ED 219 262 SE 038 765

AUTHOR Good, Thomas L.; Grouws, Douglas A.
TITLE Experimental Study of Mathematics Instruction in Elementary Schools. Final Report.
INSTITUTION Missouri Univ., Columbia. Coll. of Education.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE Dec 79
GRANT NIE-G-77-003
NOTE 236p.; Parts of appendix are marginally legible.

EDRS PRICE MF01/PC10 Plus Postage.
DESCRIPTORS Achievement; Educational Research; *Elementary School Mathematics; Grade 4; Grade 6; Intermediate Grades; Learning Theories; Mathematics; Mathematics Curriculum; *Mathematics Instruction; *Problem Solving; *Program Descriptions; Program Design; Teaching Methods
IDENTIFIERS *Mathematics Education Research

ABSTRACT

The development and field testing of two instructional mathematics programs are discussed. The general instructional program evolved from earlier research, and was field tested with 40 teachers and over 1000 fourth-grade students. Data reflected that this program had strong, positive effects on students' achievement. The second program focused on the development of instructional strategies for attempting to improve students' skills for solving verbal problems. The data base for building such a training program was considerably more limited than the base available in designing the general program. The second experiment was run with 36 teachers and other 1000 sixth-grade students. Data suggest that verbal problem-solving skills of treatment students were enhanced in comparison to that of control students. Research that examined the impact of instructional programs upon particular student types is also reported. Interactions were noted between treatment condition, teacher type, and student type. Work on interactions in the second experiment was still in progress at the time of this report. Early indications suggest low-achieving and dependent students may benefit the most. (MP)

Experimental Study of Mathematics Instruction in Elementary Schools

Thomas L. Good
Douglas A. Grouws
University of Missouri--Columbia

December, 1979

Final Report of National Institute of Education Grant NIE-G-77-003

Abstract

This report discusses the development and field testing of two instructional, mathematics programs (a general program and a verbal problem solving component). The general instructional program evolved from our earlier naturalistic research in fourth grade classrooms and from recent process-product studies. The data base for this program, although correlational in nature, appeared to be solid. However, much interpretation of the data was necessary in order to construct the first training manual and teaching recommendations. This program was field tested with 40 teachers and over 1000 fourth grade students. The data reflected that the program had strong, positive effects on students' achievement (especially mathematics skills and knowledge).

The second program training manual focused upon the development of instructional strategies for attempting to improve students' skills for solving verbal problems. The need for such an emphasis was suggested by the results from the first field study. The data base for building such a training program was considerably more limited than the base we had for designing the general program. The second field study involved the combined effects of the general training program and the problem solving program. The sample for the second experiment was 36 teachers and over 1000 sixth grade students. The data suggest that the verbal problem solving skills of treatment students was enhanced in comparison to that of control students. This was especially the case for schools that were not using open space concepts. The differences on general achievement measures were not clearly supportive. Although the raw gains of treatment students exceeded that of control students on the general achievement tests, these differences were not statistically significant.

Research that examines the impact of the instructional program upon particular types of students is also reported. The data reveal several interactions between treatment condition, teacher type, and student type. However, the implication of these interactions is not immediately clear in many instances. Work on the interactions in the second experiment is still in progress and the results of this work will help to determine the stability of certain interactions and also their application value for classroom teaching. At present, the clearest implication from the interaction work is that low-achieving and dependent students appear to benefit most from the program. However, it is important to realize that interactions may be a function of the population studied.

We feel that the program has led to the development of an instructional program that has application value in certain applied settings (especially for elementary school teachers who teach the class as a whole). We suspect that there are many ways to successfully structure mathematics learning, and the program we have developed is but one alternative. This program appears to have ecological validity. That is, teachers seem to be willing to implement the program and to continue using it. Hence, the demands on teacher time and the teaching activities called for in the program do not appear to be inappropriate from the viewpoint of teachers who have used the program.

Acknowledgements

We want to express our sincere thanks to a number of individuals who helped us in various stages of conducting, analyzing, and preparing project materials and reports: Terrill Beckerman, Sally Beckerman, Howard Ebmeier, Larry Flatt, Carol Flatt, Gail Hinkel, and Sharon Schneeberger. Terrill Beckerman and Howard Ebmeier were instrumental in all stages of the project and we want to underline their invaluable assistance.

We also want to express our appreciation for the logistical support provided by the Center for Research in Social Behavior, Bruce J. Biddle, Director. In particular, we want to acknowledge the continuing clerical assistance of three individuals who helped us in countless ways: Barbara Breen, Mary Harbourt, and Sherry Kilgore.

Finally, we want to thank two individuals from the Tulsa School District who were instrumental in helping us to conduct the research. Dr. Jack Griffin and Mr. J. W. Hosey provided us with consistent support and we appreciate their active interest in this program of research.

Table of Contents

Introduction

In 1970, the accumulated knowledge about the effects of classroom processes on student achievement was weak and contradictory. In some curriculum areas at the elementary school level the picture is now more optimistic. Within less than a decade, the literature on basic skill instruction in reading and mathematics in elementary schools has moved from a state of confusion to a point at which experimental studies can be designed upon a data base. In brief, several large scale correlational studies have produced data to illustrate that it was possible to identify some teachers who consistently obtained more student achievement than expected and that it was possible to identify instructional patterns that differentiated these teachers from teachers who were not as successful on the operational definition of effectiveness (standardized achievement).

As Brophy (in press) has noted, progress in this area can be attributed, in part, to three major factors. First, the important review works by Rosenshine and Furst (1973) and by Dunkin and Biddle (1974) helped to summarize what was known about the effects of teaching, and to clarify some of the weaknesses of extant research, and were instrumental in bringing a degree of conceptual coherence to the field. The second factor was an increased awareness o the methodological problems inherent in studying teacher effects and the concomitant willingness of investigators to begin to respond to those challenges in creative ways (e.g., Dunkin and Biddle, 1974, and the national conference sponsored in 1974 by the National Institute of Education). A third factor was the willingness of the National Institute of Education to invest in large scale process-product research and the ability of researchers to design broad scale exploratory studies. In particular, the field of teacher behavior moved its research base from that of "commitments" (Dunkin and Biddle, 1974) and the search for universal dimensions of teaching effectiveness to the study of teaching in manageable contexts. It is beyond the scope of this final report to review extant literature. General reviews of methodological advances and substantive findings of recent process-product studies can be found elsewhere (for reviews see: Medley, 1977; Rosenshine, in press; Brophy, in press; Good, 1979).

The purpose of this report is to describe our efforts to build and field test a program of mathematics instruction. Although the focus of this report will primarily spotlight our own research, it should be understood that our work has been stimulated and enriched by the work of others in the field. Also, it should be noted that portions of this report have appeared elsewhere (Good and Grouws, 1977; Good and Beckerman, 1978; Good, Ebmeier and Beckerman, 1978; Good, 1979a, 1979b; Good, Grouws, and Beckerman, 1978; Good and Grouws, 1979a; 1979b; Ebmeier and Good, 1979).

Background: The Naturalistic Study

In 1975, with the support of the National Institute of Education (NEG-00-3-0123), we completed a large observational study of teaching effectiveness in fourth grade mathematics instruction, Good and Grouws (1975). The purpose of the research was to see if it would be possible to identify teachers who were consistent (across different groups of students) and relatively effective or ineffective (in terms of student performance on the Iowa Test of Basic Skills). Furthermore, it was our intention to observe teachers who differed in effectiveness and to see if differences in their classroom behavior could be identified.

The program of research was conducted in a school district which skirted the core of a large urban city. The student population was primarily middle class, but a number of students from low and high income homes attended target schools. Teachers in the district were using the same textbook series and the district had a stable student population.

To identify patterns of behaviors that make a difference in student learning, it was considered desirable, at least initially, to focus all observation upon classroom activity during the teaching of a particular subject. Mathematics instruction was chosen because of its importance in the elementary school curriculum (reading and mathematics are commonly accepted as the subject curriculum in elementary schools) and because it was felt that more teacher and school variance would be associated with students' mathematics performance than reading performance. This assumption has now received empirical support from Coleman's analysis of data from the International Educational Achievement Study (1975).

Sample Selection

Over 100 third- and fourth-grade teachers were initially studied. The data unit for the investigation was individual students' scores on the Iowa Test of Basic Skills. Residual gain scores were computed for students on each math subtest by using the student's pretest score as a covariate (using a linear model where g = y (a + bx). Residual gain scores were computed within year and grade level. Data for teachers were then compiled by computing a mean residual gain score (from the scores for students) for year one and year two.

It was possible to identify nine teachers who were relatively effective[1] and stable on total math residual scores across two consecutive years (that is, they were in the top one-third of the sample across two years) and nine teachers who were relatively ineffective and stable across two consecutive years. During the year of observation, the high and low effective teachers were again found to maintain their relative patterns of achievement. Hence, these teachers were found to be stable across three years.

The stability of context is an important issue but one seldom discussed in teaching research literature. If the conditions of teaching are not stable, it is difficult to make inferences about teacher effectiveness. For example, if student populations are changing rapidly (changes in the school neighborhood, busing, etc.) then the lack of stability in teaching performance may be due to the fact that the conditions of teaching are changing.

Similarly, if teachers use different curriculum textbooks, it is difficult to compare their relative teaching effectiveness. The pace and order of subject matter presentation embedded in curriculum material may interact with teaching method. Hence, in our initial investigation, it was advantageous to study teachers using the same curriculum material.

Observational data were collected in 41 classrooms to protect the identity of the relatively effective and ineffective teachers. Approximately equal numbers of observations were made in all classrooms (6-7). Data were collected by two trained observers (both certified teachers) who worked full-time and lived in the target city. Prior to the collection of observational data, the observers went through a three and one-half week training program that involved the coding of written transcripts, videotapes, and live coding. Training was terminated when the two coders exhibited over 80 percent agreement on all coding categories. Each coder visited all 41 teachers and made roughly one-half of the observations obtained in a given classroom. Furthermore, all observations were made without knowledge of the teacher's level of effectiveness.

Four basic sets of information were collected in the study. First, time measures were taken to describe how mathematics instructional time was utilized. In addition to their descriptive function, these categories were designed to facilitate the testing of several hypotheses suggested by earlier experimental research on time variables in mathematics instruction (e.g., the ratio of time spent in development vs. practice activity). The second set of codings were low inference descriptions of teacher-student interaction patterns. These data were collected with the Brophy-Good Dyadic System (1970). The third set of data were high inference variables drawn from the work of Emmer (1973) and Kounin (1970). The fourth type of data coded were checklists that were used to describe materials and homework assignments (see Good and Grouws, 1975, for a discussion of reliability and copies of the instrument).

Results

During October, November, and early December, observational data were collected in the rooms of the 41 participating teachers during mathematics instruction. Process (observational) data collected during this time period were subsequently analyzed with classroom mean residual scores on total mathematics (on the Iowa Test of Basic Skills) collected during that same year. Product measures (Iowa Test of Basic Skills) were administered in October 1974 and April 1975, and classroom residuals were computed from these two data sets and associated with classroom process.

Classroom process data were analyzed with a one-way analysis of variance model to see if there were behavioral variables on which the nine high effective and nine low effective teachers differed. The significant and near significant findings (on the high and low inference variables) are presented in Table 1.

In presenting the results, emphasis is placed upon positive findings and the reporting is largely an integrative effort. This position is assumed because as we have argued elsewhere (Good and Grouws, 1977), we do not believe that separate, individual teaching acts constitute an appropriate way to conceptualize an independent variable. We feel that the importance or role of any particular teaching act is meaningful only in relationship with the degree and sequence of other teaching acts.

Hence, what follows is our characterization of the major ways in which relatively effective and ineffective teachers differed[2] in their classroom behavior during mathematics instruction. Variables that we emphasize in this presentation have two major characteristics: (a) the process variables significantly related to student achievement scores (that is high and low teachers as a group performed significantly differently on measures of the behavior as shown in Table 1, and (b) the process variables generally discriminate among high and low group membership (that is formal and informal discriminant analysis show that individual teachers within a high or low group consistently show more or less of a behavior than do teachers in the contrast group).

Before discussing these patterns, it is useful to note one context organizational finding. We use the term context because the organizational variable (whole class instruction) did not differentiate high and low classrooms categorically (both the nine high and nine low teachers utilized whole class instruction).

It is interesting to note that students in the nine high classes received their instruction as a unit. Within each of the classes they were given the same in-class assignments and identical homework assignments. Several of the teachers (but not the nine high and nine lows) included in the observational study taught mathematics to two or three operating groups.

Teachers who taught mathematics via group instruction fell in the middle of the distribution (teachers' average residual gain score on total mathematics); teachers who taught the whole class consistently appeared at the top or bottom of the distribution. These data suggest that teaching the class is not a poor or good strategy categorically. If the teacher possesses certain capabilities, it may be an excellent strategy; if not, the whole class instructional mode may not work well.

One of the necessary skills for effective whole class instruction is the ability to make clear presentations. Highs regularly exceeded lows in clarity scores. They generally introduced and explained material more clearly than did lows. Interestingly, in whole class settings, highs asked more product questions (a question that demands a single correct answer) and appeared to keep the "ball moving." However, when students did experience difficulty, highs were more likely to give process feedback (a response that not only provides a correct answer, but also suggests how that answer could be derived, than lows. In contrast, lows were more likely to ask process questions (a question that demands integration of facts, explanation) and less likely to give process feedback. It seems that highs did not focus upon process as a ritual, but rather they used process responses when student responses indicated some error (i.e., they appeared to move class discussion in a diagnostic cycle).

In brief, it seemed that high achievement teachers presented students with a clear focus of what was to be learned, provided developmental (process) feedback when it was needed, structured a common seatwork assignment for the class and responded to individual students' needs for help.

Two patterns clearly differentiated the behavior of high and low teachers. The first is that highs demanded (communicated higher performance expectations) more work and achievement from students. For example, high teachers assigned homework more frequently than did now teachers and highs also moved through the curriculum considerably faster than did lows. The assignment of more work and covering more curriculum material increases time on task and opportunity to learn and these variables have been consistently related to increased student achievement.

The second pattern of effectiveness observed was students' opportunity for immediate, nonevaluative, and task relevant feedback. Several different behavioral measures consistently demonstrated that high teachers were approached by students more than teachers in low classrooms. Presumably when students in high classrooms wanted information or evaluative input they felt free to approach the teacher. Even when the teacher dealt with the entire class in a public format, students in high rooms were able to participate by their own initiative. Students in these rooms asked the teacher more questions, called out more answers, and proportionately were asked more open questions (questions which students indicate they want to answer: they raise their hand, etc.).

In this context, student initiated behavior appears to make good sense. For example, students' call-out rates per hour were very low. Given a general population of middle class students, who possess basic skills and who have given a clear explanation of the learning task, it appears appropriate to allow them to approach the teacher as they need help. In a different context (low aptitude students, teacher does not provide clear work structure), heavy rates of student initiated behavior may indicate confusion, managerial problems, and be associated with lower rates of student achievement.

In addition to designing an environment in which students could get feedback (especially during seatwork), highs were more likely to provide developmental feedback than were lows. Also high teachers were more likely to provide nonevaluative feedback than were lows.

The data demonstrate that high praise rates do not categorically enhance learning. Indeed, in this study, praise was negatively associated with both achievement and climate.[3] Consistently, high teachers were found to praise less than low teachers. Interestingly, despite their high praise rates, lows were much less likely than highs to praise students when they approached them about their academic work. Presumably, low teachers prefer to go to students (rather than being approached by them), a strategy that proved to be ineffective in this study. High teachers were basically nonevaluative. They did not criticize or praise academic work as frequently as did low teachers. The evaluative stance of lows, coupled with their high rates of approaching students, may have interfered with learning progress as well as creating a "heavy" climate. High classrooms were regularly described more favorably by students, despite the fact that high teachers did not praise much.

Low teachers seemed to have more frequent managerial problems than did high teachers. However, the data here are not as clear as for the two clusters described above. Several measures show little difference between high and low teachers (e.g., percent of students not involved in lesson). Suggestion of discipline problems stems from the fact that lows issued many more behavioral warnings and criticisms and initiated more alerting and accountability messages. The data in the present study suggests that, in some contexts, it may be possible for teachers to communicate too many accountability and alerting messages (especially if teachers are highly evaluative).

Lows appear to have more managerial problems than highs. However the process that minimizes the behavioral disturbances are not clear. However, we suspect that teachers who communicate high performance expectations, who structure clear learning goals, and who structure systems that provide students with immediate, helpful feedback will also minimize behavioral problems.

After the project was completed, the two coders were asked to rank the 41 teachers in terms of perceived effectiveness in producing student learning gains. The coders had no trouble in identifying the relatively ineffective teachers; however, many of the high teachers were described as performing at a medium level of effectiveness. Furthermore, there was little correspondence between incorrect classifications across coders (i.e., they misclassified different teachers). Presumably (and not surprisingly) ineffective teaching is easier to distinguish from average teaching than is effective training.

Qualifications and Discussion: Background Study

Certain qualifications must be applied to these data. One basic consideration is that the general pattern of results presented must be interpreted within the operational definition of effectiveness (student performance on a standardized achievement test). The definition employed is but one way of looking at classroom progress. It is a valuable way to study classroom progress. It is a valuable way to study classrooms, but any operational definition of effectiveness imposes restraints upon the investigation <u>per se</u> and the conclusions that are drawn. Studies that link classroom process to other operational definitions of effectiveness are needed.

The data reported here describe only linear relationships between process measures and product outcomes. Subsequent work needs to center upon non-linear relations that may exist in the present data. Several findings reported in this study may be clarified and/or extended by subsequent data analysis activities. For example, praise and teacher-afforded contact may be variables that interfere with learning only if engaged in too frequently.

The data described here only report teacher behavior toward students generally. Student initiated work contact was analyzed as student initiated contact categorically. High achievers and low achievers may show different initiation rates and such behaviors may relate to effective learning. Such hypotheses were not examined in the present analysis.

Some of the advice typically given in teacher education programs has been drawn into question in this study. Praise, historically, has been a teacher behavior that has been liberally prescribed for teachers to provide to students. However, the data here indicate that too much or inappropriate praise may not facilitate learning. Other investigators also have recently questioned the desirability of high praise rates for students who are performing capably (e.g., Brophy and Evertson, 1976).

Complex questions have often been touted as superior to simple ("right-answer" oriented) questions. However, for producing achievement gains in a highly structured subject like mathematics it may be better instructional strategy for teachers to ask relatively more product than process questions. (Why engage in a diagnostic cycle unless a pupil makes an error?)

The fundamental qualification that needs to be applied to these findings and their potential for teacher training is their correlational nature. The results suggest that certain patterns of teacher behavior are associated with student achievement; however, no causal linkage can be argued.

The Present Study: Need for Experimental Inquiry

We were pleased with the fact that some consistent differences could be found between relatively effective and ineffective mathematics teachers. However, at that point we only had a description of how more and less effective teachers (in our sample) behaved differently. We did not know if teachers who did not teach the way more effective teachers did could change their behavior or whether students would benefit if teachers were trained to use new methods. We wanted to determine whether teachers could be taught these behaviors, and if such instruction and training could be used to improve the mathematics performance of students.

Building the Treatment Program

Originally, our intention had been to test two clusters of instructional behavior separately and in combination. These two clusters are presented below. The reasons (we feel compelling) for testing only a single treatment are also presented below. The description of the two clusters that appear below are taken from the original grant proposal. The ultimate modification synthesis will follow.

The first cluster we conceptualized (performance expectations and time utilization) is associated with teacher actions which are not a part of the teacher-pupil interactions in the classroom, but which are associated with the teacher's role of decision maker and efficient manager of instructional time.

The principle components of this cluster include instructional pacing, homework assignments, systematic review and maintenance, and the relationship between developmental instruction and practice activities. Before specifying how these components are synthesized to form the Performance Expectation cluster each facet is examined independently beginning with instructional pacing.

In our naturalistic study of effective teachers (Good and Grouws, 1975) there was strong data to show that effective teachers consistently moved through the mathematics textbook more quickly than did less effective teachers. This same result is supported by the work of Lundgren (1972) in Swedish schools where the term used is "steering group." In fact, many of the cross cultural achievement differences in mathematics identified by the International Study of Achievement in Mathematics (Husen, 1967) may be due to differences in opportunity to learn (Postlethwaite, 1971) which in turn may be moderated by the pace at which teachers move students through instructional material. Of course there are a number of other teacher behaviors which may be a part of effective teaching. Teacher decision making in assigning homework is one such area that has been examined in previous research efforts.

Gray and Allison (1971) in their review of writing and research on homework in mathematics found a lack of consistency in results. This may be due to the examination of a "behavior in isolation" approach taken in some studies. There are a number of studies (Goldstein, 1960; Koch, 1965) which have shown homework to be beneficial in increasing computational skills in the middle and upper grades. There were also indications in our earlier work (Good and Grouws, 1975) that this variable is worthy of further study in connection with other teacher behaviors. One such behavior is systematic review and maintenance which we now examine in more detail.

Student achievement in the various subject areas has received much critical attention during the last few years with children's computational skills receiving special scrutiny. In order to increase proficiency in basic skills many people have suggested more instructional time be spent on practice and many recent textbook series have substantially increased their emphasis on drill activities by increasing the number of problems in each text. Milgram (1969) has shown that elementary school teachers spend over 50 percent of instructional time on oral and written drill. This result suggests a different approach would be worthwhile. Especially worthy of study is the less widely acclaimed recommendation of instituting a systematic review component into mathematics programs whereby children, on a regular schedule, devote some instructional time to maintaining and increasing computational skills.

In the preceding discussions, the potential value of homework and systematic review and maintenance has been highlighted while at the same time the value of increasing the instructional pace has been advocated. The demands these suggestions make on the fixed amount of instructional time available requires decisions concerning utilization of instructional time.

The portion of class time spent on developmental activities compared to the portion spent on drill and practice activities has been studied quite extensively (Shipp and Deer, 1960; Shuster and Pigge, 1965; Zahn, 1966; Dubriel, 1977). Their accumulated evidence suggests that classes that devote more than 50 percent of the mathematics period to developmental activities performed better on achievement tests than those classes spending 50 percent or more time on drill activities. This was generally true for all ability levels of students.

In consideration of the preceding rational, one of the treatments will integrate pacing, homework, systematic review and maintenance, and attention to developmental activities in the following way. Teachers in this treatment will move through the textbook at a faster pace than usual. This increased rate will not be accomplished by skipping material but by teaching topics more quickly. Topics will be covered more quickly by keeping an emphasis on developmental work but spending comparatively less time on drill and practice work.

Drill and practice will not be slighted in this arrangement because students will be given homework assignments on a daily basis to be completed outside class. To maximize the usefulness of the homework, teachers will promptly score and return the papers to students. Teachers will also analyze five or so papers each day for systematic processing errors rather than just look at answers. This seems to hold particular promise in view of Roberts (1968) who found that students' errors are frequently systematic in nature. Teachers will also be encouraged to put variety in their homework assignments to hold interest and also to differentiate assignments to provide for differences between individuals in the class.

Finally, teachers in this treatment group will institute a systematic maintenance program by setting aside the last 20 minutes of the mathematics class period each Friday for review and maintenance. This will often involve a paper-pencil activity but will also involve review and practice type games when appropriate. This provision seems particularly appropriate when it is recalled that drill is not a way of learning but rather a process for consolidating learning that has been attained during the developmental or integrative stages of learning.

Cluster B: Instructional Focus

This cluster of teaching behaviors involves the interaction between students and teachers. The behaviors involved are associated with increasing opportunities for student feedback, emphasizing meaningful instruction, and focusing on task oriented feedback.

Our previous naturalistic research has shown that rates of feedback opportunities for students highly discriminate low and high effective teachers with increased feedback opportunities being associated with the highly effective teachers. The general instructional context in which this feedback is provided is of concern.

Much research (Dawson and Ruddell, 1955; Greathouse, 1966; and Miller, 1957) has shown that the meaningful learning of mathematics is much more desirable than rote learning on all dimensions that have been investigated including efficiency, retention and transfer. Consequently, initial instruction on new learning should focus on developing topics meaningfully, and then increasing proficiency through practice.

An obvious part of a meaningful development is providing feedback to students. Our previous research (Good and Grouws, 1975) further suggests that task oriented feedback without embellishments such as praise or criticism is associated with effective teacher style.

Increased opportunities for feedback, a focus on meaning, and task oriented feedback are integrated to form the second experimental treatment. Teachers in this treatment group will provide increased opportunities for student feedback. This may include group or individual self-checking activities, working in pairs, as well as verbalization by the teacher that she is anxious to answer questions. Further, to the extent possible (peer feedback cannot be controlled), feedback will be task focused. Finally, instruction in the early phases of new topics will be meaningful in nature.

Alteration in Research Strategy

The description of the two clusters above appeared in our original grant proposal. However, at the time when we were making arrangements to select a sample, teachers in the Kansas City area were on strike. Rather than wait until this conflict was resolved (and delay our attempts to secure a sample), we decided to seek a new sample. We were fortunate in securing the cooperation of the Tulsa School System but the process took time.

As we began to finalize our treatment plans in the summer of 1977, it became clear that a single treatment represented a more desirable plan. In part, our thinking was influenced by the fact that the preparation of three effective sets of training material would be a difficult, but not impossible task. More importantly, our thinking shifted because the more we examined available data and the more we thought about the problem, the more convinced we became that neither cluster alone would be sufficient to improve student performance in mathematics.

A final consideration was the fact that dividing the sample into four groups (Cluster A, Cluster B, combined, and control) would have necessitated separate meetings. We felt (and still do) that a strong Hawthorne control needed to be established if the field study was to be valuable. One of the most direct ways was to have a single meeting with all teachers present for a general orientation which was also attended by the associate superintendent of instruction for the school district as well as supervisory personnel and building principals. It would have been difficult for all of these individuals to have attended a single meeting.

By dividing into only two groups, it was possible to have all teachers to attend the same meeting (and to see the obvious interest of the school district) and then to allow each of the two co-investigators to work with one group following the general meeting. Although the final research plan differs from that originally planned, we believe the resultant plan was a better one.

The Treatment Program

Although we were pleased with the naturalistic findings in the sense that they provided some clear contrasts between relatively high and low gain classrooms, we were aware of the fact that these were only correlational results and that they did not substantiate the fact that these teacher behaviors caused student achievement. It could be that behaviors not studied in our observational research were more directly related to achievement (more effective teachers plan more thoroughly and because of this they are more task focused, assign more homework, for instance).

We now wanted to see if more direct linkages could be established between the behaviors that had been identified in our observational naturalistic study and student achievement. Because of the expense involved in field testing a program, we wanted it to be as comprehensive as possible. Thus, in building the training program, we integrated our results with those available from other process-product studies and those from previous experimental mathematics studies. The effort resulted in a 45-page training manual (see Appendix A).

The treatment program that resulted was stimulated by and consistent with the findings in our naturalistic study; however, it also included instructional dimensions suggested by the work of others. Some of our original thinking about ways in which teachers might attempt to improve mathematics instruction was reflected in the descriptions of the two clusters that we presented above.

Over time we clarified these ideas more fully and integrated them more broadly with other findings. Ultimately our ideas were written (and rewritten) into a training manual. It should be understood that in the writing process, research findings had to be <u>translated</u>. Emphasis is placed upon translation because descriptive statistics do not provide direct implications for how a given teacher should modify his or her behavior.

To illustrate the translation difficulty, consider the finding that "more effective" teachers move through the curriculum more quickly than do "less effective" teachers. What are the implications for a given fourth grade mathematics teacher? The call for action is not clear. Clearly, if a teacher is moving through the content at an appropriate pace, the advice to move more quickly would be dysfunctional. Formulating teaching recommendation from research findings is not a straightforward forward task. In many instances our recommendations do not represent a literal interpretation of the data. To understand the program, it is necessary to examine the training manual in detail (see Appendix A).

The major steps of the program are presented in Table 2. In brief, the program can be summarized in the following way. It is a <u>system</u> of instruction: (1) Instructional activity is initiated and reviewed in the context of meaning; (2) Students are prepared for each lesson stage to enhance involvement and to minimize errors; (3) The principles of distributed and successful practice are built into the program; (4) Active teaching is demanded, especially in the developmental portion of the lesson (when the teacher explains the concept that is being studied, its importance, and so on).

Summary From Teachers' Handbook

To further illustrate the general orientation of the program, a brief summary (that appears in the training manual) follows:

> We have asked you to do several things during the next few weeks in an attempt to improve student performance in mathematics. In the first part of this handbook we emphasized that we didn't feel that changing one or two teacher behaviors would make much difference

in student performance. We feel that the systematic application of all the behaviors discussed in this treatment program can make an important difference in student learning. The purpose of this last section is to briefly review the teaching requests we have made and to show how each of the pieces fit together into a total program.

The predevelopment portion of the lesson begins with a brief summary and a review of the previous lesson. The review (including the checking of homework) is designed to help students maintain conceptual and skill proficiency with material that has already been presented to them. Mental computation activities follow and provide an interesting bridge for moving into the new lesson.

Next comes the development part of the lesson which is designed to help students understand the new material. Active teaching helps the student comprehend what he is learning. Too often students work on problems without a clear understanding of what they are doing and the reasons for doing it. Under such conditions, learning for most students will be filled with errors, frustration, and poor retention. If student learning is to be optimal, students must have a clear picture of what they are learning; the development phase of the lesson is designed to accomplish this understanding.

The controlled practice that occurs toward the end of the development portion of the lesson is designed to see if students are ready to begin seatwork. It simply doesn't make sense to assign seatwork to students when they are not ready for it...practicing errors and frustrating experience guarantees that student interest and performance in mathematics will decline. The controlled practice part of the lesson provides a decision point for the teacher. If students generally understand the process and are able to work problems correctly, then the teacher can proceed to the seatwork portion of the lesson. If student performance on problems is relatively poor, then the development must be retaught. If students are ready to do practice work, it is foolish to delay them; similarly, if students are not ready to do development work, it is foolish to push them into it. The controlled practice part of the lessons allows the teacher to decide if it is more profitable to move to seatwork or to reteach the development portion of the lesson.

Hence, when teachers move to the seatwork portion of the lesson, students should be ready to work on their own and practice should be relatively error free. Seatwork provides an opportunity for students to practice successfully the ideas and concepts presented to them during the development lesson and carefully monitor student performance during the controlled portion of the lesson, then student seatwork will be a profitable exercise in successful practice.

The seatwork part of the lesson allows students to organize their own understanding of concepts (depend less upon the teacher) and to practice skills without interruption. The seatwork part of the lesson also allows the teacher to deal with those students who have some difficulty and to correct their problems before they attempt to do homework. If teachers actively monitor student behavior when seatwork is assigned and if they quickly engage them in task behavior and maintain that involvement with appropriate accountability and alerting techniques, then the essential conditions have been created for successful practice.

Homework is a logical extension of the sequence we have discussed. During the mathematics lesson students learn in a meaningful setting. During seatwork students have a chance to practice and deal with material they understand. The homework assignment provides additional practice opportunity to further skill development and understanding.

The above aspects of the mathematics lesson combine to give the student: (1) a clear understanding of what they are learning; (2) controlled practice and reteaching as necessary to reinforce the original concepts and skills; (3) seatwork practice to increase accuracy and speed; and (4) homework assignments which allow successful practice on mastered material (distributed practice which is essential to retention).

To maintain skills it is important to build in some review. Skills not practiced and conceptual insights not reviewed from time to time tend to disappear. Even mature adults forget material and forget it rapidly. For this reason we have asked you to provide a review of material presented the previous week each Monday and to provide a comprehensive review every fourth Monday. Such procedures will help students to consolidate and retain their learning. Finally, we have suggested that the systematic presentation of mathematics material may facilitate student learning (i.e., initial acquisition) such that you can pick up the pace a bit and we encourage you to do so if you can. Finally, when many students experience trouble, the development portion of the lesson should be repeated and students should never be asked to do homework until they are ready to do it successfully.

Field Experiment I: Method

The field testing of the instructional program commenced in the fall of 1977. With the active assistance of administrators and principals in the Tulsa Public School system, it was possible to recruit a volunteer sample of 40 classroom teachers who used the semidepartmental plan.[4] The decision was made to do the research within this organizational pattern because it afforded a classroom structure that was most comparable with the classroom organization in which the correlational research was conducted (e.g., no classrooms that were completely individualized). Choice of this structure also provided a rough control for instructional time, since teachers did not keep the same students for the entire day. Hence, for most of the teachers there was pressure to end the mathematics class at a set time, and reteaching later in the day was impossible.

Teacher Training

On September 20, we met with all teachers and all school principals who had volunteered to participate in the project. Fourth-grade teachers who taught using a semidepartmentalized structure were invited to participate in the project. (Eighty percent of the available population volunteered for the program.) Most of the semidepartmentalized schools were in low socioeconomic status (SES) areas.

At this workshop, all 40 teachers were told that the program was largely based upon an earlier observation of relatively effective and ineffective fourth-grade mathematics teachers. Teachers were told that although we expected the program to work, the earlier research was correlational and the present project was a test of those ideas. After a brief introduction, the teachers (drawn from 27 schools) and their principals were divided into two groups: treatment and control. Schools were used as the unit for random assignment[5] to experimental conditions. This was done to eliminate the difficulties that would doubtlessly follow by implementing the treatment in one class but not another in the same school.

Teachers in the treatment group were given an explanation of the program (the training lasted for 90 minutes). After the training session, treatment teachers were given the 45-page manual along with the instructions to read it and to begin to plan for implementation. In this manual definitions and rationales were presented for each part of the lesson, along with detailed descriptions of how to implement the teaching ideas.

Two weeks after the treatment began we returned to meet with treatment teachers. The purpose of this 90-minute meeting was to respond to questions that teachers had about the meaning of certain teaching behaviors and to react to any difficulties that the teachers might have encountered. Thus, the treatment consisted of two 90-minute training sessions and a 45-page manual that detailed the treatment and provided a base for teacher reference as necessary.

Control teachers were told that they would not get the details of the instructional program until February, 1978. Furthermore, they were told that it was hoped that this information might be especially useful to them then because at that time they would receive individual information about their own classroom behavior and refined information about the program itself. Finally, control teachers were told that their immediate role in the project was to continue to instruct in their own style.

Given that control teachers knew that the research was designed to improve student achievement, that the school district was interested in the research, and that they were being observed, we feel reasonably confident a strong Hawthorne control was created.[6] To the extent that a strong Hawthorne condition was created, it could be argued that differences in performance between control and treatment groups were due to the program, not to motivational variables. However, at the other extreme, it was not intended to create so strong a "press" that teachers (because of concern) would seek out information from treatment teachers or would alter their instructional style trying to guess what the experimenters wanted. If control teachers changed their instructional behavior, then differences could be due to the fact that they were using a poorly thought out or inconsistent pattern of instruction.

Process and Product Measures

The treatment program started on October 3, 1977, and was terminated on January 25, 1978. During the course of the project all 40 teachers (with few exceptions) were observed on six occasions. Observers collected information using both low- and high-inference process measures. The basic nature of the observational instruments was briefly described earlier in the discussion of the background study. For more details see Good and Grouws (1975). Reliabilities of all observational variables in the present study were good (at least .80). Students were administered the mathematics subtest of a standardized achievement test (Science Research Associates; SRA, Short Form E, blue level: K-R 21 Reliability Estimate .80) in late September and in mid-December. In mid-January students responded to a mathematics achievement test constructed by Robert E. Reys at the University of Missouri (K-R 21 Reliability Estimate .78). This test measured the content that students had been exposed to during the program (this instrument appears in-Appendix B). Furthermore, an instrument measuring student learning styles (preferences and attitude toward mathematics) was administered in September and in January (see Appendix C). The development of this instrument and the reliability of scales will be discussed later. Also in January, a 10-item attitude scale was administered (reliability .89) to assess the impact of the treatment on students' attitudes toward mathematics (see Appendix C). An instrument measuring teacher beliefs and preferences for mathematics instruction was also used in September (see Appendix D ... development of this instrument and the reliability of each scale will be discussed later).

Results of Experimental Study I

At the debriefing session in February, control teachers consistently indicated that (a) they did think more about mathematics instruction this year than previously, (b) they did not feel that they had altered their behavior in any major way, and (c) directly or indirectly they had not been exposed to program information. Hence, the Hawthorne control condition appeared to have been satisfactorily implemented.

Implementation

The second finding is that treatment teachers implemented the program reasonably well. If one is to argue that a program works or is responsible for a change, it is important to show that teachers exhibited many more of the classroom behaviors related to the treatment than did control teachers.

Fidelity of Treatment Implementation

To assess the degree of implementation of the experimental treatment, coders collected low inference data concerning the presence, absence, or duration of specific instructional events. Although data gathered were low inference in nature, and thus not very susceptible to intercoder disagreement or drift, intercoder reliabilities for each code were assessed. Results revealed 90% or better agreement on each of the variables employed to assess treatment implementation.

Given the complexity (several different behavioral requests involving sequences of behaviors) of the treatment, it is difficult to provide a single, precise statement about the extent to which the treatment was implemented. Implementation was estimated by using a summary checklist that observers filled out at the end of each observation. The information on the checklist provides good, but not total, coverage of treatment behaviors.

To illustrate the scoring procedure more concretely, one of the eight scoring definitions will be explained in detail. The following variables are relevant to that scoring procedure:

> Daily Review - the number of minutes the teacher devoted toward a review of the previous day's assignments, developed concepts of skills, etc. Coded as number of minutes.
>
> Development - the number of minutes the teacher devoted to establishing comprehension of skills and concepts in a direct manner. Coded as number of minutes.
>
> Seatwork - the number of minutes the students practiced work individually at their desks. Coded as number of minutes.
>
> Review - the number of minutes the teacher devoted toward review of any type. Coded as number of minutes.
>
> Involvement during Seatwork - the number of students clearly on task divided by the total number of students.
>
> Availability of Teacher during Seatwork - the availability of the teacher to help students with their seatwork and active demonstration on the teacher's part of a willingness to be approached for help. Coded as yes or no.

Student Accountability - actual checking of students' seatwork toward the end of the seatwork phase of the lesson. Coded as yes or no.

Homework - the assignment of homework. Coded as yes or no.

Mental Computation - the teacher asking the students to work in their heads; a problem given verbally or written on the board. Coded as yes or no.

To obtain an overall score for implementation for each teacher, the variables were first averaged across observations, then combined. Since the treatment program actually specified time parameters for daily review, development, seatwork, and total review, an adjustment was made to allow for these program specifications. Scores on these four variables represent actual minutes spent on these activities up to a specified limit. Time in excess of this limit was not credited to the score. (Limits were Daily Review = 8 minutes, Development = 20 minutes). To adjust for the scaling differences among these four scales and those that were dichotomous, each score was divided by the time limit for that variable, thus reducing the range to a minimum of zero and a maximum of one. This corresponded to the scale ranges of the remaining dichotomous variables. Finally, because the time variables were felt to be a more important indicator of implementation than the dichotomous variables, the scores were multiplied by two. An arithmetic representation of this process is presented below.

$$\text{Implementation Score} = 2(\frac{\text{Daily Review}}{8}) +$$

$$2(\frac{\text{Development}}{20}) + 2(\frac{\text{Seatwork}}{16}) + 2(\frac{\text{Total Review}}{12}) +$$

Involvement during Seatwork + Availability of Teacher during Seatwork + Student Accountability + Homework + Mental Computations

The implementation scores for the experimental treatment and control, as well as the means and standard deviations, are presented in Table 3. Results of the analysis of variance comparing control and experimental teachers presented in Table 3 indicate that the experimental teachers exhibited more of the treatment behaviors than did control teachers.

This one alternative scoring method has been presented because it is the most conservative method. However, using the checklist, eight other implementation scores were generated. Multiple definitions of implementation were explored because it is possible to score program implementation in absolute and relative ways. However, on all scoring definitions, treatment teachers were found to perform significantly more of the treatment behaviors than did control teachers. Comparisons of implementation behavior between treatment and control teachers using these other scoring definitions appear in Table 4.

Table 5 summarizes those behaviors included on the checklist which were used to estimate the degree of program implementation. For example, in 91% of the observations, treatment teachers were found to conduct a review, whereas control teachers were found to conduct a review 82% of the time. The p value associated with these percentages indicates the level of the significance of the difference between them and is also shown on Table 5.

Table 5 also reports the correlation between the frequency of occurrence of selected treatment behaviors and teacher residual gain scores on the SRA mathematics test. As can be seen, homework assignments, frequent review, and use of mental computation activity were found to correspond with favorable gains. In summary, treatment teachers exhibited significantly more of the treatment behaviors than did control teachers.

There were only 2 of the 21 treatment teachers who exhibited uniformly low implementation scores.[7] Development appears to be the only variable that teachers, as a group, had consistent trouble in implementing. The reason for the low level of implementation may be due to teachers' focusing on the many other teaching requests that were perhaps easier to implement. Alternatively, teachers might not have had the knowledge base necessary to focus on development for relatively long periods of time. Another possibility is that some of the other components required more time and preparation than we anticipated and thus development was given insufficient attention by the teachers. These issues need further study, but it is clear that the experimental series of studies in which development alone was manipulated suggest that a development component is important. More work needs to be done on the development component, and this may involve more and different types of training.

Impact on Student Performance

As can be seen in Table 6, the treatment group began the project with lower achievement scores than did the control group. The initial difference between experimental and control students was significant ($p<.001$).

These figures show that in the 2½ months of the project, the number of questions that were answered correctly by the average student in the experimental group increased from 11.94 to 19.95.

Similarly, it can be seen that in terms of national norms, the percentile rank of the experimental group increased from a percentile of 26.57 to 57.58. Such results are truly impressive given the comparatively short duration of the project. Interestingly, the control group also shows a large gain, but their gains do not match those of the experimental group.

All experimental teachers taught mathematics to the class as a whole (as requested). However, only 12 of the 19 control teachers taught mathematics to the whole class, whereas the other 7 taught mathematics to groups of students. Hence, pretest and posttest differences for control whole class and control group teachers are presented separately in Table 6. As can be seen, control group teachers started and ended the project with greater student achievement levels than did control whole-class teachers and the experimental group. However, it should be noted that the raw gain of the experimental group was much higher than that of the control teachers who taught groups of students. Furthermore, in 2½ months the experimental group virtually caught up with control teachers who used a group strategy.

Table 7 presents the results from an analysis of variance on residual gain scores comparing the performance of experimental and control groups. Irrespective of the metric used, the performance of the treatment group significantly exceeds the performance of the control group. All of the residual means show a large positive discrepancy for the treatment group. That is, the experimental group showed considerably more achievement at the posttesting than was predicted by the pretest. In contrast, the control group showed a large negative discrepancy. Table 7 also presents the results of an analysis of variance on the residual mathematics content test total scores (using SRA raw scores as the covariate). As can be seen in Table 7 the performance of the experimental group exceeds that of the control group (with and without group teachers included).

Interestingly, the favorable results reported here were also confirmed by data analyses performed by the local school district. In part, their testing interest was stimulated by the following two factors: (1) The test period was very short and (2) the SRA short form was used and does not yield separate scores for concepts and computation. To address these concerns, Mr. Stan Harrison, in the evaluation office of the Tulsa Public Schools, ran a covariance comparison using the long form of the SRA (using students' scores on the April, 1977 test as a covariate) to assess the April, 1978 performance of treatment and control students. The difference on both the SRA Concepts and Computation Tests revealed a significant difference (p<.01) in favor of students in the treatment group. These data collected by the school district in its regular testing program revealed that the experiment held up three months after the project terminated (i.e., past testing was completed and observation stopped).

Correlations between implementation scores and residual gain performance on the standardized achievement test and the mathematics content test were computed. All of the implementation definitions correlate positively with residual gain performance; however, the correlations are consistently higher between implementation scores and performance on the standardized test than on the mathematics content test.

The correlations between implementation and the content test may be due to the procedures used in constructing the content test. The plan was to assemble a test that measured the content to which most students had been exposed. The test did not measure the material that some teachers had reached. When time allows, we anticipate reanalyzing teacher logs of content coverage to see if teachers who had high gains on the standardized test were penalized by a ceiling effect on the content test.

In addition to the statistical analyses presented, it is useful to consider teachers' rank order in the distribution of residual scores. For example, within the control group, teachers who used a whole-class teaching strategy obtained both the best and worst results. Within the control group, three of the five teachers who had the highest residual means taught the entire class; however, the lowest six teachers also taught with a whole-class methodology. These results are a direct replication of our earlier naturalistic research (Good and Grouws, 1975; 1977), in which it was also found that teachers who used a group teaching strategy fell in the middle of the distribution of residual gain scores.

Examination of teachers' rank order in the residual distribution also helps to illustrate the general effectiveness of the treatment. Ten of the 12 teachers with the highest residual means were in the treatment group and none of the treatment teachers were among the five lowest teachers. However, the impact of the treatment is not even across the treatment group. Some teachers show considerably less gain than do other teachers. However, strong emphasis should be placed on the word relative because all teachers' posttest means were higher than their pretest means. Also, it should be noted that a sizeable, positive correlation (.64) was found between teachers' residual scores on the SRA test and the content test. Teachers who are high (or low) on one measure tend to be high (or low) on the other. Hence, in this particular setting the assessment coverage of the standardized achievement test appears to correspond reasonably well with the curriculum.

Student Attitude and Teacher Attitude

The experimental students also expressed more favorable attitudes toward instruction than did control students on a brief attitudinal questionnaire (see Appendix C). The mean (lower scores indicate greater satisfaction) for the control group was 18.38 and the mean for the experimental group was 17.55 ($p<.05$). The differences were statistically significant although the practical differences are nil. The data suggest, at a minimum, that the achievement gains did not come at the expense of attitudes, at least within the limits of the operational attitude measure. It would seem that emphasis upon variables like review and homework (when done in the context of meaningful and successful practice) does not necessarily lower attitudes as it is sometimes argued.

It is also important to note that feedback from the experimental teachers was supportive and indicated that teachers felt that the program was beneficial and that they planned to continue using it. We collected this information because we feel that teachers' feelings and beliefs about an instructional program are as important as teacher behaviors. Teacher behavior (whether they used the program in their classroom teaching) is an important comment upon the program, but affective reactions of teachers are equally important. That is, despite the fact that student achievement improves, teachers may choose not to continue an instructional program because it takes more time for preparation, makes the teaching act too demanding, or because it conflicts with teachers' personal definitions of what teaching should be Thus, we found the anonymous positive feedback of teachers very edifying because it gave a sense of ecological validity to the research.

The letter and response form that was sent to teachers can be found in appendix E. Twenty of the twenty-one experimental teachers responded to the confidential letter (they were provided with a self-addressed, stamped envelope and no numbers, names nor identifying information was associated with the survey instrument).

Eighteen teachers felt that all six phases of the project were either very good or good (valuable to them as teachers). The regular assignment of homework proved to be the most useful methodology, while increased pace evoked the lowest affective response from teachers. Even so, 13 of 17 teachers responding to this item thought that the increased pace stage of the project was "good."

Questionnaire responses revealed that most of the participants planned to continue using all aspects of the program, on or near the initial level recommended by the project directors. After the program had ended, 18 of 20 teachers were still conducting expanded weekly review sessions. At least 14 teachers were still implementing the prescribed development and mental computation phases 4-5 times a week, and 15 teachers continued to assign homework at least 3 nights a week.

In general, teachers thought that the mental computation, development, review, and homework phases of the program were best. Many noted a higher level of student interest in mathematics after implementation of the project. Several teachers appreciated the increased time they spent on math instruction, particularly the developmental stage.

Negative responses to the program (in response for information about weakest or most confusing parts) were infrequent, quite variable and general in nature. Three teachers had difficulty getting pupils to hand in homework; three others complained that students initially had trouble with mental computation exercises, which were unfamiliar to them. A few teachers thought the program was not flexible enough to allow for a wide range of student abilities within their classes.

In total, the affective reaction of teachers to the program was extremely positive. Their response indicated a general willingness to continue using the program and suggests that the program does not present an increased level of work that is apt to be unacceptable to the average classroom teacher.

Discussion

Given the short period of the treatment program and the relative ease of implementation, the results of this study are important. It is part of a recent trend (Anderson, Evertson, & Brophy, in press; Crawford et al) in research on teaching that is beginning to show that not only do well-designed process-outcome studies yield coherent and replicable findings, but treatment studies based on them are capable of yielding improvements in student learning that are practically as well as statistically significant. Such data are an important contradiction to the frequently expressed attitudes that teaching is too complex to be approached scientifically and/or that brief, inexpensive treatments cannot hope to bring about significant results.

Also, it is important to note that these gains were made in urban, low-income schools. That achievement increments can occur in such schools is aptly demonstrated by this project, and this experimental finding appears to be important, given the low expectations that educators hold toward inner-city schools.

It is interesting to note that the study had positive effects on both control and experimental teachers. That control teachers and their students showed marked improvement is probably due to the strong Hawthorne effect that was purposefully built into the project. Such motivation probably led control teachers to think more about their mathematical instruction, and such proactive behavior (e.g., more planning) may have brought about increased achievement. However, the presence of a strong Hawthorne control makes it possible to argue with more confidence that the resultant differences between control and treatment classes are due to the instructional program and not to motivational variables.

We are not suggesting that the instructional program used in the study is the only or best approach to take for facilitating the mathematics achievement of students. However, we are arguing that the instructional program appears to have considerable value for teachers who utilize and/or prefer a whole-class organizational pattern for teaching mathematics in the middle elementary grades. Although students at this age appear to benefit from the program, it does not follow that all their mathematics instruction should be of this mode.

As noted previously some of the individual treatment variables correlated moderatly highly and positively with student achievement; however, it must be emphasized that these variables were expressed in the context of other variables. For example, students did homework only after they had been prepared for it and had shown the ability to do the homework in teacher-supervised seatwork activity. Hence, it is difficult and perhaps misleading to overemphasize the meaning of any individual behavior. At this point the most reasponable interpretation is that the instructional treatment, when implemented, has a positive impact upon mean student achievement. The importance of particular variables can only be evaluated in subsequent studies that delete certain aspects of the instructional program. Still, it will be of some use to examine in detail all instructional behavior (especially the high- and low-inference measures that were not included on the checklist utilized in the present study to estimate implementation) in order to understand the program components that appear to be most strongly related to achievement gains and which can help determine which behaviors to delete or modify in subsequent studies.

Development is one variable that would seem to need clarification in future research. Associated with development is the need to improve the observation scale for development. This could involve trying to pinpoint behaviors that characterize development and improving the quantitative measures of this component. Another appropriate direction to pursue is the creation of reliable assessments of development along qualitative dimensions.

Continued efforts to improve and refine the entire treatment are necessary if more insight into the teaching of mathematics is to be achieved. Still, the large magnitude of the treatment effect is important and offers convincing proof that it is possible to intervene successfully in school programs. These data are consistent with other recent treatment interventions in elementary schools (Good, 1979).

Follow-Up Analysis I: What is the Source of Achievement Gains?

One form of analysis that had been outlined in the grant proposal was an attempt to determine where teachers obtain their achievement gains. We attempted to respond to this issue in two major ways. The first step was to explore data collected in the background study and the second effort was to examine new data collected in the treatment research. We will first describe our efforts to analyze data that we previously collected on the naturalistic study.

Reanalysis of the Naturalistic Study

The data base for this study was drawn from the earlier study (Good and Grouws, 1975; 1977). In review, the locale for the study was a population area that "skirted" the core city of a large metropolitan school district. In general, the school district served a middle-class population (although students from affluent and lower-class homes make up a significant part of the school population). The school district appeared to have a stable student population (e.g., the mean student IQ in the school district has been 103 for several consecutive years). One hundred and three third- and fourth-grade teachers were used in the study.

The teaching effectiveness indicator was derived from individual students' total mathematics score on the Iowa Test of Basic Skills. The test scores from the fall of the third grade were used as prescores for the third grade and tests given in the fall of the fourth grade were used as post scores. Similarly, the tests administered in the fall of the fourth grade were used as prescores for the fourth grade and post scores were obtained by fall testing in the fifth grade. These data were compiled for fall testing in 1972, 1973, and 1974.

Residual-gain scores were computed for students on each subtest by using the student's score on the prescore subtest as a covariate (using a linear model where $g = y - [a + bx]$). Residual-gain scores were computed within year and within grade level (third and fourth). Data for teachers were then compiled by computing a mean residual gain score.

Initially, 103 third- and fourth-grade teachers, within grade level, were assigned to high, middle, and low groups on the basis of their residual-gain score. Students were assigned to high, middle, and low aptitude groups on the basis of their scores on the Cognitive Abilities Test. If teachers did not have four students in each cell they were dropped from the analysis. Applying this criterion reduced the number of teachers from 103 to 81 (40 third grade teachers and 41 fourth grade teachers). Teachers that were excluded from the analysis came from all three teaching levels.

Two major analyses were then performed. In the first analysis, students were assigned to cells by dividing the entire population of third- and fourth-grade aptitude scores into three equal groups. Students in the top third were assigned to the high group and so forth. This analysis was called the absolute analysis.

The second analysis involved the assignment of students into thirds on the basis of their aptitude rank within their own mathematics class. This analysis was called the relative analysis.

A 3 (teacher competence) x 3 (student aptitude or achievement level) analysis of variance with repeated measures was computed using student residualized-gain scores (on the Iowa Test of Basic Skills) for the dependent measures for both relative and absolute analyses. Student groups were used as the analytic unit treating the aptitude groups as repeated measures for the same teachers. Teachers, consequently, were treated as a nested variable within the three levels of teacher competence. To reiterate, teachers who did not repeat across all levels of student aptitude were omitted from the analysis.

Both the relative and absolute analyses showed similar patterns of results. Significant main effects for teacher competence and student aptitude occurred (as expected). There were no significant interaction effects. However, in both of the absolute analyses the interaction approached significance, but the amount of variance explained by the interaction is very small. An examination of the means revealed that in both of the absolute analyses teachers of middle-level competence obtained less than expected amounts of residual gain from low-aptitude students. Effective teachers, as a group, did not appear to achieve their results because of the performance of one student-aptitude group. Similarly, ineffective teachers, as a group, are not especially ineffective with one aptitude group. Summaries of the variance tests are presented in Tables 8 and 9.

Discussion of Follow-up Study I

There is no general evidence in this data set to suggest that one student-aptitude group benefits most from contact with highly effective teachers or that any student-aptitude group is disproportionately penalized from being in class with less effective teachrs. Highly effective teachers as a group do a better job with students at each level; less effective teachers as a group do a relatively poorer job with all students. Although it was possible to identify a few teachers within each effectiveness group who appeared to do an especially good or poor job with a particular student group there were no consistent patterns.

Within the context of this study (e.g., a middle class school district) with a particular set of student achievement (means and variances), teacher effectiveness and student aptitude were not found to interact in any systematic way. Unfortunately, the pattern of these results did not provide clues for how to design the observational procedures for the experimental study. For example, had we found that effective teachers achieve their status because of the performance of certain students, it would have been important to determine how teachers interact with such students. Given the general nature of teacher effectiveness in this study, there were no compelling reasons to study individual students during the initial field experiment.

Follow-Up Study II: Experimental

To explore achievement patterns more fully in terms of student and teacher characteristics in the experimental study, it was deemed desirable to define teacher and student types more broadly. Much of the responsibility for this analysis was assumed by Howard Ebmeier and more details can be found in his dissertation (Ebmeier, 1978).

This study was an attempt to consider the instructional effects (associated with the experimental study) against the background of student aptitude and teacher style categories. (Details of the experimental project and population have been presented above.)

Student Measures

To develop student typologies, an instrument (Aptitude Inventory) was developed to assess characteristics which might interact with key features of the treatment program, definable teacher characteristics, and/or classroom procedures.

Two pilot tests of the instrument using low SES students were undertaken. Items that showed little variation, little stability, or caused student confusion were dropped or modified. The final instrument consisted of 37 self-report true-false questions organized into seven subscales:

1. Mental Computations - like/dislike of doing mental computations independent of pencil and paper menmonic devices.
2. Conscientiousness - forms of conscientiousness, such as completion of homework, keeping track of papers, and remembering what to do.
3. Choice - preference for choice in assignments and activities in math class.
4. Dependence - dependence on the teacher for initial structuring of the math lesson.
5. Other Orientation - like/dislike of working with other individuals to solve math problems.
6. External Motivation - dependence on external forces (such as checking of papers) for motivation in math.
7. Misbehavior - amount of trouble the child gets into in school.

To establish the stability, the instrument was administered twice (with a two-week interval) to 62 students. The procedures for these two testing sessions were identical to the administration procedures used in the main study. Written instructions for administering the inventory were given to each teacher to control for individual teacher differences. Although the instrument readability was determined by the Harris-Jacobson Readability Formula to be at grade level 2.4, teachers were instructed to read the questions to the students to overcome any possible student reading difficulties. Stability coefficients were calculated for each subscale (Cureton, 1958) and showed adequate two week test-retest reliability (the lowest stability coefficient was 0.644). Internal consistency (K-R-20) of the subscales was determined using the student data from the main study and is reported in Table 10, along with the interscale correlations.

Teacher Measures

To obtain teachers' views of the characteristics, organization, and typical activities of their classroom, a questionnaire was developed (Teaching Style Inventory). Each item was selected because of its relationship to factors that previous research had suggested to be relevant to student achievement in elementary mathematics. After numerous revisions and two pilot tests, a final version was developed which contained 73 questions divided into three sections. The first section contained 39 items assessing normal classroom procedures. Teachers indicated where they would classify their classroom on a continuous scale with regard to specific classroom practices (amount of testing, emphasis on enjoyment, etc.) The second section consisted of ten items about teachers' opinions, interests, and attitude regarding mathematics. Teachers indicated agreement or disagreement using a five-point scale for each item. The last section used a fill-in-the-blank format to obtain specific quantifiable information, such as the number of days per week math was taught in each class. This section also included several open-ended items that posed a particular instructional problem and asked teachers how they would resolve the dilemma. Seven subscales were derived from the inter-question correlation matrix.[8]

1 <u>Need for Personal Control</u> - of classroom events and rules.

2. <u>Need for Contextual Stability</u> - in the curriculum, classroom organization, and instructional pattern.

3. Degree of Individualization - of children in instruction.
4. Degree of Abstractness - using abstract concepts or using techniques or materials with which the students have little familiarity.
5. Degree of Security - feeling comfortable and secure about ability to teach math.
6. Experience - total number of years of elementary school teaching experience plus the number of years of experience teaching fourth grade mathematics.
7. Education - total number of credit hours in mathematics plus the number of graduate credit hours.

Internal consistency estimates of these subscales were determined in a manner analogous to the procedures used with the Aptitude Inventory. Reliability results for five of the subscales as well as all inter-scale correlations are presented in Table 11.

Categorical Typing of Students and Teachers

Cluster analysis was used to group students and teachers each into four types. Teachers were clustered (Barr, SAS, 1976) based on the similarity of their responses on the subscales of the Teaching Style Inventory. Because the various subscales had different numbers of items contributing to them, the scores for each teacher were standardized (mean = 0 and S.D. = 1) before they were entered into the cluster program. This prevented one subscale from exerting undue influence on the resultant clusters.

Since the number of students in this study far exceeded the maximum number of observations that can be grouped using any computer clustering program, a procedure suggested by Overall and Klott (1972) was used to cluster students. The procedure involves the clustering of random subsamples, subsequently followed by a clustering of the clusters.

A computerized random number generator (Barr, SAS, 1976) was used to produce ten random subsamples from the total sample of students (N = 1097). Variables entered into the clustering procedures included the seven aptitude factors derived from the aptitude inventory plus student sex and the pre-SRA math achievement score. The initial cluster analysis produced a total of 50 student clusters. The 50 clusters were then entered into a second order cluster analysis using within-cluster means on the various components. This ultimately resulted in four student typologies.

The statistical features of the types are shown for students (Table 12, Figure 1) and teachers (Table 13, Figure 2). Table 14 describes the resulting features of the type configuration.

Main and Interaction Effects

An analysis of variance procedure was used to test the statistical properties of the 4 x 4 x 2 factorial design. The residual scores on the SRA mathematics test served as the dependent variable. As can be seen from Table 14, all main and interactive effects among and between teacher types, student types and treatment types (control or experimental) were statistically significant. To determine the loci of the interaction effects, simple main effects were calculated and are reported in Table 15. The Newman-Keul multiple range test (adjusted for unequal N's) was used to indicate which particular student/teacher, student/treatment, or teacher/treatment pairing were causing the significant differences that were found in the simple main effect analysis. Statistically significant interactions can be summarized as follows.

1. Type one students (dependent) did significantly better with type two (experienced/unsure) and type three (educated/secure) teachers who were in the experimental treatment condition. They did significantly poorer with type three (educated/secure) teachers in the control treatment.
2. Type two students (independent) did significantly better with type three (educated/secure) teachers and significantly poorer with type four (individualized) teachers, both who were in the experimental treatment condition.
3. Type three students (low achievers) did significantly better with type two (experienced/insecure) and type three (educated/secure) teachers in the experimental treatment and poorest with type three (educated/secure) in the control.
4. Teacher type four (individualized) did worst with student type two (independent) in the experimental treatment condition. Individualized teachers did not do significantly better with any student type under either the treatment or control condition.
5. Teacher type three (educated/secure) did significantly better with student type four (high achievers) in the control but poorly with student types one (dependent) and three (low achievers), both in the control condition.
6. Type one students (dependent), who are in the experimental treatment, did best with teacher type two (experienced/unsure) and worst with teacher type one (less experienced/less educated).

7. Type two students (independent) and type three students (low achievers), who are in the experimental treatment did significantly better with teacher types three (educated/secure) and two (experienced/unsure). Independent students did poorly, on the other hand, with teacher type one (less experienced/less educated) and four (individualized).
8. Type for students (high achievers) did not do significantly better under any teacher type.

Discussion

The findings suggest that the treatment generally worked (i.e., the means in each cell tended to be in favor of the treatment group) but clearly the results of the program are more viable for certain teacher and student groups than for others.

It is also the case that the magnitude of interactions is much greater than was the case for the interactions in the naturalistic data set (see above and/or Good, Grouws and Beckerman, 1978). However, there are three major differences in these two data sets. First, the population is different (the naturalistic study is basically a middle class population and the experimental study basically involves a lower class population). The treatment dimension is also a variable that differentiates the two data sets. Finally, student aptitude is defined more broadly in the experimental study and teacher type is also measured (teacher variables were not explored in the naturalistic study).

The teacher type, instruction type, student type interactions and tentative implications have been addressed elsewhere (Ebmeier and Good, 1979). Given the small sample size in some cells, such results until replicated must be viewed as highly speculative.

Despite this important qualification, the number and magnitude of the interaction found in this study offer convincing evidence that interactions between and among student types, teacher types, and treatment types exert influence on students' mathematics achievement.

One of the more interesting findings of this study was the interactions between teacher type and treatment type. There exists a strong teacher effect in the treatment condition that is not found in the control sample. This interaction occurs for type two (experiences/unsure) and three (educated/secure) teachers but not for teacher types one and four. An examination of the mean implementation scores for the teacher types in the treatment group revealed that teacher types two and three significantly implemented more of the treatment behaviors than did teacher types 1 and 4 (Means: Type 1 = 8.48, Type 2 = 9.82, Type 3 = 9.64, Type 4 = 8.25). The data collectively suggest that teachers who implement the model get good results, yet some teacher types choose to use more facets of the model than other teacher types.

Since people will more likely adopt and internalize ideas that are consonant with their existing beliefs, one could predict that teachers who already believed in a direct instructional model or teachers who were unsure using their present instructional strategies would be most likely to implement the experimental treatment program if requested to do so. Thus, for example, type three teachers (educated/secure), who indicated they teach in a more direct manner, would be more likely to employ the experimental treatment program than type four teachers who prefer to teach using an individualized model. Similarly, type two teachers (experienced/unsure) would probably enhance the treatment because it resembles the "old" method of teaching with which they are familiar, and because they indicate they are currently insecure teaching math in the present manner. Teacher type one (less experienced/less educated), on the other hand, showed a high degree of security in teaching in the present manner, and therefore would not be likely to change without additional and more specific training in how to change.

Further studies in this area probably need to make methodological adjustments in two areas. First, the treatment program used in this research needs to be modified so that teachers who are uncomfortable or who do not understand some of the teaching requests can still accommodate them into their teaching style. Although the results presented here lend support in favor of the treatment program's general effectiveness in increasing student mathematics achievement, future studies need to include outcome measures in other diverse areas. Second, future studies of this nature need to verify by classroom observations the existence of the derived student and other teacher types. Although studies to date which placed students and teachers into typologies chiefly by pen and paper instruments have found important results, it is useful to also gather some clinical data from which explanatory theories could develop.

In an effort to explore the effects of the treatment program more fully on different types of students, Mr. Terrill Beckerman is now completing a dissertation study that forms student clusters on the basis of teacher descriptions. His attempt to use teachers' descriptions of students rather than student self-report data may qualify the findings reported above in important ways. At a minimum, the comparison of student progress in the treatment with derived and nominated student clusters will provide an interesting viewpoint. Still, all of the existing typology data that we have collected depends on reported characteristics. The long term stability of the student and teacher typologies is unknown.

Follow-Up III: A Comparison of Effective Instructional Behaviors in High and Low SES Settings

At this time we had two large data sets. One set was drawn from a relatively affluent school district which served a middle to upper middle SES population, while the other data set came from a district that served students from low income families.[9]

In the affluent school district, most of the teachers started the year with classrooms which were on or above grade level achievement as measured by standardized tests. In contrast, in the low SES sample, teachers' mean classroom achievement levels were considerably below grade level.

There are a number of similarities between the two sample populations. For example, in both cases roughtly 40 fourth grade teachers were observed teaching mathematics six times between October and December. Furthermore, pre- and post-achievement data were collected in each study making it possible to compute a mean residual score for each teacher. These scores provide an operational definition of teaching effectiveness which can be related to instructional process measures.

It is important that a large set of common process measures was coded in both studies, thus making it possible to study the impact of identical teaching behaviors in two different SES contexts. Furthermore, the two coders who collected all of the data in the high SES study were members of a four-person coder team who collected data in the lower SES study.

There are two other differences between the samples besides the general SES level of students. First, different math textbooks were used, but a content examination suggests that the differences are minor.

A more important consideration is that roughly half of the teachers in the low income schools were part of an experimental study and were asked to teach in specified ways.[10] Thus a treatment influence may have altered naturally occurring process-product relationships. However, the data are still relevant to the more generic question of <u>what processes appear to relate to achievement</u>?

Only a few of the process measures reported in this paper were part of the explicit treatment study and called to teachers' attention. (Much of the treatment dealt with lesson stages and sequences of the lesson stages rather than individual process measures.) While the differences dilute the SES comparison somewhat, the strength of the comparison still seems evident.

The major process variables used for comparisons in this research were collected with the Brophy-Good Dyadic System (Brophy & Good, 1970). In both studies, observers reported over 80% agreement on all coding categories in the system. Because of differences in class periods (some teachers taught 37 minutes; others, 45) . all of the process measures were time-adjusted to represent estimates of occurrences per hour.

Residual scores were computed for each student on the achievement test by using the student's pretest score as a covariate.[11] Data for teachers were then compiled by computing the mean residual score of their students. Teachers' mean residual scores were then correlated with each of the behavioral, process measures. These data are presented in Table 16.

Results

The most striking finding is that but few of the behavioral comparisons show important differences across the two samples. However, some minor and major differences do appear. There are very minor differences in the types of questions that appear to be most useful in the two settings. The correlations are quite low, but comparatively academically focused questions (product rather than self-reference or opinion questions) appear to be important in the low setting. Simple product questions seem to be more useful than process questions in both settings.

Another subtle difference appears in terms of teachers' reactions to students when they do not answer questions or answer incorrectly or incompletely. In the higher SES setting, it is useful for the teacher to stay with a student and work for further response from the student who gives a partially correct answer. In contrast, it seems better in the low SES classroom to maintain momentum and not to continue working with the individual student in a public setting, even when a student gives a partially correct response.

Praise seems to have an important but differential impact in the two settings.[12] In the high SES classroom, praise is negatively related to student achievement, but is positively related in the low SES setting. The climate variable shows that a relaxed, more pleasant learning atmosphere is facilitating in both settings, but a relaxed climate appears to be more important in the low SES setting.

The management of seatwork is another area where differences emerge. In the high SES setting, it appears desirable for teachers to allow students to seek them out (student-created, work-related contact); however, teacher-initiated contact seems to be more strongly associated with student achievement gains in the low SES setting.

Also, it should be noted that student involvement codes are strongly related to achievement in the low SES sample, but virtually no relationship exists in the high SES sample.[13]

Discussion

Two general findings developed from the comparison of low and high SES settings. First, no individual teacher behavior has a strong relationship with classroom achievement. All of the correlations are comparatively low, but several are significant. As Brophy and Evertson (1976) have stated previously, patterns of instructional behaviors (as opposed to individual behaviors) are more useful for describing effectiveness and providing direction for instructional programs.

The second finding is that SES differences do not appear to be as sharp as Medley (1977) suggests or as Brophy and Evertson (1976) report. However, we found four context differences in the present study and two of these seem important. Low SES classrooms demand that teachers supervise and monitor students' seatwork actively and control private interactions with students (decide which students to contact rather than allowing students to seek them out). A second finding is that positive affect and a relaxed climate is more important in low than higher SES settings.

Both of these context effects were also reported by Brophy and Evertson (1976); hence, this replication makes these data much more generalizable. Also, both of the findings are reasonably consistent with findings in developmental and social psychology. Low achieving students, compared to high achievers, have shorter attention spans and are more distractible. Hence, teacher contact (or the physical nearness of the teacher) and feedback help students to maintain task involvement. The low achievers' greater need for positive affect, and the damaging impact of negative teacher affect, can be explained in terms of low achievement students' previous history of failure in the classroom. Such students are likely to interpret neutral or ambiguous feedback as an indication of failure. Miller (1975) reports data that seem to imply that minority students have a greater need for approval. He reported that minority students are likely to derogate their own ability in failure situations and to show less tolerance for conflict (Miller, 1970).

Two weak context findings also appear in the data. The first of these, that low achievers appear to benefit from product questions, is also a finding that Brophy and Evertson report. However, the data presented here suggest that the questions should also be academically focused. That is, questions about subject matter appear to be more useful than self-reference questions (e.g., what do you like to do after school?). Frequent use of nonacademic questions may be an expression of low teacher expectations and may divert student attention from the major, substantive discussion. However, it is difficult to attach much importance to this finding since both self-reference and opinion questions occur infrequently.

The other context finding, that more effective teachers tend not to stay with low SES students in failure situations, is in direct conflict with the Brophy-Evertson data. They report that more effective teachers in low SES classrooms tended to stay with (repeating or rephrasing the question) students in failure situations. These conflicting results will only be resolved with additional research. However, some differences in the two studies may be important.

One plausible explanation for the differences between the two studies is the exclusive focus on mathematics in the present study. Given that the correctness of the answer is often more verifiable in math than in other subjects, it may be that students have greater self-evaluation capacities in mathematics. That is, their failure to respond in mathematics is because they don't know the answer (rather than because of their anxiety about speaking publicly); hence, it may make more sense to deal with students' misunderstandings privately than publicly. Also, students in the Brophy-Evertson study were younger (second and third graders) and may have had less capacity for determining whether or not they knew an answer.

The fact that ratings of student involvement correlate positively in the low SES setting but not in the high SES setting is probably due to the fact that perceived student involvement is more important in a low SES setting. Immature learners probably cannot attend to two or three things at the same time. They may not have the capacity to look out the window and watch other children on the playground and still listen to the teacher as more mature learners can. Part of the difference can probably be explained in terms of methodology. In the low SES sample, student involvement was actually counted. However, in the high SES sample it was estimated with a high inference code. We suspect that student involvement may be a better proxy for student learning in low than in high SES settings.

Teaching is complex and invariably teachers have to adjust their teaching to the particular group of students in their class. No pattern of teaching is going to apply uniformly in any setting. For example, on a probability basis it appears that low SES students need more praise than do high SES students. However, it is equally true that some high SES students will benefit from teacher praise and that some low SES students do not need high levels of teacher praise.

In general, the data presented here suggest that SES differences are real but perhaps not as great as suggested by Brophy and

Evertson (1976) and Medley (1978). However, the reader should realize that our data may represent a "minimum case" of SES differences, since student age (fourth graders) and subject matter (mathematics) were controlled.

Other age levels and/or subject areas, such as reading or social studies (which may be more affected by social influences than mathematics), might show more extreme differences. Additional research at various grade levels and in differing subject areas is needed to answer these questions. It may also be profitable to examine teacher effectiveness more intensively within a specific subject. That is, those teaching behaviors that correlate with student achievement during the introductory aspects of presenting a particular mathematics concept may <u>not</u> be the same behaviors that are useful during the consolidation phase of the unit.

Experimental Study II

Much of the research reported above was still in progress when the decision about the second experimental study had to be made (e.g., resources had to be allocated well in advance to data collection, the school district needed to be informed about the nature of the second field experiment, etc.). In retrospect, we feel our ultimate decision to shift our concern to the development of a second treatment package (verbal problem solving) and to test at an older grade level (sixth) in the same school district was an adequate but perhaps not an optimal decision.

Originally our intention had been to develop an affective treatment in the second year. However, the fact that achievement gains were not coming at the expense of students' affective reaction made it clear that there were no compelling reasons to proceed in this direction.

A second concern that was emerging from the data was the relative poor implementation of the development phase of the lesson... a key part of the experimental program from our theoretical viewpoint. However, the expense of modifying this aspect of the program (more observation, the development of video tapes, the development of new observational measures and the fact that trained observers would not be available for the second field experiment [new observers would have to be secured and trained] made the short term costs prohibitive, both in terms of cost and time).

A third possible area was the refinement of the treatment to make it more suitable for certain types of students and teachers. However, our initial work in exploring interactions (Good and Beckerman, 1978) had not provided important clues in how to do this nor had our analysis of teaching behavior in high and low SES classrooms (Good, Ebmeier, and Beckerman, 1978), although the latter study had provided some direction. Furthermore, the Ebmeier typology work was still in an early stage of data analysis. Although this early work suggested that interactions were occurring, it was too early to assess their importance or meaning. We did decide to devote resources so that Mr. Ebmeier and Mr. Beckerman could pursue their analysis of existing typology data more fully.

A fourth area of concern was that the verbal problem solving scores of the treatment students did not appear to be distinctive on the content test (see Appendix B) that Dr. Reys had designed for the first experimental study. The reliability of the instrument as a whole was fine and showed that experimental students' achievement was superior to that of control students. The reliabilities for the three subtests of the instrument (knowledge, skill, and problem solving) reflected that only the skill subtest had adequate reliability for separate analysis (and on this subtest the achievement of the treatment group surpassed that of the control group). However, in examining the means of the other two subtests, we found that treatment students appeared to do better than controls on the knowledge items but that there was little difference between the two groups on the verbal problem solving test. Obviously, it was impossible to tell whether the comparability of the two groups was real or only a function of poor reliability (e.g., too few items).

We were disappointed in this aspect of the findings because we felt that if mathematics knowledge is to be applied to "everyday" use, students need skills in this area (e.g., to compare whether the 12 oz. or 16 oz. package is the better buy). Unfortunately, the extant literature on instructional behavior and students' performance on verbal problem solving did not lead to any consistent orientation or procedure.

Given our perceived importance of attempting to understand and to possibly improve students' ability for solving relatively simple verbal problems, we decided to make a systematic development effort to develop testable instructional strategies in this area.

Treatment Program for Field Study II

Hence, we decided to shift our concern to broaden the instructional program by adding an emphasis upon verbal problem solving. The first task was to develop a training manual suggesting instructional strategies that teachers might use to influence students' verbal problem solving skills. The five techniques that teachers were requested to use were problems without numbers, writing verbal problems, estimating the answer, reading verbal problems, and writing open sentence problems. Discussion of these strategies and related research can be found elsewhere (e.g., Suydam and Weaver, 1970). Space limitations prevent an extended discussion of the rationale and procedures presented in the training manual but one brief example follows to provide some understanding of our procedural directions. The entire training manual appears in Appendix F and is available upon request (Grouws and Good, 1978).

Problems Without Numbers

The use of problems without numbers is one instructional technique for improving verbal problem solving performance (Riedesel, 1964). It provides students an opportunity to gain insight into the problem solving process by avoiding the use of numbers and thus the need to perform any computation whatever.

Example

To illustrate the method, consider the following typical problem:

> Two classes sold 100 football game tickets.
> One class sold 27 tickets.
> How many did the other class sell?
> (*Holt School Mathematics, Grade 6, p. 32.*)

This problem can easily be rephrased so that it is a problem without numbers:

> Our class and Mrs. Smith's class sold tickets.
> We know how many tickets were sold altogether
> and how many tickets our class sold.
> How many tickets did Mrs. Smith's class sell?

The teacher presents only the problem without numbers and asks the class *how* to solve it. An appropriate answer might be something like this: "I'd subtract how many tickets we sold from the total number of tickets to find how many tickets Mrs. Smith's class sold." Time permitting, the teacher should follow-up with another problem without numbers of occasionally consider the same problem only with the numbers included.

Rationale

Our reason why this technique may be effective is that it causes students to focus extensively on the method needed to solve a problem without any numerical or computational distractions. Many teachers realize that too frequently students begin doing the computation before they have really thought through the problem. In fact, some students have been known to begin computing before they have read the entire problem! Avoiding the use of numbers tends to resolve these kinds of problems. Since the strategy does not require computation, students can be exposed to a substantial number and variety of verbal problems in a short period of time.

Other Treatment II Decisions

After having made the decision to shift our concern to problem solving, it was also necessary to make three related decisions: (1) whether to test the instructional materials associated with verbal problem solving with or without the program that had been designed for the first field experiment; (2) at what grade level(s) to test the program; and (3) whether to observe or not.

It seemed more reasonable (at the time) to see if the previous gains could be maintained in knowledge and skill areas while also improving students' performance on verbal problem solving skills. Given that the program had demonstrated effectiveness within the context of that school program, it seemed more reasonable to test an expanded, comprehensive program rather than to test only a piece of the program.

The grade level decision was a relatively straightforward one. We could have tested the program at the fifth grade level and thereby gain the advantage of looking at students over consecutive years. However, the movement from school to school within the student population was relatively high. Student movement would mean that some teachers would have some fifth grade students who had been in the program as well as those who hadn't. To avoid this confusion, we decided to test the modified program at the sixth grade level. Hence, we could attempt to test the program on an "uncontaminated" population of classrooms and also we would have an older population upon which to test various questions about the program (e.g., Does it have too much structure for older students?).

The final decision we had to make concerned the role of observation in the field experiment. Limited funds, the fact that new observers had to be trained, and our interest in building a new treatment program (as well as exploring the existing typology data), collectively lead to the issue of whether to observe or not. Limited observation was a possibility.

Our decision not to observe came down to final realization that if observation remained a part of the treatment, the successful application of the program would be limited to situations where repeated classroom observation was included. Classroom observation is an expensive item and we were curious about the program... would it work without observation?

Our interest in testing the program without observation was stimulated by our awareness that observed and unobserved treatment teachers were both successful in obtaining student achievement gain in the field experiment conducted by Anderson, Evertson and Brophy, 1979. Their results suggest that the treatment had an effect upon student achievement which was not moderated by the presence of observers.

Our own observational data had suggested that most of the experimental teachers implemented the program reasonably well and that some parts of the program were better implemented than others. In general, those behaviors that were implemented most consistently involved specific requests and required no extra work on the part of the teachers. It seemed to us (at the time) that the new teaching requests being made were relatively specific although we realized that implementation might involve some extra work. Ultimately, we decided <u>not</u> to observe teachers and to see if students' achievement in several areas of mathematics could be improved without elaborate training (e.g., video tapes) or classroom observation.

Unfortunately, despite our efforts to secure an "uncontaminated" population by avoiding fifth grade classrooms, a degree of contamination was present in the design. In part, we were "victimized" by success. The school district was sufficiently impressed with the results of the first study that they wanted all fourth grade teachers to be exposed to the model. Due to this dissemination (which we helped with) as well as our own debriefing of control teachers, program descriptions of the first experimental treatment were present in most schools and hence, potentially available to sixth grade teachers. In retrospect, it might have been more profitable to have only studied the problem solving materials (which were uniquely available only to sixth grade teachers) in the second field experiment. Without observation it was impossible to determine whether sixth grade control teachers were aware and/or using parts of the treatment program.

Method for Field Experiment II

The design for this experiment was similar to that used in the first field study. The expanded program was evaluated in 36 sixth grade classrooms from elementary schools. Schools were matched on the basis of SES and then randomly assigned to treatment and control conditions. Three organizational patterns were present in the data: the same departmentalized structure (which was utilized exclusively in the first field study and which has been explained previously); math as a special subject (these sixth grade teachers taught math to several different sixth grade classes); and open classrooms (where team teaching and individualized instruction was prevalent).

The same departmental structure and math as a special subject organizational patterns seemed to be consistent with the basic data base from which the project had been developed. The open classroom structure was not. However, the school district expressed interest in including some of these classrooms into the design in order to have teachers exposed to the rationale for the directed teaching aspects of the program. We included these teachers in the design but emphasized that the treatment would be conceptual rather than operational (if the program stimulated interest in certain aspects of the program, the adaption would be left to them). Our plan was to analyze program results with open teachers included and again without these teachers. The final sample for the treatment group was math as a special su~~bject~~ (5), semi ~~some~~ departmental structure (9), open classroom (3). For the control group the final sample was math as a special subject (4), semi-departmental 10 (~~5~~), and open structure 5 (~~10~~).

Teacher Training

On October 1, 1978, we met with all teachers participating in the project. The focus of the project on the improvement of student achievement in mathematics was explained. Again, key officials from the school district were present (to create the Hawthorne conditions described in the first field experiment) and the interest of the school district was communicated. Teacher descriptive data and forms were distributed and completed. At this point the treatment teachers and control groups were divided into separate groups. The control teachers were informed that they would receive delayed feedback and that their role in the project was to teach as they had been. Treatment teachers were presented with the philosophy and details of the instructional program (1½ hours). Experimental teachers were also given the 45 page general manual and the 17 page verbal problem solving manual. Teachers were asked to implant the basic program and also to spend 10 minutes a day on verbal problem solving strategies. Two weeks after implementation of the treatment had begun, another meeting was held with treatment teachers to answer questions and to resolve any problems associated with the treatment (1½ hours). Hence, total contact with the treatment teachers was three hours.

Testing

Students were administered the mathematics subtest of a standardized achievement test (Science Research Associates [SRA]; Short Form E, blue level) at the beginning of the project (early October) and as a posttest in mid-February. Additional post measures included a 20 item research-constructed verbal problem solving test (see Appendix G: K-R 21 reliability estimate .68); a short attitude scale (see Appendix C), and the students completed basically the same typology instrument that had been used in the first field experiment (see Appendix C).

Results of Experimental Field Study II

The raw means and standard deviations for the SRA (pre and posttests) and the problem solving posttest are presented in Table 17 by treatment condition and by organizational structure. As can be seen, student performance increased from pre to post in all cases on the 40 item SRA test. Furthermore, the treatment group surpassed the performance of the equivalent control group in all cases. In terms of performance on the problem solving test, two of the three treatment groups had higher mean performance than did the equivalent control group. It should be noted that the exception, the open treatment classes, had the lowest pretest scores on the SRA.

As can also be seen in Table 17, the mean pretest SRA scores for control teachers was generally lower than was the case in the equivalent treatment group The only exception occurred in the math as a special subject classes where the pre-SRA mean scores of the treatment classes slightly exceeded that of control classrooms.

To reiterate, in terms of raw gains, it was found that the treatment group's performance was generally superior to that of the equivalent control group. What then were the effects of the formal analyses using adjusted mean scores? To pursue this question we used intact groups (i.e., teacher classroom means) because we feel that this is the most appropriate form of analysis. In comparing student performance in the post-SRA test using the pre-SRA test as the covariate (with all forms of classroom organization included in the analysis) it was found that the performance of the treatment group was not significantly higher than that of the control group ($p = .26$). When these data were reanalyzed using the student as the unit of analysis, the p-value was found to lower ($p = .13$), but again, we stress that the group mean analysis is a more appropriate form of analysis. We include this one example with student unit data to illustrate that this type of analysis yields a more favorable interpretation but we feel an erroneous one.

A similar analysis was performed on the problem solving test (using the pre-SRA as a covariate) to compare the significance of adjusted means across all treatment and control classrooms (using classrooms as the unit of analysis). This analysis reflected that the performance of the treatment group exceeded that of the control group in a way that approached significance (p = .10).

Earlier it was mentioned that we had some reservations about including open classroom teachers in the study because the program had not been designed for such settings. Hence, the analyses were repeated without open class teachers (using the class as the unit of analysis and the pre-SRA as a covariate). When open space teachers were excluded, it was found that the performance of treatment classrooms on the SRA posttest did not significantly exceed that of the control group. However, the comparison on the problem solving test revealed that the treatment group's performance was significantly superior to that of the control group (.015).

The source tables for these and the adjusted means are presented in Tables 18 through 21. Interactions between organizational type and treatment conditions were not significant and hence, are not presented in table form.

Student Affect Data

Student affect as measured by the ten item affect test was comparable before and after the treatment. The pre mean for the control group was 17.83 and 18.29 for the treatment group. At the end of the experimental program , the mean for the control group was 18.45 and 18.57 for the treatment group. These data suggest that the affective reaction was similar for both groups and that the treatment had no meaningful impact on student attitudes.

Teacher Response

We assessed the reactions of the treatment teachers to the program in a confidential fashion, two months after the program had ended (teachers were given an unmarked response sheet to return by mail). In general, their responses (the form appears in Appendix H) indicated positive acceptance of the program and an intention to continue using it. Sixteen of the seventeen treatment teachers responded to the survey.

Questionnaire Responses: Treatment Teachers

The overall affective reaction of experimental teachers (N = 16) to the program was extremely positive. Thirteen teachers felt that all eight phases of the project were either very good or good (valuable to them as teachers). The regular assignment of homework and the review proved to be the most useful methodologies, while increased pace evoked the lowest affective response. Even so, thirteen of the sixteen teachers thought that the increased pace stage of the project was either good or very good.

Questionnaire responses revealed that about 2/3 of the participants continued using all aspects of the program on or near the initial level recommended by the project directors. After the program ended, 10 teachers were still including verbal problem solving in their curricula, and 13 were implementing the prescribed development phase at least 3 times a week. Fifteen teachers continued to assign homework a minimum of 3 nights a week, and 13 were conducting weekly and monthly review sessions. In general, teachers thought that the development, verbal problem solving, review, and homework phases of the program were best. When asked about their negative responses to the program (weakest or most confusing parts), 5 teachers said they had difficulty using it with classes in which there was a wide range in student ability. Some of these teachers thought the program was particularly difficult for their low ability pupils. Six teachers thought that there was not enough time allotted on a daily basis to complete all phases of the program.

Responses of Control Teachers

At the debriefing session we provided control teachers with a copy of the program manual. Two months later we assessed their reaction to these materials (see Appendix I). We did this for two reasons. First, we wanted to see how teachers who had been exposed to the program but who had not used it would react. Were the favorable comments of experimental teachers due to the fact that they had used the program and hence, felt committed to recommend it? Also, we wanted to see how new various aspects of the program were to the control teachers. Their responses indicated that they were familiar with most parts of the material and in a couple of cases the free responses of the control teachers indicated that supervisors had advocated the use of the directed lesson to them. Seventeen of the nineteen control teachers responded to the questionnaire. Five of the 17 control teachers who responded to the questionnaire reported that they had carefully read both the general manual and the verbal problem solving manual. Five others had read both manuals quickly, and 6 had at least skimmed them quickly and thought about the highlights. Responses revealed that there was considerable correspondence between the teaching methods control teachers were already using and those requested by the program. Eight teachers were already utilizing the prescribed development and seatwork aspects of the program, and were also teaching their classes as a whole. At least five more teachers reported general overlap between the program and what they had been doing, for each category except the verbal problem solving. This is of special interest because it was in the area of verbal problem solving skills that treatment students tended to outperform control students.

In general, control teachers said that the program was not new to them, although three teachers reported that the Mental Computation was somewhat novel. Two teachers had not been using the verbal problem solving strategies previously.

Mental Computation was listed as a strength of the program by 3 of 9 teachers who responded to this item, and 4 teachers thought that the review sessions were especially useful.

Only 3 teachers listed weaknesses of the program: it was hard for low achievers; there was not enough time to complete all parts of the program daily; and it was hard to get pupils to do homework on a daily basis. Five teachers planned to continue using the verbal problem solving strategies outlined in the program, and 4 expected to continue the Mental Computation exercises. Two teachers said they would use the review, and two more planned to continue using the entire program.

Discussion of Field Experiment II

The reception of the program by the experimental teachers was excellent and this is an important result *per se*. Programs that increase student achievement but which fail to gain teacher acceptance probably have little durability. The fact that teachers indicated that they used and intended to continue using the program is gratifying, but the extent to which this verbal response matches with actual behavior is unknown. At a minimum, the program seems acceptable to teachers.

The achievement data indicate that both treatment and control groups made measurable progress. In terms of performance on the SRA posttest, the raw achievement gains of treatment classes surpassed that of control classes but these results were <u>not</u> significant. The result was not expected; in fact, based on the fourth grade study, we expected a large effect. The lack of a significant effect in this study may be due to several factors. First, the fact that control teachers reported using many of the treatment behaviors may have diluted some of the treatment effect. Second, the treatment may not have been well implemented in all treatment classrooms. Perhaps because of the addition of the problem solving strategies the teachers may have found the program overwhelming and hence, implemented only certain aspects of it. However, it should be noted that treatment teachers report good useage. Also, it could be that older students need less of the general treatment than do older students.

The results of the program on students' verbal problem solving abilities were much more encouraging. Treatment classes were found to perform better than did control classrooms on the verbal problem solving test. These results were particularly pronounced when open classes were dropped from the analysis. Hence, the program provides good data to support the contention that instructional strategies can be utilized to improve student performance on verbal problems.

The problem solving part of the treatment package seems to have potential for application in sixth grade classrooms. However, the implementation of the program needs to be verified with observation. Presently, Mr. John Engelhardt is testing the verbal problem solving strategies with another sixth grade sample. Hence, this data will be available in a few months.

The interaction of the treatment with different types of students is also being explored in a follow-up study by Dr. Howard Ebmeier. Students in this study were again clustered into four student types based on the similarity of their responses to questionnaire and achievement data, while teachers are being clustered on the basis of their classroom structure and treatment condition. When these results are completed we will be able to specify more closely the types of conditions that benefit particular students (Dr. Ebmeier is presenting these results at the 1980 meeting of the American Educational Research Association).

Dissemination

We anticipate wide dissemination of project findings and activities. Perhaps the best way to answer this question is to illustrate the dissemination activities that have been performed during the past 2½ years to present findings of research supported by National Institute of Education monies.

Professor Good has made the following project related presentations.

American Association of Colleges of Teacher Education, 1978.

American Educational Research Association, 1978.

Invitational Conference at the University of Texas, 1978.

Colloquium at Ohio State University, 1978.

Colloquium at Southern Illinois University, 1979.

Colloquium at University of Minnesota, 1979.

General presentation for teachers in the Milwaukee School District.

General presentation for supervisors and central administrative staff in Parkway School District, St. Louis, Missouri.

Professor Grouws has made the following presentations related to the project:

National Council of Teachers of Mathematics Meeting, Indianapolis, Indiana, November, 1977.

American Association of School Administrators, Minneapolis, Minnesota, Summer, 1978.

Missouri Mathematics Association for the Advancement of Teacher Training Meeting, Kansas City, Missouri, October, 1978.

Inservice training in mathematics for elementary school teachers in Milwaukee, Wisconsin, 1979.

National Council of Teachers of Mathematics. Peoria, Illinois, scheduled March, 1980.

Project Publications

Ebmeier, H. and Good, T. An investigation of the interactive effects among student types, teacher types, and type of instruction and the mathematics achievement of fourth-grade students. American Educational Research Journal, 1979, 16, 1-16.

Good, T. Teaching mathematics in elementary schools. Educational Horizons, Summer, 1979, 178-182.

Good, T. Teacher effectiveness in the elementary school: What we know about it now. Journal of Teacher Education, 1979, 30, 52-64.

Good, T. and Beckerman, T. Time on task: A naturalistic study in sixth-grade classrooms. Elementary School Journal, 1978, 78, 193-201.

Good, T. and Beckerman, T. An examination of teachers' effects on high, middle, and low aptitude students' performance on a standardized achievement test. American Educational Research Journal, 1978, 15, 477-482.

Good, T., Ebmeier, H., and Beckerman, T. Teaching mathematics in high and low SES classrooms: An empirical comparison. Journal of Teacher Education, 1978, 29, 85-90.

Good, T. and Grouws, D. Teaching effectiveness in fourth-grade mathematics classrooms. Chapter in G. Borich (Ed.), The Appraisal of Teaching: Concepts and Process. Reading, Mass.: Addison-Wesley, 1977.

Good, T. and Grouws, D. Teaching effects: A process-product study in fourth-grade mathematics classrooms. Journal of Teacher Education, 1977, 28, 49-54.

Good, T., Grouws, D., and Beckerman, T. Curriculum pacing: Some empirical data in mathematics. Journal of Curriculum Studies, 1978, 10, 75-81.

Good, T. and Grouws, D. The Missouri effectiveness project: An experimental study in fourth-grade classrooms. Journal of Educational Psychology, 1979, 71, 355-362.

Good, T. and Grouws, D. g and mathematics learning. Educational Leadership, October, 1979, 39-45.

School District Useage

Numerous school districts and individual teachers have written for information about the general training manual (availability of the verbal problem solving has not been dissiminated). To date we have honored all of these requests. Although we have no idea about the useage of the materials by individual teachers, we do know that at least 10 school districts are making some use of the program. Given that the grant has just ended, it would seem that the results have been as widely dissiminated as could reasonably be expected.

Discussion

Following each of the main studies and each follow-up study, the obtained findings were discussed. It would seem pointless to repeat those discussions. Hence, the discussion here is limited to a few brief points and qualifications that transcend specific findings.

The collective results of our work suggest that it is possible to improve student performance in important ways in urban schools. The attainment of a theoretical model that can accurately relate instructional processes to subject matter achievement is a goal that we continue to pursue. Despite the fact that a comprehensive all-inclusive theoretical structure is not yet available, the program has yielded an important set of interrelated teaching concepts and interpretable empirical findings. We feel that the set of concepts that we have characterized collectively as active teaching offers heuristic, orienting direction to teachers.

At this point there are many alternative explanations to explain why the program has had at least moderate success in the locations where it has been tested. The two explanations that presently make the most sense to us follow. We suspect that many educators are generally pessimistic about the ability of schooling to enhance academic performance. The active criticism that schools have received in the past decade may have reduced expectations for student performance. Thus, one of the reasons the instruction model works is because it strongly underlines the importance of the individual teacher.

If teachers can make a difference, active instruction arguments may provide a positive motivational source that encourages teachers to plan their days more fully, take their responsibilities more seriously, and thus fulfill their expectations (e.g., present more careful demonstrations, provide more consistent feedback, and so forth). Such proactive stimulation may lead teachers to spend more time in planning for and conducting activities related to achievement and thereby enhance student achievement, but not necessarily at the expense of other goals. Reciprocally, the clearer focus of the teacher may help students to allocate their learning time more profitably and to practice skills in a more meaningful context.

It should be noted that the proactive stimulation argument is an nypothesis, not empirical fact. Fortunately, some researchers (e.g., Peterson, Marx, and Clark, 1978) have begun to examine the relationship, if any, between teacher beliefs, classroom planning, and effects on classroom behavior and achievement.

The second explanation is that the model provides a plausible, practical system of instruction. At present, there are no data that comprehensively test the explanatory model; however, it is consistent with available data. For example, in the first field study, it was possible to produce large gains for both the control and the experimental group. Given the fact that teacher success and student progress appear to be improved by raising teacher motivation to fulfill demand characteristics (increase student achievement), the first two aspects of the explanation appear to be fulfilled. However, the resulting positive difference between control and treatment gains represents achievement growth that can be attributed to the instructional model itself. The model works, in part, because of motivational arousal, but also because the set of instructional activities for students and teachers is relevant, if not important, to conditions for learning.

Despite some of our positive findings, we do not want to imply that the methods used in our instructional program are the most desirable way to teach mathematics. It is clear that the program works for some teachers and students better than it does for others. One of the difficulties of research using mean classroom achievement as a criterion is that prescriptive statements are restricted to effect on the class as a whole. What makes sense for a given student or subgroup of students <u>may</u> be detrimental to the class as a whole and vice versa (Good and Power, 1976).

Effectiveness research has failed to deal with the subgroup-whole class issue and this seems the next important step in the research paradigm. Teaching is complex and invariably teachers have to adjust their teaching to the particular group of students in their class. Hopefully, our ongoing analysis of teacher and student type interaction with the program will yield important insights into program modifications. However, even after such modifications the existing program would still represent but one approach to mathematics instruction. The results here have little direct implication for teachers who use individualized or group instruction models. Based upon the school districts we have worked in, whole class instruction appears to be the dominant delivery system. Still there are sufficient numbers of teachers who use group based instruction (teach two or more instructional groups in the class) to make it an important research focus.

As a case in point, some of the teachers in the control condition who taught mathematics to groups of students achieved very good results. Indeed, one very important research question that needs to be answered at the elementary school level is how teachers who obtain good results using small group instruction behave in the classroom. We know from data we have collected that some teachers who teach small groups achieve better results than other teachers, but very little specific information is available on how these teachers behave. Information about effective small group instruction would seem to be a very important next step in trying to understand mathematics learning in the elementary school.

Additional refined research on the development portion of the lesson is needed at the elementary school level. We feel that the active presentation of information and careful conceptual development are important aspects that are often missing in mathematics lessons. Better ways to describe this portion of the lesson and improved conceptual descriptions of teaching strategies that can be used to enhance development are needed. Also, as we have noted previously, teachers use of the verbal problem solving strategies needs to be verified with observational data. (This work is in progress.)

Our research has focused upon fourth and sixth grade classrooms. We suspect that our conceptualization of active mathematics teaching would be relevant for at least certain types of students. More research is needed at the secondary level in order to establish support for this contention and to achieve more understanding of how teachers can structure (without over-structuring) mathematics learning. However, the little research on secondary school mathematics teaching that is available appears to support the advantages of active teaching, atleast in terms of short-term student performance (Evertson, Anderson, and Brophy, 1978; Weber, 1978).

Clearly, to answer questions about the effects of active mathematics teaching in secondary classes it will be necessary to conduct field experiments and to observe how well an active teaching model can be incorporated into classroom teaching. It will also be necessary to measure the effects of such teaching on student achievement and attitudes. If we are to understand the effects of any instructional program more fully, it may be important to involve teachers directly in the research process. Teachers have demonstrated the capacity to conduct research successfully (see, for example, Behnke and others, 1979), and the need for integrating teacher beliefs into studies of classroom effectiveness has been argued elsewhere (see, for example, Fenstermacher, 1978; and Shulman and Elstein, 1975). Hence, an important next step that we anticipate in our research program is to involve teachers directly in the design and modification of the teaching model that we plan to test experimentally in secondary classrooms.

Table 1

Significant or Near Significant Process Variables from an Analysis of Variance across the Top and Bottom Nine Teachers

Variables	p Value	$\bar{X}$ High	$\bar{X}$ Low
Number of Students	.0001	26.70	21.34
Time Teacher Taught "Whole" Class	.1001	40.47	35.83
*Time Going Over Homework	.0650	4.98	8.19
*Classroom Climate [1]	.0771	2.00	2.26
*Clarity	.0135	4.06	3.53
*Average Accountability	.0424	3.46	3.15
*Average Alerting	.0350	3.90	3.59
Discipline Question [2]	.0656	0.11	0.35
Direct Question	.0113	14.07	28.26
Process Question	.0131	2.72	7.53
Correct Response	.0533	38.70	50.98
Wrong Response	.0017	5.39	11.39
No Response	.0058	1.37	3.26
Student Response Followed by Teacher Praise	.0046	2.74	14.09
Negates Wrong	.0088	1.51	3.29
Repeats Question	.0295	1.39	2.78
Student Initiated Work Related Contact; Teacher Gives Process Feedback	.0654	4.41	1.56
Student Initiated Work Related Contact: Teacher Gives Feedback	.0004	17.65	9.30
Teacher Initiated Work Related Contact; Type Feedback Unknown	.1072	0.02	0.24
Teacher Initiated Behavior Related Contact; Teacher Gives Warning	.0081	1.75	3.37

Variables	p Value	$\bar{X}$ High	$\bar{X}$ Low
Teacher Initiated Behavior Related Contact; Teacher Gives Criticism	.0548	0.30	0.67
Total Teacher Initiated Work Related Contacts	.0383	3.01	5.96
Total Teacher Initiated Behavior Related Contacts	.0853	4.22	5.85
Total Teacher Initiated Contacts	.0129	7.23	11.83
Total Student Initiated Work Related Contacts	.0004	23.44	11.80
Total Student Initiated Contacts (Work and Procedural)	.0003	25.35	13.41
$\frac{\text{Direct Questions}}{\text{Total Response Opportunities}}$	.1089	28.13	36.54
$\frac{\text{Total Teacher Initiated Contacts}}{\text{Total Student Initiated Contacts}}$	.0058	54.10	116.41
$\frac{\text{Process Questions}}{\text{Total Questions}}$	.0518	7.44	14.56
$\frac{\text{Correct Responses}}{\text{Total Responses}}$	.0051	82.80	76.17
Total Process Feedback	.1005	6.51	3.04

* Indicates a high inference rating.

[1] This scale was reversed so the lower score on the scale implies a more relaxed learning environment.

[2] The unit used in reporting the behavioral data is frequency per hour.

Table 2

Summary of Key Instructional Behaviors*

Daily Review (First 8 minutes except Mondays)

a) review the concepts and skills associated with the homework

b) collect and deal with homework assignments

c) ask several mental computation exercises

Development (About 20 minutes)

a) briefly focus on prerequisite skills and concepts

b) focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.

c) assess student comprehension

   1) using process/product questions (active interaction)

   2) using controlled practice

d) repeat and elaborate on the meaning portion as necessary

Seatwork (About 15 minutes)

a) provide uninterrupted successful practice

b) momentum - keep the ball rolling - get everyone involved, then sustain involvement

c) alerting - let students know their work will be checked at end of period

d) accountability - check the students' work

Homework Assignment

a) assign on a regular basis at the end of each math class except Fridays

b) should involve about 15 minutes of work to be done at home

c) should include one or two review problems

Special Reviews

a) Weekly Review/Maintenance

   1) conduct during the first 20 minutes each Monday

   2) focus on skills and concepts covered during the previous week

b) Monthly Review/Maintenance

   1) conduct every fourth Monday

   2) focus on skills and concepts covered since the last monthly review.

*Teachers were also requested to slightly pick up their pace through the textbook material.

Table 3

Analysis of Variance between Experimental Treatment and Control Treatment Teachers' Implementation Scores*

Source	df	MS	F	Probability
Treatment Condition	1	13.28	4.53	0.0400
Error	37	2.93		

*Note the mean for the control group was 7.89 (S.D. 2.02) and for the treatment group 9.06 (S.D. 1.35).

Table 4

Implementation Differences in Treatment and Concrol Teachers Using Eight Different Scoring Definitions

Implementation[2] Definitions	Treatment $\bar{x}$	Control $\bar{x}$	p-Value
I	7.2	4.3	.0001
II	7.7	4.4	.0001
III	3.9	2.4	.0001
IV	3.1	1.5	.0001
V	2.4	1.4	.0001
VI	3.2	2.3	.0034
VII	8.1	5.6	.0001
VIII	8.6	5.8	.0001
SRA Residual	1.53	-1.46	.002

---

[2] The definitions used represent different linear combinations of the treatment components and subcomponents. It should be noted that correlations between each implementation score and residual achievement were computed. This data consistently show that residual achievement and implementation are positively and significantly correlated.

Table 5

Mean Percent of Occurrence of Selected Implementation Variables for Treatment and Control Group Teachers and the Correlation of These Variables with Teachers' Residualized Gain Scores on the SRA Mathematics Test

	Treatment $\bar{x}$	Control $\bar{x}$	p-Value	Correlation	p-Value
1. Did the teacher conduct review?	91%	62%	.0097	.37	.04
2. Did development take place within review?	51%	37%	.16	.10	.57
3. Did the teacher check homework?	79%	20%	.0001	.54	.001
4. Did the teacher work on mental computation?	69%	6%	.001	.48	.005
5. Did the teacher summarize previous day's materials?	28%	25%	.69	.20	.26
6. There was a slow transition from review.	7%	4%	.52	-.02	.91
7. Did the teacher spend at least 5 minutes on development?	45%	51%	.52	-.08	.65
8. Were the students held accountable for controlled practice during the development phase?	33%	20%	.20	.12	.50
9. Did the teacher use demonstrations during presentation?	45%	46%	.87	-.15	.41
10. Did the teacher conduct seatwork?	80%	56%	.004	.27	.13
11. Did the teacher actively engage students in seatwork (first 1½ minutes)?	71%	43%	.0031	.32	.07
12. Was the teacher available to provide immediate help to students during seatwork (next 5 minutes)?	68%	47%	.02	.28	.11
13. Were students' held accountable for seatwork at the end of seatwork phase?	59%	31%	.01	.35	.05
14. Did seatwork directions take longer than one minute?	18%	23%	.43	-.02	.92
15. Did the teacher make homework assignments?	66%	13%	.001	.49	.004

Table 6

Pre Project and Post Project Means and Standard Deviations for Experimental and Control Classes on the SRA Mathematics Achievement Test

I. All Treatment and All Control Teachers

	Pre Project Data			Post Project Data			Pre-Post Gain		
	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentil
Experimental									
Means	11.94	3.34	26.57	19.95	4.55	57.58	8.01	1.21	31.01
Standard Deviations	3.18	.51	13.30	4.66	.67	18.07			
Control									
Means	12.84	3.48	29.80	17.74	4.22	48.81	4.90	.74	19.01
Standard Deviations	3.12	.48	12.43	4.76	.68	17.45			

II. Control Whole Class Teachers and Control Group Teachers

	Pre Project Data			Post Project Data			Pre-Post Gain		
	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentile	Raw Score	Grade Equivalent	Percentil
Whole Class Control									
Means	11.70	3.30	25.30	16.20	3.98	43.00	4.50	.68	17.70
Standard Deviations	2.58	.40	10.15	4.96	.68	18.09			
Group Control									
Means	14.78	3.77	37.50	20.38	4.64	58.77	5.60	.87	21.27
Standard Deviations	3.14	.48	12.68	3.12	.47	11.56			

*SRA = Science Research Associates

Table 7

Analysis of Variance on Residual Gain Scores (Using Mean Teacher Scores) For Treatment and Control Teachers on SRA Test and Content Test

I. SRA Mathematics Achievement Test

A. Treatment vs. Control (Group Teachers Included)

	Treatment	Control	p-Value
Grade level scores	2.22	-2.08	.002
Percentile scores	5.67	-5.51	.003
Raw scores	1.53	-1.46	.002

B. Treatment vs. Control (Group Teachers Not Included)

Grade level scores	1.98	-3.31	.002
Percentile scores	5.11	-8.46	.003
Raw scores	1.30	-2.22	.002

II. Content Mathematics Test*

A. Treatment vs. Control (Group Teachers Included)

	Treatment	Control	p-Value
Content test	1.14	- .48	.10

B. Treatment vs. Control (Group Teachers Not Included)

Content test	1.13	- .83	.11

*Analyses using the student rather than the teacher as the unit of analysis yielded the following results: Treatment vs. Control (group teachers included), p<.008; Treatment vs. Control (group teachers not included) p .002.

Table 8

Anova: Differences in Residual Achievement for Three Levels of Teacher Competences and Three Levels of Student Aptitude--Grade 3 (Relative Analysis)

Source of Variation	SS	F Value	df	Significance
Between 3 levels of teacher competence	272	45.80	2	p<.001
Error	112		38	
Between 3 levels of student aptitude	169	18.10	2	p<.001
Between teacher competence (teacher nested) by student aptitude	28	1.48	4	.21 N.S.
Error	356		76	

Anova: Differences in Residual Achievement for Three Levels of Teacher Competences and Three Levels of Student Aptitude--Grade 4 (Relative Analysis)

Source of Variation	SS	F Value	df	Significance
Between 3 levels of teacher competence	383	42.16	2	p<.001
Error	199		37	
Between 3 levels of student aptitude	197	21.71	2	p<.001
Between teacher competence (teacher nested) by student aptitude	7.46	.41	4	.80 N.S.
Error	336		74	

Table 9

Anova: Differences in Residual Achievement for Three Levels of Teacher Competences and Three Levels of Student Aptitude--Grade 3 (Absolute Analysis)

Source of Variation	SS	F Value	df	Significance
Between 3 levels of teacher competence	291	39.02	2	p<.001
Error	142		38	
Between 3 levels of student aptitude	135	14.05	2	p<.001
Between teacher competence (teacher nested) by student aptitude	36	1.88	4	.12 N.S.
Error	365		76	

Anova: Differences in Residual Achievement for Three Levels of Teacher Competences and Three Levels of Student Aptitude--Grade 4 (Absolute Analysis)

Source of Variation	SS	F Value	df	Significance
Between 3 levels of teacher competence	373	41.74	2	p<.001
Error	165		37	
Between 3 levels of student aptitude	196	15.01	2	p<.001
Between teacher competence (teacher nested) by student aptitude	56.68	2.24	4	.07 N.S.
Error	483		74	

Table 10

Aptitude Inventory Inter-Scale Correlations (Internal Reliabilities on Diagonal)

	Mental Computations	Conscientiousness	Choice	Dependence	Other Orientation	External Motivation	Misbehavior	Pre-SRA Achievement Score
Mental Computations	0.768	0.274	-0.066	-0.149	0.153	-0.020	-0.009	0.052
Conscientiousness		0.648	-0.107	0.033	0.193	-0.140	-0.285	0.249
Choice			0.651	-0.093	-0.149	0.149	0.061	-0.086
Dependence				0.478	-0.082	-0.020	-0.075	0.030
Other Orientation					0.520	-0.226	-0.006	0.196
External Motivation						0.565	0.080	-0.300
Misbehavior							0.636	-0.094
Pre-SRA Achievement Score								0.900

Table 11

Teaching Style Inventory Inter-Scale Correlations (Internal Reliabilities on Diagonal)

Subscale	Need for Personal Control	Need for Contextual Stability	Degree of Individualization	Degree of Abstractness	Degree of Security	Experience	Education
Need for Personal Control	0.540	0.027	-0.274	-0.065	0.067	0.289	-0.109
Need for Contextual Stability		0.727	-0.518	-0.078	-0.026	-0.075	-0.009
Degree of Individualization			0.772	0.015	-0.124	0.153	-0.023
Degree of Abstractness				0.636	-0.527	-0.059	-0.175
Degree of Security					0.606	-0.180	-0.036
Experience						--	-0.026
Education							--

Table 12

Student Types Based on the Cluster Analysis: Means, Standard Deviations and F Ratios for Cluster Components

		Components								
Student Typology		Mental Computations	Conscien-tiousness	Choice	Dependence	Other Orientation	External Motivation	Behavior	Sex 1=Male 2=Female	SRA Pre-Achievement Score
One (N=388)	Mean	2.46	7.32	1.11	4.44	1.87	4.55	0.50	1.55	10.97
	S.D.	1.47	1.87	1.22	0.69	1.18	1.23	0.83	0.49	1.80
Two (N=214)	Mean	2.69	7.40	1.89	4.09	1.95	3.66	0.70	1.55	17.19
	S.D.	1.48	2.07	1.51	1.18	1.39	1.65	0.97	0.49	3.22
Three (N=344)	Mean	2.72	6.13	1.90	3.91	1.50	4.90	0.65	1.49	6.70
	S.D.	1.35	2.29	1.47	1.10	1.10	1.15	0.91	0.50	2.24
Four (N=151)	Mean	3.29	8.82	0.78	4.18	2.24	3.66	0.27	1.35	20.86
	S.D.	1.19	1.08	0.98	0.96	1.32	1.52	0.70	0.48	5.62
Total (N=1097)	Mean	2.70	7.17	1.46	4.17	1.82	4.36	0.55	1.50	12.21
	S.D.	1.40	1.97	1.33	0.97	1.22	1.34	0.87	0.49	3.01
F Ratios (3,1093 df)		12.60*	68.60*	41.08*	18.71*	14.55*	54.37*	8.95*	6.41*	1009.42*

*p<0.001

Table 13

Teacher Typologies Based on the Cluster Analysis: Means, Standard Deviations and F Ratios for Cluster Components

Teacher Typology		Components						
		Need for Personal Control	Need for Contextual Stability	Degree of Individualization	Degree of Abstractness	Degree Security	Experience	Education
One (N=13)	Mean S.D.	20.00 2.51	27.53 5.69	18.53 6.50	13.61 2.63	17.84 1.34	15.30 7.99	13.15 5.88
Two (N=8)	Mean S.D.	20.00 2.07	26.87 5.56	16.75 3.05	17.37 1.50	14.50 1.60	27.25 10.63	35.50 17.09
Three (N=8)	Mean S.D.	22.00 2.61	24.75 5.25	18.62 5.47	12.25 2.65	19.00 1.41	23.62 13.19	42.87 23.64
Four (N=10)	Mean S.D.	17.60 2.83	21.30 5.18	22.20 6.69	19.20 5.73	15.10 2.96	17.60 8.66	17.30 14.17
Total (N=39)	Mean S.D.	19.79 2.87	25.23 5.82	19.12 5.92	15.53 4.41	16.69 2.59	20.05 10.66	24.89 19.22
F Ratio (3,36)		4.53***	2.77*	1.44	7.77***	10.89***	2.94**	8.35***

*p<0.10 **p<0.05 ***p<0.01

Table 14

Newman-Keul Multiple Range Test on Differences between Student Types Under the Experimental Treatment and Control Treatment When Taught by Type Three Teachers (Educated/Secure)

		Student Type-Treatment Type								
		4. High Achievers in Control	2. Independent in Exp.	3. Low Achievers in Exp.	4. High Achievers in Exp.	1. Dependent in Exp.	2. Independent in Control	1. Dependent in Control	3. Low Achievers in Control	Critical Values
Student Type-Treatment Type	Means	4.37	3.95	3.46	2.86	2.39	-1.65	-4.27	-4.30	
4. High Achievers in Control	4.37		0.42	0.91	1.51	1.98	6.02*	8.64*	8.67*	R_2=4.12
2. Independent in Experimental	3.95			0.49	1.09	1.56	5.60	8.22*	8.25*	R_3=4.93
3. Low Achievers in Experimental	3.46				0.60	1.07	5.11	7.73*	7.76*	R_4=5.42
4. High Achievers in Experimental	2.86					0.47	4.51	7.13*	7.16*	R_5=5.75
1. Dependent In Experimental	2.39						4.04	6.66*	6.69*	R_6=6.00
2. Independent in Control	-1.65							2.62	2.65	R_7=6.21
1. Dependent in Control	-4.27								0.03	R_8=6.39
3. Low Achievers in Control	-4.30									

<0.05

Table 15

Newman-Keul Multiple Range Test on Differences between Student Types Under the Experimental Treatment and Control Treatment When Taught by Type Four Teachers (Individualized)

		Student Type-Treatment Type								
		4. High Achievers in Exp.	1. Dependent in Exp.	3. Low Achievers in Exp.	4. High Achievers in Control	3. Low Achievers in Control	2. Independent in Control	1. Dependent in Control	2. Independent in Exp.	Critical Values
Student Type-Treatment Type	Means	1.44	0.17	-1.00	-1.44	-1.85	-2.48	-2.77	-.625	
4. High Achievers in Experimental	1.44		1.27	2.44	2.88	3.29	3.92	4.21	7.69*	R_2=3.49
1. Dependent in Experimental	0.17			1.17	1.61	2.02	2.65	2.94	6.42*	R_3=4.17
3. Low Achievers in Experimental	-1.00				0.44	0.85	1.48	1.77	5.25*	R_4=4.57
4. High Achievers in Control	-1.44					0.41	1.04	1.33	4.81	R_5=4.86
3. Low Achievers in Control	-1.85						0.63	0.92	4.40	R_6=5.07
2. Independent in Control	-2.48							0.29	3.77	R_7=5.25
1. Dependent in Control	-2.77								3.48	R_8=5.40
2. Independent in Experimental	-6.25									

.0.05

Table 16

CORPELATION OF BEHAVIORAL MEASURES WITH TEACHERS' RESIDUAL IN BOTH THE LOW AND HIGH SES CLASSROOMS[1,2,3,4]

VARIABLE	CORRELATION IN LOW SES	p VALUE	CORRELATION IN HIGH SES	p VALUE
Classroom Climate	0 42	0 01	0 28	0 11
Managerial'	0 10	0 57	0 00	0 97
Total Class Time	0 10	0 56	0 18	0 32
Transition Time	− 0 27	0 12	0 11	0 55
Time going Over Homework	0 26	0 14	0 02	0 91
Review Time	0 09'	0 60	0 29	0 10
Development	− 0 14'	0 41	− 0 13	0 50
% of Student Probably Involved	0 45	0 01	− 0 07'	0 68
Student Asks Question	0 13	0 46	0 09	0 60
Discipline Type Question	− 0 04'	0 79	− 0 30'	0 08
Direct Question	0 16	0 34	− 0 08	0 65
Open Question	0 15	0 39	0 17	0 64
Student Calls Out Answer	0 01	0 95	0 32	0 06
Process Question	− 0 16'	0 34	− 0 15	0 60
Product Question	0 15'	0 39	− 0 02	0 89
Choice Question	0 14	0 42	0 15	0 59
Self Reference Question	− 0 07'	0 66	0 20'	0 25
Opinion Question	− 0 25'	0 14	0 05'	0 76
Correct Response	0 11	0 51	− 0 03	0 85
Partially Right Response	− 0 09	0 60	− 0 13	0 51
Wrong Response	0 16	0 36	− 0 20	0 26
"Don't Know" Response	0 02'	0 88	− 0 22'	0 20
No Response	− 0 12	0 47	− 0 19	0 28
Praise	0 35	0 04	− 0 18	0 67
Affirm	0 04	0 80	0 15	0 57
Summarize	− 0.39'	0 02	− 0 13'	0 50
No Feedback	0 26'	0.13	0 03	0 84
Negate Wrong	0 15	0 39	0 04	0 80
Criticism	− 0 09'	0 58	0 02'	0 91
Process Feedback	− 0 11	0 53	0 27	0 12
Gives Answer	− 0 09	0.59	− 0 02	0 90
Ask Another Student	0.30	0 08	− 0 07	0 70
Another Student Calls Out Answer	− 0 20'	0 26	− 0 15'	0 59
Repeats Question	− 0.25	0.15	− 0 07	0 68
Gives Clue	− 0 06	0.73	0 03	0 87
Asks New Question	− 0 28	0 10	0 12	0 51
Expands Student's Response	− 0 32'	0 06	0 02'	0 92
Student Initiated Work-Related Contact—Teacher Praise	0.12'	0 47	0 28	0 11
Student Initiated Work-Related Contact—Teacher Gives Process-Type Feedback	− 0 25	0 15	0 14	0 56
Student Initiated Work-Related Contact—Teacher Gives Feedback	0 00	0 99	0 37	0 03
Student Initiated Work-Related Contact—Teacher Criticizes	− 0 37'	0 02	− 0 11'	0 55
Student Initiated Work-Related Contact—Teacher Type Feedback Unknown	0 04'	0 78	− 0 21'	0 24
Right Response Followed by Teacher Praise	0 35	0 04	0.09	0.62
No Response or "Don't Know" Response Followed by Sustaining Feedback	− 0 22'	0.20	− 0 21	0.24
No Response or "Don't Know" Response Followed by Terminal Feedback	0 01	0.93	− 0.22	0.23
Wrong Response Followed by Terminal Feedback	0 22	0 21	− 0 19	0.29
Wrong Response Followed by Sustaining Feedback	− 0 16	0 36	− 0 10	0.56
Part Right Response Followed by Terminal Feedback	− 0 02'	0 88	0.11	0.58
Part Right Response Followed by Sustaining Feedback	− 0 18'	0 29	0.30	0.12
Total Response Opportunities	0 16	0 36	0.14	0 55
Total Teacher Initiated Work-Related Contacts	0 28	0 10	− 0 33	0.06
Total Teacher Initiated Behavior-Related Contacts	0 00	0 97	− 0.21	0.23
Total Teacher Initiated Contacts	0 22	0.20	− 0 33	0.06
Total Student Initiated Work-Related Contacts	− 0 06	0 70	0.37	0 03
Total Student Initiated Procedure Related Contacts	0 10'	0.55	− 0.03	0.87
Total Student Initiated Contacts	− 0 05	0.75	0 35	0 04
Total Dyadic Contacts (Student Initiated, Teacher Initiated, and Response Opportunities)	0 04	0.79	0.17	0.66
$\frac{\text{Direct Question}}{\text{Direct. Plus Open Question}}$	0 00	0 98	0 05	0.76
$\frac{\text{Direct Question}}{\text{Response Opportunities}}$	0 12	0.48	0.02	0.91
$\frac{\text{Open Questions}}{\text{Response Opportunities}}$	0.09	0.61	− 0 09	0.62
$\frac{\text{Call Outs}}{\text{Response Opportunities}}$	− 0 07	0.69	0.00	0.98
$\frac{\text{Student Initiated Work-Related Contacts}}{\text{Total Student Initiated Contacts}}$	− 0 01	0.93	0.01	0.96

---

[1] High inference variable

[2] Correlations based on variables with a low frequency of occurrence

[3] Correlations that might be contaminated by the treatment

Table 16 continued

VARIABLE	CORRELATION IN LOW SES	p VALUE	CORRELATION IN HIGH SES	p VALUE
Teacher Initiated Work-Related Contacts / Total Teacher Initiated Contacts	0.32	0.06	−0.26	0.13
Total Teacher Initiated Contacts / Total Student Initiated Contacts	0.11	0.53	−0.34	0.05
Process Questions / Total Questions	−0.31'	0.07	−0.19	0.28
Choice Questions / Total Questions	−0.06	0.70	−0.25	0.15
Opinion Questions / Total Questions	−0.16	0.35	−0.03	0.84
Product Questions / Total Question	0.25'	0.14	−0.10	0.60
Correct Responses / Total Responses	0.00	0.96	0.2[illegible]	0.15
Wrong Responses / Wrong Responses Plus No Response	0.21	0.22	0.19	0.[illegible]7
"Don't Know" / "Don't Know" Plus No Response	0.04	0.81	−0.16	0.61
% of Responses Teacher Gave No Feedback	0.08	0.64	−0.07	0.71
Student Initiated Procedure-Related Contact—Teacher Praise	0.16'	0.35	0.18'	0.30
Student Initiated Procedure-Related Contact—Teacher Gives Feedback	0.10'	0.56	−0.05	0.76
Student Initiated Procedure-Related Contact—Teacher Criticizes	−0.02'	0.87	−0.10'	0.57
Teacher Initiated Work-Related Contact—Teacher Gives Praise	0.18'	0.29	−0.14'	0.56
Teacher Initiated Work-Related Contact—Teacher Gives Process Feedback	0.24	0.16	−0.29'	0.10
Teacher Initiated Work-Related Contact—Teacher Gives Feedback	0.22	0.20	−0.25	0.15
Teacher Initiated Work-Related Contact—Teacher Criticizes	−0.06'	0.70	−0.19'	0.30
Teacher Initiated Work-Related Contact—Teacher Type Feedback Unknown	0.16'	0.36	−0.28'	0.11
Teacher Initiated Behavior-Related Contact—Teacher Gives Procedure Feedback	−0.10'	0.57	0.02	0.88
Teacher Initiated Behavior-Related Contact—Teacher Praises	0.05'	0.78	0.05'	0.79
Teacher Initiated Behavior-Related Contact—Teacher Warns Student	0.02	0.89	−0.30	0.08
Teacher Initiated Behavior-Related Contact—Teacher Criticizes Student	−0.18'	0.30	−0.05'	0.76
Wrong Response Followed by Teacher Criticism	−0.11'	0.51	−0.15	0.61
Process Feedback / Response Opportunities	−0.01	0.95	0.24	0.17
Process Feedback / Product Feedback	−0.01	0.93	0.25	0.15
Expands Feedback / Total Feedback	−0.24	0.16	−0.15	0.57
Process Feedback in Student Initiated Work Related Contacts / Total Student Initiated Work	−0.17	0.34	−0.19	0.30
Process Feedback in Teacher Initiated Work Related Contacts / Total Teacher Initiated Work Related Contacts	0.14	0.43	−0.20	0.25
Total Process Feedback	−0.10	0.54	0.16	0.61

---

[a] Most of the variable descriptors are self explanatory however some may need additional clarification as provided below For an extended discussion of these definitions and coding examples, see Brophy and Good (1970)

DIRECT QUESTION Teacher calls on a child who is not seeking a response opportunity

OPEN QUESTION The teacher creates the response opportunity by asking a public question and also indicates who is to respond by calling on an individual child but chooses one of the children who has indicated a desire to respond by raising his/her hand

PROCESS QUESTION Requires students to explain something in a way that requires them to integrate facts or to show knowledge of their interrelationships It most frequently is a "why?" or "how?" question

PRODUCT QUESTION Product questions seek to elicit a single correct answer which can be expressed in a single word or short phrase Product questions usually begin with "who?", "what?", "when?", "where?", "how much?" or "how many"

CHOICE QUESTIONS The child does not have to produce a substantive response but may instead simply choose one of two or more implied or expressed alternatives.

SELF-REFERENCE QUESTION Asks the child to make some nonacademic contribution to classroom discussion ("show and tell," questions about person experiences preferences or feelings, requests for opinions or predictions, etc)

OPINION QUESTIONS Much like self reference (except no one correct answer) except that they seek a student opinion on an academic topic (e g , Is it worth putting a man on the moon?)

NEGATION OF INCORRECT ANSWERS Simple provision of impersonal feedback regarding the incorrectness of the response and not going further than this by communicating a personal reaction to the child This can be communicated both verbally and nonverbally

Table 17

Means and Standard Deviations on Pre and Post SRA and Post Problem Solving Test by Instructional Group and by Classroom Organization

	Pre SRA		Post SRA		Pre-Post Change on SRA	Post Problem	
	$\overline{x}$	SD	$\overline{x}$	SD		$\overline{x}$	SD
<u>Control</u>	26.80	4.1	29.65	3.7	2.85	14.71	1.6
Semi	27.35	4.1	30.56	4.0	3.21	14.86	1.8
Open	25.36	2.7	27.70	2.6	2.34	14.55	.85
Special	27.26	5.9	29.78	4.3	2.52	14.56	2.1
<u>Treatment</u>	25.03	5.0	28.96	4.8	3.93	14.90	2.0
Semi	25.22	4.2	28.71	4.8	3.49	15.17	1.4
Open	20.41	1.3	26.01	4.9	5.60	13.13	3.6
Special	27.44	6.3	31.18	4.7	3.74	15.46	1.8

Table 18

Analysis of Variance Results on Adjusted Mean* Post SRA Test Scores (Using Pre SRA Scores as the Covariate) Between all Treatment and Control Classrooms

Source	DF	MS	F	Probability
Treatment Condition	1	5.58	1.31	.26
Error	33	4.27		

* Note the adjusted mean for the control group was -.38 and for the treatment group .42.

Table 19

Analysis of Variance results on Adjusted Mean* Problem Solving Scores (Using pre SRA Scores as a Covariate) Between all Treatment and Control Classrooms

Source	DF	MS	F	Probability
Treatment Condition	1	4.33	2.86	.10
Error	33	1.51		

* Note the adjusted mean for the control group was -.33 and for the treatment group .37.

Table 20

Analysis of Variance Results on Adjusted Mean* Post SRA Test Scores (Using Pre SRA Scores as a Covariate) Between Treatment and Control Classrooms With Open Classes Dropped

Source	DF	MS	F	Probability
Treatment Condition	1	.93	.29	.59
Error	25	3.17		

* Note the adjusted mean for the control group was -.18 and for the treatment group .18.

Table 21

Analysis of Variance Results on Adjusted Mean* Problem Solving Test Scores (Using Pre SRA Scores as a Covariate) Between Treatment and Control Classrooms With Open Classes Dropped

Source	DF	MS	F	Probability
Treatment Condition	1	5.45	6.77	.015
Error	25	.81		

* Note the adjusted mean for the control group was -.45 and for the treatment group .45.

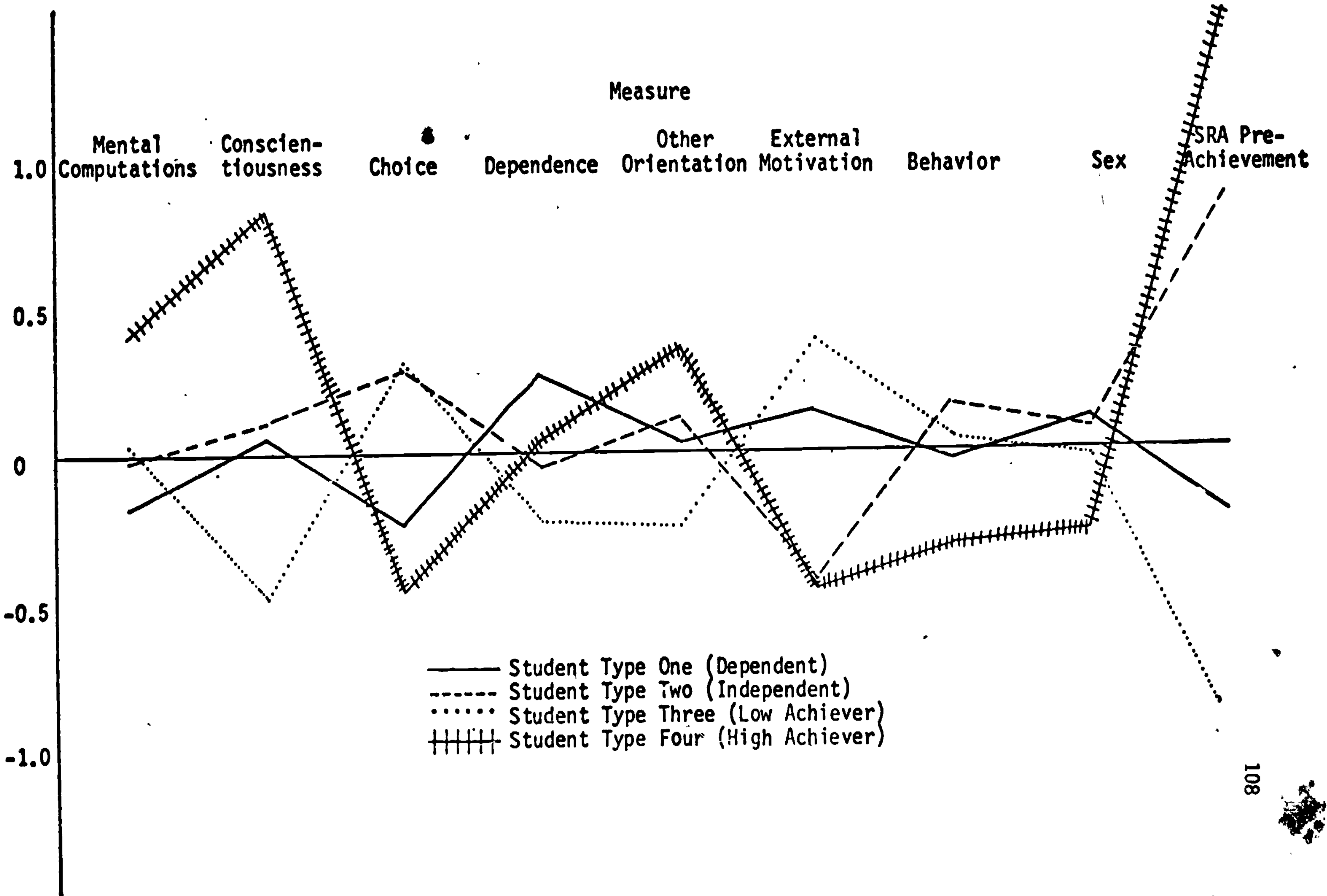


Figure 1: Graphic Representation of the Standardized Scores for Each of the Four Student Types

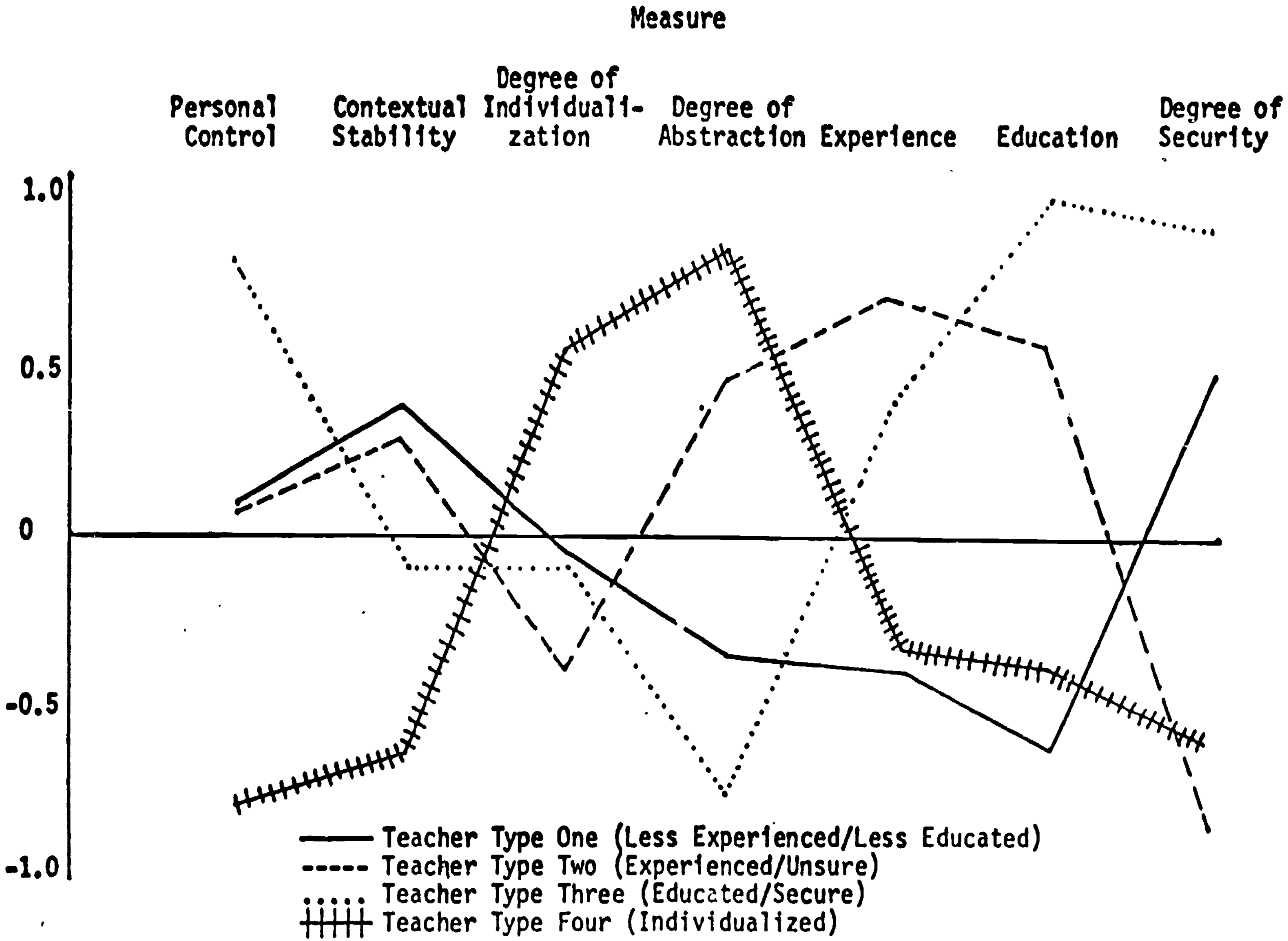


Figure 2: Graphic Representation of the Standardized Scores for Each of the Four Teacher Types

References

Anderson, L., Evertson, C. and Brophy, J. An experimental study of affective teaching in first-grade reading groups. *Elementary School Journal*, (in press).

Barr, A. J., Goodnight, J. H., Sall, J. P. and Helwig, J. T. *SAS*, 76. Raleigh, N. C. Sparks Press, 1976.

Brophy, J. Advances in teacher effectiveness research. *Journal of Educational Psychology*, in press.

Brophy, J. and Evertson, C. *Learning From Teaching: A developmental Perspective*. Boston: Allyn and Bacon, 1976.

Brophy, J. and Good, T. Brophy-Good Dyadic System. In A. Simon and E. Boyer (Eds.), *Mirrors for Behavior: An Anthology of Observation Instruments*. Philadelphia: Research for Better Schools, Inc., 1970.

Crawford, J. *et al*. An experiment on teacher effectiveness and parent-assisted instruction in the third grade. Vol. 1. Purposes, methods, and pre-test results. Unpublished manuscript, Stanford University. Program on Teaching Effectiveness, Center for Educational Research, 1978.

Cureton, E. E. Validity, reliability and baloney. *Educational Psychological Measurements*, 1958, *18*, 715-738.

Dawson, D. T. and Ruddell, A. K. The case for the meaning theory in teaching arithmetic. *Elementary School Journal*, 1955, *55*, 393-399.

Dubriel, John B. Utilization of class time in first year algebra (Doctoral Dissertation, University of Missouri--Columbia, 1977).

Dunkin, M. J. and Biddle, B. J. *The study of teaching*. New York: Holt, Rinehart, and Winston, Inc., 1974.

Ebmeier, H. An investigation of the interactive effects among student types, teacher types and treatment types on the mathematics achievement of fourth grade students. Unpublished dissertation, University of Missouri--Columbia, August, 1978.

Ebmeier, H. and Good, T. The effects of instructing teachers about good teaching on the mathematics achievement of fourth grade students. American Educational Research Journal, 1979, 16, 1-16.

Emmer, E. Classroom observation scales. Austin, Texas: Research and Development Center for Teacher Education, 1973.

Goldstein, A. Does homework help? A review of research. Elementary School Journal, 1960, 60, 213-224.

Good, T. Teacher effectiveness in the elementary school: What we know about it now. Journal of Teacher Education, 1979a, March-April 30 (2), 52-64.

Good, T. Teaching mathematics in elementary schools. Education Horizons, Summer, 1979b.

Good, T. and Beckerman, T. An examination of teacher's effect on high, middle, and low aptitude student's performance on a standardized achievement test. American Educational Research Journal, 1978, 15, 477-482.

Good, T., Ebmeier, H. and Beckerman, T. Teaching mathematics in high and low SESclassrooms: An empirical comparison. Journal of Teacher Education, 1978, September-October, 29, 85-90.

Good, T. and Grouws, D. Teaching effects: A process-product study in fourth grade mathematics classrooms. Journal of Teacher Education, 1977, 28, 40-54.

Good, T. and Grouws, D. Process-product relationships in fourth grade mathematics classrooms. Final report of National Institute of Education Grant (NE-G-00-3-0123), University of Missouri--Columbia, October, 1975.

Good, T. and Grouws, D. Teaching effects: A process-product study in fourth grade mathematics classrooms. Journal of Teacher Education, 1977, 28, 49-54.

Good, T. and Grouws, D. Teaching effectiveness in fourth grade mathematics classrooms. Chapter in G. Borich (Ed.), The appraisal of teaching: Concepts and process. Reading, Mass.: Addison-Wesley, 1977.

Good, T. and Grouws, D. The Missouri mathematics effectiveness project: An experimental study in fourth grade classrooms. Journal of Educational Psychology, 1979a, 71 (3), 355-362.

Good, T. and Grouws, D. Teaching and mathematics learning. Educational Leadership, October, 1979b, 39-45.

Good, T. Grouws, D. and Beckerman, T. Curriculum pacing: Some empirical data in mathematics. Journal of Curriculum Studies, 1978, 10, 75-81.

Gray, R. and Allison, D. An experimental study of the relationship of homework to pupil success in computation with fractions. School, Science, and Mathematics, 1971, 71, 339-346.

Greathouse, J. An experimental investigation of relative effectiveness among three different arithmetic teaching methods. The University of New Mexico, 1965. Dissertation Abstracts, 1966, 26, 5913.

Grouws, D. and Good, T. Verbal problem solving manual. Technical report number 155, Center for Research in Social Behavior, University of Missouri--Columbia, September, 1978.

Husen, T. *Internationality study of achievement in mathematics: A comparison of twelve countries*. Vol. 1 & 2. New York: John Wiley and Sons, 1967.

Koch, E. Homework in arithmetic. *Arithmetic Teacher*, 1965, *12*, 9-13.

Kounin, J. *Discipline and group management in classrooms*. New York: Holt, Rinehart and Winston, 1970.

Lundgren, U. *Frame factors and the teaching process*. Stockholm, Sweden: Almquist and Widsell, 1972.

Medley, D. Teacher competence and teacher effectiveness: A review of process-product research. The American Association of Colleges for Teacher Education, Washington, D.C., 1977.

Milgram, J. Time utilization in arithmetic teaching. *Arithmetic Teacher*, 1969, *16*, 213-215.

Miller, G. H. How effective is the meaning method? *Arithmetic Teacher*, 1957, *4*, 45-49.

Miller, N. Personality differences between the Black and White children in the Riverside school study. Paper presented at the meeting of the Society of Experimental Social Psychology, Minneapolis, Minnesota, 1970.

Miller, N. Summary and conclusions. In H. Gerard and N. Miller (Eds.), *School desegregation: A long-range study*. New York: Plenum Press, 1975.

Overall, J. E. and Klett, C. J. *Applied multivariate analysis*. New York: McGraw-Hill, 1972.

Postlethwaite, T. N. International association for the evaluation of educational achievement: The mathematics study. *Journal on Research in Mathematics Education*, 1971, *2*, 69-103.

Roberts, G. H. The failure strategies of third grade arithmetic pupils. *Arithmetic Teacher*, 1968, *15*, 442-446.

Rosenshine, B. Content, time, and direct instruction. In I. P. Peterson and H. Walbert (Eds.), *Research on teaching: Concepts, findings, and implications*. Berkeley, Calif.: McCutchan Publishing Corporation (in press).

Rosenshine, B. and Furst, N. The use of direct observation to study teaching. In R. Traver's (Ed.), *Second handbook of research on teaching*. Chicago: Rand-McNally, 1973.

Shipp, D. and Deer, G. The use of class time in arithmetic. *Arithmetic Teacher*, 1960, *7*, 117-121.

Shuster, A. and Pigge, F. Retention efficiency of meaningful teaching. *Arithmetic Teacher*, 1965, *12*, 24-31.

Suydam, M. N. and Weaver, J. F. *Using research: A key to elementary school mathematics*. Center for Cooperative Research with Schools, Pennsylvania State University, 1970.

Zahn, K. G. Use of class time in eighth grade arithmetic. *Arithmetic Teacher*, 1966, *13*, 113-120.

Addendum

Coleman, J. Methods and results in the IEA studies of effects of school on learning. *Review of Educational Research*, 1975, *45*, 355-386.

Footnotes

1. The term relatively effective should be stressed, because different criteria may yield a different sample of teachers. Effectiveness here refers to teachers' ability to produce results on standardized achievement tests.
2. Many classroom behaviors of high and low teachers were similar. For a discussion of these results, see our earlier final report (Good and Grouws, 1975).
3. One complicated and to some extent contradictory finding is that during the initial study of over 100 teachers there was no correlation between teachers' mean climate scores (affect) and students' residual mean performance in mathematics. During the observation stage the correlation between these two product measures was .50. Among the explanations we have considered in solving this anomaly is that in the initial sample students were reacting to teachers and schools generally, but during the observational study students were responding to the more specific context of mathematics. However, this suggestion is clearly an inference upon our part.
4. A semidepartmentalized structure calls for teachers to teach only two or three different subjects a day.
5. Using information provided by school officials, an attempt was made to match schools in terms of students SES, and then one school from each pair was assigned to the experimental condition. In the earlier correlational study, teachers used only one textbook. In this project, teachers used one of two textbooks, and it was impossible to match on both SES characteristics and textbooks.

6. Dr. Harris M. Cooper provided helpful advice about creating the Hawthorne condition, and we acknowledge his assistance.
7. Data from these two classrooms were left in the analysis in order to represent the most conservative procedure for estimating treatment effect.
8. Factor analysis was initially used in attempting to derive subscales. However, the derived factors were difficult to interpret. Therefore, an alternate procedure was used (see Nunnally, 1967, p. 288) in which questions are first grouped conceptually and then submitted for reliability analysis.
9. It is important to recognize that SES is a proxy variable that stands for a complex set of factors. Obviously, there are both high and low achievers and students with favorable and unfavorable attitudes toward school in both samples. The samples should be viewed as different but overlapping populations. Proportionately, in one sample, the students came from moderate and moderately high income families and achieve at moderately high levels. In contrast, in the other sample, students probably came from low income families and achieve at comparatively low levels.
10. To establish that the treatment conditions (experimental or control) did not interact with the coded variables to influence the dependent measure, a series of F tests were performed. Results indicated that there were no more than a chance number of interactions. Many of the behaviors occurred with only low frequency in the two studies. The meaning of these low frequency variables cannot be interpreted with confidence. To aid in interpretation, those measures that are

potentially contaminated by low frequency or by the treatment will be identified in the data tables.

11. In the low SES sample, the blue level for ES mathematics subtest of the SRA Achievement Test series constituted the outcome measure; whereas in the high SES study the mathematics subtest of the Iowa Test of Basic Skills was employed. A content analysis indicated that the two tests were comparable although not equivalent in terms of content items (e.g., percent of addition, subtraction, division, fractions, geometry) and level of operation (e.g., percent of multiplication, problems involving one digit, two digit, four digit numbers, etc.). In both samples there is <u>considerable</u> pre-post achievement gain on the two tests. Hence, there is test score variance in both samples that can be related potentially to measures of classroom teaching.

12. The dysfunctional effects of praise in the higher SES sample are shown more fully in an analysis of the top and bottom nine teachers (Good and Grouws, 1977). This analysis shows that teachers who got the best achievement used <u>significantly less praise</u> than teachers who got the lowest results.

13. It should be noted that involvement was coded as a high inference variable (measured on a rating system) in the high SES sample, but coded as a low inference variable (actually counted) in the low SES sample. This methodological difference might be a major part of the differential results.

Appendix A

Teachers Manual:

Missouri Mathematics Effectiveness Project

Principal Investigators:

Thomas L. Good
Douglas A. Grcuws

Members of Design Team:

Terrill Beckerman
Howard Ebmeier
Larry Flatt
Sharon Schneeberger

September, 1977

*The development of materials in this handbook were supported by Grant NIE G 77 0003 (2526-2228) from the National Institute of Education. The materials in the handbook are not to be used or cited without permission from the authors.

I. Introduction

We believe it is possible to improve student performance in mathematics in important ways. We look forward to your help and cooperation in implementing the program that we have discussed at the workshop and which is outlined again in the material that follows. Through your efforts we believe a significant difference in student performance will be made.

We do not believe that any single teacher behavior will make a critical difference in student learning, but we do feel that several behaviors in combination can make a major impact. In the material that follows, we present a system of instruction that, if followed daily, will enhance student learning.

In general, we feel that the plan should be followed each day. However, we also realize that special circumstances will force you to modify the plan on occasion. Still, it is important that you follow the daily plan as frequently as you can.

For purposes of clarity, we will discuss each part of the teaching program separately. However, once again we want to emphasize that the program works when all parts are present. To maximize your opportunity for obtaining a clear picture of the instructional program, the program is summarized in Table 1. The rationale for each part and how the pieces fit together will be discussed at a later point in the handbook.

Table 1
Summary of Key Instructional Behaviors*

Daily Review (First 8 minutes except Mondays)

- a) review the concepts and skills associated with the homework
- b) collect and deal with homework assignments
- c) ask several mental computation exercises

Development (About 20 minutes)

- a) briefly focus on prerequisite skills and concepts
- b) focus on meaning and promoting student understanding by using lively explanations, demonstrations, process explanations, illustrations, etc.
- c) assess student comprehension
  - 1) using process/product questions (active interaction)
  - 2) using controlled practice
- d) repeat and elaborate on the meaning portion as necessary

Seatwork (About 15 minutes)

- a) provide uninterrupted successful practice
- b) momentum - keep the ball rolling - get everyone involved, then sustain involvement
- c) alerting - let students know their work will be checked at end of period
- d) accountability - check the students' work

Homework Assignment

- a) assign on a regular basis at the end of each math class except Fridays
- b) should involve about 15 minutes of work to be done at home
- c) should include one or two review problems

Special Reviews

- a) Weekly Review/Maintenance
  - 1) conduct during the first 20 minutes each Monday
  - 2) focus on skills and concepts covered during the previous week
- b) Monthly Review/Maintenance
  - 1) conduct every fourth Monday
  - 2) focus on skills and concepts covered since the last monthly review

---

* Definitions of all terms and detailed descriptions of teaching requests will follow.

II. Development

Variable Description

The developmental portion of the mathematics period is that part of the lesson devoted to establishing comprehension of skills and concepts. It should be viewed as a continuum which runs from developing understanding to allowing for meaningful practice in a controlled setting. During all stages of the developmental portion of the lesson, both ends of the continuum may be present to some degree. However, in general, the comprehension emphasis with very little practice will come at the initial part of the lesson, then toward the middle of the lesson, practice with process feedback from the teacher will become quite prominent, and finally in the latter portion of the lesson there will be controlled practice with meaningful explanations given as necessary.

The role of the teacher in the first part of the lesson, the comprehension phase, is to use instructional strategies that help students <u>understand</u> clearly the material being studied that day. In this portion of the lesson emphasis is placed upon comprehension rather than rote memorization. Activities which often focus on comprehension include teacher explanations and demonstrations, and they may include use of manipulative materials to demonstrate processes and ideas, use of concrete examples in order to abstract common features, making comparisons and searching for patterns, and class discussions.

During the middle portion of the lesson, the number of questions posed to students may increase as the teacher begins to assess comprehension and provides them an opportunity to model processes already demonstrated and to verbalize the understanding they have developed. During this phase of the lesson, the teacher may decide that further explanations and demonstrations are necessary or decide that controlled practice is appropriate since students seem to understand what they are doing.

In the controlled practice phase of the lesson the emphasis is on increasing proficiency; that is, increasing speed and accuracy. However, meaningful feedback is still given as necessary or requested.

Problem

Many problems arise in math classes in which teachers give too little attention to development. Students exposed to such teaching frequently attempt to memorize rules for doing things and concentrate on mechanical skills. These rules have no meaning for the student (because developmental work was not done) and, thus, they are easily forgotten especially when new sets of rules are "learned." When students do not understand what they are doing, each new problem causes them great difficulty. Often the comment, "We haven't done any of these before" is heard. When students learn without understanding, the ability to transfer skills to new situations is greatly reduced. Other negative results such as the inability to detect absurd answers and loss of self-confidence also occur. Thus, there are many compelling reasons to include a large measure of developmental work in mathematics lessons.

Teaching Practice

Initially, the teacher should focus briefly upon prerequisite skills that students may need for the lesson. Then the major aspect of the meaning portion of the development lesson occurs: active demonstration of the concept, idea and/or skill that is being focused upon in the lesson, etc. Teachers need to demonstrate actively the process, so that students can comprehend the learning goal. You need to be cautious about moving too quickly into the assignment of problems and practice without providing students with an adequate conceptual orientation.

After the active demonstration and explanation by the teacher (and we recommend that 10 minutes minimum be spent on this), the teacher should begin to assess student comprehension. There are two primary ways to do this. First, teachers may ask oral questions. In general, we recommend that teachers generally ask brief product oriented questions. Product questions are questions that assess whether or not the student can produce the correct answer (see appendix A for a complete description). Teachers can maintain an emphasis upon meaning by frequently providing process explanations themselves after students respond ("Yes, Tina, that's right because . . .").

The second way that teachers can assess student comprehension is by having students work practice problems. However, it is important to recognize that the role of a practice problem in this stage of the lesson is not to build up student speed and accuracy per se, but rather, the goal is to allow teachers to assess student comprehension. Hence, the assignment of problems in this stage should be limited to a single, brief problem followed by teacher assessment and explanation and then the provision of another brief problem assignment. In general, this stage of the lesson can be completed in 3-5 minutes.

If your questions or assigned problems reflect a moderate degree of student difficulty, then you should repeat the meaning portion of the lesson. If possible, use different examples; however, if this is not possible, verbatim repetition of the initial portion of the lesson is better than to proceed to controlled practice and seatwork when students are confused. Such a situation guarantees that students will practice errors.

If assessment of student comprehension is largely satisfactory, *then* teachers should proceed to the controlled practice portion of the development lesson. Now, the teacher provides opportunity for students to develop increased speed, accuracy, and proficiency in completing problems of a specific type. However, the practice is still heavily controlled (unlike seatwork practice which will be discussed in the following section).

During controlled practice, teachers should assign only one or two problems at a time. *Students should not be asked to work longer than a minute without feedback about the correctness of their responses.* The reason for this is that during the controlled part of the lesson the teacher is still trying to identify and correct any student misunderstanding. Too often many students are left to watch while a few students demonstrate a problem on the board. A great deal of practice time is lost this way and often the involvement of some students in the lesson (momentum) is lost as they become engaged in side conversations and distractions.

During controlled practice exercises, teacher accountability and alerting should be *immediate* and *continuous*. By alerting, we mean teacher behaviors that remind students that they should be doing work and that it will be checked. For example, if the teacher sends 3-4 students to the board to demonstrate the problem that students have just worked at their desks, the teacher might say, "Now the rest of you do these two new problems at your desk and I'll check them in a minute." Such teacher behavior maintains student momentum. Instead of watching classmates write on the board, they have their own work to do and they are alerted to the fact that they will be held responsible for the work.

By accountability (more on this when we describe the seatwork portion of the lesson) we mean the actual checking of student responses. For example, while students put their work on the board, the teacher could look at the work of students who remain at their desks and check the problems that they were to have completed. Furthermore, the teacher can call on students to provide answers to practice problems, etc. Through such procedures, the teacher is able to assess when students are prepared to move to the seatwork portion of the lesson where they have a longer block of time for uninterrupted practice. A final important characteristic of the controlled seatwork portion of the lesson is that the practice is done in the context of meaning (e.g., the teacher is frequently providing process explanations "Yes, that's right because . . ."). Although the teacher is beginning to work for speed and accuracy, some attention is still being paid to students' understanding of the concepts, ideas, and skills that are being developed.

In summary, the development part of the lesson calls for the following teacher behavior:

(1) Review briefly and/or identify prerequisite skills.

(2) Focus upon the development of meaning and comprehension using active demonstration and teacher explanation.

(3) Assess student comprehension (ask questions/work on supervised practice).

(4) Repeat meaning portion of the lesson as necessary (using different examples and explanations if possible).

(5) Provide practice opportunities for students.

- (a) Practice should be short (one or two problems at a time).
- (b) Students should be held responsible for assigned practice problems.
- (c) Practice should be performed in a meaningful context (teacher provides frequent process explanations).
- (d) When success rate is high, move students into seatwork portion of the lesson where students have an opportunity for uninterrupted practice.

III. Seatwork

Variable Description

Seatwork refers to practice work that students complete individually at their desks. Since seatwork practice follows the controlled practice part of the development lesson, students should know the purpose of assigned problems and how to do them when they begin to work. The role of seatwork practice is both important and easy to describe. Seatwork assignments allow students to practice, on their own, problems and principles that you have just actively taught. Seatwork provides students with an opportunity for *immediate* and *successful* practice. This practice experience allows students to achieve increased proficiency and to consolidate learning. New material or review work should *not* be assigned during the seatwork portion of the lesson.

Problem

Often a great deal of time is wasted when students work on problems individually. Indeed, research has consistently shown that students show less involvement (amount of time that students actually spend working on problems) during the seatwork portion of the lesson than during the active teaching portion of the lesson. Too often teachers stop active supervision after they make the seatwork assignment. Two of the more common ways that teachers stop supervision are by doing desk work, grading or by providing extended feedback to a single student (before all students are working on the task). Such behavior virtually guarantees that teachers cannot provide the type of supervision that students need if they are to begin to work productively. The first teaching task is to get students started on the seatwork. Often students do not use seatwork time productively simply because the teacher does not obtain their attention initially.

In addition to the problem of not "demanding" students to start work, some teachers create a problem by moving from the development portion of the lesson to seatwork with such abruptness that it is not surprising that students do not begin to work immediately (e.g., four students spring to the pencil sharpener, two students search for materials, and three students begin a private conversation). Momentum needs to be maintained throughout all stages of the lesson. When momentum is lost, students are apt to take a psychological break and once momentum (student attention and involvement) is lost, it is difficult to "recapture." Teachers who end the development portion of the lesson with a controlled practice segment have done much to ease the transition from the group lesson to individual seatwork.

Teaching Request

Given that the role of seatwork is to provide opportunity for successful practice, we recommend that about 10-15 minutes each day be allotted for seatwork. Ten to fifteen minutes allows sufficient time for students to work enough problems to achieve increased proficiency but not so long as to bring about boredom, lack of task involvement, and the behavioral problems that soon follow when students are bored or frustrated. Frustration should be minimal in seatwork activity because the problems students are asked to do are a direct extension of the development part of the lesson. If practice time does not exceed 15 minutes, few students are likely to be bored.

The number of problems assigned should take most students only 15 minutes to complete. Hence, approximately 75 percent of your students should be able to complete the work within the allotted 15 minutes. In making the seatwork assignment, emphasis should be placed upon accurately working as many problems as possible within the allotted time. Low achievers who remain on task and do accurate work have done well and should know that they have done well. That is, the criterion to communicate to students is to keep working and to do as many problems accurately as they can.

To help optimize the effectiveness of seatwork, three general principles should be observed. The first principle, *momentum*, has already been discussed indirectly. By momentum we mean keeping the ball rolling without any sharp break in teaching activity and in student involvement. Teachers can achieve momentum by ending the development portion of the lesson by working problems similar to the ones that students are asked to work individually and by starting students on individual work with a simple and direct statement. "We've worked problems 1 and 3. Now, individually, at your desk do problems 5-15. Work as many problems as you can, and we'll check our work in 15 minutes. Remember doing the problem correctly is more important than speed." Following such a statement, *you should actively monitor all students*. Before providing feedback to individual students, make sure all students are engaged in the seatwork.

If some students do not begin working immediately, walk to their desks and if your physical presence doesn't initiate student work as it usually will, then quietly say something like "Frank, it's time to do the problems." After all students are working on the problems (the ball is rolling), you can then attend to the needs of individual students. In general, students should get immediate feedback and help when it is needed. Thus, it is usually reasonable to *allow* students to approach you when they have a question or problem. However, when presenting feedback to individual students, keep in mind the general principle of momentum. You have to provide feedback and conditions that allow most students to stay on task (keep working). Hence, it is not advisable to continue to provide lengthy feedback to an individual while several students are waiting for teacher feedback before they can continue to work.

Alerting is a second principle to observe during seatwork. Alerting behaviors tell students that they will be held accountable for their work. Often students engage in off task behavior because they are not alerted to the fact that they will have their work checked at a specific point in time. If students are assigned seatwork that won't be checked until the following day (or when it is not checked at all), students are not likely to be highly engaged in seatwork. A statement like, "We'll check the work at the end of the period." alerts students to the fact that there is reason to engage in productive work immediately. A statement at the beginning of the seatwork is sufficient. Repeated statements are apt to interfer with students' work concentration. Public announcements should not occur during seatwork. Once you have students working it doesn't make sense to distract them.

Accountability is the third principle to observe during seatwork. Alerting, as we noted, is a signal to students that their work will be checked. Accountability is the actual checking of the work. It is important that your accountability efforts do not interrupt the seatwork behavior of students. During the controlled practice part of the lesson (see development section), accountability is immediate. However, during the seatwork portion of the lesson, students are to be working more independently and those students capable of doing the work need time for uninterrupted practice. Public accountability needs to be delayed until the end of the lesson. A teacher's public questions during this stage of the lesson are very disruptive. For example, when the teacher asks a public question (e.g., "How many of you have done the first four problems?," "What's the answer to the second problem?," etc.) all students stop work and once momentum is lost, some students will take much time before resuming their work. Furthermore, questions like "How many of you have finished the first four?" may make students anxious and distract them from task behavior if they have not worked the first four problems. Occasionally, you may need to stop seatwork practice to correct a common misunderstanding. In general, these errors should be corrected during the development (controlled practice) phase of the lesson. Public statements (except for necessary behavioral management) should be avoided. If most students are not ready for seatwork practice, then you should stay in the controlled practice part of the lesson. Such behavior will help students develop the following attitude toward seatwork: "I can do the problems and now it is time for me to apply myself."

Perhaps the most direct and easiest way to hold students publicly accountable without disrupting seatwork is to call on individual students at the end of the lesson. Checking students' work at the end of the period also provides the teacher with a chance to spot any systematic mistakes that students are making and to correct those misunderstandings. Hence, when your students are assigned their homework, conditions should be set so that the homework provides for additional and relatively successful practice.

Specifically, we are asking you to get student involvement immediately after making a seatwork assignment. Continue to monitor and supervise all students until they are engaged in assigned work (the first minute or two). Early in the seatwork period (the first three to five minutes), be available for students when they need feedback. Toward the end of the seatwork period, try to get to the desks of some low achievers to see if they are making any systematic errors and to provide feedback as necessary. At the very end of the seatwork period, hold students accountable for their work by asking individual students to give the answer to a few of the assigned problems. This checking of answers should be very rapid and you need only check 3 or 4 of the problems (check one or two problems at the first, in the middle and at the end of the assigned work). If misunderstandings are corrected here, the homework should be a successful practice experience for most students.

When conducting the review of seatwork, it is generally advisable to call on low achievement students to provide answers only to the first few problems assigned so as not to frustrate them for failure to complete all problems, but be sure to increase seatwork expectations for these students as the year progresses.

Finally, all seatwork should be collected. This helps encourage students to work productively because they know that they are held accountable for the work assigned during seatwork. Because of the way teachers have used seatwork in the past, many students have built up the expectation that seatwork is a time to relax and waste time. Taking up the seatwork will help students to adjust to the expectation that seatwork is a time to apply themselves and to see if they can do the type of problems which will be assigned as homework. Although there is no compelling reason to grade seatwork, it is important to examine the papers to see if students are using seatwork time appropriately. If a student's work is unduly incomplete, impossible to read, etc., it would be important to mention this to the student so that he or she knows that you care about his seatwork performance.

After the seatwork is collected, the homework assignment is made. Delaying the assignment of homework helps to insure that students will do the work at a later point in time, hence, building distributed (repeated) practice into the mathematics programs. Research has consistently shown the superiority of distributed practice over mass practice in helping students to master and retain new concepts and skills.

IV. Homework

Variable Description

Mathematics homework is written work done by students outside the mathematics class period. It is usually done at home; thus, it is distinctly different from seatwork which is done within mathematics class time.

Problem

The emphasis on homework in schools over the years has varied considerably. Unfortunately, homework has been misused frequently. Sometimes the assignments were so long that students became bored and careless when working the assigned problems. No doubt some students' dislike for mathematics is in part associated with these lengthy assignments. The instructional value of long homework assignments is very questionable. If students make errors on the first few problems of the assignment, then by the end of the assignment they may have become more proficient in making those errors!

Other situations in which homework has not been used to its full potential are plentiful. In some schools homework is never given or so few problems are assigned that an excellent opportunity for distributed practice is wasted. Another undesirable situation occurs when homework is given primarily to please parents but without much attention to selecting problems and assignments that will foster progress toward important objectives. *But perhaps the most devastating misuse of homework is when children are assigned problems for which inadequate background has been developed in class.* While long assignments often lead to frustration, this latter situation always leads to frustration and ative attitudes toward the mathematics class.

Another situation which detracts from the value of homework assignments happens when the teacher fails to stress the importance and value of the problems assigned. This can be done directly by not commenting on the importance of assignments or indirectly by not scoring or collecting assignments.

If spite of these misuses of homework, homework can be an important part of mathematics learning if certain guidelines are followed. Research suggests that giving homework to students on a regular basis may increase achievement and improve attitudes toward mathematics. Short assignments have been found to be most effective and some variety in the type of homework is helpful. Also, if a teacher gives importance to the homework through oral comments and by scoring papers regularly, then students frequently respond by completing their assignments with greater care.

Teaching Request

Because of the important role that homework can play in improving student performance in mathematics, we would like to have you do the following during the study:

1. At the very end of the math class period on Monday through Thursday, give a homework assignment which is due at the beginning of the class period the following day.
2. Each assignment should require about 15 minutes of outside class time. Within this time frame, assignments will probably average about eight problems per day depending on the kinds of problems being assigned. A typical assignment is shown in Appendix B.
3. The primary focus for an assignment should be on the major ideas discussed in class that day. Also each assignment given on Tuesday and Wednesday should include one or two review problems from the current week's work.

4. Each assignment given on Thursday should be primarily devoted to review problems from the current week's work. In order for sufficient practice to be given on the material discussed on Thursday, this assignment will be a bit longer than assignments for other days and will probably take about 20 minutes for most students to complete.

5. Typically, each assignment should be scored (number correct) by another student. Papers should then be returned to their owners for brief examination. Finally, papers should be passed forward so that the scores can be recorded in the grade book.

6. The assignments given should be recorded daily in the Teacher's Log.

The short homework assignments complement seatwork by distributing practice over time without putting undue time pressure on students. Short assignments help hold student interest; adding variety to assignments is also helpful. This can be done by embedding the problems to be worked in different formats such as games, puzzles, codes, and so on. Appendix C illustrates this idea. Another component of variety might be to have students check their work. Multiplication problems can be checked by doing division, addition problems by doing subtraction, and so on. Variety can also be introduced by giving differentiated assignments. For example, some students could be given ten easy problems, while other students are given six problems of a more difficult nature.

The scoring and recording of grades on all homework assignments are designed to emphasize the importance of homework and to provide regular feedback to students and teachers regarding progress being made by each student. It is important to realize that there are a number of efficient ways to score homework other than the teacher's going through the papers individually. For instance, students can exchange papers or score their own papers. Either of these procedures is improved if students are expected to have their homework completed and ready to be scored at the very beginning of math time. Efficiency is also improved if answers are prepared in advance by the teacher in written form (transparency, blackboard) and then shown to the students. Otherwise, the teacher may need to orally repeat each answer a large number of times.

Explanation~~s and~~ reteaching the homework must be somewhat limited if adequate time for discussion and practice of new material is to be available. This should not cause too much difficulty because most student difficulties and errors should have been remediated prior to the seatwork of the previous day.

A good strategy may be to quickly have children exchange and score papers, then have children indicate by raising their hands--how many missed problems #1, #2, and so on. Then you can rapidly work the one or two problems that caused students the most difficulty. Since there are usually only a small number of homework problems to be checked and discussed, this part of the lesson should be easily completed in two minutes. Finally, note that any reteaching that is not completed can be done during the weekly review that is discussed in the next section.

In the rare event that the checking of homework reveals numerous student errors, you should reteach the previous day's lesson beginning with development, then controlled practice, then seatwork, and finally a homework assignment on the same material. Under these circumstances you should not try to cover new material due to the very limited amount of time available to develop the new ideas.

You are requested to personally score the homework that is assigned on Thursdays. There are two reasons for this. First, the information gathered from this homework is to be used to structure the weekly review each Monday. Second, the focus of student scoring is of necessity on answers rather than kinds of errors being made. It is very important, however, that regular attention be given to the procedures and processes that students are using. This is especially true when they are making errors!

In connection with the scoring of Thursday's work, each student's paper should be analyzed for systematic error patterns. Systematic error patterns refer to incorrect procedures which are consistently used on a wide range of problems. In two-digit multiplication problems, for example, a student might consistently forget to "carry" the tens digit from the initial multiplication of the units digits. According to recent,research such errors are much more common than was once realized and,thus,spending time examining homework with them in mind can be very helpful in remediating some students' difficulties with mathematics. Further examples of common computational error patterns can be found in Appendix D. Since the particular errors you find probably will not be associated with groups of students, the remediation of such errors is usually best done on a one-to-one basis.

Homework is an important component of this program and since both students and teachers devote a considerable amount of time to it, it is recommended that homework count at least 25 percent of each student's math grade and that this information be communicated to them.

Parents are interested and should be informed about what is happening in school. Therefore, it is recommended that an explanation of the homework policy to be followed during the study be sent home to parents. A letter which could be duplicated and used for this purpose can be found in Appendix E.

Homework is explicitly related to each of the other components of the study in a number of ways. With an increase in development time, it provides an opportunity to supplement the practice part of the lesson. It is structured such that practice is distributed over time and students have an opportunity to correct difficulties encountered in seatwork. The homework provides important information for structuring the specific details to be covered in the review component. It is also related to the pacing variable in that it allows some necessary work to be done outside of the time regularly scheduled for math.

V. Special Review/Maintenance*

Variable Description

Children forget. It is imperative, therefore, that ideas be reviewed and skills maintained on a systematic basis in elementary school mathematics. Reviewing ideas may involve the teacher stating and explaining properties, definitions, and generalizations and the students recalling the appropriate term or name. These roles occasionally may be reversed (where the teacher supplies a term and the students illustrate and explain), but the focus should generally be developmental in nature. That is, there should be a strong emphasis on meaning and comprehension. Similarly, skills need to be practiced with _regularity_ in order that a _high level of proficiency be maintained_. The focus should be developmental in nature; comprehension again is an important component.

Problem

When discussing children's performance in mathematics, frequently the comment is made that many have not mastered the basic skills. From this it is concluded that teachers do not spend enough time teaching basic computation. But this conclusion often is not valid because the inability to perform may _not_ be associated with the initial learning but rather with a lack of maintenance. Newly learned material is particularly susceptible to being forgotten, but even material thought to be "mastered" is sometimes lost. For example, many fourth grade teachers have had the experience in which a student seems to have mastered his basic multiplication facts, indeed, he or she can recall them with almost 100 percent accuracy but four weeks later seems to have forgotten a great number of them.

---

*The review discussed here is in addition to the brief (1-4 minute) daily review that we will discuss later in the handbook.

Teaching Request

To minimize this problem and similar problems, we are asking that you incorporate review/maintenance sessions regularly into your mathematics instruction. Regularly in the sense that each Monday you have a Weekly Review/maintenance session and every fourth Monday you have a Cumulative Review/maintenance session. The purpose of the two types of review sessions is to help students retain concepts and insights.

Weekly Review/maintenance. The following things are necessary to do if the review/maintenance component is to be implemented effectively:

1. The first one-half of each Monday's math period (roughly 25 minutes) should be devoted to review/maintenance.
2. The focus should be on the important skills and concepts covered in math during the previous week. The suggested order for covering these skills and ideas are:
    a. Those that are thought to be mastered and can be done very quickly.
    b. Those that need additional development and practice as identified from the analysis of the Thursday homework assignment.
    c. Those that need additional work (as identified during this review session).

Most of the important skills and concepts that should be reviewed can easily be identified by examining the homework assignments from the previous week. That is, these homework assignments deal with each important data or skill; thus, reviewing them will assist you in identifying important topics. It is of utmost importance that all major ideas covered during the week be reviewed. Reviewing ideas that students have "mastered" the previous week helps to guarantee that ideas will be retained. Areas in which some reteaching is definitely needed should be identified in advance by the teacher from an analysis of the Thursday homework assignment and handled during the second portion of this designated review segment.

There are many ways that this maintenance program can be successfully organized. One important attribute of any effective organizational scheme is active student involvement. In most teaching situations, it is important to avoid situations that involve only one student in checking problems because such a procedure is usually ineffective and boring to most children. This is especially true in a review situation in which students are already familiar with the problem. One scheme that we highly recommend (because it overcomes this difficulty) is one in which the teacher presents an idea or problem and then allows students to work individually at their desks until most arrive at an answer. Finally, answers are checked (children are held accountable), and someone explains or demonstrates how to arrive at the answer (in many cases by using the chalkboard at the front of the room).

Cumulative Review/maintenance. This aspect of the review/maintenance program can best be implemented in the following way:

1. Every fourth Monday the entire math period should be devoted to a cumulative review/maintenance session.
2. This review should encompass the work of the previous four weeks and thus replace the regular Monday maintenance/review session.

This session provides an opportunity to reteach ideas that have given difficulty over the past four weeks. It will be particularly useful to those students who have difficulty acquiring skills and ideas on initial exposure.

The interest in and value of this review session can be greatly enhanced by structuring it in an interesting format such as a game, contest, or quiz show.

Postscript

On occasion, it may be desirable to reschedule a review for a day other than Monday. For example, if by not reviewing on a Monday you can complete a chapter or unit, by all means do this and simply conduct your review on Tuesday. If it becomes necessary for you to reschedule a review, please make a note of it in the log so that we are aware of it.

VI. Mental Computation

Variable

Mental computation is computation that is done without the aid of pencil and paper (or minicalculator). The process is done by the most powerful computer of all, the human brain. Mental processing is often vastly different than pencil and paper calculation. For example, in pencil and paper addition situations the calculation always goes from *right* to *left*. The student asked to solve 41 + 12 on paper is going to move mechanically from right to left. However, in a mental activity (the teacher says what is 41 + 12) the student may frequently move from *left* to *right*. First, the student does something to the tens column, then to the ones column, and then combines. We feel that the inclusion of some time for mental computation each day will help students to further develop their quantitative sense, to become more flexible in thinking and in approaching problem-solving situations. Furthermore such activities help students to detect absurd answers (e.g. when checking their written computation) and make estimations that are frequently needed in daily activities.

Problem

The attention given to mental computation and mental problem-solving has largely disappeared from the modern mathematics curriculum. At one point in time much emphasis was given to mental problem solving. This de-emphasis has occurred despite some research evidence which suggests that mental practice on a regular basis appears to be related to large increases in student achievement. If students are not given some work in mental computation, then they are missing a very important way to check their work (other than the time consuming and inefficient process of completely redoing the work).

Teaching Request

We would like for you to include 3-5 minutes on mental computation activities each day at the beginning of the lesson; the predevelopment part of the lesson will be described later in the handbook. Ideally, the material presented for mental resolution would be related to the content of the material being studied. During the study of subtraction, mental computation activities should focus on subtraction. However, some units that you study in the year will not lend themselves to this form of mental processing. During such a unit (e.g. geometry) it would be useful to rotate on a daily basis with the following types of mental computation activities: addition, subtraction, multiplication, division, and verbal problems.

The following examples will give you some ideas about the kinds of problems you may present to your students. Some of the examples here may be too easy or too difficult for your students. You should try to use problems which are challenging yet accessible to most students. It is a good idea to discuss how a problem might be solved mentally before students are asked to give solutions.

For example, for a problem like 6 x 12 you might suggest thinking as follows: "6 times 12, that's 6 times 10 plus 6 x 2, that's 60 + 12, 72." Then begin giving students problems one at a time to solve like 8 x 12, 6 x 15, and so on. It is worthwhile to mention to the students that there are many ways to solve problems mentally and the way you showed is but one way. Children should be encouraged to discuss their mental computation procedures.

Further illustrations of the kinds of problems which are appropriate are given below. You should generate other types of mental computation exercises for your students as well.

Addition

(1) 75 + 77 = _____

Think: 77 = 70 + 7. First add 70 to 75 (145) then add 7 to that sum (152).

or: Rename 77 as 70 + 7 and 75 as 70 + 5. Add the tens (140), add the ones (12), then find the total of the sums (152).

(2) 97 + 8 = _____

Think: How much do I add to 97 to get 100? The answer is 3. Since 8 = 3 + 5, first I add 3 to 97, and then add 5 to the sum.

(3) 243 + 104 = _____

Think: 104 = 100 + 4. First add 100 to 243 and then add 4 to the sum.

(4) 125 + 49 = _____

Think: 49 is 1 less than 50. Since 125 + 50 = 175, 125 + 49 = 174.

Subtraction

(1) 125 - 61 = _____

Think: 61 = 60 + 1. First subtract 60 from 125, and then subtract 1 from the difference.

(2) 105 - 8 = _____

Think: First subtract enough from 105 to get 100: 105 - 5 = 100
Since 8 = 5 + 3, subtract 3 more: 100 - 3 = 97.

(3) 425 - 97 = _____

Think: 97 = 100 - 3. First subtract 100 from 425, and then add 3 to the difference. 425 subtract 100 is 325, add 3 is 328.

Multiplication

(1) 20 x 36 = _____

Think: 20 = 2 x 10. Ten times 36 is 360, and 2 x 360 = 720.

or: 2 x 36 = 72, so 20 x 36 = 720

or: 20 x 36 that's the same as (½ x 20) x (2 x 36), or 10 x 72 = 720.

(2) 4 x 17 x 25 = _____

Think: Since the product of 4 and 25 is 00, these numbers are multiplied first. Then 100 is multiplied times 17.

(3) 32 x 50 = _____

Think: The product is unchanged if I double one factor and half the other factor. Thus, 32 x 50 is the same as 64 x 25 or 1,600.

(4) 4 x 53 = _____

Think: 53 = 50 + 3. Four times 50 is 200. Four times 3 is 12. So to find 4 x 53 add 200 + 12.

Division

(1) 84 ÷ 4 = _____

Think 84 = 80 + 4. 80 divided by 4 is 20 and 4 ÷ 4 is 1, so 84 ÷ 4 is 20 + 1 or 21.

(2) 396 ÷ 4 = _____

Think: 396 = 400 - 4. Since 400 ÷ 4 = 100 and 4 ÷ 4 is 1, the quotient is 100 - 1 or 99.

(3) 250 ÷ 50 = _____

Think: 250 ÷ 50 is the same as 500 ÷ 100 which is 5.

Verbal Problems

(1) Mr. Thomas has a debt of $120. If he pays $70 of it, how large a debt will he have left?

Think: I need to find 120 - 70 = _____.
12 - 7 = 5, so 120 - 70 = 50.
50 is the answer.

VII. Instructional Pace

Variable Description

Instructional pace refers to rate. It may be thought of in terms of how quickly a class is moved through a given curriculum or in terms of how rapidly students are presented with particular topics. The pace associated with different teachers varies. Some teachers move through the curriculum faster than others.

Problem

Instructional pace may inhibit learning in several ways. At one extreme is the situation in which a teacher moves through the curriculum too quickly for learning to take place. At the other extreme is the teacher who plods along so slowly that many of the students are bored. Furthermore, some teachers, because of their slow pace, find themselves forced to cover so much material at the end of the year that they do not have time to build in the distributed practice which is essential if students are to retain the material. Research suggests that for most teachers efficiency could be improved if they increased their pace slightly. That is, there seems to be more of a tendency to procrastinate than to move forward. If the suggestions presented earlier in the manual are implemented in your teaching program, the important element of review and distributed practice should be fulfilled and you will probably be able to pick up the pace.

Teaching Request

For this variable we ask that you carefully consider your teaching behavior with respect to the instructional pace you set. Many of you will find that you can increase the pace somewhat and we ask you to attempt to do so.

The instructional strategies suggested in this study are such that if you speed up a bit too much, then you can resolve problems that arise through your regularly scheduled review/maintenance sessions.

VIII. Starting and Ending the Lesson

We have now discussed the major parts of the mathematics instructional program. Two aspects that we have not discussed explicitly are the start and end of the lesson.

The beginning portion of the lesson (Predevelopment) will have three parts: (1) a brief review, (2) the checking of homework, and (3) some mental computation exercises. We ask that all three of these activities be done within the first eight minutes of the class period. This may be difficult for teachers who slowly ease into the lesson, but it has been commonly observed that time is frequently used inefficiently at the beginning of a lesson.

The review of the previous day's lesson should begin with a brief summary by the teacher. Several sentences that briefly and concisely remind students of what they did and why, and demonstrating how to solve a single problem is usually sufficient. Next comes the checking of homework. This should proceed very quickly once students learn that when math period begins they are to have their homework on top of their desks ready for checking. Initially, it may take some time to establish this routine, but once the routine is established it should take only a couple of minutes to check homework.

The third activity, mental computation, plays two roles in the lesson structure. First, it is an important activity *per se* (see earlier section). Second, these activities can provide a smooth transition for getting students engaged in thinking about math prior to the point at which the teacher begins a new development lesson.

The ending of the lesson is a very simple procedure. After allowing students a period of time for uninterrupted practice, the teacher briefly checks pupils' work on a few problems (may call on students, ask students who got problems correct to raise their hands, etc.). This accountability procedure encourages students to apply themselves during seatwork and allows an additional opportunity to clear up misunderstanding. After checking some of the seatwork, the teacher ends the mathematics lesson by assigning the homework problems.

The predevelopment phase of the lesson should take roughly eight minutes. The exact distribution of time on review, homework, and mental computations depends upon a variety of conditions (e.g. moderate difficulty with homework vs. no difficulty) and you are asked to use your judgment. In general, we think the following situation will be most applicable: 1-2 minutes on review; 3-4 minutes checking homework; and 3-4 minutes on mental computations.

IX. Summary and Integration

We have asked you to do several things during the next few weeks in an attempt to improve student performance in mathematics. In the first part of this handbook we emphasized that we didn't feel that changing one or two teacher behaviors would make much difference in student performance. We feel that the systematic application of all the behaviors discussed in this treatment program can make an important difference in student learning. The purpose of this last section is to briefly review the teaching requests we have made and to show how each of the pieces fit together into a total program.

The predevelopment portion of the lesson begins with a brief summary and a review of the previous lesson. The review (including the checking of homework) is designed to help students maintain conceptual and skill proficiency with material that has already been presented to them. Mental computation activities follow and provide an interesting bridge for moving into the new lesson.

Next comes the development part of the lesson which is designed to help students understand the new material. Active teaching helps the student comprehend what he is learning. Too often students work on problems without a clear understanding of what they are doing and the reasons for doing it. Under such conditions, learning for most students will be filled with errors, frustration, and poor retention. If student learning is to be optimal, students must have a clear picture of what they are learning; the development phase of the lesson is designed to accomplish this understanding.

The controlled practice that occurs toward the end of the development portion of the lesson is designed to see if students are ready to begin seatwork. It simply doesn't make sense to assign seatwork to students when they are not ready for it. . .practicing errors and a frustrating experience guarantees that student interest and performance in mathematics will decline. The controlled practice part of the lesson provides a decision point for the teacher. If students generally understand the process and are able to work problems correctly, then the teacher can proceed to the seatwork portion of the lesson. If student performance on problems is relatively poor, then the development must be retaught. If students are ready to do practice work, it is foolish to delay them; similarly, if students are not ready to do development work, it is foolish to push them into it. The controlled practice part of the lessons allows the teacher to decide if it is more profitable to move to seatwork or to reteach the development portion of the lesson.

Hence, when teachers move to the seatwork portion of the lesson, students should be ready to work on their own and practice should be relatively error free. Seatwork provides an opportunity for students to practice successfully the ideas and concepts presented to them during the development portion of the lesson. If teachers consistently present an active development lesson and carefully monitor student performance during the controlled portion of the lesson, then student seatwork will be a profitable exercise in successful practice.

The seatwork part of the lesson allows students to organize their own understanding of concepts (depend less upon the teacher) and to practice skills without interruption. The seatwork part of the lesson also allows the teacher to deal with those students who have some difficulty and to correct their problems before they attempt to do homework. If teachers actively monitor student behavior when seatwork is assigned and if they quickly engage them in task behavior and maintain that involvement with appropriate accountability and alerting techniques, then the essential conditions have been created for successful practice.

Homework is a logical extension of the sequence we have discussed. During the mathematics lesson students learn in a meaningful setting. During seatwork students have a chance to practice and deal with material they understand. The homework assignment provides additional practice opportunity to further skill development and understanding.

The above aspects of the mathematics lesson combine to give the student: (1) a clear understanding of what they are learning; (2) controlled practice and reteaching as necessary to reinforce the original concepts and skills; (3) seatwork practice to increase accuracy and speed; and (4) homework assignments which allow successful practice on mastered material (distributed practice which is essential to retention).

To maintain skills it is important to build in some review. Skills not practiced and conceptual insights not reviewed from time to time tend to disappear. Even mature adults forget material and forget it rapidly. For this reason we have asked you to provide for <u>review</u> of material presented the previous week <u>each</u> <u>Monday</u> and to provide a <u>comprehensive</u> <u>review</u> every <u>fourth</u> <u>Monday</u>. Such procedures will help students to consolidate and retain their learning. Finally, we have suggested that the systematic presentation of mathematics material may facilitate student learning (i.e., initial acquisition) such that you can pick up the pace a bit and we encourage you to do so if you can. Finally, when many students experience trouble, the development portion of the lesson should be repeated and students should never be asked to do homework until they are ready to do it successfully.

The plan described above is summarized in Table 2 that follows. This table outlines the sequence and length of each lesson component in order to provide a general picture of the mathematics lesson that we are asking you to teach.

habits is minimized. If process and product questions are used appropriately, then student involvement and achievement are enhanced. If they are used inappropriately, then much instructional time is lost and errors are practiced--errors that subsequently are very hard to correct.

Request for Teaching Behavior

We feel that the presence of a few process questions in the development stage of a lesson are helpful (especially when a new principle is being introduced) because listening to a student's explanation can help teachers diagnose inappropriate assumptions, etc., that students have made. However, we believe that most of the process development can be done through teacher modeling of process explanations rather than by asking students to respond to process questions. For example, the teacher could ask, "Who can tell me what zero times seven is?" The teacher surveys the room and calls upon Bill (who may or may not have his hand up). When Bill says "zero," the teacher could respond with something like, "That's right, Bill, the answer is zero. Whenever zero is a factor, the product is always zero." By actively verbalizing and demonstrating (e.g., writing problem solutions on the board, etc.), teachers can help students to achieve process understandings in a very efficient way. Still, it is useful to ask process questions occasionally to assess student understanding. However, if asked properly, product questions can provide information that assesses the student's ability to relate ideas, transfer concepts to different situations, and understand the process sufficiently well to solve problems. Product questions can also provide all students in the class (or group) a chance to practice the computation. This is especially true when the teacher asks the question principle by saying something like "when zero is a factor the product is zero" or "zero times anything equals zero." A written example of a process question would be "7 + 3 = 10 and 3 + 7 = 10, why?" The student is expected to respond with something like "changing the position (order) of the addends (numbers) does not change the sum."

In summary, product questions are those questions that ask students to provide the right answer (how much, what, when). In contrast, process questions ask students to explain how an answer was or could be obtained (why questions).

Problem

Often when teachers think about development and conceptual work, they equate it with process questions. This is not the case. Indeed, often process questions are overused or used inappropriately. The problem with process questions is that they are sometimes ambiguous to the student (what is the teacher asking me?) and may produce an ambiguous student response even though the student understands the concept. Process questions often consume a lot of instructional time (student thinks, mentally practices the response, makes an oral response). Hence, if process questions are overused, a lot of instructional time can be wasted. If selectively used, process questions can be very valuable. For example, by asking a few process questions, teachers can see if students understand the rationale or principle upon which computational work is based and help consolidate student learning.

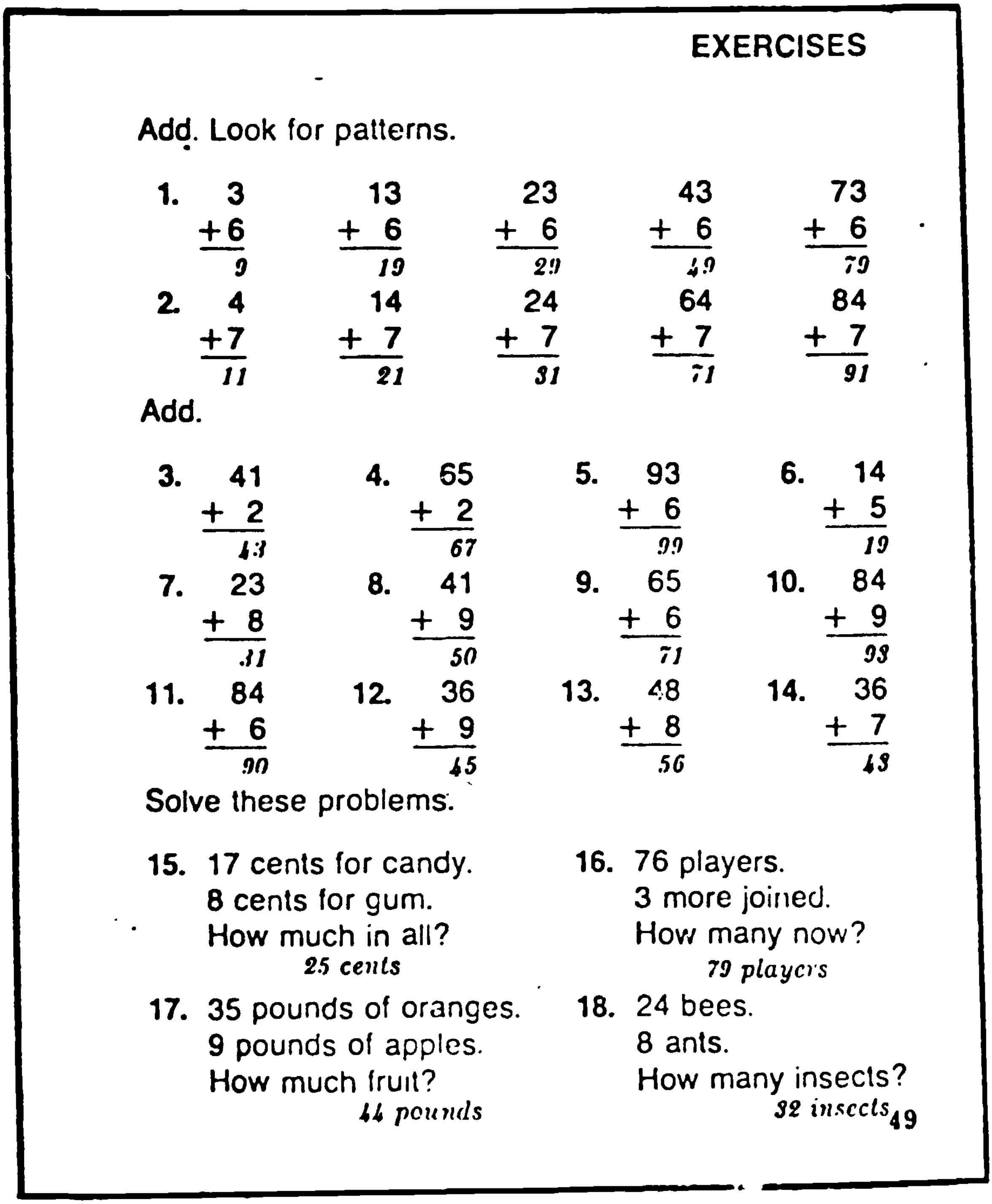
If teachers are alert to student responses, hold students accountable by asking individual students questions, and keep all students involved in the lesson, then the learning of unproductive first and then calls on a student. If a teacher names a student and then asks the question, many of the students will not perform the calculation (that's Mary's problem). Similarly, if teachers hold non-volunteers accountable on occasion, it increases the number of students who are likely to think about the problem under discussion.

Although a major goal of the development portion of the lesson is to strengthen students' conceptual understanding (why), this goal can be achieved with a heavy use of product questions. The usefulness of product questions is due to the following factors: (1) they typically elicit a quick response from the student (and quick feedback from the teacher); hence, more material can be covered in a given amount of time; (2) they provide more practice opportunity for a broader number of students; hence, a teacher's diagnosis is not limited to the responses of a few students; (3) and they help to create a "can do" attitude on the part of students (a series of quick questions that the students respond to successfully). However, it is desirable to ask process questions and enter a diagnostic cycle (reteaching) when students respond to product questions incorrectly. When students miss the same type of product questions, then it is useful to stop and review the process and ideas behind the computation. To reiterate, process questions can and do play a valuable role in successful mathematics teaching although they should not be overused.

Appendix B

Typical Homework Assignment

Reproduced below is a page from the fourth grade Holt Mathematics textbook. An appropriate homework assignment would be to assign problems #4-18 (evens). The remaining problems could be used in connection with the development or seatwork portions of the lesson. Appendix E shows how these same problems could be put in a different format and thus provide some variety in your assignments.



EXERCISES

Add. Look for patterns.

1. $\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array}$ $\quad$ $\begin{array}{r} 13 \\ +\ 6 \\ \hline 19 \end{array}$ $\quad$ $\begin{array}{r} 23 \\ +\ 6 \\ \hline 29 \end{array}$ $\quad$ $\begin{array}{r} 43 \\ +\ 6 \\ \hline 49 \end{array}$ $\quad$ $\begin{array}{r} 73 \\ +\ 6 \\ \hline 79 \end{array}$

2. $\begin{array}{r} 4 \\ +7 \\ \hline 11 \end{array}$ $\quad$ $\begin{array}{r} 14 \\ +\ 7 \\ \hline 21 \end{array}$ $\quad$ $\begin{array}{r} 24 \\ +\ 7 \\ \hline 31 \end{array}$ $\quad$ $\begin{array}{r} 64 \\ +\ 7 \\ \hline 71 \end{array}$ $\quad$ $\begin{array}{r} 84 \\ +\ 7 \\ \hline 91 \end{array}$

Add.

3. $\begin{array}{r} 41 \\ +\ 2 \\ \hline 43 \end{array}$ $\quad$ **4.** $\begin{array}{r} 65 \\ +\ 2 \\ \hline 67 \end{array}$ $\quad$ **5.** $\begin{array}{r} 93 \\ +\ 6 \\ \hline 99 \end{array}$ $\quad$ **6.** $\begin{array}{r} 14 \\ +\ 5 \\ \hline 19 \end{array}$

7. $\begin{array}{r} 23 \\ +\ 8 \\ \hline 31 \end{array}$ $\quad$ **8.** $\begin{array}{r} 41 \\ +\ 9 \\ \hline 50 \end{array}$ $\quad$ **9.** $\begin{array}{r} 65 \\ +\ 6 \\ \hline 71 \end{array}$ $\quad$ **10.** $\begin{array}{r} 84 \\ +\ 9 \\ \hline 93 \end{array}$

11. $\begin{array}{r} 84 \\ +\ 6 \\ \hline 90 \end{array}$ $\quad$ **12.** $\begin{array}{r} 36 \\ +\ 9 \\ \hline 45 \end{array}$ $\quad$ **13.** $\begin{array}{r} 48 \\ +\ 8 \\ \hline 56 \end{array}$ $\quad$ **14.** $\begin{array}{r} 36 \\ +\ 7 \\ \hline 43 \end{array}$

Solve these problems.

15. 17 cents for candy.
8 cents for gum.
How much in all?
25 cents

16. 76 players.
3 more joined.
How many now?
79 players

17. 35 pounds of oranges.
9 pounds of apples.
How much fruit?
44 pounds

18. 24 bees.
8 ants.
How many insects?
32 insects

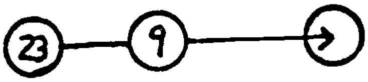
49

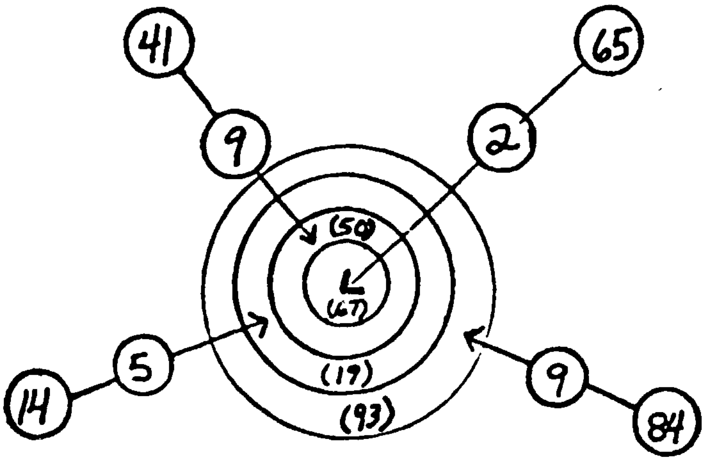
Appendix C

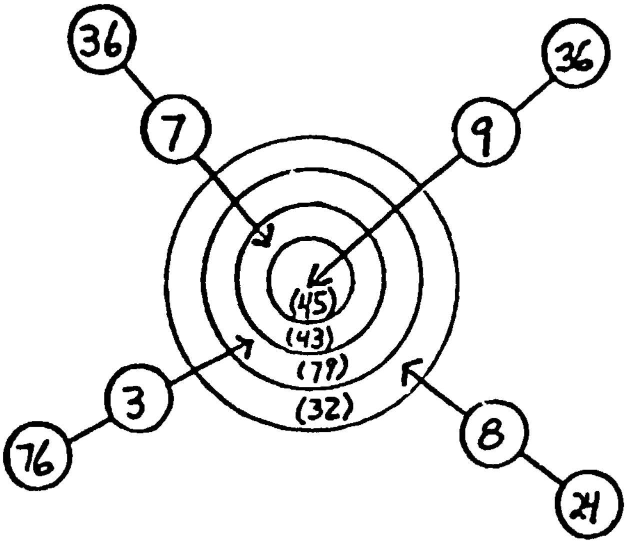
Variety in Assignments

Frequently students can be freed from the somewhat boring routine of always doing problems from the textbook as their homework assignment. The assignment shown below is an alternate to the typical row-by-row set of computation exercises found in most textbooks, yet it accomplishes the same objectives in a more interesting format. Answers for the problems are shown in parentheses.

ADD to find the missing target values. For example, 32 would be the missing value in this example:







Appendix D

Systematic Processing Errors Illustrations

A systematic processing error is an error a student consistently makes on a particular kind of problem. It is different from making random errors. Simple examples include always working addition from left to right or "borrowing" in every subtraction problem whether or not it is necessary. Other common examples are explained below.

In each of the following situations, carefully analyze the examples and try to determine the error pattern. Then check your work by reading the description of the error pattern.

Situation #1

23	34	29	38
+6	+9	+5	+4
83	124	79	78

ERROR PATTERN: In these problems the student does not add straight down a column, but rather adds the number of tens from the first number to the units from the second number. Thus, in example #1 the 2 tens are added to the 6 ones to get 8 tens.

Situation #2

53	86	95	31
-27	-39	-27	-19
34	53	72	28

ERROR PATTERN: In these problems the student does not "borrow," but rather always subtracts the smaller digit from the larger digit.

Situation #3

7	5	3	5
48	49	86	67
x59	x36	x45	x28
432	294	430	536
270	177	354	174
3132	2064	3970	2276

ERROR PATTERN: The first part of each problem, the multiplying by the ones is done correctly. However, when multiplying by the tens the crutch number recorded from the multiplying by ones is incorrectly used again. For instance, in the first example, when multiplying by the 5 tens the 7 (carried over from the 9x8) is used again when the 7 is added to 5 times 4 and the 27 is recorded.

Situation #4

4	1	3	2
26	83	38	53
x7	x5	x4	x8
422	455	242	564

ERROR PATTERN: In these problems the crutch is added before multiplying in the tens place, whereas the correct procedure is to multiply and then add the crutch. Thus, in the first example the 4 is added to the 2 and then this sum multiplied by 7. If this problem was done correctly, the 2 is multiplied by the 7 and then the 4 is added.

Situation #5

44	14	87	39
2/88	4/164	3/234	5/465
80	160	210	450
8	4	24	15
8	4	24	15

ERROR PATTERN: These problems are worked correctly except that the quotient figures are written from right to left. Consider the third example, there are 7 threes in 23, but the 7 is recorded at the extreme right, rather than above the 3.

Situation #6

32r3	78r2	94r2
9/2721	6/4250	6/5426
27	42	54
21	50	26
18	48	24
3	2	2

ERROR PATTERN: In these problems, whenever the students brings down and cannot divide, he brings down again but forgets to record a zero in the quotient.

Appendix E

Letter to Parents

August 25, 1977

Dear Parent(s):

As part of the fourth grade math instructional program this year, I will be regularly assigning some work for the students to complete at home. It should take your son or daughter about fifteen minutes to complete this homework. If you find that it regularly takes considerably longer for him/her to finish this assignment or the assignment causes other difficulties, please let me know in that I may be assigning too many or too difficult problems.

Programs in other school districts, educational research, and common sense indicate that the more a student practices important math concepts and problems, the more proficient he becomes in essential math skills. I view homework as an opportunity for the student to practice the concepts and skills that he/she has learned in class. I hope that you will encourage your son or daughter to complete every assignment to the best of his/her ability. Parental support is very helpful. Thank you for your cooperation in this matter.

Sincerely,

Appendix F

Teaching Groups in Schools Using a Departmental Organization

The emphasis thus far has been placed upon teaching mathematics to the class as a unit. We feel that many of the principles presented (the importance of development, the use of controlled practice and seatwork, accountability, etc.) will transfer to classrooms in which teachers are teaching groups of students. In applying these principles to a group situation, teachers will have to adjust them to their teaching situation.

In general, we are not enthusiastic about the use of two or more groups to teach mathematics. Three recent and major research projects have shown that third, fourth, and fifth grade students appear to benefit more from whole class instruction than they do from individual or group instruction. Although the precise reasons for these differences are unknown, we suspect that students learn less in group and individual settings because they have less direct developmental work with the teacher. Also, the extra transitions (teachers moving from group to group) probably results in the loss of time that could have been used for instructional purposes. Furthermore, student work is probably less effective when the teacher is not available to supervise work.

If the differences between groups are not great, we strongly recommend that the class be taught as a whole class. However, we understand that sometimes the differences between students in a given classroom are so great that grouping is a practical necessity.

If grouping is necessary, you should attempt to limit yourself to only twc groups because the transition and supervision problems that accompany the use of more than two groups are normally very difficult to justify.

Since teaching circumstances are so varied (sometimes the difference between two groups is moderate but in other classrooms there are vast differences between the two groups), it is impossible for us to describe a plan that would be best in all situations. Still, there are a few key things that we would like to emphasize.

First, whenever possible, we think it will be useful for you to teach the class as a group. Students learn a great deal from teacher illustrations and explanations. Perhaps the easiest way to do this in a group situation is by holding common reviews from time to time. The review might be a short-term review for the lowest group and a long-term review for the highest group.

An especially good way to conduct a common review is through the use of mental computation problems. We strongly recommend that each day of the week but Monday you use the first ten minutes of the class for review with mental computation problems. As we have noted earlier in the handbook, we feel that mental computation problems are a very important addition to an instructional program.

Second, we would like you to set aside each Monday for a review session. After spending the first five minutes on mental computation, review ideas and skills that are needed by both groups. Then involve one group in a seatwork review, then begin the developmental review with the other group. Roughly half way through the period reverse the roles; give group two a seatwork review assignment and begin an oral review with group one.

To maximize the value of this review, a homework assignment containing review problems should be given the previous Thursday. Your analysis of these papers should suggest the topics and skills that should receive emphasis in the Monday review. Besides the homework assignment each Thursday, we request that you assign homework three other days per week. Remember that these assignments are to provide brief, successful practice.

The third request is that you maximize the amount of development time for each group. The exact amount to be given to each group will necessarily vary depending on the topic being considered and the group itself; however, the importance of development work for both groups cannot be overemphasized. As you do the development work, remember the guidelines previously discussed. For instance, teacher explanations and illustrations are important, especially initially. Also, process explanations are very important and often times are related to efficient use of limited instructional time.

Finally, we ask that you implement other recommendations as regularly and consistently as you can. Little things are important (e.g., getting all students started on seatwork before doing other instructional tasks) and we hope you will carefully review the ideas presented in the handbook with an eye toward applying them in your classroom.

WEEKLY LESSON TIME TABLE

Monday	Tuesday	Wednesday	Thursday	Friday
Weekly Review (20 Min.) * * * *	Homework, * Review, Mental Computation (5 Min.)	Homework, * Review, Mental Computation (5 Min.)	Homework, * Review, Mental Computation (5 Min.)	Homework, * Review, Mental Computation (5 Min.)
* Development (10 Min.) *	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *
Seat Work (10 Min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)
Lesson Conclusion & Homework Assign. (2 Min Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion & Homework Assign. (2 Min. Max)	Lesson Conclusion (2 Min Max.)

* LESSON TIME TABLE (4th WEEK)

	Tuesday	Wednesday	Thursday	Friday
Monthly Review (45 min.) * * * * *	* Review, * Mental Computation (8 Min)	Homework, * Review, * Mental Computation (8 Min)	Homework, * Review, * Mental Computation (8 Min)	Homework, * Review, * Mental Computation (8 Min)
	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *	Developmental (20 min.) * *
	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)	Seat Work (15 min.)
	Lesson Conclusion & Homework Assign.(2 Min.Max)	Lesson Conclusion & Homework Assign.(2 Min.Max)	Lesson Conclusion & Homework Assign.(2 Min.Max)	Lesson Conclusion (2 Min. Max.)

Mathematics Content Test*

*This test was used to measure student achievement in Field Experiment I: Fourth Grade Sample.

Name ____________________

Mathematics Content Test

Teacher ____________________

School ____________________

Add

1. $$\begin{array}{r} 9 \\ 4 \\ 7 \\ 3 \\ +\ 5 \\ \hline \end{array}$$

2. $$\begin{array}{r} 58 \\ +\ 27 \\ \hline \end{array}$$

3. $$\begin{array}{r} 65 \\ +\ 34 \\ \hline \end{array}$$

4. 17 + 135 + 4 = ________

5. 9999 + 1 = ________

Subtract

6. $$\begin{array}{r} 190 \\ -\ 63 \\ \hline \end{array}$$

7. $$\begin{array}{r} 506 \\ -\ 127 \\ \hline \end{array}$$

8. $$\begin{array}{r} 65 \\ -\ 36 \\ \hline \end{array}$$

9. $$\begin{array}{r} 1476 \\ -\ 539 \\ \hline \end{array}$$

Multiply

10. 8 x 7 = ________

11. 6 x 9 = ________

12. $$\begin{array}{r} 21 \\ \times\ 4 \\ \hline \end{array}$$

13. $$\begin{array}{r} 82 \\ \times\ 3 \\ \hline \end{array}$$

14. $$\begin{array}{r} 76 \\ \times\ 7 \\ \hline \end{array}$$

Divide

15. 12 ÷ 3 = ________

16. 28 ÷ 7 = ________

Fill in the correct answer in the blank for each question.

Example: d 1. 2 + 3 = ______

a) 1
b) 2
c) 3
d) 5
e) 6

_____ 17. If 1 x ☐ = 8, the ☐ equals

a) 0
b) 1
c) 8
d) 9
e) 7

_____ 18. If you have $1 and buy a pencil for 25¢, what change do you get back?

a) 1 quarter
b) 1 quarter and 2 dimes
c) 1 half dollar and 1 dime
d) 3 quarters
e) 5 nickles

_____ 19. The "6" in 465 stands for

a) 6
b) 60
c) 65
d) 600
e) 6000

_____ 20. Two hundred seven is

a) 2007
b) 207
c) 702
d) 7002
e) 20070

_____ 21. 527 is the same as

a) 500 + 72
b) 500 + 20 + 7
c) 700 + 20 + 5
d) 5 + 2 + 7
e) 700 + 200 + [illegible]

_____ 22. 1990 may be written as

a) nineteen thousand ninety
b) one thousand nine hundred nine
c) one thousand nine hundred
d) one thousand nine hundred ninety
e) one nine nineteen

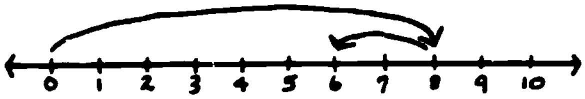
_____ 23. The largest three digit number with a "2" as one of its digits is

a) 299
b) 929
c) 292
d) 999
e) 992

_____ 24 In $165.27 the "1" stands for

a) one dime
b) one dollar
c) ten dollars
d) one hundred dollars
e) one thousand dollars

_____ 25.



This number line picture shows

a) 6 + 2 = 8
b) 8 - 2 = 6
c) 8 + 2 = 10
d) 6 - 2 = 8
e) 8 - 6 = 2

_____ 26. Which number makes 8 + 3 = 6 + □ true?

a) 8
b) 3
c) 5
d) 11
e) 17

_____ 27. Which number pairs make □ - △ = 6 true?

a) 7 and 1
b) 2 and 3
c) 4 and 2
d) 5 and 5
e) 12 and 2

_____ 28. Some coins are in a box. Doug put 73 coins in the box. Now there are 151 coins in the box. Which number sentence describes this story?

a) 73 < 151
b) 73 + 151 = □
c) □ + 73 = 151
d) 151 + □ = 73
e) □ - 151 = 73

_____ 29. Toby's mother gave him $2.00. Toby bought a movie ticket for $1.25 and a box of popcorn for 40¢ and a soda for 30¢. How much money did he spend?

a) 75¢
b) $1.30
c) $1.95
d) 5¢
e) 45¢

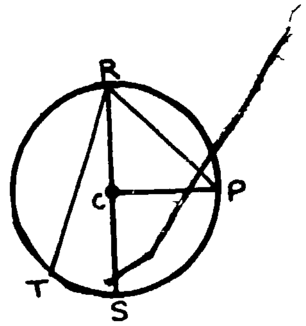
_____ 30. 348 was subtracted from the covered number. The covered number is

$$\begin{array}{r} \boxed{} \\ -\ 348 \\ \hline 380 \end{array}$$

a) 48
b) 628
c) 728
d) 720
e) 32

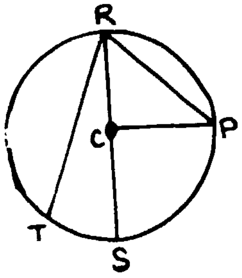
_____ 31. The radius of this circle is

a) P
b) $\overline{CP}$
c) C
d) $\overline{RS}$
e) $\overline{RP}$



_____ 32. The diameter of this circle is

a) C
b) $\overline{CP}$
c) $\overline{RT}$
d) $\overline{RS}$
e) $\overline{RP}$



_____ 33. Look at the table:

N = _____

a) 3
b) 6
c) 8
d) 9
e) 12

+3

Input	Output
0	3
4	7
7	10
6	N

_____ 34. Look at the table:

If 9 is the input, the output is:

a) 0
b) 1
c) 8
d) 10
e) 3

Input	Output
7	1
2	0
6	0
3	1
4	0
9	

_____ 35. 4 x 6 means

a) 6 + 6 + 6 + 6
b) 1 + 2 + 3 + 4 + 5 + 6
c) 6 x 6 x 6 x 6
d) 4 x 4 x 4 x 4
e) 6 + 6 + 6 + 6 + 6 + 6

_____ 36. Zero times N is equal to

a) zero
b) N
c) any number
d) 1
e) no solution

_____ 37. Which is not a factor of 12?

a) 1
b) 2
c) 3
d) 4
e) 5

_____ 38. There are 6 cans of balls. Each can has 3 balls. How may balls are there?

a) 2
b) 3
c) 9
d) 18
e) 24

___ 39. Scott had 40 cents. He lost a dime and spent the rest on pieces of candy that cost 5¢ each. How many pieces of candy did he buy?

a) 5
b) 6
c) 8
d) 10
e) 11

_____ 40. Tom planted 12 trees in 3 rows. He planted the same number of trees in each row. How many trees are in each row?

a) 15
b) 9
c) 36
d) 4
e) 12

Appendix C

Student Attitude Inventory*

---

*This instrument along with achievement data was used to cluster students into typologies (questions 1-51). This instrument was also used to assess students' global attitudes (questions 52-61), and these questions were used as an outcome proxy for students' affective reactions to mathematics instruction. The instrument was used in both field experiment I and II.

Name ____________________

Boy ________ Girl _______

Teacher's Name ______________

School's Name ______________

ATTITUDE INVENTORY

Directions:

Read each statement and decide if you usually agree or disagree with that statement. If you agree, circle the letter T for True next to the question. If you disagree, circle the letter F for False next to the question.

Please answer every question. Be sure you write your name, your sex, your teacher's name, and your school's name on this sheet. If you have a question, ask your teacher for help.

T F 1. I like to work my math problems with several other students.

T F 2. I always like to choose what math problems to do.

T F 3. I get into trouble in school about once every week.

T F 4. I do not like to work alone.

T F 5. I work harder on math problems that I know will be checked.

T F 6. I need to learn math.

T F 7. I need to be reminded often to get my math assignment done.

T F 8. I want to get good math grades just to show my friends.

T F 9. I sometimes forget to do my assignments.

T F 10. Practicing new math problems with my teacher is a waste of time.

T F 11. I do not need any practice work before I start work on new math problems.

T F 12. I can always remember what I am told to do.

T F 13. I usually finish the easy math problems but not the hard ones.

T F 14. I like my teacher to work a few example problems before I have to do a new problem by myself.

T F 15. I like to learn about math best by listening to my teacher.

T F 16. I will get good math grades this year.

T F 17. I am not good at math games.

T F 18. I usually finish my math assignments.

T F 19. I am good at working math problems in my head.

T F 20. I get into trouble in school about once every week.

T F 21. I like to do math problems in my own way.

T F 22. My teacher really wants me to get good grades in math.

T F 23. I usually do not finish my math assignment.

T F 24. Getting good grades in math is really important to me.

T F 25. I am good at working math problems in my head.

T F 26. I sometimes lose my books and papers.

T F 27. I like to have my parents help me with my math problems.

T F 28. I like to work math problems by myself.

T F 29. I like to learn about math best by reading my book.

T F 30. I always like to choose what math problems to do.

TURN THE PAGE OVER

T F 31. I like to figure out how to work a new math problem without my teacher's help.

T F 32. I will need math next year.

T F 33. Before I start working new math problems, I like to make sure I can do them.

T F 34. I like to learn about math best by listening to my teacher.

T F 35. I do not like to check my math problems.

T F 36. I like to know if a math assignment will be checked.

T F 37. It is not that important to know math.

T F 38. If I have a question in my math class, I ask the teacher right away.

T F 39. Other subjects are more important than math.

T F 40. My math teacher last year yelled at me a lot.

T F 41. I want to get good grades just for myself.

T F 42. If I find out why I made a mistake on a math problem, I usually do not miss that kind of problem again.

T F 43. I like to be able to choose what our class does in math.

T F 44. I like to have my teacher explain how to work a new math problem.

T F 45. I will get good math grades this year.

T F 46. I do not like to check my math problems.

T F 47. Getting good grades in math is really important to me.

T F 48. If I know my math problems will not be checked, I do not work on them very much.

T F 49. I like to check my math problems to see which problems I missed.

T F 50. I work harder if I know my math problems will be checked.

T F 51. I like to work math problems in my head.

Answer the following questions by circling . . .

1 if you want to answer <u>always</u>
2 if you want to answer <u>most of the time</u>
3 if you want to answer <u>sometimes</u>
4 if you want to answer <u>never</u>

Always	Most of the time	Sometimes	Never		
1	2	3	4	52.	Do you like to be in this class?
1	2	3	4	53.	Do you have much fun in this class?
1	2	3	4	54.	Do most of your close friends like the teacher?
1	2	3	4	55.	Does the teacher help you enough?
1	2	3	4	56.	Do you learn a lot in this class?
1	2	3	4	57.	Do you ever feel like staying away from this class?
1	2	3	4	58.	Are you proud to be in this class?
1	2	3	4	59.	Do you always do your best in this class?
1	2	3	4	60.	Do you talk in class discussions in this class?
1	2	3	4	61.	Are most of the students in this class friendly to you?

T F 31. I like to figure out how to work a new math problem without my teacher's help.

T F 32. I will need math next year.

T F 33. Before I start working new math problems, I like to make sure I can do them.

T F 34. I like to learn about math best by listening to my teacher.

T F 35. I do not like to check my math problems.

T F 36. I like to know if a math assignment will be checked.

T F 37. It is not that important to know math.

T F 38. If I have a question in my math class, I ask the teacher right away.

T F 39. Other subjects are more important than math.

T F 40. My math teacher last year yelled at me a lot.

T F 41. I want to get good grades just for myself.

F 42. If I find out why I made a mistake on a math problem, I usually do not miss that kind of problem again.

T F 43. I like to be able to choose what our class does in math.

T F 44. I like to have my teacher explain how to work a new math problem.

T F 45. I will get good math grades this year.

T F 46. I do not like to check my math problems.

T F 47. Getting good grades in math is really important to me.

T F 48. If I know my math problems will not be checked, I do not work on them very much.

T F 49. I like to check my math problems to see which problems I missed.

T F 50. I work harder if I know my math problems will be checked.

T F 51. I like to work math problems in my head.

Answer the following questions by circling . . .

1 if you want to answer always
2 if you want to answer most of the time
3 if you want to answer sometimes
4 if you want to answer never

Always	Most of the time	Sometimes	Never		
1	2	3	4	52.	Do you like to be in this class?
1	2	3	4	53.	Do you have much fun in this class?
1	2	3	4	54.	Do most of your close friends like the teacher?
1	2	3	4	55.	Does the teacher help you enough?
1	2	3	4	56.	Do you learn a lot in this class?
1	2	3	4	57.	Do you ever feel like staying away from this class?
1	2	3	4	58.	Are you proud to be in this class?
1	2	3	4	59.	Do you always do your best in this class?
1	2	3	4	60.	Do you talk in class discussions in this class?
1	2	3	4	61.	Are most of the students in this class friendly to you?

Appendix D

Teacher Typology Instrument*

---

*This instrument was used to cluster teachers into typologies for field experiment I.

Name ______________________________

School ______________________________

Part I CLASSROOM PROCEDURES

Please check the point within each of the following scales which most accurately describes your math class. (If you are teaching math for the first time or your present situation is very different from pervious years, please respond as you anticipate your class will be like this year.) Please respond according to what actually happens, not what you think should happen, or what you would like to have happen. There are no right or wrong answers. Please answer all the questions

1. Amount of testing

I give a math test about once every three weeks. ____ 1

____ 2

____ 3

____ 4

I give a math test at least once every week. ____ 5

2. Emphasis on enjoyment

Very strong explicit emphasis is put on having a pleasant, happy and friendly time in my math class. ____ 1

____ 2

____ 3

____ 4

Although having an enjoyable time in math is important there is little explicit emphasis on having a pleasant, happy and friendly time in my math class. ____ 5

3. Task emphasis

The importance of getting work done on time and done well is frequently stressed in my class. ____ 1

____ 2

____ 3

____ 4

Students can turn in their work when they are finished. There are no strict deadlines. ____ 5

4. Organization of tasks

Most learning tasks in this class have a step-by-step organization and sequence. ____ 1

____ 2

____ 3

____ 4

Most of the learning tasks in this class are "open-ended" or discovery oriented. ____ 5

5. Commonality

Math learning objectives are the same for all students in the class. ____ 1

____ 2

____ 3

____ 4

Math learning objectives are set for each student separately. ____ 5

6. Problems

Students are encouraged to get a lot of help with their math problems. ____ 1

____ 2

____ 3

____ 4

Students are encouraged to solve their math problems without a lot of teacher help. ____ 5

7. Help with work

Almost all help is initiated by students asking for it. ____ 1

____ 2

____ 3

____ 4

Almost all help is initiated by my seeing the need for it. ____ 5

8. Plan changing

Daily lesson plans are stable, not usually subject to change. ____ 1

____ 2

____ 3

____ 4

Daily lesson plans are changed very frequently. ____ 5

9. Different activities

Many different activities are almost always going on simultaneously during math class. ____ 1

____ 2

____ 3

____ 4

Almost all the time the students are all engaged in the same activity during math class. ____ 5

10. Evaluation standards

The same standards are used for all students. ____ 1

____ 2

____ 3

____ 4

Different standards are used for each individual. ____ 5

11. Evaluation procedures

Evaluation procedures are the same for all students in the class. ____ 1

____ 2

____ 3

____ 4

Evaluation procedures are different for each student. ____ 5

12. Oral presentation

On a typical day, I give an oral presentation for three-fourths of the math time. ____ 1

____ 2

____ 3

____ 4

I almost never give an oral math presentation. ____ 5

13. Peer help

Students frequently help one another during math class. ____ 1

____ 2

____ 3

____ 4

Students seldom help one another during math class. ____ 5

14. Instructional direction

On a typical day, I direct my attention to the math class as a group three-fourths of the time or more. ____ 1

____ 2

____ 3

____ 4

On a typical day, I teach or direct my attention to individual students (or small groups) three-fourths of the time or more. ____ 5

15. Approaches to learning

I encourage students to solve a given math problem the way I have demonstrated. ____ 1

____ 2

____ 3

____ 4

I encourage students to solve math problems any way that they desire. ____ 5

16. Conceptualization

I use conceptual ideas, such as the commutative and associative properties of addition and multiplication to teach math. ____ 1

____ 2

____ 3

____ 4

I teach math from a more practical, less theoretical point of view. ____ 5

17. Inductive-deductive approach

I present a math concepts first then illustrate that concept by working several problems (deductive). ____ 1
____ 2
____ 3
____ 4
I present the class with a series of similar problems, then together we develop concepts and methods of solving the problems (inductive). ____ 5

18. Curriculum organization

The curriculum is organized such that certain topics are repeated (but in more depth) on a regular basis throughout the year. ____ 1
____ 2
____ 3
____ 4
Once a certain topic is covered, that same topic is not covered again except during reviews. ____ 5

19. Transfer

A good deal of time (1/3) is spent trying to teach students to see similarities and differences between new and previously learned math ideas. ____ 1
____ 2
____ 3
____ 4
New topics are generally introduced with limited reference to previously learned math ideas. ____ 5

0. Practicality

Math is taught strictly as a practical subject. ____ 1
____ 2
____ 3
____ 4
Math is taught with emphasis on theory. ____ 5

21. Predictability of student pace

I can usually predict where my students will be in the math textbook in January. ____ 1
____ 2
____ 3
____ 4
I can't usually predict where my students will be in the math textbook in January. ____ 5

22. Student choice

Students have a choice as to what problems or exercises they can do for math practice. ____ 1
____ 2
____ 3
____ 4
I decide what problems the students will do for math practice. ____ 5

23. Pre-assessment

I know a good deal about my students' math abilities before or shortly after the school year starts. ____ 1
____ 2
____ 3
____ 4
It usually takes about 9 weeks before I know about my students' math abilities. ____ 5

24. Motivation

All students are rewarded in the same manner for good work. ____ 1
____ 2
____ 3
____ 4
Students are rewarded in different ways for good work. ____ 5

25. Mobility

Students seldom stay in their seats for the major part of the math lesson. ____ 1
____ 2
____ 3
____ 4
Students are generally in the same seat for the math period. ____ 5

26. Math emphasis

In my math class I emphasize the basic computational skills.

____ 1
____ 2
____ 3
____ 4
____ 5

In my math class I emphasize understanding the concepts underlying mathematics.

27. Study places

Each child works mostly at his own desk during math lesson.

____ 1
____ 2
____ 3
____ 4
____ 5

All math work is divided among a variety of places (centers) in and out of the classroom, with no "home base" seat.

28. Instructional changes

I seldom change my approach throughout the semester (such as lecture-discussion, discovery, etc.).

____ 1
____ 2
____ 3
____ 4
____ 5

I change my approach frequently (from discovery to direct telling or from another method to something different) throughout the semester.

29. Changes

The arrangement of furniture and equipment has changed every week or so, this year.

____ 1
____ 2
____ 3
____ 4
____ 5

The arrangement has changed once or not at all.

30. Rule enforcement

I enforce the classroom rules.

____ 1
____ 2
____ 3
____ 4
____ 5

Students enforce classroom rules.

31. Rule making

I make the classroom rules.

____ 1
____ 2
____ 3
____ 4
____ 5

Students make the classroom rules.

32. Reinforcement

I generally use concrete reinforcers such as stars.

____ 1
____ 2
____ 3
____ 4
____ 5

I generally use verbal praise as reinforcement.

33. Affective objectives

Appreciation of math is of high importance.

____ 1
____ 2
____ 3
____ 4
____ 5

Appreciation of math is not vital.

34. Emphasis on consumer math

Heavy emphasis is placed on consumer math.

____ 1
____ 2
____ 3
____ 4
____ 5

Little emphasis is placed on consumer math.

35. Sex differences

Boys are better in math skills. ____ 1

____ 2

____ 3

____ 4

Girls are better in math skills. ____ 5

36. Divergence from planned lesson

I try hard to stick to the lesson planned for that day during math period. ____ 1

____ 2

____ 3

____ 4

If a student raises an interesting question during the math lesson, I may change my whole lesson plan for that day and pursue the student's question. ____ 5

37. Emphasis on comprehension

Understanding the methodology of why a given method gives the correct answer is important. ____ 1

____ 2

____ 3

____ 4

Understanding the methodology is not critical. ____ 5

38. Exploration

Most of the time is spent drilling the students in math fundamentals. ____ 1

____ 2

____ 3

____ 4

Most of the time is spent exploring math-related topics. ____ 5

39. Pacing

Most math class activities require students to work at about the same pace; topics are expected to be mastered by specific times during the year. ____ 1

____ 2

____ 3

____ 4

Each student works at his or her own pace, with no timing restrictions. ____ 5

Part II TEACHER OPINION

Select the appropriate choice for each statement.

A = Agree

B = Somewhat agree

C = Undecided

D = Somewhat disagree

E = Disagree

40. ____ Teaching math makes me feel secure and at the same time it is stimulating.

41. ____ Teaching multiplication and division is more enjoyable than teaching geometry or fractions.

42. ____ In terms of teaching skill, math, in comparison to other subjects and activities I teach, is a personal strength.

43. ____ Math, in comparison to other subjects and activities I direct, is one of my lesser interests.

44. ____ Math is one of the few areas in which poor readers can do well.

45. ____ My basic function as a math teacher is to convey my knowledge of math to the students in a direct manner.

46. ____ Boys in my class have more interest in math than girls do.

47. ____ Without the assistance of a special teacher (i.e., a specialist in mathematics), the classroom teacher should not be regarded as responsible for the limited progress made by the slowest pupil.

48. ____ Individualization of math instruction seems impractical for actual classroom application.

49. ____ If resources were available, I would prefer total individualization of math instruction rather than group or whole class instruction.

50. ____ I feel I have a good sound background in mathematics.

PART III COMPLETION

51. Most of my students complete ___% or more of all the problems in their textbook associated with each lesson that is taught.

52. As of today, I have ____ students that are discipline problems.

53. When you use practice exercises to reinforce math skills, approximately what percentage are:

____ written work to be done in class

____ written work to be done at home

____ oral work or chalkboard work

____ games or puzzles that illustrate the concept

____ other

100%

54. When some students do poorly on tests or otherwise indicate that they have not understood a unit in math, what are three (3) things you do to improve the situation.

A. ____

B. ____

C. ____

55. On the average I spend about ____ minutes a day developing math concepts and skills and have the children practice these skills through homework and problems ____ minutes a day.

56. This year I teach math ____ days a week for an average of ____ minutes a day.

57. My students should have the opportunity to select and use math materials on a nonstructured basis at least ____ times a week.

58. I assign math work to be done at home about ____ times a week.

59. Sometimes students have difficulty solving story problems. Briefly describe how you help your students solve story problems. (Example. I have pupils make drawings or diagrams to help clarify the problem.)

60. When you correct students' papers, how would you describe the type of marks you most often put on the students' papers? (Example: I mark the problems that are incorrect and provide the correct answer.)

61. How often do you review material already covered? (Examples: At the end of the chapter, before vacations, etc.)

62. When I assign students math story problems, I go over the vocabulary in the problem and point out what new words mean about ____% of the time.

63. Before I start presenting the math lesson for the day, I spend about ____ minutes going over the previous lesson.

64. The students in my class make use of or manipulate concrete educational equipment (such as blocks, compasses, rulers, etc.) to aid in understanding math concepts about ____ times a week.

65. I move the students into new material when I feel that all but about ____% of the students are ready.

66. During the year when you start a new math unit that is especially difficult, what do you do differently? (Example: I present the material more slowly than normal and I assure the students they can handle the new material.)

67. Given my present objective and methods of teaching, I feel the ideal class size in math would be ____(number) students and that the maximum number I could teach and still do a good job would be ____(number) students.

68. How many years (including this year) have you taught math to fourth grade students?

_____years

69. How many years (including this year) have you taught in an elementary school setting?

_____years

70. How many hours of college credit in math have you completed (including math methods courses)?

_____hours

71. How many hours of graduate college credit (including courses you may presently be enrolled in) have you completed beyond the B.A. or B.S. degree?

_____hours

72. When math assignments are checked, what percentage would fall into the following categories?

_____ I check the students' papers.

_____ An aide checks the students' papers.

_____ Students check their own work.

_____ Students check each other's work.

100%

73. If you had your choice, what type of ability in math would you prefer to teach? (Check one.)

_____ mostly high ability

_____ mostly average ability

_____ mostly low ability

_____ a mixture of abilities

Follow-Up Letter Mailed to
Experimental Teachers in Field Experiment I

UNIVERSITY OF MISSOURI-COLUMBIA

Graduate School

Center for Research in Social Behavior

111 East Stewart Road
Columbia, Missouri 65201
Telephone (314) 882-7888

May 4, 1978

Dear Participating Teacher:

Your help and cooperation in the project was excellent. Apparently the program has worked well in most classrooms. Enclosed for your information is our first report. Later we'll be able to provide you with a more detailed report.

We hope that the program proved to be helpful to you in organizing and presenting mathematics lessons.

We would like to ask you a few questions about the program. However, since we promised not to bother you again for information, we are prepared to pay you an additional $5 for your time in completing the form (wish it could be more).

You may have liked or disliked the program in its entirety or there may have been certain parts that seemed especially good or bad. The following set of questions are designed to solicit your reactions.

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

__ my reaction to the entire program
__ my reaction to the review phase of the lesson
__ my reaction to the development stage of the lesson
__ my reaction to the seatwork stage of the lesson
__ my reaction to the homework stage of the lesson
__ my reaction to the use of mental computation problems
__ my reaction to the increased pace suggestion

II. Classroom behavior after the project

Some teachers in the project will decide to continue to use the program; other teachers will decide to discontinue it; and yet others will continue some parts of the program but not other aspects of it. Would you please describe your current practice by choosing the appropriate alternative for each question that follows.

I conduct a short review

__ 4 or 5 times a week
__ 3 times a week
__ 2 times a week
__ 1 time a week
__ not at all

I include mental computation questions in my mathematics lessons

__ 4 or 5 times a week
__ 3 times a week
__ 2 times a week
__ 1 time a week
__ not at all

I conduct a lesson with a large development stage

__ 4 or 5 times a week
__ 3 times a week
__ 2 times a week
__ 1 time a week
__ not at all

On the average I spend

__ minutes on development in each of my mathematics lessons

In an average week I now assign homework

__ 4 or more nights a week
__ 3 times a week
__ 2 times a week
__ 1 time a week
__ not at all

I still conduct an expanded review weekly

__ yes
__ no
__ not at all

I still conduct a large review monthly

__ yes
__ no

The best part of the program in my opinion was:

The weakest or most confusing part of the program was:

Was the recommendation and explanation of development clear? Please explain:

Sincerely yours,

Thomas L. Good

Thomas L. Good
Professor of Education

Douglas A. Grouws

Douglas A. Grouws
Associate Professor

TLG:DAG/sjk
Enclosures

Verbal Problem Solving Teachers' Manual*

Principal Investigators:

Douglas A. Grouws
Thomas L. Good

September, 1978

---

*This manual was used in field experiment II along with the manual developed for field experiment I (see Appendix A).

Verbal Problem Solving

Introduction

There are many reasons for teaching students mathematics and different people stress different reasons as they testify to its importance. On one thing, however, there is universal agreement: mathematical problem solving is of paramount importance! This agreement stems from the fact that many real world problems are most easily solved by expressing and treating them mathematically. An important step toward developing problem solving ability in students is to help them gain competence in solving verbal problems. By verbal problems we mean those problems which are commonly referred to as "story problems" or "word problems." These are the problems that are traditionally found in contemporary student mathematics textbooks.

In the past, instruction on verbal problem solving has amounted to little more than the teacher solving a few sample problems in front of the class and then asking students to solve similar problems on their own. Usually such instruction is grossly inadequate; students do not understand the assignment and are not able to do the problems successfully. Because of such poor presentation many students develop a permanent dislike for these problems. This situation is particularly unfortunate because research has shown that there are a number of instructional strategies that can be used to improve student problem solving performance significantly. The remainder of this manual is devoted to describing techniques that can be incorporated successfully into daily instructional practice. When these techniques are used systematically we believe that students' ability to solve verbal problems will show steady progress.

In particular, it is important to include some work on verbal problem solving each day. Too often verbal problem solving is taught only three or four times a year as a special topic. However, it is only the day to day brief but systematic exposure that will allow students to become proficient in solving mathematical problems.

Problems Without Numbers

The use of problems without numbers is a very effective instructional technique for improving verbal problem solving performance. It provides students an opportunity to gain insight into the problem solving process by avoiding the use of numbers and thus the need to perform any computation whatever.

Example

To illustrate the method consider the following typical problem:

> Two classes sold 100 football game tickets.
> One class sold 27 tickets.
> How many did the other class sell?
> (Holt School Mathematics, Grade 6, p. 32)

This problem can easily be rephrased so that it is a problem without numbers:

> Our class and Mrs. Smith's class sold tickets.
> We know how many tickets were sold altogether
> and how many tickets our class sold.
> How many tickets did Mrs. Smith's class sell?

The teacher presents only the problem without numbers and asks the class how to solve it. An appropriate answer might be something like this: "I'd subtract how many tickets we sold from the total number of tickets to find how many tickets Mrs. Smith's class sold." Time permitting, the teacher should follow-up with another problem without numbers or occasionally consider the same problem only with the numbers included.

Rationale

The specific reasons why this technique is effective are difficult to isolate. One reason for its effectiveness may be that it causes students to focus exclusively on the method needed to solve a problem without any numerical or computational distractions. Many teachers realize that too frequently students begin doing the computation before they have really thought through the problem. In fact, some students have been known to begin computing before they have read the entire problem! Avoiding the use of numbers tends to resolve these kinds of problems. Since the strategy does not require computation, students can be exposed to a substantial number and variety of verbal problems in a short period of time.

Implementation

This technique should be used frequently as part of a comprehensive effort to improve verbal problem solving skills. It seems especially effective if teachers create the problems to be used by recasting verbal problems found in the student textbook. It is also helpful if the problems are written down and ready for presentation prior to the beginning of the math period. This allows efficient use of the available instructional time.

Writing Verbal Problems

Research has shown that when students create and write verbal problems, their problem solving ability improves. Certainly a comprehension of what constitutes a problem is necessary in order to succeed at writing problems, and this is turn may be a vital component in learning to solve verbal problems.

Example

There are a variety of interesting formats that a teacher may use when having students write verbal problems. One method is to supply data and ask students to make up their own problems based on this information. For example, the data might consist of a football team roster like the one below.

Number	Player	Position	Year	Height	Weight
11	Anderson, Bill	Quarterback	9th	5'8"	155
24	Baker, Burt	End	8th	5'7"	140
17	Brunson, Jim	Quarterback	8th	5'5"	135
..	....	....	...	....	...
..	....	....	...	....	...

To illustrate the kinds of problems that may be written, the teacher could supply examples like the following which range from the easy to the complex:

> Bill Anderson and Jim Brunson are both quarterbacks
> on the Memorial Junior High School football team.
> Bill weighs 155 pounds and Jim weighs 135 pounds.
> Bill weighs how much more than Jim?
>
> There are three quarterbacks on the Memorial team.
> Jim weighs 135, Bill 155, and Sam 130.
> What is the average weight of the quarterbacks?
>
> All 33 players on the Memorial team are going on the
> bus to the away game with Fulton Junior High.
> Highway 24 is the shortest way to Fulton, but the
> Mason Creek bridge on this route limits loads to
> less than five tons.
> The bus with the driver weighs 3200 pounds.
> Will the bus loaded with the players be too heavy
> to use Highway 24?

After students have had some experience writing problems, the teacher may allow them to make up problems by supplying their own data from situations that are of interest to them. Placing some restrictions on the problems to be written will help to keep this activity consistent with the operations and kinds of numbers currently being studied. For example, a teacher might want to restrict the problems written to those that can be solved by division of whole numbers or to those involving addition and subtraction of fractions.

Rationale

The value of having students create verbal problems is closely tied to their simultaneous development of the ideas of information given, information to find, and a link or path from the former to the latter. Writing a problem requires attention to all three components. In the early stages of this development a student may only consider the given aspect and write a "problem" like:

> Suzi has 9 packages of baseball cards.
> There are 12 cards in each package.

As students progress in their ability to comprehend what constitutes a problem and thus the ability to write problems, there is likely to be some transfer to those situations where students are presented with problems to be solved. This transfer may be in the form of recognizing what is given, what is to be found, or that the task is to build a bridge or link between the two. The importance of this transfer is emphasized by the number of times we have all heard the comment: "I really don't know where to begin." If teachers regularly have students write verbal problems, they should hear this question much less frequently.

Implementation

This technique can be closely tied to instruction on any of the basic operations (addition, subtraction, multiplication, and division) as well as most other topics, including measurement and geometry. Students may be asked to write problems in class, as part of a homework assignment, or both.

Allowing students to solve one anothers' problems often stimulates their interest. Contests based on ideas like "stump the teacher" and "problem of the week" also add variety and interest.

Estimating the Answer

Students who estimate the answers to verbal problems before they attempt to solve them seem to make important gains in the ability to correctly solve problems. Use of this technique is not difficult, yet the payoff from using it can be substantial.

Example

Students can be asked to estimate the answer to any verbal problem. Consider this problem:

> Janet picked 17 daisies for each of her
> classmates. She had 38 classmates.
> How many daisies did she pick in all?
> (Holt School Mathematics, Grade 6, p. 63)

Students may estimate the answer to be 600 by formally thinking of the product 15 X 40, or by informally thinking of 15 sets of 40. Another estimate might be 700 by thinking that the answer will be somewhat less than 20 X 40. Each of these estimates is close enough to the exact answer of 646 to serve the desired purpose. Of course, students may estimate the answer in an entirely appropriate way that is very different from the formal and informal methods mentioned here. A discussion of the methods used to estimate a particular answer can be very enlightening for students and teachers alike. In particular, such discussions provide an excellent learning experience for those students who have a poor concept of what is involved in the estimation process.

Rationale

The benefits derived from using the estimation strategy may be due to several factors. In order to estimate the answer to a problem a student must comprehend, at least in an intuitive way, what the problem is about. This is an important first step in solving a problem. A reasonable estimate of the solution also suggests and eliminates certain computational procedures. For instance, in the previously cited example the operations of addition, subtraction, and division are ruled out quickly since there is no way they can operate on the numbers in the problem (17 and 38) so that the result will be anywhere close to a reasonable estimate; in fact, such operations would not even yield a three-digit number!

Another factor which may contribute to the value of estimation is that it provides a safeguard from absurd answers and thus provides a means of detecting computation errors. Although there may be other reasons why the estimation technique is so effective, suffice it to say that the results are generally very positive.

Implementation

The estimation technique is easy to use and should be used in two distinct situations. First, it should be used regularly as an instructional method, perhaps by being a part of a regular rotation among other problem solving methods. Second, once students are acquainted with the idea, they should be required to make and record an estimate of the answer for every verbal problem they solve. Teachers are responsible for soliciting and discussing estimates for all problems worked orally in class. They should also monitor seatwork and homework to insure that students are estimating answers in these situations too.

One successful approach to monitoring is to have students record their estimates and then identify them by underlining them. Exact answers are then either circled or underlined twice.

It is important to emphasize again that discussion of the various methods of making an estimate for a specific problem is an ideal learning situation for those students having difficulty with this technique. Teachers can also foster the initial development of this ability by thinking aloud as they make their estimates as part of work done in front of the class.

Providing practice in rounding numbers and doing mental computation is also beneficial. A teacher must emphasize that in order for an estimate to be helpful it must be carefully made and not a "wild guess." Teachers can best do this early in the year by frequently modelling (thinking out loud) and clearly demonstrating to students how to make estimates.

One final thought to keep in mind as you do estimation work is that estimating can be informal in nature and need not rely on formal calculation, either written or mental. Recall that the product of 20 and 40 can be thought of informally as 20 groups of 40, and the approximate result gained from relying on one's quantitative sense is usually accurate enough to serve the desired purposes outlined in this section.

Reading Verbal Problems

The inability to read verbal problems is a definite factor in the difficulty many students have in learning to solve verbal problems. Thus a sustained effort to overcome reading problems is necessary in order to improve verbal problem solving ability significantly.

Example

There are many facets to the reading process that must be taken into account in the instructional process. To read well a student must be able not only to "string words together," but also to comprehend these words. Consider this problem:

> The Great Pyramid was originally 481 feet tall.
> The Great Pyramid was as tall as a building of
> how many stories, if you use 12 feet per story?
> (Addison Wesley, Investigating School Mathematics,
> Grade 6, p. 141)

There are many kinds of reading-related difficulties associated with verbal problem solving. An initial difficulty in the example problem might be with recognition of words like "Pyramid" and "building." Another difficulty, associated with a higher level of thinking, might be recognizing a word but not associating it with its appropriate meaning. In the example problem a student might incorrectly think of the word "stories" as being a collection of narratives rather than a measure of the height of a building. Finally, even if the words and their meanings are correctly discerned there is sometimes difficulty with general comprehension. Among other things the student must realize what information is given and what is to be determined.

Rationale

If a student cannot read a problem he is going to have great difficulty solving it. We now examine a method for handling these reading-related problems.

Implementation

There are two goals to be worked on jointly. First, assistance must be given to students to help them overcome their reading problems. Progress on this goal is oriented toward a long term solution to the problems, which in turn will result in better problem solvers. The second goal is to provide practice in solving verbal problems which circumvent reading difficulties. This is done by the teacher reading problems aloud, using tape recorders, and so on. The second goal insures that improvement in verbal problem solving will not have to wait until the reading difficulties are remediated which in many cases may involve a considerable period of time.

Several things must be done as part of our *regular mathematics instruction* regardless of the particular topic being studied in order to reduce the possibility of later reading difficulties. Terminology must be given special attention. Whenever a new term is introduced it must be written on the board, carefully pronounced first by the teacher then by the students, and then its meaning must be carefully discussed. This discussion should include both examples and nonexamples of the concept and also distinguish between the mathematical meaning of the word and any nonmathematical uses of the word. For example, the word "plane" has a special mathematical meaning quite different from everyday use where it might designate an airplane or a hand tool.

Whenever verbal problem solving is the *main topic for a lesson* the teacher must take direct steps to deal with reading problems. This means that all problems presented in the development part of the lesson and the first problem in any seatwork assignment must be carefully read aloud by the teacher or a student and important words and ideas discussed. An example of how this is done is described later in this section. Students must also be given reading assistance on more than the first seatwork problem. A teacher could effectively make use of audio recordings of the problems, or provide reading assistance as needed and requested during the seatwork time.

Special attention to reading problems alone should be included periodically during the daily portion of the mathematics period which is devoted to problem solving. This may involve teachers and students alternately reading problems, with a discussion of each problem after it is read. For example, in the problem:

> Waves as high as 112 feet have been reported
> on the "high seas." If each floor of a building
> is 14 feet tall, the wave would be as tall as a
> building with how many floors?
> (Addison Wesley, Investigating School Mathematics,
> Grade 6, p. 109)

several meanings of the word "wave" could be discussed, and attention would also be given to identifying the two pieces of information given and what needs to be determined to solve the problem. Problems to be read may be collected from the textbook, teacher and student written problems, and problems from older textbooks which are no longer in use in the school district. In order to focus primarily on reading, especially reading for meaning, problems read and discussed need not be solved. This allows for many problems to be considered in a short period of time.

Progress on reading difficulties should result from the above mentioned suggestions. Of course, progress can also be expected from students due to their regular reading instructional program. Certainly it is quite appropriate for mathematical material to be used as part of this instruction. Finally, not all students will benefit to the same degree from the attention to reading problems, but it will be a valuable experience for some students.

Anyone who has taught verbal problem solving is aware that reading problems which hinder verbal problem solving do not appear in isolation. How many times have you read a problem to a nonreader and he still could not solve the problem? For this reason attention to reading is only one of the many important techniques that must be given regular attention in instruction on problem solving.

Writing an Open Sentence

Many potential benefits of mathematics learning are realized when mathematics is used to model physical situations, because it is in this way that mathematics is used to solve everyday problems. The simplest situation where this takes place is where an open sentence is written to represent a verbal problem involving a minimal number of conditions. Verbal problems can often be solved without going through this step but some research has shown that developing the ability to translate problem conditions into mathematical sentences is related to improved problem solving performance.

Example

To illustrate how an open sentence can be used to model a verbal problem or a real world situation, consider this example:

> Nine classes in the school gave a total of $1,080 to the Book Fund. Each class gave the same amount. How much did each give? (Holt School Mathematics, Grade 6, p. 87)

This problem can be translated into the open sentence $9 \times \square = 1080$. The answer to the problem is then found by solving the open sentence using informal means such as estimation or by formal calculation of the quotient 1080 divided by 9.

Rationale

As with many other successful techniques, this technique probably gains much of its power by forcing the student to read carefully and to come to grips with the meaning of the problem. That is, it is necessary to determine how the given information pieces relate to one another in order to write an appropriate open sentence. Another reason for the usefulness of this method is that it reduces memory load in complex problem situations. Given a complex problem involving many conditions it is difficult, if not impossible, to mentally remember, manipulate, compare and contrast the given conditions. If on the other hand, these conditions are represented in the form of a collection of open sentences the task becomes much more manageable. For example, the following problem is difficult to solve without the use of open sentences.

> In order to put a fence around two adjacent sides of a rectangular lot 38 feet of wire is needed. The area of the lot is 217 square feet. What is the length of each side of the rectangle?

Let x and y represent the length of the two adjacent sides. Then $x + y = 38$ and $xy = 217$. From the first sentence we know $y = 38 - x$ and when we substitute this into the second sentence we have $x(38 - x) = 217$. This can now be solved by trial and error or more formal means, but in either case getting the problem into manageable form involved writing open sentences (equations).

Implementation

This problem solving method should be taught to all students. Hopefully most students will already have had prior exposure to the technique and thus only periodic review will be necessary in order for them to use the technique as they solve verbal problems. The periodic review can be part of a rotation among other techniques described in this manual and like the others can be done daily using a small part of the mathematics class period.

When providing practice on translating verbal problems to open sentences several important ideas need to be remembered. First, there will be a tendency for students not to write an open sentence for the very simple problems. The typical comment will be "I already know how to do it!" The teacher must persevere and require that sentences be written in most cases because only in this way will the skill be learned and the student develop the capability to apply the skill in more complex situations. Attempts to translate difficult problems involving many conditions to open sentences without considerable practice on simple problems usually ends in failure.

Second, teachers must be aware that several different open sentences may equally well model the same verbal problem. In the example already described the sentence $1080 - 9 = \square$ could be used as a model quite appropriately. It's likely, however, that many students will think of the problem multiplicatively and write the sentence $9 \times \square = 1080$. Either sentence is acceptable and both lead to a correct solution. Finally, make students aware that any one open sentence may model a large number of situations that seem perceptually different but are alike structurally.

Several other suggestions may be of help. When this technique is being used as just a small segment of the lesson (e.g., 10 minutes) it will be desirable frequently to have students only write the open sentence that goes with a problem and not continue to find the exact solution. Also, for those students who have had little work with using open sentences in this way it will be easier if the initial translations involve simple problems. Using problems from textbooks at lower grade levels is often a good idea under these circumstances.

Appendix G

Verbal Problem Solving Post Test*

*This instrument was used in field experiment II to compare the relative progress of treatment and control students.

Name____________ ________

School_______________________

Teacher_______________________

1. Doug and Tom collected basketball cards. Doug had 6 cards. Tom had 8. How many cards did the two boys have altogether?

____

2. If each box has 8 pencils, how many pencils in 4 boxes?

____

3. Last year Kate sent 32 Valentines. She bought 15 of these cards and made the rest herself. How many cards did she make?

____

4. Jane had to do 9 problems on Monday. On Tuesday she had to do 13, on Wednesday 16, and on Thursday 21. How many problems did she have to do for these days?

____

5. David wants to save 30 nickels. He can put 5 nickels in each row of his nickel-card. How many rows of nickels will he need?

____

6. A lake is 450 miles from our home. If we go 240 miles the first day, how many miles from the lake will we be?

______

7. If candy bars are 20¢ each, how many can you buy for 80¢?

______

8. On Monday 321 tickets were sold. On Tuesday 433 tickets were sold and on Wednesday 125 tickets were sold. How many tickets were sold on these 3 days?

______

9. Each week for 5 weeks, Robert had to learn 15 new spelling words. How many new words did he have to learn during those 5 weeks?

______

10. Carol had 14 pens. She gave Suzi 2 and Heather 4. How many does Carol have now?

______

11. The park owned 250 acres of land. Of these, 156 acres were in a woodland and not open to visitors. How many acres of the park could be visited?

____

12. The children in our school drink 305 pints of milk each day. How many pints of milk is that for a 5-day week?

____

13. How much will a dozen apples cost if 3 apples cost 30¢?

____

14. A rope is 9½ feet long. It is cut into 2 pieces. One piece is 4 feet long. How long is the other piece?

____

15. How much can we spend for the class party? The parents gave us $5.00 and 25 children brought a dime each.

____

16. Jim found 25 golf balls. He will keep 10 and give 1/3 of what is left to each of 3 friends. How many will each friend get?

17. After Halloween candy bars were on sale for 7¢ each, and gum was on sale for 5¢ a pack. Doug spent 43¢ on gum and candy bars.

How many packs of gum did he buy? _____

How many candy bars did he buy? _____

18. Tom saw birds and cats at the zoo. He made a puzzle for his sister. He said, "30 heads and 80 feet. How many birds? How many cats?"

Answer: _____birds _____cats

19. David is typing page numbers on his report. On the first page he types a 1. On the next page a 2 . . . on page 10 he hits the "1" and the "0" keys . . . and so on. There are 24 pages in his report. How many times will he have to hit the typewriter keys?

_____times

20. Doug bought a bike for $30 and sold it for $40. Then he bought it back for $45 and sold it again for $50. How much profit did he make altogether? ______

Appendix H

Follow-Up Letter to Assess the Reaction of Experimental Teachers to the Program in Field Experiment II

UNIVERSITY OF MISSOURI-COLUMBIA

Graduate School

Center for Research in Social Behavior

111 East Stewart Road
Columbia, Missouri 65201
Telephone: (314) 882-7888

Dear Participating Teacher:

Your help and cooperation in the project were excellent. I hope that the program, or some parts of it, was useful to you. Your reaction to the program (its strengths and weaknesses) is important information and I would like your candid and confidential opinion of it (there is no reason to sign the questionnaire).

I would like to ask you a few questions about the program. However, since we promised not to bother you again for information, we are prepared to pay you an additional $5 for your time in completing the form (wish it could be more).

You may have liked or disliked the program in its entirety or there may have been certain parts that seemed especially good or bad. The following set of questions are designed to solicit your reactions.

I. Reaction to the project

When responding to these questions, please use the following scale: 1 = very good; 2 = good; 3 = so-so; 4 = of little value; 5 = of no value.

___ my reaction to the entire program
___ my reaction to the review phase of the lesson
___ my reaction to the development stage of the lesson
___ my reaction to the seatwork stage of the lesson
___ my reaction to the homework stage of the lesson
___ my reaction to the use of mental computation problems
___ my reaction to the increased pace suggestion
___ my reaction to the verbal problem solving material

II. Classroom behavior after the project

Some teachers in the project will decide to continue to use the program; other teachers will decide to discontinue it; and yet others will continue some parts of the program but not other aspects of it. Would you please describe your current practice by choosing the appropriate alternative for each question that follows.

I conduct a short review

___ 4 or 5 times a week
___ 3 times a week
___ 2 times a week
___ 1 time a week
___ not at all

I include mental computation questions in my mathematics lessons

___ 4 or 5 times a week
___ 3 times a week
___ 2 times a week
___ 1 time a week
___ not at all

I conduct a lesson with a large development stage

___ 4 or 5 times a week
___ 3 times a week
___ 2 times a week
___ 1 time a week
___ not at all

I include work on verbal problem solving

___ 4 or 5 times a week
___ 3 times a week
___ 2 times a week
___ 1 time a week
___ not at all

On the average I spend

___ minutes on development in each of my mathematics lessons

In an average week I now assign homework

___ 4 or more nights a week
___ 3 times a week
___ 2 times a week
___ 1 time a week
___ not at all

I still conduct an expanded review weekly

___ yes
___ no
___ not at all

I still conduct a large review monthly

___ yes
___ no

The best part of the program in my opinion was:

____

____

____

____

____

The weakest or most confusing part of the program was:

____

____

____

____

____

Was the recommendation and explanation of development clear? Please explain:

____

____

____

____

____

Sincerely yours,

Tom

Thomas L. Good
Professor of Education

TLG/sk
Enc.

Appendix I

Follow-Up Letter to Assess the Reaction of Control Teachers to the Program in Field Experiment II

UNIVERSITY OF MISSOURI-COLUMBIA

Graduate School

Center for Research in Social Behavior

111 East Stewart Road
Columbia, Missouri 65201
Telephone (314) 882-7888

Dear Participating Teacher:

It was good to have a chance to finally provide you with details of the project in our February meeting. Since the program was derived from methods that classroom teachers were already using, you may have already been performing many aspects of the program.

We would like to ask you a few questions about the program. However, since we promised not to bother you further, we are prepared to pay you an additional $5 for your time in completing the form (wish it could be more). Feel free to respond as you see fit. Your response is confidential and there is no need for you to sign the letter.

I. Reading of Project Material

I realize that you have had the project materials for only a short period of time and you're in the midst of a busy year. Please choose the response that best represents the extent to which you have studied the manual:

The general 45-page manual:

___ 1. I have read it carefully.
___ 2. I have read it quickly.
___ 3. I have skimmed it quickly and thought about the highlights.
___ 4. I have paid very little or no attention to it.

The 17 page manual on verbal problem solving.

___ 1. I have read it carefully.
___ 2. I have read it quickly.
___ 3. I have skimmed it quickly and thought about the highlights.
___ 4. I have paid very little or no attention to it.

II. We realize that because fourth grade teachers in your school were using the program and because the ideas in the program are general ones that you may already have been using some of the ideas prior to obtaining the treatment manual in February. Please indicate which aspects of the program were already part of your classroom teaching.

1 = great correspondence between what I was already doing and the program request.
2 = general overlap between what I was already doing and the program request.
3 = some overlap between what I was already doing and the program request.
4 = little if any overlap between what I was already doing and the program request.

___ teaching the class as a whole
___ verbal problem solving strategies
___ development
___ seatwork
___ homework
___ mental computation
___ broad review and weekly review
___ daily review

III. How new were the five verbal problem solving strategies (1 = it was a new strategy; 2 = it was somewhat new; I had used somewhat similar ideas; 3 = not new at all; I was already doing this?

___ problems without numbers
___ writing verbal problems
___ estimating the answer
___ reading verbal problems
___ writing an open sentence

IV. Before receiving treatment manual, on an average day roughly how much time, if any, did you spend in various parts of the lesson?

___ review
___ development
___ problem solving
___ mental computation

V. In general how new was the program to you; what strengths and weaknesses appear to you after reading the program materials?

VI. Do you plan to use any aspects of the program in your teaching? If so, which parts?

Again, my thanks for your considerate and prompt response to these questions. A self-addressed envelope is enclosed for your convenience.

Sincerely yours,

Tom

Thomas L. Good
Professor of Education

TLG/sk
Enc.