DOCUMENT RESUME

ED 215 911 SE 037 755

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TITLE	The Evolution of Children's Mental Addition.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	82
GRANT	SED-80-21521
NOTE	12p.
AVAILABLE FROM	Paper presented at the Annual Meeting of the American Educational Research Association, New York, NY, March 18-23, 1982).
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Addition; *Cognitive Processes; Educational Research; Elementary Secondary Education; *Mathematics Education
IDENTIFIERS	*Mathematics Education Research; *Mental Computation

ABSTRACT

Students in grades 1, 4, 7, and 10 were tested in a two-part investigation of simple and complex mental addition (with college students as a reference point). One session involved a normal reaction time task in which children made true/false judgments about a series of addition examples. The other session involved a verbal protocol interview, the results of which are compared to the time-based results for an examination of children's mental addition performance. The hypothesized shift from counting to memory retrieval seemed to occur at about fourth grade, although older students continued to rely on immature processing for certain example types. A subgrouping of the fourth graders by mathematical ability was possible, yielding a strong effect on the reaction time measure. (Author/MNS)

The Evolution of Children's Mental Addition

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ABSTRACT

Students in grades 1, 4, 7, and 10 (college group added for comparison) were tested in a two-part investigation of simple and complex mental addition. One session involved a normal reaction time task, in which children made true/false judgments about a series of addition facts. The other session consisted of a verbal protocol interview, the results of which are compared to the time-based results for an examination of children's mental addition performance. The hypothesized shift from counting to memory retrieval seemed to occur at about fourth grade, although older subjects continued to rely on immature processing for certain problem types. A subgrouping of the fourth graders by math ability was possible, yielding a strong effect on the reaction time measure.

Paper presented at the meetings of the American Educational Research Association, New York, March 19-24, 1982. The research reported here was partially supported by a National Science Foundation Grant, SED-8021521, awarded to the first author, and formed the basis for an M.A. thesis submitted to the Cleveland State University by the second author. We are indebted to the Lakewood, Ohio school system for their generous assistance in the conduct of this research. Address correspondence to the first author, Department of Psychology, Cleveland State University, Cleveland, OH, 44115, or the second author, Department of Psychology, Carnegie-Mellon University, Pittsburgh, PA, 15213.

The major objective of the research being described is to investigate the cognitive processes and structures which children display as they perform mental arithmetic tasks. That is, we are concerned with the variety of strategies which children use and invent as they "do arithmetic in their heads", the role that memory retrieval plays in the increasing sophistication of children's arithmetic performance, and the underlying mental representations of the number facts. For the project under consideration here, we investigated simple and complex mental addition across elementary and secondary grade levels, in order to trace the evolution of these processes and structures (see also Ashcraft & Fierman, in press; Fierman, 1980).

Our perspective in this research is that of the information processing paradigm in psychology, a time-based examination of performance in which the relative contributions of strategies, such as counting or estimating, and memory retrievals can be determined. The theoretical framework within which we operate involves a continuum of arithmetic processing models, reliance on counting and other such strategic processes early in the grade school years (e.g., Groen and Parkman's "min" model, 1972), followed by a shift to memory retrievals and heuristic processing beginning at about the fourth grade level (e.g., Ashcraft and Stazyk's "network retrieval" model, 1981; see Ashcraft, in press, for a review).

In our studies of children's mental addition, we have elaborated on the usual information processing paradigm in order to capture the richness of children's cognitive processes. That is, we have not simply presented arithmetic problems to the subjects in a reaction time task. Instead, we have combined the reaction time measure with a separate interview session, presenting stimulus problems for tape-recorded data collection. These verbal protocols are then scored for frequency of strategies and methods, and are analyzed jointly with the time-based measure. The power of this technique is that the normal statistical examination of the children's performance can be supplemented with a measure of the child's own reliance on counting, memory retrieval, "principled solutions", and so forth.

Students in grades 1, 4, 7, and 10 (and college for a reference point) were tested in the combined reaction time / verbal protocol tasks. While there were some indications that the verbal task induced a somewhat different sort of processing than would normally be the case (particularly with 10th graders), for the most part the spoken solutions provided a useful window on the constellation of strategies used by these children.

The overall RT effects on the whole range of true problems are presented in the first graph.

FIGURE 1 HERE

Plotted across the x-axis is the size of the problem, whether it had a single digit sum (small), a double digit sum up through 18 (medium), or a sum larger than 18 (large). We have added the RT function for college students from an earlier study (Ashcraft & Stazyk, 1981) merely for comparison purposes. One striking feature of the graph is the absolute magnitude of the RTs for children in grade school. First graders require over 6 seconds for large problems, and slightly over 3 seconds for even the simple, one-digit sum problems drawn from the whole number facts. Bear in mind here that these problems were presented for true/false verification; in other words, the correct answer was shown along with these problems, it did not have to be produced in any fashion. Obscured by the collapsing of problems into the three size categories, but present in more detailed analyses, is a replication of several earlier developmental findings; in particular, that fourth graders yield RT functions which resemble older children's performance, and that later changes in performance tend to involve overall speeding of the times, rather than changes in pattern or slope.

We have conducted multiple regression analyses on the RT data, as is customary in this area of research, and have found a general pattern on the whole number facts which reconfirms the earlier statement; there appears to be heavy reliance on counting strategies early in grade school, but a shift in performance to memory retrieval around the 4th grade level, a shift which is then followed by overall speeding up of the latencies later in school.

By expanding the range of problems which were tested, and also by expanding the range of predictor variables which were assessed in regression, we seem to have uncovered a new and potentially important pattern of significance. Let me introduce this result with a short explanation. In their recent book, Resnick and Ford (1981) discussed several studies of children's difficulties in arithmetic which were conducted in the early part of this century. In particular, a study by Wheeler in 1939 ranked the whole number facts of addition in terms of difficulty; Wheeler assessed difficulty by means of a classroom-based arithmetic game, followed by tests of the facts to see which ones had been mastered at which points during practice. We coded Wheeler's difficulty ranks as a percentages, and included the variable in our regression analyses.

In our grade by grade regression analyses, we found almost exactly the same three predictor variables being selected, with age differences reflected in the size of the slope and intercept values. In particular, RTs in all grades are well predicted (over 60% of the variance accounted for) by three variables: the size of the problem, whether it's small, medium, or large; the Wheeler difficulty variable, applied to the addition of the two numbers in the one's or unit's column; and thirdly, a variable indicating that a carry operation was or was not required. We will return to the significance of this Wheeler difficulty measure in a moment; for now, notice that most models of arithmetic performance claim that one or another structural variable, such as the sum or the smaller addend, is theoretically meaningful, whereas the Wheeler difficulty variable is not a structural variable, but instead is a measure of subjective difficulty.

We turn now to the results of the interviews, tape recorded protocols of the children as they solved a subset of the problems out loud. In this phase, we tested both production problems, where the child had to supply the answer to the problems, and verification problems, in which an answer is presented. After trying several different coding and categorizing procedures, we settled on the following scheme (at least for today's presentation). A child's production protocol was scanned to try to determine whether a problem was solved by counting or by fact retrieval process. A child was scored as having used a certain procedure if we could identify at least two trials on which the strategy appeared in the spoken protocol. We used a rather rigorous criterion for this scoring; for a counting solution, the child had to indicate something like the following in his protocol: "4 plus 3, that's 4..5.6.7." For a fact retrieval score, the child would not only say something like "4 plus 3 is 7", but also had to say something to the effect that "I know that from memory", or "I know that in my head." As it turns out, the data are essentially unchanged if the scoring criterion is one rather than two occurrences of the statement.

FIGURE 2 HERE

The second graph presents the frequency with which children across grades used counting or fact solutions; the left half shows the frequencies for the simple whole number facts with sums less than 10. Not surprisingly, 80% of the first grade sample showed evidence of using a counting strategy; this percentage of course drops dramatically across the later grades. What is peculiar here is the frequency information on the right half of the graph, showing performance to more complex problems like 13 + 5. For these problems, not even first graders use any obvious counting strategy — in direct disagreement with their verbalized performance on simple problems, first graders here seemed to use fact retrieval for the one's column part of the problem.

Our second scoring method for protocols involved solutions to the verification problems in which an answer is presented and the subject was asked "Tell me how you know if it's right or wrong." Since the children's responses to the correct problems were largely redundant with what we've just presented on the production task, we will focus only on the incorrect problems in this task.

FIGURE 3 HERE

The third figure summarizes the protocols in terms of the three common strategies used on the verification problems. A Sum strategy indicates that a child considered the entire correct sum of a problem in his decision; for example, one fourth grader was given the problem 15 + 13 = 16, and responded "Add 5 plus 3 is 8, and you add 1 plus 1 is 2, and that's wrong." In other words, the child generated the entire sum, then compared it to the sum presented in the problem. A Component Sum strategy indicates that the child generated the sum to the one's column addition, and used this as the evidence; for the problem 19 + 11 = 29, one child merely said "9 plus 1 is wrong", meaning that 9 and 1 don't sum to 9. Finally, a global strategy indicated a judgment that the problem was wrong without any particular addition fact having been mentioned; for example, for the problem 16 + 11 = 7, one first grader said "It's false because 16 is higher than 7 and 11 is higher than 7, so how can it be true?"

We were surprised by two aspects of the data here. First, the very young children showed a remarkable flexibility in their solution methods

— first graders, for example, used the global strategies quite frequently, suggesting a much richer knowledge about arithmetic than they are normally given credit for. We say even greater flexibility among 4th graders; they selected freely from a real smorgasboard of strategies — beginning with the one's column then shifting to the 10's, beginning with 10's then 1's, using a doubling strategy for ties like 14 + 14, converting problems like 9 + 4 to 10 + 3 to capitalize on the 10 as a referent, and so on. Secondly, we were surprised at how rigid and inflexible the older subjects' protocols were, especially the tenth graders' solutions. As an example, one tenth grader required only 1165 msec to the problem 14 + 14 = 28 during RT verification, but in the protocol session indicated the following as his solution strategy: "Take 4 and 4, that's 8, and 1 and 1 is 2, so that's 28". We have seen so many of these lengthy protocols for 10th graders, even for the simple whole number facts, that we're creating a new category for them; these are "blackboard solutions", that is, solutions a child might verbalize at the blackboard while working the problem for a math teacher. We think they bear little if any relationship to the older child's actual processing of the arithmetic problem under more normal conditions.

Finally, the RT and protocol results we are presenting appear to have some useful educational implications. The fourth grade teacher involved in this project made sure, without our knowledge, that we tested children from all three of the math ability groups in her classroom. When we discovered this, we decided to capitalize on the subgroupings we had been provided. Consequently, we have broken down our RT results in terms of the three math abilities levels found in that classroom; Figure 4 presents these interesting effects.

FIGURE 4 HERE.

The math ability factor was significant in this analysis, but did not interact with any other variables, as the virtually parallel functions suggest. If these curves were added to figure 1, then the two lower ability groups would fall above that 4th grade function, with the advanced ability group falling below — in other words, the advanced group seems significantly faster in performance than the less accelerated students, but not as fast as the average 7th grader. Finding these ability effects on RT was unusual, we felt, and of enough importance that we decided to report them, their post-hoc status notwithstanding.

These results suggest four major conclusions to us. First, it does indeed seem that children's mental addition shifts from rudimentary counting-based performance to performance based on fact retrieval. While this is not a surprising conclusion in one sense (after all, "memorized" facts for the basic operations characterize the emphasis of most basic arithmetic curricula), it is somewhat surprising in the empirical sense. No other research we are aware of has demonstrated older children's reliance on fact retrieval, although this conclusion has been demonstrated with adults (e.g., Ashcraft & Stazyk, 1981; Stazyk, Ashcraft, & Hamann, in press).

Secondly, we consider the protocol results, and especially the "blackboard solutions", that were presented. We suggest that certainly by the 10th grade level, and probably much earlier, mental addition has not only shifted to a memory-based retrieval process, it has also become a largely automatic, as opposed to conscious, process. As such, the requirement to verbalize leads to lengthy rationalization which does not reflect underlying mental operations (Ericcson & Simon, 1981). A related conclusion is based on the pattern of significance found in the regression analyses. These patterns suggest that the shift from counting to memory retrieval is well underway, if not virtually completed, by the fourth grade level.

A third conclusion, perhaps less obvious than those above, is that even older children continue to rely on less-than-mature processes for certain types of problems, those involving 9's, carrying, 0's, and so forth. Despite what must surely be great familiarity, a typical seventh grader still will report solving a problem like 9 + 6 in the following fashion: "Well, that's like 10 + 5".

A final conclusion speaks to the scientific and educational significance of this research. By examining children's performance in an "on-line" timed fashion, and by comparing this performance to that in the verbal interview session, we find noteworthy discrepancies between what the child was taught to do and what the child actually does. The impact of teaching certain facts by rule alone (for example, "anything times zero equals zero") is revealed in the time-based measure (Stazyk et al., in press), and has surprising consequences for other mathematical processing. We advocate the use of joint interview and timed measures for the mutual explication of each measure, for further study of the possible math ability-to-RT relationship, and for investigations of the evolving richness of children's arithmetic performance.

References

Ashcraft, M.H. The development of mental arithmetic: A chronometric approach. Developmental Review, in press.

Ashcraft, M.H. & Fierman, B.A. Mental addition in third, fourth, and sixth graders. Journal of Experimental Child Psychology, in press.

Ashcraft, M.H. & Stazyk, E.H. Mental addition: A test of three verification models. Memory & Cognition, 1981, 9, 185-196.

Ericsson, K.A. & Simon, H.A. Verbal reports as data. Psychological Review, 1980, 87, 215-251.

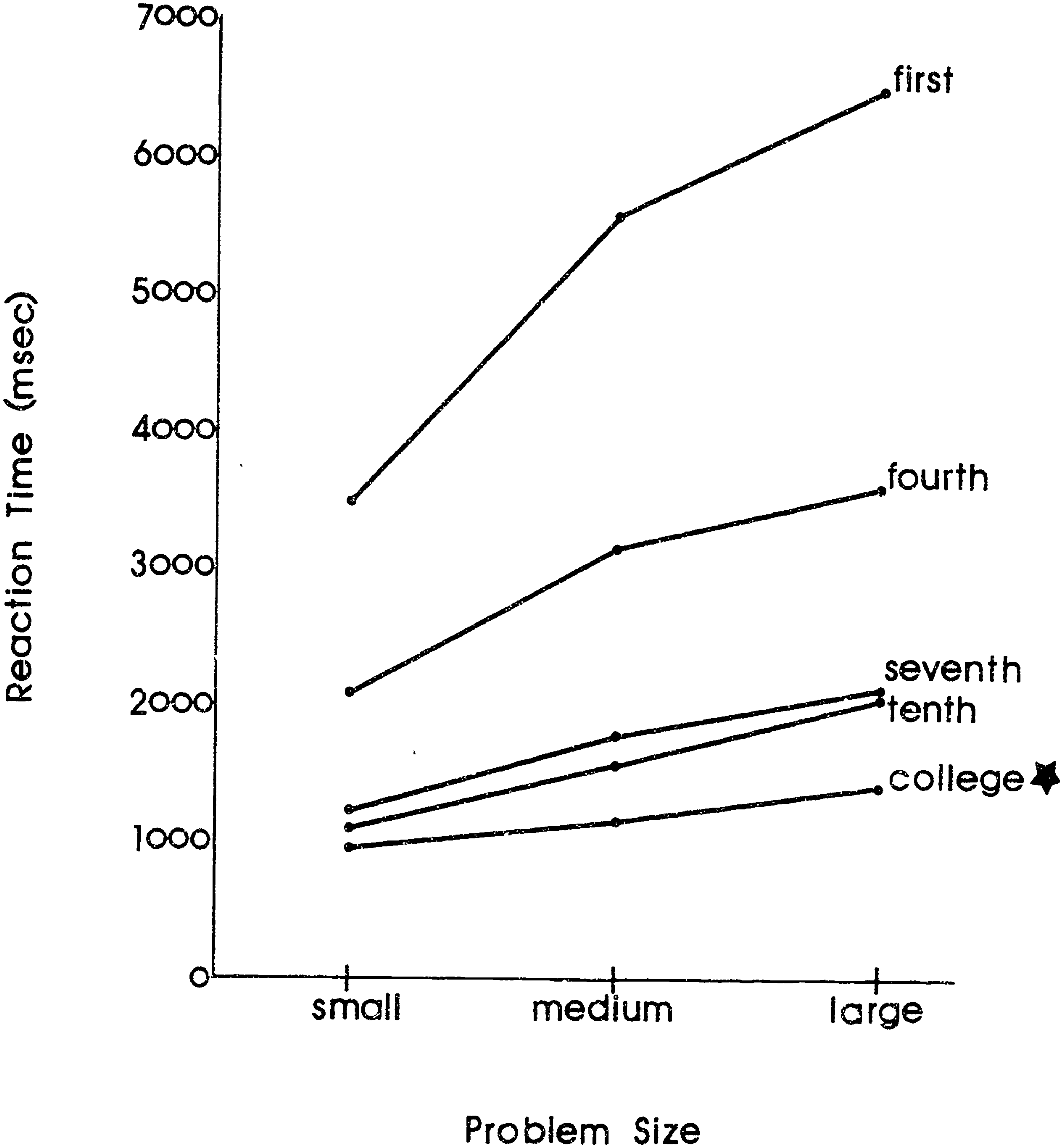
Fierman, B.A. Developmental Mental Addition: A Test of Two Models and Two Methods. Unpublished M.A. Thesis, Cleveland State University, 1980.

Groen, G.J. & Parkman, J.M. A chronometric analysis of simple addition. Psychological Review, 1972, 79, 329-343.

Resnick, L.B. & Ford, W.W. The Psychology of Mathematics for Instruction. Hillsdale, N.J.: Erlbaum Associates, 1981.

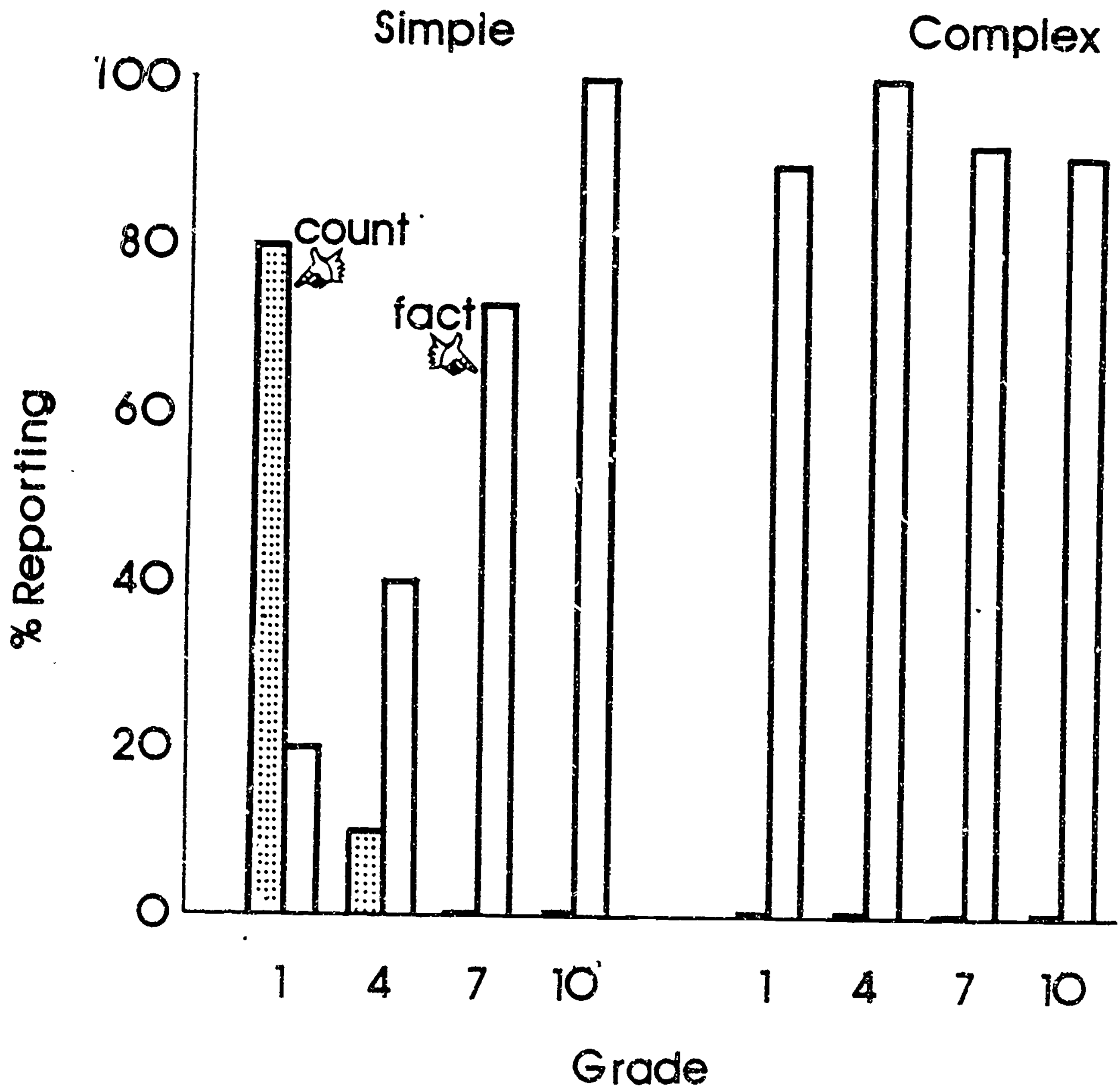
Stazyk, E.H., Ashcraft, M.H., & Hamann, M.S. A network approach to mental multiplication. Journal of Experimental Psychology: Learning, Memory, and Cognition, in press.

Wheeler, L.R. A comparative study of the difficulty of the 100 addition combinations. The Journal of Genetic Psychology, 1939, 54, 295-312.



Reaction Time (msec)
7000
6000
5000
4000
3000
2000
1000
0
first
fourth
seventh
tenth
college
small
medium
large
Problem Size

FIGURE 1



Simple
Complex
100
80
60
40
20
0
count
fact
% Reporting
1
4
7
10
1
4
7
10
Grade

FIGURE 2

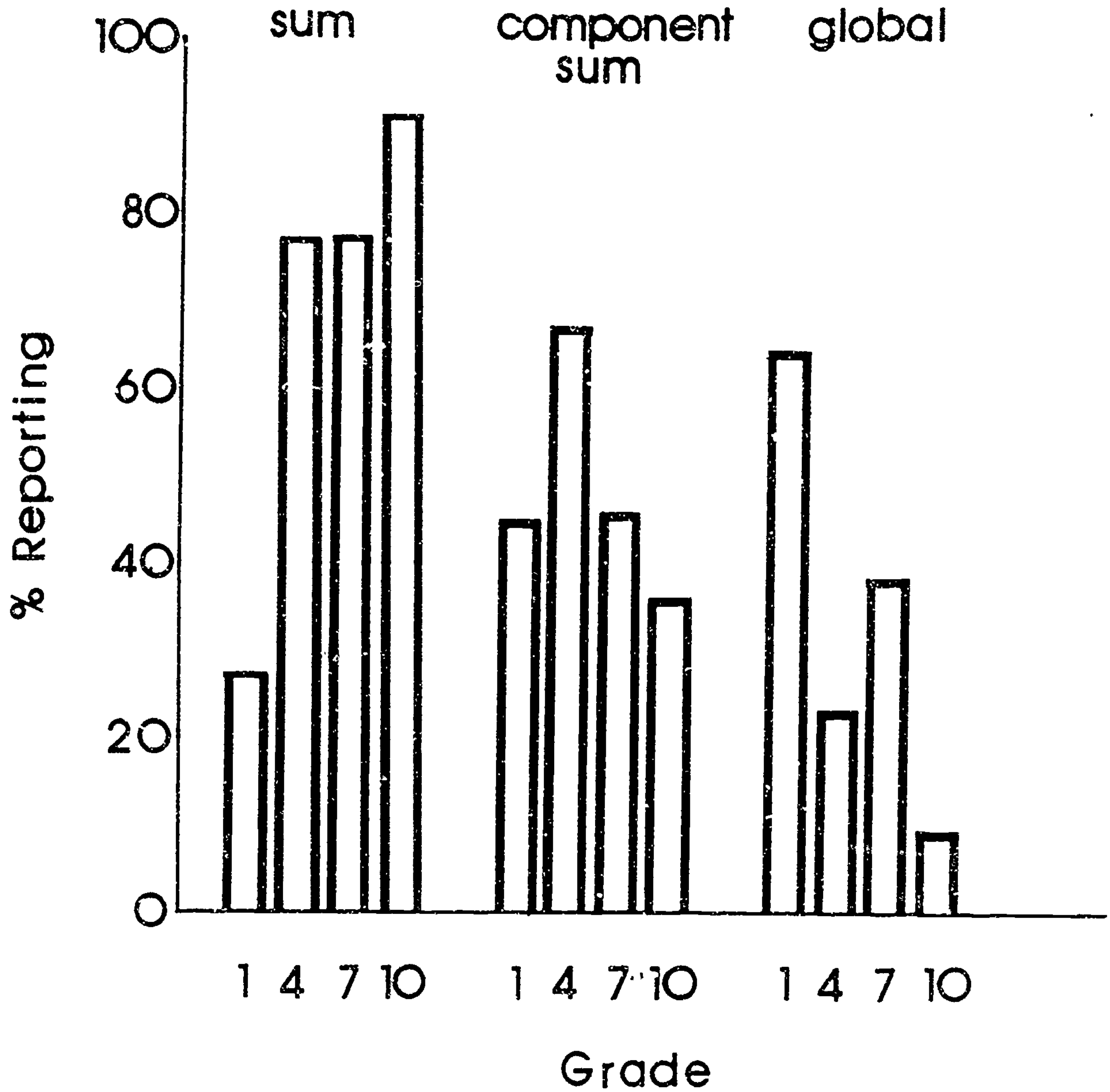

sum
component sum
global
100
80
60
40
20
0
% Reporting
1 4 7 10
1 4 7 10
1 4 7 10
Grade

FIGURE 3

4th Grade

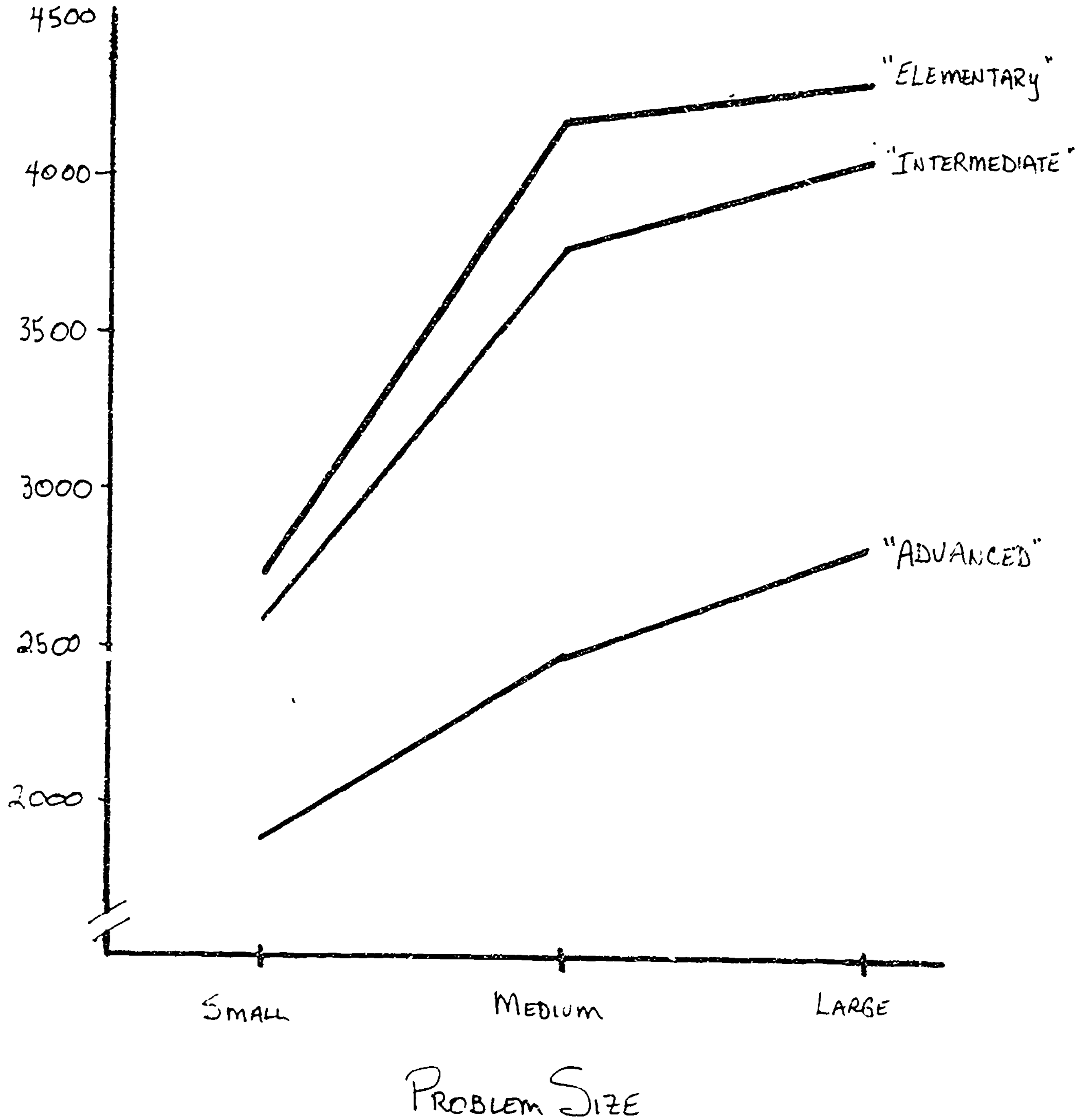


FIGURE 4