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ABSTRACT

This study was designed to investigate the effectiveness of a heuristic approach to problem-solving instruction. The 330 subjects were randomly assigned to either a Problem Solving, Calculator Problem Solving, or Control group. The two experimental groups studied problem-solving heuristics applied to nonroutine problems for 18 weeks, after which 102 subjects of 3 ability groups were randomly selected for post testing. In an interview setting they responded to five-selected problems. Seven subjects were identified as case studies. Results showed that the experimental groups had significantly higher problem-solving success and used more strategies. The Calculator Problem Solving subjects were found to make fewer computational errors and required significantly less time in problem solving. (Author/MP)

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CALCULATOR USE AND PROBLEM SOLVING STRATEGIES OF GRADE SIX PUPILS

Final Report

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Grant No. SED-791914

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Strategies of Grade Six Pupils

Final Report

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April 1982

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As many know, successful projects reflect the work of the secretarial staff. We are particularly indebted to Barbara Bloch for her many valuable contributions, including the preparation of this Final Report.

Grayson H. Wheatley
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April 1982

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Introduction

There is a concerted effort to infuse problem solving into the school mathematics curriculum. One impetus for this movement is the recommendations of the National Council of Teachers of Mathematics presented in An Agenda for Action (NCTM, 1980). The number one recommendation of the report was "Problem solving should be the focus of school mathematics in the 1980's." Judging from the many publications and conference presentations devoted to problem solving it is an idea whose time has come. Thus, while the mathematics curriculum emphasis in the 1970's was back to basics, the emphasis in the 1980's promises to be problem solving.

Although there is much interest in problem solving being expressed by practitioners and researchers alike, many important questions exist. Polya's four steps have been widely accepted but do not provide specific directions for teaching problem solving (Polya, 1962). In particular, little is known about effective instructional methods at the elementary school level. The 1980 yearbook of the National Council of Teachers of Mathematics, Problem Solving, provides some guidance for teachers but only serves as a beginning to understanding the teaching of problem solving.

During the 1970's, attention was given to the processes students use in problem solving. Researchers have studied the use of process variables and their relation to problem solving success (Kantowski, 1977; Lucas, 1974; Schoenfeld, 1979; Webb, 1975, 1979; Zweng, 1979). These studies have provided useful information which has led to the formulation of several instructional methods for teaching problem solving (Lester, 1980). Among those methods listed by Lester are: 1) have students solve many problems without intervention by the teacher; 2) teach specific skills, e.g., draw diagrams, translate word problems to number sentences using word clues; and 3) teach heuristic strategies. For this study, the heuristic approach was chosen as the instructional method.

While the teaching of heuristics has proven effective at the university level (Schoenfeld, 1979), little is known about applications at the elementary school level. It has been recommended by Krulik and Rudnick (1980), LeBlanc (1977), Meiring (1980), Polya (1974), and Schoenfeld (1980). Schaaf (Note 2) developed an instructional program designed to teach heuristic strategies. Based in large part on Schaaf's instructional model, this study investigated the effectiveness of such a heuristic approach to problem solving instruction.

Dramatic sales and low prices have stimulated many professional groups to call for experimentation with the use of calculators (Higgins, 1974; NACOME, 1975; NCTM, 1974; NCTM, 1980). Studies of calculator use generally have taken the form of having one group of pupils use calculators for a fixed period of time and another group study the same material without a calculator followed by testing for achievement differences. While this type of research is essential and valuable, there is a need to understand the impact of calculator use on the learning of specific domains.

With the increased use of calculators in elementary school mathematics, problem solving of elementary school pupils may be enhanced. Since in problem solving it is often necessary to perform difficult and lengthy computations, the act of switching into the computational mode may detract from using effective problem solving heuristics. Thus, availability of calculators may facilitate the development of problem solving skills. As Suydam (1978) states,

> The calculator's relationship to problem-solving is a question of vital concern. Although the research in Suydam's 1976 report for the NSF shows conflicting reports about calculator effects on problem solving, all of the research. . . had the common element that the calculator was adjuct to units in problem solving--it was not incorporated into a specific

problem solving strategy. This appears to be the best hope for meaningful use of the calculator--by incorporating it into a specific strategy (pp. 10-11).

In solving problems without a calculator, two contrasting modes of thought are usually employed. They are (1) deciding how to solve the problem and (2) performing the necessary computations. In devising a plan the individual must synthesize ideas, try several approaches and in general keep a global perspective. On the other hand, paper-and-pencil computations necessary to solve the problem require rule oriented behavior. It is suggested that the extensive time devoted to computations may inhibit the use of problem solving strategies by focusing undue attention on the mechanics of computation. In fact, the pupil may see the task as (1) quickly deciding what to do and (2) computing to "solve the problem" (Suydam and Riedesel, Note 3). However, if pupils realize that computations can be performed quickly and accurately on a calculator, they may be able to focus on devising a plan and evaluating a derived result. Finally, the calculator may become a tool for thinking whereby different strategies are possible.

Purpose

The purpose of this study was to compare the problem solving success and strategy use of sixth grade pupils as influenced by calculator use. The following research questions were posed:

1. Is a heuristic strategy approach to problem solving instruction effective with sixth grade pupils?
2. After learning to solve problems with the aid of a calculator, do elementary school pupils
   a. employ a wider range of problem solving strategies?
   b. differ in their strategy choices?
   c. require less time in problem solving?
   d. solve more problems correctly?
3. Is there a differential effect of problem solving instruction and calculator use on pupils of varying ability?

In thirteen empirical studies with elementary school pupils using calculators, nine studies reported achievement results favoring the calculator group, one favoring the noncalculator group, and three reported mixed results (Allen, 1976; Borden, 1977; Campbell, 1976; Hawthorne, 1976; Jones, 1976; Nelson, 1976; Schafer, 1975; Wheatley, Shumway, Coburn, Reys, Schoen, Wheatley, & White, 1979; White & Shumway, 1977). Where differences were found, they were on tests of concepts, reasoning, or problem solving.

In a large scale study (n=1386, five sites) the impact of calculator use in elementary school mathematics was assessed (Wheatley, et. al., 1979; Shumway, White, Wheatley, Reys, Coburn, & White, 1981). Relevant to the present study, there were no significant treatment differences in mathematics achievement or attitudes. In particular, the noncalculator and calculator groups performed at the same level on the applications section (word problems) of the Stanford Achievement Test. While the calculator group did not score significantly higher, it must be recognized that the treatment did not emphasize problem solving. Furthermore, the level of overall calculator use approximated "first time use" in a school. With more emphasis on calculator use in problem solving, differences might be observed.

Kasnic (1977) reports a study of calculator assisted problem solving. The sixth grade pupils worked through problems in two 50 minute sessions and were then posttested on problem solving. There were no treatment differences attributable to calculator use. He did find that the calculator assisted the low ability problem solvers to solve more problems correctly. The study was compounded by a school effect; each treatment was in a different school. Kasnic recommends that future studies of this nature have longer treatment periods.

A study was conducted by Wheatley (1980) using a design similar to the one used in the present study with 50 sixth grade students. Over an eight week period one class of sixth grade pupils studied problem solving without calculators while another class taught by the same teacher studied problem solving with the aid of calculators. Wheatley found that (1) the calculator group used significantly more problem solving processes, (2) the calculator group made significantly fewer computational errors, and (3) there were no differences in problem solving scores or time for completion of the task. The results suggest that problem solving is enhanced by calculator use.

The think-aloud technique has been used successfully to study problem solving processes (Dalton, 1974; Days, 1977; Hollander, 1973; Kantowski, 1974; Kilpatrick, 1967; Paige & Simon, 1966; Schwieger, 1975; Webb, 1975). Recently, Ericsson and Simon (1980) put forth a strong rationale for using the think-aloud technique. They argue that verbalized data should be considered as valid and reliable research data. They further posited that the think-aloud process does not interfere with the subjects' thinking. However, they do point out that "Some heeded information may not be vocalized when task directed processes take priority and interrupt the verbal, encoding and production processes." p. 225. This suggests a low interference level when using the think-aloud technique.

While, at times, the child may not be aware of the processes being used and thus not be able to verbalize his or her reasoning, much can be learned using the think-aloud technique.

PROCEDURE

Sample

The subjects for this study were 330 sixth grade pupils drawn from a midwestern city of 50,000. The people of the city represent a bread spectrum of socioeconomic levels. The achievement level of the pupils in the sample was near the fiftieth percentile on national norms. Eighteen classes from seven elementary schools and ten teachers were represented. The teachers had from one to 37 years of teaching experience.

Instrumentation

A Problem Solving Test (PST) was constructed to assess the problem solving performance of pupils. Based on the work of Wheatley (1980), five problems were selected. The problems were piloted with comparable subjects prior to the scheduled interviews. An attempt was made to select problems which could be solved by a variety of strategies. The five problems used in thc interviews are shown in Appendix A.

All five problems were presented to all subjects in the order shown. The criteria for problem selection is given below.

1. The problem can be solved in more than one way.
2. The problem requires more than one step to

solution.

3. The problem requires arithmetical computations.

4. No simple algorithm is directly applicable.

5. The problem is challenging but comprehensible by sixth grade pupils.

The subject interviews using the PST were conducted by two two-person teams over a four week period in February and March of 1981 immediately following the 18 week treatment phase. The think-aloud-technique was utilized. One experimenter acted as an observer, recording any pertinent information while the other experimenter presented the tasks and questioned the subject. Each interview session was tape recorded for later analysis. A coding form developed for the purpose was used to record strategy use, computational errors, calculator use, and nine other categories of observed behavior. The Interview Codng Form is found in Appendix B. The 16 interview ratings are listed and defined in Appendix C. Immediately following each interview the experimenters discussed the coding. Selected sections of the tape recorded protocols were played as necessa y to agree on the ratings to be assigned.

The Iowa Problem Solving Pretest and Posttest, Forms 561 and 562 (Schoen and Oehmke, 1980) were administered to all classes. The pretest was given during the first treatment week and the posttest was administered during the eighteenth treatment week. The pretest results were used to classify subjects by ability. A Math Attitude and a Calculator Attitude Inventory (shown in Appendix D) was given on the same day as the Problem Solving Tests. These inventories provided a measure of the pupils attitude toward math and calculators. The results of the Problem Solving pretest provided data for classification of subjects by ability.

Design

Initially, the 18 classes were randomly assigned to the three treatment groups, Calculator Problem Solving (CPS), Problem Solving (PS), and Control. After school began, one class was shifted from the PS to the CPS group to compensate for ability grouping at one school. The number of subjects in each treatment group was, CPS = 150, PS = 75, and Control = 105. In September, just prior to the treatment phase, the Iowa Problem Solving Test (Schoen and Oehmke, 1980) was administered to all subjects. On the basis of the IPSP data, three problem solving ability levels were determined. The number of subjects in each treatment by ability cell is shown in Figure 1.

A decision was made to have 102 pupils (34 from each treatment group) respond to the PST. It was felt that 34

Ability

Treatment	Low	Middle	High	
CPS	29	65	56	150
PS	20	36	19	75
Control	35	53	17	105
	84	154	92	330

Figure 1. The number of subjects in each Treatment X Ability cell.

Treatment

Ability	CPS	PS	Control	
High	9	9	9	27
Middle	16	16	16	48
Low	9	9	9	27
	34	34	34	102

Figure 2. The number of subjects in each Treatment X Ability cell taking the PST.

in each treatment group was adequate and this was the maximum that could be tested with the PST. It was deemed important to have balanced representation from the three ability groups. Further, it was decided to over sample the middle ability group. These decisions led to the design shown in Figure 2.

The PST was administered as a posttest to the 102 selected students. The scores from the PST were analyzed using a 3 (Treatment) X 3 (Ability) analaysis of variance. If the assumption for analysis of variance were not met, Wilson non-parametric procedure was used (Wilson, 1956) to analyze the data.

The IPST was administered to all subjects in the study. Complete data was available for 330 subjects. The data were analyzed using a 3 (Treatment) X 3 (Ability) analysis of covariance with the pretest as a covariate.

Treatment Materials

The problem solving materials were assembled and developed by the research team. The Techniques of Problem Solving, Deck D, containing 200 problems on cards was available in each of the CPS and PS classrooms (Greenes, Immerzeel, Ockenga, Schulman, Spungin, 1980). Teachers were encouraged to have pupils complete as many cards as feasible in the time available. A copy of Keystrokes:

Exloring New Topics, was available in each of the CPS classrooms (Reys, Bestgen, Coburn, Marcucci, Schoen, Shumway, Wheatley, Wheatley, & White, 1980). One challenge problem was delivered each week on a large poster to further stiumulate problem solving activity. Additionally, problems were selected from other sources. Of particular value were the cards developed in the Iowa Problem Solving Project (Immerzeel, Note 1) and the Lane County Mathematics Project (Schaaf, Note 2). Weekly sets of problems were prepared for each of the 18 treatment weeks. The first two weeks were devoted to introductory activities. During the next five weeks, five problem solving strategies were taught, one each week using materials provided by the researchers. The five strategies taught were (1) Guess and Test, (2) Draw a Diagram, (3) Make a List, (4) Simplify, and (5) Look for a Pattern. Later, a lesson on Write an Equation was taught. Other specific lessons were suggested to teachers in subsequent weeks. The last half of the treatment period was devoted to problem solving without focus on any specific strategy. Teachers were directed to encourage strategy use. Discussions of the strategies followed problem solving sessions.

Calculators were provided for each child in the CPS classes. After careful study of the available calculators, a decision was made to purchase the Sharp EL 211. This model has an eight-digit LCD display, the four arithmetic functions, percent, memory, square root, and automatic shut-off. The calculator worked quite well in the study. Failure rate was less than one percent and the batteries lasted for the duration of the treatment.

Description of the CPS And PS Treatments

The Calculator Problem Solving Group and the Problem Solving Group experienced similar problem solving activities. The only difference between the two groups was the use of calculators. The same problem solving materials were distributed each week to the CPS and PS classes. Where calculator activity sheets were provided for the CPS classes, similar activities were provided for the PS classes. In preparing the PS worksheets, references to calculators were removed. Every effort was made to provide the same experiences for both groups except for the use of calculators.

In addition to the project materials distributed each week, all classes studied from their adopted mathematics textbook (Scott, Foresman Series). Teachers were asked to plan approximately two days per week for problem solving. The actual time devoted to problem solving varied from class to class but most teachers did spend the prescribed time on problem solving. Some teachers spent more than the two days per week on problem solving.

The classes in the Control Group did not study any of the problem solving materials. Teachers were asked to teach from their text as prescribed by the school administration.

Weekly plans were developed for classes in the CPS and PS treatments. These plans included the following topics.

Weeks 1-2	Exploratory activities, introductory problem solving, and calculator activities (for the CPS Group)
Week 3	Introduction of the Guess-and-Test strategy
Week 4	Introduction of the Draw-a-Diagram strategy
Week 5	Introduction of the Make-a-List strategy
Week 6	Introduction of the Simplify strategy
Week 7	Introduction of the Look-for-a-Pattern strategy
Weeks 8-9	Mixed Practice on all strategies
Week 10	Introduction of the Write-an-Equation strategy
Weeks 11-18	Mixed practice on all strategies

In teaching the strategies, teachers would present a problem, discuss it with the class, demonstrate the use of that particular strategy, and after providing time for problem solving, have class discussions in which strategy use was discussed. Each week when a new strategy was introduced, the strategies previously introduced were reviewed and applied. During the week that a particular strategy was introduced, problems which lent themselves to the use of that strategy were included in the problems distributed for use. Throughout the treatment, pupils were encouraged to select the strategy(s) they wished to use on any given problem. Pupils were encouraged to be exploratory and use a variety of problem solving strategies. Since each teacher was ultimately responsible for the instruction in his/her classroom, there was variability in the treatment implementation. A copy of the Flow-Charted Schedule of Activities is shown in Appendix E.

Case Studies

During September, 1980, seven students from the CPS and PS Groups were identified as case study subjects. Students who appeared to be active and able to communicate were selected. During the treatment phase each of the case study subjects was observed weekly by a project staff member. Information was recorded on the Case Study Observation Form shown in Appendix F. Only the project staff (not classroom teachers) were aware that case studies were being prepared. These seven subjects were automatically included in the subsample selected to take the PST.

Inservice training

Three inservice training sessions, each one and one-half hour in length, were conducted for teachers of the CPS and PS classes. The first session was held on the second day of the school term. This session presented a rationale, overview of the study, and explanations of problem solving strategies. During this session, CPS and PS teachers met separately with a project director. A Teachers Resource Packet was distributed. It contained the following items.

1. project goals
2. procedures
3. time schedules
4. problem solving guidelines
5. explanations of problem solving strategies
6. sample problems
7. ways of utilizing calculators
8. teacher log forms
9. student and teacher record sheets.

Classroom observations

The classes in the CPS and PS groups were visited at least once a week by a project staff member. During the visits, the staff member observed students, talked with the teacher if time permitted, interacted with students, and delivered materials for the following week (selected problems and one challenge problem).

The observations provided the opportunity to determine the level and nature of treatment implementation, observe case study subjects, gain valuable impressions about the problem solving process and help the teacher implement the treatment as designed.

RESULTS

The Problem Solving Test

The scores of the 102 subjects selected to take the PST were analyzed using a 3 X 3 (Treatment by Ability) analysis of variance when the statistical assumptions were met and by a Wilson's nonparametric analysis of variance otherwise. The assumptions were not met for the variables Looking Back, Computational Error, Time, Calculator Frequency, and Calculator Use. In order to compare the performance of the CPS and PS groups, planned comparisons were used. In a similar manner, planned comparisons were used to compare the average score of the CPS and PS groups to the Control group. The cell and margin means with standard deviations are shown in Tables 1 and 2.

Strategy Use. The two main effects were significant and there was no interaction. As shown in Table 3, the treatment effect was highly significant with an F value of 44.1. The CPS and the PS group means were each significantly greater than the Control group means while the CPS and PS difference was not significant. The ability group means paralleled the corresponding means on the IPSP Test. The differences were significant but less dramatic than the treatment means.

Table 1

Treatment X Ability Means and Standard Deviations for PST Variables

	Ability								
	High			Middle			Low		
Variable	CPS	PS	Control	CPS	PS	Control	CPS	PS	Control
Strategy Use	12.78	13.00	4.67	10.13	11.19	3.44	7.89	8.78	3.22
	(4.63)[a]	(4.21)	(2.60)	(2.90)	(3.39)	(2.80)	(3.33)	(5.36)	(2.64)
Success	22.44	18.00	8.00	14.63	12.63	3.00	8.78	10.78	3.11
	(7.89)	(9.15)	(5.59)	(7.82)	(6.28)	(4.44)	(6.40)	(9.24)	(3.18)
Estimation	3.33	3.78	1.00	2.63	2.63	1.44	2.67	2.44	1.11
	(2.06)	(1.45)	(1.32)	(2.06)	(1.59)	(1.21)	(2.12)	(2.13)	(1.05)
Representation	7.22	7.11	3.44	5.50	6.25	3.56	4.00	5.78	3.22
	(1.56)	(1.45)	(1.42)	(2.48)	(1.48)	(2.22)	(1.00)	(2.05)	(1.72)
Conditions	3.67	2.78	1.78	2.81	2.25	1.00	1.89	2.22	0.78
	(1.23)	(0.97)	(1.09)	(1.28)	(0.86)	(0.73)	(0.93)	(1.30)	(0.67)
Organization	4.78	3.44	1.67	2.94	3.66	1.19	2.44	1.78	1.11
	(2.22)	(2.30)	(1.41)	(2.24)	(2.31)	(1.28)	(1.33)	(1.30)	(1.69)
Plan	8.00	7.44	3.89	5.69	6.50	4.50	4.33	6.00	5.11
	(1.58)	(1.01)	(1.90)	(2.41)	(1.03)	(1.90)	(1.94)	(1.87)	(0.93)
Looking Back	4.00	3.22	0.78	3.00	2.56	1.06	3.22	3.00	0.44
	(3.35)	(2.86)	(0.97)	(2.00)	(1.90)	(1.34)	(1.86)	(2.29)	(0.73)
Confidence	7.67	7.78	4.00	6.31	6.88	4.38	5.11	5.67	5.11
	(2.06)	(1.39)	(2.18)	(1.58)	(1.59)	(2.34)	(1.69)	(2.29)	(1.76)
Persistence	7.11	7.56	5.56	6.81	6.94	4.31	6.56	5.89	4.00
	(2.15)	(1.88)	(1.51)	(1.56)	(2.21)	(2.15)	(1.42)	(2.03)	(2.60)
Computational Error	0.56	2.56	0.44	1.50	2.44	1.69	2.56	4.44	2.33
	(1.13)	(1.42)	(0.73)	(2.42)	(2.37)	(2.58)	(3.25)	(4.28)	(3.57)
Correct Solution	9.67	7.33	3.22	5.94	5.13	1.25	3.44	3.44	0.89
	(5.39)	(5.07)	(2.33)	(3.64)	(3.81)	(2.05)	(2.51)	(4.39)	(1.45)
Correct Method	12.00	10.67	4.89	8.06	7.50	1.75	5.33	6.67	2.22
	(4.00)	(4.47)	(3.48)	(3.87)	(2.88)	(2.52)	(4.00)	(4.00)	(2.11)
Time	32.78	31.89	26.22	29.63	40.56	17.75	27.44	41.33	15.44
	(15.33)	(12.17)	(8.60)	(7.28)	(16.41)	(5.16)	(8.46)	(19.71)	(5.46)
Calculator Frequency	10.67	2.44	4.22	9.19	1.81	2.25	4.11	3.67	0.33
	(5.27)	(5.08)	(4.92)	(4.68)	(2.71)	(3.13)	(4.29)	(4.03)	(1.00)
Quality of Calculator Use	7.00	2.00	2.33	6.00	1.50	1.69	2.89	2.56	0.33
	(3.24)	(4.00)	(2.69)	(2.73)	(2.31)	(2.52)	(2.71)	(2.65)	(1.00)

Note. The number of subjects in each treatment group was 34.

[a]Standard deviations in parentheses.

Table 2

Treatment Means on PST Variables

Variable	CPS (34)[a]		PS (34)		Control (34)		Total (102)	
Strategy	10.2	(3.9)[b]	11.0	(4.4)	3.7	(2.7)	8.3	(4.9)
Success	15.1	(8.9)	13.6	(8.2)	4.4	(4.9)	11.0	(8.8)
Estimation	2.8	(2.0)	2.9	(1.7)	1.2	(1.2)	2.3	(1.8)
Representation	5.6	(2.2)	6.4	(1.7)	3.4	(1.9)	5.1	(2.3)
Used all Conditions	2.8	(1.3)	2.4	(1.0)	1.1	(0.9)	2.1	(1.3)
Organization	3.3	(2.2)	3.0	(2.2)	1.3	(1.4)	2.5	(2.1)
Plan	5.9	(2.5)	6.6	(1.4)	4.5	(1.7)	5.7	(2.1)
Looking Back	3.3	(2.4)	2.9	(2.2)	0.8	(1.1)	2.3	(2.2)
Confidence	6.3	(1.9)	6.8	(1.9)	4.5	(2.1)	5.9	(2.2)
Persistence	6.8	(1.7)	6.8	(2.1)	4.6	(2.2)	6.1	(2.2)
Computational Error	1.5	(2.5)	3.0	(2.9)	1.5	(2.6)	2.0	(2.7)
Correct Solution	6.3	(4.5)	5.3	(4.4)	1.7	(2.2)	4.4	(4.3)
Correct Method	8.4	(4.6)	8.1	(3.9)	2.7	(2.9)	6.4	(4.6)
Time	29.9	(10.1)	38.5	(16.4)	19.4	(7.5)	29.3	(14.2)
Calculator Frequency	8.2	(5.3)	2.5	(3.8)	2.3	(3.6)	4.6	(5.4)
Calculator Use	5.4	(3.2)	1.9	(2.9)	1.5	(2.3)	3.0	(3.3)

[a]See Table 1.

[b]Standard deviations in parentheses.

Table 3

Analysis of Variance on the PST Variable
Strategy Use

Source	df	MS	F
Treatment (T)	2	549.088	44.110 ***
Ability (A)	2	83.810	6.733 **
A X T	4	8.212	.660
Error	93	12.448	

** p< .01
*** p< .001

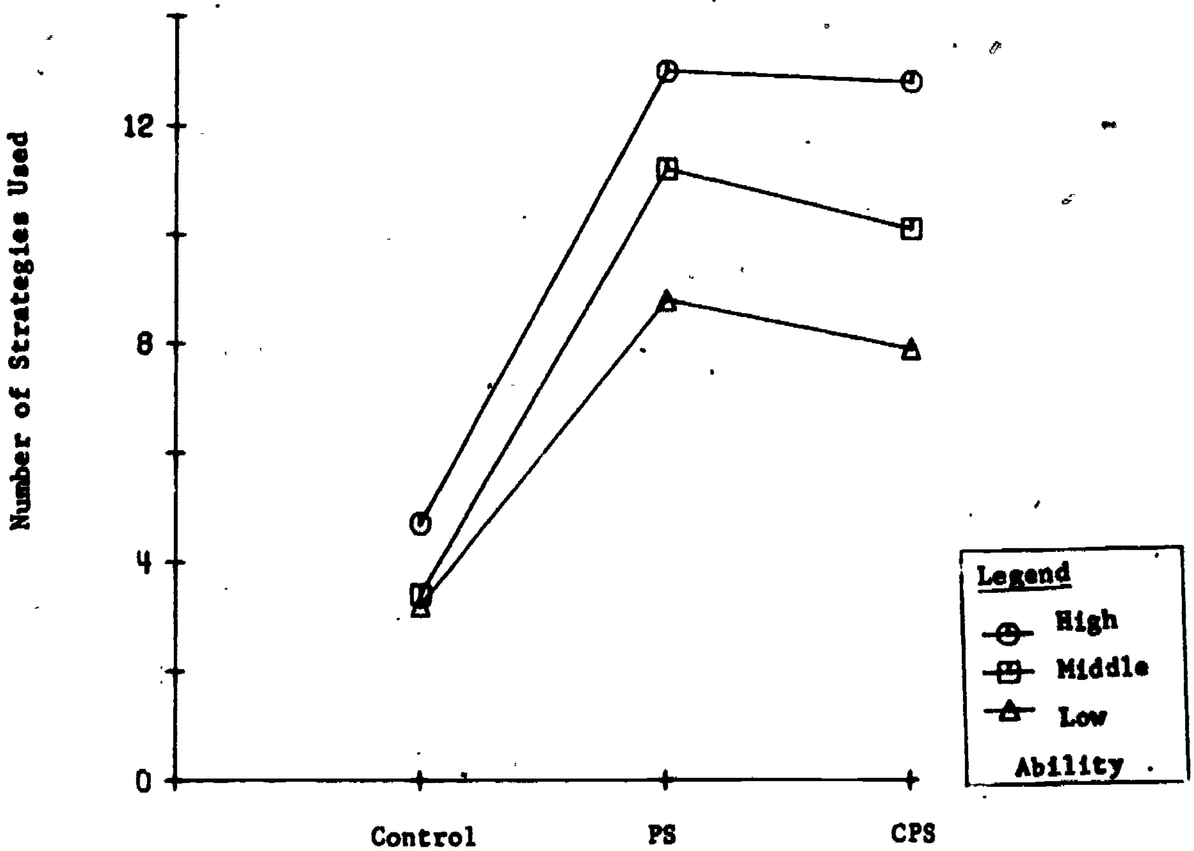

Figure 3. Ability by Treatment Interaction for Frequency of Strategy Use.

Table 4

Analysis of Variance on the PST Variable
Success

Source	df	MS	F	
Treatment (T)	2	1154.775	24.884	***
Ability (A)	2	538.110	11.596	***
A X T	4	49.117	1.058	
Error	93	46.406		

*** p< .001

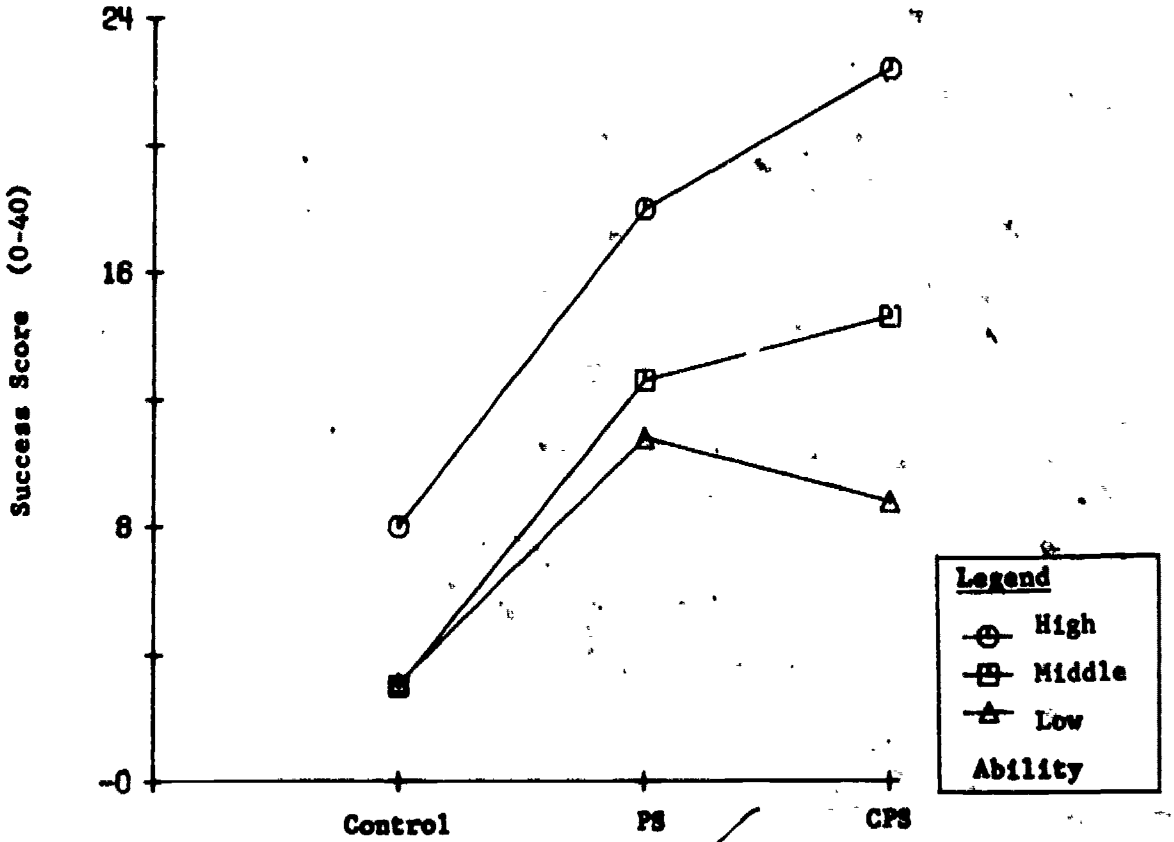

Figure 4. Ability by Treatment Interaction for Success.

<u>Success.</u> The two main effects were significant and there was no interaction. As shown in Table 4, the Treatment effect was highly significant with an F value of 24.9. The CPS and the PS group means were each significantly greater than the Control group mean while the CPS-PS difference was not significant. The Ability effect was significant with higher ability groups havirg higher means. The within treatment ability differences were somewhat greater for the CPS group than the PS or Control groups. This observation is important because the same pattern was noted on other variables.

<u>Estimation.</u> As can be seen in Table 5, only the Treatment main effect was significant (F = 10.5). The CPS and PS group means were each significantly greater than the Control group mean, while there was no difference in the means of the CPS and the PS groups. The high ability PS group had a higher mean than the CPS group in an absolute rather than a statistical sense. It may be that high ability students learn to estimate better without a calculator.

<u>Representation.</u> The two main effects were significant and there was no interaction (see Table 6). The treatment effect was highly significant with an F value of 22.6. The CPS and the PS group means were each significantly greater than the Control group mean while the CPS-PS difference was

Table 5

Analysis of Variance on the PST Variable
Estimation

Source	df	MS	F
Treatment (T)	2	29.686	10.454 ***
Ability (A)	2	3.000	1.056
A X B	4	2.121	.747
Error	93	2.840	

*** p< .001

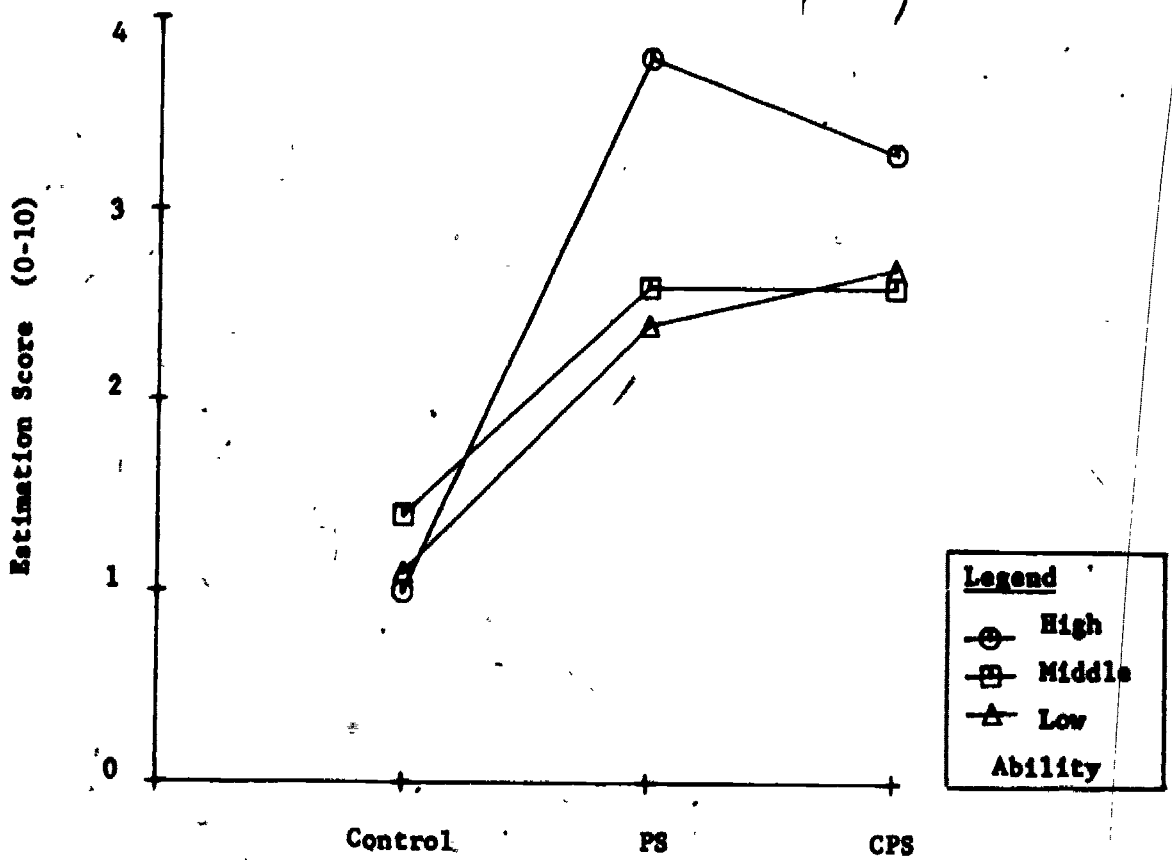

Figure 5. Ability by Treatment Interaction for Estimation.

Table 6

Analysis of Variance on the PST Variable
Representation

Source	df	MS	F	
Treatment (T)	2	77.029	22.619	***
Ability (A)	2	17.129	5.03	**
A X T	4	5.389	1.582	
Error	93	3.406		

** p< .01
*** p< .001

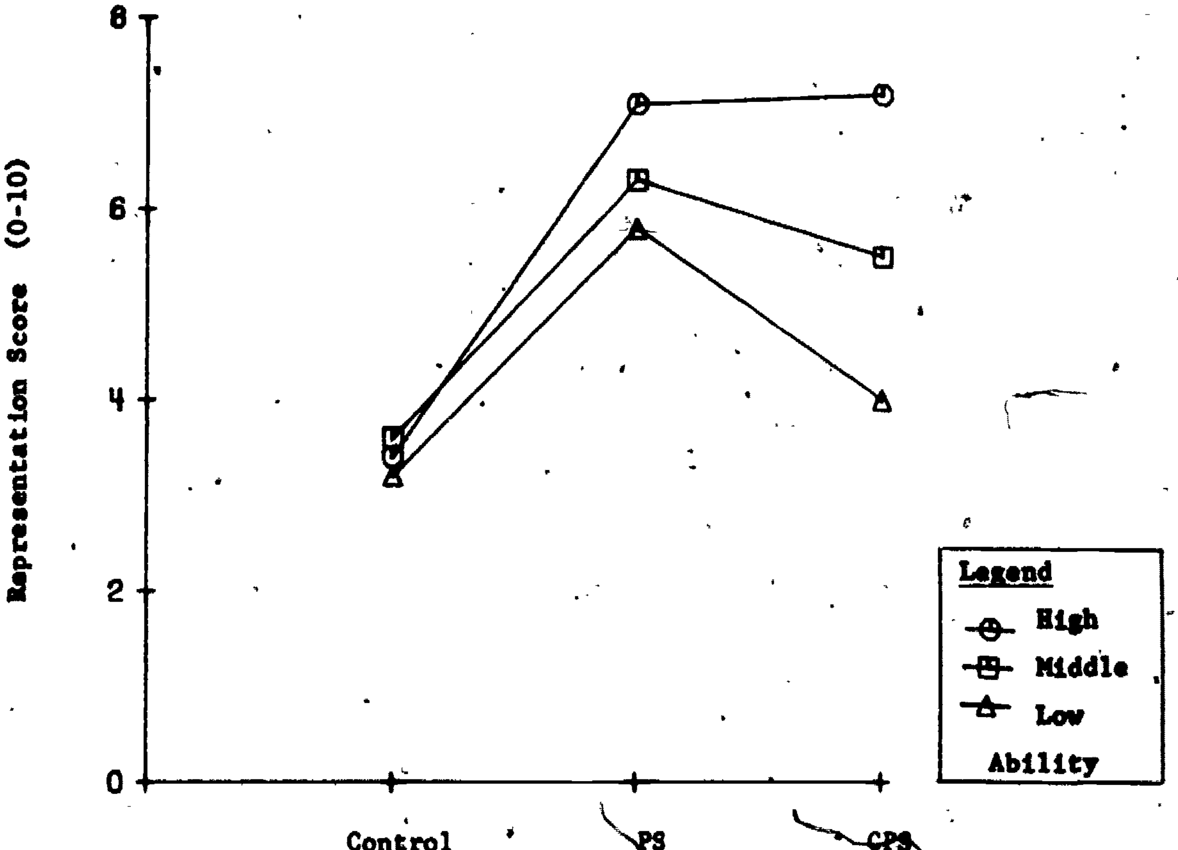

Figure 6. Ability by Treatment Interaction for Representation.

not significant. The Ability effect was significant ($p < .01$) with higher ability groups having higher means. Figure 6 shows one interesting pattern. The low ability CPS group mean was lower than the low ability PS group mean. This will be discussed in the next chapter.

Used all Conditions. The two main effects were significant and there was no interaction (see Table 7). The Treatment effect was highly significant with an F value of 24.1. The CPS and the PS group means were each significantly greater than the Control group mean while the CPS-PS difference was not significant. As shown in Figure 7, higher ability groups had higher means.

Organization. The two main effects were significant and there was no interaction (see Table 8). The Treatment effect was highly significant with an F value of 11.2. The CPS and the PS group means were each significantly greater than the Control group mean while the CPS-PS difference was not significant. The ability differences were relatively small in the directions one would expect.

Plan. There was a significant interaction of Treatment and Ability (see Table 9). This interaction resulted from a reverse ordering of the means for the ability groups in the Control group (see Figure 9). The pattern of means for the PS and CPS groups was similar to that for the previous variables. The Treatment main effect

Table 7

Analysis of Variance on the PST Variable
Conditions

Source	df	MS	F	
Treatment (T)	2	24.980	24.062	***
Ability (A)	2	8.677	8.358	**
A X T	4	9.88	.952	
Error	93	1.038		

** $p < .01$
*** $p < .001$

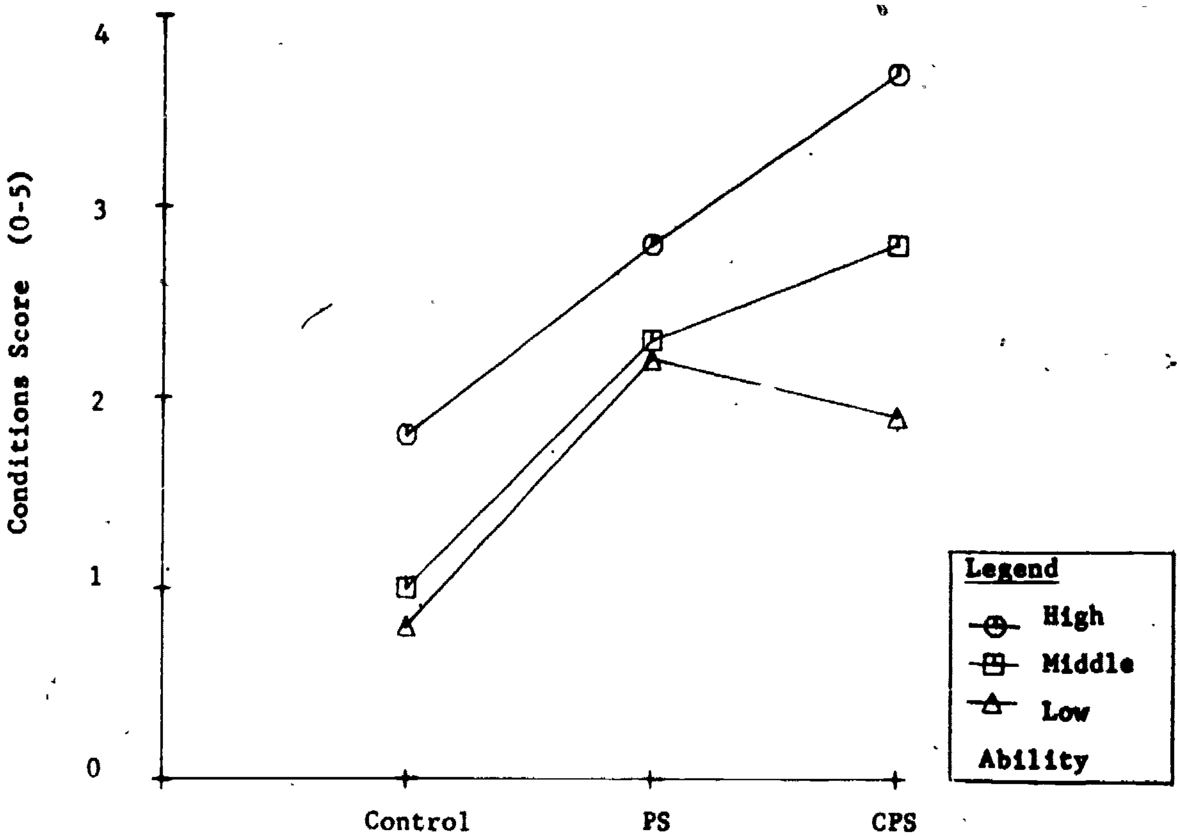

Figure 7. Ability by Treatment Interaction for Conditions.

Table 8

Analysis of Variance on the PST Variable
Organization

Source	df	MS	F	
Treatment (T)	2	39 647	11.250	**
Ability (A)	2	15.568	4.417	*
A X T	4	4.306	1.222	
Error	93	3.524		

* p< .05
** p< .01

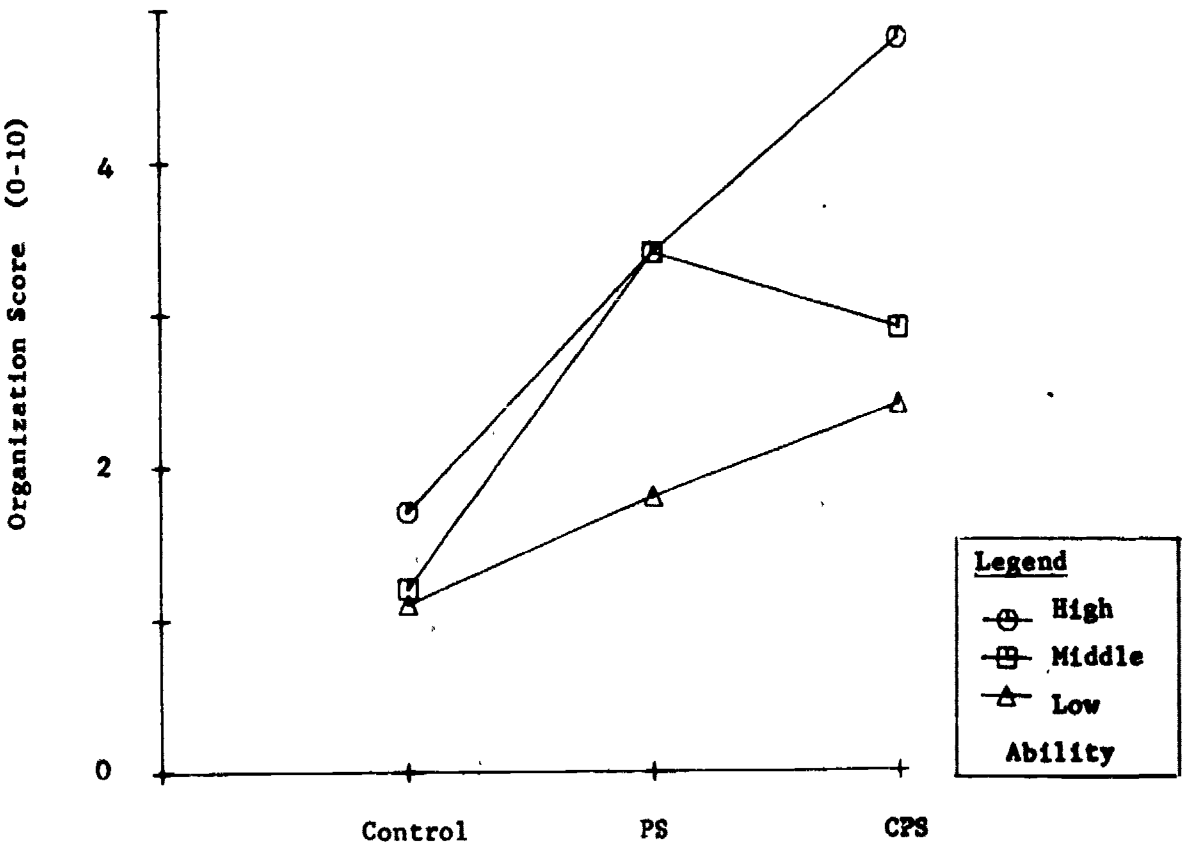

Figure 8. Ability by Treatment Interaction
for Organization

Table 9

Analysis of Variance on the PST Variable
Plan

Source	df	MS	F	
Treatment (T)	2	39.775	13.237	***
Ability (A)	2	12.037	4.006	*
A X T	4	13.725	4.568	**
Error	93	3.005		

* p<.05
** p< .01
*** p< .001

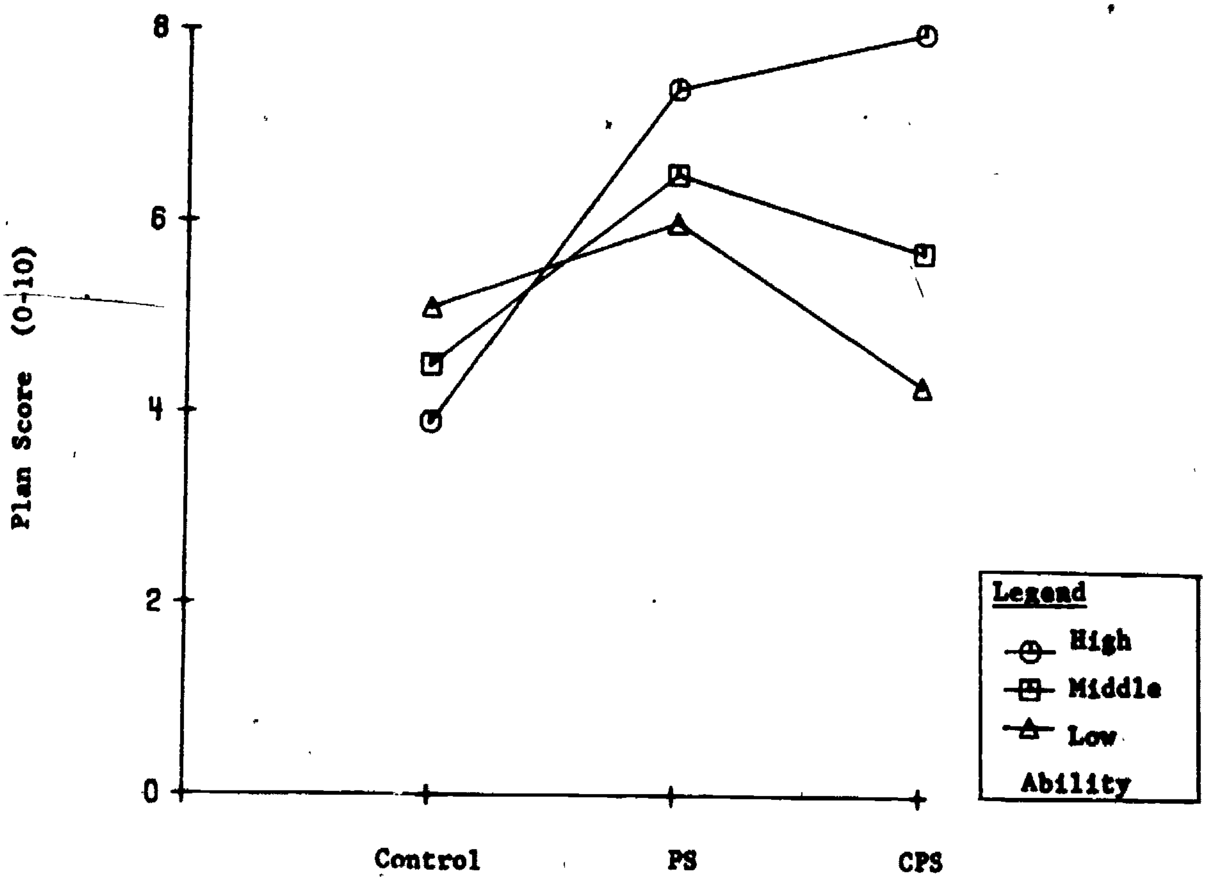

Figure 9. Ability by Treatment Interaction for Plan.

F was 13.2 with the CPS and PS group means higher than the Control group mean.

Looking Back. The Wilson ANOVA showed a pronounced Treatment effect (see Table 10). Figure 10 shows that the CPS and PS group means were much greater than the Control group mean. There were no Ability effects.

Confidence. There was a significant Treatment main effect (see Table 11). The CPS and PS group means were significantly greater than the Control group mean while the CPS and PS did not differ. Although the Ability main effect was not significant, the predicted ability differences were observed in the PS and CPS groups but in the Control group they were neutralized by a reverse ordering of the ability means (see Figure 11).

Persistence. There was a significant Treatment main effect (see Table 12). The CPS and PS group means were each significantly greater than the Control group mean while the CPS and PS means did not differ. There were no Ability or interaction effects.

Computational Error. Because the Computational Error scores were not normally distributed, parametric ANOVA procedures could not be used. Wilson's ANOVA procedures showed a significant Treatment effect but no Ability or interaction effects (see Table 13). The PS group made

Table 10

Wilson's Two-Way Analysis of Variance on the PST Variable Looking Back

Source	df	χ^2	
Treatment (T)	2	24.267	***
Ability (A)	2	.180	
A X T	4	2.639	
Total	8	27.087	

*** p< .001

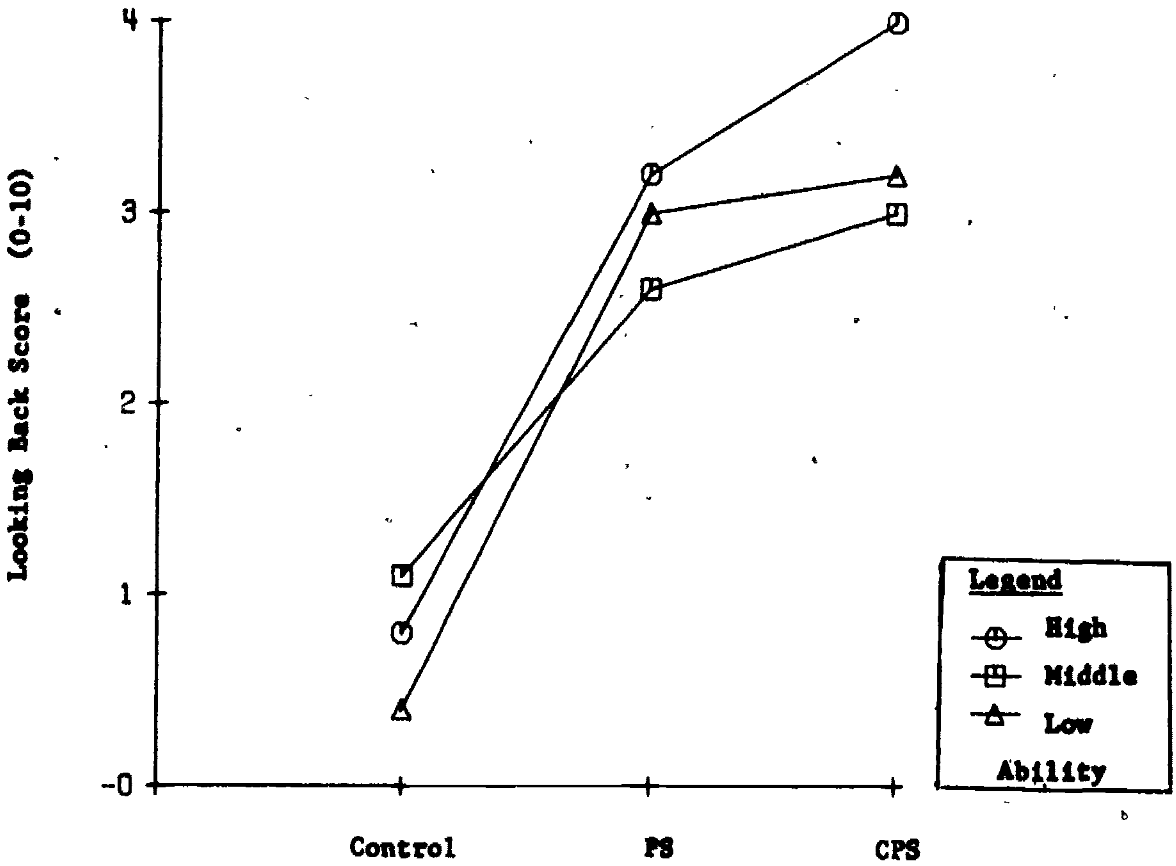

Figure 10. Ability by Treatment Interaction for Looking Back.

Table 11

Analysis of Variance on the PST Variable
Confidence

Source	df	MS	F	
Treatment (T)	2	51.775	14.405	***
Ability (A) n	2	9.497	2.642	
A X T	4	9.132	2.541	
Error	93	3.594		

*** p< .001

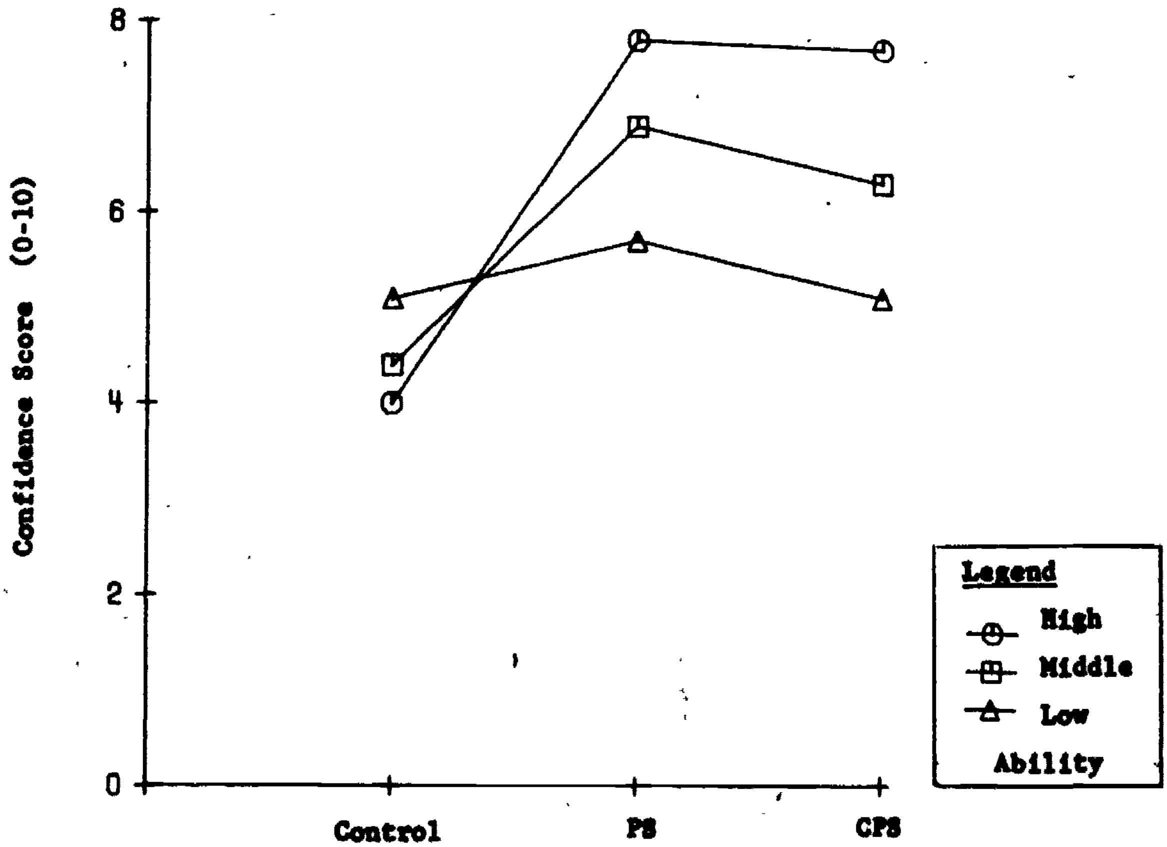

Figure 11. Ability by Treatment Interaction
for Confidence.

Table 12

Analysis of Variance on the PST Variable
Persistence

Source	df	MS	F	
Treatment (T)	2	58.127	14.800	***
Ability (A)	2	10.807	2.752	
A X T	4	1.348	.343	
Error	93	3.927		

*** p< .001

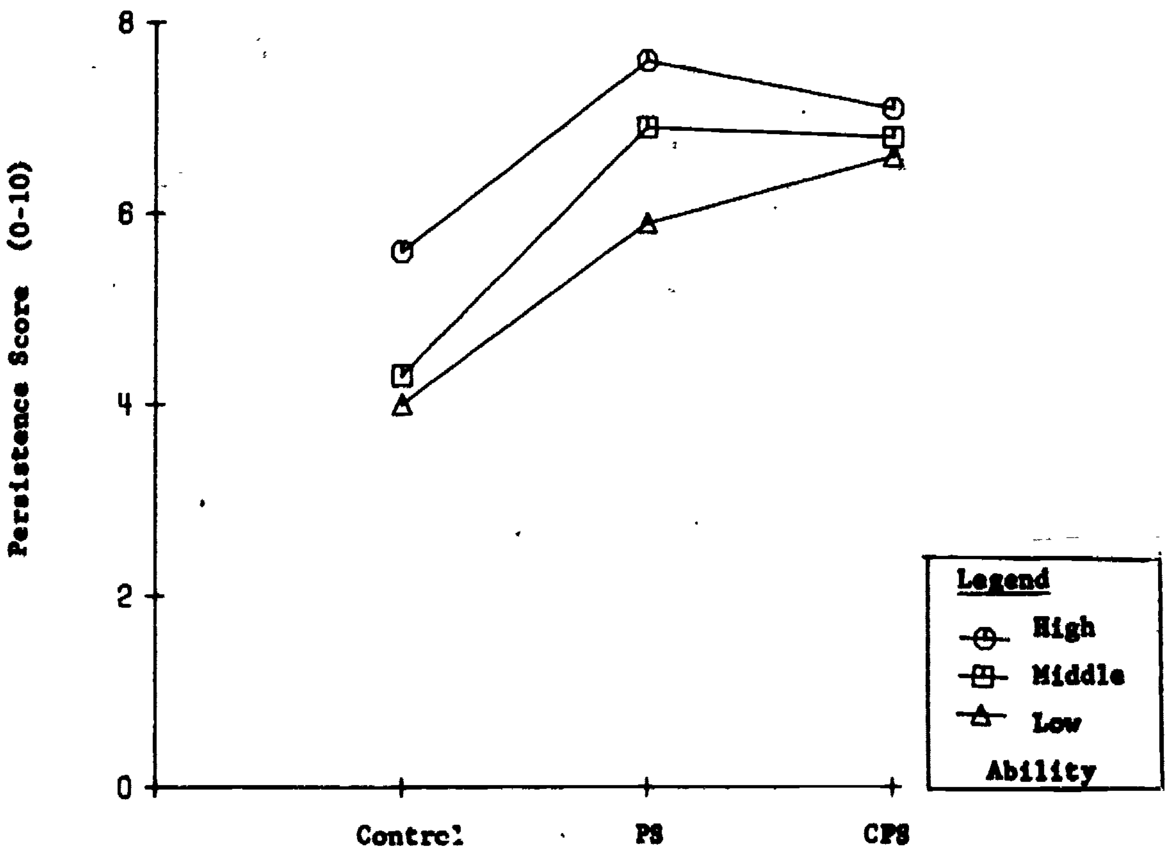

Figure 12. Ability by Treatment Interaction for Persistence.

Table 13

Wilson's Two-Way Analysis of Variance on the PST Variable Computational Errors.

Source	df	χ^2
Treatment (T)	2	12.584 **
Ability (A)	2	2.783
A X T	4	3.744
Total	8	19.111

** $p < .01$

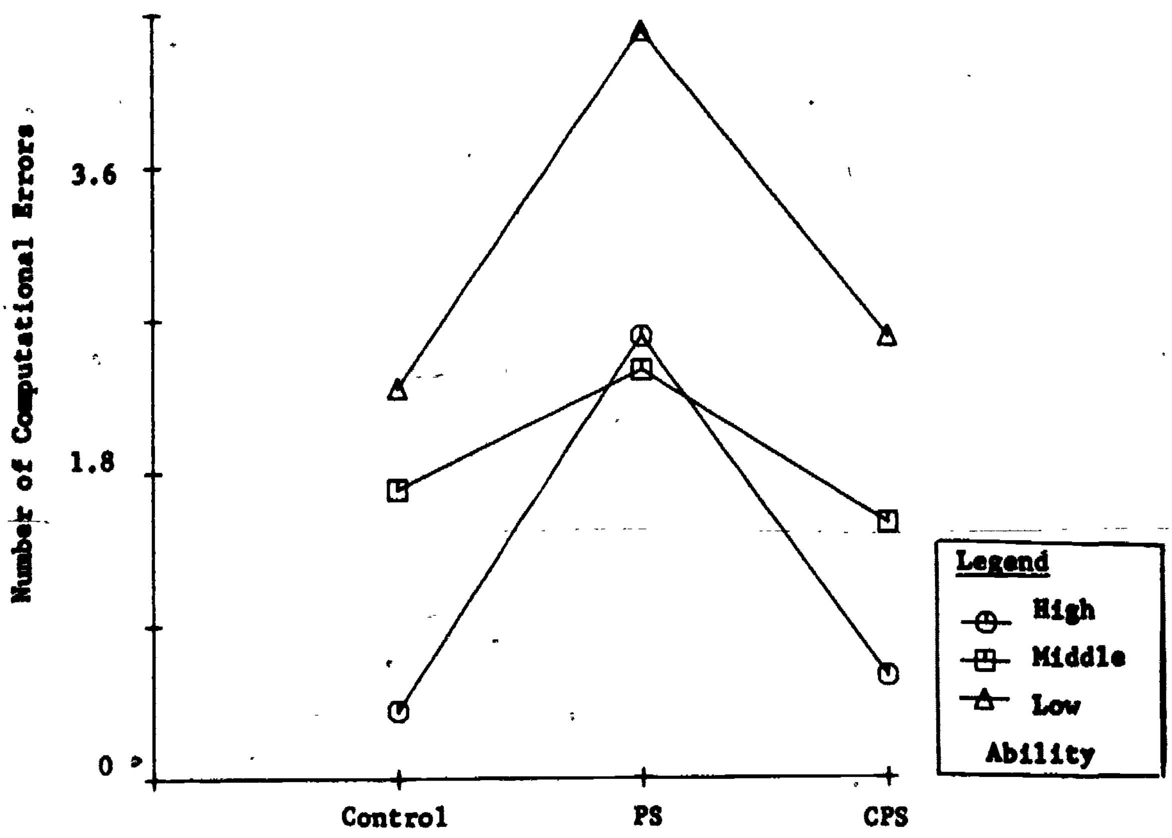

Figure 13. Ability by Treatment Interaction for the Number of Computational Errors.

Table 14

Analysis of Variance on the PST Variable
Correct Solution

Source	df.	MS	F	
Treatment (T)	2	197.922	15.565	***
Ability (A)	2	120.168	9.450	***
A X T	4	8.941	.703	
Error	93	12.716		

*** p< .001

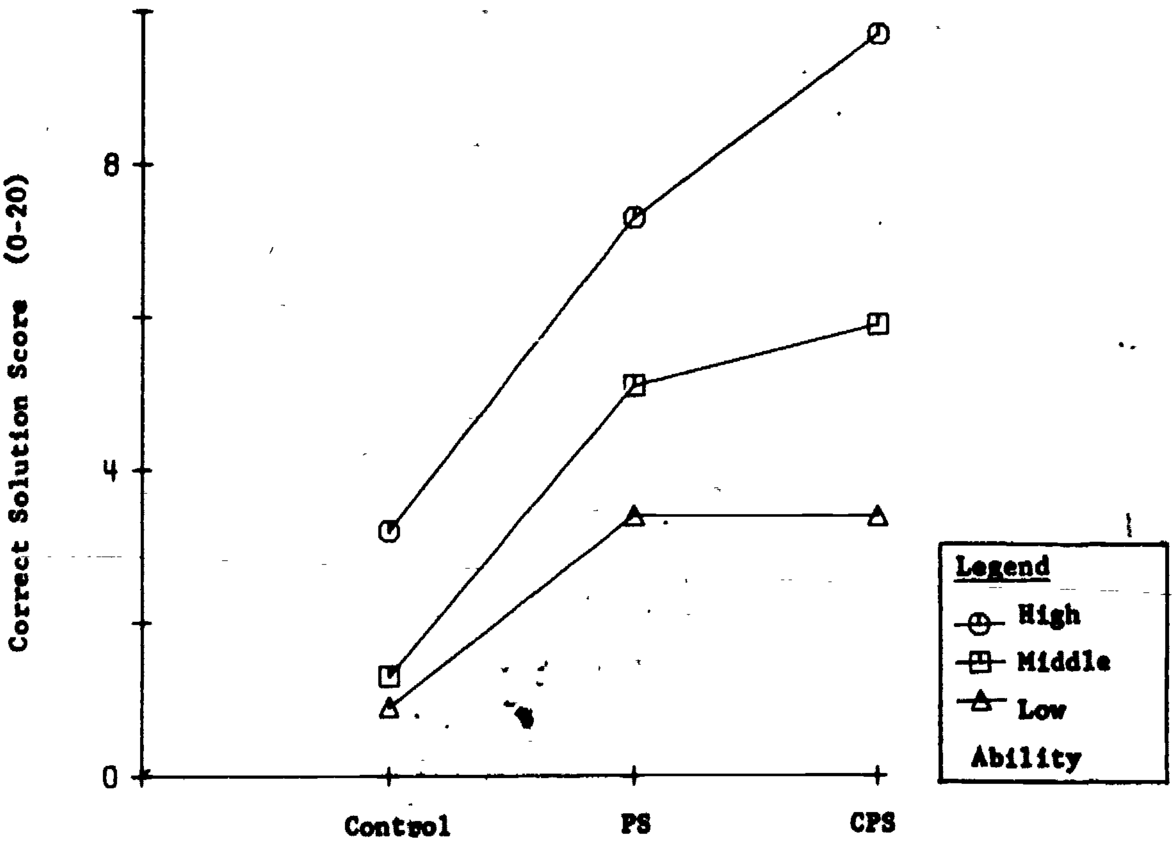

Figure 14. Ability by Treatment Interaction for Correct Solution.

Table 15

Analysis of Variance on the PST Variable
Correct Method

Source	df	MS	F
Treatment (T)	2	348.951	28.966 ***
Ability (A)	2	151.391	12.567 ***
A X T	4	10.864	.902
Error	93	12.047	

*** p< .001

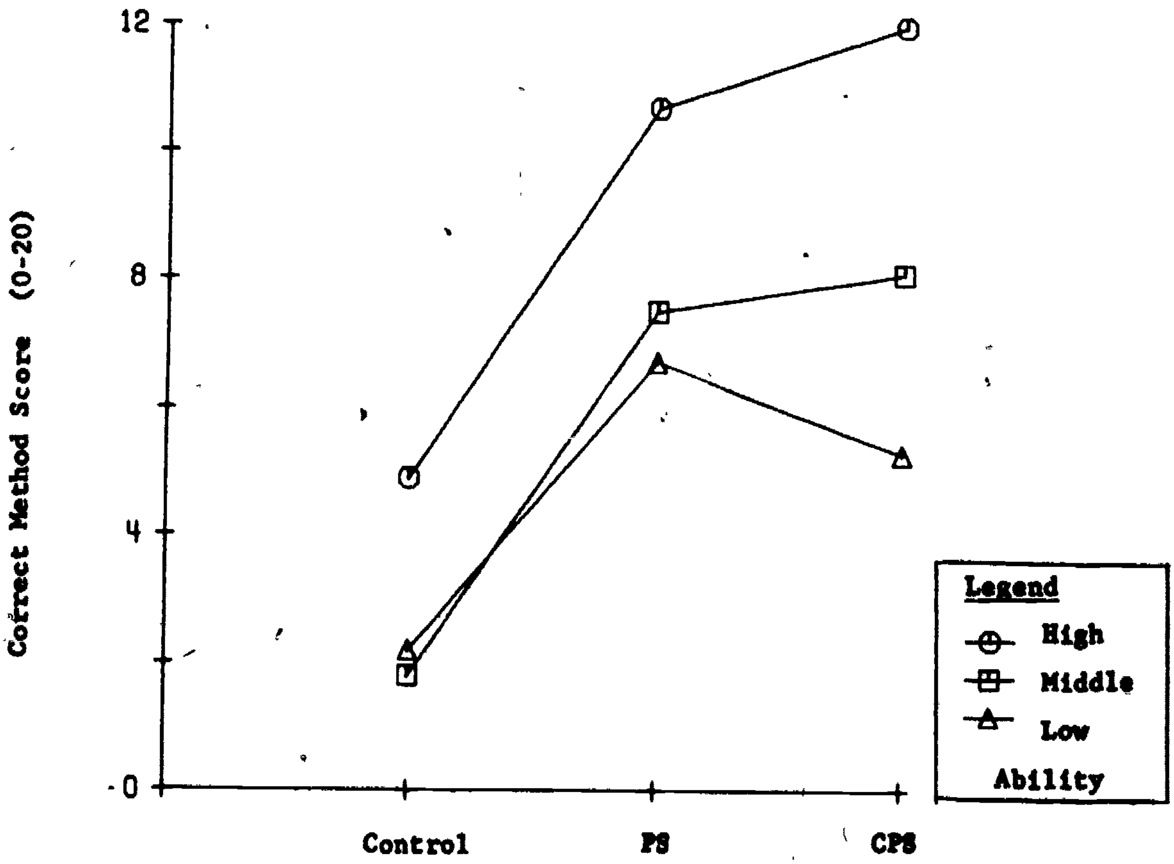

Figure 15. Ability by Treatment Interaction for Correct Method.

Table 16

Wilson's Two-Way Analysis of Variance on the PST Variable Time

Source	df	χ^2
Treatment (T)	2	24.284 ***
Ability (A)	2	2.642
A X T	4	10.322
Total	8	37.248

*** p< .001

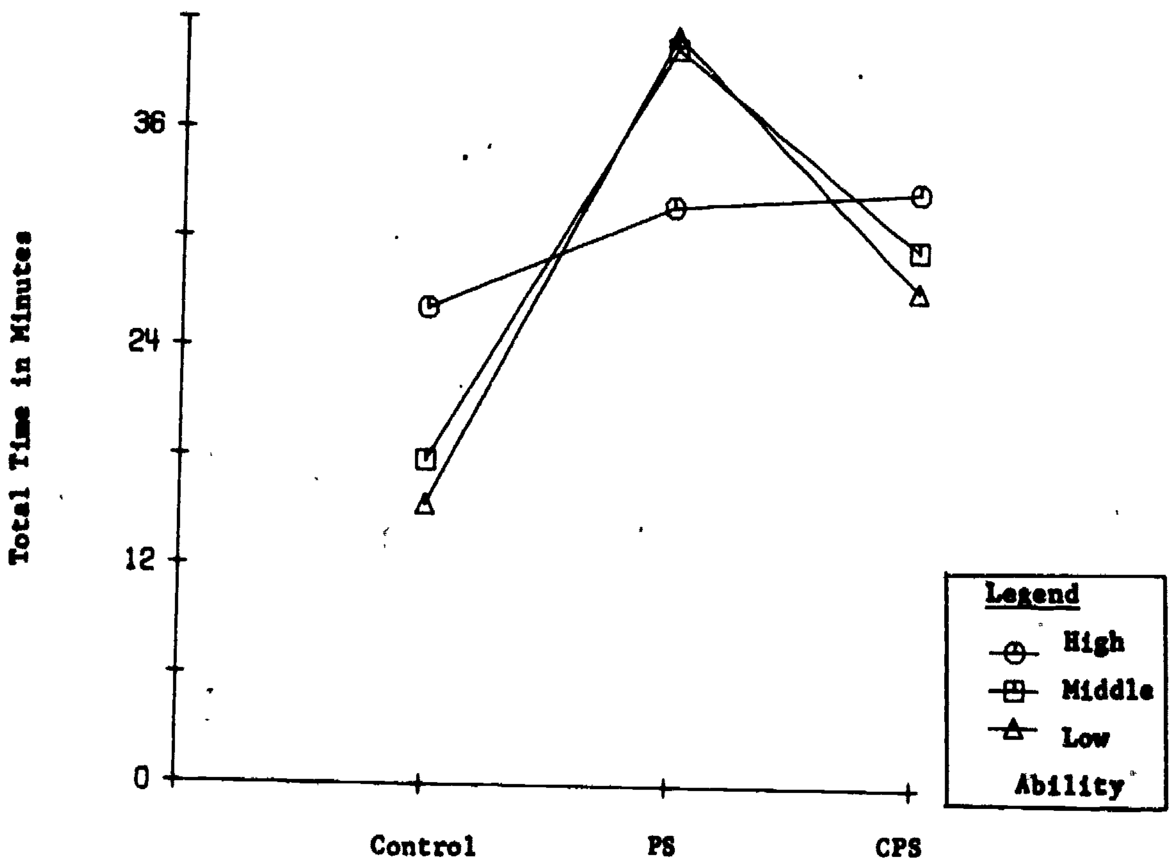

Figure 16. Ability by Treatment Interaction for the Total Time in Minutes.

significantly more errors than either the CPS or Control group. In both the Control and CPS groups the Ability group differences were large in the predicted directions (See Figure 13). The number of computational errors of the PS, CPS, and Control groups were, respectively, 3.0, 1.5, and 1.5.

Correct Solution. The Success score was the sum of the Correct Solution and the Correct Method scores. As can be seen from Figures 14 and 15, the results for both Correct Solution and Correct Method were quite similar and paralleled the results for Success.

Time. The Time variable was examined using Wilson's ANOVA. There was a significant interaction effect (see Table 16 and Figure 16) resulting from a disordinal performance of the high ability PS group. There was little difference between the means of the high ability groups across treatment. The results for the low and middle ability groups were nearly identical within treatment, while the PS subjects took longer than either the CPS or Control subjects.

Calculator Frequency. Wilson's ANOVA revealed a Treatment by Ability interaction (see Table 17). As can be seen in Figure 17, this interaction resulted from the disordinal performance of the ability groups in the PS treatment. In both the Control and CPS groups, the higher

Table 17

Wilson's Two-Way Analysis of Variance on the PST Variable Calculator Frequency

Source	df	X^2
Treatment (T)	2	17.949 ***
Ability (A)	2	.391
A X T	4	8.165
Total	8	26.505

*** $p < .001$

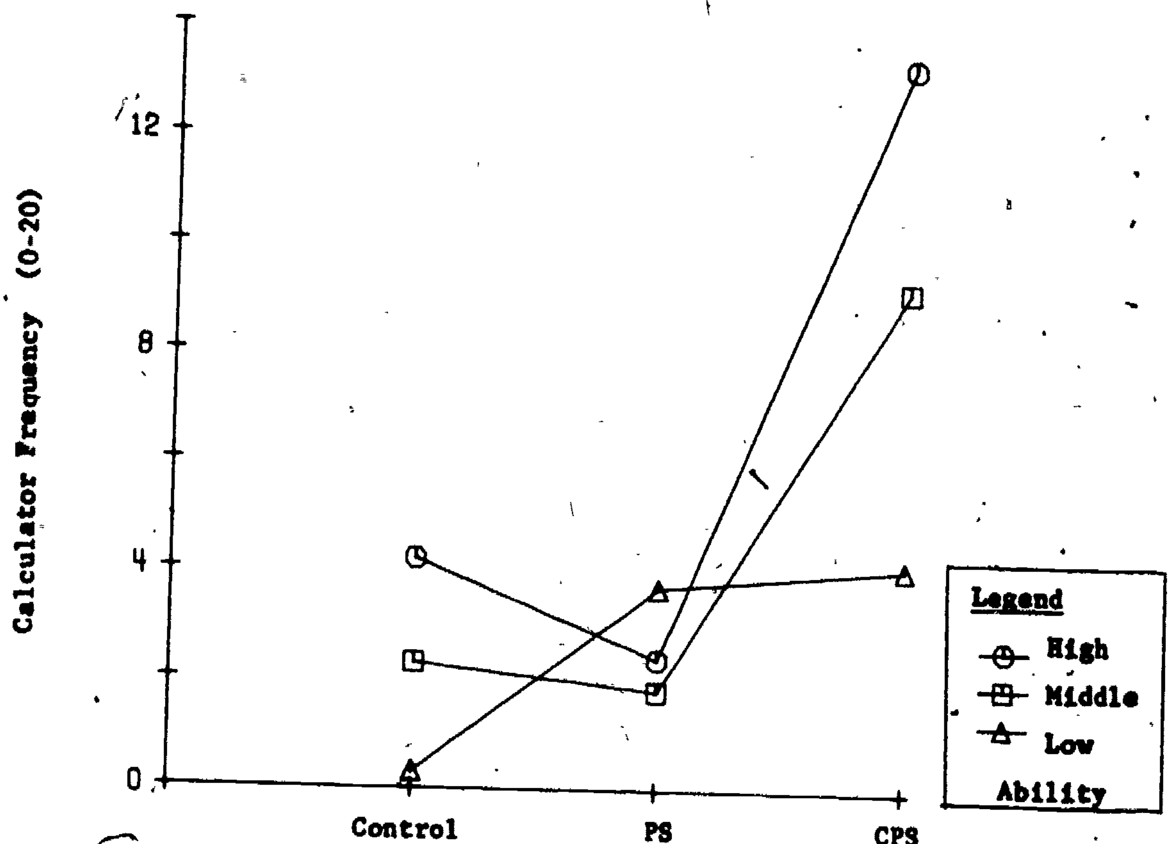

Figure 17. Ability by Treatment Interaction for Calculator Frequency.

Table 18

Wilson's Two-Way Analysis of Variance on
PST Variable Quality of Calculator Use

Source	df	χ^2	
Treatment (T)	2	19.419	***
Ability (A)	2	.290	
A X T	4	6.961	
Total	8	26.670	

*** $p \leq .001$

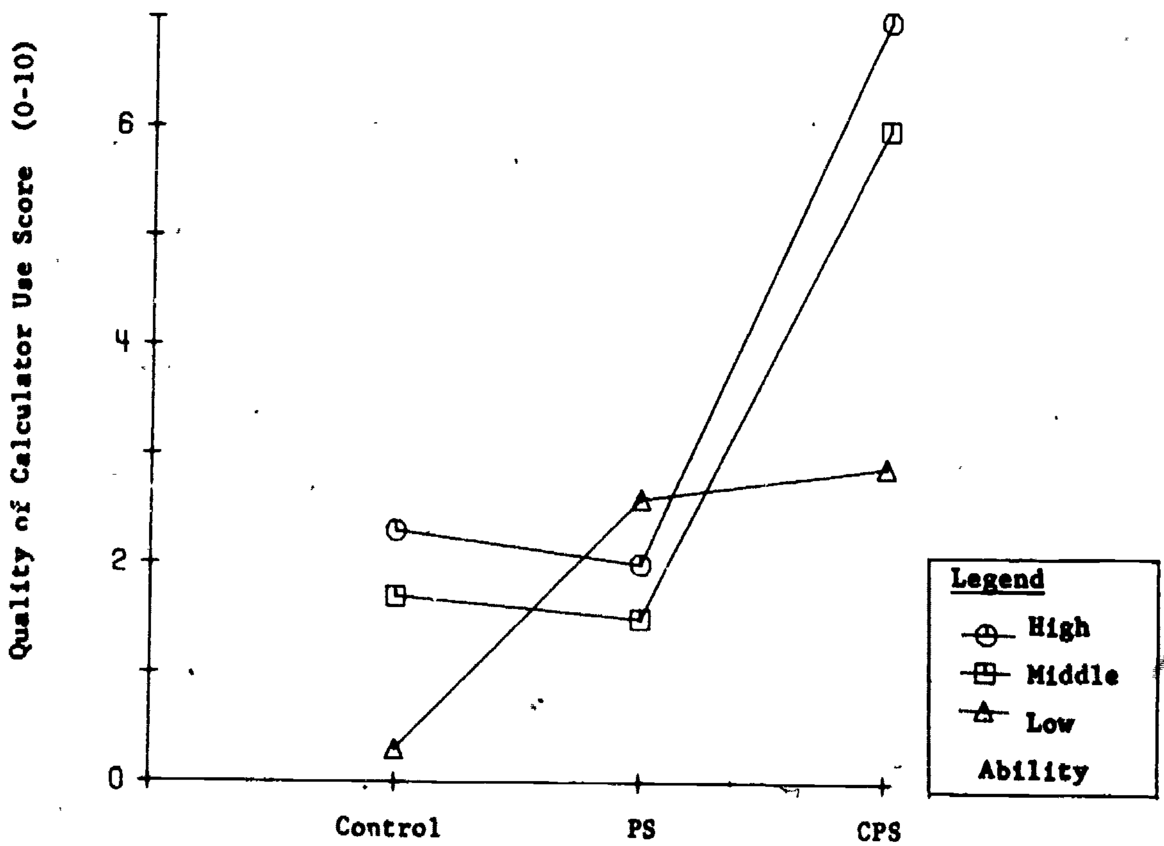

Figure 18. Ability by Treatment Interaction for Quality of Calculator Use.

Table 19

Intercorrelations of 16 Interview Ratings (n=102)

Variable

	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1. Strategy	.79	.57	.74	.75	.67	.65	.67	.60	.61	-.04	.71	.80	.52	.37	.42
2. Success		.55	.74	.85	.58	.69	.70	.55	.66	-.22	.96	.96	.37	.48	.51
3. Estimation			.65	.43	.33	.56	.61	.57	.40	-.11	.54	.55	.17	.30	.36
4. Representation				.60	.54	.84	.66	.59	.64	-.06	..9	.74	.39	.25	.30
5. Used all Conditions					.49	.52	.57	.38	.60	-.12	.75	.84	.44	.47	.52
6. Organization						.53	.48	.41	.39	-.06	.55	.60	.24	.25	.22
7. Plan							.58	.60	.48	-.06	.63	.68	.19	.20	.22
8. Looking Back								.46	.58	-.06	.70	.66	.47	.35	.36
9. Confidence									.42	-.06	.53	.52	.15	.29	.35
10. Persistence										.06	.62	.64	.50	.28	.32
11. Computational Error											-.31	-.13	.27	-.32	-.31
12. Correct Solution												.87	.30	.49	.50
13. Correct Method													.41	.43	.46
14. Time														.14	.16
15. Calculator Frequency															.90
16. Calculator Use															

the ability, the more often the calculator was used. However, in the PS group, the low ability subjects used the calculator most frequently It is interesting to note that while low ability Control subjects made almost no use of the calculator, the low ability PS group made more use of the calculator than either the middle or even the high ability group. Yet the low ability CPS group made relatively little use of the calculator compared to the other CPS ability groups. The Treatment effect was quite pronounced with CPS subjects using calculators much more frequently than either of the other two treatment groups (8.9 vs. 2.6 and 2.3). Figure 17 shows that there were large mean differences between the ability groups within the CPS group with frequency of use parallelling ability. This will be discussed at length in the next chapter.

Quality of Calculator Use. The results for this variable are quite similar to frequency of use (see Table 18 and Figure 18). Evidently these two variables were measuring the same subject characteristics.

Interrelationship of variables on the PST

An examination of the intercorrelation matrix of the 16 PST variables showed that most variables were highly correlated (see Table 19). A factor analysis of the 16 PST variables was performed using a varimax rotation. The results are shown in Table 20. Three factors resulted. It is clear that the variables loading on factor one, Success, Conditions, Representation, Estimation, Looking Back, Organization, Confidence, Persistence, and Plan are measuring the same general attribute. Factor One could be called a problem solving performance factor. Table 19 shows that these variables were indeed highly correlated. Factor Two could be called a time factor. Note that persistence tends to load on this factor; not an unexpected result. The third factor represents a calculator effect factor. Calculator Frequency and Calculator Use do not load on either of the other two factors.

Table 21 shows regression analyses using Success as the dependent variable and the other 13 variables as independent variables (Correct Solution and Correct Method were omitted because they are components of Success). All possible subsets were considered in the analysis (McCabe, 1978). The best single predictor of Success was Conditions, accounting for 56% of the variance. The best combination of five predictors were Conditions, Representation, Looking Back, Confidence, and Computational Error which together accounted for 86% of the variance.

Number of strategies used

In an attempt to understand the nature of strategy use, several analyses of strategy use were performed. The

Table 20

Factor Analysis of 16 PST Variables
After Varimax Rotation

Variable	Factor 1	Factor 2	Factor 3
Strategy	.79	.24	.34
Success	.85	.39	.18
Estimation	.65	.19	-.02
Representation	.88	.06	.16
Used all Conditions	.65	.41	.33
Organization	.62	.11	.13
Plan	.86	-.01	-.04
Looking Back	.70	.21	.27
Confidence	.65	.15	-.03
Persistence	.60	.17	.42
Computational Error	-.11	-.37	.36
Correct Solution	.80	.40	.10
Correct Method	.83	.32	.24
Time	.25	.06	.84
Calculator Freq.	.19	.93	.06
Calculator Use	.23	.92	.05

Table 21

Regressions Analyses of Interview Ratings with Success as the Dependent Variable

Variables in Regression	R
One at a Time	
Conditions	.73
Strategies	.62
Representation	.55
Looking Back	.50
Plan	.47
Persistence	.43
Organization	.34
Confidence	.31
Estimation	.31
Calculator Use	.26
Calculator Frequency	.24
Time	.14
Computational Error	.05
Five at a Time	
Conditions Representations Looking Back Confidence Computational Error	.86

Table 22

Average Number of Different Strategies Used per Subject on All Problems

	Treatment Group		
	CPS	PS	Control
Mean	3.47	3.56	1.76
S.D.	1.02	1.28	1.22

Table 23

Number of Subjects Using Each Strategy on Problem One

	Treatment Group			
Strategy	CPS	PS	C	Total
Guess and Test	20	23	3	46
Draw a Diagram	0	1	0	1
Make a List	1	1	0	2
Simplify	0	0	0	0
Look for a Pattern	2	4	0	6
Write an Equation	0	1	0	1
Others	1	1	0	2
Total	24	31	3	58

Note. The number of subjects in each treatment group was 34.

Table 24

Number of Subjects Using Each Strategy on Problem Two

Strategy	Treatment Group CPS	PS	C	Total
Guess and Test	5	2	1	8
Draw a Diagram	[illegible]	0	0	1
Make a List	24	22	10	56
Simplify	0	0	0	0
Look for a Pattern	31	29	20	80
Write an Equation	0	0	0	0
Others	4	4	0	8
Total	65	57	31	153

Note. The number of subjects in each treatment group was 34.

Table 25

Number of Subjects Using Each Strategy on Problem Three

Strategy	Treatment Group CPS	PS	C	Total
Guess and Test	20	16	5	41
Draw a Diagram	0	0	0	0
Make a List	1	1	1	3
Simplify	0	1	0	1
Look for a Pattern	2	0	1	3
Write an Equation	0	0	0	0
Others	3	10	3	16
Total	26	28	10	64

Note. The number of subjects in each treatment group was 34.

Table 26

Number of Subjects Using
Each Strategy on
Problem Four

Strategy	Treatment Group CPS	PS	C	Total
Guess and Test	2	0	0	2
Draw a Diagram	12	12	4	28
Make a List	0	1	0	1
Simplify	0	1	0	1
Look for a Pattern	0	0	0	0
Write an Equation	0	0	0	0
Others	5	3	0	8
Total	19	17	4	40

Note. The number of subjects in each treatment group was 34.

Table 27

Number of Subjects Using
Each Strategy on
Problem Five

Strategy	Treatment Group CPS	PS	C	Total
Guess and Test	29	31	20	80
Draw a Diagram	1	2	0	3
Make a List	16	21	2	39
Simplify	0	0	0	0
Look for a Pattern	4	6	0	10
Write an Equation	0	0	0	0
Others	6	2	1	9
Total	56	62	23	141

Note. The number of subjects in each treatment group was 34.

mean number of different strategies used by each subject on the five problems of the PST was computed for each treatment group. Results are shown in Table 22. The CPS and PS groups used a greater variety of strategies than the Control group while there was no difference between the CPS and PS groups. The CPS and PS groups averaged more than three different strategies used on the PST while the Control group averaged less than two strategies.

Tables 23-27 show the number of subjects using each strategy on each problem. It can be observed that the CPS and PS groups consistently used more strategies than the Control group irrespective of the particular problem.

Results on the Iowa Problem Solving Project Test

The analysis of covariance on the IPSP posttest scores with IPSP pretest as a covariate resulted in a Treatment F value of 4.61, significant at the .01 level (see Table 28). The IPSP posttest means were 20.4, 19.3, and 17.7 for the CPS, PS, and Control groups respectively. Using Newman-Keul's post hoc analysis on difference of means, it was found that the CPS mean was greater than the Control group mean but all other differences were nonsignificant. However, the PS-Control comparison approached significance at the .05 level. The analysis for the 102 subjects taking the PST is shown in Table 29. Thus, not only did the Success score on the PST show treatment differences but the

Table 28

Analysis of Covariance of IPSP Posttest Scores for the Total Sample with Pretest Scores as a Covariate

Source	df	MS	F
Treatment (T)	2	114.734	9.385 ***
Ability (A)	2	2.166	.177
A X T	4	17.165	1.404
Error	320	12.225	

*** $p < .001$

Table 29

Analysis of Covariance on IPSP Posttest Scores for the Interviewed Subjects with Pretest Scores as a Covariate

Source	df	MS	F
Treatment (T)	2	61.56	4.61 **
Ability (A)	2	12.98	.97
A X T	4	3.13	.23
Error	92	13.36	

** $p < .01$

IPSP results substantiate this finding. Taken together the two results provide strong support for the statement that the problem solving treatment was effective in improving problem solving performance.

Case Studies

Case Study Identification and Data Collection

The case study subjects were identified during the first three weeks of school. In order to obtain information about students for case study identification, students in groups of five to seven, were interviewed informally about summer vacation, hobbies, interests, and school. Students were also observed in mathematics class during regular visits. Specific notes were made about the responsiveness and participation of each child. Based on the interviews and observations, seven students were identified as case study subjects. Criteria for case study selection were:

1. Only students in the PS or CPS treatment group were considered.
2. Both boys and girls were represented.
3. The case study subjects were able to communicate orallv to teachers, observers, and other students.
4. Each of the high, middle, and low ability groups were represented.

Only the investigating team knew of the selection of case study subjects. Teachers, parents, or students were not informed of this aspect of the project until May, 1981.

The case study subjects were observed each week during the treatment phase of the project. An observation form was developed for use during these observations. Every effort was made to observe closely and to talk with the case study subjects in a group or individually during each visit.

The teachers were asked to keep a semester log of observations and comments. Information in the teachers' log related to case study subjects was combined with the observational data recorded by the investigating team in preparing the case study reports.

At the end of the treatment phase of the project, each case study subject was interviewed to obtain responses to specific questions. A list of questions for interviews with case study subjects, parents, and teachers was written and compiled for use at the end of the observation period. These questions were the basis for discussion, yet the questions were not posed in a fixed order. A member of the investigating team conducted an interview with the subject, the present teacher, the former teacher, and one parent. Direct quotes were recorded during the interview.

The case study subjects were included in the sample of subjects to take the PST. Sample work was also collected where possible.

Each case study subject folder included:

1. Completed observational forms
2. Pretest and posttest data
3. Attitude data

4. Anecdotal data

5. Subject interview question responses

6. Parent/teacher interview responses

The above data constituted the information for this phase of the final report. Figure 19 summarizes the case study identification and data collecting process.

Description of the case study subjects

Paula is a typical sixth grade girl who has a strong drive to be popular with her peers. She is pleasant, kind, and most cooperative. She is a high average achiever who receives good grades yet does not score as well on achievement tests. She has always been rated superior by her teachers on school tasks and daily assignments. She excels in carrying-out tasks by following explicit directions. Paula's parents are both active in community affairs, and her social posture in school reflects her parents' social concerns. She strives for peer approval. Often it was difficult to observe Paula because she was hesitant to share her work. She did not want to make mistakes or to be seen using a "wrong" method.

Bob is a below average student who has strong family support. He is a typical boy with interest in baseball,

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* The names of the case study subjects have been changed to protect their anonymity.

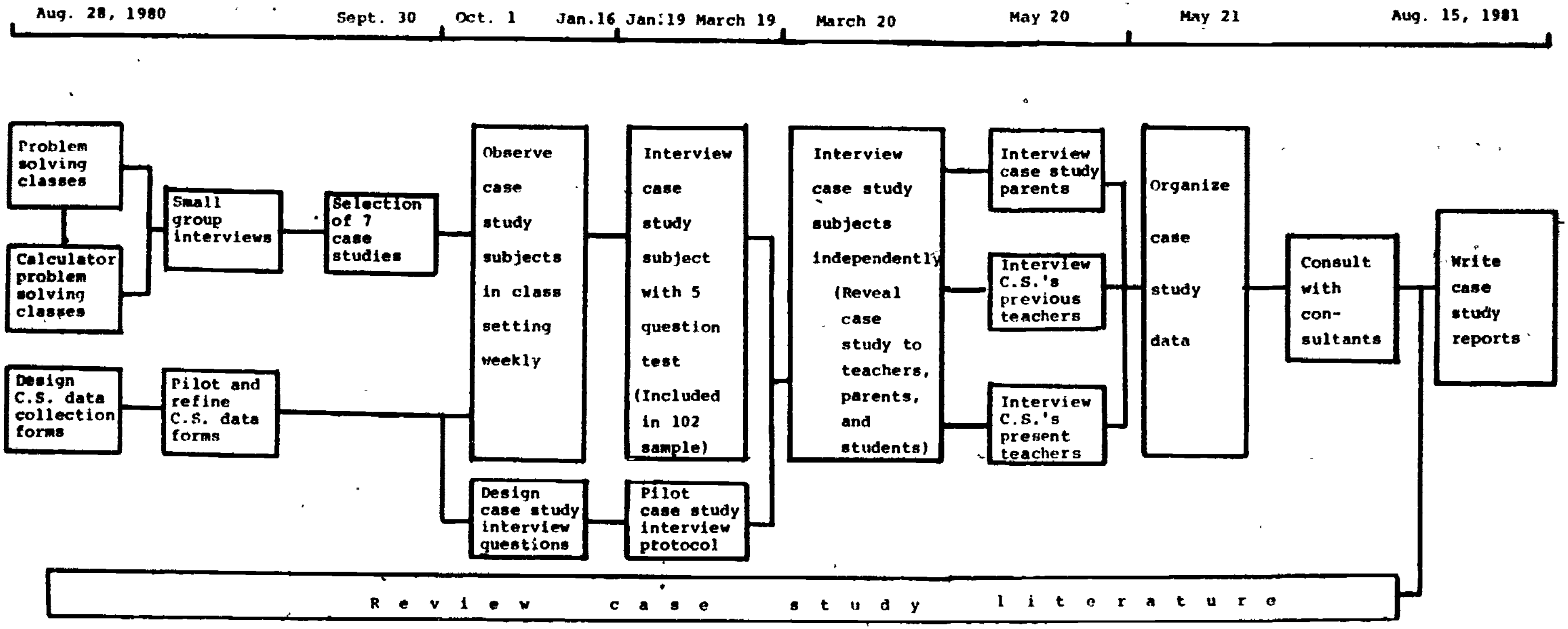


Figure 19. Case study identification process and data collection process

basketball, and camping. He could be classified as a plodder; as one who works dilligently. Reading and math have presented him problems throughout his school years. Although he is a cooperative student and tries to please, he is not enthusiastic about school work.

Bill is a quiet, sensitive boy who achieves low average results. He is serious about school and tries to please his teachers and parents. Bill has reacted negatively to a younger sibling who receives much attention due to a medical problem. Bll blossoms with success and support. Because of his sensitivity and passivity, he is not well accepted by his peers. Without peer acceptance, he chooses to play with younger children or girls. He is known to be a follower or a loner in school. Bill provided us with some insightful data throughout the study.

Tom is a bright-eyed, active, alert boy who responds to a challenge; the bigger the task, the harder he works and achieves. He has presented his teachers and parents problems in previous years due to his poor achievement and work habits. Tom is classified as an under-achiever. He has not become responsible for his own learning and thus is unable to direct his efforts toward positive results. He has a stable family structure providing him much support. Tom provided us much insight into how a bright student copes and survives in school.

Harry is a slow student who has a history of poor school achievement and poor study habits. In previous years his composite rankings on national achievement tests range between the 10-20 percentiles. (Current year - 17th percentile.) He also has had family problems which cause him much difficulty; problems like re-adjustment to a new father, step brothers, and step sisters. Very little interest and genuine concern for him is exhibited by his family. Harry is labeled a trouble-maker in school and has required much teacher attention. Because of his attention getting behavior, his presence in the classroom is usually noted. In order to keep him on task, Harry was often isolated from the class. He has energy which needs to be directed. His sixth grade performance, school attitude, and self-concept changed during the year.

Sue is an alert, attractive, and mature girl who might be classified as gifted or talented, yet she has a poor achievement record in mathematics. Math has presented many problems for her and she was able to vocalize much insight to her response to math instruction in school. Sue is an avid reader, an artist, a musician, a poet, and a world traveler. She has many friends and relates well to persons of all ages. Her parents are both professionals who treat her very adult-like and insist on her being independent.

Sam is a high average student who achieves average results in school. He is known to be an inconsistent worker; yet at times he is insightful, organized, and logical. He performs very well when motivated. His attention span and performance record are directly related to his motivational level; he "turns on" or "tunes out" depending on his interest in the task. Sam relates well to his family. They are frustrated with his inconsistent school record. Many times Sam is placed in a "do-or-else" situation before he tackles a task. He is a natural leader providing the direction and support for others. He finds difficult tasks challenging and rewarding.

Response to Problem Solving

Students. Generally speaking, the case study subjects were initially cautious and hesitant in approaching problem solving. As they learned strategies to use in problem solving, however, their attitudes changed. Sam and Tom seemed to respond most positively to problem solving. They were very successful and became quite confident with problem solving experience. On the other hand, Paul and Sue seemed much more comfortable with the standard textbook approach to mathematics. Even though not a strong student generally, Harry slowly came alive to mathematics as he learned problem solving strategies.

Bob and Bill did not seem to respond as favorably to problem solving. It is interesting to note that none of the three students in the PS Groups responded enthusiastically. While calculator use was a likely influence, the teacher variable should not be ignored. The teachers of Tom, Harry, Paula, and Sam were enthusiastic and effective. Listed below are the student comments.

"I like problem solving."

"I can solve problems."

"I feel good when I solve problems."

"Problem solving makes you think."

"The problems were interesting and challenging."

"I share my problem solving problems with Mom and Dad."

"I like to talk about the problems."

"The problems were fun; it was neat trying to figure out those hard problems."

"I like problem solving. I like to think harder. I liked both easy and hard problems."

"I worked harder last semester than I ever did before in school, because it was fun."

"Everybody in class learned during problem solving time. Nobody learned when we did book work."

"Problem solving makes math real."

"For the first time I remember what I learned earlier. I never will forget the problem solving."

"I love problem solving. It was challenging. I've learned a lot."

"My grades went up during problem solving."

"There would have been no order in our math class this year without the problem solving. We wanted to get the problems done so we worked."

"Math has not been my favorite suject prior to sixth grade because I couldn't remember when tc do what. I was forced to decide when to add/subtract or etc. all by myself this year. Last year I depended on my teacher to tell me what to do."

<u>Teachers.</u> The teacher's response to problem solving was quite varied. Of the ten teachers involved, six were openly enthusiastic about the problem solving approach. The others were supportive but seemed more comfortable with the textbook approach to mathematics instruction. Two teachers in particular became strong advocates of teaching problem solving. All teachers were very cooperative. Teacher comments aabout teaching problem solving are given below.

"My kids like the problem sheets."

"First time I've had no discipline problems. They get so involved that they don't have time to get in trouble."

"The problem solving homework is always done. If there is any work not completed, it definitely is not the math problem solving."

"I teach a concept when I need it to solve a problem."

"Students have to read critically to do a problem. It helps their reading comprehension."

"We now use the problem solving strategies a lot in science and social studies."

"The TOPS cards are really great. All the kids like them."

"I hope we continue with problem solving instruction. I still have lots to learn about it."

"I'm going to keep my materials - I'll share them, but I won't give them away."

Parents. The parents of the case study subjects were interviewed to obtain their perspective of the project and its impact on their child. The parent response was unanimously positive. As a result of the problem solving experience the parents saw their child as more confident, more motivated, more inquisitive, and more willing to talk about school work. These attitudes were shared by all parents interviewed. It is as though previously children had viewed school as a chore, an undesirable task that had to be done in contrast to seeing problem solving as challenging, worthwhile, and fun. The parents were very pleased with these observed changes in their child. The following comments characterize the parents' feelings about problem solving.

"So happy to see my child respond to learning. What a change."

"His school work has improved in all areas."

"He is asking questions again. It seems he has found a need for learning."

"He's less dependent on us at home. He does his math homework first."

"She's become aware that math is real and is useful."

"She has shared her work with us. This is a real first. She asks us about different problems. Many problems we can't do, yet she is able to explain what and how to approach the problems. Wished we had had problems like this when we were in school."

"My son has worked hard this year - more than ever. I think it was because he knew that he must 'dig' to solve these problems. He really wanted to do well."

"My son is so proud of his math grades this year. I'm very proud that he feels this way."

"I'd like to have my child continue with calculators in school. If one year can make this big a change in my child, I'm anxious to have more of the same instruction."

"Guess I'd have to say, I was against my child using a calculator in September. In January I'm saying the opposite. The entire project has been good and helpful for my child."

"So happy to see my son interested in school. This has been his best year in school. Math has always been his poorest subject. This year he has gotten A's and B's and seems to understand it. I want to thank you both very much for helping turn him on to math."

"My child has demanded less help this year. He says he can do it himself."

"My child has shown more interest in learning this year - what a welcomed change."

"I hope problem solving teaching continues in school. My child is much more confident about his abilities."

"The dinner table has been the center of much problem solving discussion in our house."

When questioned about their child's participation in the project, the parents unanimously expressed approval and volunteered favorable comments.

Response to Problem Solving Materials

Students. There were marked diff rences in pupil response to textbook use and problem solving materials. Case study students pointedly contrasted the two:

"Problem solving made math real. In problem solving if you don't understand the book, you just ask the teacher. In problem solving we discuss it, think about it, and then try to solve it before we ask the teacher for help. Problem Solving made me think for myself."

"Problem solving is more interesting."

"In problem solving we do a variety of things. In the book, we just do page after page of 3 x 2 = 6."

"In problem solving we choose which problems we wanted to do and how we did them. The teacher always tells us which page in the textbook, which problem, and what we need to do to complete it. I fall asleep in class and at home doing my math homework."

"The problems are interesting, challenging, and real in problem solving. In the book, they are boring. I never get it finished."

"My parents and I solved my problem solving problems together at dinner. We al enjoyed them."

"My parents say I ask more questions now. I also ask for less help."

"I'd choose problem solving to do over the text because the text is not interesting. I don't have to think or explore. In problem solving I didn't know where to begin - that's the fun part. It became a challenge - Can I do it? I like to explore."

"When we'd go to the book, we all got low grades. Half the time I woudn't do my homework, it was too boring. They have boring examples. Teacher says, 'Flip to page --, do number 1-5.' No reason to do the work except to get a grade."

"Book problems are simple - our problem solving problems were hard - I had to think."

"I like problem solving more than book work. I like to think harder. You have to think hard to do problem solving. I looked forward to the new problems."

"As soon as our class went back to working in our books, the grades went down. We were bored. There are boring examples and problems in the book."

Teachers. Teachers were more hesitant than students to contrast treatment materials with textbook materials. Most expressed a renewed interest in teaching math and enjoyed the interesting problems. Using the problem solving materials made several teachers uncc fortable and uneasy;

teachers seem to adjust more slowly than the students. It was difficult for teachers to adjust to the freedom of exploration, the group strategy, and calculators in the classroom (CPS classes only). Evaluation of pupil work was a problem for some teachers. They were somewhat uncomfortable without definite grades on each assignment. All teachers remarked about the increased interest in mathematics. Comments of teachers follow.

"I'm just beginning to learn what and how to teach the problem solving." (December, 1980)

"It was harder for me than for the kids."

"Hope our achievement scores are good in May."

"I felt like I didn't know what each kid was doing or what he knew."

"I had trouble grading the students."

"Where can I get some of these problems for next year?"

"The challenge problems were a real challenge for me."

"I still use the textbook to teach the basics."

"I can't believe how long we spend on one of those problems."

Response to Group Work

Students. Different teaching methods were encouraged during the treatment phase. Group work was emphasized in all PS and CPS classes. Students responded favorably to the group appr neir comments were:

"Most of the problem solving time we worked in groups. I liked to work in groups because we could help each other. I helped them and then they helped me."

"I felt less pressure when working in groups."

"It was fun to argue about the problems. I was wrong sometimes."

"We learned to question each other."

"We had to explain to the group what we were doing."

"We discussed the problems together and explained them to the class."

"I like using the overhead to explain the problems."

"My teacher gave us extra points if the group could find two different ways to solve the problem."

"We helped the teacher solve several problems. He didn't have the answer."

"I love group work because it is fun to discuss the problem. Everybody has ideas about it. I can lead the group or follow. I like to help kids with their work. It makes me think harder when I have to explain it to someone."

"Each group worked as a team. If you got all the problems, your team got points for the day. I liked my team although our team never won a contest. I like to work in groups."

<u>Teachers.</u> Having students work in small groups was new to most teachers. They usually had their students working independently when they were not presenting to the entire class. The teachers' response to group work was varied. Some teachers took to it immediately but others were not comfortable with students in groups. They seemed to feel less in control. One teacher with many years of experience did not know how to use group work. But after he was assisted in trying it, he used it regularly from then on. Specific comments made by teachers follow.

"I've never used group work before. I like it, yet I feel better when I have the kids working independently."

"I just used group work one day a week."

"I like the involvement of group work."

"Kids have to help each other."

"It seems the children respond well to group work. Maybe they like to talk."

"They have learned to help each other."

"I don't know if I feel comfortable with group work yet. I usually use an interest center for my group work."

"My students surprised me with their problem solving in the groups."

"How do I grade them? I usually give the whole group the same grade."

"I use the team approach giving the team points for its work."

"I have to learn to adjust with the students. Believe me, they've done a great job! I'm very proud of their math scores on the SRA Tests this year."

"My kids have done well. They've jumped two grade levels in math this year."

"I managed to get all the work done, but is was a struggle."

Response to Challenge Problems

Students' responses to difficult problems, especially the math challenge problems, were very surprising. Repeatedly, case study subjects of cross ability levels commented about the desire, excitement, and challenge of hard problems. Their comments follow.

"I like the Math Challenge Problems. They were hard and challenging. I like the 'Fly' one the best. I got most of them. I liked the challenge problems because they made me think the most."

"I like all the challenge problems. I got all but one; I like them best. I had to get them to satisfy myself - I couldn't stand not to get one."

"I like the hard problems the best. They make me think more."

"I got all the Math Challenge Problems. My teacher helped me with two of them. I got 60 points for doing the math challenge problems alone. Could have gotten 80 points if he had not helped me with two."

"The harder the problems, the harder I worked. The challenge problems were my favorites."

"The problems were interesting; some were hard. I love the hard ones; more challenging and fun to figure out. It took me a long time to do some of them, but it didn't seem long because I got involved in the problem. The longest I worked on one problem was one hour. Sometimes I had to come back to it."

Response to problem solving strategies

We were particularly interested in whether the subject recognized and used the problem solving strategies. We were quite impressed with the manner in which the students spoke about the strategies. They knew the names of the strategies and spoke freely about strategy use although most students had a favorite. Among the favorite strategies were Guess and Test, Make a List, and Look for a Pattern. Statistical and observational data substantiated that problem solving strategies were learned and utilized by the case study subjects. Their comments follow:

"I like to use the strategies to help me. My favorite one is Guess and Test, because it is fun to see how close you can get and to try to get close in a few steps. Simplify is my least favorite because I don't understand it. I use Make a List the most - I can use it with just about any kind of problem."

"I like Draw a Diagram the best. I helps me see what I do. I do not use Simplify. I can't break it down; I don't know why. My favorite one is Look for a Pattern."

"My favorite strategy is Make a List because it's easy to see what you're doing. Guess and Test is my least favorite because it takes too long, then I can't see the pattern. I use Draw a Diagram the most. It helps me see what I did. It helps me to draw pictures of problems."

"My favorite strategy is Look for a Pattern because it's fun to try to figure out what the problem is all about. It's like a game. My least favorite strategy is Simplify - I don't know what to do. I use Draw a Diagram the most. I like to see what the answer is going to be."

"My favorite strategy is Guess and Test because it is easy for me. My least favorite strategy is Simplify. It makes more work."

"I use Make a List and Look for a Pattern more than the others because I like to organize what I'm doing.

I write more down on paper with Make a List, but I get more right when I do it."

"The strategies sure helped me to see it more clearly. In math problems when I got stuck, I'd try one of those strategies to begin. Sometimes it worked, sometimes it didn't. I kept trying until one would work."

"I used several of the strategies on the achievement test this year."

"In science I use many of the strategies."

"Draw a Diagram was helpful in Social Studies."

"Look for a Pattern is very useful in all my subjects. I use it when playing games."

"I taught my parents how to use Guess and Test and make a game out of it."

"I try to make as few guesses as possible because it takes so long."

"I learned to ask myself questions like, 'How would I do it?' What strategy to use,' What I need to solve it, and What would I use to solve it?'"

"I learned the strategies - they were easy for me."

"I learned different methods of solving problems (Guess and Test, Make a List, Draw a Diagram, etc.). The strategies helped me to see it more clearly."

"I use problem solving in my other subjects. I used the strategy Draw a Diagram in Social Studies yesterday to solve a problem."

"I used the problem solving strategies on a test recently. I learned to ask myself, 'How would I do it?' What strategy to use? What do I need to solve the problem and what strategy would I use to solve it?' I still know the strategies. I learned to apply what I learned and I remember it."

"When I don't know what to do, I just try lots of things. I use all the strategies."

Response to Calculator Use

Students. Four of the case study subjects were in the CPS group and thus experienced calculator use in problem solving. They were uniformly enthusiastic about using calculators. They seemed to feel more confident in approaching a problem when the calculator was available. It also seemed to foster exploration. That is, if the student had no ideas where to begin, they might just try anything using the calculator. Often this suggested a next step and consequently led to understanding the problem and ultimately to a solution. Student comments regarding calculators were most positive. Their comments were:

"I think I could get hooked on calculators."

"I like using a calculator in math class."

"I like to play with the calculators."

"I got more problems right using the calculator."

"I like to get the work done fast."

"I got more work done using the calculators."

"The calculator works fast. I get more done in a shorter time."

"The calculators helped me to tell if I was close to the right answer."

"I take the original problem, estimate in my head, then estimate on the calculator to see if I was right."

"I'd like to use calculators on tests, too."

"Calculators made math easier for me."

"I like using the calculator and doing those hard problems."

"I like to play calculator games, especially the strategy games. They are really fun."

"I use the memory all the time."

"I like to push 'clear' when I get confused and start all over. I don't like to start over without the calculator."

"Estimation is easy for me with the calculator. My estimation has improved."

"I get more answers correct."

<u>Teachers.</u> While the response to calculator use varied among teachers, it was quite positive; most teachers felt

that calculator-aided problem solving was a beneficial unit of instruction. One teacher was uncomfortable with the new technology but after the semester of use the others were enthusiastic about the use of calculators. Most were cautious at the beginning.

The teachers reported fewer discipline problems and more enthusiasm for mathematics when calculators were used. Their comments were:

"First time I've used a calculator in school. I was apprehensive at first, but I really feel it has a place. The slower kids could do some of the long computations which otherwise they could not attempt."

"I was worried about long division and learning the facts. The kids didn't seem to suffer from the calculator use."

"I like using the calculator. My kids enjoy using it, too."

"It has not caused them any problems that I can see."

"There definitely is more interest when we use calculators than when we do not."

"I was afraid we'd lose many calculators, but we only lost two. It was surprising how much care the students gave the calculators."

"It helped my bright kids, too. I thought they'd become lazy and not want to do the hard work when we weren't using the calculators.

"I think the kids were thinking the calculators would solve all their problems, but it didn't. They can't just push the buttons."

"I liked seeing the slow kid at least attempt a problem. It definitely gave him some confidence."

"My kids can't wait until they get to use the calculators next semester."

"I don't think my students really learned to utilize the calculators to the fullest in this semester. I know I didn't."

Parents. Initially, parent reaction to calculator-use in sixth grade math class varied from neutral to negative. However, at the end of the treatment phase, parents were uniformly positive about calculator use in problem solving. When parents were questioned at conference time about their child's use of calculators, initially they responded apprehensively. However, after the teachers demonstrated calculator use in problem solving to the parents they became quite supportive. The investigating team visited one PTA Group and met with the sixth grade parents in the project. Comments below reflect parent support and the change of attitude toward calculator use.

"I didn't think the calculator should be used when my child was still learning how to add, subtract, multiply, and divide. Yet, he/she was able to do those problems with the calculator that he wouldn't even attempt due to the lack of long computations. His basic facts have improved throughout the year even with the calculator. He sure loves to use the calculator."

"My son's grades have improved since he is able to use the calculator. He has gotten more problems correct, thus a better score on daily work.

"My daughter would definitely choose working with the calculator.

"The first thing my daughter asked for this Christmas was a calculator like she had in school. Where can I buy one? Should I buy another model or is the model she is using adequate?"

"Seeing my child enjoy math is really a delight. I know the calculator was a definite plus for her. I hope all teachers are learning how to use the calculators in schools."

"We keep a calculator by the dinner table for our evening dinner problems which my son brings home. He is better with those problems than we are."

"I was strongly opposed to my child using a calculator in school until I saw how much he/she enjoyed and improved using it. He seems to have a better grasp and desire to learn. He says math is easy for him this year."

The statement below reflects one CPS teacher's response to the project.

"The problem solving calculator project was stimulating, fun, challenging and most rewarding. I look forward to math class because I never know what my students will do that day. I no longer worry about running out of materials and keeping the students out of trouble. Time passes quickly and I have less discipline problems. The kids are busily engaged into thinking and discussing problems in the context of problem solving. Yes, sometimes it gets a little noisy, but I realize that they can control it within the groups. I find the interest generated carries over to other daily activities. Many children enjoy bringing in problems from home, altering the ones we have, and even at times generating other problems. Our math class involved English, Social Studies, Science, Health, Reading, and Language at the same time. Make a List proved very helpful for all the children. I believe Guess and Test was their favorite and that Simplify was the most difficult strategy.

"This project has definitely had a major impact on my teaching. I no longer can teach from the text presenting the computations per se. I have seen what sixth graders can do and will do if given the opportunity."

Individual analyses of case study subjects

Paula, being highly motivated to please and to be socially accepted, approached problem solving cautiously. This was the first time she had been presented with math tasks that were problems rather than exercises. Previously, she could complete her work quickly and then proceed with the next assignment. She was programmed to complete daily assignments in routine fashion, achieving excellent results. This behavioral pattern did not facilitate her work in calculator problem solving. She lost confidence and was reluctant to discuss or share her work. She would cover her written work when we would walk by her desk. When asked about the problems, ~~she would~~ remark that she was thinking. All during the year Paula was exposed to a relaxed, open classroom where small group instruction was the predominate teaching strategy. As the study progressed, she capitalized on her out-going personality and became a productive group member. She also learned that it was all right to make mistakes and there often would be false starts in the exploration stage of problem solving. Once she realized that she was not expected to solve the problem in a prescribed way, she relaxed and tackled the problems. Both teachers and parents remarked about her insecurity and uncertainty early in the year.

Paula's progress was related to her change of focus and purpose. Her previous goal was to follow explicit directions carefully and accurately. This had brought good results. Paula was forced to develop different response patterns to problem solving.

Paula's parents focused on her desire to lead. She had a strong desire to lead the group, but was not always recognized as the group leader. She learned to respect the opinions of others and to work cooperatively. The group work allowed Paula to improve her relationship with peers.

Her parents questioned the teacher about continuing the new approach in seventh grade. They wanted Paula to continue studying problem solving. They were able to notice changes in her response to problem situations around the house. She became more confident in her own problem solving approaches. Yet she did not become an outstanding problem solver.

Paula is a pleasant, cooperative, and alert student that any teacher would enjoy having in class. She has the ability to adapt to an exploratory setting and make excellent progress. Paula commented about how hard she worked to keep up with the class. This was the first time in school where she had been expected to "think" rather than just "compute."

Although Paula has the innate ability to become a superior problem solver she has such a strong desire to please that she could once again become a "dependent thinker" if placed in a teacher dominated classroom.

Bob was quite a puzzle to the investigating team. He never complained or questioned assignments. He worked diligently and independently on assigned problems. He would repeat the same set procedures on every problem. He did not break out of a computational mode of thought, thus he showed little growth in problem solving. He would start to explore, then quickly resort to thinking about adding, subtracting, multiplying, and dividing. His major problem was that he had not learned to "think." He had a strong desire to please the teacher. His scores on daily computations were good, yet he was unable to apply his knowledge in problem situations. He relied heavily on the teacher to conceptualize the problem for him. After conceptualization, he was able to carry out the plan.

Bob's reading was slightly below grade level. His teachers and parents thought he might be limited by his reading performance, yet we noted that he was able to read the test questions.

Bob expressed enjoyment for math and problem solving this year, yet we were unable to observe change in his behavior as a result of the treatment. The reasons for this appear to be:

* A deep-rooted computational mode of thought.

* Inability to adjust to a new approach.
* The need for much one-on-one assistance.
* Need for constant support.
* Lack of motivation for task.
* Hesitancy to explore.

In the many weeks of observation, we saw little change in Bob's problem solving performance.

Bill's progress was marked by great intensity of effort. Since he was a quiet, reserved student, the observers utilized a more direct route of investigation. While he did not volunteer information easily or openly he responded openly when asked a direct question. He rarely asked for assistance from his teacher. Bill responded well to a structured classroom where students worked individually. However, once group work was introduced, he showed interest in sharing. During the classroom transition periods he wasted much time waiting to be directed. Throughout the year he never took a leadership role. As the year progressed he did contribute to the group with key thoughts and questions.

Bill was very grade conscious and felt he had to get at least B's in all subjects. He commented that his teacher did not grade them on problem solving per se and he felt freer during that time of day than any other. As I chatted with his father, I commented about Bill's grade consciousness. He said he was not aware of this internal pressure, yet he believed Bill was not working up to his abilities and could do better.

Of all the 102 subjects, Bill spent the longest time of any subject completing the PST. He worked at the PST for 85 minutes, never once showing signs of tiring. This intensity was characteristic of his behavior throughout the year. He showed very little emotion for any task, yet he remarked that math had been really fun this year.

Learning problem solving approaches for Bill came slowly. Yet his teacher reported that he was able to apply some strategies in science and social studies.

Bill learned how to explore a problem but experienced difficulty formulating a plan and bringing closure to the task. Once he returns to a rule-oriented, structured curriculum he may once again become passive. He needed a longer time to become an active, independent learner.

Tom showed great academic growth and emotional maturing during the treatment period. Although Tom is an alert, curious boy, he had been labeled a lazy, mediocre student. He had been unable to focus on completing a task, and been unable to take responsibility for his own learning.

After observing Tom weekly from September to May it was obvious that changes were taking place. At the beginning of the year he was inattentive and mischievious in class. He had to be prodded to do his math work, yet he reported that math was easy for him. A few weeks later, he openly displayed enthusiasm for math (problem solving). He is the type of person that needs to see a reason for what he is doing. As the study progressed he became an excellent problem solver. He commented many times that he liked doing the work quickly. He hesitated to use the Guess and Test strategy because it took too long, yet when other strategies failed, he would resort to Guess and Test.

He thoroughly enjoyed working with the calculator. Tom's family are stable business people in the community who see the importance of calculators. Tom said, "Why not use calculators in school? You'll use them when you take a job!" His perception of calculator use far excelled most students. He was one of the students who used the calculator whenever possible and complained when the calculator was not available.

Tom is very observant. He was often the first one to notice any minor change in classroom management or problem format (even print size or type). On several occasions he identified a problem as having the same structure as a previously solved problem. He would say, "This problem is like ..." His greatest growth came in emotional stability and learning posture. He took a productive, active role in group work and became a group leader. He learned to work with peers and direct his energies toward the goal.

Tom showed much satisfaction and pride throughout the project. He shared much of his experience with his parents. His mother was the PTA President who visited Tom's math class. At the November PTA meeting she invited all parents to drop by his classroom and learn what is going on in sixth grade math. She gave a personal testimony about her son's changed performance and attitude.

Tom's personal interview and questionnaire data revealed that he thoroughly enjoyed problem solving and calculator use. He repeatedly thanked us for helping him and for providing calculators. His concluding comment on a questionnaire was, "I thank you for letting us use the calculators. I also wish that we could keep on using them, but I guess we can't. I had lots of fun working with the calculator. THANK YOU VERY MUCH."

Harry provided much information about a low ability student's response to the CPS treatment. His overall school performance was in the 17th percentile. For the first time this unattentive, mischievious boy was able to feel good about his mathematics performance. Of all the students in the study, Harry had to put forth more effort to achieve a "turn-around." Since he was in need of constant guidance and supervision, there is some concern about his continuing along constructive paths. He even expressed concern about doing well in junior high school. His math performance was much higher than his performance in other subjects. His teacher was surprised but pleased with his work. The following comments reveal his reactions and feelings about sixth grade math and the CPS project.

"I like problem solving."

"I like to work in groups. When I get stuck, I ask someone beside me for help. I help them too when they get stuck. It's lots more fun to work in groups."

"I worked harder last semester (fall, 1980) than I ever did in school before."

"I don't like pages and pages of book problems. The problems were fun - it was neat trying to figure out those hard problems. First time I was given any hard problems. When I had nothing to do, I just got a notebook and paper and tried to figure the problems. When I finished a problem the kids didn't think I could do they'd say, 'I can't do that problem - how can you do it? I don't think you got that problem right.' They'd try to prove me wrong. Sometimes we'd work on it together. I'd show them how I did the problem; they never thought I could get it done."

"I like using the calculator. My grades

went up when I used the calculator. When we don't use the calculator, my grades go down."

"I want to continue using the calculator because it helped me get more problems right and better grades. First time my friends thought I could do math right."

Harry's teacher remarked about the impact the CPS project had on him:

> Harry finds school work very difficult. He has trouble paying attention and getting his work done. He showed a renewed interest in school as a result of the 'strokes' he received in math this year. The calculator was a big factor in his getting the problems done and done correctly since he did not know his basic facts well. To my surprise he tackled all the challenge problems, completing all but one. His classmates did not trust him at first, but learned to listen when he offered help. He was a better follower than a leader. He worked best in a group, as he needed the direction and support of other students. His parents rewarded him with a new bike for his good grades in school. It was a real joy to see this kid change - he's a different boy.

In summary, Harry's case shows that even a low achiever can be successful in problem solving. A student ranking nationally at the 25th percentile scored at the mean for the CPS group (14) on the problem solving posttest interview. He demonstrated a grasp of problem solving and made excellent use of the strategies.

Sue was one of the most valuable case study subjects. Her response to school and problem solving were definite and notable. Sue is a bright, well adjusted child. Actually, she is quite mature for a sixth grader. This stems in part from a family background of social consciousness and affluence. Although mature, she is not always self-confident. She is very pleasant and attemps to please adults. She is the type of student a teacher would depend on for many responsible tasks.

Yet having noted these many positive attributes, Sue did not demonstrate mastery of the problem solving process. She was slow to learn the strategies; in fact, she never mastered the technique of exploring a problem.

Sue was not as successful in problem solving as many other students. Although she was a bright hard working student, she was quite slow learning to use the strategies. This could have been, in part, a result of the relatively poor strategy instruction in her classroom. Yet others in that classroom were successful in using the strategies in problem solving. Sue liked to be told explicitly what to do. She performed complex long division expertly but seemed lost in deciding what to do on a problem. Being generally successful in school, she seemed frustrated by her ability to solve problems. Even though she tried hard, problem solving never became easy for her. She did make progress and was able to solve many problems, yet she did not display the insights observed in other successful problem solvers. She liked problem solving and worked diligently at the problems whenever that was expected. Yet she continued to struggle with the more difficult problems. If she was in a small group she would wait for others in her group to determine the approach. Once she had direction, she would work enthusiastically to arrive at the answer. She was quick to ask for help, expecting the teacher to tell her how to solve the problem. She did not have confidence in her ability to solve problems. She has the ability to be an excellent problem solver. It seems that she could not overcome her passiveness. Her desire to please was strong and her confidence was low. This combination explains her willingness to work at the problems but with little success.

Even though Sue was cooperative, her comments during a post session interview indicated she was not happy to perform textbook tasks. She commented that, "Math was boring." "I very seldom get my homework done." "I fall asleep before I get the problems done." "Never been able to remember what I learned yesterday." "I don't know why I have to learn fractions." Sue described her response to math textbook work as "flipping through the pages and doing examples 1-10."

Sue's verbalization and perception of the project expressed during the interview was phenomenal. She began to view math as a thinking and learning process. She was able to perceive many of the changes within herself as well as in her classmates and teacher. Being able to express her thoughts so adult-like was quite unusual.

Three weeks after the school year ended, she sent us a poem which she wrote summarizing her sixth grade problem solving experience. It is surprising that a twelve-year-old could comprehend and express the thrust and impact of the project so clearly. Her poem follows:

The S ition That Has Never Lost Its Class

This is a story about a class
Who had a terrible time dealing with math.
Decimals and fractions bored them to tears,
Percentages and graphs were pains in the rear.
The teacher thought the kids were all deaf and dumb
All the class did was twiddle their thumbs.
This went on for days and days
There seemed to be no hope no way.
Until one day the teacher walked into the room
Smiling in the middle of the class' gloom.

There was a small box she clutched in her hand
A box and a rubber band.
She started talking with a laugh
And told the kids they were going to enjoy math.
The kids showed their doubts through their looks
You couldn't enjoy math with their math books!
Said the teacher, "These are Story Problem
Solving Cards."
"They won't be easy, they will be hard."
"Problem Solving!" the class groaned.
"We don't need to solve their problems
We've got enough problems of our own."
"But these problems will be fun - just wait.
You'll never get far at this kind of rate."
Two weeks passed and the teacher was right.
The kids now loved math with all their might.
Every kid could now choose what he wanted to do
There were 200 cards colored in red, white,
and blue.
They could solve their problems by guessing
and testing,
Drawing diagrams, looking for patterns, simplifying,
and making lists.
There was so much of a choice in math it was
absolutely bliss!
The kids could work in groups or work alone.

They could do the cards at school or do them at home.
Then there were challengers sheets
The problems were interesting, challenging,
and neat.
Now Problem Solving Cards weren't absolutely perfect
Nothing ever is.
Kids still got frustrated
And there were problems they missed.
But over all the cards were great!
They put kids into situations they'd soon have
to face.
The cards taught them how to balance money and
how to read a map.
They taught them how to triple a recipe and
other things kids have to learn to adapt.
It showed them how much the world was run by math
if you really looked
So much more math then you'd ever find in a math book.
And because the kids now enjoyed math time
Their grade averages began to climb.
So the moral of this story is merely
Get a kid interested and he will improve clearly!

From the very beginning, Sam responded favorably to problem solving. As the year progressed, he became confident and self assured. Throughout the treatment phase he was among the three best problem solvers in his class.

As noted earlier, Sam was an inconsistent worker who could achieve excellent results when motivated, while at times he performed as a low-average student. His entire posture in math class changed as he became motivated by problem solving. The observers were impressed with his insights, his leadership qualities, his decision making abilities, and his persistence. He tackled problem solving aggressively, becoming an intent, serious, persistent, exploratory, and successful problem solver. The aspect of problem solving which he enjoyed the most is explained by his comments: "In problem solving I didn't know where to begin - that's the fun part. It becomes a challenge - Can I do it? I like to explore."

Sam's inconsistent behavior throughout the year can be explained in that situations and activities which required only routine responses forced him to take a different approach to school. He saw the project tasks as PROBLEMS and thoroughly enjoyed wrestling with them. He commented about the Challenge Problems because he was forced to explore and struggle to find a solution path.

Sam did not like activities which demanded routine memorization or application of rules. His general progress in school was evaluated by teachers as C or C- work. However when it came to problem solving, his response was totally different; he was enthusiastic, task oriented, and creative. He became one of the best problem solvers in the class.

Sam displayed and used excellent mental estimation skills. This was particularly evident in the testing situation. His pleasant, easy-going, personality allowed him to relate to his peers quite well. He was very sensitive to their needs and could assess his group role easily. His remarks about small group work were:

> I love group work because it is fun to discuss the problems. Everybody has ideas about it. I like to help kids with their work. It makes me think harder when I have to explain it to someone.

Sam thoroughly enjoyed using the calculator in school and profited from the calculator experience. Although initially his parents did not fully support calculator use in schools, they became convinced that Sam's calculator problem solving experience was the highlight of his sixth grade year. Sam's comments reflect his feelings about calculators: "I think I could get hooked on calculators. I still need to use my mind when I use the calculator."

Having observed Sam in many situations, the investigators conclude that Sam's inconsistent overall school performance and poor attendance record from previous years can be directly related to the type of school tasks. When presented with problems in problem situations, he responded actively and aggressively. If the task became routine or practice-like, he resorted to his passive self. Sam's response was interesting but perplexing at times.

Discussion

This study investigated the problem solving performance of sixth grade pupils under several conditions. There were three treatment groups, a Problem Solving group (PS) that experienced problem solving activities in addition to their regular mathematics program, and a Calculator Problem Solving group (CPS) that used the same supplementary problem solving materials as the PS group but in addition, had calculators available, and a Control group that studied mathematics from their regular grade level text. Three problem solving ability levels were formed using scores on the Iowa Problem Solving Project Test.

In order to assess treatment effects, three measures were used; a Problem Solving Test, the Iowa Problem Solving Project Test, and case studies. The Problem Solving Test was administered in an interview format; two experimenters recorded information as they observed the subjects solve five nonroutine mathematics problems. A rating scale was constructed for coding the pupils responses. The sixteen ratings are defined in Appendix C. The IPSP Test was group administered using alternate forms, in a pretest-posttest design. Seven subjects from the PS and the CPS groups were identified for careful observation throughout the 18 week treatment phase.

Treatment Effects

When the results of both the PST and the IPSP are taken together, it is apparent that the subjects studying problem solving, with or without calculators, performed at a much higher level than those subjects in the Control group. On the PST, the Success scores of the Control, PS, and CPS groups were 4.4, 12.4, and 14.3, respectively. Not only are these differences statistically different, they are educationally significant. Further, the PS and the CPS means were significantly different from the Control groups means on all 16 PST ratings. Each of the ratings will now be discussed.

Strategy Use

The PS and CPS groups used nearly three times as many strategies as the Control group on the five problems. Not only did they use more strategies, they used a greater variety of strategies. This finding is of particular importance. Mathematics educators have searched for ways of helping pupils solve word problems. Over the past 30 years the wanted-givens approach has predominated. The National Assessment results as well as findings from many studies suggests that this approach has been ineffective. In this study, the heuristic approach to problem solving was found to be effective in helping sixth grade pupils of all ability levels learn to use problem solving strategies. Use of problem solving strategies was accompanied by significant increases in problem solving performance. Knowledge of heuristics seemed to change the way students perceived the problem solving task; they no longer expected to apply a rule but realized they could explore and understand the problem. When they performed a computation, it was not with the expectation that the answer would necessarily be obtained but that information be obtained that would lead to understanding the problem and ultimately to a solution. They expected to make several exploratory moves in reaching a solution. They also realized that certain tools in the form of problem solving strategies were available. This was apparent from their comments during the PST interviews. They would say, "I think I'll try Guess and Test." or "This looks like a Make a List problem."

Most generalizations about the problem solving process have been based on studies with older subjects, many at the college level. It may be the case that sixth grade pupils approach problem solving in a manner different from older students. Certain differences in the problem solving behaviors of the subjects in this study need to be recognized. For these students, the use of exploratory methods seemed quite important. While students typically sit and stare at the problem or give up immediately because they "Don't know what to do.", a student with an exploratory mind set does not expect to know what to do at first. In fact, he/she may not even understand the problem. This raises an interesting point. When faced with a problem, understanding the problem may be the most important part of the problem solving process and may not be achieved until near the end of the solution process. Based on the observations in this study, the following steps in problem solving of sixth grade pupils are suggested.

Steps in problem solving

1. Explore
2. Formulate a tentative plan
3. Carry out the proposed plan
4. Try another approach as needed.
5. Look back

Our observations suggest that for sixth grade students, Polya's first two steps run together. Rarely does a student understand the problem before he/she devises a plan. The problem solving strategies serve as tools for exploring which lead to understanding and the evolution of a solution plan. Exploration seems critically important for sixth grade pupils and the strategies are the tools for this exploration.

The use of the Draw a Diagram strategy was rarely observed during the PST. This may have resulted from the choice of problems. However, Draw a Diagram was clearly appropriate and needed for problem number four, yet only two pupils used it. While there may have been some unidentified confound, the use of this strategy was unpredictably low.

If, in fact, elementary school pupils do not spontaneously use Draw a Diagram as a problem solving strategy, instruction in this heuristic must be carefully considered. It may be that students need longer and specifically designed instruction in order to become proficient in the use of this strategy. On the other hand, cognitive level or spatial ability may play a role in use of Draw a Diagram. Yet, mathematics educators agree that few heuristics are as powerful as Draw a Diagram. Certainly, we must study the teaching and learning of this strategy. Pupils may not draw diagrams because of the contrasting mode of thought required. The theory of hemispheric specialization suggests that Draw a Diagram would elicit more right hemisphere processing than other strategies.

Estimation

The treatment subjects demonstrated more than twice as many instances of estimation during problem solving. While subjects in the treatment groups were, on occasion, encouraged to estimate, little instructional time was devoted to this skill. It is likely that increased estimation resulted from exploration and use of specific strategies. If one explores, it is for a purpose and the work is evaluated. Furthermore, Guess and Test as well as Look for a Pattern implicitly encourage estimation. Thus the increased estimation seemed to be a byproduct of teaching the strategies rather than a result of direct instruction in estimation.

Representation

Building a mental representation of a problem seems central in problem solving. Evidently, studying and using problem solving strategies facilitate this constructive process. Subjects in the treatment groups showed evidence of having a mental representation of the problem more frequently than those in the Control group. Often, persons do not consider a problem as solved until they have a mental representaion. Otherwise, there is cognitive dissonance. Festinger (1957) has shown that a basic drive of man is the reduction of cognitive dissonance. Problem representation is an important topic for problem solving researchers to consider.

Used All Conditions

This rating was a measure of whether subjects used all the explicit conditions in the problem. The Conditions score of both the CPS and PS groups was more than twice that of the Control group. Thus, the treatment had a profound effect on sixth grade students' use of Conditions. One might say that the treatment subjects read more carefully. There were no differences between the CPS and PS groups.

Problem five of the PST illustrates the meaning of this variable. Many subjects found a combination of coins with a value of $1.85 but ignored the condition that there were 16 coins. Some treatment subjects initially made this error but frequently discovered their oversight by re-reading the problem during the solution process.

This variable had the highest correlation with the Success score of any of the variables, indicating that using all the conditions was a critical factor in problem solving success. Considering the importance of this variable, the fact that the treatment was effective assumes even greater significance for the practitioner. Future research in problem solving should consider this variable.

Organization

It is not clear why PS and CPS subjects appeared more organized. It may have been a function of their greater production. Yet one could argue that with greater production there is more opportunity for disorganization. In general the treatment subjects were not well organized even though they were more organized than the Control subjects. It is possible that using heuristic strategies induced an organization effect. Maybe the subjects found they were more successful when they organized their work. The treatment teacher may have been effective in encouraging the students to organize their work. Because of the low level of organization exhibited by the subjects in this study, the results on this variable should be viewed with caution.

Plan

As subjects were solving the five problems of the PST, the two observers were looking for evidence of a plan. Some subjects proceeded to perform a series of activities which seemed unrelated, there was no evidence of a plan. Others, in contrast, seemed to know where they were going, what they would do next. However, their steps were not programmed from the beginning as in rule applicaton but the next step depended on the results of previous moves. This latter behavior was considered as evidence for planful behavior.

The analysis of variance on the Plan variable revealed a Treatment X Ability interaction. This was a complex interaction caused, in part, by the reverse ordering of the planning scores of the three ability groups for Control subjects; the highest abiliy group having the lowest planning scores. Actually the three treatment means (H, M, L) were quite similar. The spread of ability group means in the PS and CPS groups was much greater. This pattern of mean spread was observed on other variables, Success, Conditions, Calculator Frequency, and Calclator Use. Another reoccurring pattern was the lower performance of the low ability CPS group in comparison to the PS low ability group. This suggests that the low ability subjects using calculators did not plan as well as those low ability subjects studying only heuristic strategies.

Looking Back

There was a dramatic difference in the looking back behavior among the treatment groups compared to the Control. The Control subjects rarely looked back while the subjects in either treatment group looked back frequently. This is a particularly incouraging result. Teachers have implored their mathematics students to label their answers, consider the reasonableness of the answer, and in general reflect on their result. This approach has met with little success. Teaching heuristic strategies had the effect of stimulating looking back. We can only speculate on the reasons for this. Since CPS and PS subjects had a higher Representation score it is likely that they had a better understanding of the problem. Thus when they arrived at a solution, they were interested in seeing whether the answer matched their expectations. Other explanations are certainly possible and should be sought.

The higher mean score of the CPS group compared to the PS group is attributable to the performance of the high ability CPS group; they scored quite a bit higher than the other groups. It is not surprising that the high ability group looked back more frequently than other ability groups but their performance was in contrast to the high ability PS group. It appears that the calculator encouraged more looking back. This suggestion is tentative and must be verified by other studies before being accepted.

Confidence

As each student finished each problem an estimate of confidence was obtained. The measure of confidence, based partially on the subjects self-report, was essentially a judgment decision. There may have been experimenter bias operating. There was a Treatment X Ability interaction for the Confidence variable. The low ability subjects did not differ in their confidence ratings irrespective of treatment group. For the other ability groups, the subjects in the CPS and the PS groups appeared more confident than Control group subjects. There was no difference between the Confidence scores of the CPS and the PS groups. However, there was a difference between the scores of the male and female subjects overall with the males appearing more confident ($p < .01$). This was the only one of the 16 ratings on which there was a significant sex difference.

Persistence

Persistence was, in part, a jugment decision. There was a degree of objectivity since Time influenced the rating. Subjects were rated high on persistence if they explored the problem fully to a solution. Some subjects just quickly performed a computation and wrote the answer obtained. They would be rated low on persistence. The PS and CPS groups had significantly higher persistence ratings than the Control group. Thus, it appears as though the problem solving training contributed to greater persistence on the part of the subjects in this study.

Computational Error

More computational errors were made by the PS group than either the CPS or the Control groups. The PS group had the most computational errors because they were the group that performed a large number of computations without a calculating device. The fact that they still performed relatively well in problem solving suggests that they were able to recover from many of these errors. The CPS and Control groups made about the same number of errors but for very different reasons. The Control group made few computational errors because they performed few computations relative to the other groups. On the otherhand the CPS group made few computational errors because they performed most of their computations on the calculator, especially complex computations. As might be expected, the low abiliy subjects made more computational errors irrespective of the treatment group membership.

Time

The Treatment X Ability interaction for the Time variable resulted from the performance of the high ability group; they took about the same time irrespective of treatment group. For the other ability groups, the PS group took longer than either the CPS or the PS group. They took longer than the Control group because they did much more work. The time difference between the PS and the CPS groups for the middle and low ability groups would have to be attributed to calculator effect. The high ability PS subjects did not re ure more time than the CPS subjects; they were able to compute with paper-and-pencil efficiently while the other ability groups in the PS Treatment required more time to complete the problems.

Viewed as a whole, these findings suggest that the calculator is helpful to students learning to solve problems. Pupils are able to solve more problems in a given period of time and thus obtain more problem solving experience.

Correct Method

The correct method score paralleled the Success score and the Correct Solution score. Since these two variables yielded the same results there is nothing to be gained by breaking Success into the two components. In this study it would have been sufficient to have coded just Correct Solutions and not include a Correct Method score. This might not be the case in other studies.

Correct Solution

The Correct Solution score paralleled the Correct Method score and the Success score.

Calculator Use

The pupils that used calculators during the treatment phase made much greater use of calculators than students that did not have them. All subjects had a calculator available during testing with the PST. Yet many failed to use it, even when they were struggling with a complex computation. Many were brushing the calculator with their computing hand as they worked. Even though the week before testing the PS and Control subjects were trained to operate a calculator, they did not make as much use of it during testing as the subjects in the CPS group. This suggests that an extended period of time is needed to incorporate the calculator as a thinking tool.

In studying Figure 4 it became apparent that there was a greater spread in the performance of the three ability groups within the CPS treatment group compared with the PS and Control groups. The distribution of the ability group means suggests that calculator experience led to a more divergent performance of the three ability groups within the CPS treatment group. This pattern can be generalized by saying that pupils benefited from calculator experience in relation to their ability; the higher the ability, the more they benefited from calculator use. The more capable CPS subjects showed concomitantly higher problem solving performance.

When the low ability CPS subjects' performance is viewed in relation to the other ability and treatment groups, a potential calculator interference hypothesis is suggested. Rather than profiting from calculator use, the low ability subjects in this study seemed to be adversely effected by using calculators. This finding is in sharp contrast with the statement by some that calculators will be particularly beneficial to low ability students because of their weak computational skills. These results suggest that, on the contrary, at least in the short term, calculators alone will not make problem solvers of low achieving pupils.

A longer treatment period could produce different results: In this study the pupils used calculators for only half of a year. This may have been too brief a time for slow learners to incorporate the calculator as a problem solving tool. In fact, an interference hypothesis seems tenable. That is, the low achievers may have had difficulty learning two new things at once; calculators and problem solving strategies. The task complexity may have been too great for these students. The low achievers did profit from the treatment: they outscored both the middle and high ability subjects in the Control groups. The interference hypothesis is further supported by the group performance patterns for other ratings, particularly, Strategy Use, Representation, Conditions, and Plan. Low achievers may need more time to assimilate new methods. Over a longer period of time, calculators may in fact be facilitative for low ability students. They can learn to use problem solving strategies and when they learn to use calculators, the combined effect should be helpful.

Interrelationship of PST Variables

The factor analysis of the 16 PST variables revealed that most of the variables loaded on the same factor. The exceptions were Computational Error, Time, Calculator Frequency, and Calculator Use. This could be interpreted to mean that the variables loading on the major factor were all measuring the same thing; general problem solving ability. Subjects that used many and varied strategies were the same subjects that built a mental representation of the problem, made a plan, estimated, looked back, were confident, were persistent, and were therefore successful. The cluster consisting of Strategies, Representation, Conditions, Plan, and Looking Back were highly interrelated.

Ability Group Differences

For each of the three ability groups, the CPS and PS groups showed greater problem solving success compared to the Control group. At the beginning of the study, several teachers expressed concern that problem solving was too difficult for their students and that they would not be able to solve the problems. As the semester progressed, it became apparent that this was not the case. In fact, those same teachers became outspoken in support of the problem solving activities for the low ability students. In the PS group there was little difference in the mean performance of the low and middle ability groups.

Summary

Teaching sixth grade pupils to solve problems using specific heuristics was clearly effective. These pupils, whether they used calculators or not, became much more proficient in solving nonroutine problems. They had more problems correct, used more strategies, and in general were more aware of their thinking as they approached a difficult problem. Generally, they were more confident and exercised more options in attacking a problem. While not all pupils in the treatment groups learned to solve mathematics problems efficiently, pupils of all ability levels showed progress; it is not only the high ability child that can become an effective problem solver. However, several important questions remain. Will this level of problem solving performance be maintained over time? Will it effect scores on standardized achievement tests? Will the training transfer to standard textbook problems or, more generally, be reflected in future academic performance? Other studies should consider these questions.

Certain problem solving heuristics were more easily learned by sixth grade pupils than others. Students in this study quickly picked up on Guess and Test, Look for a Pattern, and Make a List, while not using Draw a Diagram as frequently. Great difficulty was encountered in learning to solve a simpler problem, probably because one must understand the abstract structure of the problem to use this heuristic. Quite surprisingly, sixth grade pupils rarely wrote an equation in solving problems. While elementary school children can be taught to write equations in response to specific stimuli, the evidence from this study suggests that they do not assimilate equation writing as a problem solving heuristic.

It appears to the investigators that a key to learning to solve mathematics problems is developing an exploratory mind set. Children seem to learn a rule oriented mind set in school which inhibits successful problem solving performance. Once a child learns that it is acceptable to explore a problem, to make a decision, and to decide how to proceed, he or she is well on the way to becoming a successful problem solver. This observation may not generalize to older persons.

The effects of calculator use during problem solving were less obvious. The hard data show that children using calculators required significantly less time to solve problems and made fewer computational errors. It appears that considerable time (a semester or longer for some students) may be required to incorporate calculators as a tool in problem solving. Over the 18 weeks of this study the low ability pupils did not seem to learn to use calculators fluently. In fact, the calculator seemed to interfere with their problem solving performance. The high ability problem solvers profited most from calculator use.

There seems to be great potential in improving the problem solving performance of elementary school pupils. Teaching problem solving heuristics is one effective way to build problem solving competence. By becoming good problem solvers, children may be acquiring knowledge which can be useful in a broad range of activities. As Greeno (1980) argues, the thought processes needed in solving nonroutine problems is not different from the processes necessary to solve well structured problems. Thus problem solving ability may improve a person's ability to function effectively in a broad range of endeavors.

Reference Notes

1. Immerzeel, G. Problem solving with calculators. University of Northern Iowa: Price Laboratory School, 1977.

2. Schaaf, O. Lane County mathematics project. Eugene, Oregon: Lane Educational Services District, 1978.

3. Schoenfeld, A. Episodes and executive decisions in mathematical problem solving. Paper presented at the annual meeting of the American Educational Research Association, Los Angeles, 1981.

4. Suydam., & Riedesel, C. Interpretive study of research and development in elementary school mathematics. Three volumes. Final Report. The Pennsylvania State University, 1969.

References

Allen, M. B. Effectiveness of using hand-held calculators for learning decimal quantities and the metric system (Doctoral dissertation, Virginia Polytechnic Institute and State University, 1976). (University Microfilms No. 76-17, 090)

Borden, U. L. Teaching decimal concepts to sixth grade students using the hand held calculator. (Doctoral dissertation, University of Northern Colorado, 1976). Dissertation Abstracts International, 1977, 37, 4192A. (University Microfilms No. 76-29, 734)

Brandeau, L. and Easley, J. Understanding the realities of problem solving in elementary school with practical pointers for teachers. Columbus, Ohio: ERIC Clearinghouse for Science, Mathematics, and Environmental Education, 1979.

Campbell, P., & Virgin, A. E. An evaluation of elementary school mathematics programs utilizing the mini-calculator. New York: Board of Education, 1976. (ERIC Document Reproduction Service No. SE 022 352)

Dalton, R. M. Thinking patterns in solving certain word problems by ninth grade general mathematics students: An exploratory study in problem solving (Doctoral dissertation, University of Tennessee, 1974).

Dissertation Abstracts International. 1975, 35, 5526B. (University Microfilms No. 78-03, 213)

Days, H. C. The effects of problem solving structure on the processes used by concrete-and-formal operational students to solve verbal mathematics problems (Doctoral dissertation. Purdue University, 1977). Dissertation Abstract International. 1978, 38, 6038A. (University Microfilms No. 788-03, 213)

Driscoll, M. Research within reach. Reston, VA: National Council of Teachers of Mathematics, 1981.

Fennema, E. (Ed.). Mathematics education research: Implications for the 80's. Alexandria, VA: Association for Supervision and Curriculum Development, 1981.

Festinger, L. A theory of cognitive dissonance. Evanston, IL: Row Peterson, 1957.

Furnival, G. M. All possible regressions with less computation. Technometrics, 1971, 16, 499-512.

Greenes, C., Immerzeel, G., Ockenga, E., Schulman, L., & Spungin, R. Techniques of Problem Solving. Palo Alto, CA: Dale Seymour Publications, 1980.

Hawthorne, F. S. & Sullivan, J. J. Using hand-held calculators in sixth-grade mathematics lessons. New York State Mathematics Teachers' Journal, 1975, 25, 29-31.

Higgins, J. L. Curriculum report, Volume 4. Reston, VA: National Association of Secondary School Principals, 1974.

Hollander, S. K. Strategies of selected sixth graders and working verbal arithmetic problems (Doctoral dissertation, Hofstra University, 1973). Dissertation Abstracts International, 1974, 34, 628A. (University Microfilms No. 74-7, 896)

Jones, E. W. The effect of the hand-held calculator on mathematics achievement attitude and self concept of sixth grade students (Doctoral dissertation, Virginia Polytechnic Institute and State University, 1976). Dissertation Abstracts International, 1976, 34, 1387A. (University Microfilms No. 76 19, 885)

Kantowski, E. L. Processes involved in mathematical problem solving (Doctoral dissertation, University of Georgia, 1974). Dissertation Abstracts International, 1975, 36, 2734A. (University Microfilms No. 75-23, 764)

Kantowski, M. Problem solving. In E. Fennema (Ed.), Mathematics education research: Implications for the 80's. Alexandria, VA: Association for Supervision and Curriculum Development, 1981.

Kasnic, M. The effect of using hand-held calculators on mathematical problem solving ability among sixth-grade students (Oklahoma State University, 1977). Dissertation Abstracts International, 1978, 38, 5311A.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: An exploratory study (Doctoral dissertation, Stanford University, 1967). Dissertation Abstracts, 1968, 28, 4380A. (University Microfilms No. 68, 6442)

Kilpatrick, J. Research on problem solving in mathematics. School Science and Mathematics, 1978, 78, 189-192.

Krulik, S. & Reys, R. (Eds.). Problem solving in school mathematics. Reston, VA: National Council of Teachers of Mathematics, 1980.

Krulik, S. & Rudnick, J. Problem Solving: A handbook for teachers. Boston: Allyn and Bacon, Inc., 1980.

Krutetskii, V. The psychology of mathematical abilities in schoolchildren. Chicago: The University of Chicago Press, 1976.

Lesh, R., Mierkiewicz, D., and Kantowski, M. (Eds.). Applied mathematical problem solving. Columbus, Ohio: ERIC: Clearinghouse for Science, Mathematics, and Environmental Education, [illegible].

Lindquist, M. (Ed.). Selected issues in mathematics education. Reston, VA: National Council of Teachers of Mathematics, 1980.

McCabe, G. P., Jr. Evaluation of regression coefficient estimates using α-acceptability. Technometrics, 1978, 20 (2), 131-139.

Meiring, S. Problem solving: A basic mathematics goal. Columbus, OH: Ohio Department of Education, 1980.

National Advisory Committee on Mathematical Education: Overview and analysis of school mathematics grades K-12.

New York: Conference Board of Mathematical Sciences, 1975.

National Council of Teachers of Mathematics. An agenda for action: Recommendations for school mathematics of the 1980's. Reston, VA: National Council of Teachers of Mathematics, 1980.

National Council of Teachers of Mathematics. Newsletter, December, 1974.

Nelson D. W. Effects of using hand calculators on the attitude and computational skills of children in grades four through seven (Doctoral dissertation, Arizona State University, 1976). Dissertation Abstracts International, 1976, 37, 3382A. (University Microfilms No. 76-27, 135)

Paige, J. & Simon, H. Cognitive processes in solving algebra problems. In D. Kleinmuntz (Ed), Problem solving: Research method and theory. New York: John Wiley and Sons, 1966.

Polya, G. Mathematical discovery: On understanding, learning, and teaching problem solving (vol. 1). New York: John Wiley and Sons, 1962.

Schafer, P., Bell, M. S., & Crown, W. D. Calculators in some fifth-grade classrooms: A preliminary look. Elementary School Journal, 1975, 76, 27-31.

Schoen, H., & Oehmke, T. A new approach to the measurement of problem-solving skills. In S. Krulik and R. Reys, Problem solving in school mathematics. Reston, VA: National Council of Teachers of Mathematics, 1980.

Schnur, J. & Lang, J. Just pushing buttons or learning? -a case for minicalculators. Arithmetic Teacher, 1976, 23, 559-562.

Schoenfeld, A. Explicit heuristic training as a variable in problem solving performance. Journal for Research in Mathematics Education, 1979, 10, 173-187.

Schoenfeld, A. Teaching problem-solving skills. American Mathematical Monthly, 1980, 87(10), 794-805.

Shumway, R. (Ed.). Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics, 1980.

Schwieger, R. D. A component analysis of mathematical problem solving (Doctoral dissertation, Purdue University, 1974). Dissertation Abstracts International, 1974, 35, 3308A. (University Microfilms No. 74-26, 777)

Shumway, R., White, A., Wheatley, G., Reys, R., Coburn, T., and Schoen, H. Initial effects of calculators in elementary school mathematics. Journal for Research in Mathematics Education, 1981, 12(2), 119-141.

Simon, H. Problem solving and education. In D. T. Tuma & F. Reif (Eds.). Problem solving and education: Issues in teaching and research. New York: Lawrence Erlbaum Associates, 1980.

Skemp, R. Intelligence, learning, and action. New York:

John Wiley & Sons, 1979.

Spencer, J. N. Using the hand-held calculator in intermediate grade arithmetic instruction (Doctoral dissertation, Lehigh University, 1974). Dissertation Abstracts International, 1975, 35, 7048A. (University Microfilms No. 75-10, 376)

Sutherlin, W. N. The pocket calculator: Its effect on the acquisition of decimal estimation skills at intermediate grade levels (Doctoral dissertation, University of Oregon, 1976). Dissertation Abstracts International, 1977, 37, 5663A. (University Microfilms No. 77-04. 762)

Suydam, M. State-of-the-art review on calculators: Their use in education. Columbus, Ohio: Calculator Information Center, 1978. ERIC Document Reproduction Service No. ED 167426.

Webb, N. L. An exploration of mathematical problem solving processes (Doctoral dissertation, Stanford University, 1975). Dissertation Abstracts International, 1975, 36, 2689A. (University Microfilms, No. 75-25, 625)

Webb, N. Processes, conceptual knowledge, and mathematical problem solving. Journal for Research in Mathematics Education, 1979, 10(2), 83-93.

Wheatley, C. Calculator use and problem solving. Journal for Research in Mathematics Education, 1980, 11, 323-334.

Wheatley, G., Shumway, R., Coburn, T., Reys, R., Schoen, H., Wheatley, C., & White, A. Calculators in elementary schools. Arithmetic Teacher, 1979, 27, 18-21.

Wilson, I. A distribution free test of analysis of variance. Psychological Bulletin, 1956, 53, 96-101.

Zweng, M. The problem of solving story problems. Arithmetic Teacher, 1979, 27, 2-3.

APPENDICES

APPENDIX A

The Problem Solving Test (PST)

NAME ______________________________

PROBLEM NUMBER 1

STUDENT NO. ______________________

THE SUM OF TWO NUMBERS IS 33.
THEIR DIFFERENCE IS 15.
WHAT ARE THE TWO NUMBERS?

ANSWER

NAME ____________________

PROBLEM NUMBER 2

STUDENT NO. ____________

TIM IS READING A 216 PAGE BOOK.

THE TABLE BELOW SHOWS HIS READING SCHEDULE.

HOW MANY DAYS WILL IT TAKE TIM TO READ THE BOOK?

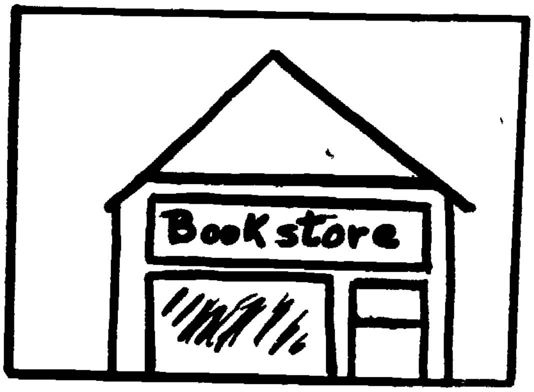
DAY	PAGE NUMBER AT END OF DAY	
DAY 1	COMPLETED PAGE	12
DAY 2	COMPLETED PAGE	29
DAY 3	COMPLETED PAGE	46
DAY 4	COMPLETED PAGE	63
.		.
.		.
.		.

ANSWER [ ]

NAME ______________________

PROBLEM NUMBER 3

STUDENT NO. ______________



NOTEBOOKS COST \$1.57 EACH.

HOW MANY CAN YOU BUY WITH \$20.00?

HOW MUCH CHANGE WILL YOU HAVE LEFT?

ANSWER

NAME ______________________

PROBLEM NUMBER 4

STUDENT NO. ______________

PAUL HAS A SWIMMING POOL IN THE SHAPE OF A RECTANGLE.
IT IS 31 FEET LONG AND 23 FEET WIDE.
THERE IS A WALKWAY 3 FEET WIDE AROUND THE POOL.
WHAT IS THE LENGTH OF A FENCE AROUND THE WALKWAY?

ANSWER

NAME ______________________

PROBLEM NUMBER 5

STUDENT NO. ______________________

MARK GETS A WEEKLY ALLOWANCE.
HE ALWAYS GETS 16 COINS WHICH TOTALS $1.85.
EACH WEEK HE GETS ONLY NICKELS, DIMES AND QUARTERS. EACH WEEK HE GETS A DIFFERENT COMBINATION OF COINS. FIND AT LEAST TWO DIFFERENT COMBINATIONS MARK CAN GET.

CAN YOU FIND OTHER COMBINATIONS?
EXPLAIN YOUR ANSWER.
IF SO, WRITE THE COMBINATIONS YOU FIND.

ANSWER

APPENDIX B

Interview Coding Form

CODING FORM

Student No. ____________________

Problem No. ____________________

Date of Interview ____________________

Boy

Girl

CORRECT STRATEGY USED AND VARIETY OF STRATEGIES USED

Type of Strategy	No. Used	No. Used Correctly
Guess and Test		
Draw a Diagram		
Make a List		
Simplify		
Breaks into Parts		
Look for Pattern		
Write an Equation		
Other/Others		
TOTAL		

USE OF CALCULATOR

Computational Error Score

Frequencies	Total

Correct Solution Score

Correct Method Score

Strategy 1. ____

Success 2. ____

Estimation 3. ____

Representation 4. ____

Used All Conditions 5. ____

Organization 6. ____

Plan 7. ____

Looking Back 8. ____

Confidence 9: ____

Persistence 10. ____

Computational Error 11. ____

Correct Solution 12. ____

Correct Method 13. ____

Time 14. ____

Calculator Frequency 15. ____

Calculator Use 16. ____

COMMENTS:

Time: ____________________

Was subject's picture taken?

Yes No number ____________________

APPENDIX

Definition of Interview Ratings (PSI)

Definitions of Interview Ratings

1. Strategy (0-)
The Strategy Score is the sum of the number of strategies tried and the number of strategies used correctly for each problem. The list of strategies include Guess and Test, Draw a Diagram, Make a List, Simplify, Look for a Pattern, Write an Equation and a category named Others.

2. Correct Solution (0, 2, 4)
   - 0 The subject arrives at an incorrect solution.
   - 2 The subject arrives at one of the correct solutions on a two-part problem.
   - 4 The subject arrives at the correct solution to the problem.

3. Correct Method (0, 4)
   - 0 The subject does not use an appropriate method.
   - 4 The method or processes used were correct and complete except possibly for computational error(s).

4. Success (0, 4, 6, 8)
The Success Score is the sum of the correct solution score and the correct method score for each problem.

5. Estimation (0, 1, 2)
The Estimation Score is a measure of observed estimation use on each problem.
   - 0 No estimation is observed.
   - 1 Some evidence of an attempt to estimate is observed.
   - 2 The subject makes reasonable estimates.

6. Representation (0, 1, 2)
The Representation Score measures the extent to which a subject developed some understanding in his mind of the problem's structure.
   - 0 The subject shows no evidence of building a representation; subject merely operates on the given numbers without understanding the problem.
   - 1 Subject develops a representation, but it is not complete and correct.
   - 2 Subject develops an appropriate representation that leads to the correct solution method.

7. Conditions (0, 1)
The Conditions Score measures the extent to which the conditions of the problems are used.
   - 0 The subject fails to use all the conditions in the problem.
   - 1 The subject utilizes all the necessary explicit and implicit conditions in the problem.

Definition of Interview Ratings (continued)

8. Organization (0, 1, 2)
The Organization Score measures the extent to which a subject arranged and ordered his/her written work on the problem.
   - 0 Written work lacks any organization. If numbers are written, they are in a haphazard way.
   - 1 Written work is partially organized. Student attempts to order his/her work in some useful way.
   - 2 Written work is well organized with sequencing, ordering, and labeling of parts.

9. Plan (0, 1, 2)
   - 0 There is no evidence that a plan was formulated.
   - 1 There is evidence of some degree of planning but the subject does not have an over-all plan.
   - 2 The subject works purposefully, following an orderly set of steps and/or trials.

10. Looking Back (0, 1, 2)
The Looking Back Score measures the subject's attempt to verify or check the step(s) of the solution process.
   - 0 The subject makes no attempt to check computations or processes.
   - 1 Some evidence of an attempt to look back is observed. Subject checks one or more computations.
   - 2 Subject considers the reasonableness of the answer and the procedures used.

11. Confidence (0, 1, 2)
The Confidence Score measures the degree of confidence the subject has in the process and the solution itself.
   - 0 No confidence. Perhaps he/she cannot arrive at a solution or his/her answer is more of a guess.
   - 1 The subject arrives at a solution but still feels unsure about his/her work.
   - 2 Subject seems sure that he/she has the correct solution.

12. Persistence (0, 1, 2)
   - 0 Subject gives up easily; stops without success.
   - 1 Subject works on the problem for awhile and then either gives up or just writes any number as the answer.
   - 2 Subject probes deeply into the problem, even if unsuccessful.

13. Computational Error (0-)
The Computational Error Score is the sum of the computational errors in the problem.

14. Time (0-)
The Time Score is the number of minutes the subject spent on the problem from the moment he/she begins to read the problem aloud to the moment he/she wrote an answer in the box and ceased to work on the problem.

Definition of Interview Ratings (continued)

15. Calculator Frequency (0, 1, 2, 3, 4)
    The Calculator Frequency Score is the number of computations performed on the calculator.
    - 0 No calculator use
    - 1 1-3 uses
    - 2 4-7 uses
    - 3 8-11 uses
    - 4 12 or more uses

16. Calculator Use (-1, 0, 1, 2)
    The Calculator Use Score is a measure of the quality of calculator use.
    - -1 The calculator misled or confused the subject.
    - 0 No use of the calculator
    - 1 The calculator is used for only a few computations.
    - 2 The calculator is used to perform most complex computations and it played an important role in the solution process.

APPENDIX D

Attitude Inventories

NAME_______________

Example: For each pair of words below place an X on the blank that best tells how you feel about--

SNOW

like ____:____:____:____:____ hate
cold ____:____:____:____:____ hot
work ____:____:____:____:____ play

Directions: For each pair of words below place an X on the blank that best tells how you feel about--

CALCULATORS

bad ____:____:____:____:____ good

sad ____:____:____:____:____ happy

boring ____:____:____:____:____ exciting

jump in ____:____:____:____:____ hold back

hard ____:____:____:____:____ easy

more ____:____:____:____:____ less

Directions: Read each question and fill in the space below your answer.

1. Is there at least one calculator in your home?

YES NO

2. Are you allowed to use a calculator at home?

YES NO

3. Do you think you would do better in math if you used a calculator?

YES NO DON'T KNOW

4. Do you have a calculator of your own?

YES NO

NAME______________

Example: For each pair of words below place an X on the blank that best tells how you feel about--

SNOW

like ____:____:____:____:____ hate
cold ____:____:____:____:____ hot
work ____:____:____:____:____ play

Directions: For each pair of words below place an X on the blank that best tells how you feel about--

MATH

bad ____:____:____:____:____ good

sad ____:____:____:____:____ happy

boring ____:____:____:____:____ exciting

jump in ____:____:____:____:____ hold back

hard ____:____:____:____:____ easy

more ____:____:____:____:____ less

Appendix E

Schedule of Activities

Schedule of Activities for
Fall Semester, 1980 and
Spring Semester, 1981

Teacher
Orientation
Aug. 25. Week 1

PS Group
5 classes
Pretest Wk. 1-2
Aug. 25 - Sept. 12

CPS Group
7 classes
Pretest Wk. 1-2
Aug. 25 - Sept. 12

Control Group
6 classes
Pretest Wk. 1-2
Aug. 25 - Sept. 12

Introduction to
calculator
Week 1-2
Aug. 25 - Sept. 12

Teaching
problem solving
Weeks 3-7
Sept. 15 - Oct. 17

P.S. with Calculator
treatment
Teaching techniques
of P.S. Weeks 3-7
Sept. 15 - Oct. 17

6th grade
math
curriculum

18 weeks

Sept. 15 - Oct. 17

Mixed practice with
techniques of P.S.
Weeks 8-17
Oct. 20 - Jan. 9

Post Testing
Week 18
Jan. 12 - 16

Semester II

Interview Students
34 from each of the three treatment groups
Weeks 19 - 27
Jan. 19 - March 19

Interview

APPENDIX F

Case Study Observation Form

Student ______________________ Date ______________________

Teacher ______________________ School ______________________

Evaluator

(1 low, 5 high)

COMMENTS:

1. Motivation 1 2 3 4 5
2. Grasp of P.S. 1 2 3 4 5
3. Strategies used:

   G&T DD ML S P

4. Strategies used correctly:

   G&T DD ML S P

5. Other strategies used (list)?

   ____

6. Looked back? Yes ___ No ___
7. Used calculator:

   How?

   How much?

8. Time on task ______________%
9. Was the work organized?
10. Follow directions?

    1 2 3 4 5 NA

11. Interaction with teacher

    1 2 3 4 5 NA

12. Interaction with peers

    1 2 3 4 5 NA

Classroom atmosphere, class activities, other pertinent information.