yourself or to see done that would probably help kids learn mathematics yet would cost just a small amount of money? If so, describe it briefly and indicate what it would probably cost.

6th: I'd like set of c-rods and some type of counter (e.g., abacus)

3rd: In my 5 years of teaching my biggest problem was trying to accumulate good and varied manipulatives. Everything I have I bought, made or stole with the exception of this year when I used the balance of education funds. My own personal belief is kids have not played with enough manipulatives before they come to school which is one reason math may be hard for them. Every year I get 4 or 5 children who can't count past 5, and who have no idea what 4 is. These children do not necessarily have a low I.Q. just very limited experiences.

4th: A calculator, $40.

4-5: A calculator, $40.

5th: A calculator, $35.

2nd: PTA is making balances for my group and also providing more flash card materials. I will make a number line for each desk this summer.

7th: Develop discipline, scholastic curiosity, and humility in students. No cost.

5th: Manipulative aids for the four basic operations as well as basic theory. Manipulatives on fractions or algebra.

5-6: I would like to see a class being taught mathematics on an individual basis. I don't know if this is cost related.

8th: Metric materials, graph paper. $2/child

2nd: I'd like to have a pocket calculator available--just a loan. Not sure how appropriate for 7 year olds, but...

1st: More worksheets of the Lore Rasmussen type--a sense of fun about them.

3rd: Graduate students available in our learning center with pooled math resources from all the teachers,manipulatives, accumulated materials, etc., for use as needed.

4th: Workshop to give teachers confidence in making their own manipulative materials. When teachers make their own materials, they have to think about basic concepts and skills.

2nd: The availability of $50 to buy any material I need for use in teaching math.

IIIA continued

3rd: $125-$250 (depending on number and cost--could be shared) electronic calculators for the classroom. (I've used yours!)

4th: A place in which all new things (books, other printed materials, manipulatives, etc.) are kept on display where I could go to look, examine, touch, play with, and then decide what to make, duplicate, or buy; or just to stimulate ideas.

4-5: Have lots of dittoed sheets, one per day per child, with basic math word problems.

Lrng Lab: Initiate a project with a group of youngsters with the discovery of math used in everyday life. E.g., baseball, shopping for their parents (measure, $, etc.) games--such as horseshoes--involving measurement, observing skilled workers, etc. Project culmination would be a written log of such observations over a 3 month period. Pre & post attitude tests would be administered. ($100 for field trips, supplies, and games.)

2nd: I'd like to see a program built around found objects or recycled objects. ($0)

Lrng Lab: I think students learn subjects that teachers feel competent to teach. No price tag can be placed on that!

7-8: Don't know cost--but would like materials such as described in II-3 above [workcards, workbooks, drill sheets, etc.].

Learning Disability: I'd like to make Montessori's long division set. (I have some of the materials.) I need more materials for measurement and transition to metric.

III. How can we help?

A. Is there anything you would like to do yourself or to see done that would probably help kids learn mathematics yet would cost just a small amount of money? If so, describe it briefly and indicate what it would probably cost.

Probably cost $________. Description:

See compilation of responses on preceding two pages.

B. Would you be interested in attending any of the following Mathematics curriculum workshops that are being offered at the University of Chicago this summer (see attached course descriptions) under either of the following conditions:

1) for half-tuition ($200) and full credit?
2) for no cost with no credit?

| | | Yes, no, maybe | 1) credit<br>2) free |
|---|---|---|---|
| June 24-July 12 | Applications of Mathematics: 11am-2pm | Yes 5<br>Maybe 2 | |
| July 15-Aug 2 | Learning and Teaching Basic Mathematics with Manipulative Materials: 1pm-4pm | Yes 6<br>Maybe 3 | |
| Aug 5-Aug 23 | Correlated Activities for Teaching Basic Mathematics and Science: noon-3pm | Yes 5<br>Maybe 5 | |

C. The course on learning and teaching using manipulative materials may be offered from about September 1 to December 10 a couple of days per week after school. If so, would you be interested in taking it:

1) For about $200 with full credit? Yes, No, Maybe
2) For no cost and no credit? Yes, No, Maybe

Yes 14
Maybe 14

D. Would you be interested in joining other Hyde Park teachers in a discussion/seminar about what we might cooperatively do to improve the learning of mathematics in local elementary schools? If so, indicate by a "yes" or a "no" below what times and subjects would interest you. (This is not a commitment, just an indication of interest.)

Yes 23
Maybe 3

When?

*~~July 15-Aug 2, maybe~~ a couple of mornings per week ________

*~~August 5-23, maybe~~ a couple of mornings per week ________

*During the Autumn, starting about September 15, maybe once a week for a couple of hours ________

What?

*Diagnosis and remediation of mathematics learning problems ______

*Better use of the great variety of manipulative materials and games and printed materials now available. We would examine them and discuss how to use them and how to work them into the everyday classroom experience. ______

*Other (you name it) ______

IV. Some information about yourself:

A. Where have you taught? (Most recent first)

School　　Grade level　　Number of years

| Years of Experience: | Blank | 1-2 | 3-4 | 5-6 | 7-8 | 9-10 | 11-15 | 16-20 | 21-25 |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 4 | 5 | 3 | 4 | 4 | 9 | 5 | 4 |

mean = 10.63

B. Where will you be teaching next year? ______

C. What mathematics courses or mathematics methods courses have you had?

1) In high school: ______

2) In college:

| No. of Math. Meth. Cses. | | | | | | Number of Mathematics Courses in College | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Blank | 0 | 1 | 2 | 3 | 4 | Blank | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7+ |
| 2 | 10 | 21 | 4 | 2 | 0 | 2 | 17 | 10 | 7 | 1 | 0 | 0 | 2 | 0 |
| mean = .95 | | | | | | mean = 1.05 | | | | | | | | |

D. B.A. degree: Where ______________________________

When ____________ Major ______________

E. College work beyond Bachelor degree: ______________________________

____________________________________________________________

____________________________________________________________

In appreciation of your taking the time to complete this survey, we would like to give you one of the following books as a gift. The books must be ordered in bulk and then sent to you so please list an address at which you can be reached during the summer and indicate your preference as to which book you'd like.

Name: ______________________________

Summer address: ______________________________

______________________________

______________________________

Preference of book wanted: ______ Second choice if that not available: ______

Books

1. Nuffield, Pictorial Representation 1 - designed to help teachers of children between age 5 and age 10, deals with graphical representation in its many forms.
2. Nuffield, Beginnings 1 - deals with the early awareness of both the meaning of number and the relationships which can emerge from the everyday experiences of measuring length, area, capacity, and time.
3. Nuffield, Computation and Structure 3 - suggests an abundance of ways of introducing children to multiplication so that they will understand what they are doing rather than simply follow rules.
4. Nuffield, Graphs leading to Algebra 2 - develops the use of coordinates, open sentences and truth sets, deals with the graphical aspects of these statements, introducing graphs of inequalities, intersection of two graphs, and graphs using integers.
5. Dienes, Building Up Mathematics - An introduction to the constructive approach to the teaching of mathematics, mostly at the primary level but with mathematics usually in the secondary school curriculum. The use of concrete materials of many kinds is explained in detail.
6. Dienes, Teaching Logic to Young Children - An exploration of the terms and operations involved in set theory and a good discussion of materials and techniques that may be used to teach them to elementary school children.
7. Curriculum Guidelines for North York (Canada) Elementary Schools

   7a. K-6 - A series of very clearly defined and stated educational goals for the mathematics curricula for each of the grades. The objectives themselves give innumerable suggestions of teaching techniques in all areas since they are primarily centered in concrete and activity oriented behaviors.

   7b. for Junior High School - same as above for grades 7 to 9 with a good mathematics library list and an excellent annotated bibliography of books and audio-visual materials.
8. Bell, Mathematical Uses in Our Everyday World - many problems using genuine data and information from many sources, mostly accessible with only arithmetic skills.

Appendix E: Report on Teacher Center Activities

This is part of M. S. Bell, Explorations into Ways of Improving the Elementary Mathematics Learning Experience, a report on a project supported by NSF grant PES 74-18938. The report of the Center staff is reproduced here.

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# REPORT ON TEACHER CENTER COMPONENT OF NSF GRANT

Prepared by Chris Brown
David Messerschmidt
Susan Carpenter

May 27, 1975

## 1) Teacher Center Support Services

The Hyde Park Teacher Center provides a number of support services in mathematics (and other areas) for Chicago teachers. The Center has an extensive collection of math books, curricula, and manipulative materials. The math file includes curriculum suggestions, classroom activities, sample worksheets, and duplicating materials. Homemade math games and puzzles are displayed, and raw materials (cardboard, markers, laminating materials, wood blocks, spinner, disks, etc.) are available to duplicate them. Finally, the Center staff are available to advise, stimulate and support teachers in new approaches to teaching math. The Center also has regularly scheduled Saturday workshops which are frequently on math topics.

## 2) Special Workshops

March 8, 1975 Workshop from 10-4, conducted by

Barry Hammond, of Winipeg Teacher Center (formerly Co-Director of Teacher Center in Hyde Park)

64 people attending, 22 from Hyde Park

The Bartlett Studio was set up.as a mathematics laboratory with Teacher Center mathematics materials.

The morning session focused on the algebra of integers. The objective was to enable teachers to gain a better understanding of the basic concepts of associativity, commutativity, identity and inverses through the solving of selected problems (Problems are enclosed). The afternoon session continued with problems demonstrating the algebra of permutations and of rectangles. This was followed by a lively discussion of the uses of various mathematics materials -- i.e., workbooks and worksheets, activity cards, games and manipulatives in the classroom and of the integration of mathematical operations into the total curriculum.

April 12, 1975 from 10-4, Workshop for Intermediate and Upper Grades conducted by Gordon Clem, Headmaster of St. Thomas Choir School, New York City and Chairperson of Mathematics Section of NAIS

30 persons attending, 11 from Hyde Park

The participants worked at solving a variety of logic and attribute games; and games including finding functional relationships through games such as What's My Rule and Towers of Hanoi. There were ongoing informal discussions of how to introduce and use these problems and games in the classroom. (See enclosed problem sheets.)

3) Mathematics Lending Library

Mathematics Lending Library at the Teacher Center provides a resource of manipulative materials which are generally not available to classroom teachers. The library consists of a classroom set of Cuisenaire Rods, three sets of Relationshapes, chip trading materials, and Cuisenaire Cubes, Rods, and Solids. During the year each of the materials was introduced to the teachers in the Math Workgroup with suggestions for class activities. Teachers were then able to borrow the materials from the library and try them in their own classrooms. At the next session of the workgroup, teachers would discuss what they did with the materials and the reactions of children in their classrooms. The follow-up discussions provided a forum for both discussing specific classroom practices and creating new uses for the materials.

4) Math Workgroup

A group of intermediate grade math teachers met biweekly on Thursdays throughout the year. The group's leaders were

David Messerschmidt and Chris Brown, with assistance from Susan Carpenter of the Teacher Center staff. Attendance ranged from 2 to 10.

a) Purposes of the Workgroup

The purposes of the groups were:

1) to introduce manipulative material in the teaching of math;
2) to provide a discussion and support group for the teachers involved in experimenting with new math class organizations and new techniques for teaching math;
3) to develop new ideas and materials for teaching math concepts.

b) Sessions

The sessions were held from 4:00-5:45 on Thursdays. The general format for each session was:

1) game activity or introduction of new material by group leaders;
2) discussion of use of materials, and general math programs, in individual classrooms;
3) in-depth discussion or development of a particular topic;
4) questionnaire evaluating that session.

| Session Number | Date | Topics |
|---|---|---|
| 1 | Oct. 3 | Surface area with Cuisenaire Rods<br>Course Outline |
| 2 | Oct. 17 | Poker Chip Exchange game<br>Record keeping in the classroom<br>Examination of books used in different classrooms |
| 3 | Oct. 31 | Magic Squares |
| 4 | Nov. 14 | Place Value Activities<br>Self-Rating of math classes (see attached list) |
| 5 | Dec. 3 | Straw Geometry<br>Christmas decoration polyhedrons<br>Maps of where math materials are kept in classrooms |
| 6 | Jan. 16 | Clock Math<br>Teacher- and student-made word problems |
| 7 | Jan. 30 | Relationshapes<br>Attribute games |
| 8 | Feb. 13 | Fraction strips<br>Integrating fractions and decimals<br>Fraction Circles |

| Session Number | Date | Topics |
|---|---|---|
| 9 | Feb. 27 | Geometry in cardboard, with Jim Bottomley |
| 10 | March 13 | Fraction operations with Cuisenaire Rods |
| 11 | March 20 | Fractions boxes |
| 12 | May 1 | Geometry -- Miras and Symmetries |
| 13 | May 15 | Geometry -- Pattern Blocks<br>Mirrors |
| 14 | May 22 | Geometry -- Angles, Protractors, and Compasses |

c) Evaluation of Workgroup

The plans for evaluating the math working group seem, in reflection, almost grandiose. Several methods were discussed, among them, taping each session, providing questionnaires to participants about their background and interest in mathematics, visiting classrooms with a checklist on materials and operation, teacher self-reports on room inventories and methods.

All of the methods required a stable group of at least eight or more people working in classrooms who were willing to commit themselves to a full year of meetings. As the size of the group dwindled, it became useless to pursue evaluation in the way it had first been conceived.

Thus, the evaluation of the Math workgroup is subjective. The leaders feel that there was considerable growth in the regular pariticpants, particularly in the areas of mathematical understanding, and in the ability and willingness to try new techniques. There were many new games and techniques employed in the classrooms of the participants.

Comments of a Teacher Center staff member on the repercussions of the math group on some of its members:

Participating in the Thursday bi-weekly math workgroup has had consequences far beyond that of increasing the participants' understanding of mathematics and willingness to try out new materials and methods in the classroom. As a result of the children's responses to the introduction of new mathematics materials, some of the teachers have begun to take a different view of how a classroom can be organized and to move away from dependence on paper and pencil, textbooks and workbooks. In the case of one teacher, the children's enthusiastic responses to the Exchange Game stimulated her to continue to introduce new materials into the classroom. She has gone on to building furniture for her classroom out of tri-wall. Her pleasure in teaching has grown and her expectations of the children have changed. There is no doubt that her classroom is now a more alive, interesting and comfortable place as a result of her working with the math group.

d) Perspective and Critique on Math Workgroup

Although the Math Workgroup provided considerable positive assistance to the participants, the sparseness of ongoing attendance limited its total impact on the schools. There were several realizations that came out of the Workgroup.

The leaders felt that the course was too long; 5 or 6 weeks of weekly meetings would have been preferable. Also, the sessions (bi-weekly) were too far apart to provide much continuity.

A more specific definition of the purposes and expectations of the Math Workgroup would have led to more productive sessions. Whether the Workgroup was a course, with materials and ideas to be presented by the leaders, or a workshop directed by the participants, was an unresolved issue.

Both leaders and participants had positive feelings about the Workgroup's accomplishments and expressed interest in continuing, on a modified basis, next year. It was agreed, however, that a larger pool of teachers in the immediate Hyde Park area should be identified to participate.

HOW DO I RUN/RATE MY MATH CLASS? Checklist for teachers developed by Teacher Center Math Workgroup -- Winter, 1975

1) Groupings -- one group or many
   -- by ability or by interest

2) Materials -- variety -- different levels
   -- amount -- use and adaptation

3) Physical arrangement of class -- moveable desks
   -- provision for small groups
   -- set for whole class lessons
   -- location of materials

4) Time use -- teacher-led
   -- small group
   -- working with individuals
   -- students in small groups by themselves

5) Student attitude toward math (tied in with individualization)

6) Visual Aids and stimuli on walls -- Roman numerals
   -- posters
   -- charts and graphs
   -- etc.

7) Record keeping -- by teacher
   -- by students

8) Time use -- students -- in books
   -- manipulatives
   -- games and other activities

9) Independent applications -- student projects
   -- discussions of real life and everyday math situations

1882 Hyde Park Herald 1974
chicago's oldest community newspaper

Oct. 23, 1974

## Reception at Teacher Center

Local school administrators were invited recently to a reception at the Teacher Center, 1400 E. 53rd st. so they could learn more about the operation of the facility. The center provided teachers with materials and sponsors many workshops so educators can better their teaching skills.

1882 Hyde Park Herald 1974
chicago's oldest community newspaper

Jan. 1, 1975

# Teacher Center News

**By RUTH NEDELSKY**

We hope that you had a fine holiday. The Teacher Center staff scattered East, West and South. Joan went to New England, Susan to Iowa, and Mary to North Carolina. Ruth stayed in Chicago and enjoyed children and grandchildren. The Center re-opens on Thursday January 2; hours as usual 2:30 to 5:30. The Center will be open Saturday from 10 to 4, but there will be no workshop.

### Workshops

An exceptionally interesting series of workshops has been planned for the winter quarter. Unless otherwise noted, the workshops are held from 1 to 3 p.m. in the Center, 1400 E. 53rd st.

January 11—Displays-Workshop leader-Carol Brindley, teacher, University of Chicago Nursery school. We have had so many requests for ideas on bulletin boards that it was decided to devote a workshop to the general topic of displays. Various kinds of displays and their uses in the classroom will be discussed. Special attention will be given to ways of displaying children's work.

January 18 and 25, February 22 and March 1—A series on Language Development in Young Children. Workshop leader Carole Harmon, teacher at Ancona Montessori school.

February 1—Small Worlds Workshop leader Karen Hermann, teacher at Co-op #3. Karen will demonstrate how primary children can construct miniature worlds out of all kinds of scraps.

February 8—Mathematics Workshop Leader-Barry Hammond, former Co-director of the Center and now in Winipeg.

Barry is returning to Chicago to conduct an ALL DAY workshop (10-3). Circle this date on your calendar. More details about the precise nature of the workshop will appear in this column at a later date.

February 15—Woodworking-Workshop leader-Duncan MacLaren. Another ALL DAY workshop. Details later.

### Practicum for New and Experimenting Teachers

The practicum will again be meeting on Wednesdays from 4-6 at the Center.

The practicum is designed especially for new teachers. We extend a most cordial welcome to all first and second year teachers to visit the class and see if it will be helpful to you. (other teachers interested in discussing aspects of life in the classroom are also welcome).

The focus of the practicum in the winter quarter will be on groupings within the class room.

We will start by looking at the kinds of activities that children can best do in small groups of threes, fours and fives. The class sessions will be geared to helping teachers think about and plan for, in very concrete ways, particular activities that they would like to carry out in their own classrooms, such as using cuisinaire rods, doing map studies, making a mural, playing math games. The class meets over coffee and cheese and crackers; you can relax after a hard day at school in good company and share ideas on common problems.

### Mathematics work group

The math work group will again be meeting at Center on alternate Thursdays from 4-6. The first session will be on January 16. Both Susan and David are going to Boston to attend the Educational Arts Association Regional Conference, January 10, 11, 12. There is room in the group for four or five more persons.

* * *

We would like to find ways of increasing our usefulness to local schools. We will be sending a questionaire to all the teachers in the neighboring schools and another to the principals. We urge you to take a few minutes to fill out the questionaire and return it to the Center. If you have not been in to see us yet this school year, why not start the New Year with a visit to the Center?

1882 Hyde Park Herald 1974
chicago's oldest community newspaper

# News from the Teacher Center

Sept. 25, 1974

**By RUTH NEDELSKY**

If you come to the workshop on Saturday, September 28, conducted by Dave Messerschmidt, you will have the chance to play the people pieces game.

People pieces, that attribute game, pattern blocks, color cubes. these are some of the games that are fun for children to play, yet, at the same time, stimulate them to think logically.

The games require children to sort, to classify, to categorize, to compare, to see and create pattenrs. The best way to understand the appeal these games have for children is to play them yourself and see what you learn in the process.

At the workshop, you will be able to play the games and to discuss with your fellow players what your children could do with them and how you could use them in your classroom.

These games can be used on different levels. Primary children can use the people pieces in one way and fourth and fifth graders can use the pieces to play more complex and sophisticated games. Even within the same classroom, kids will find different ways of sorting and patterning. Such games provide the experiences out of which grows the child's ability to deal with abstractions, so necessary to his acquiring a concept of number and an understanding of basic math operations. (as well as concepts in all other fields).

It is possible to make simple, inexpensive versions of some of these games so that you can have a number of them for use in your classroom. There will be time for workshop participants to make games to take back to their schools. All this is further proof that learning need not be a bore or a chore for either teacher or children. October begins with a round of activities.

On October 1, from 3 to 4:30 the Center is having open House and a coffee hour for principals of neighboring schools. We at the Center are eager for school administrators to become familiar with the kinds of curriculum materials and educational services that we offer.

Then, on Wednesday, Oct. 2 at four o'clock, the Practicum for new and experimenting teachers will begin.

The main purpose of the Practicum is to help teachers with the problems that arise in the classroom. Teaching requires a number of skills that can only be learned on the job, yet rarely is help available in this learning, leaving most new (and experimenting) teachers are strictly on their own. The practicum will serve as a meeting ground, a place where teachers can pool information and ideas and return to the classroom, refreshed. There is a five dollar registration fee, payable to Loop college, which is jointly sponsoring the Practicum.

We expect the practicum to continue throughout the year. as three eight-week sequences, fall, winter and spring. Call 955-1329 if you are interested in joining the course.

On Thursday, Oct. 3, at four o'clock, the first session of an intermediate math work-group, conducted by Dave Messerschmidt and Chris Brown, two fifth grade teachers will be held. The group will meet for two hours every other Thursday.

A variety of specific math materials and activities for the teaching of basic math concepts will be discussed during the sessions; the teachers will then try out some of them in their classrooms and report back to the group on their successes and failures.

There is still room for three or four more persons in the group. Call 955-1329 if interested.

NEWS FROM THE CENTER
1400 E. 53rd Street
Chicago, Illinois 60615
(312) 955-1329

We hope all of you have managed to avoid the winter doldrums! Our spirits have been lifted by our refurbished workroom—we painted it during our last staff workday and are finding it a much more pleasant place to work in. We've also had fun purchasing new equipment—we now have a new hotplate, a vacuum cleaner, a second large laminating machine, and a new ditto machine. The laminating machine is still in the box, since we don't have space to set it up. We are negotiating with the Y for more extensive use of Bartlett, and we're hoping to be able to spread out a bit soon.

Staff and board members have been busy getting out our funding proposal and making plans for the summer and the coming year. Also, on some Mondays following our regular staff meeting, we are having staff development sessions in which we explore a particular material or idea in depth. This means that we are even less available than usual to work with teachers on Mondays. We have hired David Dunning, a U. of C. graduate student, to man the Center during Monday open hours.

## WORKSHOPS

We have a number of good workshops planned for the end of winter and the beginning of spring.

| | |
|---|---|
| Saturday<br>March 8<br>10 to 4 | If you thought you missed Barry Hammond on February 8, you didn't! He'll really be here on March 8, doing an all-day math workshop. We plan to set up Bartlett as a classroom and fill it with all sorts of math materials. |
| Saturday<br>March 15<br>10 to 3 | Wooded Island workshop with David Carlovsky. The workshop will start out at the Center and then move to Wooded Island. Mapping, plant and animal identification, survival, and some history will be explored with reference to the island. |
| Saturday<br>March 22<br>1 to 3 | Workshop on probability with Chris Brown, who teaches fifth grade and is one of the leaders of the math group which meets at the Center. |
| Saturday<br>April 5<br>1 to 3 | Workshop with Elvie Moore on explorations of ways to use movement in the classroom. |
| Saturday<br>April 12<br>10 to 12<br>1 to 3 | Math workshop with Gordon Clemm, headmaster of the Choir School of St. Thomas Church, New York City, and chairperson of the NAIS Math Committee. There will be both morning and afternoon sessions, and participants are invited to attend one or both. |
| Saturday<br>April 19<br>1 to 3 | Elizabeth Hollander, a city planner, will give a workshop on "What's Under the City—An Exploration of the City for Primary Children." Part of the workshop will take place outside—participants will discover everything they can about the city in a block. |
| Saturday<br>April 26<br>1 to 3 | Workshop on the middle school with Pat Gardner. Pat, who teaches at an alternative middle school in Hyde Park, will discuss open classroom at the junior high level, apprenticeship programs, integrated curriculum, and upper level language arts. |

## SCHEDULE

Spring Vacation. The Center will close from Monday, March 24, through Saturday, March 29. It seems to make sense for us to take our break then and be open during public school vacations in April.

Tuesday evenings. Although staff members leave at 5:30 on Tuesdays and return for the 7 to 9 open hours, anyone who wants to stay through and work is welcome to do so.

## MEMBERSHIPS

The support we're getting through memberships is really gratifying--we now have 140 members contributing over $1500 of support. We'd like to remind everyone that library privileges are now extended to members only. Also, members should keep in mind that our annual meeting and party will be coming up in May.

## SUMMER SCHOLARSHIPS

This year the Center will again offer scholarships to enable teachers to attend summer workshops. Last summer several teachers received scholarships of up to $250 and attended workshops in this country and in England. These teachers have shared their experiences by giving workshops at the Center throughout the year. At this point, we're gathering information about various workshops and drawing up the specifics of the program. The application deadline will be May 10. Drop in at the Center or call us for more information.

## OTHER NEWS

Susan is setting up a file of information on various tests teachers must take, such as certification exams. The file will tell about experiences with the tests and give pointers on taking them. If you have information to add to the file, or if you're seeking information, please contact Susan.

Several individuals at the American Friends Service Committee would like to meet regularly with the intention of developing peace studies curricula for the elementary school child. They invite participation by other teachers who see a necessity for educating young children about the prerequisites for a peaceful world. Anyone interested may call Rich Weston at AFSC during the day (427-2533), Marti Weston nights (241-6614), or Alice Walton (WI 5-1744).

If you haven't been to the Center lately, come in and enjoy our bright workroom and our new equipment. We'll look forward to seeing you!

| | |
|---|---|
| Joan Bradbury | Sandy Lang |
| Carol Brindley | Hannah MacLaren |
| Susan Carpenter | Mary Mathias |
| Sharon Feiman | Ruth Nedelsky |

# TEACHER CENTER CALENDAR
March-April, 1975

The Center is at 1400 E. 53rd Street, Chicago, 60615. Phone: 312-955-1329

Hours: weekdays 2:30-5:30; Saturdays 10-4; Tuesday evenings 7-9

| monday | tuesday | wednesday | thursday | friday | saturday |
|---|---|---|---|---|---|
| March 3<br>Center closed—staff workday | 4 | 5<br>4:00 class | 6 | 7 | 8<br>10 to 4: math workshop with Barry Hammond |
| 10<br>Center open; staff in meeting | 11 | 12<br>4:00 class | 13<br>4:00 math group | 14 | 15<br>10 to 3: Wooded Island workshop with David Garlovsky |
| 17<br>Center open; staff in meeting | 18 | 19<br>4:00 class | 20 | 21 | 22<br>1 to 3: workshop on probability with Chris Brown |
| 24<br>Center closed for spring vacation--March 24-29 | 25 | 26 | 27<br>Center closed for spring vacation--March 24-29<br>4:00 math group | 28 | 29 |
| 31<br>Center open—staff in meeting | April 1 | 2<br>4:00 class | 3 | 4 | 5<br>1 to 3: movement workshop with Elvie Moore |
| 7<br>Center closed; staff workday | 8 | 9<br>4:00 class | 10<br>4:00 math group | 11 | 12<br>10-12 and 1-3 math workshop with Gorden Clemm |
| 14<br>Center open; staff in meeting | 15 | 16<br>4:00 class | 17 | 18 | 19<br>1 to 3: workshop on explorations of the city for primary children with Elizabeth Hollender |
| 21<br>Center open; staff in meeting | 22 | 23<br>4:00 class | 24<br>4:00 math group | 25 | 26<br>1 to 3: workshop on the middle school with Pat Gaarder |

## Appendix F: Activities Generated by the "Everyman Mathematics Development Group" *

Reproduced here are the edited activities as they were distributed to participants in the working groups. For details on these working groups, see 3.8 in the main report.

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*This is an appendix to M. S. Bell, *Explorations into Ways of Improving the Elementary Mathematics Learning Experience*, a report on a project supported by NSF grant PES 74-18938.

## EVERYMAN MATHEMATICS DEVELOPMENT GROUP

### Participant List

| Name | School | Grade |
|---|---|---|
| Siter Mary Jane Adams | St. Thomas Apostle | 4, 5, 6 |
| Pamela Ames | U. of Chicago | Teacher Education |
| Ann Barrish | ? | |
| Max Bell | U. of Chicago | Teacher Education |
| Katherine Blackburn | Kenwood High School | 9-12 |
| Warren Crown | Governors' State Univ. | Teacher Education |
| Molly Day | U. of Chicago Lab. School | 2 |
| Georgine Ezre | Phoenix School | 1-2 |
| Dorothy Freedman | U. of Chi. Lab. School | 2 (Assistant) |
| Sherye Lynn Garmony | Louis Wirth School | 7, 8 |
| Patricia Gerling | St. Thomas Apostle | 7, 8 |
| Nancy Hanvey | Kozminski School | 1 |
| Janet Kobrin | U. of Chi. Lab. School | 1-3 |
| Kaye Letaw | Lewis-Champlin School | 5-8 (remedial lab.) |
| Donna Marlin | Kozminski School | 3 |
| Joyce Martin | Lab. School, U. of Chi. | 2 |
| Mary Mathias | Family Free School | K-3 |
| Cillie McGlory | Louis Wirth School | 6 |
| Susan McKamey | Louis Wirth School | 6, 8 |
| Mike McNamee | U. of Chi. Lab. School | K (Assistant) |
| David Monk | U. of Chi. student | |
| Kate Morrison | U. of Chi. Lab. School | 2 (Assistant) |
| Roase Nayer | Harvard-St. George | 2 |
| Blythe Olshan | Metro High School | 9-12 |
| Dan O'Neill | Bret Harte School | Principal |
| Mary Page | Kozminski School | 1 (consultant) |
| Judith Petzold | Bret Harte School | 7 |
| Carol Samuels | U. of Chi. Lab. School | 2 |
| Pauline Schafer | St. Thomas Apostle | 4-6 |
| Sidney Shirley | ? | |
| James E. Schultz | U. of Chicago (Visiting Prof.) | Teacher Education |
| Joyce Smith | Woodson School | 4-6 |
| Susan Smolinske | Lindbloom Tech. H. S. | 9-12 |
| Sr. Elizabeth Marie Stoltz | St. Thomas Apostle | 7, 8 |
| Jay Sugarman | U. of Chi. Lab School | 2 (assistant) |
| Marilyn Thompson | Bret Harte, St. Thomas | Consultant |

## EVERYMAN MATHEMATICS DEVELOPMENT GROUP

### Activities Related to Measure Concepts
### (1/14/75 Sessions at the University of Chicago)

Participants

Sister Mary Jane Adams
Pamela Ames
Max Bell
Warren Crown
Dorothy Freedman
Sherye Lynn Garmony
Patricia Gerling
Janet Kobrin
Kaye Letaw
Donna Marlin
Joyce Martin
Mary Mathias
Susan McKamey
Mike McNamee
David Monk
Kate Morrison
Rose Nayer
Blythe Olshan
Dan O'Neill
Mary Page
Judith R. Petzold
Carol Samuels
Pauline Schafer
James E. Schultz
Joyce Smith
Susan Smolinske
Sister Elizabeth Marie Stoltz
Jay Sugarman
Marilyn M. Thompson

## PROBLEMS AND ACTIVITIES HAVING TO DO WITH SEVERAL KINDS OF MEASURES

### Matching sizes, ordering, objects, refining notion of "bigness"

Preschool K Choose several pots with lids. Mix them up. Find the lid that fits each pot. Put the pots in order from biggest to smallest. How did you decide which is biggest? Is it tallest? or fattest? or does it hold more? or does it have the widest lid? Can you put the pots one inside another?

NOTE: "Big"and "small" are not specific enough for most measure activities; also, they depend on context. "More" or "less" and similar words are frequently useful.

### Looking for measure problems, pervasiveness of measure

3-6 Make a list of things you see (in the room, as you walk to school, in the playground, etc.) that can be measured. How would you measure each of them?

4-6 Look through a magazine. Find as many pictures as you can that show something about measure. Try grouping the pictures several ways: e.g., types of things being measured or types of measuring instruments. What categories make most sense? Was it easy or hard to find pictures? Discuss the reasons for this.

### Introducing the ideas of "unit" and "measure"

1-3 Measure with non-standard units. e.g., length with a spoon, area with square floor tiles, weight with pennies or paper clips. Compare results when different units are used.

This might be done before introducing a new "standard" unit. Also, it may combat the feeling of some children that length _is_ inches, weight _is_ pounds, etc.

4-6 Pretend that you do not have any of the standard measuring devices. How would you measure: head sizes, weights, quantities of food, land area, quantities of lumber, etc.

| | | |
|---|---|---|
| 3-5 | Measure a given object in several ways e.g., a ball, for its weight, diameter, circumference, volume, texture, bounce, etc. | Observe that many different measures can apply to the same object. |
| 6-8 | Describe the apparent size of the sun in as many ways as you can. E.g., "You can ring the sun with a notebook paper hole if it is held 1 foot from you." How would these descriptions differ for a Mercury Man or a Pluto Man? | |

<u>Estimating</u>

| | | |
|---|---|---|
| K-8 | For most measure activities, estimate the result before measuring. For each new unit, do you find your "guesstimates" getting closer as you measure more things? | NOTE: As a result of these exercises, students frequently have a better feeling for the unit involved, improve their estimating skills, and show more interest in the actual measuring. |
| 1-3 | Guess, and then count, the number of marbles in jars of various small sizes, (i.e., not beyond counting abilities). | |
| 2+ | Guess beans in a jar, etc. Then find a good approximation to actual number using ways proposed by class members; e.g., 1) Everybody take a "handful," count, add the results. 2) See how close if only one persons handful is counted, then multiplied by number of handfuls. 3) Measure out a small amount; count how many in it; measure to see how many of the small measures in the large. 4) Weigh a small amount; count; weigh the total. 5) Spread evenly on a large sheet of paper marked with a grid. Count how many covering one section, etc. | |

### Knowing when to count, measure, or estimate

| | | |
|---|---|---|
| 3-7 | To measure these, would you 1) Count a number? 2) Take a measuring device and use it to get a number? 3) Judge or estimate or guess a number?<br>Hairs on head, distance to the moon, length of a car, weight of a shoe, | NOTE: Answers will vary depending on purpose, i.e., Parking a car in a parking space (estimate) vs. building a garage for a car (measure). |

### Choice of units

| | | |
|---|---|---|
| 3 | What kinds of things can you measure in feet? in gallons? in pounds? Which would you use to measure: milk for the lunchroom? table? tree? water in a pool? | NOTE: Answers may vary as above. |
| 2-3 | How many miles long is this room? How many inches from your home to school? How many tons do you weigh? | Playfully done, not to get actual answers. Youngsters should readily see the inappropriateness of the units. |
| 3-4 | Measure the room length with yellow Cuisenaire rods. Measure the width of the desk in yards. Etc. Discuss the problems with these tasks. | |
| 1-3 | Find the length of this room using one of several sticks or a bodylength. Compare several answers. Why are they different? If you wanted to order carpeting as long as the room, which number would you write on the order form? Why? | |

### English to metric conversions

| | | |
|---|---|---|
| 2-8 | If children have had lots of practice working with non-standard as well as standard units, the concept behind unit conversion shouldn't be a problem. However, to facilitate immediate approximate conversion, children should have lots of experience in expressing the same measurement in both English and metric units. | |

Activities involving metric conversions might include relabeling household items (cups, pails, cereal boxes, etc.) in metric units. Also personal measures such as weight and height.

Conversions along with judgement about choice of units

6 You are in the advertising business. Choose the best statement for your purposes:

"My shampoo contains $\left\{\begin{array}{l}.03\text{ gal}\\ .1\text{ pint}\\ 2\text{ ounces}\end{array}\right\}$ more."

"Our route is only $\left\{\begin{array}{l}.2\text{ hours}\\ 12\text{ minutes}\\ 720\text{ seconds}\end{array}\right\}$ longer and it's many times prettier."

6 Is this a biased headline? If so, how could it be said to be "fair"? Headlines-- 1) Rocky owes \$.9 million taxes 2) Days are getting shorter by 2 1/2 nano-seconds a year

6 Find some numbers that are used in newspaper articles. What size numerals are used? What types of units are used? Why do you think those units were chosen?

"Longer than" and "shorter than" (Ordinal Measurement)

K-1 Use cuisenaire rods, sticks, or any other easily comparable graduate series of manipulations. Say: "Choose any rod (stick,etc.) Which rods are longer? Which are shorter? Which are the same?" After child has made three piles you might ask "Which of the longer rods is closest (next longer, next bigger) to the one you chose. Which is next shorter? Notice how the child decides which are longer or shorter. If he picks at random, ask him to show you how he knows that a rod is shorter or 1 longer. If a child puts the ends of two blocks at the same point to compare lengths he is probably ready for more sophisticated measuring.

Transitivity of ordinal measurement.

1-2 When children have successfully manipulated rods in longer and shorter categories, ask: 1) Is the green block longer than the red block? 2) Is the black block longer than the green block? 3) Is the black block longer than the red block?

Measuring heights of block structures

K This can be done in non-standard units (cuisenaire rods, erasers, etc.; finding something that is as tall as the structure) or in standard units.

1-2 Measurements with cuisenaire rods could be expressed numerically. If this is done, make sure that children understand that the white rod is the unit of measure.

Measuring distance by steps.

K Use giant steps, baby steps and middle-sized steps. Count the number of steps. How do they differ?

Measuring distance - arriving at a standard measure

1-2 Find the length of the room by starting in the corner and a) Pace off the distance along the wall, putting one foot just in front of the other. How many"feet"?

b) Cut out tracing of child's own foot on cardboard. Measure along same wall using cardboard foot as measure. How many "feet"?
c) If different people in the class have done this exercise, compare answers. Why are they different? What can we use for measuring that will give us the same answer?

Comparative Measure with different units

K A collection of pairs of shoes belonging to older and younger members of a childs family can generate comparative measures of the childs foot with different sized shoes. He will probably want to try walking in bigger shoes. Suggest walking heel-toe and counting how many steps it takes to walk across the room in his own shoes, his mothers, his fathers, etc.

Cutting specified lengths

K Cut the pdece of paper (cardboard,wood, etc.) to make a bed for the doll (alligator horse, car, or any toy available). Try to make the bed just as long as the toy, not too long, not too short. After children have cut out the beds, ask them to check and see if their toy "just fits" on the bed.

Measuring circumferences of body parts

K-1 Use pieces of string, ribbon, or strips of paper to measure the circumference of wrists, head, chest, waist, etc. This activity is probably best done in pairs where each child measures his partner. Have scissors available so string can be cut to the exact length. If paper is used it can be torn to length. For younger children only one or two measures need be taken.

1-2 Each child can hang strings lengthwise and order them by length. Geometric variation: Children can display length of string in any shape--circle, line, curved line etc. "This is my waist (head, etc.)"

1-2 Measure several body parts. Child may order his own circumferences by size. Try to find out which circumferences change. How far can you expand your chest? What happens when you flex your arm muscle?

2-6 Use length of string to find circumference in inches. If children have had graphing experience try making some graphs to show the relationship between head size and wrist size, etc.

Measuring length and height.

K-1 Children can measure each others heights, length of thumb, arm etc. Measurements can be taken in non-standard units (how many blocks high, long etc.).Keep a chart in the room to record the information. Children might like to make scatter graphs showing the frequencies of certain heights, arm lengths, etc.

2-6 Make graphs showing the relationship between two measurements.

Measuring large objects with non-standard units

K-1 How long is the blackboard? (rug, table top, etc.) Is it longer than the eraser? (piece of chalk, pencil, your hands; feet, etc.) How many erasers long is it?

1-2 Use a variety of objects to measure. Results will vary. Discuss why answers differ might follow. Why should there be a standard unit of measure?

Choosing an appropriate unit of measure

2-4 When children are comfortable using rulers and yardsticks, design a chart as follows.

| length of | in inches | feet | yards |
|---|---|---|---|
| chest | | | |
| table | | | |
| room | | | |
| hall | | | |
| shoe box | | | |
| crayon | | | |
| etc. | | | |

Which unit was "easiest" for measuring a specific object. Which gave the quickest answer. Which gave the most exact answer. When and whether would you use a certain unit of measure to describe a given object?

Measuring length with standard or non-standard units

3-4 Measure a very long object (rope, spool of thread, etc.) that can be laid out in the hall. Making a long paper chain is a good individual or class activity. Object can then be measured with a variety of standards. (inches, feet, meters, inches per foot, etc.)

Indirect measure using the trundle wheel

2-3 Trundle wheels are easy to make - if you have hammers and nails the class can help put one together. (See instructions in the Math Lab.) After children have measured the circumference of the wheel, use it to measure long distances such as the length and width of the playground or the distance from one end of the hall to the other. Children count "clicks" to determine how many times the wheel has gone full circle.

Finding a way to measure the path of a moving animal

3-4 If you have small animals (rats, turtles) in the class devise a "race" between two of them. Trace the path each takes, then measure and compare distances.

Comparing estimation and measurement

5-6 Estimate the distance from the pencil sharpener to the floor, the top of the desk to the floor. Then measure with a ruler or yardstick to see how close you were.

Discuss situations in which exact measurements are preferable to estimation. Is measure ever exact? Also, why a certain unit was used in a child's estimation and/or measurement. Did they differ? Why?

## WEIGHT

### Comparing weights

K-1 Which is heavier? How can you find out?

K-1 Weigh yourself on the weighing scale whenever you like. Is your weight always the same? Check

2-3 Check your weight on several consecutive days--or once a week on the same day. Keep track of your results and graph them. Check your weight at the beginning and end of your school day. What changes do you notice, if any? Why does your weight seem to change?

NOTE: Many children will need help reading the scales. To catch the variations, they should be able to read to the nearest pound (if that is enough).

### Balance scale - equalization of weight

K-1 Use paper clips, rubber bands, pine cones, feathers, poker chips, etc., to make a pan balance level.

### Non-standard measures - balance scale

4-6 Make a balance scale and a set of your own weights using three units: paperclip, marble, reading book. Weigh objects in the classroom

# LIQUID MEASURE

## Non-standard units

2 Choose four jars of different sizes and label them A, B, C, and D. Choose another set of three jars (numbered 1, 2, and 3). How many of jar 1 does it take to fill each of the lettered jars? Write the results in the chart.

NOTE: For younger children jars 1-3 should be smaller than jars A-D so that whole numbers can be used (e.g., jar A is between 3 and 4 of jar 2 or nearly 6 of jar 3). Older children don't need this restriction.

| | 1 | 2 | 3 |
|---|---|---|---|
| A | | | |
| B | | | |
| C | | | |
| D | | | |

Do the same for jar 2. Also for jar 3.

2-3 Take several paper cups (all the same size). Fill one half full and label the cup 1/2. Fill one to the top and label it 1. Also make cups containing 1/4, 1/3, etc. "cups" of water. Keep this set of measuring cups. Which do you think is more: one half cup or two thirds cups? How could you find out? (Don't use the water in your measuring cups.) Can you be sure of your answer?

## Conversions among standard measures of liquid capacity

1-2 Find containers that hold each of these amounts: 1/2 pint, 1 cup, 1 pint, 1 quart, 1/2 gallon, 1 gallon. Find the relationships among these. e.g., How many pints in a quart? How many cups in a gallon? etc.

2 Use cups of several different sizes. (Have at least three of each size.)
RECIPE: Fill one cup to the top with flour. Fill a second cup (same or different size) one quarter full of salt. Fill a third cup one fourth full of water. Mix the ingredients.
Experiment with several sets of three cups, using the same instructions each time. Can you find a set of three cups for which the mixture has the consistency of play dough?

NOTE: Dough should turn out when all three cups are same size. However careless measuring could result in wrong consistency. Also other proportions of these ingredients can work, so kids may find that 3 different sizes works.

## Standard units - cooking

1 Make a simple recipe.
2-3 Make a half recipe or a double recipe.
Don't forget to halve or double everything.
2-3 Use an oven thermometer to measure temperature of the oven. How much can you vary the temperature and still have it turn out OK?

3 Make candy using a candy thermometer.

3 Use an English recipe with English measures. How would you change other recipes to English recipes?

Measuring larger quantities

2 If you wanted to conserve water, would you take a shower or a bath? Which do you think takes less? How would you measure it? Measure to find which is less. Compare your answer with others. Why might some say the bath is less and others, the shower?

NOTE: Individual preferences (such as longer shower or deeper baths) will affect the results. Also size of bath tub.

Estimation and measure

2-3 Use several differently shaped boxes and enough cubic inch counting blocks to fill the largest one. 1) Predict which box would hold the most, next, etc. Also predict how many each would hold. 2) Fill each box with the blocks. Can you determine exactly how many blocks are in the box without counting them all? Does it help to know how many blocks in the height, length and width of the box? 3) How do the results compare with the predictions?

3-4 4) After doing this several times, can you find the exact number of blocks without filling the box? How do you do it?

Volume, non-standard units

2 Use several containers that can hold water, including an almost flat one (e.g., a cookie sheet). Predict their order from largest (i.e., holds most) to smallest. Let the smallest be the unit. 1) Predict and then 2) measure the number of "units" in each of the other containers. How did the results compare to the predictions? Try using one of the other containers as the "unit." Is it easier or harder to do now? Why? Can you find a way to use water to find the volume of solid objects? Try to find how much water is equal in volume to a 1-cubic-inch block. Then, how many cubic inches of water in each of the above containers? And how many cubic inches of water in a standard measuring cup?

Area-defining a square inch as a unit of measure

3-4 Give students paper marked off in square inches. How long are sides of each square? (1 inch). It's a square inch. Cut out square inches and use them to cover notebook, desk, etc.

Finding area with geoboards

6-8 Geoboards can be used in a variety of activities. See activity sheets in the file cabinets in the Math Lab.

## OTHER MEASURES

### Comparative sizes

Preschool, K Preschool children might enjoy trying on shoes, hats, gloves, shirts, etc. of different sizes. "Is is too big or too small?" How much too big (small) is it?" Some children might notice sizes on labels. "Which is bigger? Size 2 or size 6?" (Sizes are tricky so don't emphasize them too much.) Encourage the use of terms other than "big" or "small."

### Unit conversion using foot length and shoe sizes

2-4 Use a shoe store measure to measure feet, then try measuring feet in inches or centimeters. Chart or graph the relationship between the two measures.

### House numbers as measurements

3-6 ∝ See what children can come up with be studying the house numbers on their streets. Observations might include: each is different, they get bigger, (maybe) they increase in a regular way, the first part of the number is the same for the whole block, in the next block the first part of the number changes, and the last part repeats.

5-6 Older children can work with maps to discover what relations house numbers have to layout of the city. (In Chicago the first part of the house number denotes number of blocks from the center of the city.)

### Measuring temperature

1-2 As children come in from outdoors, discuss the temperature in their terms. (cold, hot, warm, freezing) Check the outdoor thermometer for temperature and match their description to the thermometer reading. When this is done and recorded over a number of weeks, the relationship between temperature and its measurement should become more comprehensible.

### Measuring heartbeats

2-4 Work in pairs, one using a stethoscope to listen to the other's heartbeat. Use a clock with a second hand to measure heartbeats per minute. Try the exercise two or three times to see if number of heartbeats per minute remains the same from one minute to the next. When does rate of heartbeat change? What happens to the rate of heartbeat when you hop on one foot for a minute?

4-6 Why do we use time and counting to measure heartbeat? What other things use time and something else in a measurement?

Measuring "mood"

2-4 When children have had lots of experience with more/less relationships and have worked with the number line, Ask: How many of you are happy? sad? Very happy? Very sad? So-so? Lets say "0" is very sad, "10" is very happy. As the numbers get larger they mean less sad and more happy. Where would you place yourself on the scale? Children can rate themselves every day. Then talk about how their mood differed throughout the week, month.

4-8 What makes this type of measurement different from others you know? When you rate yourself "6", does that mean you feel just as happy as someone else who rated himself "6"? Does "10" mean twice as happy as "5"?

Measuring time

1-3 Discuss the difference between a long time and a short time, soon and later, etc. Why do we need a standard unit of time? What are some of the units we use for time? (days, weeks, hours, etc.) What are some of the things we use to help keep track of the time? (clock, calendar, sun, seasons)

Using time and distance as a measure of speed

3-5 Run races and time them. At first children will probably specify a distance that is the same for every contestant. Later change the distance for some of the runners. Does time (number of minutes, seconds) still tell who was fastest? What do you need to know besides time to measure speed?

Manipulating data gathered from time measurements in foot races

5-8 Record each students time. See how many ways data can be used to convey something about the class. E.g., Use a bar graph to generate the running profile for the class. Separate data into tall-short categories (dividing the class at the middle) and compare profiles, averages, medians, etc. with that of each other and the whole class.

Measuring reaction time as an average

3-6 (Students should feel comfortable with averages.) A group of students holds hands in a circle. One student holds a stop watch in his left hand. As he starts the watch with his left hand, he simultaneously uses his right hand to squeeze the hand of the person on his right. This person in turn squeezes the hand of the person on his right and so on until the last person stops the watch. Divide the total reaction time by the number of students, thus obtaining an average. Further variations might include reducing number of participants. What happens to the average? Other impulse-reaction situations could be explored using a single student or a group.

Measuring rate of growth

2-4 Plant seeds (use large seeds - beans, popcorn, raw peanuts) in paper cups. When seeds have sprouted, begin taking measurements of height of plant at regular intervals - perhaps every two or three days. Measurements can be made with rulers, or strips of centimeter graph paper cut to plant height. Record each measurement. How much did the plant grow in the first period? The second? etc.

3-4 Graph the results. Note that graph should be a line graph since growth is continuous.

Pervasiveness of measure

4-6 Ask: What radio station do you listen to? Where do you find it on your dial? What do those numbers mean?

Measurement of price (price in hours worked, inflation and purchasing power through a comparison of incomes and cost of consumer goods in 1900 and today)

7-8 The Sears 1902 catalog is a good reference for cost of consumer goods at the turn of the century and is easily obtained through most bookstores. Income information can be gathered from history books or government documents. For comparison, have a current

Sears catalog on hand, along with current data concerning average incomes. After selecting items from both catalogs and comparing their prices, some of the following questions might be asked:
What is the cost of each in terms of work? (i.e., how long did a man have to work to buy a pair of shoes in 1900? Today?)
How does the average workers purchasing power compare with that of the worker of 1900?
Given the data, what can we say about inflation? (How is the decline in the value of the dollar offset by the rise in incomes?)

# REFERENCES

Isaac Asimov The Realm of Measure (Fawcett Co.Paperback) Very fine.

Chapter III in M. Bell Mathematical Uses and Models in Our Everyday World (SMSG)

David Greenhood Mapping (U. of Chicago Press, Paperback). Find stuff on maps and mapping adaptable for youngsters from about 4th grade on.

Booklet #16 of More Topics in Mathematics for El. Teachers (NCTM Yearbook 30 or 31) Good down to earth discussion of measurement. See Schültz for more detail.

See Schultz for activities with hypsometer and other technical measuring devices for upper grades.

Lauren Woodby San Pablo Bay activity. This is a good one! See Schultz for details.

NOTE: Please share with us references to other good material you know of, especially material with fresh ideas..

An attachment to Appendix F has been omitted because print is not legible.