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ABSTRACT

The cognitive processes involved in the human ability to understand and use positional notation (i.e., place-value) were investigated in a series of psychological experiments. Although the tasks used in all studies were very simple, usually only requiring the tested individual to identify the larger of two numbers as quickly as possible, a number of unintuitive results were obtained. For example, in two digit number comparison, skilled adults' decision times are affected by the ones-place digit even when the tens-place digit is logically sufficient to determine the correct response (e.g., is 48 less than or more than 55?). However, when the number of places is a sufficient cue to answer the question (is 735924 more or less than 5000?), insignificant place information is not extracted. Memory for numbers was also shown to be affected by place-value information. Surprisingly, speed of understanding the relative value of numbers is slowed when the numerical value conflicts with the relative physical size of the numerals. Overall, several physical and symbolic factors strongly influence our ability to understand, manipulate, and remember numbers. (Author)

Final Report

Number Place-value Information: Task Analyses and Development

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Task Analysis and Development

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Abstract

The cognitive processes involved in the human ability to understand and use positional notation (i.e., place-value) were investigated in a series of psychological experiments. Although the tasks used in all studies were very simple, usually only requiring the tested individual to identify the larger of two numbers as quickly as possible, a number of unintuitive results were obtained. For example, in two-digit number comparison, skilled adults' decision times are affected by the ones-place digit even when the tens-place digit is logically sufficient to determine the correct response (e.g., is 48 less than or more than 55?). However, when the number of places is a sufficient cue to answer the question (is 735924 more or less than 5000?), insignificant place information is not extracted. Memory for numbers was also shown to be affected by place-value information. Surprisingly, speed of understanding the relative value of numbers is slowed when the numerical value conflicts with the relative physical size of the numerals. Overall, several physical and symbolic factors strongly influence our ability to understand, manipulate, and remember numbers.

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Although it is a relatively recent invention in mathematical history, the use of positional notation in number systems is now considered a fundamental skill and a prerequisite to understanding basic mathematics. It is essential to understand positional notation, i.e., the meaning of place-value in numbers, in order to carry out basic calculation, accurately represent quantities, and to approximate the results of exact calculation. Use of positional notation is so fundamental that most adults are only dimly aware that nonpositional numbering systems exist. Only the example of Roman numeral systems remain as an archaic example of the complexities involved in carrying out computation in nonpositional systems.

Nevertheless, we have little knowledge of the psychological processes involved in our ability to comprehend and use positional notation in mathematical representation. We expect children at a very early age to understand and use place-value representation of numbers with little knowledge of what the final form of the mastery of that skill should be. The problem is most evident in attempting to approximate the results of calculation. Omission of a crucial zero or misplacement of a decimal point is an easy computational mistake but we expect even the least skilled adult to know that 200 + 200 is not equal to 40, or 40,000.

The primary goal of the research supported by the National Institute of Education and summarized here was to investigate the psychological processes and skills that affect our ability to represent and compare basic numerical information. Most of the investigation was concerned with empirical studies of skilled adults ability to judge the relative magnitude of multidigit numbers rapidly and accurately to determine which cues and abilities most affect speed and accuracy. These investigations also inspired a number of related inquiries concerned with children's knowledge of place-value information, comparison of quantities, spatial representation, memory for numbers, and sex differences in mathematical ability.

Because the technical details, results and procedures are available in other reports (see list beginning on page 9) this Final Report will describe the basic questions and results obtained in the various studies. It will be less formal, more speculative, and presented in less technical language than the more formal reports. The report is divided into two main sections. The first section concentrates on the issues most fundamental to the thrust of the originally proposed research and directly supported by the grant. The second section describes related issues that developed during the course of the research and inspired by the grant research. Finally, the report concludes with a list of all the papers, publications, and theses produced by grant-supported or grant-related personnel during the term of the project. Each item is annotated by an abstract or summary written by the authors of the paper.

Central Issues

The primary goal of the grant research was to produce empirical data and theoretical interpretations of the psychological processes involved in the representation and manipulation of the meaning of large numbers. In this context large refers to numbers whose meaning cannot be conveyed by a single digit or symbol. In the simplest case this would obviously refer to two-digit numbers and the type of information rapidly extracted from the presentation of two-digit numbers. Multi-digit numbers with more than two digits bring more complicated processing to bear as will be demonstrated below and require more sophisticated mathematical knowledge.

A number of basic decisions had to be made at the beginning of the research. In all instances we tried to use the simplest case or the most fundamental process as the target of our investigations. Most of our research was conducted with skilled adults, i.e., college students, rather than children because we were interested in the competent use and representation of number rather than the acquisition or development of number representation. Some later studies investigated children's use of place-notation but the primary emphasis was on skilled use rather than acquisition. Similarly, the experimental paradigm concentrated on simple tasks, e.g., comparison of numbers, rather than the use of numbers in more complicated settings such as calculation, or other manipulation. Nevertheless, even with these restrictions a number of surprising and unintuitive results occurred.

Two-digit Number Comparison

Consider the simplest case of multidigit number comparison in which a presented number is to be compared to the number 55 held in memory. The subject's task is to decide whether the presented number is less than or greater than 55. A fundamental question concerns the relative contribution of the ten's place and the one's place digit to the individual's decision. In a series of experiments Hinrichs, Yurko, and Hu (1981) examined the use of place cues in comparing two-digit numbers. Some of the results were very surprising and unexpected.

First, for the benefit of the reader not familiar with the past literature in this area, it should be pointed out that the comparison of even single-digit numbers is not an instantaneous decision. Moyer and Landauer (1967) showed that the time required to decide which is the larger of two single-digit numbers increases as the difference between the two numbers increases and as the magnitude of the smaller of the two numbers increases. In other words, it is easier to decide that 6 is larger than 2 than to decide that 4 is larger than 2, and it is easier to decide that 6 is larger than 4 than to decide that 8 is larger than 6. The time required to make these decisions ranges from

400 msec for the easiest to about 800 msec for the most difficult. While it was expected that the basic pattern of differences would be maintained in comparing two-digit numbers it was uncertain how subjects would combine the information in the two single-digits in order to compare the two-digit numbers.

If two-digit numbers are processed as two separate digits, the tens place digit would be expected to be examined first--both on the basis of left-right reading habits and the more significant quantitative information contained in the left-most digit. In most cases, a decision would be made simply on the basis of the value of the tens place digit. The ones place digit would need to be processed only when the comparison number and the standard number agree in the tens place digits. For example, any number that contains a "2" at the tens place can be judged to be smaller than 55, regardless of the value of the ones place digit. Decision time would decrease as the difference between the tens digit and 5 increases, but within each decade the decision time should be constant, indicating no influence of the ones digit.

On the other hand, if two-digit numbers are encoded as single integrated values and compared as quantities, similar to single-digit comparisons, then the ones place digit should affect comparison time with a continuous decrease in reaction time as distance increases. The central characteristic for present purposes is that there should be no special change in number comparisons across decade boundaries. That is, in comparing numbers with a standard of 55, although a change occurs in the tens digit in moving from 40 to 39, the expected reduction in reaction time is less than the reduction from 41 to 40.

Note the mathematical and psychological assumptions entailed in each of these interpretations. The first view assumes that subjects are efficient and extract only the information necessary to make the decision. If the tens place digit is sufficient to answer the question, it is processed and not the ones place number. The second interpretation asserts that both quanitites in a two-digit number are evaluated and understood before the comparison can be made. While it is less efficient in the sense that some irrelevant information is extracted in most cases it does suggest that more complete processing of all the numerical information presented.

The results of several experiments clearly reject the first, place-value interpretation of two-digit number comparison. Decision times are clearly influenced by the magnitude of the ones place number. Only in a severely constrained experimental situation were subjects able to ignore the value of the ones place digit in comparing numbers. Consequently at least with two-digit numbers it appears that skilled adults do not analyze the number into two components treating each separately, but rather treat the number as a whole and evaluate it as a unit.

Multidigit Number Comparison

The complexity of the process of comparing large numbers increases as the number of digits in the number increases. It seems unlikely that all of the constituent numerals will be evaluated in multidigit number comparison. Consider the case in which individuals must decide if a presented number is less than or greater than 5000. In addition to evaluating each numeral in each place in the multidigit number, another more potent cue for number value is present--the number of places in the number. One does not need to extract the meaning of each individual digit to decide that a seven-digit number is larger than 5000. Hinrichs, Berie, and Mossell (1982) demonstrated that the number of places in the presented number is a very powerful cue permitting a very rapid and accurate number comparison when the number of places markedly differs from the standard value. However, the first-place digit in a three- or five-place number does affect decision time when the comparison standard is 5000. When the comparison is made very difficult, by using a standard of 5555, a roughly serial comparison of individual digits occurs, but nonsignificant digits did affect decision time. That is, subjects appeared to process the value of numerals beyond the place value necessary to make the comparison. Again, there was considerable evidence for a holistic, nonanalytic evaluation of large multidigit numbers.

Retention of Place-value Information

The number comparison paradigm suggests that skilled individuals are influenced by place-value information in extracting numerical information from presented displays. Do subjects also use place-value information in the retention of multidigit numbers? In several memory experiments, Hinrichs and Novick (1982) showed that subjects do remember the most significant digit in a four-digit number better than the least significant digit. The results were considerably different than those usually obtained in serial recall experiments where bowed serial position curves are usually obtained. The difference appears to be that in most memory experiments item-by-item or digit-by-digit processing is emphasized. When subjects are given the option of encoding number sequences as if they were large numbers, they appear to use place value cues to represent and retain those numbers. Very different results were obtained when the subjects were instructed to treat the numbers as either prices or as telephone numbers. Coding the numbers as prices enhances place-value information and produced what we called magnitude encoding; labeling the numerals as telephone numbers reduced place-value cues and encouraged what we called nominal encoding. Our results suggest that it is possible for subjects to encode multidigit numbers in two very different fashions, one emphasizing relative magnitude information and useful for remembering approximate values and the other emphasizing digit-by-digit recall and used when arbitrary information must be recovered. Interestingly, the level of exact retention in the two situations appears to be identical; it is only when partial recall is examined that the differences of the two types of encoding emerges.

Use of Sign Information in Number Comparison

An issue closely related to the processing of particular places in large numbers is the processing of sign information. In deciding whether -3 is larger or smaller than +2 subjects must evaluate both the value of the number and the sign. Unlike the results for strict place-value comparison our results suggest that sign and number information are processes separately. Most surprisingly, the order of processing sign and number information appears to be the opposite of that that would be predicted by the most efficient interpretation. In the case where the sign of the two numbers are different, it is not logically necessary to process the value of numbers in order to decide which of the two is larger. However, our data strongly suggest and are most consistent with the interpretation that subjects process number information before they consider the sign of the number. The source of this inefficient bias in processing sign numbers is unknown but may be related to the fact that negative numbers are only rarely encountered in daily activities so that the sign of individual numbers rarely needs to be considered.

Size-value Congruity and Number Comparison

Another source of irrelevant information in the comparison of numbers that may influence the speed and efficiency in comparing numbers is the mere physical size of the number. In several experiments we have demonstrated the surprising result that the mere size of a number influences subjects' ability to choose the numerical larger value. When presented with two single-digit numbers, subjects will be slower in deciding that 5 is larger than 3 when the 3 is physically larger than the 5, and will be faster when the 5 is physically larger than the 3, compared to equal-size numerals. This size-value congruity effect suggests that subjects--even highly skilled adult subjects--are very sensitive to the physical characteristics of presented numerical information. It should be further noted that the congruity effect is asymmetrical. Physical size influences the extraction of numerical information far more than numerical information affects the extraction of physical size. The congruity effect resembles the Stroop effect (referring to the difficulty of naming colors printed as a different color name, e.g., the word blue printed in red). However, the effect is exactly the opposite of the Stroop effect. Whereas the stroop effect refers to the interference of a symbolic value (color word) on the identification of a physical value (color), the numerical size-value congruity effect refers to the interference of a physical cue (size) on the extraction of a symbolic value (number).

The size-value congruity effect might be applied as a teaching aid. Students having difficulty mastering the use of place-value cues, especially in calculation, might be presented with numbers with physical cues correlated with the place-value cues. In other words, the most significant place value would be associated with the largest numeral and the least significant with the smallest. Students could be taught to associate the physical size cue with the place-value cue and to disassociate the numerical value from the physical size or position. Such an aid might reduce the often observed tendency of children to subtract the smaller valued numeral from the larger numeral regardless of position.

Representation of Numbers

Much of the research we have conducted concerned with number representation has been consistent with the view that numbers are encoded internally as magnitudes analogous to physical quantities. One of the most speculative investigations concerning the form of this magnitude representation of number was conducted in a Ph.D. dissertation by Hu and concerned the psychophysical comparison of numbers and numerosity (dot patterns) in a reaction time task. Subjects were asked to choose as rapidly as possible the larger numeral or the larger number of dots in a display. The dot patterns were either randomly arranged or had an associated cue of length or density which could be used to determine the relative number of dots. The surprising results of the research was that the models that accounted for the use of the length cue in numerosity judgments also perfectly accounted for number comparison in the numeral displays. At the risk of the usual statistical peril of accepting the null hypothesis, we were unable to detect any difference between the models that account for dot numerosity judgments and number comparison. Our tentative conclusion is that some form of length representation may underlie the magnitude encoding of numbers.

Related Issues

In the course of conducting projects directly related to the goals of the grant a number of side issues were explored. Most of these projects were initiated by students and were not grant supported, however, they are loosely related to the overall goals of the NIE sponsored research and illustrate how funded research can lead to an increase in related but unfunded investigations. The topics range from use of rules in estimation to sex differences in algebra, problem-solving and spatial abilities.

Development of Numerosity Judgments

In her Ph.D. dissertation, Berie extended Siegler's (1976) rule assessment model to the understanding and development of children's estimation processes. Siegler has characterized children's knowledge of basic physical systems (i.e., balance scale problems) in terms of rule systems which change as the children's knowledge matures. Berie took a similar approach in characterizing children's knowledge as they attempt to estimate the number of dots in a dot pattern. In particular, she examined how length and density cues influence estimates of numerosity in children and adults and how the subjects used rules and rule combinations in making their estimations.

Berie found that the use of a single cue to make estimations decreased with age. From adolescence to adulthood there was an increasing sophistication in the combined use of cues to make correct estimations. As assessed independently by an attentional preference task, there was an concomitant increase in preference for complex perceptual cues. As is true in other aspects of our research, young children were characterized by an overwhelming reliance upon length cues with use of other cues becoming evident much later in the developmental sequence.

Strategies in Solving Algebra Word Problems

In another dissertation, Herring examined two different approaches to solving algebra word problems. One approach called the direct translation method, emphasizes a rote mechanical translation of verbal statements into their algebraic equivalents. The alternative approach, the schema method, requires individuals to recognize problems as instance of a prototype and to retrieve from memory the appropriate method of solution. Herring found that both methods of solution increased accuracy over a no training control, but that schema- trained subjects were faster and more knowledgeable in their solutions.

Although not an essential concern to his research, Herring's results have implications for estimation processes in mathematics generally. Where both direct translation and schema methods can and do produce correct computation, the development and use of a schema representation for problem solving may lead not only to faster solutions, but more useful approximations of solutions. Extending Herring's results to our other number comparison paradigms, it may be the case that subjects use a schema for number representation to approximate answers to direct computational problems. The use of magnitude representation of numbers may provide a useful schema to compare number values rapidly and to judge the reasonableness of computed answers.

Sex Differences in Spatial Ability and Algebra Problem Solving

No one can long investigate mathematical cognition without soon confronting the issue of sex differences in mathematical abilities and understanding. Two aspects of this issue were considered in undergraduate honor theses with unexpected outcomes.

Guedet and Senchak attempted to investigate differences in spatial ability as a function of sex role identification as distinct from gender. Using the Bem Sexual Role Inventory, Senchak's hypothesis was that self-identification will predict spatial ability better than gender per se. Although sex differences in spatial ability are well documented in the literature and although Senchak used two spatial ability tasks that have previously identified male/female differences in spatial ability, she found no sex differences in her study. Failure to obtain sex differences in spatial ability was very surprising, but may be related to contemporary social-cultural changes. Senchak noted that most of the literature establishing sexual differences in spatial ability is quite old, mostly conducted in the 1950's and 1960's. It may well be the case that her more recent investigation parallels a shift in educational practices, cultural attitudes and social expectations for equal male/female abilities. Her results strongly suggest that a reexamination of some of the older literature on spatial abilities is in order.

In an associated study, Guedet examined the influence of sexual stereotypes in algebra word problem-solving. Male and female college students were asked to solve algebra word problems that were either presented in a stereotypic male form or a stereotypic female form. The hypothesis was ease of solving the problems would interact with subjects' self-identified sex roles. In terms of expectations, the results were again somewhat surprising. Females appeared to be unaffected by the form of the problem, while males solved significantly more problems presented in the masculine form than in the feminine form. Both males and females were more likely to attribute solution failures to the difficulty of the problems (external attributions) when cross-sex forms were used.

An Annotated List of
Papers, Publications, Presentations, and Theses
supported by or related to
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Berie, J. L. Judgment of numerical inequality: Use of sign information. Masters thesis, University of Iowa, 1978.

Three models of processing signed numbers were proposed and tested. The models include a composite model in which sign and number are treated as a single unit and two transformational models in which sign and number are processed separately in a serial manner. Obtained results clearly supported the unintuitive number-sign transformational model in which number meaning is extracted before sign information is evaluated.

Berie, J. L., & Hinrichs, J. V. Size-value congruity in the development of numeral comparisons. Midwestern Psychological Association, Detroit, May, 1981.

First, third and fifth graders chose either the larger numerical value or the larger sized numeral in pairs of single-digit numbers, varying in physical size and numerical value. Differences in physical size slowed value judgments whereas numerical value had minimal effect on judgments of size. The magnitude of the size-value congruity effect increased with age.

Berie, J. L., & Yurko, D. S. Comparing numerals and number words: Distance effects in same/different judgments. Psychonomic Society, Phoenix, November, 1979.

In contrast to previous number comparison research, the present investigation used a successive same/different procedure in order to evaluate two major interpretations of the representation and processing of numeric information. Distance effects were observed for both response time and errors. Additionally, same judgments were faster than different judgments. These results were consistent with an "internal psychophysics" interpretation, but not with the semantic encoding model.

Guedet, P. Sex differences in mathematical problem solving as function of sex role appropriateness of problem content. Honors thesis, University of Iowa, 1981.

College students, 133 males and 83 females, participated in an experiment designed to test the hypothesis that problem-solving performance should be facilitated when problem language and context are appropriate to the solver's sex role. Two forms of ten multiple-choice algebra word problems were constructed, one in a masculine context and one in a feminine context. The Bem Sex Role Inventory was utilized as the measure of sex role identification. Subjects were divided on the basis of their scores into eight sex by sex-type groups including androgynous, masculine, feminine, and undifferentiated. Approximately half of the subjects in each sex by sex-type group were given the masculine form of the problems; the other half were given the feminine form. The dependent measures included mean number of correct solutions, mean number of internal and mean number of external causal attributions, and mean solution time. Results did not support the hypothesis as stated; no significant interaction effects of sex role type and sex-appropriateness of problem emerged. However, results indicated significant gender differences. Males solved more problems than females overall. While females' performance did not differ across forms, males solved significantly more problems on the masculine form than on the feminine form. Females made more internal attributions for failure than males on the average. Males and females both made more external attributions on the cross-sex form. No differences emerged with respect to mean solution times.

Herring, R. D. Solving algebra word problems: A comparison of instructional strategies and problem characteristics. Ph.D dissertation, University of Iowa, 1981.

The process of solving algebra word problems provides a test of the instructional implications of current theories of cognitive psychology. Three preliminary studies and a main experiment investigated the effects of instructional strategy and problem identification on speed and accuracy of solving problems.

The effects of two instructional strategies (training methods) were compared. The first was the direct translation method in which subjects translated written problems into algebraic equations, solved these equations, and computed answers. The second strategy was the schema method in which subjects recognized problems as instances of prototypes of problems, retrieved solution methods from memory for each of the appropriate types of problems, and applied these solution methods to derive answers. Problem identification and classification were manipulated in two ways: (a) labeling problems by their correct solution methods, and (b) manipulating the problems' cover stories.

The main experiment was conducted with 86 beginning college students selected by pretests to have good algebra skills, but only fair ability to solve algebra word problems. These subjects received either direct translation or schema training, or were assigned to a no-training control group. Training subjects attended four sessions in which they worked on programmed instruction booklets. In a final test, half of all subjects received problems which were labeled by their correct solution methods, and the remaining subjects received problems which were not labeled. The pairing of cover story and solution method was varied for each subject by presenting half the problems with cover stories and solution methods which had been paired during training, and half the problems with cover stories and solution methods which had not been paired during training. All training and test materials are reproduced in the dissertation appendices.

Results showed that instructional strategy and problem cover story both affected learners' problem solving, but only instructional strategy affected problem classification. In the problem solving task, learners in the training groups solved a greater number of problems than subjects in the control group. The subjects in the two training groups correctly solved an equal number of problems, but the subjects in the schema group solved them more quickly than subjects in the other groups. Problems that had familiar pairings of cover story and solution method were more frequently and quickly solved than problems that had unfamiliar pairings of cover story and solution method. In the problem classification task, subjects in the schema group classified problems most frequently by correct solution method.

The type of training method produced different problem solving performance and classification by a sample of undergraduates of average mathematical ability. Both training methods improved solution accuracy, but the schema method produced faster solution times, and increased the number of classifications that were based on the mathematical aspects of the problems.

Hinrichs, J. V. Psychophysical judgment of numerosity compared to number judgments. Workshop on Children's Mathematical Cognition, Learning Research and Development Center, University of Pittsburgh, September 22, 1978.

Hinrichs, J. V., Berie, J. L., & Mosell, M. K. Place information in multidigit number comparison. Memory & Cognition, in press.

How do we use place information in a number comparison task involving multidigit numbers? In four experiments, subjects identified stimulus numbers as larger or smaller than the number 5000 in a choice reaction time task. As the difference between the number of places in the stimulus number and the number of places in the standard decreased, response time and errors significantly increased. When the number of places was held constant, the type of numeric information extracted depended on the value of the standard (5000 or 5555). In some cases, irrelevant place information affected choice time.

Hinrichs, J. V., Berie, J. L., & Yurko, D. S. Multidigit number comparison: Use of place-value information. Psychonomic Society, Phoenix, November, 1979.

Two aspects of positional notation, the value of the most significant place and the number of places, were investigated in a series of studies using a number comparison task. Consistent with analog models of comparative judgment, differences in absolute numerical value predicted decision times better than specific place-value cues.

Hinrichs, J. V., Yurko, D. S., & Hu, J.-M. Two-digit number comparison: Use of place information. Journal of Experimental Psychology: Human Perception and Performance, 1981, 7, 890-901.

Are two-digit numbers processed as two separate digits or as a single value? Two-digit numbers from 11 to 99 were compared to a standard (50 or 55) held in memory. Decision time decreased as numerical distance from the standard increased. The ones-place digit significantly affected decision time even when the tens-place digit was logically sufficient to select the correct response. The one digit could be ignored only when standard was placed at a decade boundary (50) and the range of tested numbers was small (±10). The results suggest that two-digit numbers are compared as integrated quantities with comparison time decreasing as a logarithmic function of the absoluted difference between the two numbers. A place-value effect occurred only when the presented number was in the same decade as the standard. In that case, the comparison process appears to shift to an evaluation of the ones-place digit.

Hinrichs, J. V. & Novick, L. R. Memory for numbers: Nominal versus magnitude information. Under review.

Across three paired-associated learning experiments, the recall of four-digit number responses to word stimuli favored the most significant digit over the least significant (magnitude encoding) rather than exhibiting typical bowed serial position functions (nominal encoding). Only with instructions emphasizing exact recall of each individual digit did recall functions revert to bowed curves. The results were interpreted as evidence for two kinds of number coding in semantic memory.

Holt, C. S. Temporal aspects of numerical inequality judgments for two-digit numbers. Honors thesis, University of Iowa, 1981.

Two-digit number comparisons have been suggested to follow a single-digit analog process (Hinrichs, Yurko, & Hu, 1981). If comparisons of two-digit numbers were made within specific decades, the various decades should follow the increasing RT function of absolute numerical values demonstrated for single-digit numbers (Parkman, 1971). Subjects were presented with number pairs from the 20s, 50s, and 80s decades as well as similar comparisons between decades around the three target decades. The results of the present experiment suggest a more complex process than the single-digit analog by itself. Support is provided for a construct in which individual digits are initially encoded separately and subject to the task demands, then re-encoded into single quantities for final comparison. Place-value cues are also suggested in which the individual digits in the ten's place are compared before re-encoding, as are the one's place digits.

Hu, J.-M. Comparison of numbers and numerosity: A developmental study of psychophysical judgment. Ph.D dissertation, University of Iowa, 1978.

Reaction time studies with adult subjects suggest a quantitative basis for their number concepts; i.e., numbers appear to be internally encoded as analog magnitudes and comparisons among them are made in a manner analogous to psychophysical judgments. A limited number of developmental investigations have also replicated these results with children as young as kindergarteners. However, very little evidence concerning children's psychophysical judgments has thus far been documented in the literature in order to enable closer evaluation of the relationship between number processing and psychophysical comparisons. Hence, the present thesis was conducted with a primary empirical interest in systematically exploring children's psychophysical judgments of numerosity with stimuli varying in physical arrangements, in addition to their judgment of numerical inequality.

Two experiments were conducted. In Experiment 1, kindergarteners, second graders and adults were tested with a judgment of numerical inequality task (choose the larger single-digit number) and three judgment of numerosity inequality tasks (choose the display with greater number of dots) differing mainly in the arrangement of stimulus dots: (a) Under the length condition, pairs of vertical arrays of unequal numbers of dots were presented with length as an additional inequality cue. (b) Under the density condition, pairs of vertical arrays equal in length but varying in linear density were presented. (c) Under the random pattern condition, two dot patterns were presented with the dots randomly distributed over a 3 x 6 position matrix. Only second graders and adults were studied in Experiment 2 with the judgment conditions varying between subjects: three different groups of subjects at each age level were tested under either the number, length, or random pattern conditions.

In both experiments even the youngest children were clearly capable of numerosity judgments with arrays containing as many as eight dots, regardless of their physical arrangement. No major age trend was observed: numerosity judgments in the length condition, particularly for pairs of small physical difference, were relatively easier than in the density or the random pattern condition for both children and adults. A logarithmic relationship between decision time and numerical inequality, similar to that observed with numerosity judgments, was also obtained for all subjects, replicating previous adult data and suggesting an analog comparison process.

Regression analyses indicated a close relationship between number comparison and the psychophysical judgment of length. Consistent failure to obtain significant differences in the slope parameters of the decision time functions for the length and the number conditions suggests that the functional relationship for number comparison is probably isomorphic to the psychophysical comparison of line length. Therefore, the pattern of the results suggests no qualitative differences in the process employed for number judgments versus numerosity judgments by both children and adult subjects. In addition, the striking similarity in performance under the number and the length conditions argues for a functional isomorphism between the cognitive representations of these two types of information.

Hu, J.-M, & Hinrichs, J. V. Comparison of number and numerosity: A developmental study of psychophysical judgment. Midwestern Psychological Association, Chicago, May, 1978.

Kindergarteners, second graders and college students were asked to judge both numerical inequality and the relative numerosity of dot patterns varying in physical arrangements. Reaction time patterns for all subjects indicated a closer psychophysical relationship between number and dot patterns varying in length than other arrangements (density or random).

Mosell, M. K. Place information in number comparison. Honors thesis, University of Iowa, 1978.

The storage and processing of place information was examined in a number comparison task involving one- to seven-place numbers. Fourteen undergraduate students, eight females and six males, identified stimulus numbers as larger or smaller than the number 5000 in a two-choice reaction time task. It was found that as the difference between the number of places in the stimulus number and the number of places in the standard decreased, RT and errors significantly increased. In addition, when the value of the first digit was altered so that the numerical difference between the stimulus number and the standard became smaller, RT increased in three-place numbers and decreased in five-place numbers. The interaction between the number of places within the stimulus number and its first-digit value was significant. A strict place-difference model of number comparison was rejected. The application of Moyer and Landauer's (1967) analog model was extended to a broader range of numbers.

Senchak, M. T. Sex roles and the sex difference in visual-spatial ability. Honors thesis, University of Iowa, 1981.

To assess sex differences in visual-spatial ability and the influence of sex roles on spatial ability, 216 college freshmen completed the Card Rotation Test, the Hidden Figures Test and the Bem Sex Role Inventory. Contrary to expectations, there were no sex differences on either spatial task. The BSRI was able to differentiate subjects on the basis of sex type, but the sex role classifications did not vary significantly with spatial ability. Socio-cultural change was discussed as possible explanation for the lack of sex differences in spatial ability.

Yurko, D. S. Size-value congruity and comparative numerical judgments. Masters thesis, University of Iowa, 1978.

Three models of the size-value congruity effect were proposed and tested. The size-value congruity effect refers to influence of irrelevant size cues on the processing of the value of numerical stimuli; when size and value are congruent, i.e., both relatively large or both relatively small, decision time is faciliated, but when the size and value are incongruent, decision time is increased. The results were inconsistent with a semantic encoding interpretation (Banks, 1977) and a response competition explanation (Paivio, 1975). A simple normalization model was also unable to account for the observed data. The best interpretation at present appears to be a parallel processing model with information from each decision accumulator "leaking" over to affect the other.

Yurko, D. S., & Hinrichs, J. V. Judgment of numerical inequality: Size-value congruity. Midwestern Psychological Association, Chicago, May, 1978.

College students chose either the larger or the smaller numerical value in pairs of single-digit numbers, varying in physical size and value. Compared to equal-size controls, incongruent size-value pairs were slower, but larger-smaller psychophysical functions did not differ. Congruent pairs were faster, and numerical value interacted with type of decision.