DOCUMENT RESUME

ED 218 100 SE 038 101

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TITLE An Application of Mathematical Groups to Structures of Human Groups. Applications of Finite Mathematics to Anthropology and Sociology. Modules and Monographs in Undergraduate Mathematics and Its Applications Project. UMAP Unit 476.
INSTITUTION Education Development Center, Inc., Newton, Mass.
SPONS AGENCY National Science Foundation, Washington, D.C.
PUB DATE 80
GRANT SED-76-19615-A02
NOTE 30p.

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS *Anthropology; *College Mathematics; *Groups; Higher Education; Instructional Materials; *Learning Modules; *Mathematical Applications; *Mathematical Models; Models; Problem Solving; Sociology; Supplementary Reading Materials
IDENTIFIERS *Group Theory

ABSTRACT

This module is designed for students with a high school algebra background. The goal is to present the elements of the group idea, primarily by way of a geometric model, and to see its application to the study of kinship relations within certain human groups. The material opens with a presentation of clans in a hypothetical society in an early stage of development. It is noted that the application of mathematics to social-science questions is not nearly as "clean" as in the physical sciences, but models of the type used have been helpful to those who study the structures of complex human organizations. Exercises are included, with answers provided at the conclusion of the module. (MP)

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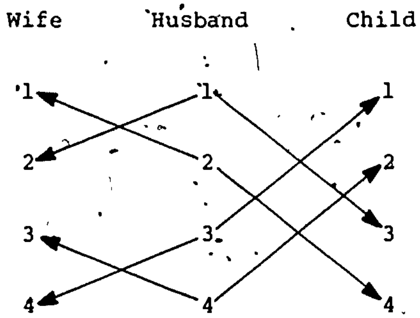
UNIT 476

MODULES AND MONOGRAPHS IN UNDERGRADUATE MATHEMATICS AND ITS APPLICATIONS PROJECT

AN APPLICATION OF MATHEMATICAL GROUPS TO STRUCTURES OF HUMAN GROUPS

by Roger Carlson



APPLICATIONS OF FINITE MATHEMATICS TO ANTHROPOLOGY AND SOCIOLOGY

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AN APPLICATION OF MATHEMATICAL GROUPS
TO STRUCTURES OF HUMAN GROUPS

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Intermodular Description Sheet: UMAP Unit 476

Title: AN APPLICATION OF MATHEMATICAL GROUPS TO STRUCTURES OF HUMAN GROUPS

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Review Stage/Date: III 10/15/80

Classification: APPL FINITE MATH/ANTHROPOLOGY & SOCIOLOGY

Prerequisite Skills:
1. High school algebra.

Output Skills:
1. To learn the elements of the group idea, primarily by way of a geometric model, and to see its application to the study of kinship relations within certain human groups.

The author would like to thank Paul Mullinix, a graduate student at the University of Missouri, Kansas City, for preparing the answers to the exercises for this unit.

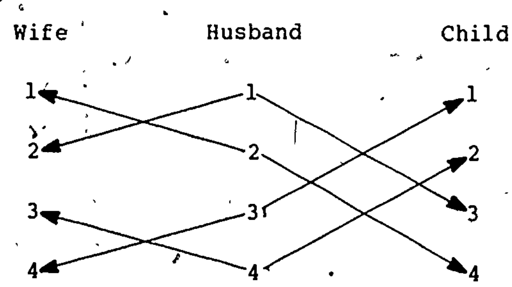
The Project would like to thank James J. Kaput of Southeastern Massachusetts University; Bernice Kastner of Montgomery College, Tacoma Park, Maryland; and Peter Lindstrom of Genesee Community College, Batavia, New York, for their reviews, and all others who assisted in the production of this unit.

This material was prepared with the partial support of National Science Foundation Grant No. SED76-19615 A02. Recommendations expressed are those of the author and do not necessarily reflect the views of the NSF or the copyright holder.

1. INTRODUCTION

This module presents the idea of a mathematical group and shows an application to the structure of human groups.

Many societies in an early stage of development are divided into sets of families called clans, and marriage and descent depend upon clan membership. For example, imagine a society divided into four clans, which we'll identify by 1, 2, 3, and 4. In the diagram below the arrows pointing to the left show the permitted marriages of this society while the arrows pointing to the right show the clan membership of the offspring.



We see that if a man belongs to clan 1 then he must marry a woman from clan 2. Their children will belong to clan 3. We can gather further information from this diagram. For example, a man from clan 1 would have grandchildren in clans 1 and 2. That is, his son would be in clan 3 and thus his son's son would be in clan 1. (See diagram.) But his daughter, who is in clan 3, would have to marry a man from clan 4, and their child would be in clan 2. In the same, rather tedious way, we could trace out the clan of any person's relatives and thus discuss a variety of questions that are of interest to anthropologists. Will our clan 1 man have some relative in each of the four different clans? (This would give the society some cohesiveness). What types of relatives are allowed to marry? For instance, can cousins marry according to the diagram above? We will introduce a neat mathematical way of attacking questions like these using (mathematical) group theory.

Abstractly, the diagram above shows that wives' and children's clans are merely a re-ordering, or permutation, of the numbers 1, 2, 3, and 4. That is, if we put the husbands in the order 1, 2, 3, 4 then the wives will be arranged 2, 1, 4, 3 and the children will be arranged as 3, 4, 1, 2. Our main example of a mathematical group is based on the different arrangements of 1, 2, and 3 that may be obtained by geometric transformations of a triangle with vertices labeled 1, 2, and 3. A study of this group will

give us the tools for looking at some anthropological questions. A great deal more can be found in the book An Anatomy of Kinship by H.C. White.

The application of mathematics to social-science questions is not nearly as "clean" as in the physical sciences. In particular, we do not expect to find any human societies as neatly arranged into clans as described here; nor are the rules of marriage and descent precisely and absolutely followed. The "law" of marriage is certainly different in its force than the "law" of gravity! Nevertheless, models of this type have been helpful to those who study the structures of complex human organizations.

2. AN EXAMPLE OF A MATHEMATICAL GROUP

2.1 Labeled Triangles

Draw an equilateral triangle, and label each vertex with one of the numbers 1, 2, and 3. Label each vertex with a different number. Your triangle is labeled in a particular way. Before going on, think about this: In how many ways could your triangle have been labeled (using only the numbers 1, 2, and 3 and using each number only once)? Write your answer here _____.

There are actually six ways of labeling the vertices of a triangle. To see this, pick any vertex and note that it could be labeled with any one of the three numbers. Once this vertex is labeled, there are two numbers left for the next vertex. And when two vertices have been labeled the third vertex can be labeled only in one way -- because only one number is left. Thus, in all, there are

$$(3 \text{ ways}) \times (2 \text{ ways}) \times (1 \text{ way}) = 6 \text{ ways}$$

of labeling the vertices of a triangle.

Exercise 1. (i) According to the argument above, how many ways are there to label the vertices of a square? (ii) How many ways of labeling the vertices of a regular n-sided polygon?

2.2 Rotations

All six ways of labeling an equilateral triangle are shown in Figure 1. For our discussion we will call triangle (a) in Figure 1 the "standard triangle." Stop a moment and make a standard triangle for yourself. Cut it out of cardboard or heavy paper and label it just as in Figure 1 (a). Be sure to label both sides of each vertex with the same number because later you will need to turn the triangle over.

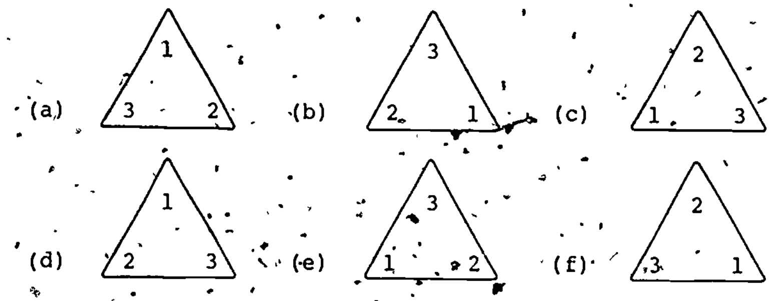


Figure 1. Six ways to label a triangle.

Each of the triangles in Figure 1 can be obtained from the standard triangle by simple transformations of the standard triangle. To see this, take your triangle and put it into the standard position of Figure 1(a) and rotate it about its center. You can see that Triangle (b) is obtained by a clockwise rotation of 120 degrees. Triangle (c) is obtained by rotating Triangle (b) clockwise through 120 degrees which is the same as rotating Triangle (a) through 240 degrees. In fact, any one of the triangles (a), (b), or (c) can be obtained from any one of the other triangles (a), (b), or (c) by a rotation about its center.

Exercise 2. (i) Through how many degrees must Triangle (a) be rotated in order to obtain Triangle (a)? (ii) How many answers are there to Question (i)? (iii) Through how many degrees must Triangle (a) be rotated in a counterclockwise direction to get Triangle (b)? (iv) Triangle (c)? (v) Triangle (a)? (vi) Can Triangles (d), (e), and (f) be obtained from each other by rotation?

2.3 Flips

Although Triangles (a), (b), and (c) can be obtained from one another by rotation, none of Triangles (d), (e), or (f) can be obtained in this way from (a) or (b) or (c). The reason for this is that in all of the triangles (a), (b), and (c) the numbers 1, 2, and 3 are in the same relative order. Reading the labels of the vertices clockwise in any of these triangles we always get the order 1-2-3 for these numbers, whereas for Triangles (d), (e), and (f) the order is 1-3-2. To change the relative order of the labels we have to "flip" the triangle over (like flipping a coin). Again put your triangle in standard position, pick it up and flip it over on the axis through vertex 1. The vertex labeled "1" keeps its position at the top but vertices "2" and "3" are interchanged giving triangle (d). Can Triangle (e) be obtained from Triangle (a)? Notice that Triangles (a) and (e) have the same label for the right-hand vertex. Thus a flip through this vertex will give (e) from (a).

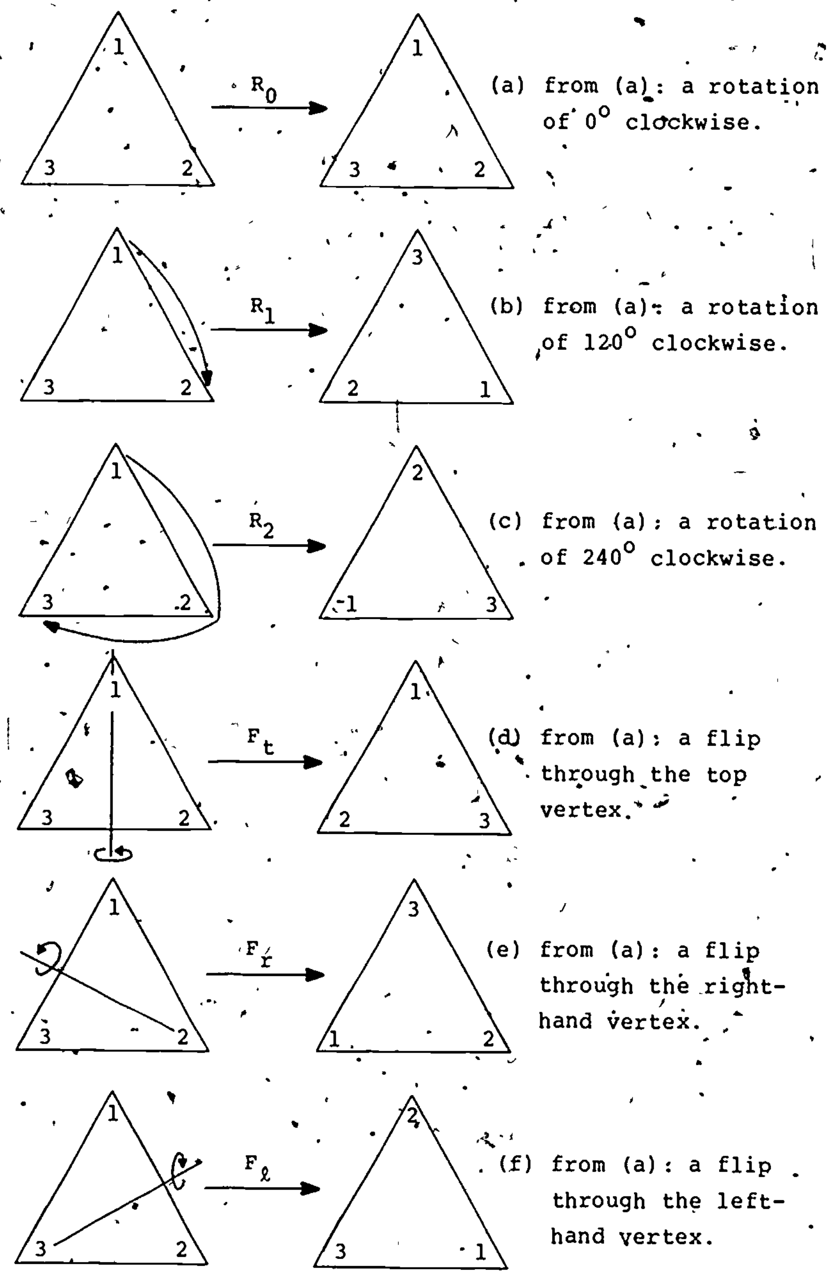


Figure 2. Operations on triangle (a) to get triangles (a), (b), ..., (f).

(Or (a) from (e)!) In the same way, you see that Triangle (f) can be obtained from (a) by flipping through the vertex labeled "3."

The results of rotating clockwise and flipping the standard triangle to get each of the triangles in Figure 1 are shown in Figure 2. To make Figure 2 complete note that we have used a rotation of zero degrees (also 360 degrees, 720 degrees ...). Each of these operations is given a suitable label: R for rotations; F for flips.

Exercise 3. (a) Make a figure like Figure 2 showing how these operations transform Triangle (b)--that is, use Triangle (b) for the "standard." (b) Make a figure like Figure 2 using Triangle (d) as the "standard," and complete Table 1.

TABLE 1

Results of Using Triangle (d) as "Standard"

Operation on Triangle (d)	Resulting Triangle
R_0	
R_1	(f)
R_2	
F_t	
F_r	(c)
F_ℓ	

It should be clear from Exercise 3 that any of the labelings in Figure 1 can be taken as the standard labeling and that by use of the six operations -- three rotations and three flips -- any other triangle of Figure 1 can be obtained.

2.4 Successive Applications of Rotations and Flips

It is now natural to ask what would happen if, beginning with any standard triangle, we apply the operations of Figure 2 successively, mixing flips and rotations. For example, what happens to Triangle (a) if we rotate it through 240 degrees and then flip it through the top? The rotation takes Triangle (a) into (c) and a flip of (c) through the top takes it to (f). From Figure 2 we also see that (f) can be obtained from (a) by a flip through the left-hand corner. Thus the two operations R_2 and F_t, performed in this order, are equivalent to the single operation F_ℓ. Because of certain analogies with ordinary multiplication

it is customary to call F_ℓ the product of R_2 and F_t and write the equivalence:

$$R_2 \times F_t = F_\ell.$$

Notice that although we used Triangle (a) as our standard here -- because we had to have some triangle to work with -- the same result would be obtained by using any other standard. For example, beginning with (d) R_2 gives (e) and then F_t gives (b). But (b) is obtained from (d) by F_ℓ.

Other equations (equivalencies) can be obtained by successive application of the six operations. For example, a flip through the top of Triangle (a) gives Triangle (d), and if this is then flipped through its right-hand corner we arrive at Triangle (c). But Triangle (c) can also be obtained from (a) by a rotation of 240 degrees, so:

$$F_t \times F_r = R_2.$$

Rotating and flipping your triangle will enable you to find the product of any two operations as above. There are 36 different products in all, but there are only six different answers since no matter what we do to a triangle -- by way of rotating and flipping -- we always get one of the six positions of Figure 1. Before going on you should do a few of these. For example, those given in the exercise below.

Exercise 4. Using Triangle (a) as the standard, verify the following results:

(i) $R_1 \times R_1 = R_2$ (ii) $R_0 \times F_t = F_t$

(iii) $F_t \times F_t = R_0$ (iv) $F_t \times R_2 = F_r$.

(v) Repeat (i) through (iv) above using Triangle (d) as the standard.

2.5 The Multiplication Table

Table 2 gives the product of each of the operations. The row headings in Table 2 indicate the first operation and the column headings indicate the second operation. Thus from Table 2 we see that R_1 followed by F_t is F_r. (Look in the row headed R_1 and in the column headed F_t.) This table is analogous to a multiplication table for numbers. There is, however, one very important difference between this kind of product and products of numbers. If you look in the row headed F_t and the column headed R_1 you find F_ℓ not F_r. That is,

$$R_1 \times F_t \neq F_t \times R_1$$

and we see that the product of operations on labeled triangles is a non-commutative product. Table 2 shows other non-commutative products. In fact, the only commutative products shown in Table 2 are those involving two rotations or the same flip performed twice. (e.g. $F_t \times F_t = R_0$). This is one major difference between this type of product and products of numbers. There are, however, quite a few similarities between these two products which we discuss below.

Exercise 5. (i) Verify the entries in Table 2 that have not already been calculated above filling in the blanks where necessary. (ii) For each operation X, use Table 2 to find an operation A so that $X \times A = R_0$. (iii) Do X and A always commute? (That is, if $X \times A = R_0$ is $X \times A = A \times X$)?

2.6 Table 2 as a Mathematical Group

The operations on labeled triangles represented in Table 2 are an example of what is called a group in mathematics. Though we are not concerned here with the general theory of groups we point out the main properties of Table 2 (and operations on triangles) that make it a group. Multiplication of rational numbers (excluding zero) is also a group operation and we will use this example too in our discussion.

TABLE 2

Products of the Six Operations Performed Successively

Second Operation

	R_0	R_1	R_2	F_t	F_r	F_ℓ
R_0	R_0	R_1	R_2	F_t	F_r	F_ℓ
R_1	R_1		R_0	F_r		F_t
R_2	R_2	R_0	R_1	F_ℓ	F_t	F_r
F_t	F_t	F_ℓ	F_r	R_0	R_2	R_1
F_r		F_t	F_ℓ	R_1	R_0	R_2
F_ℓ	F_r	F_t	F_ℓ		R_1	R_0

1. R_0 is a very special operation. R_0 leaves every triangle the same, so for any operation X, $X \times R_0 = R_0 \times X = X$. The number 1 plays the same kind of role in the multiplication of positive rational numbers. R_0 and 1 are called the identity for their respective products or groups. Every group has an identity.

2. Given an identity for a product it is natural to ask whether every element has an inverse. In the group of positive rational numbers the inverse of any number is its reciprocal -- the inverse of 3 is 1/3, the inverse of 5/2 is 2/5. An operation A is the inverse of an operation X, if and only if $X \times A = A \times X = R_0$, (the identity). In Exercise 5 (ii) you found that each of the operations of Table 2 has such an inverse. These are probably easier to find by thinking of the geometry of the labeled triangle: What do we have to do after any operation to get back to the "standard?" After any flip, the same flip will get us back, so the inverse of F_x is just F_x. After rotating through 120 degrees, a rotation of 240 will get us back to standard position and conversely. So, R_1 and R_2 are the inverses of each other. Every operation here has an inverse operation -- just as every positive rational number has a reciprocal. In a group every element must have an inverse.
3. A group must be a closed set, which is to say that the product of any two elements of the group is also an element of the group. In terms of Table 2, closure is easily checked by noting that the only thing that is inside the table (as a product) is one of the six operations. The set of rational numbers is closed under multiplication, since the product of two rational numbers is always a rational number.
4. Finally, we mention another property of groups: the product operation for a group must be associative. That is, if we take any three operations -- say, F_t, F_r, and R_1 then the product is the same no matter which product we do first. Thus, we have, for example

$$(F_t \times F_r) \times R_1 = F_t \times (F_r \times R_1).$$

The order of the factors within the product must of course remain the same. In words, F_t times F_r and then this product times R_1 must be the same as F_t times the product of F_r times R_1 (performed first). This fact can be checked for all the possible products in Table 2.

2.7 Other Examples

1. The set of positive and negative integers together with zero is a group under the operation of addition. The identity is zero and the inverse of an integer is its negative. (The inverse of -5 is +5.)
2. "Clock arithmetic" is a group under "clock addition." Three o'clock plus 11 hours is two o'clock, etc. Three o'clock minus 4 hours is eleven o'clock. Just counting

hours, there are 12 elements in the group; the identity is 12 hours; and the inverse of x is 12 - x.

2.8 Summary

The operation on labeled triangles defined here is one of many examples of a group. There are six possible labelings of an equilateral triangle and any one of these can be obtained from any other by a rotation (including a rotation of zero degrees) or a flip through a vertex. Taking these operations -- not the triangles -- we formed a group by successive application of any two operations to a triangle. (Any triangle may be used for the "standard.") We called the successive application of two operations the product, and wrote $R_1 \times F_t$ to mean the operation that is equivalent to a rotation of 120 degrees followed by a flip through the top corner. Table 2 gives all the results of such products. There are similarities between this product and the multiplication of rational numbers, but there is an important difference: $R_1 \times F_t \neq F_t \times R_1$, thus giving an example of a non-commutative group. For this group, R_0 is the identity (compare "1" for rational numbers under multiplication). We computed the inverse of each operation X as the operation that when multiplied by X gives R_0.

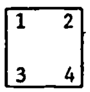
Exercise 6. Verify that R_1, R_2 and F_r associate. That is,

$$(R_1 \times R_2) \times F_r = R_1 \times (R_2 \times F_r).$$

Exercise 7. Show that every operation for the equilateral triangle can be given by successive use of only two operations R_1 and F_t. Thus R_1 and F_t can be used to generate the group. What other pairs can be used in this way?

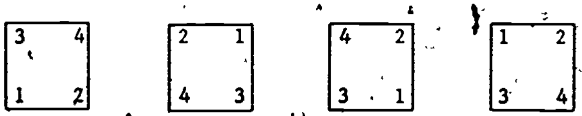
Exercise 8. Explain your answer to Exercise 8 in terms of the geometry of the triangle.

Exercise 9. Make a square and label it like the one below.



Develop the group of rotations of your (the) square.

Exercise 10. Consider, in addition to the rotations above, four flips of the square as shown below.



F_h F_v F_ℓ F_r

Develop the group of these eight operations on the square.

Exercise 11. Suppose you had a six-hour clock. Construct the multiplication table for the group of this clock.

Exercise 12. Compare the multiplication table for the six-hour clock and the multiplication table for the operations on the triangle. In particular, is the clock-table commutative?

Exercise 13. A subgroup is a subset of a group that is itself a group. Is the set consisting of R_0, R_1, R_2 a subgroup for the operations on the triangle? Are there any other subgroups?

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3. ANOTHER METHOD OF COMPUTING PRODUCTS

All of the calculations so far were done by rotating and flipping an actual triangle or by looking at Figure 2. In many examples of groups concrete models like the labeled triangle are a little harder to construct. We therefore introduce another method for computing the product of two operations.

To calculate the product of two operations we introduce a notation that shows what happens to each of the labels when the triangle is transformed. When we look at what happens to 1, 2, and 3 when the standard triangle (a) is rotated through 120 degrees we find that:

1 takes the place formerly held by 2

2 takes the place formerly held by 3

3 takes the place formerly held by 1.

This is written as follows:

$$R_1 = \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix}.$$

We can say to ourselves: "1 goes to 2, 2 goes to 3, and 3 goes to 1 under R_1." From Figure 2 we see that in the same way each of the six operations can be written

$$R_0 = \begin{pmatrix}1 & 2 & 3\\ 1 & 2 & 3\end{pmatrix} \quad R_1 = \begin{pmatrix}1 & 2 & 3\\ 2 & 3 & 1\end{pmatrix} \quad R_2 = \begin{pmatrix}1 & 2 & 3\\ 3 & 1 & 2\end{pmatrix},$$

$$F_t = \begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 2\end{pmatrix} \quad F_r = \begin{pmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{pmatrix} \quad F_\ell = \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & 3\end{pmatrix}.$$

Note that in this way of denoting an operation the numbers 1, 2, and 3 are always on the top row in the order 1, 2, 3. The second row gives the number that takes the place of the number above it. With this scheme, we can now calculate a product by following the "history" of the numbers. For example, the product R_1 times F_t would be written down in this fashion

$$R_1 \times F_t = \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix},$$

and the result of the product can be given by reading from left to right: "1 goes to 2 and 2 goes to 3, so 1 goes to 3."

$$R_1 \times F_t = \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} = \begin{pmatrix}1&2&3\\3&&\end{pmatrix}$$

Then, "2 goes to 3 and 3 goes to 2, so 2 goes to 2." Lastly, "3 goes to 1, 1 goes to 1 and so 3 goes to 1." So finally,

$$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} = \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix},$$

or, as before, $R_1 \times F_t = F_r$. Of course we get exactly the same product as before. This is not a new product, it is just another way of computing the same product.

It may help to have another example here.

$$F_r \times F_t = \begin{pmatrix}1&2&3\\3&2&1\end{pmatrix}\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} = \begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} = R_1.$$

Thus: "1 goes to 3 and 3 goes to 2, so 1 goes to 2;"
"2 goes to 2 and 2 goes to 3, so 2 goes to 3;"
"3 goes to 1 and 1 goes to 1, so 3 goes to 1."

Exercise 14. (i) Do two more products this way and check your answers using Table 2. (ii) Consider two operations A and B on the numbers 1, 2, 3, and 4 as given below. Compute A × B and B × A.

$$A = \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix}$$

$$B = \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix}$$

(iii) Compute the same products as in (ii) above defining A and B by operations on a labeled square.

(iv) Use this method of calculation to find the inverses of R_1 and F_t. Thus to find the inverse of R_1 write

$$\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix}\begin{pmatrix}1&2&3\\ & & \end{pmatrix} = \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$$

and fill in the blanks. Do the same for F_t.

---

4. AN APPLICATION TO KINSHIP STRUCTURES

> "...modern society still leaves the kinship bond as the dominant institution of simpler societies, fulfilling many of the welfare functions of the modern state, -- controlling ... the performance of work, the gratuitous rendering of economic sevices, the kinds and incidents of property, and ... the organization of work in the household." (Stone, p. 122).

Every society has some rules governing the types of marriages that are permitted and the types of marriages that are prohibited. For example, no society allows a parent to marry a child; very few societies allow brother-sister marriages; and most industrialized societies prohibit marriages between cousins. Of course, when we say that a certain type of marriage is "prohibited" we mean that in one way or another it is "against the law;" some type of punishment is supposed to follow a prohibited marriage. For several reasons these rules of marriage prohibition have interested anthropologists. Anthropologists have found that when the marriage rules become complicated -- as we'll illustrate below -- the mathematical theory of groups can be used to simplify their analysis. It is even possible to make mathematical calculations that will reveal prohibited marriages between relatives who are quite remote.

Among societies whose economy is based on hunting and fishing, the rules of marriage and descent are often quite explicit. Each person belongs to a marriage group, called a clan, and the members of one particular clan can only marry the members of another particular clan. The clan membership of the father will then determine that of the children and thus any relative's clan can be given by tracing through the family "tree."

For what follows we make the following assumptions regarding clans and kinship:

1. The society is divided into a definite number of distinct clans. Every person belongs to one and only one clan.
2. Clan membership determines marriage type. A man of clan A can only marry a woman of clan B and in no case may A be the same as B. (Brother-sister marriages in this broad sense are always prohibited).
3. Clan membership determines descent. The child of a man in clan A will always belong to a particular clan X. Children whose fathers are in different clans will themselves be in different clans.
4. All other kinships are given by rules 1, 2, and 3 above.

4.1 The Wife Transformation

Suppose that a society has only four clans and that we number them 1, 2, 3, and 4. As a specific example suppose that the rules of the society specify that

a man from clan 1 can only marry a woman from clan 2
a man from clan 2 can only marry a woman from clan 4
a man from clan 3 can only marry a woman from clan 1
a man from clan 4 can only marry a woman from clan 3.

Regarding these rules as transformations of the labels 1, 2, 3, and 4 we can write, as above:

$$W = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix},$$

where W stands for a man's wife. Thus, each relation to a man, e.g., wife or sister's husband, can be represented by a permutation. We can now calculate with W and see what meaning the results would have in terms of the marriage rules. For example,

$$W \times W = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

What does this mean? Clearly this is not the clan of a man's wife's wife, but it is the clan of a man's wife's brother's wife. That is, a man in clan 1 would marry a woman from clan 2, her brother would also be in clan 2 and his wife would then be from clan 4. So, 1 goes to 4 under W × W, as shown in the calculation above. In the same way clans 2, 3, and 4 are transformed to 3, 2, and 1 as shown by the above product.

We will be especially interested in obtaining the inverse of some of the transformations given here. Note that since W gives the clan of man's wife, W^{-1} (the inverse of W) gives the clan of a woman's husband. We could write H for the husband transformation, but W^{-1} will do as well. First we need to note that the identity transformations for four clans is given by

$$I = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

(Compare this to R_0 for the equilateral triangle.) Notice that since brothers and sisters are always in the same clan, the letter I can be used as a brother transformation or as a sister transformation and we will occasionally write B or S instead of I. So, in the calculation above we could have written wife's brother's wife = W × B × W. But B = I so that W × B × W = W × I × W = W × W. The transformations B and S (for brother and sister) need not be written just as the number 1 need not be written in a multiplication problem. They are sometimes helpful, however, in writing down a complicated relationship.

Coming back to the calculation of an inverse, to find W^{-1}, we recall that the inverse of any element of a group is the transformation that brings us back to I. Thus, since W^{-1} denotes the inverse of W we must have

$$W \times W^{-1} = I.$$

Since W and I are known we can solve for W^{-1} by finding the labels a, b, c, and d so that

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 & 4 \\ a & b & c & d \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}.$$

Tracing through the product we see that since 1 goes to 2 under W, 2 must go back to 1 under W^{-1}. Therefore, b = 1. Similarly, since W takes 2 to 4, W^{-1} must take 4 back to 2 and so d = 2. Continuing in this way we have:

$$W^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}.$$

Exercise 15. (i) Compute $W^{-1} \times W$. (ii) A man can have two types of brothers-in-law: his wife's brother and his sister's husband. What transformations give these relationships? (It may help to use S and B here and then substitute I.) (iii) Do the calculations for (ii). (iv) Suppose a transformation T is given by

$$T = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}.$$

Calculate T^{-1} as above. (v) Compute T × T.

4.2 The Child Transformation

All the relationships among members of a society divided into clans can be given if we know the rules of marriage and descent. Thus, for the present example, suppose that we are also given:

the child of a man in clan 1 is in clan 4
the child of a man in clan 2 is in clan 3
the child of a man in clan 3 is in clan 2
the child of a man in clan 4 is in clan 1.

Or, more compactly,

$$C = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

Calculating as before, the inverse of C is given by

$$C^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}.$$

Note that in this particular example, $C = C^{-1}$ and therefore $C \times C = I$. The interpretation of the product $C \times C$ is that it gives the clan of a child's child. Since descent is through the male this would give the clan of a man's son's children. Notice that this is not the clan of a man's daughter's children. Their clan would be given by $C \times W^{-1} \times C$, where W^{-1} gives the clan of the daughter's husband.

Exercise 16. Compute $C \times W^{-1} \times C$. Do the calculation in two ways, first as $(C \times W^{-1}) \times C$ and then as $C \times (W^{-1} \times C)$. Be sure that you get the same answer both times because group product is associative!

We mentioned above that a man has two types of grandchildren; son's children and daughter's children. As an illustration of the use of the mathematics of groups consider the following question: What would the rules of a society be like if these two types of grandchildren belonged to the same clan? That is, what is implied by the equation

$$C \times W^{-1} \times C = C \times C \ ?$$

Our calculations will be easier to follow if we drop the "times sign," as we often do in algebra. So consider the equation

$$CW^{-1}C = CC \ .$$

First, multiply both sides of this equation on the left by C^{-1} and use the fact that CC^{-1} is I. Thus,

$$C^{-1}(CW^{-1}C) = C^{-1}(CC)$$

$$(C^{-1}C)(W^{-1}C) = (C^{-1}C)C$$

$$IW^{-1}C = IC$$

$$W^{-1}C = C \ .$$

Now multiply both sides on the right by C^{-1} to get

$$W^{-1}I = CC^{-1} = I$$

or, finally,

$$W^{-1} = I \ .$$

All of this calculation comes down to this: The two types of grandchildren belong to the same clan only when $W^{-1} = I$. That is, only when the clan of a woman's husband is the same as her own clan. In this case she could marry her brother but brother-sister marriages are never allowed in I.

this discussion. Only where brother-sister marriages are permitted will two types of grandchildren belong to the same clan.

4.3. First Cousin Marriages

The two types of grandchildren discussed above are actually cousins. More precisely, they are cross cousins, meaning that their parents are brother and sister. We have seen that cross cousins will never belong to the same clan. We now look at a slightly more complicated question: can cross cousins (of different sex) marry? To answer the question we must realize that there are two types of cross cousins. The two types are shown in Figure 3 below which gives part of the family "tree" for cousins. In Figure 3 we use the symbol Δ to denote a male, and the symbol 0 to denote female. Marriage is indicated by a horizontal line, descent by a vertical line, sibship by an equals sign.

(a) (b)

Figure 3: Family "trees" for cross cousins.

Thus in Figure 3(a) the boy on the left is related to the girl on the right by the fact that his father is her mother's brother. In 3(b) his mother is her father's sister. We will see that the relationship of the two types of cross cousins give different equations and hence that the rules permitting the marriages can vary from society to society. We deal with each in turn.

Figure 3(a). Patrilineal cross cousins. We can write the equation relating the cousins in 3(a) by beginning with the boy and tracing the "tree" until we arrive at the girl. For each step of the relationship we write down the appropriate transformation and form the product. Thus the boy's father's (C^{-1}) sister (S) is the girl's mother. Descent is through her husband (W^{-1}) and the boy's cousin is the child (C) of this man. Thus we can write $C^{-1}SW^{-1}C$ or

$$C^{-1}W^{-1}C$$

for the father's (sister's) husband's child. If these people can marry then this transformation must be the same as the wife transformation. We thus have:

$$C^{-1}W^{-1}C = W$$

as the equation that corresponds to the question, can cousins of type 3(a) marry? This equation can be simplified a little by multiplying both sides on the left by C. This gives

$$CC^{-1}W^{-1}C = CW \quad \text{or} \quad W^{-1}C = CW.$$

To answer the question for any particular society we need to calculate these two products and see if they are the same. For our example,

$$W^{-1}C = \begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} = \begin{pmatrix}1&2&3&4\\2&4&1&3\end{pmatrix},$$

while

$$CW = \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\2&4&1&3\end{pmatrix} = \begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix}.$$

Since the right sides of these equations are not alike, marriage of this type is not permitted in the example given here. In other cases such marriages may be permitted depending on the particular rules W and C.

Figure 3(b). Matrilineal cross cousins. From Figure 3(b) we see that the boy's father's wife is the sister of the girl's father. In other words the girl is the boy's wife's (brother's) child. We write $C^{-1}WBC$, or

$$C^{-1}WC,$$

since B = I. If they are permitted to marry then $C^{-1}WC$ must be the same as W. We therefore have

$$C^{-1}WC = W$$

as the equation for the marriage of matrilineal cross cousins. Multiplying both sides on the left by C we have

$$WC = CW.$$

The cousins of Figure 3(b) are permitted to marry only in societies in which C and W commute in the mathematical sense. The relevant calculation for this example shows

$$WC = \begin{pmatrix}1&2&3&4\\2&4&1&3\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix}$$

$$CW = \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\2&4&1&3\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\3&1&4&2\end{pmatrix}$$

and marriages of this type are permitted for this example. For other types of societies (defined by different rules W and C) such marriages may not be permitted.

5. SUMMARY

We have shown by discussing a hypothetical example that the kinds of calculations used in the mathematical theory of groups can be applied to certain transformations involving human groups; namely, the kinship relations generated by the rules of marriage and descent in primitive societies. Few societies have only four clans, although both the Kariera and Tarau groups of Australia can be presented by simple models of this kind. (See Appendix 2 of White's book listed in the Readings below). With more than four clans the calculations become more complicated but the principles remain the same. Here we have looked at one question that is of interest to anthropologists: Under what conditions may cross cousins marry? We saw how this question can be translated into a mathematical equation with the result that: if for some society $W^{-1}C = CW$, then patrilineal cross cousins are permitted to marry, if CW = WC then matrilineal cross cousins may marry. Similar questions -- involving for example the marriage of second cousins -- can be put into a similar mathematical form.

Exercise 17. The Kariera system is given by

$$W = \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} \qquad C = \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}.$$

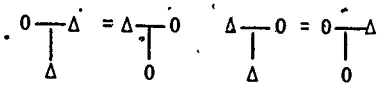
Calculate $W^{-1}C$, CW, and WC and see whether cross cousins can marry in this system.

Exercise 18. For the Tarau system W and C are given by

$$W = \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} \qquad C = \begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}.$$

Discuss cross cousin marriage for this example.

Exercise 19. Parallel cousins are defined by the diagram below. ("parallel" because the parental siblings are of the same sex.)



Write the equations for the condition that parallel cousins may marry and hence show, in fact, that parallel cousins may never marry (in societies of the type discussed here).

Exercise 20. In calculating with transformations there is a temptation to do this:

$$C^{-1}WC = W$$
$$C^{-1}CW = W$$
$$W = W.$$

As a general rule this is wrong! Why?

Exercise 21. Translate W and C of the text example into transformations of a labeled square. Use a labeled square to calculate the cross-cousin marriages.

Exercise 22. A member of a primitive tribe is reported to have said -- by way of explaining the marriage rules of his group -- If I marry my sister, I won't have anyone to hunt with. How were hunting parties made up in his group?

6. READINGS

Levi-Strauss, C. The Elementary Structures of Kinship, Beacon Press, Boston, 1969.

Stone, J. Social Dimensions of Law and Justice, Stanford Univ. Press, Palo Alto, 1966.

White, H.C. An Anatomy of Kinship, Prentice-Hall, Englewood Cliff, N.J., 1963.

7. ANSWERS TO EXERCISES

1: (i) (4 ways)(3 ways)(2 ways)(1 way) = 24 ways

(ii) (n ways)(n-1 ways)...(2 ways)(1 way) = n! ways
where n! = n(n-1)(n-2) ... 3.2.1

2: (i) K.360° K = 0, 1, 2, 3, ... i.e., any multiple of 360°

(ii) countably many

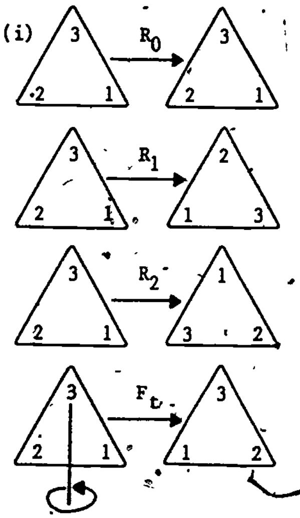
(iii) 240°

(iv) 120°

(v) same as (i)

(vi) yes, e.g.,
(e) from (d) by a clockwise rotation of 120°
(f) from (e) by a clockwise rotation of 120°,
(d) from (f) by a clockwise rotation of 240°, etc.

3: (i)

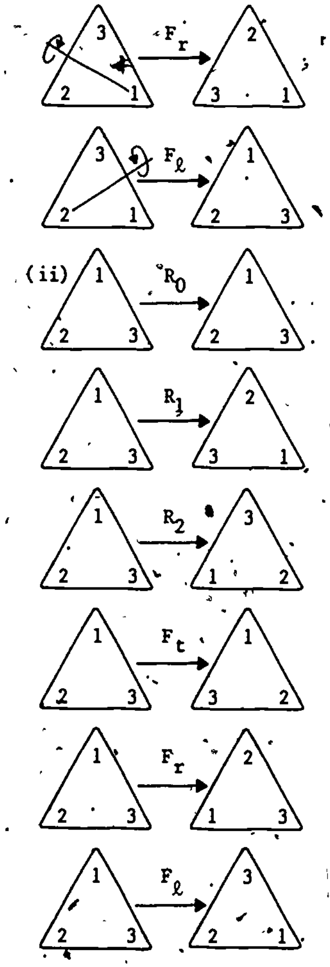


(b) from (b)

(c) from (b)

(a) from (b)

(e) from (b)



(f) from (b)

(d) from (b)

(ii) (d) from (d)

(f) from (d)

(e) from (d)

(a) from (d)

(c) from (d)

(b) from (d)

R_0 (d)
R_1 (f)
R_2 (e)
F_t (a)
F_r (c)
F_ℓ (b)

4: (i) $(a)\xrightarrow{R_1}(b)\xrightarrow{R_1}(c)$ $(a)\xrightarrow{R_2}(c)$

(ii) $(a)\xrightarrow{R_0}(a)\xrightarrow{F_t}(d)$ $(a)\xrightarrow{F_t}(d)$

(iii) $(a)\xrightarrow{F_t}(d)\xrightarrow{F_t}(a)$ $(a)\xrightarrow{R_0}(a)$

(iv) $(a) \xrightarrow{F_t} (d) \xrightarrow{R_2} (e)$ $(a) \xrightarrow{F_r} (e)$

(v) $(d) \xrightarrow{R_1} (f) \xrightarrow{R_1} (e)$ $(d) \xrightarrow{R_2} (e)$

$(d) \xrightarrow{R_0} (d) \xrightarrow{F_t} (a)$ $(d) \xrightarrow{F_t} (a)$

$(d) \xrightarrow{F_t} (a) \xrightarrow{F_t} (d)$ $(d) \xrightarrow{R_0} (d)$

$(d) \xrightarrow{F_t} (a) \xrightarrow{R_2} (c)$ $(d) \xrightarrow{F_r} (c)$

5: (i) $R_1 \times R_1 = R_2$

$R_1 \times F_r = F_\ell$

$F_r \times R_0 = F_r$

$F_\ell \times F_t = R_2$

(ii) $R_0 \times R_0 = R_0$

$R_1 \times R_2 = R_0$

$R_2 \times R_1 = R_0$

$F_t \times F_{[illegible]} = R_0$

$F_r \times F_r = R_0$

$F_\ell \times F_\ell = R_0$

(iii) yes

6: $(R_1 \times R_2) \times F_r = R_1 \times (R_2 \times F_r)$

$R_0 \times F_r = R_1 \times F_t$

$F_r = F_r$

7: $F_t \times F_{[illegible]} = R_0$

$R_1 \times R_1 = R_2$

$R_1 \times F_t = F_r$

$F_t \times R_1 = F_\ell$

R_1F_t R_1F_r R_1F_ℓ R_2F_t R_2F_r R_2F_ℓ F_tF_r F_tF_ℓ F_rF_ℓ

8: F_t followed by F_t leaves the triangle unchanged (R_0)

R_1 followed by R_1 is the same as a rotation of 240° (R_2)

R_1 followed by F_t is the same as F_r

F_t followed by R_1 is the same as F_ℓ

9:

	R_0	R_1	R_2	R_3
R_0	R_0	R_1	R_2	R_3
R_1	R_1	R_2	R_3	R_0
R_2	R_2	R_3	R_0	R_1
R_3	R_3	R_0	R_1	R_2

10:

	R_0	R_1	R_2	R_3	F_h	F_v	F_ℓ	F_r
R_0	R_0	R_1	R_2	R_3	F_h	F_v	F_ℓ	F_r
R_1	R_1	R_2	R_3	R_0	F_ℓ	F_r	F_v	F_h
R_2	R_2	R_3	R_0	R_1	F_v	F_h	F_r	F_ℓ
R_3	R_3	R_0	R_1	R_2	F_r	F_ℓ	F_h	F_v
F_h	F_h	F_r	F_v	F_ℓ	R_0	R_2	R_3	R_1
F_v	F_v	F_ℓ	F_h	F_r	R_2	R_0	R_1	R_3
F_ℓ	F_ℓ	F_h	F_r	F_v	R_1	R_3	R_0	R_2
F_r	F_r	F_v	F_ℓ	F_h	R_3	R_1	R_2	R_0

11:

	1	2	3	4	5	6
1	2	3	4	5	6	1
2	3	4	5	6	1	2
3	4	5	6	1	2	3
4	5	6	1	2	3	4
5	6	1	2	3	4	5
6	1	2	3	4	5	6

12: yes

13: yes $(R_0, R_1, R_2, F_t, F_r, F_\ell)$

(R_0, R_1, R_2)

(R_0, F_t)

(R_0, F_r)

(R_0, F_ℓ)

14: (ii) $A \times B = \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} = \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$

$B \times A = \begin{pmatrix}1&2&3&4\\2&1&4&3\end{pmatrix} \times \begin{pmatrix}1&2&3&4\\4&3&2&1\end{pmatrix} = \begin{pmatrix}1&2&3&4\\3&4&1&2\end{pmatrix}$

(iii) $R_2 \times F_n = F_v$

$F_h \times R_2 = F_v$

(iv) $\begin{pmatrix}1&2&3\\2&3&1\end{pmatrix} \times \begin{pmatrix}1&2&3\\3&1&2\end{pmatrix} = \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$; inverse of R_1 is R_2

$\begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} \times \begin{pmatrix}1&2&3\\1&3&2\end{pmatrix} = \begin{pmatrix}1&2&3\\1&2&3\end{pmatrix}$; inverse of F_t is F_t

15: (i) $W^{-1} \times W = \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix} \times \begin{pmatrix}1 & \ldots\\ 2 & \ldots\end{pmatrix}$ [illegible] $= \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix} = I$

(ii) wife's brother: $WB = WI = W$

sister's husband: $SW^{-1} = IW^{-1} = W^{-1}$

(iii) $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 1 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 1 & 3\end{pmatrix}$

$\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix}$

(iv) $\begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix}$

$T^{-1} = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} = T$

16: $(C \times W^{-1}) \times C = \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

$= \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 1 & 2\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix}$

$C \times (W^{-1} \times C) = \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

$= \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 4 & 1 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 1 & 4 & 2\end{pmatrix}$

17: $W^{-1} \times C = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

$C \times W = \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

$W \times C = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 3 & 4 & 1 & 2\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 4 & 3 & 2 & 1\end{pmatrix}$

Since $W^{-1}C = CW$ and $CW = WC$, cross cousins may marry in this system.

18: $W^{-1} \times C = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$

$C \times W = \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 4\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$

$W \times C = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix} \times \begin{pmatrix}1 & 2 & 3 & 4\\ 1 & 2 & 3 & 3\end{pmatrix} = \begin{pmatrix}1 & 2 & 3 & 4\\ 2 & 1 & 4 & 3\end{pmatrix}$

Since $W^{-1}C = CW$ and $CW = WC$, cross cousins may marry in this system.

19: $C^{-1}BC = W$

$BC = CW$ (multiplying on the left by C)

$C = CW$ (since $B = I$)

$I = W$ (multiplying on the left by C^{-1})

$W^{-1}W = W$ (since $I = W^{-1}W$)

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$W^{-1} = I$ (multiplying on the right by W^{-1})

$W^{-1} = I$ implies that a woman could marry her brother which is not allowed in this discussion.

$C^{-1}WSW^{-1}C = W$

$WSW^{-1}C = CW$ (multiplying on the left by C)

$WW^{-1}C = CW$ (since $S = I$)

$C = CW$ (since $WW^{-1} = I$)

$I = W$ (multiplying on the left by C^{-1})

$W^{-1}W = W$ (since $I = W^{-1}W$)

$W^{-1} = I$ (multiplying on the right by W^{-1})

Again, a woman could marry her brother, which is not allowed.

20: WC may not equal CW, i.e., the product may not be commutative.

21: $W = R_1$ $W^{-1} = R_3$ $C = R_2$

$W^{-1}C = R_3 R_2 = R_1$

$CW = R_2 R_1 = R_3$

Since $W^{-1}C \neq CW$ patrilineal cross cousins may not marry.

$CW = R_2 R_1 = R_3$

$WC = R_3 R_2 = R_1$

Since $CW \neq WC$, matrilineal cross cousins may not marry.

22: Hunting parties are composed of brothers-in-law.

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STUDENT FORM 1

Request for Help

Return to:
EDC/UMAP
55 Chapel St.
Newton, MA 02160

Student: If you have trouble with a specific part of this unit, please fill out this form and take it to your instructor for assistance. The information you give will help the author to revise the unit.

Your Name____________________ Unit No.______

Page______ ○ Upper ○ Middle ○ Lower	OR	Section______ Paragraph______	OR	Model Exam Problem No.______ Text Problem No.______

Description of Difficulty: (Please be specific)

Instructor: Please indicate your resolution of the difficulty in this box.

○ Corrected errors in materials. List corrections here:

○ Gave student better explanation, example, or procedure than in unit. Give brief outline of your addition here:

○ Assisted student in acquiring general learning and problem-solving skills (not using examples from this unit.)

Instructor's Signature____________________

Please use reverse if necessary.

STUDENT FORM 2

Unit Questionnaire

Return to:
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Name________________________________ Unit No.______________ Date______________

Institution______________________________ Course No.______________________

Check the choice for each question that comes closest to your personal opinion.

1. How useful was the amount of detail in the unit?

____Not enough detail to understand the unit
____Unit would have been clearer with more detail
____Appropriate amount of detail
____Unit was occasionally too detailed, but this was not distracting
____Too much detail; I was often distracted

2. How helpful were the problem answers?

____Sample solutions were too brief; I could not do the intermediate steps
____Sufficient information was given to solve the problems.
____Sample solutions were too detailed; I didn't need them

3. Except for fulfilling the prerequisites, how much did you use other sources (for example, instructor, friends, or other books) in order to understand the unit?

____A Lot ____Somewhat ____A Little ____Not at all

4. How long was this unit in comparison to the amount of time you generally spend on a lesson (lecture and homework assignment) in a typical math or science course?

Much	Somewhat	About	Somewhat	Much
____Longer	____Longer	____the Same	____Shorter	____Shorter

5. Were any of the following parts of the unit confusing or distracting? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Paragraph headings
____Examples
____Special Assistance Supplement (if present)
____Other, please explain______________________________________

6. Were any of the following parts of the unit particularly helpful? (Check as many as apply.)

____Prerequisites
____Statement of skills and concepts (objectives)
____Examples
____Problems
____Paragraph headings
____Table of Contents
____Special Assistance Supplement (if present)
____Other, please explain______________________________________

Please describe anything in the unit that you did not particularly like.

Please describe anything that you found particularly helpful. (Please use the back of this sheet if you need more space.)