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ABSTRACT

Mathematics education research from fourteen studies is abstracted and critiqued in this publication. Six cover aspects of instructional practice, four deal with points in learning theory, two with problem solving, and one each with calculators and sex differences. Research in mathematics education as reported in CIJE and RIE from October to December 1981 is also noted. (MP)

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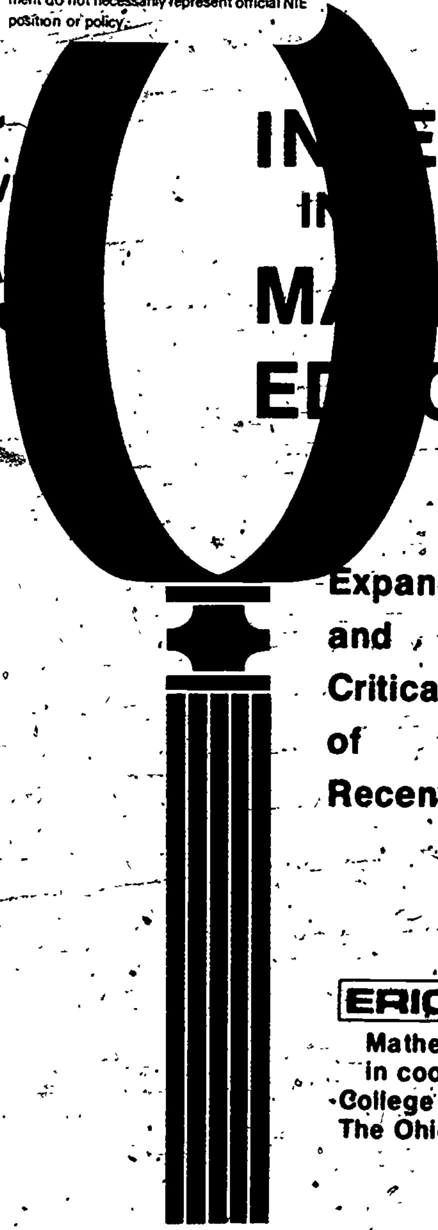

INVESTIGATIONS IN MATHEMATICS EDUCATION

Expanded Abstracts and Critical Analyses of Recent Research

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Spring 1982

Armstrong, Jane M. ACHIEVEMENT AND PARTICIPATION OF WOMEN IN MATHEMATICS: RESULTS OF TWO NATIONAL SURVEYS. Journal for Research in Mathematics Education 12: 356-372; November 1981.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

Few previous studies have controlled for students' mathematics course-taking histories. Therefore, the current study was undertaken (using the results of two national surveys) to determine sex differences in mathematics achievement when course-taking was controlled. Sex differences in spatial ability were also considered.

2. Rationale

The two surveys used in the study were: (1) Women in Mathematics Project and (2) National Assessment of Educational Progress. Previous research is cited which tends to show slightly better achievement for males in mathematics. Work by Fennema and others has suggested that the achievement differences might be explained by differential participation. Evidence seems to show significantly higher participation in mathematics for males over females.

Previous studies which have attempted to determine the affect of differential course-taking on achievement have produced inconsistent results. An alternative cause for achievement differences might be spatial ability. This is taken into consideration in the study.

3. Research Design and Procedures

The Women in Mathematics Survey: A 90-minute questionnaire was developed to assess students' mathematics achievement and attitude as well as to collect background information on matters such as course-taking history. The results here focus only on achievement and participation. Achievement was assessed via items from standardized tests. Participation was determined by asking students which courses they had taken or expected to take at the high school level. Samples were drawn from 13-year-olds and high school seniors. The 13-year-old sample included 82 schools for a total of 1 352 students. The senior-year sample included 71 schools and a total of 1 788 students.

The NAEP Mathematics Assessment: This study was conducted during the 1977-78 school year using a large national probability sample of approximately 75 000 students. Students were tested at ages 9, 13, and 17. A more detailed description of NAEP can be found in Procedural Handbook: 1977-78 Mathematics Assessment (NAEP, 1980).

4. Findings

Sex Differences in Achievement: In general, males at the senior-year level outperformed females on the various aspects of mathematical achievement. At the 13-year-old level, the females tended to do better than males. This same general pattern existed for both the Women in Mathematics Survey and the NAEP data.

Sex Differences in Participation: The pattern for both the NAEP assessment and the WIM Survey suggests that males and females participate in basic mathematics courses at roughly the same rate. At higher levels, enrollment seems to favor males.

Sex Differences in Achievement Within Participation Levels: At nearly every level of participation, men have some sort of achievement advantage over women. Apparently, achievement differences are not solely a function of differences in participation.

5. Interpretations

Achievement: Thirteen-year-old females start high school mathematics programs with at least the same mathematical abilities as their male counterparts. They usually have an edge in computation and spatial ability. By the end of high school, males have passed females in most scales and have erased the previous female edge in computation and spatial ability.

Participation: There appears to be somewhat higher participation by males as the level of course increases.

Achievement Within Levels of Participation: It seems that achievement differences are not solely a function of differences in participation. Males seem to have an advantage at nearly every level of participation. Also, the differences do not appear to be related to spatial visualization.

Possible explanations of sex differences may be due to out-of-school experiences and/or personality differences not previously considered. This warrants further investigation.

Abstractor's Comments

(1) The topic is an important one that has received much interest in the recent professional literature in mathematics education.

(2) In terms of format, the consideration of spatial ability seemed like something of an afterthought.

(3) Except for the comment in (2), the report is well-written.

(4) The suggestion that future research might look at (1) out-of-school experience and (2) personality variables seems feasible.

(5) I appreciated the author's willingness to make definite statements in the conclusions section. For example,

> It is clear that achievement differences are not solely a function of differences in participation. (p. 371)

Behr, Merlyn J. and Wheeler, Margariete Montague. THE CALCULATOR FOR CONCEPT FORMATION: A CLINICAL STATUS STUDY. Journal for Research in Mathematics Education 12: 323-338; November 1981.

Abstract and comments prepared for I.M.E. by J. D. GAWRONSKI, San Diego County Department of Education.

1. Purpose

This study was designed to determine if young, school-age children could use the "counter button" (equals key) of a calculator for performing counting activities associated with basic facts of addition and subtraction.

2. Rationale

Counting strategies such as "count all", "count-on-with tally", and "count-on-without-tally" to perform additions have been identified by other researchers. Strategies such as "count-back-with-tally" to perform subtractions have also been observed. This study attempts to extend that research by studying whether children can use a calculator to perform counting tasks. If calculator use does augment "count-with-tally" behaviors, it may be possible to facilitate a child's acquisition of addition and subtraction concepts.

3. Research Design and Procedures

Kindergarten and first-grade children who were teacher-identified as high, middle, or low ability in mathematics were the subjects of this study. At each grade level, five children were randomly selected from each classification. The results of the Purdue Test of Conservation were used to classify 3, 4, and 8 kindergarten children and 11, 1, and 3 first-grade children as conservers, transitionals, and non-conservers, respectively.

Calculator familiarization and 16 assessment tasks in correspondence, number permanence, correspondence with discrepancy, and transformation were individually presented to each child in two 25- to 35-minute videotaped interviews. Researchers alternated responsibilities of interviewer and recorder between sessions.

4. Findings

The children had no difficulty reading numerals on the calculator display and learned "with reasonable ease" to "program" the calculator to be a "counter". More than 85 percent of the children at each grade level were able to use the calculator to maintain a one-to-one correspondence between an oral count and a calculator count. On number permanence tasks, the oral count of chips remained unchanged for 14 kindergarteners and all 15 first graders, and the calculator count of chips remained unchanged for 10 kindergarteners and all 15 first graders. The results of the correspondence with discrepancy tasks indicated that approximately two-thirds of the 120 responses gave evidence that the calculator was perceived as a counter. In the simple addition and subtraction tasks "most children at each grade level recognized that the interviewer-introduced transformation produced an inequality and transformed the calculator count so that an equivalence was re-established."

On the simple inverse transformation tasks, at least 40 percent of the children at each grade level gave satisfactory explanations. The last two tasks were the complex addition/subtraction transformation tasks. The results indicate that complex addition is easier than complex subtraction; the comparative difficulty of the two tasks seems to disappear with the older, first-grade children.

5. <u>Interpretations</u>

The researchers indicate that the data suggest that most kindergarten and first-grade children appear to perceive a calculator count as a permanent indicator of the cardinality of a set and are apt to use a calculator as a facilitator of oral counting. By inference, the children might use the calculator as a counter to facilitate internalization of basic fact addition and subtraction. The researchers are "cautiously optimistic about encouraging curricular research that would employ the calculator as a counter to facilitate a child's acquisition and internalization of counting strategies appropriate for addition and subtraction."

Abstractor's Comments

The sequence of tasks used in this study is carefully described and presented. The experimental activities of pretesting, videotaped interviews, and interviewing/recording seem reasonable and well-suited to the purpose of the study. It is important to emphasize here that this was not a comparative study to determine if one method of instruction was "better than" another. This study was "to examine whether young, school-age children perceive successive punches of the counter button of a handheld calculator as a means for counting with tally." But I wonder why? And where does this research lead? Is it motivated by a need to be prepared for the pencil-less, paper-less environment of the future when we will rely on electronic devices? Or are we looking forward to the day when the plot of Isaac Asimov's Feeling of Power comes true and we need to deal with the technical [student] who discovers calculator displays can be replicated on paper with pen or pencil? It doesn't appear to me that the research sheds any light or adds any new insights to what we know about initial learning of addition and subtraction skills. The research was certainly well conducted and generally well described. This report, however, would have been improved with a more substantive rationale.

Bell, Alan; Swan, Malcolm; and Taylor, Glenda. CHOICE OF OPERATION IN VERBAL PROBLEMS WITH DECIMAL NUMBERS. Educational Studies in Mathematics 12: 399-420; November 1981.

Abstract and comments prepared for I.M.E. by KAREN A. SCHULTZ, Georgia State University.

1. Purpose

The purpose of the study was to address four questions: (1) the suitability of the general diagnostic teaching method, (2) the adequacy of the use of the conceptual map to find misconceptions, (3) the effectiveness of the use of cognitive conflict and general teaching strategies, and (4) the effectiveness of calculator-enriched teaching materials to improve understanding. Learning choice of operation in verbal problems with decimal numbers was the medium through which the four questions were considered.

2. Rationale

a. Though substantial work has been reported on the use of conceptual maps to identify level of difficulty and describe typical misconceptions and strategies, less work exists on teaching based on this information.

b. There is little or no guidance in elementary or secondary school textbooks in learning how to choose the operation in problem solving. Related research has focused on the effect of the difficulty of certain features of a problem, but not on the conceptual schemes children have. The presence of decimal numerals in problems has been shown to be particularly troublesome.

3. Research Design and Procedures

Exploratory interviews. About 20 pairs of students, aged 12 to 16, within the normal range of ability were given problems presented in three-page booklets, with pages 2 and 3 displaying relevant hints, such as using easier numbers or diagrams. Discussion between students was encouraged. When students could not solve a problem given the booklet hints, other hints were offered. The uses of easier numbers, estimation, and drawing diagrams were of particular interest to the investigators. About half of the interviews were observed by a second person and some were videotaped. Audio recordings were made of all the interviews.

Diagnostic testing. A diagnostic test based on the interview results was designed. It contained the following nine areas:

Area I	Choice of operation
Area II	Understanding of place value and ordering for decimals
Area III	The assumption that 'multiplication makes bigger' must be resisted
Area IV	Division must be recognized as non-commutative
Area V	Division symbols such as $a\overline{)b}$, $a \div b$ must be read in the correct direction
Areas VI, VII.	Misleading words 'more' and 'times' must be resisted
Areas VIII	Awareness of size of units
Area IX	Explanation of a check of correctness. (p. 409)

There were 38 items in the test, 15 of which were taken from the test "Place Value and Decimals" developed by the Chelsea project, Concepts in Secondary Mathematics and Science (CSMS) (Hart, 1980). The test was administered to 27 fifteen-year-olds of below-average ability in a middle-class secondary school. Data from 18 students are reported.

Teaching experiment. The same 18 students were given eleven 70-minute lessons after the diagnostic test was administered as a pretest. The lessons and activities addressed the concepts in the nine Areas and

> ... concentrated on one-step multiplication and division problems, involving decimal numbers, and in supposedly very familiar contexts. ... [The] teaching material used rich situations, and also games, a mixture of teacher-posed and pupil-generation questions. Feedback was sometimes through the game situation (the opponent wins or challenges), sometimes from consistency, sometimes by trial and improvements. (p. 402)

Immediately following the teaching intervention, the diagnostic test was given again as a posttest and repeated three weeks later as a delayed posttest.

4. Findings

The percentage of students reaching criterion (about 3/4 of the items) in each Area for pre-, post-, and delayed posttests was reported. The most improvement occurred in Area II, both in immediate and delayed posttesting. Areas III and VII had immediate and delayed posttesting results that remained the same. Moderate to dramatic decreases in understanding from immediate to delayed posttesting were indicated in Areas IV, V, VI, and VIII. Areas I and IX showed moderate increases from immediate to delayed posttesting.

5. Interpretations

Though the authors felt that the effectiveness of their diagnostic procedure was difficult to assess, they did conclude that the teaching based on it was "notably successful" (p. 419). They expressed a need for more research on (a) choice of operation, (b) teaching strategies especially using easy numbers and diagrams, and (c) the separate effects of certain strategies.

Abstractor's Comments

This work is interesting and important from several perspectives. The focus on the diagnostic procedure in the context of a teaching experiment has the potential of offering qualitative information about the sensitive process of diagnosis and prescription and subsequent learning. Developing the diagnostic test on the basis of the data collected through interviews increased the likelihood of producing an instrument in tune with areas of confusion. By the time the teaching materials and plans were developed on the basis of the diagnostic data, the teachers (experimenters) were very sensitized to the needs of the learner.

The experimenters are to be commended for engaging in research that makes it possible to get in touch with some of the qualitative aspects of planning for instruction based on diagnosis. Valuable information was eked out about misconceptions arising from decimal experiences. However, following are some concerns about clarification and completion of reporting that would have given the reader more needed information about both decimals and the diagnostic procedure.

1. The terms cognitive map, cognitive conflict, and general diagnostic teaching method were used without definition. The first two terms were not further explained in the report after being used in posing the questions addressed by the study. Therefore, the results of the study in the context of using cognitive maps and cognitive conflict are vague.

2. It is not clear why pairs of subjects were interviewed. Also, it is not clear what is meant by saying the subjects were in the normal range

of ability. The reader is left wondering on what basis this was determined. Finally, regarding subjects, it is not known if the students interviewed were from the same school and class as those who were diagnosed and taught. Since the interviewing phase was in preparation for the diagnostic phase, this information could influence the reader's impression of the results and conclusions of the teaching experiment.

3. It was not explained why some of the interviews were conducted with a second observer and some were not. Also, why were some interviews videotaped and some not? It might be of interest to know if those interviews which did not have a second observer did not also get videotaped.

4. How many problems were given during each interview? Why were they multiple choice in format? Did each student or each pair of students do each problem? Examples given did not specify this information.

5. How long was each interview? How many sessions was each pair of students interviewed? How long did it take to conduct all the interviews? These questions and those posed in (4) above are important for purposes of replicating this study as well as interpreting the results.

6. Finally, the title suggests that the focus of the study is choice of operation. This, in light of the four questions delineated in the introduction of the report, is misleading.

Rererences

Hart, K. Secondary School Children's Understanding of Mathematics. Centre for Science Education, Chelsea College, University of London, 1980.

Bloom, Robert B. and Bourdon, Linda. TYPES AND FREQUENCIES OF TEACHERS' WRITTEN INSTRUCTIONAL FEEDBACK. Journal of Educational Research 74: 13-15; September/October 1980.

Abstract and comments prepared for I.M.E. by ARTHUR F. COXFORD, The University of Michigan.

1. Purpose and Rationale

The empirical psychological evidence indicates that feedback correcting errors is more effective than feedback that identifies incorrect responses and that both feedback techniques are more effective in increasing academic achievement than feedback which identifies correct responses. In light of this information, the authors sought to discover the feedback techniques commonly used by experienced elementary teachers.

2. Research Design and Procedures

One hundred eighty-three elementary teachers who were enrolled in a one-week graduate course dealing with classroom and behavioral management were used as the sample. The 183 experienced teachers (average of 8.3 years) were considered to be "representative" of the experienced teachers within a 60-mile radius of the course-offering institution. Each teacher was given a sheet of paper containing "Doug's" responses to 16 addition facts and instructed to "grade Doug's paper as you usually grade your students' mathematics papers." The papers were done anonymously and were not graded, but they were discussed in a total class setting.

3. Findings

The authors each independently evaluated the papers and identified seven categories of feedback used. The interrater coding reliability was 100 percent. The categories, along with the relative frequencies found, were:

Right only	19.7%
Wrong only	20.8%
Right/Wrong	16.9%
Redo Wrongs	16.4%
Corrective	8.7%
Teacher Assist	9.3%
Diagnose/Prescribe	8.2%

4. Interpretations

The authors noted the following: (1) Noncorrective feedback was identified nearly three times as often as corrective feedback; (2) Only one third of the users of noncorrective feedback used the most effective 'wrong only' techniques; (3) Only 8.2% diagnosed many of Doug's errors as similarly based; (4) No general feedback technique had wide use; (5) Teacher feedback techniques did not mirror the techniques found to be effective in empirical work.

Abstractor's Comments

The study is obviously not a tight experimental one because the sample was not random, the participation was voluntary, and no recognizable experimental design was used. Rather, it was a status study with a group of subjects who happened to be available and whom the authors considered to be typical, if not above average. As a status study, the work has importance for teachers and teacher educators. Its findings illustrate that the majority of the teachers participating used less than the most effective feedback techniques available. It seems reasonable to conjecture that they did as they did because of lack of knowledge. If that is the case, then the teacher educators who were their instructors may not have been aware of the empirical bases of their scholarly field--or else they would have taught the importance of effective feedback technique.

To be sure to have great confidence in the findings, one would have to replicate the study with better control procedures. However, we should not hide in academics when we learn that teachers and teacher-trainers are not performing well. Rather we should take it upon ourselves to teach what is clear from our scholarly base.

Brassell, Anne; Petry, Susan; and Brooks, Douglas M. ABILITY GROUPING, MATHEMATICS ACHIEVEMENT, AND PUPIL ATTITUDES TOWARD MATHEMATICS. Journal for Research in Mathematics Education 11: 22-28; January 1980.

Abstract and comments prepared for I.M.E. by GEORGE M. STANIC, University of Wisconsin at Madison.

1. Purpose

According to the authors, the primary purpose of the study was to investigate the relationships between pupil attitudes toward mathematics and mathematics achievement among assigned ability groups. The four hypotheses were:

1. There will be significant differences in pupils' attitudes toward mathematics among district-determined ability groups.
2. There will be significant differences in mathematics ability among district-determined ability groups.
3. There will be significant differences in pupils' attitudes toward mathematics among teacher-determined ability levels within district-determined ability groups.
4. Selected attitudes scales will correlate with measures of mathematics ability.

2. Rationale

After citing a number of studies which have dealt with pupil attitudes toward mathematics, the authors state that varied results have been produced in investigations which have attempted to relate pupil attitudes to mathematics achievement. The authors assume that their use of the Mathematics Attitude Inventory is sufficient to overcome what they see as a major problem with recent research on attitudes toward mathematics -- an overreliance on "home-grown" attitude instruments.

3. Research Design and Procedures

The sample selected for the study included 714 seventh-grade mathematics students from five junior high schools "representing a mixture of socioeconomic backgrounds in a suburban community." The authors state that this age group was chosen because grades six and seven "have been shown to be critical grades in the development of mathematical attitudes."

The attitude variables were measured using the Mathematics Attitude Inventory (MAI), which includes the following scales: attitude toward teacher, value of mathematics in society, anxiety toward mathematics, self-concept, enjoyment of mathematics, and motivation. Values for the achievement variables were determined using scores on the applications, concepts, and computations scales of the California Test of Basic Skills (CTBS); the total score was also used.

The MAI was administered in mid-winter; the CTBS scores were from administrations of the test during the previous school year. The authors state that the CTBS scores and the teacher recommendations were the criteria for the ability grouping. Questionnaires given to the mathematics teachers for each class were used to determine the trimester of mathematics in which the students were enrolled; the mathematics ability level of the class (high, medium, low); and the three highest and three lowest achieving students in each class.

The data used to determine between-group differences were summarized using mean scores on the six attitude variables and four achievement variables for each district-determined ability group; the mean scores across all three ability groups were also given for each variable. The data used to determine within-group differences were summarized using mean scores on the six attitude variables for each teacher-determined ability level within each district-determined ability group. Data were then analyzed to determine whether significant differences in scores existed between district-determined ability groups (in terms of attitude and achievement) and within district-determined ability groups (in terms of attitude). The specific tests and test statistics were not given for the analyses mentioned above. Other analyses were also undertaken. A discrimination analysis was done to determine attitudinal variables that relate to ability grouping; a correlation matrix of the six attitude variables and the four CTBS measures of mathematical ability was constructed; and a canonical correlation analysis was used to compare attitude and achievement scores.

4. Findings

Significant differences ($p < .01$) were found between district-determined groups on all attitude variables except motivation (with the largest differences found on self-concept) and on all four achievement variables. Within-group attitude differences (based on teacher-determined ability levels) were also significant on all attitude variables except motivation. The results of the discriminant analysis to determine the relationship between attitude and ability grouping "suggest that a combination of pupils' attitudes toward the teacher, mathematics self-concept, and enjoyment of mathematics are correlates of pupils' assigned ability level in mathematics." The authors state that the correlation matrix "highlights the relationship between mathematics self-concept and anxiety toward mathematics (r = .40)* and also between mathematics self-concept and achievement in mathematics (r = -.30)*. As self-concept decreases, anxiety increases (r = -.44)." Finally, as a result of the canonical correlation analysis used to compare attitude and achievement scores, the authors state that "measures of mathematics self-concept, anxiety, and enjoyment are correlates of math achievement" (with canonical correlations of r = .44 in the first discriminant function and r = .18 in the second discriminant function).

5. Interpretations

The authors claim that the "general overlapping trend" of mean self-concept and anxiety scores across ability groups may be a result of "the learning context." That is, being a low student in a high group "may raise anxiety, undermine self-concept, and thus affect achievement." The authors suggest that this is a reason for students "trying to fail" in order to be placed in a lower group.

The authors further suggest that mathematics teachers should attend to "self-concept enhancement" and "anxiety reduction," that activities "perceived as more enjoyable might reduce anxiety," and that self-concept improvement may aid anxiety reduction. Based on their findings, they also advocate that special attention be devoted to students in middle-level classes, particulary to the "low-ranked" students in those classes. Finally, the authors

*There is a discrepancy between the text and the matrix table. See abstractor's notes.

claim that pupil attitudes toward the teacher may be very important in forming pupil attitudes toward mathematics.

Abstractor's Comments

This study is related to the matrix of other studies which attempt to relate attitude and achievement. In terms of the conceptual contributions of this particular study, the authors came up with a number of statistically significant conclusions; however, the practical significance of these conclusions must be questioned. A general difficulty lies in their synonymous usage of "achievement" and "ability," which is reflected in the above description of their study. A second problem comes in implications which may be drawn from a correlational analysis. The correlations are, in some cases, trivial and, in all cases, not sufficient to imply causal relationships. For instance, there is certainly an important relationship between self-concept and achievement; but suggesting that teachers "attend to self-concept enhancement" is a little misleading. An example of a more specific conceptual difficulty is the attempt to explain students who "try to fail" in a few sentences. Certainly, more discussion is necessary to provide the basis for such claims. Finally, given the background research on attitudes and achievement in mathematics mentioned in the introduction, the authors do little to compare and contrast their results with the results of other research.

Methodologically, the authors might claim that the additional information they have provided in regard to the use of the MAI is valuable. But, this immediately brings up questions of validity and reliability. The authors chose MAI because of the recommendation given MAI by Aiken and because of MAI's multidimensional nature. It would have been helpful to have had more information regarding the validity and reliability of the instrument. In addition, the use of the previous year's CTBS scores needs to be explained further, expecially since the authors state that "the unit of analysis was ability levels as they currently existed."

A number of questions must be raised about the research design. First, the generalizability of results depends on appropriate sampling procedures. Although the sample represented "a mixture of socioeconomic backgrounds," we must ask how representative the chosen suburban community is of other types of communities and how it was selected. In addition, we are not told specifically whether all seventh graders from all junior high schools in the community were chosen, or whether sampling was involved within the population of seventh graders in the community.

There are other research design issues. Students were told to think "only of their present mathematics class" on the MAI; more discussion about the rationale and feasibility of this would have been helpful. On the questionnaires, the teachers recorded the ability level of each class as a whole. Are there any possible problems in assuming these ratings would always be the same as the general "district" determinations? Within-class groupings were based on the ratings by the teachers of their three highest and three lowest achieving students. What criteria did different teachers use to make this decision? The between-group decisions were based on relatively large group N's; the within-group decisions were based on N's for the low achievement levels which were much smaller than the middle achievement level N's. What effect did these group sizes have on the results? Finally, we must ask how critical values for the significance tests were chosen, whether decisions were made before the tests were done, and exactly what the significance tests were.

Many of the research design problems may actually have been problems of the quality of the written report. Much more discussion, especially of the techniques of statistical analysis, would have been very helpful. In addition, referring specifically to the correlation matrix table, two issues exist. First, the enjoyment of mathematics scale of the MAI correlates significantly ($p < .01$) with all the other MAI scales. This may say something about the MAI generally, or it may lead to other discussion. Yet this is not mentioned in the text. Second (as was mentioned in the findings section), there is either a misprint or the authors misinterpreted their table. The correlation of .40 mentioned in the text does not refer to the relationship between self-concept and anxiety as the authors state, but to the relationship between self-concept and the total achievement score on the CTBS. Similarly, the -.30 correlation refers to anxiety and total achievement, not to self-concept and achievement as the authors state in the text.

Some final comments must be made about the current state of research in this area and about future research. It would seem that the first major goal of this research would be to establish some sense of conceptual clarity.

The discussion of attitudes and their relationship to achievement is certainly important. But what aspects of this discussion require philosophical clarification rather than clarification through research? What aspects of this discussion can be translated into important, non-trivial research questions? This, it seems, is the first major task of researchers working in this area.

In terms of the research efforts themselves, more work is certainly necessary to develop valid and reliable instruments and techniques of measurement. Ethnographic case studies, structured individual interviews, and the "methode clinique" may help to reformulate this area of research and direct it into more fruitful avenues of further research.

Cohen, Martin P. and Walsh, Sister Margaret. THE EFFECTS OF INDIVIDUALIZED INSTRUCTION ON LEARNING AND RETENTION OF A GEOMETRY UNIT IN JUNIOR HIGH SCHOOL. International Journal of Mathematical Education in Science and Technology 11: 41-44; January-March 1980.

Abstract and comments prepared for I.M.E. by F. JOE CROSSWHITE, The Ohio State University.

1. Purpose

The purpose of this study was to test the relative effectivenesss of two modes of instruction -- individualized and traditional -- for promoting learning and retention of selected geometrical concepts.

2. Rationale

Reviews of research on individualized instruction reveal conflicting results. These may have evolved, in part, from the use of nonequivalent control group designs within which experimental variables were inadequately controlled. Also, the question of retention has not been adequately considered.

3. Research Design and Procedures

One hundred nineteen students from three seventh- and two eighth-grade classes homogeneously grouped by ability were randomly assigned by ability to either the individualized or traditional treatment. The individualized group employed a series of 20 SRA Computape lessons supplemented by worksheets designed specifically for reinforcement and practice of basic concepts and skills in geometry. The individualized treatment emphasized small-group work as well as independent work and self-evaluation. The traditional group lessons were primarily expository and consisted of oral and written drill and required assignments from the textbook. The experiment consisted of five 45-minute periods per week for approximately six weeks. One teacher handled the individualized group and another the traditional group. An achievement test was administered immediately following instruction and a retention test three weeks later with no intervening instruction in geometry.

4. Findings

The study found no significant differences due to mode of instruction, and no interaction between ability and mode of instruction for either the achievement or retention measures.

5. Interpretations

a. The authors identify the use of two teachers, a private school population, and student absenteeism as factors that may have limited or altered some of their conclusions. However, they feel the design and procedures (e.g., randomized equivalent control group, six-week duration) and research strategies (e.g., incorporating the variables of ability, the retention measure) they employed represent fertile leads for future research -- and they suggest that other studies "may yet demonstrate more clearly the effectiveness of individualized instruction."

Abstractor's Notes

The authors appropriately identify the use of two different teachers -- one for the individualized group and one for the traditional group -- as a limiting factor in their study. Indeed it is! In the absence of significant differences, it might be easy to overlook this severely limiting factor in the research. Clearly differences in the teacher variable might have been offered as an alternative explanation for any significant differences that might have been found. It is surprising that those who try to be careful in other ways would not make some effort to control for the teacher variable. In the absence of such control, any conclusions are suspect.

Davis, Robert B. and McKnight, Curtis. THE INFLUENCE OF SEMANTIC CONTENT ON ALGORITHMIC BEHAVIOR. Journal of Mathematical Behavior 3: 39-89; Autumn 1980.

Abstract and comments prepared for I.M.E. by THOMAS O'SHEA, Simon Fraser University, British Columbia.

1. Purpose

"The central goal was to clarify the notion of 'understanding'" (p. 39). This goal was pursued through the examination of students' thought processes in solving a single arithmetic task, that is, the subtraction problem $\begin{array}{r} 7{,}002 \\ -\ \ 25 \\ \hline \end{array}$.

2. Rationale

First, there is evidence that many elementary school students do not understand the arithmetic they are studying. For example, one study cited dealt with the inability of fifth graders to shade squares on graph paper to suggest the multiplication problem 4 x 5. Second, there is a lack of agreement on what it means to "understand" what one is doing. To illustrate, the correct application of the algorithm "invert and multiply" applied to the problem 1/3 ÷ 1/2 does not demonstrate an ability to relate correctly the real situation and mathematical representations of real situations.

The authors believe that:

> Students need to learn to deal with mathematics in both of the two basic modes:
> 1. as a meaningless set of symbols that are manipulated according to explicit rules;
> 2. as meaningful symbols, where the translation between real-world problems and the abstract mathematical representation of these problems is an essential part of the task. (p. 42)

3. Research Design and Procedures

During the school year 1979-80, third- and fourth-grade students in three schools were individually interviewed, the focus of each interview being the child's attack on the subtraction problem $\begin{array}{r} 7{,}002 \\ -\ \ 25 \\ \hline \end{array}$. Interviews were audiotaped and each student's written work was preserved. Previously, the researchers had found that the error which regularly recurred in this problem was:

$$\begin{array}{r} 7,002 \\ -\ 25 \\ \hline 5,087 \end{array}$$

That is, students ignored the "middle zeroes" in the minuend when using the standard "borrowing" algorithm.

The authors suggest that there are at least five areas of relevant knowledge which should alert students to the likely presence of an error:

1. Approximate size: an answer of 5,087 is incorrect because 7002 less 25 should still be close to 7000.
2. Making change: if the numbers were interpreted as dollars it would not be fair to exchange a one-thousand dollar bill for ten one-dollar bills.
3. Dienes's MAB blocks: if children have used MAB blocks they should be aware of what trades necessarily must occur.
4. Easier number heuristic: if the child can solve a simpler subtraction problem such as 702 - 5, the solution to 7002 - 25 should follow.
5. Mental arithmetic: procedures apart from the usual pencil-and-paper algorithm may be used. For example, if the child is aware that 7000 - 25 is 6975 then, in the case where we have 2 more, the child should realize that 7002 - 25 = 6977.

For each child interviewed it was established that:

1. the student could subtract correctly using the standard algorithm, provided there were no zeroes in the minuend, and
2. the student made the "skipping over zeroes" error in the problem under investigation.

Systematic exploration of the five relevant areas of knowledge was carried out using the following procedure:

1. Could the student deal with similar problems with smaller numbers, for example, 702 - 25? If so, did the student wish to change the answer to the original problem?
2. The student was presented with a written list of multiples of 1000, from 1000 to 10,000, and was asked which would be nearest to 7002 - 25. After consideration of size, the student was again given the option of changing the original answer.

3. The interviewer asked the student to solve the problem mentally, and tried to lead the student through a sequence of problems, such as 99 - 3 = ?, 99 - 4 = ?, 100 - 4 = ?, etc. If the student obtained the correct answer to the problem in this way, he or she was asked which answer was correct, the written one or the one obtained by mental arithmetic.
4. Did the student understand money, particularly the exchange of one one-hundred dollar bill for ten tens? The student was asked the relevance of this to the problem and whether he or she wished to work further on the given problem.
5. Was the student familiar with Dienes's MAB blocks? If so, the student was asked to represent 7002 - 25 using the blocks. Then the student was invited to revise the original solution if desired.

4. Findings

No student could use approximate size as an indicator that something had gone wrong, even after suggestions from the interviewer. None could see the relevance of "making change" as a guide to judging the correctness of the algorithm. Children showed considerable familiarity with MAB blocks; but did not apply this knowledge to recognize the subtraction error. No student could apply the easier number heurisitic. When asked to try the problem in their head, all students initially envisaged the problem using the same standard algorithm they would ordinarily write. Children who correctly determined the answer mentally were asked to choose which answer was correct --the written one or the mental one. Nearly all chose the written (incorrect) one.

Overall, no student interviewed drew on any of the relevant knowledge, or, as the authors put it, this "semantic knowledge", to correct the subtraction algorithm.

Over half of the article is devoted to one specific interview with Marcia, a fourth-grade student who had been given remedial instruction by her teacher to overcome this specific problem. At points in the transcript the authors interject interpretations of the child's thinking patterns as she struggles with each of the five areas of relevant knowledge.

5. Interpretations

The authors point out that Marcia was quick and accurate in arithmetic facts, that she had had experience with MAB blocks, and that, in general, her use of standard algorithms was accurate and reliable. They consider her typical of the fourth-grade students in her school, and maintain that her error of "skipping over intermediate zeroes" is entirely typical of fourth graders. On this particular task, there is no effect of semantic knowledge on algorithmic behavior.

The authors suggest that remediation for this specific problem requires that the algorithm be drawn out of manipulation of the MAB blocks. The student can then be shown exactly where and why her algorithm is flawed. Finally, the authors would instruct her specifically in using relevant knowledge to guide her work with algorithms.

The authors suggest that movements such as "back to basics" and computer assisted instruction in the form of "drill and practice" may ultimately prove harmful;

> They seek to improve student performance by simplification--*but the simplification they seek may leave large numbers of students with impoverished cognitive resources that will handicap them in the long run.* (p. 78, italics in original)

Abstractor's Comments

I approached the task of abstracting this article with feelings of trepidation on two accounts. The first was the length of the article--fifty pages. Happily, the clarity of the authors' writing style and the lack of jargon alleviated this problem considerably. I was quickly drawn to the essence of the problem and followed the interview with Marcia with a great deal of interest. It is a pleasure to read a research article without having to analyze each sentence to determine what exactly the author is trying to say.

The second doubt stemmed from the fact that over half the article consists of a transcript of an extensive interview with one of the students. In such a case, one is immediately at a disadvantage in critiquing the design of the study. What is there to critique? In this particular case, my main concern is that, although the authors took pains to point out that they could not generalize to other schools, they do intimate their findings apply to a much broader population. They state: "We have, however, worked with a few children from three other schools, representing a total of three different school systems, and see only unimportant variations in the key phenomenon we wish to report" (p. 75). Since the authors use their findings to reach a rather serious indictment of the entire pedagogical program, the numbers interviewed in Marcia's school and in the others would help one to judge how powerful their inductive argument might be.

The findings are disturbing. Children of this age working on this particular problem cannot bring knowledge outside of the standard algorithm to bear when, in a sense, the algorithm is pushed to its limit, or perhaps more accurately, when the student is pushed to the limit in applying the algorithm. But, before we despair, more investigation needs to be done. For example, the authors point out that the idea of approximate size of numbers may not be useful to children of this age since they may not have enough experience with the size of large numbers to be able to apply the knowledge appropriately. Then the question arises: would older students be able to make effective use of this type of "semantic knowledge"? Is there a developmental aspect which would alleviate the problem through the passage of time? A similar study cutting across the intermediate grades would indicate whether and to what degree the phenomenon persists.

Immediately following the article is an editorial note containing implications based on this article and two other reports. The suggestion is made that "correct answers written on paper cannot be taken as conclusive evidence of learning" (p. 88). Reactions to such a statement will differ according to the role of the reader in the educational system. Mathematics educators may react by concurring, and arguing that in the best of all possible worlds students should be able to carry out calculations quickly and accurately, and understand exactly what they are doing at each step. School classroom personnel, on the other hand, may be dismayed to find that, even though their students have all the external trappings of knowledge, they are being asked for much more from their students and from themselves.

Nibbelink, William H. and Witzenberg, Harvey G. A COMPARISON OF TWO METHODS FOR TEACHING YOUNGER CHILDREN TO TELL TIME. School Science and Mathematics 81: 429-435; May-June 1981.

Abstract and comments prepared for I.M.E. by GAIL SPITLER, University of British Columbia.

1. Purpose

Two instructional strategies for teaching youngsters to read a circular clock face were compared. The two instructional strategies being compared are referred to as Methods A and B. Method A "followed the traditional sequence: telling time on the hour, on the half hour, on the quarter hour, on the five minute readings and finally on any reading" (p. 429). Method B followed the sequence: on or after the hour using only the hour-hand, reading any minute where a rim with each minute marked circumscribed the regular clock face, reading any minute where a rim with each five-minute interval marked circumscribed the clock face, and, finally, reading any time with no minute marks numbered.

2. Rationale

Reading a clock face is an important skill which is often ignored in the present curriculum. Despite the fact that most people do learn to read a clock at one time or another, the problem for mathematics educators is to find the strategy by which this skill is acquired efficiently. As the circular clock will continue to exist along side digital clock faces, the reading of the circular face remains an important skill. Further, it is argued that the "Time-distance feature built into the regular clock is psychologically significant and may explain why many adults prefer it over a digital time piece" (p. 431).

3. Research Design and Procedures

After completing three pilot studies, the main study was undertaken. Six grade two classrooms from similar middle-class areas were used in the main comparison. One class in the first school was assigned Method A and the other two classes, Mathod B; the opposite assignment was used in the other school. The instruction occurred over eight days, about half an hour a day. A pretest was administered the day previous to the commencement of the instruction and a posttest the day after the instruction terminated. "Both tests contained 28 items requiring children to record the time shown by a clock, 14 items with minute reading a multiple of five and 14 items with minute reading not a multiple of five" (p. 432).

Data from two other groups were also collected. One group was composed of two grade two classrooms in which 2 to 4 instructional periods followed the instructional sequence of a textbook. The fourth group was a second-grade class of learning disabled students where the instructional sequence followed was Method B.

To control the "teacher variable", very detailed lesson plans, scripts, and equivalent story lines, which were used to motivate the unit, were developed.

4. <u>Findings</u>

ANCOVA using the pretest scores to adjust the posttest scores revealed that Group B (Method B) outperformed Group A (Method A) ($p < .06$). Further analysis of the data revealed that the difference between the two groups was specifically due to differences on items for which the reading was not a multiple of five ($p < .01$). "Boys did better than girls for both methods, with $p < .01$ for the posttest for Method B" (p. 433). No differences were found among high, middle, and low mathematical ability groups.

5. <u>Interpretations</u>

Based upon the results of the three pilot studies and the results reported here, the authors conclude that Method B should be the instructional strategy employed for teaching children to read the circular clock face until a more effective strategy can be found. Specifically, the recommended sequence is a slightly altered version of Method B so as to include telling time on the half hour and quarter hour.

<u>Abstractor's Comments</u>

The authors are to be commended for studying an area of the curriculum which is too often ignored. While the telling of time seems to be significant to both parents and teachers, it has largely been ignored by researchers. The article outlines an interesting and creative approach to the problem. In addition, the article may serve as food for thought for others who wish to study this area. Certainly the "traditional" approach to the teaching of clock reading is fraught with logical inconsistencies. For example, the traditional instructional sequence begins with the hour as the basic unit of measure and then deals with fractions of that unit often before children have the appropriate fraction concepts. We also include "before" and "after the hour" at the same time, thus asking youngsters to read backwards on a dial which never operates with a backward motion. Next, the "traditional" approach may examine reading the minutes which are multiples of five. At this stage we have changed the basic unit of measure from the hour to the minute, but we do not deal with the minute per se, only with sets of five minutes. Finally, the sequence concludes with reading any minute, which was surely one of our main objectives in the first place. Entirely left out of the "traditional" sequence are all of the other "before the hour" readings; for example, 8:53 is 7 minutes before 9. Clearly, a more consistent approach can and should be developed. Nibbelink and Witzenberg may have developed such an approach, but they have not presented a convincing argument for the superiority of Method B over the traditional method.

In attempting to convince the reader of the efficacy of Method B, the authors have employed a standard experimental paradigm. However, as an experimental study this work has overwhelming shortcomings. Generally, it is unfortunate that much curriculum development activity cannot find its way to publication unless it is framed as an experimental study, a form which seems particularly inappropriate for reporting the insights gained as one carefully and thoughtfully develops instructional materials, strategies, and sequences.

As an experimental study, the major weakness lies in the fact that the posttest which is the measure of the effectiveness of the treatments is not a fair measure of the two treatments. Recall that the posttest contained 14 items requiring the children to read a clock set at a minute which was not a multiple of five and 14 items in which the minute was a multiple of five. The rationale for these two sets of items is not supplied in the article. Surely, if one can read any minute, then one can read minutes which are multiples of five, which is substantiated by the data in that both groups averaged about 12 on the 14 items related to reading a multiple of five. Also recall that the significant difference on the posttest resulted from the fact that Group B children were better able to read minutes which were not multiples of five.

The unstated model of efficiency used by the authors is

$$\text{Efficiency} = \frac{\text{content taught or learned}}{\text{time for instruction}}$$

which requires that one experimentally control the equivalence of either the numerator or denominator of this ratio. In this particular study, neither can be considered to be equivalent. Consider the content taught in each method:

Method A	Method B
Telling time:	Telling time:
1) On the hour	1) On the hour or after the hour
2) On the half hour	2) On any reading with each minute marked
3) On the quarter hour	3) On any reading with only five-minute marks
4) On the five-minute reading	4) On any reading with no minute marks
5) On any reading	

If we assume that the instructional time was controlled, then the content taught is not equivalent in that the students were not tested on their ability to tell time on the half or quarter hour, topics which were part of Method A. Alternatively, if we assume that the content taught (and tested) was controlled, then the instructional times devoted to the content of the of the posttest are not equivalent. As the authors admit, "Method A devoted relatively more time to minute readings which were multiples of five, while Method B devoted relatively more time to minute readings which were not multiples of five" (p. 433). It can be argued that the students in Group B had significantly more instructional time devoted to the content of the posttest, a difference which is further heightened since skill of reading time to any minute subsumes the skill of reading multiples of five, while the converse is not true. Since the authors have chosen to use the ratio referred to above as their model for efficiency, and since neither the numerator or denominator can be considered equivalent, they cannot claim that Method B is more efficient than Method A.

Peck, Donald M. and Jencks, Stanley M. CONCEPTUAL ISSUES IN THE TEACHING AND LEARNING OF FRACTIONS. Journal for Research in Mathematics Education 12: 339-348; November 1981.

Abstract and comments prepared for I.M.E. by DONALD J. DESSART, The University of Tennessee, Knoxville.

1. Purpose

Hundreds of children in grades 6, 7, and 9 were interviewed for the purpose of determining the extent of their conceptual understandings of fractions. A sample of 20 of these students was selected at random from 60 students of a typical sixth grade. The records of the interviews of these 20 students were summarized and reported in this article.

2. Rationale

Clinical studies in which researchers emphasize the qualitative aspects of the observations of children working mathematically over the statistical aspects has gained popularity in recent years. Such studies are extremely time-consuming, but often provide insights that might be investigated at a later time and on a wider scale using a more complex statistical design.

3. Research Design and Procedures

The 45-minute interviews of the children which were videotaped consisted of two parts. During the first part, which lasted about 20 to 25 minutes, the children worked with physical materials for the purpose of acclimating them to the interview process. This activity consisted of multiplication exercises with whole numbers using graph paper and paper strips. One of the paper strips represented a multiplier, the second strip represented the other multiplier, and the area thus included represented the product. Of the 20 children interviewed, 19 of them were able to determine products using this procedure.

This initial activity was followed by a discussion of fractions in which each child was requested "to show what simple fractions, such as 1/3, 1/4, or 2/5 would look like" (p. 339). Following their responses to this question, the children were asked to compare fractions, to add fractions, and to explain the rationale for their actions. On some occasions, questions were also asked about other operations with fractions.

4. Findings

Each child was asked to draw sketches of the fractions 1/2, 1/3, and 1/4. Of the 20 children, nine drew appropriate pictures (usually circles or pies) and 11 were unable to draw correct sketches. Of these 11, all understood that the sketch involved showing that an object should be subdivided into a number of pieces corresponding to the denominator of the fraction, but they didn't indicate that the pieces should be of the same size.

Of these 11 students, only two were able to compare fractions using a rule. However, these two were unable to relate the results of the rule to sketches. Of the same 11 students, four were able to use correctly the common denominator method of adding fractions, but they could not rationalize the method. The remaining seven students misapplied rules for addition of fractions in a variety of ways (e.g., 2/5 + 1/3 = 3/8).

Of the nine students who drew appropriate sketches of fractions, four were unable to compare fractions and were also unable to use sketches of fractions to help in the addition of fractions. The most common error was of the type, 2/3 + 1/4 = 3/7.

Of the nine students who drew appropriate sketches of fractions, five students were able to compare fractions correctly by both sketches and rules. Of these five students, two were able to add fractions using both rules and sketches and three were able to add fractions using the common denominator rule, but were unable to justify the rule with sketches.

5. Interpretations

The researchers emphasized the following conclusions of their study:

a. Nearly all of the children related a fraction to the subdivision of an object into a number of parts corresponding to the denominator of the fraction, but fewer than half realized that the parts should be of equal size.

b. Most students used pies or circles as the object to be subdivided.

c. Many children did not relate operations with fractions to the manipulation of physical materials in their environment.

d. Many children gave correct examples of specific fractions, but could not generalize the meaning of fractions; e.g., they could sketch 1/2 or 3/4, but not 3/5.

e. Many children who did not understand a generalized concept of fraction were unable to operate with fractions, i.e., perform comparisons and addition of fractions.

f. Almost all students were rule-oriented; that is, they searched for rules to apply in operating with fractions, but they seldom were able to justify the use of these rules using physical materials.

Abstractor's Comments

Clinical studies often uncover facets of children's knowledge that can seldom be obtained in other ways, and thus the pursuit of such studies should be encouraged. This particular study involved hundreds of videotaped interviews. It seems clear that a short journal article can hardly do justice to the hours of observations by the researchers. Nevertheless, based upon the report, a number of questions seem to arise. Perhaps, these questions were dealt with more adequately in the more complete report.

First, there is the problem of the adequacy of the questions to elicit from the students their actual knowledge. For example, if a student is asked "... to show what simple fractions ... look like" (p. 339), he or she might be very perplexed as to what is an adequate response. Should one respond with a picture, a fractional numeral, an operation, or what?

Furthermore, we should be critical of children's over-reliance upon rule-oriented behavior. But, on the other hand, if abstract questions are posed, then abstract responses seem appropriate. One might wonder what the responses of the children would have been had they been presented a concrete situation calling for the use of fractions. For example, if an actual pie were brought into the interview room with five children present and one of the students was asked to cut the pie so that each child received the same amount of pie, one might wonder if the responses of the children would have been different. Following the cutting of the pie, one might ask, "What fraction of the pie did Jane receive? What part did Dan and Jane together receive?" Furthermore, if one pie were given to Dan, Jane, and Bill and another to John and Sue, would Dan or would Sue receive the larger piece?

The relationships of the responses of the children during the interviews to their responses on written achievement tests might be revealing. Even graduate students complain of "stage fright" during oral examinations that doesn't seem to be present during written examinations. The interview process can be threatening in spite of the conscientious efforts of interviewers to relax their subjects.

The authors' conclusion that children should be exposed to more instruction involving physical materials is a sound one whether or not it is justified by this study. The use of Cuisenaire rods, pattern blocks, base ten blocks, the classroom clock, money, and other materials of the children's environment are essential to effective instruction on fractions. Unfortunately, their use is probably neglected in many classroom learning situations.

Pennington, Bruce F.; Wallach, Lise; and Wallach, Michael A. NONCONSERVERS' USE AND UNDERSTANDING OF NUMBER AND ARITHMETIC. Genetic Psychology Monographs 101: 231-243; May 1980.

Abstract prepared for I.M.E. by JOE DAN AUSTIN,
Rice University.
Comments prepared for I.M.E. by JOE DAN AUSTIN and by SIGRID WAGNER,
The University of Georgia.

1. Purpose

The study "attempts to assess the implications of nonconservation [of number] for a child's use and understanding of number and arithmetic" (p. 235).

2. Rationale

Perhaps one of the most widely held beliefs about a child's understanding of number seems to be that when a child fails on Piagetian conservation of number tasks, the child cannot meaningfully understand counting and arithmetic. Thus, it seems logical and is often argued that instruction in arithmetic should be delayed until the child has mastered conservation of number. Several research studies have tended to support the belief in the central role of conservation of number for understanding arithmetic. These studies tend to show that while nonconservers may demonstrate some paper-and-pencil proficiency in arithmetic, they usually have a very limited ability to understand or apply the knowledge to physical situations. However, some research suggests that conservers and nonconservers seem to profit equally from arithmetic instruction even when the child's ability to apply arithmetic knowledge to physical situations is considered. Thus, there is a need to study in more detail how fundamental conservation of number is to a child's understanding of number and arithmetic.

3. Research Design and Procedures

The sample for this study consisted of 130 third graders from relatively disadvantaged areas. The students were mainly black and from lower income families. The students were from two schools, one rural and one city.

To ensure that all students understood the relational terms used in the conservation tasks, an initial screening task was given. Students simultaneously placed one bead in one jar and one bead in a second jar. Students were included in the sample only if they knew there were the same numbers of beads in each jar even though the beads in the jars could not be seen, could justify that the numbers of beads were the same, and could make a specified jar have more beads than the other jar.

To determine which children could conserve number, two parallel rows, each with 13 beads, were placed in front of a child, with the beads in the rows lined up opposite to show one-to-one correspondence. After agreeing that the two rows had the same number of beads, the child watched as the experimenter lengthened one row by spreading out the beads. The child was asked if there was now the same number of beads in each row. If the child said there was still the same number in each row and could justify this answer, the child was classified as a conserver. If unable to do this task, the child was retested another day. If the child failed the task the second time, he or she was classified as a nonconserver. If the child passed the task the second time it was administered, he or she was classified as an equivocal nonconserver.

The nonconservers were given additional conservation and counting tasks. Some conservation tasks attempted to minimize the visual conflicts; e.g., using one row of beads or moving the beads but keeping the length of the row the same. Both conservers and nonconservers were given a 30-item paper-and-pencil test on addition, subtraction, and multiplication. Each child in the two groups was given problems with physical objects, using problems correctly solved on the paper-and-pencil test by the student. These problems attempted to assess whether the child understood addition and subtraction in physical situations. The equivocal nonconservers were given no tests other than the Screening task and the Conservation of Number task.

4. Findings

Seven of the original 130 students failed the Screening task and were deleted. From the Conservation of Number task, there were 45 nonconservers, 12 equivocal nonconservers, and 66 conservers.

When only one row of 13 beads was used and then lengthened, 19 of the 45 nonconservers failed to know that the row still had the same number of beads. When beads in the one row were moved but the length was not changed, the number of failures was reduced to 10. When the experimenter did the same experiment but used "counted number" instead of "same number", only 3 children failed to conserve.

In the Static Counted Row task, children were to count two fixed rows of circles and answer whether the rows had the same number of circles. Lengths of the two rows were equal only when the rows had an unequal number of circles. Eleven of the 45 nonconservers answered in ways that contradicted their counting. However, of the 11 most made only one such error.

On the paper-and-pencil computation test, the nonconservers had significantly fewer correct answers ($\bar{X} = 20.6$) than did the conservers ($\bar{X} = 22.9$) at the 0.01 level. However, the difference in the two averages was less than the differences between the city ($\bar{X} = 23.5$) and rural ($\bar{X} = 20.6$) children. Impressionistic data indicated no differences between the conservers and nonconservers in their use of finger counting and other tally systems and in use of carrying and borrowing. "Conservers and nonconservers hence did not differ qualitatively either in their ability to use more abstract computational procedures or in their need to rely on concrete tallies" (p. 240).

To measure whether a child understood addition and subtraction problems that were correctly solved on the pencil-and-paper test, two addition and two subtraction tasks were given. The results were as follows:

	Addition			Subtraction		
Group	Pass both	Pass one	Pass neither	Pass both	Pass one	Pass neither
Nonconservers (N = 45)	33	7	5	35	4	6
Conservers (N = 66)	60	4	2	57	7	2

The "vast majority of nonconservers performed very well" (p. 240) on the addition and subtraction tasks. Significant differences favored the conservers on the addition tasks ($\chi^2 = 4.86$, $p < .05$). No significant differences were found on the subtraction tasks ($\chi^2 = .85$, $.30 < p < .50$).

5. Interpretations

The results of this study indicate that nonconservers do have some problems with number and arithmetic, especially when conflicting clues are present. However, "they can still have a very serviceable understanding of number and arithmetic. Full conservation of number or recognition of invariance is, thus, not a fundamental cornerstone for mathematical development. Its importance would seem to have been considerably exaggerated over the last two decades" (p. 241).

Abstractor's Comments (1)

This is a very interesting study on a question of fundamental importance to any classroom application of Piaget's theory of number, namely, how fundamental is conservation of number to a child's understanding of arithmetic. Logically, it seems that this is fundamental, but do the empirical data support this logic?

The study has several important positive aspects. The authors present a very nice literature review and argument against the view that conservation is fundamental to a child's understanding of any arithmetic instruction. It also seems a plus that two repeated testings were made before labeling a child a nonconserver. Such a procedure would seem to give some evidence on the reliability of the classification procedure. Also, the variety of tasks used in the study was very nice. Some tasks also replicated some previous research.

This reviewer had several questions about the research study that seem likely to be important when interpreting the results. These questions include the following:

1. On the conservation task, why were not conservers tested twice? If equivocal nonconservers could first fail and then pass the task, could not some students labeled conservers first pass and then fail the task? Such testing might make the differences between conservers and nonconservers clearer.

2. For the 30-item computation test, what was the reliability? Was this a multiple-choice test? Was the test timed?

3. On the conservation tasks using only one row of beads, were all 45 nonconservers given each of the tasks or only those 19 nonconservers who failed to conserve when one row of beads was lengthened?

4. On the Static Counted Rows tasks, how many such tasks were given? Knowing that most (how many?) of the 11 nonconservers who failed these tasks only missed one task is insufficient information.

For the statistical analyses reported, it seems hard to view the analyses as not supporting the case that conservation of number strongly relates to computational ability. Two of the three statistical tests that compare conservers and nonconservers are significant and favor the conservers! The author's attempt to explain away the computational test significance using city and urban school data and impressionistic data would hardly seem to justify why the statistical test was even done.

Finally, it would be nice to have a more composite picture of the nonconservers' responses. For example, some nonconservers pass one task and some pass another. How many nonconservers pass both tasks?

In spite of these comments, the authors make an important point concerning the relation of conservation of number to a child's understanding of arithmetic. Specifically, if there are nonconservers who can do and meaningfully understand arithmetic tasks, then these nonconservers would seem to negate at least in part the argument that no arithmetic instruction should be given nonconservers.

This reviewer feels that this study should be read and replicated. It raises some interesting questions that would seem to require additional research. For example, if nonconservers can master and understand arithmetic tasks, how should arithmetic instruction be modified for these children? Does this instruction do better than providing experiences that will help the child develop first a conservation of number? The study raises some important issues and questions. Additional research would, however, seem necessary before the resolution of these issues will be possible.

Joe Dan Austin

Abstractor's Comments (2)

This article reports an interesting study with a thought-provoking rationale. The authors have transcended the old chicken/egg debate over which should first -- skill or understanding -- and have conjectured that children who fail to conserve number may, nevertheless, exhibit a fairly sophisticated understanding of the concept of number simply because their system of logic differs from that of an adult. The authors essentially reiterate this idea when they further observe that a foundational analysis of concepts may yield structures very different from the mental structures that evolve in the course of development.

Their point is an important one, and it is entirely consonant with the Piagetian theory which serves as a backdrop to their study. Somewhat ironically, it is this same point which suggests that implications of Piaget's findings for mathematics teaching should not be overdrawn, because Piaget's tasks are based to such a large extent on a logical analysis of content structure. The authors of this paper implicitly acknowledge this latter observation in their discussion of conservation tasks, when they contrast the kinds of transformations that occur in conservation tasks with the kinds of transformations that occur in the context of simple arithmetic operations. True conservation tasks involve the transformation of an irrelevant, but potentially relevant, attribute to produce a "conflicting cue", as the authors call it. Certainly, conservation tasks measure an aspect of the understanding of a concept, if understanding is assumed to imply comprehension of the language typically used relative to the concept. But, conservation tasks may measure such depth of understanding that failure to conserve may not directly interfere with performance on tasks that involve more ordinary kinds of transformations.

The purpose of the reported study was to show that children who do not conserve number may still have some understanding of number concepts and may be able to apply this understanding in concrete situations. The authors' results were not strong, and it would have been better to match the conserving and nonconserving groups on general ability in order to nullify the effect of the ability factor as much as possible. Nevertheless, several of the tasks were interesting, and some of the results were intriguing. The authors' primary contribution, however, resides in their basic premises.

Sigrid Wagner

Schadler, James. A COMPARISON OF THE USE OF ACTIVE GAME LEARNING WITH A CONVENTIONAL TEACHING APPROACH IN THE DEVELOPMENT OF CONCEPTS IN GEOMETRY AND MEASUREMENT AT THE SECOND GRADE LEVEL. School Science and Mathematics 81: 365-370; May-June 1981.

Abstracts and comments prepared for I.M.E. by EDWARD J. DAVIS, The University of Georgia.

1. Purpose

This study compared the effectiveness of an active game teaching strategy with a conventional teaching strategy.

2. Rationale

Many elementary schools use an adopted mathematics textbook. This book, in many cases, becomes the curriculum. Does this have to be the case? Can students learn as much or more from approaches besides following the teacher's manual? In particular, does the medium of active games and movement experiences provide a viable alternative to closely following a commercial text and using its suggestions for manipulatives and practice?

3. Research Design and Procedures

All second-grade children at an elementary school comprised the population. They were divided into 8 groups.

Groups 1 and 2 were taught using Method 1 (following the teacher's guide for a commercial text) and also using Method 2 (active games - movement experiences).

Groups 3 and 4 were control groups - no content in the study was taught to them.

Groups 5 and 6 were taught using Method 1.

Groups 7 and 8 were taught using Method 2.

The treatment consisted of 15 daily lessons. The content was Chapter 3 of the 1972 edition of the second-grade text published by Houghton Mifflin Company. Chapter 3 deals with lines, curves, points, line segments, linear measurement, time, liquid measurement, sets, and the fraction one-half.

Method 1 followed the specifications in the teacher's manual as closely as possible; this included use of manipulative materials, workbook pages, and a review. Tests developed by Houghton Mifflin for this chapter were used as pretests, posttests, and a retention test six weeks later.

4. Findings

There were no significant differences between the mean achievement scores on the commercially developed tests between the groups of children taught by Method 1 and those taught by Method 2. Children taught by Methods 1 and 2 did have significantly higher achievement scores than children in the control groups. These findings held for both posttest and retention test scores.

5. Interpretations

The hypothesis that there was no difference between the mean scores of the two treatment groups was accepted at the .01 level of confidence. However, the author feels that one must interpret this as a plus for the active game approach -- it is at least as effective as conventional instruction. As additional evidence supporting some continued use of the active game medium, he points to the enthusiasm of the students and teachers in the game medium. And also to the fact that the actual mean gain scores from pre- to posttesting for the game group were slightly higher than the mean gain scores of the conventional group.

Abstractor's Comments

It seems relevant to pose some questions at this point:

- Would an active (movement-oriented) game approach work just as well for a unit of work that was not predominately geometric in nature?
- How much of a positive influence did the novelty of a game approach have? Would this influence wane?
- Were the students randomly assigned to treatment groups and how many students were in each group?
- Does the author feel that a game approach should be considered as an occasional mode of instruction or as the primary medium of instruction?

It would have been helpful if the author would have specified the active game mode of instruction in some detail (only references are given). Apparently, no paper-and-pencil activities were included. A description of one or two games, and the content these games were to convey, would have been welcome. Was it the case that the active game approach did not involve any pictorial representation? This would seem to handicap the written test performances of children in this medium of instruction.

While mean achievement scores are given for posttests and retention tests, it should be asked if these scores represent satisfactory achievement. What was the maximum possible test score? What expectations did teachers or test publishers set for student achievement? Sample test items and student performance on these items would help in this regard.

The author is to be commended for doing research in a "real" context, i.e., in actual classrooms and using regular teachers following a commerical text. I view the outcomes as evidence to encourage teachers to depart from a textbook format on occasion. Such departures can add variety and reality to the mathematics classroom. I do not see evidence to conclude that an active game approach would be viable on a widespread basis.

Schunk, Dale H. MODELING AND ATTRIBUTIONAL EFFECTS ON CHILDREN'S ACHIEVEMENT: A SELF-EFFICACY ANALYSIS. Journal of Educational Psychology 73: 93-105; February 1981.

Abstract and comments prepared for I.M.E. by A. EDWARD UPRICHARD, University of South Florida.

1. Purpose

The purpose of this study was to test hypotheses from self-efficacy theory as it related to children's performance with the distributive division algorithm.

2. Rationale

Self-efficacy theory predicts that different modes of influence change behavior in part by creating and strengthening self-percepts of efficacy. Perceived self-efficacy, judgments about one's capability to perform given activities, influences people's choice of activities, effort expenditure, and persistence in the face of difficulties. Higher perceived efficacy leads to greater sustained involvement in activities and subsequent achievement.

3. Research Design and Procedures

Three sets of hypotheses were tested in this study. The first set of hypotheses related to fostering the development of arithmetic skills (division) and self-efficacy by providing subjects with modeling, guided performance, corrective feedback, and self-directed mastery (Bandura, 1977). Treatments involved two instructional modes, didactic and cognitive modeling. The second set of hypotheses concerned the effects on achievement of effort attribution for success and difficulty provided during the process of competency development. For half of the subjects within each of the two instructional treatments, the experimenter periodically ascribed the subjects' successes to sustained effort and their difficulties to insufficient effort. Ascribing past achievement outcomes to effort is hypothesized to have motivational effects (Weiner, 1977, 1979; Weiner et al., 1971). The third set of hypotheses focused on the relationship of self-efficacy or accuracy of self-percepts to subsequent achievement. The dependent measures were arithmetic skill, persistence (time on task), and self-efficacy (judgment).

> It was hypothesized that compared with didactic instruction, cognitive modeling would result in higher arithmetic achievement, persistence, self-efficacy, and accuracy of self-appraisal. Effort attribution was expected to lead to higher achievement, persistence, self-efficacy, and accuracy of self-appraisal in the modeling treatment but not in the didactic treatment. (p. 95)

The sample consisted of 56 middle-class children ranging in age from 9 years 2 months to 11 years 3 months, with a mean of 9 years 10 months. There were 33 males and 23 females. The sample was drawn from five elementary schools. Teacher judgment was used to identify children who displayed low arithmetic achievement, persistence, and self-confidence. Those children identified were then individually administered the formal preassessment, which consisted of an arithmetic performance test and an efficacy judgment scale.

The arithmetic pretest consisted of 18 division problems graded in level of difficulty. Twelve were considered training problems (1-digit and 2-digit divisors) and six were generalization problems (3-digit and 4-digit divisors). Each division problem was presented on a single page, with time spent on each problem recorded as a measure of persistence.

Children's pretest level of efficacy was measured after the division performance test to insure familiarity with the different types of problems. Eighteen pairs of division problems (increasing in level of difficulty) were presented to each child for 2 seconds each. For each pair of problems, children were asked to judge on separate efficacy scales their capability to solve that type of problem.

Subjects were those children who failed to solve at least four pretest division problems. They were randomly assigned to one of four conditions of 12 subjects each (modeling-attribution, modeling-no attribution, didactic-attribution, didactic-no attribution) or to a non-treated control group of 8 subjects. All instructional treatments consisted of three 55-minute training sessions, each on a separate day. Each training session had three phases: instructional and division strategies (10 minutes), practice with learned strategies (35 minutes), and directed mastery (10 minutes). All training was administered individually and focused on solving division problems using the distributive algorithm.

Treatments were distinguished by the mode of instruction given during the instruction phases, the format of corrective feedback for conceptual errors occurring during the practice phases, and whether effort attribution was provided for successes and difficulties during the practice phases. For example, in the cognitive-modeling treatment children observed an adult model solving division problems contained in the explanatory pages of the training packet and verbalizing aloud the solution strategies used to arrive at the correct solutions. Also, corrective modeling was provided when children encountered conceptual difficulties. In the didactic treatment children studied the same explanatory pages on their own, after which they worked the practice problems. When children encountered conceptual difficulty, they were referred to the relevant section of the explanatory pages and told to review them. For children assigned to the modeling- and didactic-attribution conditions, the trainer attributed their successes to high effort and their difficulties to low effort.

Posttreatment assessment was conducted within a week after training. The procedures were identical to those used in the pretreatment assessment except that efficacy judgments were collected before the division performance test. The division posttest was an alternate form of the division pretest ($r = .92$). Metropolitan Achievement Test scores in mathematics were also obtained for each child to determine whether mathematical ability was related to children's response to treatment.

4. Findings

> The self-efficacy scores were analyzed using a median split; judgments higher than 55, which indicated at least moderate assurance, were scored as efficacious, whereas those lower than 55 were scored as inefficacious. Persistence was defined as the number of seconds children worked each problem. Performance-test problems were scored as correct if children correctly applied the division operations at each solution stage or made a small computational error but otherwise used the correct operations. (p. 97).

Posttest scores (training problem) were pooled across the four treatments and compared with pretest scores using the t-test for correlated scores to assess the overall effects of treatment. This later analysis yielded differences which were significant and reliable for division accuracy ($p < .01$), for persistence ($p < .01$), and for self-efficacy ($p < .01$). That is, children who received treatment judged themselves more efficacious, persisted longer, and solved more problems. The controls showed no significant differences except for less persistence.

The posttest measures (accuracy, persistence, self-efficacy) were analyzed using multiple-regression procedures. Variation in posttest self-efficacy judgments was significantly affected by pretest self-efficacy ($p \leq .01$); more efficacious children at the outset tended to remain so. Variation in posttest persistence scores was significantly affected by pretest persistence ($p \leq .01$) and posttest self-efficacy ($p < .05$). Efficacy ($p < .01$), persistence ($p \leq .01$), pretest accuracy ($p < .01$), the modeling-didactic variable ($p < .05$), and the MAT score ($p < .05$) each accounted for a significant increment in the explained portion of variance in posttest accuracy.

Since posttest accuracy is a function of multiple influences, a causal model involving four variables (treatment, self-efficacy, persistence, and accuracy) was identified and tested using path analysis. Results using this model reproduced the original correlation matrix except for the correlation between treatment and accuracy: reproduced $r = .05$; original $r = .23$. Thus, the model was rejected.

To provide a more stringent test of the relationship between self-percepts of self-efficacy and accuracy (training problems), the level of congruence between these two factors was calculated by comparing each posttest efficacy judgment with the subsequent accuracy score on the problem of the comparable form and difficulty. Two measures of incongruence were also computed: overestimation and underestimation. The didactic treatments showed less congruence on the posttest than on the pretest ($p < .01$; $p < .05$) and didactic-attributional children overestimated more on the posttest than on the pretest ($p < .03$). There were no significant changes for modeling groups.

Multiple regression procedures were also applied to the posttest data. The percentages of the total variation in the posttest data accounted for by these predictors were 31% (25% adjusted) for congruence, 28% (22% adjusted) for overestimation, and 9% (1% adjusted) for underestimation. Congruence was significantly affected by both the instructional-treatment and attribution-within-modeling variables. Modeling children showed significantly higher congruence than didactic children ($p < .01$), whereas modeling children receiving attribution showed higher congruence than those not receiving attribution ($p < .05$). No significant differences were found due to the attribution-within-didactic variable.

5. Interpretations

a. This study demonstrates that treatment procedures providing problem-solving principles, practice on applying the principle, corrective feedback, and self-directed mastery were effective in developing skills and enhancing a sense of efficacy in children who had experienced profound failure in mathematics.

b. Cognitive modeling was more effective than didactic instruction in promoting skills development.

c. Greater gains in self-efficacy and persistence as a result of modeling did not receive support.

d. That attributing successes and difficulties to effort should influence self-efficacy, persistence, and skill accomplishment for modeling children failed to receive support. (This finding suggests the need for caution in the use of effort attribution to correct cognitive deficiencies.)

e. Perceived efficacy was an accurate predictor of arithmetic performance across levels of test difficulty and modes of treatment. Modeling children showed significantly higher congruence than didactic children, whereas modeling children receiving attribution showed higher congruence than those not receiving attribution.

f. Regardless of treatment, children with greater mathematical ability respond better to training.

Abstractor's Comments

Mathematics learning was not the focus of this research; rather, mathematics learning was used as a vehicle to study self-efficacy theory. The context of the study is more psychological than content-specific. As such, the results have implications for achievement in general.

This research report was well-organized and well-written. I was impressed with the way the author articulated the relationships between theory or past research and judgments made about design, implementation procedures, and interpreting results. However, there are a number of points or issues I would like to address relative to this work.

1. Self-efficacy is enhanced by information conveyed through such different treatment modalities as actual performance, modeling, and systematic desensitization. The treatments in this study included actual performance and modeling but not systematic desensitization. In working with children who display low arithmetic achievement, persistence, and self-confidence, one would predict that systematic desensitization would be a more effective treatment component than modeling. How does cognitive modeling address mathematics anxiety?
2. The results indicate there is not a significant effect for effort attribution across treatments. This finding might be related to locus of control. Children who believe that the locus of control is external might not be affected as much by effort attribution as those children who believe that locus of control is internal. Locus of control issues were not examined by the investigator.
3. It is stated that the children experiencing cognitive modeling out-performed those experiencing the didactic treatment on the division posttest. While the means of the cognitive modeling groups were higher than the means achieved by the didactic groups, it is not clear how the investigator directly tested this hypothesis.
4. Given the age of the subjects participating in this study and their mathematics ability, I am not sure didactic instruction as defined here is an appropriate treatment. I must be wrong! I was surprised that the subjects instructed under either treatment condition performed as well as they did on the division posttest, given instructional time. It might be the case that individual instruction is efficacious in enhancing children's self-perceptions of their capabilities to do division.
5. Although the specific analysis of generalization scores were not presented, the investigator could have briefly addressed these results in the discussion section. Generalization results would be of interest to mathematics educators.

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Abstract and comments prepared for I.M.E. by PAUL C. BURNS, The University of Tennessee, Knoxville.

1. Purpose

The purpose of this study was to investigate the differences between four class sizes (16, 23, 30, and 37) and their effects on:

a. teacher expectations;

b. the attitudes and opinions of participants (students and teachers);

c. student achievement in reading, mathematics, composition, and art;

d. student self-concept; and

e. a variety of classroom process variables (teacher-pupil participation, pupil participation, and method of instruction).

2. Rationale

The contextual framework within which this investigation was conducted includes these aspects of previous research:

a. The issue of class size has long attracted the interest of the educational community, as illustrated by the Glass and Smith (1979) report.

b. Literature relating to class size has been interpreted in various ways.

3. Research Design and Procedures

Sixty-two classes of students in the fourth and fifth grades from 11 schools in three school districts in Metropolitan Toronto participated in this study. Teachers had at least two years of teaching experience and had expressed a willingness to participate. Students from all socioeconomic levels were included in the sample, but there was a higher proportion of students from the lower socioeconomic categories (52 percent) in the sample than represented in the total elementary student population (44 percent).

In the first year of the study, fourth-grade teachers and students were randomly assigned to classes four sizes (16, 23, 30, or 37 students). The student assignments were stratified by sex and by ratings of academic performance. For the second year, the same teachers and students were assigned in a like manner to Grade 5 classes, with the conditions that students not be in a class size of 16 or 37 for both years of the study and that teachers who taught classes of the two larger sizes would instruct classes of the two smaller sizes, and vice versa.

Each year of the study, opinions and attitudes of teacher and student participants were collected. Standardized achievement tests, a self-concept scale, and an art and composition measure were administered to students. Observations of classroom process variables were made.

One teacher questionnaire was administered prior to their being informed of their assigned class size. This questionnaire obtained background information about the teachers and their expectations for each of the proposed class sizes. A second questionnaire surveyed teachers' opinions toward their assigned class size each of the two years. Two forms of a semantic differential scale were completed by participating teachers relating to "My Classroom" and "The Pupils I Teach". Questionnaires measured students' attitudes toward specific subjects of instruction, the classroom environment, their contact with teachers and peers, and their general satisfaction in school. A semantic differential used with pupils described "My Classroom".

Four subtests of the Canadian Tests of Basic Skills were administered: vocabulary, reading comprehension, mathematics concepts, and mathematics problem solving. The New York Self-Concept Inventory measured pupils' academic self-concept. Samples of students' art were collected and these samples were rated on a developmental scale. Students' compositions on the topics "Dreams" (first year) and "Wishes" (second year) were assessed by a five-point rating scale.

Observations of classroom process variables were made with the Toronto Classroom Observation Schedule (TCOS). The following variables were investigated with the TCOS: (1) teacher-pupil interaction; (2) pupil participation; (3) pupil satisfaction; (4) method of instruction; (5) subject emphasis; (6) physical conditions; (7) use of educational aids; and (8) classroom atmosphere. For both years of the study, trained observers used the TCOS for eight half-day visits to each participating classroom. In addition, the observers used an instrument called "Indicators of Quality" during five 20-minute visits to each class. The Indicators of Quality checked four aspects of classroom activity: (a) individualization; (b) interpersonal regard; (c) creative expression; and (d) group activity.

The study generated data from 16 classes with class sizes 16 and 23, and 15 classes each of class sizes 30 and 37. Differences were assessed by a one-way analysis of variance with the class serving as a unit of analysis. For the student data, the variability due to year of the study and teacher was first removed using a multiple linear regression technique and an analysis of variance was performed using the "residuals". For the observational data, means of each variable were first compared with proportions tests; then a one-way analysis of variance by class size was conducted. When a significant difference was found, pseudovalues were calculated and a similar analysis of variance was performed on the pseudovalues. If an analysis of variance of either type of data resulted in a significant overall effect due to class size, paired contrasts or range tests were conducted.

4. Findings

The following results were reported relative to the various components of the study:

A. Teachers' Affective Measures

1) Prior to the study, teachers' responses showed that 94 percent of the positive expectations were directed toward the smaller classes, and 91 percent of the negative expectations were toward the larger classes. Following the study, it was noted that teachers' opinions matched their expectations. (That is, teachers expected that individualization would be greater in smaller classes and they restated that this practice had occurred.) Teachers who went from a "large" to a "small" class size were significantly ($p < .001$) more likely to report that they liked the smaller class size and reported a higher personal energy level. They also believed that pupils contributed more, paid better attention, and were more satisfied with the smaller classes.

2) On the semantic differential ratings, teachers in class size 16 rated "My Classroom" significantly more positively ($p < .01$) than those in class sizes 30 and 37.

B. Observation of Classroom Process Variables

1) There were numerous variables on the Classroom Observation Schedule unaffected by class size.

2) Significant differences between class sizes were detected in the following variables ($p < .05$): proportion of pupils addressed as individuals; lecture by teacher; supervision by teacher while pupils were working; and proportion of written aids used.

3) There were no significant effects of class size on the "Indicators of Quality" scores.

C. Student Affective Measures

1) There was no significant difference due to class size on the Attitudes Toward School Scale, the Semantic Differential Scale, or the Self-Concept Scale.

D. Student Achievement Measures

1) There were no significant differences attributable to class size for art, composition, vocabulary, reading, and mathematics problem solving. For mathematics concepts there was a significant overall effect due to class size, favoring the class size 16 ($p < .05$).

5. Interpretations

The following interpretations are pointed out by the investigators:

a. Manipulating class sizes experimentally resulted in few changes in classroom functioning in the fourth and fifth grades.

b. Of the dependent variables examined, the ones that tended to show differences due to class size were teachers' opinions and attitudes. Teachers believed the smaller class sizes to have many advantages over the larger classes, especially in terms of possible individualization. Teachers reported they made changes to adjust to the different class sizes, but these perceptions did not receive much support from the observational and student outcome data.

c. Teachers did not alter the proportion of their time spent interacting with the whole class, with groups, or with individual pupils.

d. Individual pupils were addressed more frequently by teachers in the small classes, but there were no corresponding differences in the total amount of time the teachers spent talking to individuals. It seemed that pupils in the smaller class sizes had more individual interactions with their teachers because a constant amount of time for individual interactions was being distributed among fewer pupils.

e. Observational data indicated virtually no changes in methods of instruction used by teachers in the different class sizes. The investigators quoted other researchers in stating that teachers generally do not take advantage of the opportunity afforded by small classes to individualize their instructional procedures; a considerable amount of instruction in small classes is still whole-class oriented.

f. Standardized measures for students' academic achievement showed a significant class size effect only for mathematics concepts; students in class size 16 had higher scores than their peers in class sizes 30 and 37. There was no significant difference in other academic achievement. The researchers state that the argument that performance in endeavors such as art or composition would be more sensitive to class size effects than the other achievement areas was not supported.

g. There were no class size effects for students' attitudes toward school and for their self-concepts.

h. Changing class size did not result in any observed differences in pupils' participation in classroom tasks.

Abstractor's Comments

The impact of class size on achievement in mathematics is a topic worthy of consideration, and other related variables such as were explored in this study should be examined. Research in the area of class size has produced conflicting and varied results.

The investigators describe clearly their purposes, rationale, research design and procedures, and findings. Readers may likely wish for a more straight-forward statement of the hypotheses they were testing. Also, a further word of explanation about assigning students by "ratings of academic performance" might have been helpful. The information provided about the instruments used is appreciated. The discussion closely followed the findings of the study. Some more straight-forwardly stated conclusions from the study might have been helpful.

A few implications could have been suggested and a few specific suggestions would be most helpful to classroom teachers. The investigators do compare their results with other researchers, particularly when pointing out some questionable parts of the Glass and Smith analysis. We are left with a need for further research which attempts to manipulate experimentally instructional procedures for different class sizes. The investigators suggest this when they state that "class size could be appropriately altered in different situations by redistributing students and time and by changing instructional techniques."

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Abstract and comments prepared for I.M.E. by ROBERT B. ASHLOCK, RTS Graduate School of Education, Jackson, Mississippi.

1. Purpose

To probe the validity of the assumption that horizontal number sentences are pedagogically desirable in helping children master the skills of problem solving. Questions were raised as children's responses were examined.

2. Rationale

The assumption that horizontal number sentences can be used to help children master the skills of problem solving is widely accepted. But Weaver (1976) wonders if some of our instruction with young children is too mathematically sophisticated for them. Other researchers point to increasing evidence that difficulty with missing addend subtraction is a developmental one.

3. Research Design and Procedures

Three subtraction verbal problems were presented to each of 502 mid-year third-grade children in 24 classrooms in Montgomery County, Maryland. For each problem, children were asked to write a number sentence and then solve the problem. Verbal problems had either a missing addend or a comparison interpretation. The first contained a basic fact, the second involved a two-digit problem with no regrouping, and the third required regrouping.

Children's answers were classified as "correct" or "incorrect", while number sentences were classified as "appropriate canonical", "appropriate noncanonical", or "inappropriate". Appropriate canonical sentences contained the sign of the operation to be used in computing the answer, while appropriate noncanonical sentences needed to be transformed before they could be computed. Inappropriate number sentences did not model the problem.

4. Findings

Rates of occurrence reported for different response categories include the data in Table 1.

Table 1

Rates of Occurrence in

Different Response Categories

	Problem 1	Problem 2	Problem 3
Percentage of Appropriate Canonical Sentences	43	48	45
Percentage of Appropriate Noncanonical Sentences	33	30	16
Percentage of Students Writing Inappropriate Sentences	24	22	39

Student responses for Problem 1 are reported in detail. Within each response category, the number of responses is reported for each different number sentence that was written. Children's responses for Problem 2 and Problem 3 are not reported, but are said to be similar to responses for Problem 1.

Examination of responses raised the following questions regarding use of number sentences by young children for problem solving:

Question 1: Do children use the horizontal number sentence in solving problems?

Many of those students who were able to find a solution were not able to construct a number sentence for the problem.

Question 2: Does the horizontal number sentence encourage an incorrect statement of the problem?

Inappropriate number sentences were written as follows: 24% for the first problem, 22% for the second, and 39% for the third.

Question 3: Does the horizontal number sentence mislead the student about the operation to be used to solve the problem?

A large number of students apparently thought of the problems as addition problems, and wrote appropriate noncanonical sentences that were <u>not</u> transformed into subtraction sentences.

Question 4: Does the horizontal number sentence mislead students about the answer to the problem?

One-third of the students who wrote a correct addition sentence (appropriate noncanonical) for the first problem were not able to identify the answer, though they had already found it. Their answer "occurs as the number following the equality symbol in the horizontal number sentence." Also, a large number of students did not use the place-holder notation, which seemed to contribute to the difficulty they experienced in locating the answer in their completed number sentence.

Question 5: Does the student translate the appropriate noncanonical statement of the problem into a canonical or computational format?

Children had greatest difficulty in translating noncanonical sentences into forms leading to solution. While 69% of the students who wrote the third problem in canonical form solved it correctly, only 40% of those writing the noncanonical form did so. Weaver (1976) also found these transformations to be very difficult for third-grade pupils. Furthermore, other researchers indicate that first and second graders have difficulty translating number sentences into the equivalent forms needed to solve the problems.

Question 6: Does the horizontal number sentence encourage computational errors?

Children had difficulty subtracting two-digit numbers presented in the horizontal format.

Question 7: What is the effect on a child's confidence in solving verbal problems when the work he does to solve a problem produces an answer which is consistent with his intuitive solution?

It was distressing to note that correct answers acquired through informal procedures were not always supported by the number sentences that had supposedly produced them. Children would arrive at an answer and supply it in their number sentence, but not recognize it as the solution to the problem.

5. Interpretations

The author questions the pedagogical desirability of having youngsters use horizontal number sentences in early work with verbal problems, as did Weaver (1976). She also notes that there is evidence that difficulty with missing addend subtraction is a developmental one for children. Questions for further research that are raised include:

- Should we require that all noncanonical forms be translated into canonical or computational format?
- Should we require that the canonical form of the sentence be constructed from the beginning?
- Should we require the vertical format for verbal problems as a first step in the symbolization of the problem?

Abstractor's Comments

The use of horizontal number sentences with children in the primary grades certainly needs further study, and the author's probes are welcome. When reading the report, we should remember that she is probing. Her questions were raised by the data; the data were not collected to investigate the questions empirically.

The exploration provides further evidence that many children merely "push symbols around", they do not use or respond to written number sentences meaningfully. However, the investigation itself would have been cleaner if all three problems were missing addend problems, for difficulty of computation has been confounded with type of problem.

It is reported that children had difficulty subtracting two-digit numbers presented in the horizontal format, but the report does not make it at all clear that children understood they could use conventional algorithms for the actual computation. If children thought they had to do subtraction with regrouping by merely looking at a number sentence, another variable was introduced.

Question 7 and the discussion which follows are confusing as they appear in the report. Perhaps what is intended is the effect of solutions which are inconsistent with intuitive solutions.

Both in the rationale and in the conclusion of the study, there are references to increasing evidence that difficulty with missing addend subtraction is a developmental difficulty, but in her discussion the author does not specify how what she is observing may be developmental in nature.

Studies of academic achievement by young children often assume that whatever most children find easiest to do at a point in time should be taught first. What children have actually learned, and how this interacts with the tasks at hand, are not examined, nor are long-term effects.

If it can be assumed that children in the study had learned to think of addition as "putting together" and subtraction as "taking away", then it is not surprising that children found it difficult to assign subtraction to putting together situations.

It is also true that large numbers of children actually learn to think of equals as "results in an answer of ___." Therefore, it is not surprising that children writing noncanonical statements had difficulty locating the answer in their own statements.

It is encouraging to see highlighted the difficulty children encounter with transformations. The reviewer is reminded of Wilson's (1967) research with fourth-grade children in which children taught to write the canonical form from the beginning (wanted-given approach) had greater achievement than those who were taught to write appropriate noncanonical statements and then transform them so they could compute (action-sequence approach). In the Wilson study, able students were actually confused by being required to write noncanonical number sentences.

The questions posed for further research at the close of the report are a fruitful product of the study.

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