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ABSTRACT

One of five volumes intended to help teachers of mainstreamed handicapped students, the book presents twelve papers on teaching mathematics. Eight papers address instructional techniques for basic skills and problem solving: "Modalities--One Technique to Mainstream in Mathematics Instruction" (G. Rossi); "Computation Errors--Are We Treating A Symptom and Not the Cause?" (J. Webb, L. Webb); "An Individualized Approach for Low-Achieving Labelled and Nonlabelled Junior High Mathematics Students--A Longitudinal Report" (R. Uhl); "Nonmathematical Diagnostic Variables" (T. Denmark); "Mathematics Means Manipulatives--Teaching Number Concepts To Young Learning-Disabled Children" (M. [illegible]yers); "Teaching Mathematics to Visually Handicapped Students" (E. Binstock); "TIPS--Techniques in Planning for Handicapped Students in Regular Class Mathematics" (C. Thornton); and "Teaching Mathematics to LD Adolescents" (R. Riley, F. Reisman). Part Two contains four papers concerning mathematical tools for independent living: "Reverse Mainstreaming with Microcomputers in Mathematics" (B. Iossi); "The Student With Exceptional Education Needs and the Calculator" (K. Dietrich-Allen, H. Kepner, Jr.); "Banking Mathematics for the Classroom with EMH Pupils" (G. Rice); and "The Mechanics of Telling Time" (E. Gramuska). (CL)

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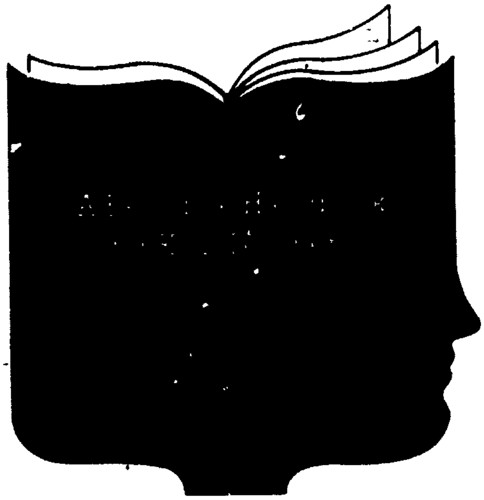
Ellen M Brockmann Editor

Teaching
Handicapped
Students
MATHEMATICS



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Note

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FOREWORD

Prepared by the

NEA Committee on Education of the Handicapped

Public Law 94-142, The Education for All Handicapped Children Act, the major federal education legislation for providing a free appropriate education for all handicapped children, must be in compliance with Section 504 of the Rehabilitation Act of 1973. Part D of Section 504 states, in part:

> The quality of the educational services provided to handicapped students must be equal to that of the services provided to nonhandicapped students, thus, handicapped students' teachers must be trained in the instruction of persons with the handicap in question and *appropriate materials and equipment must be available.*

This federal regulation is supported by NEA policy. Point (e) of NEA Resolution 79-32, Education for All Handicapped Children, reads:

> The appropriateness of educational methods, materials, and supportive services must be determined in cooperation with classroom teachers

In the context of federal education policy and NEA policy, members of the NEA Committee on Education of the Handicapped have reviewed *Teaching Handicapped Students Mathematics*. Members of the Committee are teachers of English, social studies, mathematics, special education, and science, who teach both general and handicapped students in elementary and high school

The Committee cannot emphasize too strongly the importance of teachers of regular and special education working together. The Committee would also like to urge both groups of educators to use these publications in teaching content areas to handicapped students. Members of the Committee were particularly pleased that teachers wrote these materials, in an effort to successfully teach the handicapped in the least restrictive environment, Because of their firsthand knowledge of proper teaching strategies, teachers are the best source of information to aid their colleagues.

The NEA supports P.L. 94-142 because the Association is committed to education processes which allow all students to become constructive, functioning members of their communities. To this end, when handicapped students are appropriately placed in classrooms with nonhandicapped students, teachers need instructional strategies which provide for individual learning differences. This is not new. However, most regular education teachers have not been trained, as mandated by law, in pre-service or in-service experiences to work with students with handicapping conditions. Teachers are eager to carry out the mandate of the law, but they may shy away from or even object to teaching these students because of this lack of training.

The so-called "mainstreamed" classroom presents new challenges to regular classroom teachers because of the added responsibility of teaching students with handicapping conditions. It is particularly important, therefore, to understand the student with a handicapping condition as a whole person in order to emphasize this commonality among all students.

NEA Committee on Education of the Handicapped

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EDITOR'S INTRODUCTION

As more handicapped children are being moved into regular mathematics classrooms, their need increases for mathematical materials. The task of developing appropriate materials seems enormous and may discourage some teachers from promptly accepting handicapped students into their classrooms and math laboratories. The primary purpose of this book is to give some suggestions and encouragement to teachers committed to helping the handicapped pupil.

In this book, we will simply "touch upon" some of the instructional techniques for basic skills and problem-solving and the mathematical tools for independent living that teachers throughout the country have found helpful. You will find many helpful points, but you the imaginative teacher must decide what will work best for your own classroom. As you work with handicapped children in your school, you will begin to see that as members of your class they have many more things in common than differences. Stressing the sameness of the individual sets the stage for the handicapped child becoming a bona fide member of the mathematics class.

The Editor

Ellen Mary Brockmann is a fourth grade teacher at the Washington Park School, Totowa, New Jersey Her selections for this book represent materials which she feels are especially pertinent and practical for mathematics classroom teachers

1. MODALITIES: ONE TECHNIQUE TO MAINSTREAM IN MATHEMATICS INSTRUCTION

by Geraldine Rossi

Geraldine Rossi points out that adapting instructional style to the modality preference of the student facilitates mathematics learning. The author discusses auditory, visual, and tactile kinesthetic modalities. She presents many instructional ideas to assist the teacher in meeting the math modality needs of the handicapped student. The author is an Associate Professor of Education at Salisbury State College, Maryland.

With the advent of PL 94-142, elementary school teachers will need to acquire new skills to mainstream children with special needs into their classrooms. One diagnostic skill needed relates to information processing and learning style. It is essential for children with special needs to process mathematics information efficiently. A teacher should be able to use certain formal assessment techniques or observe in an informal way the behaviors which indicate how each child processes stimuli while learning mathematics.

One phase of this processing involves how children prefer to receive stimuli and give back information. For example, some children appear to need a picture or diagram (visual stimuli) to understand a concept, when others can understand the same concept by listening to an audiotape describing that concept (auditory stimuli). Some children prefer to learn the multiplication tables using flash cards (visual stimuli). Some prefer singing along with a record on multiplication tables (auditory stimuli), and some want to form the multiplication facts using blocks or Cuisenaire rods (tactile-kinesthetic stimuli). When instruction is organized and presented according to preferences, children may attend to relevant stimuli or notice important cues in a mathematics lesson. In this manner, adapting instruction to fit needs will facilitate learning.

Children receive stimuli or input through their five sensory channels or modalities and from these same five modalities can give back information or output. The five sensory modalities are visual, auditory, tactile-kinesthetic, olfactory (smell), and gustatory (taste). In discussing modalities, preferred and weaker modalities are usually mentioned. The input channel through which a person readily processes stimuli is referred to as the preferred modality. The input channel through which a person less readily processes stimuli is referred to as the weaker modality. A child's preferred modality of input is not necessarily related to the strongest acuity channel. For example, a child with a hearing loss may find the auditory modality is the preferred modality for processing information. It is not always readily apparent to a teacher which modality is a child's preferred one at the outset.

A child's learning style can be described in terms of the preferred modality as suggested by the Maryland State Department of Education, Division of Instructional Television in their program, "Teaching Children with Special Needs." Martha H Hopkins (1978) also stressed the need to consider a child's learning style in terms of preferred modality. To give a complete and accurate diagnosis of a child's learning style, she maintains a teacher must be able to determine a child's preferred modality. As Hopkins relates, there is no formal means to diagnose modalities' strengths and weaknesses at this time. She has, however, developed a checklist form.

A research study, Daiyo Sawada and R I Jarman (1978), in the field of mathematics education suggests a relationship between modality matching and mathematics achievement. Sawada and Jarman conducted a research study with male fourth graders on information matching concerning auditory and visual sensory modalities. They analyzed the relationship of students' mathematics achievement. They found that for children with low intelligence, as measured by the Lorge-Thorndike Intelligence Test, auditory-auditory matching ability was a good predictor of their mathematics achievement. In other words, when children of low IQ (71-90) were presented a stimuli pattern of 100 cycle tones and had to select a comparison pattern of tones, they did as well on that task as they did on a mathematics achievement test produced by the Edmonton public school system. When a child of low IQ scored low on this achievement test, the child did not do well on the auditory-auditory matching task They also found that the mathematics achievement of high IQ (111-130) children seemed to be uniformly dependent on all four modality matching abilities, input auditory-output auditory, auditory-visual, visual-auditory, and visual-visual This uniform dependence seemed to all but vanish with children of medium IQ (91-110) Thus the relationships appear to change with changing IQ levels a finding that suggests modality matching is a good candidate for use in making decisions concerning individualizing mathematics

Every child would benefit from instruction stressing a variety of modalities The modalities that a classroom teacher would be most concerned about would be auditory, visual, and tactile-kinesthetic Tables 1, 2, and 3 should help the teacher become more acquainted with the diversity of possible assessment techniques and instructional strategies Their format and some of their material have been adapted from a model developed by the Maryland State Department of Education, Division of Instructional Television

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TABLE 1
THE AUDITORY MODALITY

POSSIBLE BEHAVIORS		POSSIBLE TECHNIQUES		
Pupil who MAY BE strong auditorily will		The teacher may utilize these		
Show the Following Strengths	**Show the Following Weaknesses**	**Formal Assessment Technique**	**Informal Assessment Technique**	**Instructional Technique**
Follows oral instructions very easily [4]	Loses place in visual activities [4]	Present statement verbally, ask pupil to repeat [4]	Observe pupil reading with the use of finger or pencil as a marker [4]	Provide audio tapes of story problems. Verbally explain arithmetic processes as well as demonstrate. Use oral story problems [5]
Appears brighter than tests show pupil to be [4]	Ordinal numbers- Where is start position?[5]	Tap auditory pattern beyond pupil's point of vision. Ask pupil to repeat pattern [4]	Observe whether pupil whispers or barely produces sounds to correspond to pupil's reading task [4]	Use mental arithmetic strategies [5]
Performs well verbally [4]	Writes E for 3, 3 for 8, 2 for 5, or 6 for 9 [5]	Provide pupil with several words in a rhyming family. Ask pupil to repeat pattern [4]	Observe pupil who has difficulty following purely visual directions [4]	Utilize work sheets with large unhampered areas [4]
Can orally repeat a sequence of numbers or a sentence [5]	May not be able to learn sets or groupings (closure or figure-ground) [3]	Provide pupil with several words in a rhyming family. Ask pupil to add more [4]	Observe during "travel" use an oral version [5]	Use lined wide spaced paper [4]
	Makes visual discrimination errors [4]	Present pupil with sounds produced out of pupil's field of vision. Ask pupil if they are the same or different[4] (such as seven, eleven, nine, ninety)[5]		Allow for verbal rather than written responses [4]
	Has difficulty with written work, poor motor skill [4]			Tape record important parts of the lesson for review. Use some oral testing [5]
	May not be able to discriminate differences or similarities in size and shape [6]			Use visual discrimination activities [6]
	May have difficulty in relating size of an object to the appropriate container [2]			Use tile to make number sentences and orally read [1] $\boxed{2}$ $\boxed{+}$ $\boxed{3}$ $\boxed{}$ $\boxed{5}$
	Cannot visually place numbers in a sequence as instructed [5]			Use tangrams and geoboard in visual discrimination activities [6]
				Use buzzer board, stick to clap out operations [6] 2 + 3 = 5 (2 claps) (3 claps) (5 claps)
				Use song or poems to aid retention, stress rhythmic counting [4]

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[1] Davidson Films, Inc.
[2] Gibson (1977)
[3] Mann and Suiter (1975)
[4] Maryland State Department of Education (1973)
[5] Salisbury State College Students and Rossi (1978)
[6] Simpson-Cahill and Pulsford (1979)

TABLE 2
THE VISUAL MODALITY

POSSIBLE BEHAVIORS		POSSIBLE TECHNIQUES		
Pupil who MAY BE strong visually will		The teacher may utilize these		
Show the Following Strengths	**Show the Following Weaknesses**	**Formal Assessment Technique**	**Informal Assessment Technique**	**Instructional Technique**
Skims reading material[4]	Has difficulty with oral directions[4]	Give lists of words which sound alike. Ask pupil to indicate if they are the same or different[4]	Observe pupil in tasks requiring sound discrimination, i.e., seven, eleven[5]	Force to focus on missing numbers or operations[1] 3 ___ 4 = 7
Reads well from picture clues[4]	Asks "What are we supposed to do" immediately after oral instructions are given[4]			Trace new materials[1]
Follows visual diagrams and other visual instructions well[4]		Ask pupil to follow specific instructions. Begin with one direction and continue with multiple instructions[4]	Observe to determine if the pupil performs better when the pupil can see the stimulus[4]	Show examples of arithmetic functions[4]
Scores well on group tests[4]	Appears confused with great deal of auditory stimuli[4]			Flowchart algorithms[5]
		Show pupil visually similar pictures. Ask the pupil to indicate whether they are the same or different,[4] 3 E or 6, 9[5]	Observe pupil's eye movement during lesson[6]	Allow a pupil with strong auditory skills to act as another child's partner[1]
Performs nonverbal tasks well[4]	Has difficulty discriminating between words with similar sounds[4]		Observe pupils write and say number sentences[5]	
				Allow for written rather than verbal responses. Use overhead projector and films[5]
Does well with flash cards[5]	Has difficulty in retaining an auditory sequence of numbers (memory sequence[3]	Show a pupil a visual pattern, i.e. block design or pegboard design. Ask pupil to duplicate[4]		
Understands blackboard explanations[5]				Use multiple choice questions on a test[5,6]
	Has difficulty with story problems that require mental arithmetic rapid oral drills[3]			Illustrate basic concepts with slides using pupils from class[2]
				In solving problems allow them to draw pictures[5]
				Keep oral math to a minimum, avoid oral "math bees"[6]
				Stress pictures and diagrams when explaining concepts, operations and generalizations[5]

[1] Davidson Films, Inc

[2] Gibson (1977)

[3] Mann and Suiter (1975)

[4] Maryland State Department of Education (1973)

[5] Salisbury State College Students and Rossi (1978)

[6] Simpson-Cahill and Pulsford (1979)

TABLE 3
THE TACTILE KINESTHETIC MODALITY

POSSIBLE BEHAVIORS		POSSIBLE TECHNIQUES		
Pupil who MAY BE strong tactile-kinesthetically will		The teacher may utilize these		
Show the Following Strengths	**Show the Following Weaknesses**	**Formal Assessment Technique**	**Informal Assessment Technique**	**Instructional Technique**
Exhibits good fine and gross motor balance [4]	Depends on the "guiding" modality or preferred modality since tactile-kinesthetic is usually a secondary modality [4]	Ask pupil to walk balance beam or along a painted line [4]	Observe pupil in maneuvering in classroom space [4]	Utilize manipulative objects in performing the arithmetic function, provide buttons, packages of sticks felt numbers, etc [4]
Exhibits good rhythmic movements [4]	Weaknesses may be in either the visual or auditory mode [4]	Set up obstacle course involving gross motor manipulation [4]	Observe pupil's spacing of written work on a paper [4]	Have pupil write the exercise in large movements, i e in air, on chalkboard, on newsprint, utilize manipulative numbers to write a problem [5]
Demonstrates neat handwriting skills [4]		Have pupil cut along straight, angles and curved lines [4]	Observe pupil's selection of activities during free play, i e , does pupil select puzzles or blocks as opposed to records or picture books [4]	Call pupil's attention to the feel of the numbers [5]
Manipulates puzzles and other materials well [4]		Ask child to color fine areas [4]	Observe pupil using objects to solve problems [5]	Stress sand paper numbers, rope numbers cuisenaire rods, geoboards, most mathematics manipulative aids [5]
Identifies and matches objects easily [4]				
Counts well with fingers [5]				

[1] Davidson Films, Inc
[2] Gibson (1977)
[3] Mann and Suiter (1975)
[4] Maryland State Department of Education (1973)
[5] Salisbury State College Students and Rossi (1978)
[6] Simpson-Cahill and Fulsford (1979)

2. COMPUTATION ERRORS: ARE WE TREATING A SYMPTOM AND NOT THE CAUSE?

by June R. and Leland F. Webb

The Webbs advocate direct classroom mathematics instruction so that handicapped students can increase their confidence as independent learners. The Webbs also discuss the general computation problems a teacher might find in a regular mathematics classroom. Their thesis is that the classroom teacher must become a student of how the handicapped learner perceives the mathematical rules, before offering a remediation plan. The authors are both at California State College, Bakersfield. June Webb is Associate Professor of Special Education, and Leland Webb is Professor of Mathematics and Mathematics Education.

THE HISTORICAL INSTRUCTIONAL CONTROVERSY IN SPECIAL EDUCATION

Since the late sixties there has been a growing controversy in Special Education between (1) basic process instruction and (2) direct academic skill instruction in the skill areas of mathematics and reading (Lerner, 1976). To individuals outside the professional area of Special Education, this may be a completely unfamiliar controversy.

Basic process instruction is a major historical development in Special Education. Basic process instruction states that if a student is having problems in learning the complex academic skills of reading and mathematics, the cause may be deficits in basic learning processes such as visual figure-ground discrimination, visual imagery, auditory memory, auditory sequencing, *etc.*, (Lerner, 1976). As a result of this cause-and-effect theory between deficits in basic learning processes and problems in learning complex academic skills, many clinicians in Special Education began developing elaborate remedial programs to help students learn and practice basic learning processes rather than spending time on direct instruction. The conclusion of the basic-process instruction theory is that if the student's basic learning process deficits are remediated through these systematic programs, then the student's difficulty in learning the more complex academic skills of reading and mathematics will be eliminated or at least reduced.

Starting in the late sixties reports of systematically controlled research studies to question the cause-and-effect relationship between remediating basic learning processes and improving learning in complex academic subjects (Wiederholt, Hammill, & Brown, 1978). The positive results of individual students which were obtained clinically could just as likely be attributed to a number of indirect factors. The two most powerful alternative explanations were (1) that the success during the easier basic-process instruction had positive effects on the students' attitudes toward school and learning, and (2) that the students were older intellectually and neurologically after a period of basic process instruction and, therefore, were at a higher state of readiness to learn the complex academic skills.

On the other side of the controversy, authors have written about the necessity of direct instruction in complex academic skills, which is the second theory of instruction referred to at the beginning of this chapter. A major strategy of direct instruction with handicapped students is task analysis, which is a way to subdivide complex skills into smaller parts to make learning more manageable and individualized for a student with learning problems (Bateman, 1967 and 1974). The authors of this chapter advocate direct instruction for teaching academic skills. This position does not advocate totally abandoning basic-process instruction because such teaching emphasizes success and time for maturation. However, direct instruction in academic content areas must also be part of a balanced special education program. This chapter proposes to go beyond task analysis to an even more individualized teaching strategy which can help handicapped students learn basic mathematical computational skills and increase their confidence as independent learners.

TWO BASIC PREMISES ABOUT THE NATURE OF LEARNING

There are two basic premises about the nature of learning which underlie this strategy for teaching computational skills. First, all learners attempt to make the complexity of what they are learning more manageable by forming rules (Smith, 1975). In the case of a bright student these rules seem to develop easily, accurately, and in a systematically organized structure. However, for a student having learning problems the rules he or she forms may take a great deal of effort to formulate, be inaccurate in that they work sometimes but not other times, and be inadequately organized for usefulness to the learner. Second, a learner's confidence and willingness to be an independent learner is in direct proportion to the learner's confidence in his or her ability to form the necessary rules to manage the complexity of the learning task and to avoid being overwhelmed (Ginsberg, 1977) Remember, children are born into a world of "buzzing, blooming confusion" (William James), and their job intellectually is to create gradually a set of internal rules to understand the external complexity

THE LEARNING OF COMPUTATIONAL SKILLS AND DIAGNOSIS OF COMPUTATIONAL SKILL ERRORS

Keeping the above assumptions in mind, let' now limit our focus to the learning of computational skills Some students seem to acquire speed and accuracy in computation as easily as learning to breathe, while others painfully languish or drown in the sea of numbers It is this latter group that this chapter hopes to bring into better focus Here is an example of a computational skill recently demonstrated by an elementary school pupil

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 729 \\ +345 \\ \hline 91614 \end{array}$$

The answer is obviously wrong We would all mark the sum incorrect How many of us would investigate further? Do we help this student improve computational skills by giving more problems of the same type, in which to make the same error? Or is it in fact an error in the thinking of the pupil (i e , the ability to form rules)? This may not be a simple careless error but rather a conceptual one which goes much deeper, one which deals with a student's ability to understand the addition algorithm, the step-by-step procedure or rule for addition. Is there more than one error present? As a teacher, are you able to identify the conceptual procedure that the student may be using in the problem? How might you make sure you understand precisely what the student is doing? (See the answer at the end of the chapter, footnote one) What is the most accurate way to find out what the student is doing?

Learning computational skills is an important foundation skill for all students So when a student is having problems it is essential to be able to diagnose on an individual basis how each student's unique internal set of rules works to produce his or her pattern of computational errors Indeed, the student is the only expert on how he or she is thinking, and as teachers, our first job is to become a student of how this unique learner understands the world Armed with this diagnostic perspective, we are better able to plan a remedial program to help the student recognize incorrect rules and create correct ones, thereby increasing the student's confidence and independence in computation

Therefore, the unique component of this teaching strategy is that we, as teachers, must first become a student of how a troubled learner understands the world This diagnostic strategy is an active constructive process of accurately creating another person's perspective Our goal is to seek the troubled learner's confirmation that we have indeed accurately described how he or she is thinking To obtain a feel for this diagnostic process, active participation is essential Hence in the next portion of the article, active participation is required on the part of the reader

DIAGNOSTIC PROCESS ACTIVITIES

On the following pages you, the reader, will be asked to diagnose the patterns of students' computational errors in addition, subtraction multiplication, and division Not only will you be asked to diagnose the pattern of error, you will have the opportunity to analyze the computational error and suggest a strategy for remediation Remember active participation is a necessity for complete understanding of the entire process of diagnosing and remediating computational errors

Before starting, several guidelines need to be established

1 You will be given several challenges on which to work
2 Each challenge deals with an actual conceptual computational error made by students
3 Each challenge contains only one conceptual error You are to diagnose the one conceptual error in each challenge. When you work through the errors you will note that the student's rule for solving the problem often results in many correct answers In fact in many cases 50 percent or more of the answers will be correct even when using the wrong rule[1]
4 Following the challenges, "potential" answers to the components of diagnosing and remediating will be provided Your answers may be as valid as those provided, even though they may be different
5 Be sure to write down all your answers Participate actively by writing in the text or on a separate sheet of paper

Challenge Number 1 Identify this computational error pattern

1	64	2.	23	3	5	4	33	5	58
	+8		+71		+72		+46		+4
	18		94		14		79		17

Can you calculate the "answer" to make sure you understand the computational error pattern?

1.	52	2	20	3	79
	+7		+63		+3

Below, describe the computational problem as you perceive it. Exactly what do you think the student is doing?

Your result should be as follows

1	52	2	20	3	79
	+7		+63		+3
	14		83		19

The student is missing only those problems which have a two-digit and a one-digit number In these cases the digits are added together When two two-digit numbers are used, the addition appears correct, but none of the problems of this type require regrouping (carrying) It is possible that a problem in place value exists.

What strategies would you, as the teacher, employ to help the student? Describe one instructional activity which you feel would help correct this computational error

Two possible solutions follow. Yours may be different

1. Use manipulative materials such as popsicle sticks to show bundles of ten and single popsicle sticks Have the student collect the units and tens and record each category.
2. Use semiconcrete materials and draw a place value chart

Tens	Units (Ones)
5	2
+	7
5	9

T	U
2	0
+6	3
8	3

T	U
7	9
+	3
8	2

Also, it would be helpful to ask the student what he or she is doing and to explain the procedure Are the tens and ones identified as being different in value?

Challenge Number 2 Identify this computational error pattern

			2		4		25
1	261	2	35	3	467	4	493
	+328		+92		+389		+841
	589		19		7120		119

Can you calculate the "answer" to make sure you understand the computational error pattern?

1	444	2	779	3	242
	+325		+481		+589

Below, describe the computational problem as you perceive it Exactly what do you think the student is doing?

Your answer should be as follows

			38		2
1	444	2	779	3	242
	+325		+681		+589
	796		1118		7113

The student is reversing the usual algorithmic procedure, disregarding place value We read from left to right, so why not add that way! Addition is performed left to right and left digit is recorded when more than 1 digit results, with the right digit being carried Note that the procedures that do not involve carrying are correct; this sort of procedure in which some problems are correct and others are not might result in the teacher concluding that the student is merely careless

What strategies would you, as the teacher, employ to help the student? Describe an instructional activity which you feel would correct this pattern of error

Two possible solutions follow

1 Use a bank with coins or a game board to help the student understand place value.

H	H	T	U
	7	7	9
	6	8	1

TH=Thousands
H=Hundreds
T=Tens
U=Units (Ones)

A trading process using coins or base ten blocks can be employed in the process.

2. Estimate or approximate the sum before commencing the solution to the problem In problem 2, for instance, the sum is greater than 1300.

Challenge Number 3. Identify this computational error pattern.

$$1.\ \begin{array}{r} \overset{41}{5\cancel{6}4} \\ -126 \\ \hline 428 \end{array} \qquad 2\ \begin{array}{r} \overset{51}{\cancel{6}37} \\ -366 \\ \hline 271 \end{array} \qquad 3.\ \begin{array}{r} \overset{611}{\cancel{8}88} \\ -499 \\ \hline 299 \end{array} \qquad 4\ \begin{array}{r} \overset{411}{\cancel{6}32} \\ -499 \\ \hline 93 \end{array}$$

Can you calculate the "answer" to make sure you have found the computational error pattern?

$$1.\ \begin{array}{r} 784 \\ -394 \\ \hline \end{array} \qquad 2\ \begin{array}{r} 841 \\ -255 \\ \hline \end{array}$$

Below, describe the computational problem as you perceive it Exactly what do you think the student is doing?

Your results should be as follows.

$$1.\ \begin{array}{r} \overset{61}{\cancel{7}84} \\ -394 \\ \hline 390 \end{array} \qquad 2.\ \begin{array}{r} \overset{611}{\cancel{8}41} \\ -255 \\ \hline 496 \end{array}$$

Subtracting with regrouping or renaming once creates no problem, but where renaming more than once is introduced the student's algorithm is incorrect. It is a good thing that the "crutches" are shown, because it makes it easy to see what the problem is. Can you think of a strategy to help the student?

Several possible solutions follow:

1. Use a place value chart. Discuss what we need to trade in tens for ones, then record the action; then trade hundreds for tens, recording the action
2. Use base-10 blocks for a concrete representation of the problem, repeating what is in "1" above.
3. Use popsicle sticks.
4. Use an abacus.
5. Use money.
6. Use expanded notation:

$$\begin{array}{rcl} & & \qquad \overset{130}{} \\ 841 & = & \overset{700}{\cancel{800}} + \overset{\cancel{30}}{\cancel{40}} + \overset{11}{\cancel{1}} \\ -255 & - & 200 + 50 + 5 \\ \hline & & 500 + 80 + 6 = 586 \end{array}$$

Note Adding to check the problems where errors have been made will only verify the error in algorithm, so this should not be done.

Challenge Number 4. Identify this computational error pattern

$$1.\ \begin{array}{r} \overset{11}{633} \\ \times\ 5 \\ \hline 3165 \end{array} \qquad 2.\ \begin{array}{r} 411 \\ \times\ 65 \\ \hline 2465 \end{array} \qquad 3.\ \begin{array}{r} \overset{32}{\cancel{8}97} \\ \times\ 43 \\ \hline 3581 \end{array} \qquad 4.\ \begin{array}{r} \overset{1}{315} \\ \times 283 \\ \hline 695 \end{array}$$

Using this procedure, can you calculate the "answer"?

$$1\ \begin{array}{r} 492 \\ \times\ 44 \\ \hline \end{array} \qquad 2\ \begin{array}{r} 973 \\ \times 617 \\ \hline \end{array}$$

Below, describe the computational problem as you perceive it:

Your results should be as follows.

$$1.\ \begin{array}{r} \overset{3}{492} \\ \times\ 44 \\ \hline 4968 \end{array} \qquad 2\ \begin{array}{r} \overset{2}{973} \\ \times 617 \\ \hline 5491 \end{array}$$

This is a complicated process the student has developed. It is a combination of the addition algorithm and the multiplication algorithm. Each column is considered as a separate multiplication. When the multiplier has fewer digits than the multiplicand, the left most digit of the multiplier continues to be used.

What strategies would you, as the teacher, employ to help the student? Describe at least one instructional activity which you feel would help correct this pattern of error.

Here are several solutions.

1. Use the distributive property by rewriting the problem into two problems:

$$\begin{array}{r} 492 \\ \times\ 44 \\ \hline \end{array} \rightarrow \begin{array}{r} 492 \\ \times\ 40 \\ \hline ? \end{array} + \begin{array}{r} 492 \\ \times\ 4 \\ \hline ? \end{array}$$

2 If this abstract example is not clear, use the geometric method which is a semiconcrete method

$$\begin{array}{r|c|} & 492 \\ \hline 40 & (492 \times 40) \\ + & \\ \hline 4 & (492 \times 4) \\ \hline \end{array}$$

3 Mask the multiplier's digits so only one digit is showing at a time Complete the multiplication as a partial product

$$\begin{array}{r} 973 \\ \underline{\times 617} \\ 6811 \\ 9730 \\ \underline{403800} \\ 420341 \end{array} \qquad \begin{array}{r} 973 \\ \underline{\times 7} \\ 6811 \end{array} \quad + \quad \begin{array}{r} 973 \\ \underline{\times 10} \\ 9730 \end{array} \quad + \quad \begin{array}{r} 973 \\ \underline{\times 600} \\ 403800 \end{array}$$

If the student still does not understand, go back to addition and subtraction to see if the student understands those computational processes Then try some simpler multiplication problems

Challenge Number 5 Identify this computational error pattern

$$1 \quad \begin{array}{r} 342 \\ 2\overline{)684} \end{array} \qquad 2 \quad \begin{array}{r} 322 \\ 3\overline{)176} \end{array} \qquad 3 \quad \begin{array}{r} 212 \\ 4\overline{)942} \end{array}$$

Can you calculate the "answer"?

$$1 \quad 2\overline{)276} \qquad 2 \quad 3\overline{)429}$$

Below, describe the problem as you perceive it

Your results should be as follows

$$1 \quad \begin{array}{r} 133 \\ 2\overline{)276} \end{array} \qquad 2. \quad \begin{array}{r} 113 \\ 3\overline{)429} \end{array}$$

Even incorrect algorithms sometimes produce correct answers The student is ignoring place value and treating each digit as a "ones" In addition, the student is merely dividing the smaller digit into the larger, as well as ignoring the remainder. Notice, also, that no work is shown

What strategies would you, as the teacher, employ to help the student? Describe at least one instructional activity which you feel would help the student

Two possible solutions are

1 Teach the student, with objects, to keep a step-by-step record of the division process One could use base-10 blocks Stern blocks, Cuisenaire rods, or bundles of sticks Complete a simpler problem such as 54 ÷ 3 = ? While doing the problem with manipulatives write down the algorithm abstractly

$$1 \quad \begin{array}{r} 1 \\ 3\overline{)54} \\ \underline{3} \\ 2 \end{array} \qquad 2 \quad \begin{array}{r} 1 \\ 3\overline{)54} \\ \underline{3} \\ 24 \end{array} \qquad 3 \quad \begin{array}{r} 18 \\ 3\overline{)54} \\ \underline{3} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

2 Use the scaffolding or estimating quotients For example

$$\begin{array}{r|l} 4\overline{)77} & \\ \underline{40} & 10 \\ 37 & \\ \underline{36} & \underline{9} \\ 1 & 19 \end{array} \qquad \begin{array}{r} 19\text{ R}1 \\ 9 \\ 10 \\ 4\overline{)77} \\ \underline{40} \\ 37 \\ \underline{36} \\ 1 \end{array}$$

The five challenges you have worked through are but a few examples of the myriad of types of conceptual computational errors that students make Each of the above problems is an actual student error As mentioned these five challenges are but the tip of the iceberg Perhaps some of your students have made some of these types of conceptual errors For an excellent presentation of additional computational errors, the reader is invited to read Ashlock (1976)

The purpose of this chapter is to attempt to sensitize the reader to the fact that the acquisition of computational skills by students requires that teachers see students' computational errors not as random or careless, but as a rich resource material for diagnosing the students' incorrect conceptual rules With this individualized and diagnostic understanding of the troubled learner, the teacher can begin direct remedial instruction to help the student increase computational accuracy and speed

The above challenges all resulted from a student conceptually misusing a given algorithm or rule. This type of error is the most common mistake that students make. More drill will not correct it. Diagnosing and converting conceptual errors is a task not only for elementary school teachers, but also for junior and senior high teachers, as more and more secondary school teachers are teaching remedial mathematics classes to students who make conceptual errors.

SPECIFIC STEPS IN DIAGNOSIS AND REMEDIATION

Specific steps in the diagnosis of computational skills will help teachers ascertain each student's unique level of understanding. First, the teacher can start with a clear, honest explanation to the student, such as "In order to help you learn subtraction, I need to understand first how you are thinking when you work the subtraction problems. I need you to teach me how you are thinking. You are the teacher and the best expert of how you are thinking, and I am your student." In the next step analyze the student's written problems (as you have already experienced) and interview the student to complete the investigative diagnosis. During the interview you need to listen carefully in order to reconstruct the student's rules. Try not to teach during this diagnostic phase. You must get confirmation from the student that you accurately understand how the student thinks when working on the problem, in this case subtraction, before going on to the remediation phase. This diagnostic phase, if well done, demonstrates that you value the student enough to invest your genuine attention and that you believe in the student's ability to formulate increasingly more accurate rules to learn computational skills.

The remedial phase also has two steps. First, the teacher needs to help the student see how the student's own rule may work some of the time but not all of the time. Both the sense of trust developed in the diagnostic phase and the support of the teacher will help the student give up the old rules and experiment with new ones. The second and essential step of the remedial phase is the structuring of activities for the student to "overlearn" the new rules so that the student can become increasingly independent and self-confident in thinking and solving problems.

THE SPECIAL SIGNIFICANCE OF THE DIAGNOSTIC AND REMEDIAL PROCESS FOR HANDICAPPED STUDENTS

This diagnostic and remedial strategy is important for all learners. However, handicapped learners are in a double bind when they make repeated conceptual errors in computation. This double bind comes from the fact that they not only suffer the consequences of conceptual errors but are also more likely to make conceptual errors in the first place.

The first reason for these increased conceptual errors is that a significant proportion of handicapped students are characterized by the hyperactivity syndrome. This syndrome includes short attention span, talking excessively, being argumentative with friends, siblings, and classmates, impulsive and driven motor behaviors such as fidgeting and roaming, easily distractable by external sensory stimulation, and impulsive shifts of attention from one idea and interest to another (Lerner, 1976). As a result of a combination of these characteristics, many handicapped children will tend to create bizarre or incomplete rules even when provided with effective instruction. Second, the double bind is further reinforced if a rule is at least partially successful. The student will not take the initiative to find another rule unless a supportive teacher is there to help. There is still a third facter which tends to reinforce the double bind. When repeatedly told they are wrong, these students increasingly lose their initiative to formulate learning rules and trust their own thinking. Hence, in the future the troubled learner will simply assume a passive and random set of reactions, believing that learning is too overwhelming to cope with. It is not unusual for handicapped students to refuse to attempt a computation problem unless the teacher is sitting nearby and confirming the accuracy of each small step before proceeding further.

The double bind is a repeating negative cycle for handicapped learners. The key to reverse this negative cycle is not repeated drill but, rather, individual diagnosis to learn the current inadequate rules of the troubled learner and remediation to help the learner create new, accurate rules. As the learner becomes more self-confident in thinking and in creating concepts, the learner starts to generate the positive repeating cycles of success.

A FINAL EMPHASIS

As a final emphasis to the reader to correct not only computational errors but also to diagnose and remediate them, the writers leave the reader with one more student challenge.

Diagnose and suggest strategies for remediation of the following conceptual error.

1	4	2	2	6	3
	9	3		12	6

3 $\frac{3}{8} = \frac{1}{4}$ 4 $\frac{2}{10} = \frac{1}{5}$

5 $\frac{3}{8} = \boxed{}$[2] 6 $\frac{4}{3} = \boxed{}$[3]

As teachers we want to help students learn If we treat conceptual errors in mathematics merely as careless student errors, we are treating only the symptoms As dedicated teachers we need to dig deeper to treat the cause and not just what appears to be a symptom The skills to both diagnose student errors and remediate defective computational rules are a dual goal to which all teachers should aspire It is hoped that this chapter has sensitized the reader to that end

Footnotes

[1] The student has place-value problems plus more The student is adding when the number on top is larger than on the bottom but sees a need to borrow when the number on top is not larger than the one on the bottom One wonders what the student would do with the following problem

$$\begin{array}{r} 258 \\ +572 \\ \hline \end{array}$$

The best strategy to determine what a student is thinking is to ask!

[2] $\frac{1}{3}$

[3] $\frac{2}{1}$ or 2

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3. AN INDIVIDUALIZED APPROACH FOR LOW-ACHIEVING LABELLED AND NONLABELLED JUNIOR HIGH MATHEMATICS STUDENTS: A LONGITUDINAL REPORT

by Robert A. Uhl

Robert Uhl interjects the variable of class size as an important element in the mainstreaming of children into junior high mathematics classes The author teaches mathematics at the P J. Jacobs Junior High School in Stevens Point, Wisconsin

Jarvis (1964) documented that the wide range of individual arithmetic differences per grade level increase from grade level to grade level while Kyte and Fornwalt (1967) showed that the rate of mastery and retention of mastery for arithmetic skills differs for each student. Traditionally, students were allowed a fixed amount of time to learn a particular unit or skill The result is a variation in the achievement level attained

Becher, Engelmann, and Thomas (1971) suggest etiology may be less important than the academic environment in which the student is placed Schools are designed to build successively year after year upon skills acquired by the student in previous years If at any point a student has not acquired the appropriate prerequisite skills, failure is likely The authors further state that a history of failure may promote expectations of failure which in turn make actual failure more likely The noncategorical approach submits that the organization of teaching is more important than the sorting of learners by labels (Gillespie, Miller, & Fiedler, 1975 Lilly, 1977).

The simplest and most versatile special academic environment which can replace failure with success is the tutorial method of instruction Bausell, Moody, and Walzl (1972) have demonstrated that one-to-one instruction results in greater mathematical learning than does classroom instruction. An experiment by Moody, Bausell, and Jenkins (1973) studied the effects of various student-teacher ratios (1:1, 2:1, 5:1, and 23:1) on students' learning The greatest loss in learning occurs as the instructional ratio changes from a tutorial setting of 1:1 to the smallest group setting of 2:1. Losses continue as the ratio increased. These studies strongly suggest the need for providing tutorial instruction (1:1) or the smallest group setting economically possible for educationally handicapped students who have fallen far behind their classmates.

A more economical means of maintaining a similar one-to-one approach with each student consists of individualized instruction However, the effectiveness of individualized instruction is questioned by Hirsch (1976), Miller (1976), and Schoen (1976, 1976) in their reviews of research In general, individualized instruction is defined in terms of being taught under an individualized system emphasizing (1) curriculum based on a specific set of behavioral objectives, (2) content divided into small units, (3) "self-paced" arrangement *i.e.*, students proceeding through the materials at their own rate, (4) students learning independently, and (5) pre- and postcriterion-referenced tests The teacher's role was that of manager, record keeper, individual tutor, and curriculum developer In summary, the research studies reviewed by Miller indicate (1) no significant differences on a norm-referenced basis in mathematics achievement among the individualized and traditional approaches, (2) the duration of individualized instruction increases as the achievement average decreases, (3) individualized instruction has a limited effect on student attitudes, and (4) minor support for individualized instruction benefiting students of low ability

Even though individualized instruction seems to have little effect on student attitudes, Beck (1977) did find that students in grades one through eight do possess definable attitudes in mathematics, science, social studies, and reading language. Beck's research concluded (1) students' mean attitude toward each of the four content areas (including mathematics) for grades one through eight are positive, (2) across the eight grades science is the best liked subject and mathematics the least liked, and (3) even though student attitudes are positive they decrease each year as the grade level increases.

Research exemplifying the successful and practical use of learning principles in the classroom to alter social and academic behavior has been more positive (O'Leary & O'Leary, 1972, 1977) These authors have documented how a student's behavior can be changed by manipulating observable preceding and consequent events in the classroom.

Smith and Lovitt (1976) used reinforcement contingencies to increase the students' arithmetic computational proficiency, but reinforcement contingencies were not effective in the acquisition of skills. The ramifications of this research are overlooked by many classroom teachers. Namely, reinforcement contingencies initially may not be effective because the desired social or academic behaviors are not yet in the repertoire of the student. First, the desired behavior must be learned, then reinforcement contingencies can serve to motivate, increase, and maintain satisfactory levels of performance.

In this study, junior high school students with exceptional educational needs for learning basic arithmetic skills were placed in an Individual Math Program (IMP). The purpose of the IMP was to help categorical and noncategorical students who had previously failed to meet the arithmetic requirements of a developmental school curriculum to which their chronological age had assigned them. Categorical students were students labelled by the school psychologist as educable mentally handicapped (EMH), learning disabled (LD), or emotionally disturbed (ED). Noncategorical students (NC) included those students having no etiological label. The IMP was designed (1) as a mainstreaming model to teach categorical students developmental arithmetic skills and to bridge the gap between a categorical resource center and the traditional departmentalized mathematics classroom, and (2) as a remedial model for teaching developmental arithmetic skills to noncategorical students who had failed in the traditional departmentalized mathematics classroom.

The purpose of this chapter is to characterize the IMP and report on data collected during the four longitudinal studies. Each longitudinal section lasted three academic school years: grades seventh, eighth, and ninth. Treatment variables in the IMP were combined in an attempt to maximize student success in learning basic arithmetic skills.

METHOD

Subjects. Junior high school students were selected for the IMP based upon a needs assessment for remediation of basic arithmetic skills. Students with a two-year or more discrepancy between chronological age and actual functioning level in basic arithmetic skills and consistently receiving low D's and F's in the traditional classroom were scheduled for the IMP. Discrepancies were measured by norm-referenced tests, criterion-referenced tests, and staff intuition. Class size ranged from a minimum of 12 students to a maximum of 14 students, with an average of 13 students per class. To maintain a flexible student schedule, all three grade levels were integrated in each class period, *i.e.*, nongraded. Enrollment in the IMP fluctuated from 104 to 78 students, which was 10 to 7 percent of the student body. Staff positions for the IMP were taken voluntarily. It was necessary that each teacher be willing to follow the structure of the IMP and desire to work with low-achieving arithmetic students. A discussion of the traditional classroom will not be presented. The traditional classroom is teacher-centered, teacher-paced with common tests given at the same time to all students, and has a student-teacher ratio of 25:1 or more.

Acquisition of Skills. Environmental conditions which affected students' behavior in the IMP were classified as (1) conditions for the acquisition of skills and (2) conditions for the proficiency of skills. Conditions established to maximize student acquisition of skills were: (1) individualized instruction with a student-teacher ratio of 13:1, (2) pre- and postcriterion-referenced testing, (3) a developmental curriculum, (4) programmed materials with immediate feedback and periodic reviews, (5) establishment of clear rules and objectives, and (6) student correctors.

Criterion-referenced pretesting was used to identify individual student arithmetic deficits. Arithmetic skills and concepts in which the student was deficient constituted the student's various units of curriculum. This diagnosis was initiated with an assessment of second-grade arithmetic skills and continued through a developmental skills continuum until concluding with ninth-grade general math skills.

The student began with a pretest. If the pretest results were acceptable (100-98 percent for an A, 97-95 percent for an A-, 94-92 percent for a B+, 91-89 percent for a B, and 88-86 percent for a B-), the student received a grade for that particular arithmetic skill and skipped the accompanied unit of work. The process was continued by progressing to the next sequential unit of curriculum with its pretest. Again, if the pretest results were acceptable the student skipped the work and was programmed for the next pretest with its unit of curriculum. If and when the pretest results were not acceptable, 85 percent or less, the student was programmed for that particular unit of work in order to develop a certain arithmetic skill. Upon completion of the unit of work, the student took a posttest. If the posttest results were acceptable, the student progressed to the next unit of curriculum. However, if the posttest results were not acceptable, the student was reprogrammed until the required level of achievement for that arithmetic skill was acceptable.

Developmental materials were the source of these units of curriculum. Three developmental series were adapted and programmed. Each developmental series was organized into units of curriculum which were programmed with immediate feedback available. Each unit was broken down and arranged into logically sequenced small steps. Each step or page of the unit provided information, required the student to respond to the information, and gave feedback to the student regarding the correctness of his her response Curriculum topics consisted of the 400 basic arithmetic facts, telling time, making change, measuring in inches, and understanding and operating with whole numbers, fractions, decimals, percents, and story problems Materials also included programmed reviews on a periodic spiralling continuum Developmental materials included (1) for seventh and eighth graders *Mathematics for Individual Achievement* by Houghton Mifflin, broken into 125 units of curriculum for levels three through seven, and (2) for ninth graders Spectrum Mathematics Series by Laidlaw Brothers and Programmmed Math Sullivan Associates Program by McGraw-Hill, broken into 100 units of curriculum for levels three through nine There was also a variety of supplementary materials available if the student had difficulty with the units of curriculum adapted from the three developmental series Supplementary materials were interchangeable from grade level to grade level depending upon the student's needs Supplementary materials included (1) addition, subtraction, multiplication and division flash cards with stop watch, (2) visual sequential multiplication tables with stop watch, (3) measurement cards with developmental rulers (4) Time Teller I and II by Teech-Um Company, (5) Change Maker by Creative Teaching Press, (6) Fraction Rods by Creative Publications, (7) Computational Skills Development Kit by SRA, (8) Merrill Mathematics Skill Tapes by Merrill Company, (9) Wollensak Tapes by 3M Company, (10) five tape players by Califone, (11) three calculators by Monroe (12) four Digitors by Centurion, and (13) one Classmate 88 by Monroe With these developmental and supplemental materials, students were able to work at their own developmental speeds and levels So that poor readers were not handicapped, all materials were screened to insure sufficiently low reading levels

Social rules and academic objectives were clearly stated to all students Students knew what behaviors, social and academic, were acceptable and what were not acceptable Also, minimal academic standards were arbitrarily set for the IMP as a group and for each individual student to insure minimal academic progress Minimal acedemic standards included (1) Each student must complete five or more units of work during math class per nine-week grading period depending on the student's ability If not completed then the student must complete the remaining units of work during study hall or after school in order to receive a grade (2) No student can be absent for any reason from math class for more than nine times per nine-week grading period If absent ten times or more, then the student must make up all class periods absent during study hall or after school in order to receive a grade And, (3) Each student must memorize the 400 basic arithmetic facts within the first nine-week grading period If not, then the student must continue to work on them during study hall or after school until mastery, in order not to take time away from the other units of curriculum to be learned

Students were instructed individually on a 13:1 student-teacher ratio in their individually prescribed units of curriculum Student correctors were needed as a necessary means of providing the teacher with more individual instructional time per student Student correctors were used to correct most pre- and posttests However all tests were graded by the teacher One student corrector was managed by the teacher per period Student correctors were obtained on a voluntary basis during their study halls Student volunteers were screened by staff members

Proficiency of Skills Conditions established to maximize student motivation and proficiency of skills were (1) students charting and evaluating their own academic progress (2) teachers monitoring student progress daily (3) positive reinforcement, and (4) punishment Positive reinforcement consisted of presenting token reinforcers, free-time activity reinforcers, and social reinforcers All reinforcers were intrinsic to the classroom

Token reinforcers were in the form of letter grades Students received a letter grade of an A or B for each unit of curriculum completed This was possible since the student was not graded on any unit of curriculum until either the pre- or posttest results were acceptable Initially token reinforcement was accomplished by testing at a lower level than the student was functioning, *i e*, second-grade arithmetic skills Therefore each student experienced success at the start From this starting point each student progressed to his or her own academic functioning level and continued to develop more arithmetic skills Letter grades reflected the student's individual progress rather than the grade-level expectations appropriate for their chronological age

Free time reinforcers consisted of a menu of free-time activities including (1) a library pass, a pass to another classroom, or the hall pass, (2) reading a book, magazine, or comic, (3) using math games, puzzles, or a calculator, (4) writing a note to a friend, (5) listening to a tape, (6) helping the teacher by running errands or tutoring another student, and (7) helping the math department secretary Students received free-time reinforcers after the completion of a predetermined number of units of curriculum (ratio schedule) Free-time reinforcers gave the student a free-period instead of the scheduled math class Students also received free-time reinforcers on a variable schedule as a surprise for appropriate social academic behavior In shaping a student's behavior both social and academic behaviors were of primary concern

Social reinforcers included a pat on the back, a smile, praise, social interaction from the teacher, answering a student's question, and positive contact with parents 'phone call, letter, or conference Social reinforcers were given for appropriate arithmetic or social behavior Social reinforcers were also paired with all other reinforcers so that social reinforcers would become more meaningful For any given student the effectiveness of different reinforcers varied greatly However, effective reinforcers of some form did exist for most students The central idea of reinforcement was to catch the student exhibiting the appropriate behavior and reinforce it

Students received punishment for incorrect arithmetic or inappropriate social behavior which they were able to control but chose not to manage Punishment was in the form of response cost, removal of reinforcers, and soft reprimands Response cost was the process of letting an inappropriate behavior occur, but making the behavior become so costly that the student found it not worthwhile to continue Examples included keeping the student after school one minute for every minute he or she refused to do arithmetic work or keeping the student after school one minute for each time the student disturbed another student Removal of reinforcers occurred when the student exhibited inappropriate social behavior or arithmetic errors Reinforcers were reinstated when appropriate behavior recurred Soft reprimands were also given by the teacher for inappropriate social behavior or arithmetic mistakes

The academic procedure of the IMP was monitored by both teacher and student in order to chart the student's arithmetic progress Criterion-referenced pre- and posttest results on each unit of the curriculum prescribed for the student were recorded by the teacher in the grade book and by the student in their individual folder on a developmental-skills continuum checklist Also, each day the teacher tallied the number of pages of curriculum completed by the student and recorded this data in the grade book under the current date

With this structural design, the student's behavior was affected by two types of environmental conditions The acquisition conditions provided the necessary stimulus control so that appropriate student behavior was most likely to occur The proficiency conditions of positive reinforcement as well as teacher and student charting of progress provided the necessary response, contingent upon appropriate student behavior, to increase the frequency of appropriate student behavior The proficiency condition of punishment provided the necessary response, contingent upon inappropriate student behavior, to decrease the frequency of inappropriate student behavior

RESULTS

Data gathered to evaluate the IMP was generated by the Metropolitan Achievement Test and an adapted form of the Revised Math Attitude Scale (Aiken, 1963) The intermediate form of the Metropolitan in mathematics designed for grades four, five, and six was selected for its content validity By inspection of items it was determined that the intermediate form covered a majority of the units of curriculum in the IMP The attitude scale was constructed by Likert's method of summated ratings from one to five for ten items connoting negative attitudes and ten connoting positive Results of the four longitudinal sections were evaluated by eyeballing the tabulated averages

See Table 1 for mean grade equivalents, gains, and losses on the four longitudinal studies as tested by the Metropolitan Achievement Test Each longitudinal section lasted three years covering seventh, eighth, and ninth grade Only those students enrolled for the duration of the study were included The first longitudinal study started in 1973 and concluded in 1976 with 19 ninth graders (17 NC, 1 LD, and 1 EMH student), the second study started in 1974 and concluded in 1977 with 13 ninth graders (7 NC, 5 LD, and 1 EMH student), the third study started in 1975 and concluded in 1978 with 17 ninth graders (10 NC, 5 LD, 1 EMH, and 1 ED student), and the fourth study started in 1976 and concluded in 1979 with 18 ninth grades (10 NC 7 LD, and 1 EMH student) Table 1 also gives the percentage of students successfully returned during each longitudinal section to the traditional departmentalized mathematics classroom from the IMP

See Table 2 for mean attitudinal raw scores, gains, and losses on the four longitudinal studies as tested by the Revised Math Attitude Scale

From Table 1 on a norm-referenced basis it can be concluded (1) Students gained an average of two academic years for every three years in the IMP (2) Mean yearly academic gains were relatively stable during each longitudinal section There was no substantial increase or decrease in the mean yearly gains as the duration of the IMP continued over a three year period (3) Summer losses and gains between seventh and eighth grade, and again between eighth and ninth grade were minimal ranging from a long-term retention loss of two months to an inexplicable gain of two months

On a criterion-referenced basis it can be concluded (1) Students were provided with needed arithmetic tasks at their own developmental level and were given the time necessary to master the skill Thus, students experienced academic success and received passing grades for units of curriculum completed at their developmental level of performance If these students had remained in the tradi-

TABLE 1
MEAN GRADE EQUIVALENT SCORES, MEAN GAINS, AND PERCENTAGE OF STUDENTS RETURNED FOR FOUR LONGITUDINAL SECTIONS

Grade Level	1973-1976 NC=17 LD= 1 EMH= 1	1974-1977 NC=7 LD=5 EMH=1	1975-1978 NC=10 LD= 5 EMH= 1 ED= 1	1976-1979 NC=10 LD= 7 EMH= 1
7.0	5.3	5.2	4.8	5.1
$\bar{x}$ Gain	+.7	+.8	+.5	+.9
7.9	6.0	6.0	5.3	6.0
Summer	-.1	-.1	0	0
8.0	5.9	5.9	5.3	6.0
$\bar{x}$ Gain	+.7	+.5	+.9	+.8
8.9	6.6	6.4	6.2	6.8
Summer	0	+.2	.2	.1
9.0	6.6	6.6	6.0	6.7
$\bar{x}$ Gain	+.5	+.7	+.9	+1.1
9.9	7.1	7.3	6.9	7.8
Total Gain	+1.8	+2.1	+2.1	+2.7
$\bar{x}$ Gain	+.6	+.7	+.7	+.9
% Return	32%	36%	33%	31%

TABLE 2
MEAN ATTITUDINAL RAW SCORES AND MEAN GAINS FOR FOUR LONGITUDINAL SECTIONS

Grade Level	1973-1976 NC=17 LD= 1 EMH= 1	1974-1977 NC=7 LD=5 EMH=1	1975-1978 NC=10 LD= 5 EMH= 1 ED= 1	1976-1979 NC=10 LD= 7 EMH= 1
7.0	Not	62	70	67
$\bar{x}$ Gain	Avail	+8	+0	+5
7.9	able	70	70	72
Summer		-4	+1	+0
8.0	67	66	71	72
$\bar{x}$ Gain	+4	-4	4	-4
8.9	71	62	67	68
Summer	-1	+0	+5	+6
9.0	70	62	72	74
$\bar{x}$ Gain	-7	+4	-7	5
9.9	63	66	65	69
Total Gain	-4	+4	-5	+2

tional classroom and performance remained the same, they would have received low D's and F's. (2) Table 1 further indicates that from the four longitudinal sections 32 percent, 36 percent, 33 percent, and 31 percent of the students respectively were successfully returned to the traditional departmentalized mathematics classroom

From Table 2 it can be concluded: (1) Mean yearly attitudinal raw scores were positive for each longitudinal section ranging from 62 to 74. The neutral point of the Revised Math Attitude Scale is a raw score of 60 (2) Mean yearly attitudinal raw scores were relatively stable during each longitudinal section. There was no noticeable increasing or decreasing trend in the mean yearly raw scores as the duration of the IMP extended over a three year period

Results of the four longitudinal sections were possibly minimized because approximately one third of the better performing students from each section were successfully returned to the traditional departmentalized mathematics classroom and their progress could not be included

DISCUSSION

Comparing results of the IMP with research by Miller on individualized versus traditional instruction and Beck on school attitudes reveals conflicting findings (1) On a criterion-referenced basis there was a significant difference in arithmetic achievement between the individualized and traditional approach Students in the IMP experienced success in arithmetic and these same students in the past had experienced failure in the traditional classroom (2) On a norm-referenced basis, as the duration of individualized instruction increased, the achievement average did not decrease but remained stable Students in the IMP averaged seven months growth per year (3) Attitudinal averages in the IMP were positive and as the duration of the IMP increased the attitudinal averages did not decrease but remained stable. Therefore, it can be concluded that low-achieving math students in junior high school can experience success, progress at a stable rate, and maintain a positive attitude if not in the traditional classroom at grade-level skills to which their chronological age has assigned them, then in a traditional classroom where their math skills have assigned them In summary, these three conflicting points as indicated by Miller provide more than minor support for individualized instruction benefiting math students of low ability

IMPLICATIONS

Why does previous research on individualized instruction not reveal more positive results when compared to the traditional approach? A possible reason for this is that previous research dealt with only one variable method of instruction Therefore, all other components were held constant including class size Glass and Smith (1978) present a convincing study indicating that average student achievement increases as class size decreases In their achievement study, it was shown that more than thirty percentile ranks exist between the achievement of a pupil taught individually and a pupil taught in a class of 40 The typical achievement of students in instructional groups of 15 and fewer is several percentile ranks above that of students in classes of 25 and 30 They also found that for every student by which class size is reduced below 20, the class's average achievement improves substantially more than for each student by which class size is reduced between 30 and 20 In a more recent study Glass and Smith (1979) extended their earlier work by examining the relationship between class size and other outcome measures Their research concluded (1) Class size affects the quality of the classroom environment. In a smaller class there are more opportunities to adapt learning programs to the needs of individuals (2) Class size affects pupils' attitudes In smaller classes pupils have more interest in learning (3) Class size affects teachers In smaller classes their morale is better they like their pupils better, have time to plan and diversify, and are more satisfied with their performance

If the research had compared the usual student-teacher ratio (20 1 and more) in the traditional approach with a smaller student-teacher ratio in the individualized approach (20 1 and less), then both methods could have performed as designed and a more accurate assessment could have been made The traditional method entails group instruction with less personal student contact and the individualized method involves individual instruction with more personal student contact Individualized instruction is the concept but, a smaller class size is the means by which the concept can be implemented to its fullest expectations If research had differentiated class size based upon the method of instruction, then individualized instruction might have paralleled the data collected in the IMP

Further research needs to be conducted in the area of class size versus method of instruction A possible disadvantage of the individualized approach is that one cannot teach as many students at one time A distinct advantage of the individualized approach is that it is designed to work with each individual student on different skills depending upon need, until their mastery and regardless of the time required This may be a possible reason why Miller found minor support for individualized instruction with students of low ability

In order to increase academic gains in the IMP in the future, it is recommended that the amount of student-teacher contact time be increased by (1) decreasing the number of students or (2) hiring a qualified teacher aide

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4. NONMATHEMATICAL DIAGNOSTIC VARIABLES

by Tom Denmark

This chapter should serve as a catalyst in helping the mathematics teacher find the factors that cause learning difficulties. It offers the educator a number of instructional techniques for children who have problems in dysgraphia, aphasia, auditory perception, perseveration, overloading, and overattention. The author is Professor of Mathematics Education at Florida State University, Tallahassee.

Many students who have one or more learning disabilities experience considerable frustration in their efforts to acquire essential mathematical concepts and skills. In many cases a student's difficulty in learning mathematics can be attributed to mathematical factors such as deficiencies with prerequisite concepts and skills, misconceptions about the meaning of definitions or symbolism, the continued utilization of inefficient algorithms, and the inability to transfer acquired concepts and skills to different problems. A student's deficiencies with strictly mathematical variables are, in some cases, further compounded by affective factors such as lack of motivation and poor self-concept Both mathematical and affective variables must be considered in the process of diagnosing students' difficulties in learning mathematics. In addition, for students with a specific learning disability, the diagnostic process should assess the effects that the specific learning disability has on the learning of mathematical concepts and skills. The purposes of this chapter are (1) to discuss how certain specific learning disabilities might have a negative effect on the learning of mathematics and (2) to suggest instructional techniques which could assist a student in his or her efforts to learn mathematics

DYSGRAPHIA

Students who have acquired the skill of copying mathematical symbols (numerals, operational and relational signs) and are unable to write these symbols spontaneously in the context of completing a written assignment may have a dysgraphia problem. For example, it is sometimes quite evident that a student has memorized a basic fact, but the student will spend 10 seconds or more thinking about how to write the answer. Taken individually, these slight delays in providing a written response usually do not have an adverse effect on the student's performance, but, cumulatively, the small delays can prevent the student from completing an assignment, especially a timed exercise. Thus, the student can be easily discouraged in his or her efforts to demonstrate proficiency in performing mathematical tasks. In other cases a dysgraphic student may know the correct answer to a problem, but the written response is incorrect because of an omitted symbol, for example, 347 is written as 34, 3+4=7 is written as 347, or tenths is written as ten. Also, the apparent error might be the result of reversing the order of the symbols, for example, 243 is written as 234, or 3+4=7 is written as 34+=7, or the misformation of a symbol, for example, 3+4=7 is written as 3×4=7.

When working with a dysgraphic student, as is the case with any student with a specific learning disability, the teacher's initial efforts should be directed toward maximizing the chances of the student successfully completing a task. Thus, exercises which require a written response should initially be kept to a minimum. This can be done by modifying conventional exercises to multiple-choice questions so that the student will only have to use a convenient symbol to mark the correct answer. Alternately, the student may be allowed to use plastic or cardboard symbols to construct the answers.

APHASIA

One characteristic of aphasic students is that they have considerable difficulty in expressing themselves orally They may be very slow in answering a question, especially if a complete sentence is the expected response. Or their responses may be incorrect or nonsensical statements which reflect the inadvertent substitution of one term for a related one. For example, rather than saying that 8 and 12 are *multiples* of 4, the response might be that 8 and 12 are *factors* of 4 or that 8 and 12 are *multiply* of 4. In any case a student's responses convey an impression to the teacher, the other students, and the student as well that the student does not understand. Moreover, an aphasic student may be hesitant to ask questions during the explanation phase of a lesson Thus, the student enters the practice phase of the lesson with an incomplete understanding of the concept or process, and thereby may practice or even perfect faulty procedures which must be unlearned and which can confound the learning of other topics.

One method of minimizing oral communication problems is through the use of a nonverbal demonstration technique It is not necessary for the teacher to speak even to present the problem Rather, a problem is presented, the solution is demonstrated, and then a similar problem is given to the students After a student writes a solution to a problem or completes one step of a multistep solution, a smile from the teacher can mean a correct response or a shake of the head can indicate that the response was not correct and that someone else should offer a response Gestures are an effective way of indicating whether the response is close to being right or if it is really off target After correct responses have been given to several problems, then the teacher can offer an oral explanation The advantage of this technique is that the students have a basis for interpreting the oral explanation of the concept or process. Questions asked during an oral explanation should initially require only a one-or two-word response, and students should be given practice exercises in which they are asked specific questions like those that could be asked in the context of an oral explanation. For example, a question like, "Two-thirds and one-half are ______?" Or, the question may be multiple choice, if the students are unable to give a free response

AUDITORY PERCEPTION

Many students are unable to distinguish between similar-sounding words or phrases For example, a student may not hear the difference between "ten" and "tenth " Failing to perceive this distinction, a student may wonder why the decimal place immediately to the right of the decimal point and the second place to the left of the decimal point have the same name Or, the confusion between ten and tenth could be the explanation of why some students say, "236 27 rounded to the nearest tenth is 240 " There are numerous pairs of sound-alike terms in mathematics, for example, odd and add, divide and divisor Not being able to distinguish between the pronunciations of these words is one reason why some students don't understand the meanings of certain questions and explanations Since most mathematics instruction is presented in an oral mode, it is essential that students adequately develop their auditory perception

One of the more common errors in subtraction is for the student to avoid regrouping steps by simply always subtracting the smaller number from the larger number, for example, 54-26=32 Students who make this error frequently do not hear the difference between "4 take away 6" and "4 taken away from 6." When students learn to distinguish between phrases of this type, they frequently recognize the need for the regrouping step and no longer make this common subtraction error One technique for helping students to develop their abilities to distinguish between similar sounding words and phrases is to conduct and oral drill, where the teacher says a word or phrase and then asks a student to repeat what the teacher said A variation of this drill is to have the student say a word or phrase, and then the teacher repeats the student's statement The teacher should sometimes substitute an incorrect word, for example, "multiply" for "multiplier," and the student then decides whether or not the teacher is correct At first only simple words or short phrases should be used in these exercises, then later the words and phrases should be incorporated into complete sentences Other types of discrimination exercises which involve both auditory and visual skills should be used with students who have problems in auditory perception For example, the students are given a written list of paired terms The teacher says one of the terms in each pair, and the students mark that term on their papers

PERSEVERATION

Some errors that students make are the result of the continuance of a certain behavior to an exceptional degree or beyond a desired point This is called perseveration For example, when asked to show a set of six blocks the student begins counting and fails to stop at six The result is a set with more than six blocks In certain cases, this behavior is the explanation for incorrect answers to addition and subtraction problems Perseveration is also the reason why some students count a set of tens and ones as all tens Once they begin counting by tens, they seem to be unable to switch from counting by tens to counting by ones Another example of perseveration is the way students write certain numerals, for example, 33 written as 3333 Problem exercises which contain more than one operation are likely to foster perseveration For example, if the first four items are addition problems and the next four items are subtraction problems, there is a tendency for many students to treat some of the subtraction problems as addition problems There are several explanations for this particular behavior, one of which is that some students tend to perseverate an action once it is begun.

One method for helping a student learn to control a tendency toward perseveration is to encourage the student to slow down the rate of work Encourage the student to stop after each step and think about what to do next Another technique involves drill exercises in which the student is required frequently to switch from one response to another Such exercises may require the student only to read or write a sequence of symbols, or the exercises may involve several operations arranged in a random order

OVERLOADING

In many instructional settings students are required to assimilate stimuli received through two or more senses. The most common format for presenting a mathematical lesson is to utilize some form of graphics, such as a blackboard or textbook, supplemented by an oral explanation. In these situations the student must assimilate both visual and auditory stimuli. Thus, if the student is unable to assimilate the stimuli simultaneously from both sources, it is unlikely that the student will comprehend the presentation. Similarly, if a student is required to utilize two or more senses to demonstrate his or her performance on a given task, for example, to read a problem aloud before solving the problem, a valid assessment of the student's ability to perform the task may not be obtained.

Situations requiring the integration of stimuli from more than one source cannot be entirely avoided in the day-to-day teaching of mathematics. However, if it appears that the interaction of stimuli from various senses is having a negative effect on a student's performance, then the instructional procedures should be modified so that the student is required to attend to only one source of stimulus at a time. For example, rather than requiring a student to read a graphics display while listening to an oral explanation, allow the student sufficient time to read the graphics and then provide the accompanying oral explanations.

OVERATTENTION

When a complex graphics display is used in the presentation of a mathematical topic, some students will fix their attention on a specific feature of the display. For example, a common method of constructing a diagram to illustrate that one-half is equivalent to two-fourths is to divide a figure, for example, a square, into halves by a solid line and to use a broken line to divide each half. If a student's attention is focused on either the solid or broken lines, or on the figure itself, then statements relating to the partitioning of the figure into two or four parts will not be understood by the student. Consequently, this graphics display is not an effective aid for illustrating the relationship between the two fractions. A fixation on certain features of a display that results in mathematical error also occurs in the presentation of problems. For example, in the problem 3+34, a student's attention can be fixed only on the numbers, that is, the student does not see the operation sign. Realizing that there is a problem to be solved and not seeing the operation sign, the student is likely to rely on an arbitrary rule for determining the operation. For example, if there is a large number and a small number, you multiply. In this way, the student's answer would be 102. The use of arbitrary rules for determining the operation is frequently the explanation for wrong answers, which would be correct for another problem involving the same numbers.

One technique for lessening the effects of overattention on a specific feature of a display is to emphasize the various aspects of the display by means of different colors or shadings. For example, in the display of computational problems one color could be used for numerals and another color for the operation sign. Another technique is to present a display and have the students identify various features of the display. In the case of the diagram for showing the relationship between one-half and two-fourths, have the students identify at least the following details: an outline of a square, a solid line through the center of the square, a broken line which cuts the solid line in half. To assist students in identifying a specific feature of a display, a second display without the feature can be shown. The task would be to compare the two displays, that is, point out both similarities and differences.

CONCLUSION

Due to the brevity of this chapter it is not possible to discuss each and every specific learning disability. Dyslexia and visual perception, for example, have not been discussed. Nor, is it possible to provide a comprehensive coverage of how each specific learning disability could affect the learning of divers mathematical concepts and skills. Rather, the primary objective here is to provide a catalyst for encouraging teachers of handicapped students to consider a broader spectrum of factors which could affect their students' acquisition of mathematical concepts and skills.

5. MATHEMATICS MEANS MANIPULATIVES: TEACHING NUMBER CONCEPTS TO YOUNG LEARNING-DISABLED CHILDREN

by **Mary M. Myers**

Mary M. Myers presents a number of manipulative activities for young learning-disabled children. After trying these ideas, your whole class will enjoy doing the lessons, not only the handicapped youngsters. The author teaches learning-disabled/language-delayed students in the Fairfax County, Virginia, public schools.

In order to gain a concept of numbers and discover relationships among numbers, specifically numbers zero through twelve in this chapter, a child needs to explore a variety of manipulatives in a variety of experiences. The activities described in this chapter advocate a concrete approach for teaching quantity understanding to the young handicapped child in the regular classroom

Structured activities are grouped sequentially into the following subgroups: one-to-one correspondence, counting, introducing number symbols, and number-symbol association Several activities, all on the concrete level, are suggested for each subgroup. Some of the manipulatives and materials, for instance, egg cartons, are included in each subgroup with some modifications to show the sequential steps in learning number concepts. New manipulatives and materials are also included in each subgroup to maintain interest

In addition to the primary objective of teaching number concepts, the activities provide other benefits All are easy to make using common household or school objects. Most all the activities build fine motor and visual-motor skills since they require pincer grasp and eye-hand coordination. The activities also touch on other math concepts such as patterns, volume, sets, money, shapes, space, and classifying. Many of the activities can be completed independently once the child learns how to "play" with the manipulatives involved. Finally, because the activities use manipulatives, exploration, and movement, they are intrinsically motivating.

One-to-One Correspondence

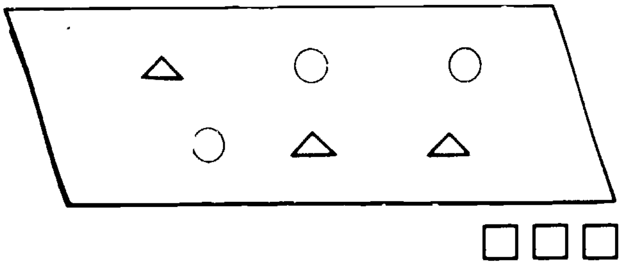
One-to-one correspondence is the concept that in given sets, one member from one set can be paired with one member from another. Such matching allows a child to discover if the sets have equivalent amounts. Matching is easier when the sets are equivalent and when the members are concrete. Thus one-to-one activities should start on this level. Gradually the child can learn to pair more abstract members from nonequivalent sets.

1. Cups and Straws

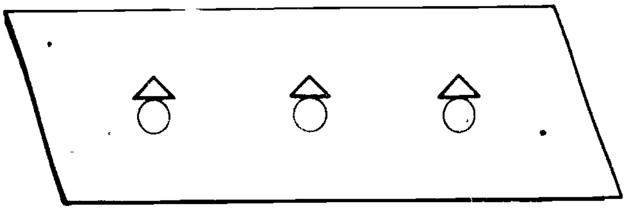
Materials	Equal number of paper cups and straws
Procedure	The teacher puts cups and straws on the table and asks the child if there is a cup for each straw The teacher then asks the child to show how he or she knows The teacher may have to prompt, "Does this cup have a straw? Show me " or "Give each cup a straw "
Variation	The teacher puts an unequal number of cups and straws on the table. The teacher asks if each cup has a straw and for the child to show an answer. The teacher then asks if there are more cups or straws

2. Attribute Blocks

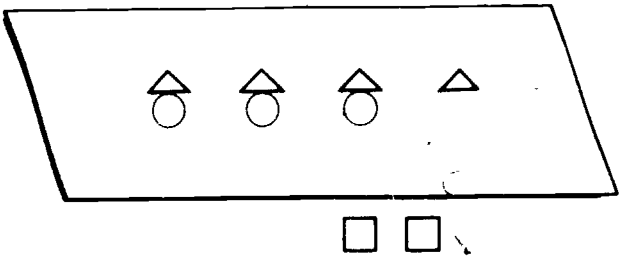
Materials	Attribute, color, or shape blocks Construction paper
Procedure	The teacher holds up a block and asks the child its shape, color, and size. Upon response, the teacher gives another child a block. The teacher goes around the table until each child has an equal number of each type of block, for example, three triangles, three circles, and three squares. The teacher then gives each child a sheet of construction paper. The teacher tells each child to put two types of blocks on paper, for example "Put the triangles and the circles on your paper."



The teacher asks if each circle has a triangle and to show the answer

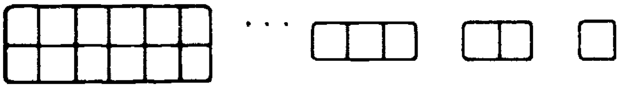


Variation The teacher distributes unequal numbers of each type of block. The child pairs designated blocks and indicates that there is not an equal matching.



3. *Egg Cartons*

a Materials Egg cartons cut apart to show sections one to twelve

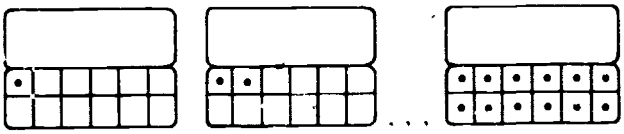


Seventy cube blocks in large box

Procedure The teacher tells the child to give each egg section a block

b Materials 13 cardboard egg cartons intact
Magic marker
Cube blocks

Procedure The teacher draws one dot on the bottom of egg sections to show sets "zero" to "twelve."



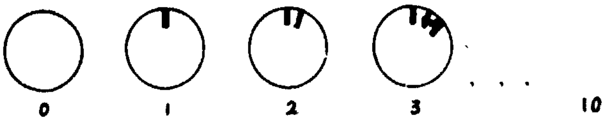
The teacher asks the child to show that there is a block for each dot, demonstrating a one-to-one correspondence to twelve

Variation With markers, the teacher makes sets of dots different colors, for example, set one with red, set two with green, set three with blue, *etc.* The child must match color as well as number, for example, three blue blocks on the three blue dots. By color coding, blocks may not match up to the dots allowing the child to recognize inequivalent sets.

4. *Clothespin Wheels*

Materials Eleven tagboard or cardboard circles five inches in diameter
Magic marker
Can of 55 clothespins

Procedure The teacher draws lines around the edge of each circle to make sets "zero" to "ten."

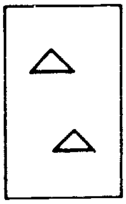


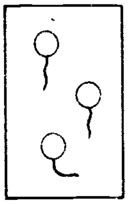
The teacher asks the child if there is one clothespin for each "spoke" and to show a response.

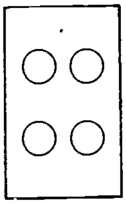
Variation The teacher provides approximately 55 clothespins.

5. *Blocks and Pictures*

Materials	Box of cube blocks or poker chips Picture counting cards (five-by-eight inches)







Procedure	The teacher places cards face down in a pile on the table. The child picks a card and puts one block on each picture to show one-to-one correspondence

Counting Objects

Rational counting involves the recitation of numerals in correct sequence as well as the assignment of each number to an object or motion, that is, a pairing of the number word with an object or motion.

1. Objects in One-to-One Correspondence. The teacher can encourage the child to count objects after completing one-to-one correspondence activities. The child counts straws, cups, attribute blocks, cube blocks, clothespins after pairing the objects

2. *Grab Bag*

Materials	Paper or cloth bag Small counters, blocks, chips, buttons Paper cups
Procedure --	The teacher puts counters in the bag and gives each child a paper cup The child reaches into the bag with one hand and grabs as many counters as possible. The child puts the objects on the table and counts them. If correct, the child recounts objects into the cup Continue until each child has a full cup.

3. *Attribute Blocks*

Materials	Blocks colored, shapes, or multi-attribute Pie tins
Procedure -	The teacher dumps blocks in center of table and gives each child a pie tin The teacher then tells each child to look for a specific type of block or tells all the children to look for one type of block, for example, "When I say 'go' put all of the circles in your pie tin." On signal, each child puts specified blocks, one at a time, in the tin. After each child has found all the specified blocks, he or she takes a turn counting the blocks by removing them one at a time from the tin If correct, the child recounts the blocks back into the main pile Play until each child finds each type of block

4. *Mystery Cartons*

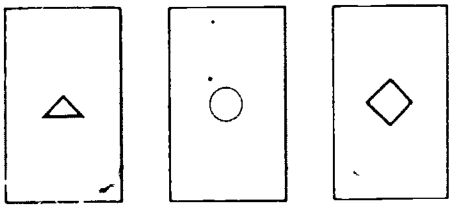
Materials	Egg cartons Box of counters, blocks, chips
Procedure	The teacher tells the children to put their heads down and hide their eyes While their heads are down, the teacher puts some counters into each carton, one per egg section The teacher then closes the lids and tells the children to put their heads up Each child picks a carton, the teacher may provide different colored cartons so that each child has to request a color. The first child opens a carton and counts the objects If done correctly the child recounts the objects back into box After the first round, the teacher may let another child hide the blocks in each carton
Variation	The teacher lets the children watch as the teacher puts counters into each carton. The children may count with the teacher while putting objects in cartons. The teacher closes the lids. Each child then picks an egg carton, and the teacher asks the child to guess how many objects are in the carton. The child then opens the carton and counts the objects to verify

5. *Making Ones, Twos, Threes . . .*

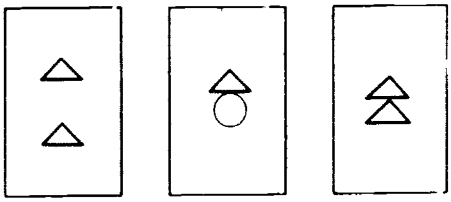
Materials Five-by-eight inch index cards
Counters parquetry shapes, cube blocks, unifex cubes, etc.

Procedure The teacher spreads out several index cards on the table and puts out a box of one type of counters. The teacher tells the child to show different ways of making a number by putting that many objects on each card The child counts out that number on each card Some examples are

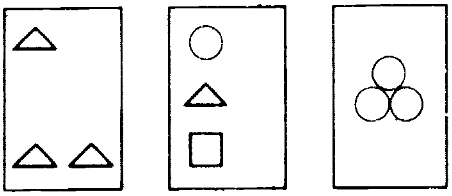
Making ones



Making twos

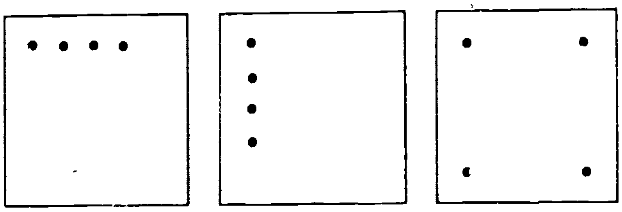


Making threes



The teacher encourages different patterns and may have another child check by counting the objects on the cards.

Variation The child makes quantities with pegs on pegboards. For example, the teacher tells the child to show different ways of making "four "

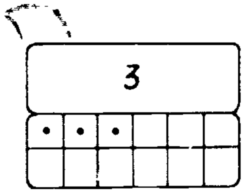


Introducing Number Symbols

Number symbols should be introduced in connection with the concept of the number

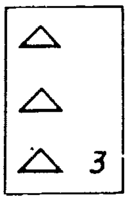
1. One-to-One Correspondence and Counting Activities. After the children have explored with these activities, the teacher can introduce the number symbols to those activities

a Egg Cartons The teacher writes the number symbol on the inside lid of each carton to correspond with the number of dots

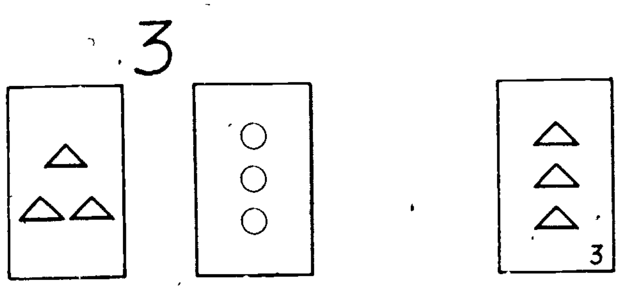


b Clothespin Wheels The teacher writes number symbols in the center of the wheel to correspond with the number of lines.

c Blocks and Pictures The teacher writes the number symbol in the corner of each card to correspond with the number of pictures

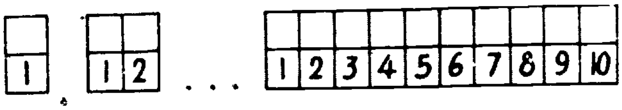


d Making Ones, Twos, Threes After telling the child to count to a certain number on the cards, the teacher puts that number symbol on the table or on each card



2. Number Grids

Materials	Number grids made from tagboard Box of counters cube blocks
Procedure --	The teacher makes number grids (one to ten).



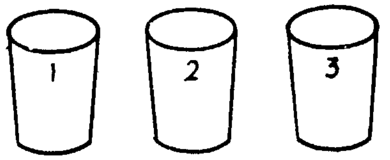
The teacher puts a grid and blocks on table and tells the child to put a block in each box while counting the blocks. The teacher shows the child where to start After the blocks are on the grid, the teacher tells the child to recount the blocks from left to right with a finger.

Number-Symbol Association

Number-symbol association is the association of the quantity of items in a set with the corresponding number symbol. The skill of number-symbol association can be shown in two ways: (1) given the number symbol, the child produces that number of objects, and (2) given a set of objects, the child produces or points to the number symbol.

1. Number Cups

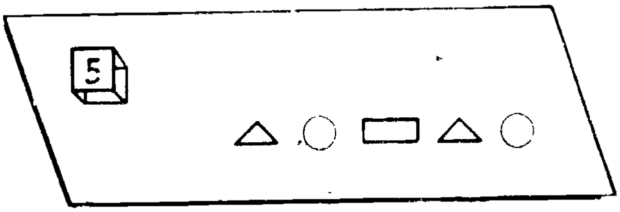
Materials	Paper or plastic cups (stackable) Magic marker Box of small counters: color chips, ½ inch blocks
Procedure	With magic marker, the teacher writes one number symbol on each cup



The teacher stacks the cups and puts them and the counters on a table The first child picks the top cup, reads the number symbol, and counts that number of objects into the cup To check, the next child empties the cup and recounts the objects Then that child picks the next cup In this way, continue until all cups have been filled Once the child learns how to play, child can fill cups independently The teacher or another child can check

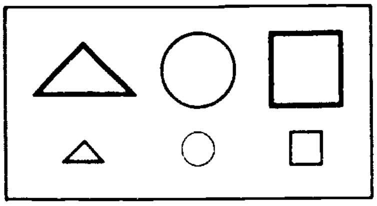
2. Roll the Die

Materials	Die with number symbols Box of counters attribute blocks, cube blocks, etc Sheet of construction paper
Procedure	The teacher puts counters on the table and gives the first child construction paper and a die. Child shakes the die and drops it on the paper Child reads number on the die. Child counts out that number of objects onto the paper Child recounts the objects with a finger to double-check a response If correct, the child keeps the counters and passes the paper and die to the next child



Continue in same manner with other children until counters are all distributed.

When the game is over, the teacher can collect blocks by attributes, for example "Give me all the red blocks," or "Give me all the squares" Or the teacher can let children build with the blocks before letting them put the blocks away in designated slots



Variation

Materials	Dice with dots (the teacher makes large dice with sandpaper dots) Number symbol cards
Procedure	The child roles dice with dots and counts the number of dots on the dice The child selects that number symbol card from table In this way, play until all number symbol cards have been taken

3. *Egg Cartons*

Materials	13 cardboard egg cartons Large box of cube blocks Magic marker
Procedure	The teacher writes a number symbol ("0" to "12") on the inside of each egg carton lid. The teacher stacks the cartons and put them and the blocks on table. Child picks top carton, reads number symbol, and puts that many blocks in the carton (one per egg section) In this way, continue until all cartons are completed.

Once a child learns how to play, the child can complete cartons independently Another child or teacher can check

4. *Clothespin Wheels*

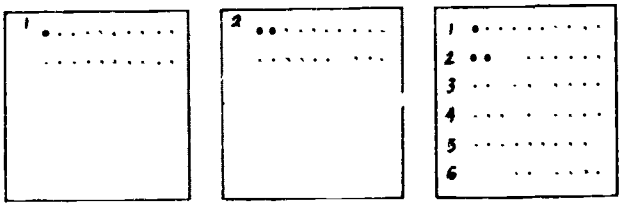
Materials	11 tagboard circles, five inches in diameter Magic marker Clothespins
Procedure	The teacher writes a number symbol in the center of each circle. The child places that many clothespins on each wheel.

5. *Store*

Materials	Pictures of food or empty food containers Magic marker Box of play pennies or poker chips
Procedure	With marker, the teacher writes one number symbol on the back of each food item The teacher displays food items on table. The first "customer" requests food, turns it over, reads number symbol, and gives the teacher that number of pennies or chips If correct, the child keeps food item Continue until all food is gone

6 *Pegboards*

Materials	Pegboards Pegs Magic marker
Procedure	The teacher writes one number symbol on each pegboard or one number symbol by each row on pegboard The child places that many pegs into holes



6. TEACHING MATHEMATICS TO VISUALLY HANDICAPPED STUDENTS

by Elizabeth Thompson Binstock

Elizabeth Thompson Binstock offers excellent hints for working with the visually handicapped child. She has observed that the blind or partially sighted child needs extra time to deal with problems, lots of manipulative materials, and a well-ordered environment. The author is Associate Professor for Special Education and Management at Lesley College in Cambridge, Massachusetts.

A wide range of children come under the heading "visually handicapped." For instance, there are children with some limited central sight, children who can see only out of the corners of their eyes, children who one saw but now are blind, and children who have never seen. These variations can be confusing for a classroom teacher trying to plan math lessons for a child with visual problems. In this chapter, therefore, I will first focus on general planning for any child with limited sight. I will then make some specific suggestions for dealing with different types of visual limitations.

GENERAL SUGGESTIONS FOR TEACHING MATH TO ANY VISUALLY IMPAIRED CHILDREN

There are some general rules of thumb which are helpful to keep in mind when planning for any child with a vision problem. First, a very careful check should be made of the child's real understanding of various concepts, such as *over, under, inside, length, triangle*. One of the pitfalls in teaching blind or partially sighted children is that they can be very verbal and use words appropriately without having a real grasp of their true meaning. Therefore, it is important to supply the student with many concrete experiences which can help build concepts. For instance, playing with clay can help the child understand about *[illegible]*, *ball*, and *long*. Manipulatives, such as Dienes blocks, attribute blocks, geoboards, unifix cubes, and Cuisenaire rods, are useful to the whole class without modifications. Braille labels can be added to other manipulatives, such as balances or chip trading activities. In addition, there are specific concrete materials which are useful for visually handicapped students. For instance, abacuses can make handy counting frames, and there are small portable ones distributed by places like the Howe Press at Perkins School for the Blind in Watertown, Massachusetts. Braille yardsticks and similar materials can also be ordered from places specializing in meeting the needs of blind people.

A second rule of thumb is that directions should be given verbally and clearly, and repeated as often as necessary. They should also be given in writing—either in Braille or large type, whichever the child uses. And, thirdly, it is important that the child can feel secure in knowing that the environment is firmly in place and won't be moved around unexpectedly. In the larger sense of the physical environment, this means that it's wise not to keep changing the location of the furniture. All the same, the child should be given a seat which has clear marks for getting there a wall, or a strip of carpet, for instance. In terms of the math environment, it means that materials should be easy to locate, pleasant to touch, and unlikely to go rolling across the room if a table on which they sit is bumped into.

TEACHING THE BASIC OPERATIONS

As I have already indicated, teaching strategies, both formal and informal, should rely heavily on manipulatives. Addition and subtraction, for instance, can be taught by using a strip of wood or pegboard which has dowels glued into the holes at equal intervals, forming a single row. Be sure to leave plenty of space between the pegs and have handy a supply of washers which will slide over the pegs. For an addition problem, like "5 + 6," the student can be asked to put five washers on the first five pegs, and then to put six more washers on the next six pegs. The total can then be counted. For the problem "14-6," the student can put fourteen washers on the pegs and then remove six. Needless to say, this approach works best for the addition and subtraction of small numbers.

For larger addition and subtraction problems, a rectangular board, divided into two columns and three rows, is more helpful. In this case, the top row can be used to represent the top line in an addition problem. Metal washers are used to represent the "tens" and rubber washers represent "ones." Thus, if the problem is "43 - 21," the top row is set up with four large metal washers in the upper left-hand corner, representing the "tens" place, and three smaller hard rubber washers in the upper right-hand corner, representing the "ones'" place. Underneath the "4," in the second row, will be two large metal washers, underneath the "3" in the second row, will be one hard rubber washer. The child then proceeds to fill in the answer on the bottom line, which may be marked off from the two top lines by a strip of masking tape.

The advantage of having different sized washers is that there is a continual reminder of place value. When it becomes necessary to borrow from the "tens" column, the student can change a large washer for 10 smaller ones, which then can be piled onto the "ones" peg.

Multiplication and division problems require larger boards, with more rows and columns. For these operations, it may be wise to number the rows and columns in Braille by using rounded nail heads, starting with the upper left-hand corner as row 1, column 1. For multiplication, the child is asked to make three rows, with four rubber washers in each row, and then to decide on an answer by counting the total number of washers. In this case, there is only one washer per peg. For division, the child is asked to divide twelve washers evenly between four rows. Again, only washer per peg.

Another manipulative which is useful when teaching multiplication or division is a plastic egg carton. You can either use the bottom of the whole carton or cut the bottom into smaller units: three cups, four cups, six cups, and so on. Dried beans can serve as your counters and be distributed equally into the egg cups. For multiplication you can have the child place three beans in each of four cups and count them, for division you can give the child six beans and ask that she or he place the same number of beans in each of three cups with none left over. Both this exercise and the pegboard exercise described earlier are helpful as a way to encourage students to struggle with the concepts and keep track of the numbers. Once the students understand the technique, they are likely to evolve their own systems for keeping track of information.

THE PARTIALLY SIGHTED CHILD

It has been my experience that some partially sighted children are highly dependent on their sight for information, while others are much less likely to rely on it as a primary sense. If your student bends down to the paper or manipulatives, notices colors, likes to paint, seems to strain in order to see what is going on, then you probably are dealing with a visual learner and should provide appropriate activities. Place the child close to any demonstration area and the board, and provide printed examples of math written large as well as visually clear manipulatives. Then stand back and assess your results. The visual learners are likely to be relieved that you are acknowledging the importance of sight to them, and they often rise to the opportunity.

If the student is not a visual learner, you should rely heavily on auditory and tactile clues, using the techniques which have been mentioned earlier as well as the ones which I will talk about in the following sections. The thing to remember with a partially sighted child is that some form of sight is available as an additional checking mechanism and that limited pieces of information about the visual world are available. Do not, however, make the mistake of assuming that more is accessible than act[illegible] is. Making sense out of the world at a distance is very [illegible] for a visually handicapped child, most [illegible] understood comes from very close examp[illegible]

THE TOTALLY BLIND CHILD, BLIND FROM BIRTH

The Problem. A child who has never seen anything has a very different sense of the world from someone who has learned through looking. Information has come in as bits and pieces, learned through the ears or fingertips. This means that blind children will be likely to have a much more fragmented sense of groups and numbers and how they relate to each other than sighted children. The conversation of these children may mislead you into thinking that they know more than they actually do, for they often have learned the correct words for things like *square, under,* or *addition* without having a concrete sense of what it is. These children are the ones most likely to have a lack of basic concepts.

Strategies to Address the Problem. It is a particularly good idea to give the child blind from birth many opportunities to handle objects while exploring math concepts. Sorting like objects into groups is one useful approach. Where possible it helps to make use of real-life activities: sorting candies for a party, dealing out Braille-marked cards for a math card game, counting out sheets of paper for groups who are working together, and counting out food pellets for the gerbils. These could be used as either multiplication or division exercises. For example: "How many straws will we need if we give two straws each to 15 students?" or "We have 10 candies and 5 students. How many can each person have?" Handling lots of shapes which are all triangles or squares is another way to help the child generalize from specific examples.

THE TOTALLY BLIND CHILD, WHO ONCE HAD SIGHT

The totally blind child, who did not lose sight until after the age of three or four, is likely to retain some fragmentary sense of the sighted world and of how things in the world relate to each other The older the child was before becoming blind, of course, the more the previous experience will be to call upon He or she may have a much better grasp of concepts like *length, group, under, beside* than the child who has never seen It is crucial to figure out some informal ways of determining whether the child understands these words. If you ask the child to place something under the chair, you should ask yourself whether any failure is due to groping around or a lack of understanding It is, therefore, wise to devise a number of informal tests over time, so that you can get an accurate profile of the gaps in the child's insights

SUMMARY

The central points to keep in mind when teaching visually handicapped children are

- Don't be misled by their verbal fluency into assuming they understand things they are, in fact, unclear about
- Give them many opportunies to test their environment
- Give them lots of manipulatives to work with
- Give them a well-ordered setting
- Give them time enough to wrestle with any concepts they are learning

7. TIPS: TECHNIQUES IN PLANNING FOR HANDICAPPED STUDENTS IN REGULAR CLASS MATHEMATICS

by Carol A. Thornton

In all of her lectures and writings, Carol A Thornton takes the common sense approach She fully realizes that the teacher's planning time is at a minimum and, therefore, gives tips that are simple and financially within the reach of every classroom teacher The author is an Associate Professor of Mathematics at Illinois State University, Normal

Typically, handicapped students are more like than different from their peers What they really need in regular class mathematics sessions is mathematics teaching at its very best The teacher's knowledge of and sensitivity to any handicapping condition will help determine necessary instructional adaptations

If a child has a hearing loss, for example, teachers would check for eye contact before saying anything of importance Key words or assignments would be written as spoken, preferably on an overhead if the child can speechread These students, like those with auditory perception or memory difficulties, need high visual and kinesthetic stimuli Others, including those with visual impairments and visual perception or memory difficulties, may require color coding and extra auditory-kinesthetic reinforcement

Often only a slight modification of an activity or assignment is necessary to make it appropriate for students using a lapboard or wheelchair desk At times it may be necessary to limit the number of written problems to make an assignment reasonable for some physically handicapped or learning disabled students. While careful sequencing, small step size, and provision for overlearning are important for all students, these approaches are necessary for children with learning difficulties

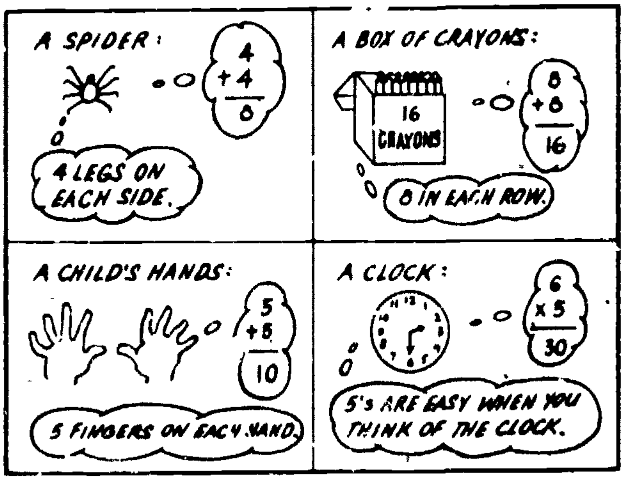
Without a lot of extra time and planning, regular class teachers can do much to adapt mathematics instruction to meet special needs. Specifically, the following TIPS may help These suggestions include some of the more effective ways of meeting the needs of handicapped students in the mainstream. The examples given should serve as prototypes for applying the techniques to other content topics

TIP 1

Use Visuals and Manipulatives to Illustrate New and Important Ideas

Handicapped children, like their peers in regular class mathematics, are basically concrete in their thinking As a general rule, the use of simple or familiar objects to illustrate facts and ideas will promote both understanding and retention

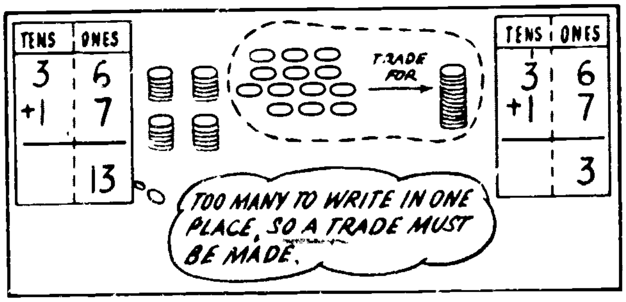
Example 1: Use familiar objects to portray certain basic facts



Example 2: Use stacks of ten and single popsicle sticks to illustrate renaming in 2-digit computation Children with motor difficulties or vision impairments may find it difficult to band the bundles of tens Chips of one color may be preferred 10-stacks (glued together) and extra singles

In whole number addition, just two big ideas prevail

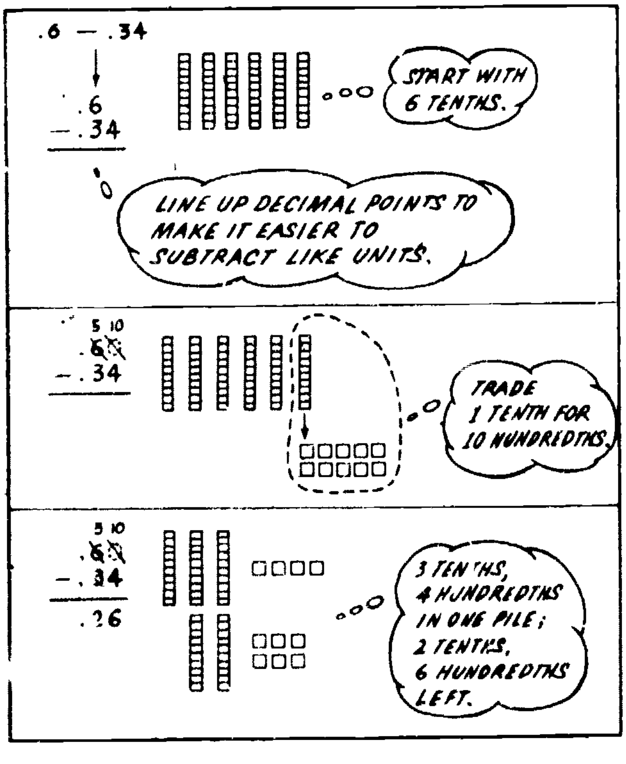
- add like units
- when there are too many of some unit to write in one position, a "10 for 1" trade must be made



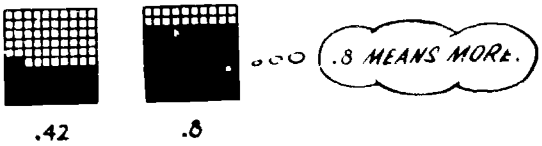
Example 3: Use graph paper squares, strips and hundredths to illustrate decimal subtraction

Again, there are just two big ideas

- Subtract like units
- If there are not enough of some unit to do the subtraction, a "1 for 10" trade must be made



Example 4: Shade graph paper with 100 squares to help in decimal comparison

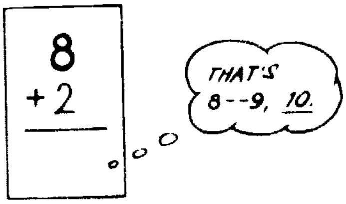


TIP 2

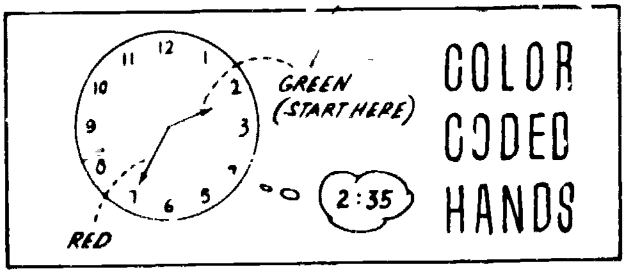
Color Code to Focus Attention and Cue Response

This technique is particularly helpful to students with visual perception or memory problems Teachers can use colored chalk or marking pens during introductory teaching sessions and make examples available to individual students The familiar red-green stoplight colors work well for students with intact color vision for these colors Green is the cue to "Go" or "Start here " Red means "stop " Blue or other colors can be used for intermediate steps

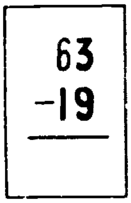
Example 1. Count on from the greatest addend[1] This technique has been used successfully with all types of students, including trainables Beyond simple introductory work for addition, there is never any need for a child to count from 1 to find answers to simple addition facts The "8" in the illustration is green



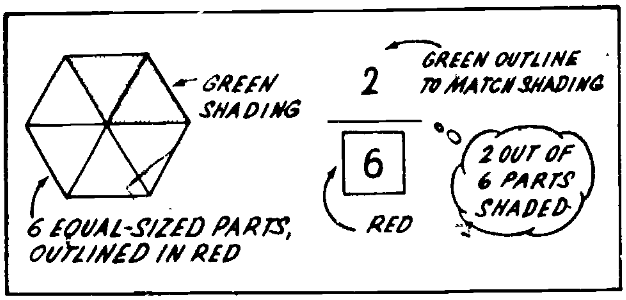
Example 2 Clock times first the hour THEN the minutes after



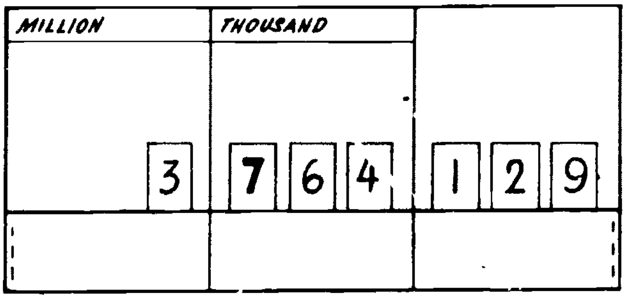
Example 3: Multistep computation Color ones digits green to mean "start here "



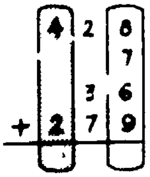
Example 4: Basic fraction meaning (also helps child read and write fraction correctly numerator first (on top), then denominator)

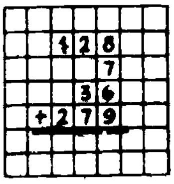


Example 5: Reading the larger numbers Color code each triple within a period green-blue-red At the vertical line, the child is cued to give the "family" name[2] This technique can be extended to reading of decimals



Example 6: Aligning numbers in columns for computation Train the child to use a highlighter to color shade the columns Alternately, use square centimeter paper



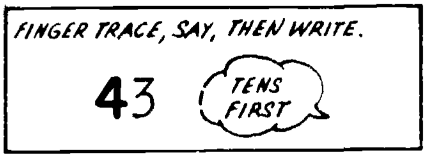


Example 7: Mixed addition-subtraction problems Some children perseverate tend to use the first problem on a page as a model for completing all others If the first problem is addition they add all remaining problems. If the first problem involves renaming, they rename in all problems whether or not this is actually needed The behavior is compulsive rather than just carelessness. Circling all addition problems green and doing these first, before turning to the subtraction problems on the page often helps Finger tracing the sign before computing is also effective

TIP 3 Allow Children to Finger Trace or Use Other Tactile Cues

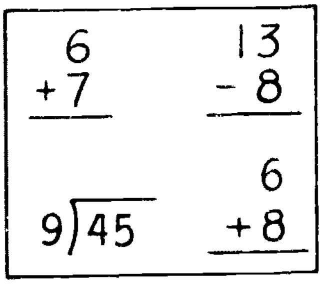
In some cases seeing is not enough More total involvement is required so that the responses a child is capable of are given

Example 1: Number reversals (e g , 43 for 34, 6 for 9) Have sample numbers available for reference. Use texture along with color and auditory cues to emphasize one part of a numeral The "4" of 43, for example, might be drawn with a green marker, then retraced with glue When allowed to dry, the glue leaves a ridge over which a child can finger trace



Example 2: Basic fact answers

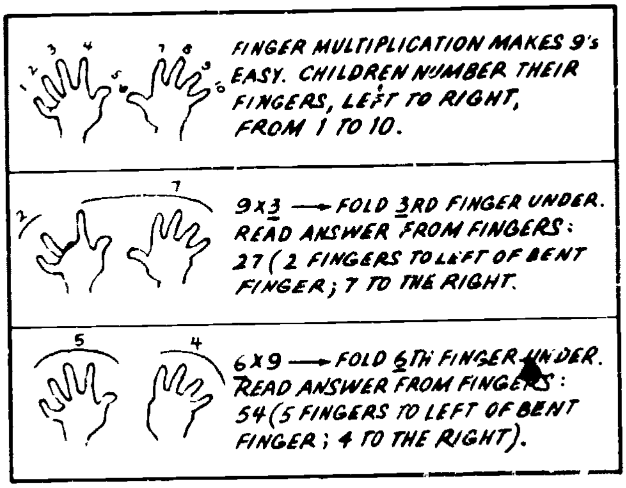
a Some children can give basic fact answers orally, but hesitate or blank out when required to write answers These students can be prompted to finger trace the problem say the answer quietly to themselves, then write it



b. To promote retention for children with visual-memory difficulties, have them finger trace and quietly say both problem and answer. Then suggest they close their eyes, picture and say the problem, and give its answer again. The answer side of a flash card deck can be used for this purpose.

Example 3: Fact errors in computation. Some children know basic facts in isolation but miss them in larger computational problems. Finger tracing the troublesome fact or writing it to the side often triggers recognition.

Example 4. Fingers for multiplication 9's



FINGER MULTIPLICATION MAKES 9's EASY. CHILDREN NUMBER THEIR FINGERS, LEFT TO RIGHT, FROM 1 TO 10.

9x3 → FOLD 3RD FINGER UNDER. READ ANSWER FROM FINGERS: 27 (2 FINGERS TO LEFT OF BENT FINGER; 7 TO THE RIGHT.

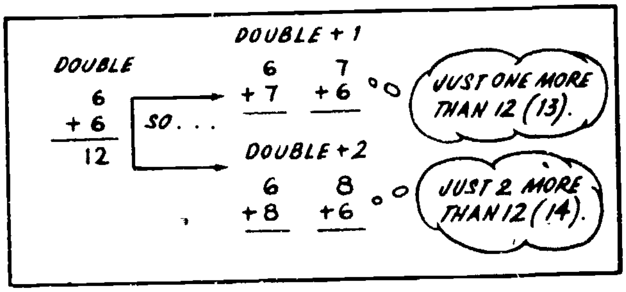
6x9 → FOLD 6TH FINGER UNDER. READ ANSWER FROM FINGERS: 54 (5 FINGERS TO LEFT OF BENT FINGER; 4 TO THE RIGHT).

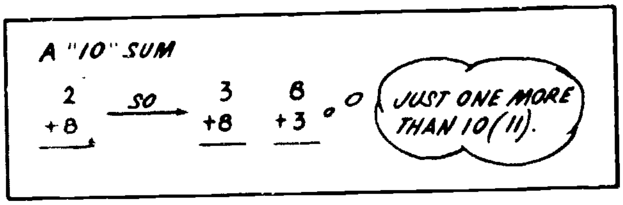
TIP 4

Capitalize on Patterns and Other Associations to Promote Retention or Understanding

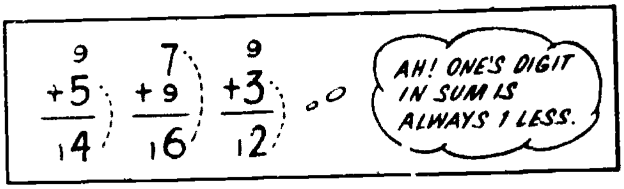
Children with problems in abstract reasoning, memory, or other related learning areas can frequently be helped by carefully structured instruction which uses patterns and relevant associations.

Example 1: Addition facts related to doubles or known "10" sums: using easy facts to help with harder ones.



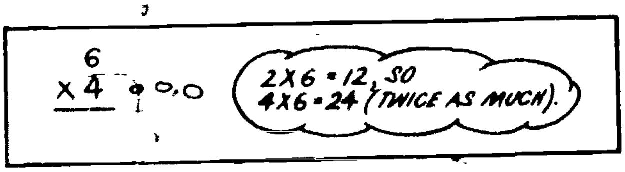


Example 2: Addition 9's and a pattern

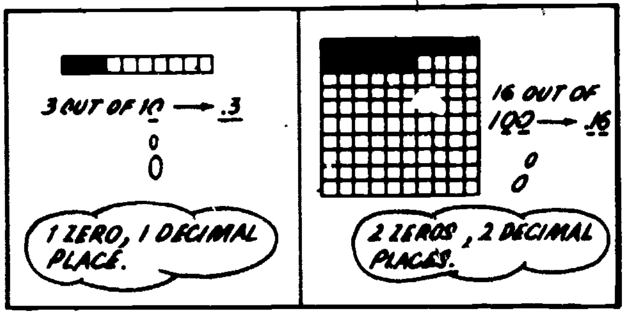


Some children use different patterns. For example, 10 + 4 is 14, so 9 + 4 is 1 less (13). Or, for 9 + 4, they may think of "taking 1" from 4 and "giving" it to 9 to make 10 + 3 (13).

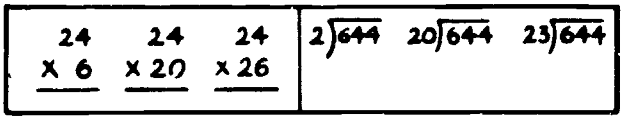
Example 3: Harder multiplication facts having even products: see what half will do!



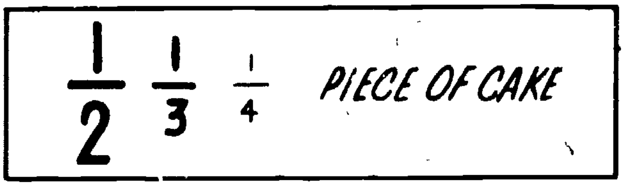
Example 4: Meaning to decimal notation. The model and underscoring help children relate one decimal digit to "tenths," two decimal digits to "hundredths."



Example 5: Easy buildup to harder problems.



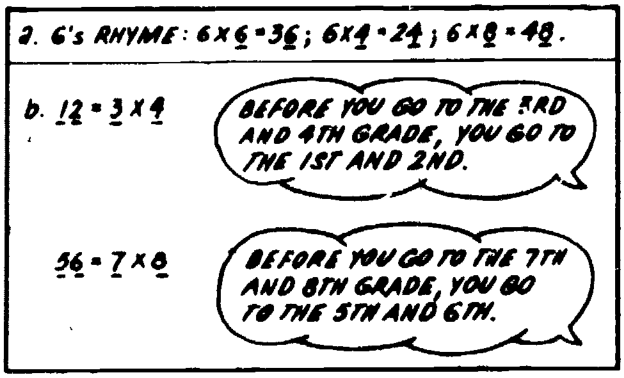
Example 6: Size cues and fraction comparisons.



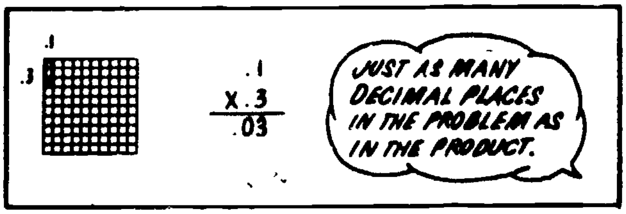
TIP 5 Use Auditory Cueing

Children with visual handicaps and difficulties in visual perception or memory generally require a high degree of auditory reinforcement.

Example 1: Basic facts



Example 2: Multiplication of decimals



Example 3: Sequence in carrying out a computation.

TIP 6 At Times, Assign Fewer Problems and Minimize or Eliminate Copying from the Board

Some children, due to high distractibility, hyperactive tendencies, or frustration do not complete assignments. In these cases it may be necessary to.

- Provide fewer problems per page.
- Create several standard formats for worksheets and provide black construction-paper masks which blot out all but one fourth or one third of a page at a time.
- Cut the worksheet into fourths or thirds and assign only one small section at a time

Similar techniques are appropriate for some students with physical, visual perception, or vision impairments. If visual-motor difficulties are severe, it may be necessary to

- Limit written problems
- Provide special lined paper or masks to mark problems which are copied.
- Require no board copying
- Provide nonskid rubber sheeting upon which writing paper and textbooks can be placed.[3]

When children tend to lose their place on a written problem page, teachers can:

- Clearly define the problem with heavily outlined boxes.
- Train students to keep one finger on the problem while calculating.
- Have students put a chip or "x" at the problem

TIP 7 Carefully Sequence Instruction in Small Steps, with Adequate Provision for Practice and Review

This approach is critical for students with learning difficulties, since extra developmental and practice time is necessary for both their understanding and retention of the concepts and processes. Breaking instruction into small, meaningful segments makes learning possible rather then overwhelming for these students.

Example 1: Basic addition facts. (Throughout, emphasize that "turn arounds" work. For example, 3 + 6 = 9; 6 + 3 = 9.)

Step 1. Count ons (For sums less than 10, emphasize counting on from the greater addend. Later this idea would be extended to all facts having 2 or 3 as an addend.)

Step 2. Sums equal to 10. (If necessary, have children "get the feel" from their fingers.)

Step 3. Doubles. (Mastery is critical before moving to step 4. Use auditory and visual cueing to help.)

Step 4. Facts related to doubles: Doubles + 1, Doubles + 2.

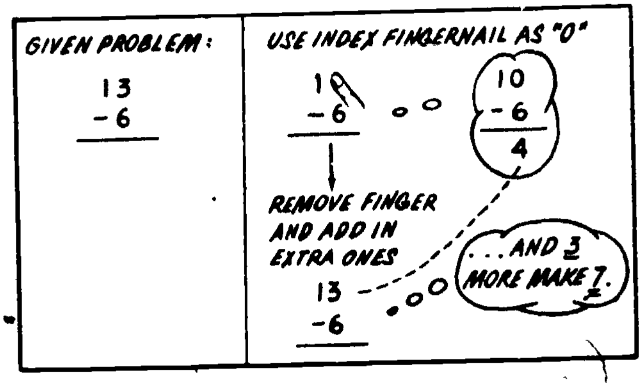
Step 5. Addition 9's. (Use patterns to help.)

Step 6. Only 3 (of the 100) facts left: 4 + 7, 4 + 8, 5 + 8. This sequence has been used successfully with all types of students, including learning-disabled children and educables.

Example 2: Basic subtraction facts: teen minuends.

Step 1. Be sure of these prerequisites:
  a. Children realize that 14 is 4 greater than 10; 16 is 6 greater, 13 is 3 greater, and so on.
  b. Children can subtract any number from 10.

Step 2. Now use 10 to help with teen minuends



Example 3: Basic multiplication facts. (Again emphasize the commutative of each fact throughout the sequence).

Step 1. 2's (Link to addition doubles.)

Step 2. 5's (Link to money, time on the clock.)

Step 3 9's (Use patterns or hands to help.)

Step 4. 9's and 1's.

Step 5. Only 15 facts left to learn:
  - Five are perfect squares (3×3, 4×4, 6×6, 7×7, 8×8);
  - ten others (3 × (4,6,7,8); 4 × (6,7,8); 6 × (7,8); 7×8). For these, refer to the suggestions of TIP 4 (example 3) and TIP 5 (example 1).

TIP 8 Work for Mastery One Step at a Time

Teachers can often anticipate common trouble spots in computation. Troublesome to the average child, these can be disastrous to students with severe learning difficulties who typically have a low frustration level. Teachers can maximize success experiences for these students by specifically focusing on rough spots. The one-step approach illustrated below has proved highly effective with many learning-disabled and other students. In each case, it is unnecessary to work the whole problem until the difficult step is mastered.

Example 1: Long Division Anticipated problem: difficulty in multiplying sideways. On worksheets children are asked just to perform the next step: that of multiplying.

$$\boxed{\begin{array}{r} 2 \\ 37\overline{)888} \end{array}}$$

Example 2: Renaming in subtraction of fractions. Anticipated problem: showing 13 8 rather than 11 8 after renaming. One-step assignment: just rename. One would of course, use materials to illustrate this renaming.

$$\boxed{\begin{array}{r} \overset{5}{\cancel{6}}\,\frac{\overset{13}{\cancel{3}}}{8} \\ -2\,\frac{7}{8} \\ \hline \end{array}}$$

Example 3: Division of fractions Anticipated problem: inverting the wrong fraction. To focus on inverting the divisor, children carry out just THE NEXT STEP of the given problem

$$\boxed{\frac{3}{4} \div \frac{1}{2} =}$$

Example 4: Decimal multiplication or division. The only new step placing the decimal point. Provide problems that are "worked" except for correct placement of the decimal point in the answer.

$$\boxed{\begin{array}{r} 2.5 \\ \times .03 \\ \hline 75 \end{array} \qquad \begin{array}{r} 18 \\ .22\overline{)3.96} \\ 22 \\ \hline 176 \\ 176 \\ \hline \end{array}}$$

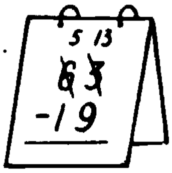
TIP 9

Provide a Sample Problem or a Cryptic Summary to Help Children Who Confuse or Forget the Sequence of a Computation

Example 1: Visual directional cues in sample problem.

$$\boxed{\begin{array}{r} \xleftarrow{\qquad} \\ 63 \\ -19 \\ \hline \end{array}}$$

Example 2: Flip chart for a sample problem (separate page for each step)



Example 3: Sample problem either started or completed at the top of worksheets

$$\begin{array}{r} 2\,9\,9\,0 \\ \cancel{3}\cancel{0}\cancel{0}\cancel{1} \\ -1284 \\ \hline \end{array}$$

Example 4: Cryptic summary the short of it for long division

1. Divide
2. Multiply
3. Subtract
4. Check
5. Bring down

TIP 10

Actively Involve Students During Instruction

- Make sure students clearly understand learning goals. Then give them a means to monitor progress made (e.g., use of a personal bar graph)

- Encourage students to describe situations which apply the mathematics to common, real-life uses

- Call on students to verbalize personal understanding of a concept or process

Intervention techniques such as these foster independence in learning and using mathematics an important goal for all handicapped students.

Footnotes

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[1] The October 1979 issue of *Arithmetic Teacher*, pp 6-9, makes some excellent suggestions for helping students count on from a given number

[2] If the child's learning disability is visual closure, use digits of one color, but underscore each triple This will help the child see the number as a whole rather than as disjoint digits. The underscoring will help trigger the recognition to "read as a 3-digit number " Then add the family name

[3] Dycem for example, can be purchased at orthopedic supply stores

8. TEACHING MATHEMATICS TO LD ADOLESCENTS

by John F. Riley and Fredricka K. Reisman

The authors suggest the use of the Reisman-Kauffman Specific Learning Disabilities in Mathematics Checklist as an aid in helping handicapped students select the mathematics they must learn for their chosen vocations in life. John F. Riley is an Instructor in Elementary Education and Fredricka K. Reisman is Professor and Division Chairperson of Elementary Education at the University of Georgia, Athens.

The term "learning disabled" (LD) is generally accepted as involving academic performance that is at least two years below expected achievement when other handicapping conditions are not apparent. Adolescence refers to the period commencing with puberty, about eleven or twelve, through eighteen or twenty years of age. This discussion focuses on mathematics instruction for learning-disabled adolescents.

In order to understand the mathematics needs of LD adolescents, a teacher must be familiar with normal adolescent development and with the unique characteristics of young people displaying the learning disabilities that influence mathematics acquisition. These two issues are outlined next.

NORMAL ADOLESCENT DEVELOPMENT

Smith and Payne (1980) provided a summary of descriptions of adolescents as follows:

> ...persons at this stage of development make a gradual shift of social orientation away from the family to the peer group (Pollard and Geoghehan, 1969).
>
> Youngsters in this group begin to seek in earnest to develop an identity of their own (Erikson, 1963).
>
> ...the adolescent begins to attain physical and social maturity and she he moves from a simply conforming person to a more self-governing individual It is at this stage that age-mates and models assume a position of great importance in the lives of these youngsters.
>
> ...the primary lessons of youth during this period are social and emotional, rather than intellectual ...specific tasks associated with adolescent development are. 1. achieving new and mature relations with age-mates of both sexes, 2. achieving a masculine or feminine social role, 3. accepting one's physique and using the body effectively, 4. achieving emotional independence of parents and other adults, 5. preparing for marriage and family life, 6. preparing for an economic career, 7. acquiring a set of values and an ethical system as a guide to behavior—developing an ideology, 8. desiring and achieving socially responsible behavior (Havighurst, 1972).

Compton (1978) distinguished between early adolescents, ages ten to fourteen, and late adolescence, and used the term "transescent" a word coined by Donald Eichhorn (1965). This distinction underlies reasons for establishing middle schools rather than high schoolish junior high organizations. Compton suggested that a general characteristic of a program based on the nature of the transescent should involve instruction that focuses on facets of a theme so that "distinctions between the content fields would be blurred or non-existent (as they are in real life)."

Adolescent theory has asserted that this period is one of "storm and stress." Terms such as "identity crisis" and "generation gap" are related to the storm and stress phenomenon. However, empirical studies have failed to support the storm and stress condition.

Coleman's (1978) research led him to formulate a "focal theory" of adolescence. According to his theory, concern for the traditional issues of adolescence, *e.g.* sex-appropriate role behavior, achievement of a sense of identity, and personal commitment to some ideology, set of values, occupation, or life-style, peak at different times, allowing many adolescents to deal with them one at a time. Those adolescents who have to deal with several crises simultaneously are the ones most likely to have problems. For Coleman the resolution of one crisis (or the completion of one stage) is not necessary for beginning another. In addition, there are no fixed boundaries between stages or crises, they are not linked to any age or developmental level, and the sequence in which they may be encountered is not immutable. Regardless of the theory one accepts, the classroom teacher is faced with dealing with individuals who are undergoing tremendous changes, both psychologically and physically.

LD ADOLESCENTS

Jacks and Keller (1978) stated that "adolescence does not miraculously bypass the child with a learning disability." The LD adolescent carries a double burden, suffering from being "special" and "different" at a time when conformity and peer acceptance are most important. He or she may become a nonconformist by circumstance rather than by choice. The pressures and experiences of adolescence are the most crucial ones confronting these students—more crucial, according to the teachers, than success in schoolwork. In addition, the adolescent with a learning disability has seldom developed the coping strategies needed, a situation due in part to low self-esteem.

Lower self-esteem among LD adolescents has been found by Rosenberg and Gaier (1977). Scores on the social self-peer subscale of the Coopersmith Self-Esteem Inventory showed a significantly more negative self-image for LD adolescents than for "normal" adolescents

Lerner (1976) discussed the effect of a child's educational experience and stated that persistent learning problems and negative attitudes toward learning are often accompanied by emotional and social problems.

Epstein and Cullinan (1979) stated that LD adolescents fail "to achieve one or more developmental-educational goals to an acceptable extent within an acceptable period of time." These goals include social participation, intellectual competence, community contribution, and career preparation.

Wiig and Semel (1974) found that when LD adolescents were given tasks involving comparatives and spatial and temporal relationships, they showed deficits in auditory comprehension, logical processing, and semantic coding.

Reisman and Kauffman (1980) identified generic influences on learning mathematics. These are grouped into four major classes: cognitive, psychomotor, sensory and physical, and social and emotional. They then present instructional strategies that may be applied to various topics included in the K-12 mathematics curriculum. The instructional strategies were developed in relationship to generic influences on learning that include the following:

- Cognitive Influences on Learning Mathematics
    - Rate and Amount of Learning
    - Speed of Learning in Relationship to Mathematics Topic
    - Ability to Retain Information
    - Need for Repetition
    - Ability to Learn Symbol Systems
    - Size of Vocabulary
    - Ability to Form Relationships, Concepts, Generalizations
    - Ability to Attend to Salient Aspects of a Situation
    - Use of Problem Solving Strategies
    - Ability to Make Decisions and Judgments
    - Ability to Draw Inferences and Conclusions
    - Ability to Abstract and to Cope with Complexity
- Psychomotor Influences on Learning Mathematics
    - Perceptual-Motor Impairment
- Sensory and Physical Influences on Learning Mathematics
    - Sensory Limitation
    - Low Vitality
    - Fatigue
    - Physical Impairment
- Social and Emotional Influences on Learning Mathematics
    - Degree of Independence
    - Attention Deficits
    - Motivation
    - Anxiety
    - Coping with Exceptionality

Reisman and Kauffman (1980) developed a checklist for evaluating a student in terms of generic influences on learning mathematics. This checklist is made up of three columns which are described below

- The first column includes specific learning disabilities in mathematics. These are grouped as Reasoning, Problem Solving, Orientation, Motor Performance, Attention, Perception, and Affect.
- The second column on the checklist identifies mathematics topics that are most affected by the particular generic influence on learning.
- The third column summarizes instructional strategies most appropriate for either circumventing weaknesses or facilitating the strengths of a learner.

A portion of the Reisman-Kauffman Specific Learning Disabilities in Mathematics Checklist (SLDM) is reprinted here as Figure 1 The purpose is to show that

FIGURE I
INSTRUCTIONAL STRATEGIES FOR SLDM: CHECKLIST

SLDM (Check box if displayed by student)	Related Mathematics Topic	Instructional Strategy (IS)
☐ Does not appropriately sequence occurrences or objects	Sequencing Order Seriation Computation Seriation extended	Present small amounts chunking Incorporate redundancy Use cues Provide complex and or subtle sequencing activities Use structured algorithms
☐ Inability to make choices and decisions	Judging and Estimating Computation	Control number of dimensions that define a linear sequence Emphasize patterns Incorporate incompleteness to activate creative potential Reinforce attention to a relevant dimension Use pupil questions Use structured algorithms
☐ Does not make appropriate inferences from data and draws inappropriate conclusions	Judging and Estimating Cause-effect	Provide complex or subtle sequencing activities Incorporate incompleteness to activate creative potential Reinforce attention to a relevant dimension Point out relevant relationship

instructional strategies may be developed for teaching mathematics to adolescents who display deficiency in one or more generic influences on learning - especially in those factors related to learning mathematics

The following section is a description of an LD adolescent for whom all three boxes on the SLDM Checklist were marked. Note that his weak areas in mathematics are apparent in the second column entitled "Related Mathematics Topic."

AN LD ADOLESCENT

Johnny, age 12, exhibited characteristics associated with LD adolescents. For example, he failed to achieve the first two goals discussed above by Epstein and Cullinan. His social participation was immature, his expected achievement was at least two years greater than his actual achievement He displayed some basic mathematics weaknesses including a lack of understanding of place-value relationships, poor computation skills, inability to perform selected measurements (*e.g* time, using a ruler, measuring capacity), and interpret graphs. Johnny was hyperactive and distractible[1] He displayed excess physical activity in relationship to the demands of a situation. He had a short attention span, and he attended to irrelevant stimuli in learning tasks. This distractibility appeared to interfere with his ability to engage in sequential and analytical thinking. These behaviors also put the other students off and inhibited his socialization with his peers In addition his written computations were messy and he appeared spatially and temporally disoriented. It seemed as if Johnny were like some powerful animal, trying to fight its way out of an entangling trap. If he could stop struggling against himself, stop repeating the mistakes he had made, he might be able to capitalize on some strengths he had and begin to work his way free.

And Johnny did have some strengths. He had a strong motivation to work He was likable, could see humor in situations, and received reinforcement in athletics

Johnny's strengths in mathematics were well developed He did well in those areas that afforded him success Of particular note was the area of fractions, which was subsequently used by his teacher as a vehicle for helping him to improve his peer relations as well as his sequential and analytical thinking.

In conclusion, the selection of those topics that will best serve the needs of an LD adolescent must be made in relationship to both academic and vocational considerations. Selection of topics must be based on a double criterion that mathematics which LD adolescents can learn and that which they should learn.

Footnote

[1] Kauffman (1977 p. 146), summarized in Reisman and Kauffman (1980), defined the following terms

Hyperactivity or *hyperkinesis* involves excessive motor activity of an inappropriate nature

Distractibility is the inability to selectively attend to the appropriate or relevant stimuli in a given situation or overselectivity of attention to irrelevant stimuli

Impulsivity is disinhibition or a tendency to respond to stimuli quickly and without considering alternatives

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9. REVERSE MAINSTREAMING WITH MICROCOMPUTERS IN MATHEMATICS

by Betty Iossi

Reverse mainstreaming is a creative idea for schools that house the microcomputers in special education classrooms in accordance with funding requirements. Betty Iossi's unique reversal role certainly has many pluses besides acquiring new mathematics skills, such as respect of classmates, self-esteem, and getting to know schoolmates in a familar environment. Formerly a teacher of special education in the Ridgewood, New Jersey, public schools, the author teaches in the Redwood City Public Schools in California

Two students playing "Baseball" are intent on the computer's screen in front of them. One is a fifth-grade girl from the regular class down the hall. The other is a neurologically impaired (NI) boy who is assigned to this self-contained special education classroom with other NI students his age. He is teaching her to use the PET Commodore Microcomputer. This is not the usual procedure for integrating handicapped students with their nonhandicapped peers. Instead of this special education student being mainstreamed out of his classroom, a fifth grader from the regular class is coming to his class to receive instruction. What is occurring is reverse mainstreaming.

The PET, as it is fondly called, is helping to make this process a smooth one. The strategy was to give priority training on the PET to the NI students. Not only were they thrilled about acquiring this new skill, but it also gave them a degree of prestige within the school. Once the NI students demonstrated the ability to run a program independently, they became the teachers, tutors, and helpers of the fourth, fifth, and sixth graders roles they know well from being on the receiving end of such services. The handicapped students express good feelings about having friends come to their classroom for a change. It raises their self-esteem. They feel important knowing how to interact with a computer and gain the respect of their classmates as well. Knowing about computers is a much sought after and desired skill. Nonhandicapped students have the opportunity to get to know a handicapped student and have the experience of being inside a special education classroom.

The role of the PET is to supplement rather than supplant the regular math curriculum. The current math programs offer different formats to practice previously taught math concepts. The PET is not used to teach new concepts. For example, the fourth graders had been taught the procedure for long division. The commercial PET tape entitled "Divide" was introduced at that point. A long division problem comes on the screen. At each step of the process, the correct digit must be typed in the precise place

$$1 \quad \begin{array}{r} * \\ 4\overline{)844} \end{array} \qquad 2. \quad \begin{array}{r} 2 \\ 4\overline{)844} \\ * \end{array} \qquad 3. \quad \begin{array}{r} 2 \\ 4\overline{)844} \\ \underline{8} \\ * \end{array}$$

Wherever the flashing star appears, the student must type in the correct digit. The process continues in this fashion until the problem has been solved. When an error is made, the computer indicates so immediately, and will not proceed until the correct answer has been typed in. If, after several attempts, the answer is still wrong, the PET flashes what it should be with a brief explanation. The students are actively involved in that they type the numerals and sometimes the operations, in order to activate the machine. Immediate feedback is received, and corrections must be made on the spot. Motivation is high. Students unknowingly are getting rapid drill practice on their facts when they think they are just playing a math game.

The PET offers the opportunity for individualization of the math program as well. The programs commercially available offer a range of difficulty levels from drill on basic facts to problems requiring conversion of fractions to decimals. Within the programs there are sometimes varying degrees of difficulty, also. The favorite, "Baseball," for instance, offers a choice of three levels of difficulty and operations addition, subtraction, and multiplication. So, within a classroom, one team of players may be reviewing basic level 1 addition facts, while others may be working problems at the level 3 multiplication setting, which requires multiple digit products.

Several of the math games can be played by two players. This encourages socialization within a structure. The rules are clear and the parameters of the game well defined. The computer acts as the referee or umpire. In the game of "Baseball" the PET occasionally makes a play at first base after the batter has made a hit, thus causing an out This chance factor provides a more interesting game for the less able student, who might otherwise have to wait a long time for a turn at bat

Selection of appropriate tapes is important during the teaching stages. Two tapes have been especially good for this. They are played in a cooperative manner. Both players work toward matching the skill of the computer, which involves strategy One is called Hurkle. A Hurkle is hidden in a grid on the screen. The players take turns guessing the coordinates of the Hurkle. After each guess, the PET gives directional clues such as "Go northwest," or "Go south " It reinforces the concepts of directionality using x and y coordinates, in addition to encouraging deductive reasoning. If the Hurkle is not found after five tries, it reveals its location.

Another good program to use during the initial teaching stages is called Hanoi. The students work cooperatively to move a series of discs from one pile to another. Only one disc may be moved at a time, and a larger disc may not be placed on a smaller one When the task has been completed, the PET indicates the solution using the fewest possible moves. The players try to match the skill of the PET The most difficult level requires juxtaposition of seven discs on three poles

HANOI - LEVEL 1

SUMMARY

One way of dealing with handicapped and nonhandicapped children within one classroom is through reverse mainstreaming, whereby students from the regular classrooms come in to the special education classroom The personal microcomputer is an effective way of doing this. Teaching their peers has enhanced the self-esteem of the NI students, developed motivation for facts drill, and fostered an atmosphere conducive to socialization Careful selection of tapes is important for the success of the program It requires little supervision from adults

The following is a partial annotated list of appropriate math programs for use by handicapped and nonhandicapped students in the classroom

1. Hurkle - use of grid coordinates
2. Rounding - to 1's, 10's, 100's, 1 10, and 1 100 places
3. Snoopy - introduction to negative numbers
4. Baseball - basic addition, subtraction, and multiplication facts
5. Dart - involves some estimating skills with basic operations
6. Divide - step-by-step procedure for long division
7. Arith - regrouping in addition, subtraction, and multiplication
8. MI DII - combining multiplication and addition skills (3x9+5)
9. Add - instruction in carrying
10. Hanoi - realigning discs

10. THE STUDENT WITH EXCEPTIONAL EDUCATION NEEDS AND THE CALCULATOR

by Kathryn Dietrich-Allen and Henry S. Kepner, Jr.

Kathryn Dietrich-Allen and Henry S. Kepner, Jr., have a fresh outlook for all teachers of mathematics in their use of the calculator by blind, paralyzed, deaf, retarded, and nonverbal students, and particularly by students with learning disabilities. Among the latter group, the authors focus on those with short-term memory problems, visual distractions, lack of eye-hand coordination, and emotional disturbances. The authors show that the calculator will allow the exceptional student to experience the exhilaration that comes from solving problems correctly. The tougher the problems, the greater the satisfaction and feeling of independence for the handicapped youngster. Kathryn Dietrich-Allen is a former gradate student and Henry S. Kepner, Jr., is an Associate Professor in the Department of Curriculum and Instruction at the University of Wisconsin-Milwaukee.

To date little has been written about the use of calculators by students with exceptional needs. Most articles on calculators have stressed enrichment and exploration activities. It is the authors' contention that the calculator may be the most important mathematical tool for many students with exceptional needs. For it is the calculator that can allow individuals to assume a more normal, independent life as an adult.

Chandler (1978) summarizes an earlier study by Browing which finds "Arithmetic to be the most critical of basic academic skills important to the working retardate." This argument is based on the fact that so much of daily life deals with problem solving, time, money, measurement, counting, and basic computations. To live a life with the least restrictions imposed, competence in a variety of arithmetic skills and applications is necessary.

Capps and Hatfield (1977) point out that at a rate of 40 minutes per day for arithmetic instruction, the typical educably mentally retarded student spends one sixth of [illegible]e in grades one through eight trying to master the computational algorithms for whole numbers. Even with this great expenditure of time, exit performance usually does not exceed a fourth-grade level. At this level of performance, the student shows little confidence or ability to handle applications.

For students with a variety of physical handicaps, the calculator allows the performance of computations which can be difficult to master with the standard paper-and-pencil procedures. For students with a variety of emotional disabilities, the calculator can help minimize the frustrations students experience in trying to perform complicated, sequential procedures, such as long division or multidigit subtraction with regrouping.

For all students, elementary mathematics has three components: basic concepts, computational skills, and applications. The extent of the use of calculators will vary depending on a student's difficulties. For many special students, the introductory concepts, like addition or multiplication, and the basic facts, like 7+3 and 6 x 5, can be mastered without depending on calculators. However, the mastery of multidigit computations without a calculator may never be realistic. For such students an emphasis on concepts, practice with basic facts, and calculator performance of algorithms is appropriate. This use of calculators can save student success and self-esteem.

For some students the basic facts, like 7 x 5, seem impossible to master. The calculator provides a highly acceptable and accessible tool for these students. In the past the use of charts or tables was allowed. Now the calculator, a tool used by most adults, can be employed without drawing attention to the individual's memory deficiency.

From studies done with regular education students, Rudnick and Krulik (1976) and Suydam (1979) report that there is no decrease in other areas of mathematical learning for students who use the calculator. Thus, what we know so far suggests that the use of calculators by exceptional students may provide great gains with minimal loss.

The extensive time now spent on computational facts and algorithms can be applied to improving understanding of mathematical concepts and applications. Of special importance are numeration concepts, estimation skills, geometric properties, and consumer skills. Frequently these are omitted because of the time spent on computational performance.

While some teachers express concern about a student's depending on calculators, the calculator actually provides the opportunity or student independence in a mainstream setting and later as an adult. Schnur and Lang (1976) found that simply having the calculator present, with no specific instruction at all, was enough to aid a group of students with special needs in learning how to compute without a calculator. Carpenter, et al (1980) found that only 46 percent of 13-year-olds and less than half of 17-year-olds correctly computed 28)3052 without a calculator. However, one half of the 9-year-olds obtained the correct result using a calculator. Most of the 9-year-olds had only minimal exposure to the division concept. Thus, the building of computational proficiency by means of the calculator can parallel concept development.

In seeking to reinforce basic facts, calculator-like devices such as "The Little Professor," "Dataman," and the "Digitar" provide important drill experiences. As one source of drill, these devices are extremely effective. The device provides instant feedback to the student in a consistent, nonthreatening way. Many students find it easier to be told repeatedly of errors by a machine than by a teacher, classmate, or parent. The tireless, consistent electronic-drill device provides a constructive opportunity for students who frequently become disruptive in groups where they receive inadequate attention or cannot succeed.

The ultimate goal of mathematics instruction for all students is problem solving. The traditional curriculum in the United States has placed computational mastery as a prerequisite to problem solving. Hence, many students with special needs have seldom been placed in the appropriate setting for problem solving in mathematics.

In the typical mathematics word-problem context, students are expected to produce the correct answer. This expectation has two parts: (1) decide on the appropriate arithmetic procedure and (2) perform the calculation correctly. For numerous students the selection of the appropriate procedure was not recognized because errors in computation led to a wrong answer.

Jaggard (1977) observed that many low-achieving seventh and eighth graders "freeze" in solving problems because of past failures in computation skills. Once freed from these failures, these students were able to become problem solvers. Use of the calculator does not guarantee success in problem solving, but it allows the student to focus on the major issue—problem solving. The calculator is available for trial-and-error, generation of hypotheses, and checking of conjectures.

Nationally there is a rapid acceptance of the calculator as a valuable mathematics tool for general mathematics students who are weak in computation. No other teaching strategy or device has improved student performance and self-concept for students who have suffered years of computation failure. This reprieve from certain failure in mathematical settings cannot be overlooked for students with special needs.

Many studies, e.g., Ockenga (1976), Gawronski and Coblentz (1976), Sullivan (1976), Schnur and Lang (1976), have found calculators to be motivating for students. This appears true over long, as well as short, periods of time. The calculator is becoming a universally accepted tool in our society. In the second national mathematics assessment (NAEP, 1979, p. 77), 86 percent of 17-year-olds reported that they or their family owned a calculator.

Beisse, Brougher and Moursund (1976) point out that there are three major modes of dealing with arithmetic in our lives: mental, paper and pencil, and the calculator. Need we expect every student to become proficient in each mode? For students with special needs, one mode may be sufficient for the tasks at hand.

PRACTICAL CONSIDERATIONS

In using calculators, attention to special needs must be given. Thus, in a given situation one student might be encouraged to use a calculator while another is discouraged. A word about individual differences may be appropriate to the students involved. Remind students that we continually try to minimize individual difficulties through the use of hearing aids and glasses, as well as calculators.

Many students with physical and perceptual problems have great difficulty in writing out numerical exercises and all the intermediate steps required in paper-and-pencil procedures. These difficulties almost insure student failure because of the tedious task of recording results.

For most of these students, the calculator should have large keys, i.e., typewriter size, and a large digital display. The small pocket calculators are hard to operate for students with a variety of motor and perceptual difficulties. Although more expensive, calculators with a paper tape are necessary for some students. The capacity to check entries for errors, such as digit reversals, to examine intermediate results, and to keep a copy of the final result are crucial.

For students with perceptual and short-term memory difficulties, a calculator which displays digits from left to right may be necessary. For these students the entering of 937.25 can be very confusing. On the standard calculator, the "9" first appears at the right edge of the display, "where the '5' should be!" In any case practice in entering and reading numerals with the calculator is necessary.

CONCLUSION

It is important that teachers do not equate performance on computational procedures with thinking in mathematics To consider a mathemathical problem and devise a plan for its solution requires thinking To analyze a consumer problem and set up a method of solution also requires thinking. However, to perform a long division exercise requires a memorized procedure based on training, not thinking. Most adults in this country have memorized a paper-and-pencil procedure How would they do if someone denied them a pencil? The calculator is an alternative to these procedures It will not remove the need for student thinking and reasoning in dealing with quantity. It is present to perform an operation chosen by the user

Calculators and computers have much to offer exceptional education students both now and in their adult lives A blind person uses a talking calculator, a paralyzed, nonverbal student communicates with a stylus and a computer, and a deaf individual visually reads telephone messages recorded by computers. The calculator is the first computational device available to all

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11. BANKING MATHEMATICS FOR THE CLASSROOM WITH EMH PUPILS

by Gayla Rice

Banking mathematics is necessary for all handicapped students who wish to acquire independent living skills. Gayla Rice presents a very realistic unit on banking. She further adds the personal elements of joy and satisfaction by holding a class auction wherein the pupil making the successful bid must pay by check. The author is a Teacher Coordinator at Dickson High School, Dickson, Oklahoma

The major responsibility of mathematics teachers with Educable Mentally Handicapped (EMH) high school students in their classes is to prepare them to live independently in our society. Because of this, my math program has been organized to include the skills necessary for survival and everyday living. One of the most important skills, the handling of money, has necessitated a large math unit on basic banking competencies, *i.e.* checking accounts. By following a certain sequence of activities, we have succeeded in motivating students and making them excited about writing checks and keeping correct checking account balances. It should be noted here that in order to succeed, the student must have the ability to use basic addition and subtraction processes

Activity 1

Before actually getting into the manual procedure for opening and maintaining a checking account, there are many general areas that need to be discussed with students Therefore, to introduce the unit on banking the teacher should have an understanding of banks as the first objective. The beginning discussion should center around the following outline.

1. What is a bank (definition)?
2 Why use a bank?
3 Choosing a bank
    a. Location
    b. Services offered
    c. Choosing between a state or federal bank

Activity 2

Before students can write checks, they need to know how to read and spell numerals. The teacher should hand out a sheet with the spelling words and make flash cards for the students to use for drill. Students should also be encouraged to make their own flash cards. Success has also been achieved by allowing the student to write the words on a chalkboard using repetition as the key factor.

Activity 3

The final step in learning to spell the numerals is a taped lesson for individual use The teacher can record a spelling test for the student to take when ready The taped lesson should have the students check their work and then report to the teacher with the test results when finished.

Activity 4

Another activity in the sequence to introduce banking will be a guest from a local bank. The bank representative should agree to talk to the students about banking services and be willing to discuss what banking problems might arise and how they are handled. The representative should also explain in a step-by-step procedure how to open a checking account To reinforce the discussion the teacher will review the steps with the students after the visitor leaves

Activity 5

The next activity involves going on a tour of a local bank. This can be arranged with a bank in the area. The person in charge of taking the class through the bank will show the students what happens to a check when it is written and how it eventually ends up back with the writer While on this tour they will also see the various departments of the bank and have each one explained to them in relation to how it helps the customer.

Activity 6

This activity will start with a discussion and review of how to open a checking account and will provide practice in filling out a form to open a checking account. For this activity the student will need a worksheet showing what most forms will ask and providing a space to fill in the needed information

Activity 7

The next step in the progression is to discuss the purpose of a deposit slip. The students will be shown how to fill out a deposit ticket. By asking local banks the teacher may obtain deposit tickets for the students to use for practice. There are different kinds of deposit tickets, so it is wise to let them practice on different kinds in order to see that the procedure is basically the same regardless of how the slips look.

Activity 8

This activity is a continuation of the one prior but was made into a separate activity because it involves night deposits. Since night deposit slips require more information from the customer, the students should be able to practice filling these out separately.

Activity 9

Before students can understand how to balance their checkbooks, they have to know exactly what the word "balance" means in relation to checking accounts. To explain this, the word "balance" should be used with different meanings to see how they relate to each other. In this way students will come to understand the meaning of "balance" in a checkbook.

Activity 10

The next activity will begin by giving each student a check register. The purpose of check registers should be discussed and the students given concrete examples showing where to put the balance of their accounts. Each student will make up his or her own problems and practice entering the date, check number, to whom the check is written, and the amount of the check. The student will also enter deposits and get the register in balance. This is one of the most important activities in the banking unit so quite a lot of time will be spent here until the student understands the procedure. These registers may also be obtained from a local bank.

Activity 11

Next, the students will be given blank checks for practice. These may be obtained from a local bank. One good way to preserve a blank check is to paste it on cardboard and cover it with clear contact. Then the student may use a crayon to practice writing a check and wipe the plastic clean when finished.

Activity 12

The next activity involves practice in writing checks from a different angle. Students are given worksheets on which there are checks written incorrectly. The student must examine them and determine what is wrong with each check.

Activity 13

The last exercise should start with a discussion of how the bank keeps a tally on the balance of an individual's money and how everyone should do the same to make sure that his or her balance agrees with the bank's records. The bank statement is then introduced with an explanation of its use. Students will then be shown how to mark off which checks are recorded on the bank statement by the bank and how to get their check registers in balance with the bank. Blank bank statements may be obtained from a local bank.

The typical EMH student needs much reinforcement to retain the information and skills introduced. To motivate these students to use banking skills, we devised a plan whereby the students could use the checking process everyday. First, each student was given a checkbook containing blank checks and a check register. The teacher started a ledger sheet on each student in order to keep up with each balance. Daily work was graded as it was turned in, and an amount of money affixed to it. At the end of each day the student was allowed to make a deposit slip and an entry in his or her checkbook recording the amount of money earned. Money was also given ~~or taken~~ away for appropriate or inappropriate behavior. After recording the money each day the student would bring his or her checking balance up to date.

To provide more incentive, at the end of every two weeks the teacher would hold an auction of items such as candy, pencils, jewelry, perfume, *etc.*, and the students would bid against each other for the items they wanted. In order to pay for the item the student would then write a check for it and proceed to bring his or her check register up to date. To insure accuracy, the teacher would check his or her balance against that of each student at the end of every month.

Overall, this plan for teaching banking mathematics has been very successful. It is the hope of this author that this plan will be helpful in some other high school classroom with EMH students. Only by teaching our students basic math needed for everyday living will we be able to integrate them successfully into our society.

12. THE MECHANICS OF TELLING TIME

by Edwina Gramuska

Knowing how to tell the correct time is a very important mathematical skill, needed by the handicapped child for independent living. Edwina Gramuska has set some very practical guidelines for the educator who teaches a unit on time to the educable mentally handicapped child. The author is the School Placement Committee Chairperson for Cheraw Elementary School, Cheraw, South Carolina. (The author would like to express her thanks to Developmental Learning Materials, Inc., for the Moving Up in Time Kit #344, *without which she could not have formulated her program.)*

According to South Carolina State Law an Educable Mentally Handicapped (EMH) student's I.Q. shall not be higher than 70 on any given standardized intelligence test. As an EMH Resource Teacher in Cheraw, South Carolina, my students' I.Q.'s range from 50-70 with their chronological ages ranging from 10-13. All my students are being mainstreamed for the first time, which gives them the added incentive to master practical yet difficult math skills. Another interesting observation about this group is their need to use pencil and paper in addition to receiving a grade for their effort It is mandatory to monitor their daily progress and success.

Two very important guidelines we live by in our classroom are

1. Never say I can't
2. Success is fun and meant to enjoyed.

Never let a standardized score alone limit your thinking about an individual's ability. Push hard, love intensely, and lend a helping hand; the results will amaze you.

A content area of mathematics I've found both interesting and extremely challenging is teaching an EMH child how to tell time. The practical application is necessary and obvious. And the deeper level of being able to blend the concrete and the abstract is a great cognitive accomplishment for the EMH student.

Before beginning a specific program, we teachers discussed the need for learning how to tell time. When the students don't feel it is necessary or can't see its relevance, introducing the concept might be futile. Shortly before Christmas we began talking about the possibility of delving into such a demanding, "time"-consuming unit. After Christmas and Santa's delivery of many watches, emotions ran high and the time was right to begin our in-depth study.

The first step I took was to have a record sheet run off for each student. Then, to lay the foundation for this unit I used the *Moving Up in Time Kit* #344 by Developmental Learning Materials, Inc. In actuality the Kit is designed to be used on an individual basis as a reinforcer, but I was able to adapt it quite easily to my program. The introduction in the kit is divided into four categories, color coded in green We discussed the first three categories as a group activity.

a *Special Times* - 32 cards of this set are designed to introduce the child to the concept of time as it affects his or her life.

b. *Everyday Events* - The picture cards help the child to become aware of significant everyday events. Some examples are waking up, eating lunch, reading, and playing.

c *Holiday Happenings* - This section contains 10 picture cards relating to various yearly holidays such as Christmas, Thanksgiving, and the Fourth of July.

d *Seasonal Settings* - This early aspect of the program proved to be the most difficult for each class. At this point we deviated from the basic program.

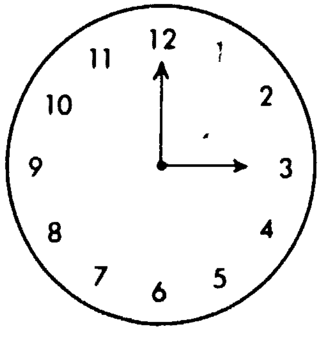
I brought in pictures most commonly connected with each of the four seasons. Through drill and daily copying activities, the students learned the names of the seasons. Each child was then given a particular season and an old magazine. The period's assignment was to find pictures related to his or her particular season. They were labeled and hung. We did this each day until each student had an opportunity to do each season at least once.

Now that the foundation had been laid we were ready to begin the mechanics of telling time We used the alarm clock in the classroom to discuss the hands, the hour, the minute, the direction they move. Each student then made a clock out of a paper plate. This tactile activity helped even the slowest child to feel the difference between the long and short hand, the direction in which they move, and how important it is to write the numbers in a certain place and not arbitrarily on the face Every clock was unique and a reflection of individual ability and personality In addition they've proven to be great fun.

We were now ready to begin Red Section "The Hour " In order for each child to work simultaneously every card was run off on a purple ditto (two on a page). They were stapled together according to letter A, B, or C Each set was put in a folder, labeled, and kept near my desk This section provides practice in reading the time from a clock as indicated by the position of the hands on the clock face. Also included in section C are those cards on which the student draws the hands on the clock to indicate the time as stated below the clock face The average EMH student could finish one set during a 50-minute period

Each day before actually working on the skill introduced by a particular sheet, I would outline it on an overhead transparency The students responded better to this mode than if I merely wrote the identical information on the board This is a question-and-answer period, the moment of discovery Each child takes a turn writing an answer on the transparency By the time the sheets are handed out, most of the students feel comfortable with the concept in addition to being quite proficient

Here is an example of a typical daily activity The students enter the room, get their folders, tablets, and pencils Each pupil then sits in an assigned seat On the transparency there is a sample which would look like this



The hour hand points to 3.
The minute hand points to 12.
It is 3 o'clock

The students will draw the clock and write the sentences in their tablets Each one will take a turn identifying the hour hand, the minute hand, and the actual time Together we make a generalization about o'clock Following this I will present a number of similar exercises, so that each student has an opportunity to complete at least one for the entire class When I feel sure that the majority understands the concept, a student hands out the work-activity sheets I am free to float around and give individual explanations when needed Each student then receives an individual record sheet with the following directions "The student is to check off each card as he or she works it by crossing out the number in the coppesponding box " This type of check system has proven to be extremely rewarding in its own right The student sees and feels progress as each becomes filled with yet an additional accomplishment

Previously I mentioned that paper plates were a great source of fun We would allow a few minutes at the end of each class period for telling-time bees Each student wrote his or her name on the board, then found a comfortable, private seat somewhere in the room I'd stand in front and call out a time such as 8 00 Each student positioned the hands on the paper clock. Everytime someone is correct, he or she receives a check Five checks equals one star to be placed on our star chart which is used to work towards game day on Friday It is an excellent way to reward success immediately and inexpensively

Another dimension I've added to the program is a written quiz after each color-coded section The results have been amazing As a result of the basic program and the enrichment activities, almost every child passes Any student who fails, repeats only troublesome areas and not the entire section

An award certificate is presented for mastery of a particular concept The principal has attempted to work closely in offering the students positive reinforcement for a job well done When a student has completed the entire program or an equivalent of 10 certificates, he or she receives a free Coca Cola form It's an added incentive not only to complete the program but also in the least amount of time in order that a new mountain may be climbed and another summit reached

I have described in depth one phase of the program I will now briefly outline the remainder of the cards, because all have been taught in basically the same fashion The Half-Hour (Blue) also provides three types of exercises using the same format as the Red Section The Quarter Hour Past (Yellow) and Quarter Hour To (Brown) help the student understand the quarter hour in the same fashion The Minute After The Hour (Purple) and the Minutes To The Hour (Orange) are designed to help the student read the five-minute intervals in a conversational tone Travel Time (Gray) refers to the five-minute intervals as they may be indicated on plane, train and bus schedules Although these activites may seem boring and routine, my experiences have shown that consistency and repetition will reap numerous successes

AFTERWORD

by Ellen Mary Brockmann

The importance of mainstreaming into the regular mathematics class goes beyond the needs of handicapped students, as it demonstrates the truth of a very old and important educational postulate -- good teaching is individualized. Mainstreaming, like all good teaching, requires the teacher to diagnose, understand, and respond to the myriad individual capacities, needs, interests, and concerns found in any group of children. Educators who serve the individual needs of regular classroom children can do the same for the handicapped youngsters.

Teacher attitudes will probably determine, as much as any other variable, whether or not mainstreaming will work successfully. For the temperament of the instructor will eventually shape all aspects of the mathematics program in the classroom.

It is the hope of all who contributed to this book that you, the creative teacher, share your joy and knowledge of mathematics with all of your students thus improving the quality of their lives through a greater utilization of mathematical skills and technology.

RESOURCE CENTERS

Below is a partial list of organizations that provide printed materials for the teacher interested in mainstreaming, the handicapped, or mathematics. There are many, many more. These, however, constitute a good starting point for whoever wishes to go further into the subject. Each organization will supply a publications list upon request.

Accent on Living
P.O. Box 700, Bloomington, Illinois 61701
Accent's special publications will bring you topics about the handicapped usually not found elsewhere.

Association for Children with Learning Disabilities
4156 Library Road, Pittsburgh, Pennsylvania 15234
ACLD and its state affiliates work directly with school systems in planning and implementing programs for early identification and diagnosis, as well as remediation in resource and special classroom situations.

Closer Look
U.S. Department of Education, Bureau of Education for the Handicapped, Box 1492, Washington, D.C. 20013
Closer Look has reading lists to help you learn more about children and youth who have handicaps.

Council for Exceptional Children
1920 Association Drive, Reston, Virginia 22091
The CEC produces up-to-date material on the handicapped.

Educational Resources Center
1834 Meetinghouse Road, Boothwyn, Pennsylvania 19061
ERC is devoted to current educational practices which illustrate the broad concept of the least restrictive alternatives in educational settings.

National Council of Teachers of Mathematics
1906 Association Drive, Reston, Virginia 22091
NCTM publishes books ranging from teaching methods and study techniques to tests and contests, from computer-assisted instruction to do-it-yourself teaching aids. It also serves as publisher for *Arithmetic Teacher*, *Mathematics Teacher*, and *Journal for Research in Mathematics Education*.

National Education Association
1201 16th Street, N.W., Washington, D.C. 20036
NEA publishes books and produces audiovisual materials for all areas of in-service training. The publications focus on individual content areas and are for teachers of all grade levels, from elementary through higher education. NEA also publishes *Today's Education*, with a special edition for mathematics teachers.

Training and Resource Directory for Teachers Serving Handicapped Students K-12
Office for Civil Rights, 330 Independence Avenue S.W., Washington, D.C. 20201
This directory has been compiled to alert teachers in the regular classroom to resources that will assist them in accommodating students with handicaps.

Prepared by
Ellen Mary Brockmann

NEA Resolution adopted by the NEA Representative Assembly

B-25. Education for All Handicapped Children

The National Education Association supports a free, appropriate public education for all handicapped students in a least restrictive environment which is determined by maximum teacher involvement However the Association recognizes that to implement Public Law 94-142 effectively

a The educational environment, using appropriate instructional materials, support services and pupil personnel services, must match the learning needs of both the handicapped and the nonhandicapped student
b Regular and special education teachers, pupil personnel staff, administrators and parents must share in planning and implementing programs for the handicapped
c All staff must be adequately prepared for their roles through in service training
d The appropriateness of educational methods, materials and supportive services must be determined in cooperation with classroom teachers
e The classroom teacher(s) must have an appeal procedure regarding the implementation of the individualized education program, especially in terms of student placement
f Modifications must be made in class size, using a weighted formula scheduling and curriculum design to accommodate the demands of each individualized education program
g There must be a systematic evaluation and reporting of program developments using a plan that recognizes individual differences
h Adequate funding must be provided and then used exclusively for handicapped students
i The classroom teacher(s), both regular and special education must have a major role in determining individual education programs
j Adequate released time or funded additional time must be made available for teachers so that they can carry out the increased demands placed upon them by P L 94 142
k Staff must not be reduced
l Additional benefits negotiated for handicapped students through local collective bargaining agreements must be honored
m Communications must be maintained among all involved parties
n All teachers must be accorded by law the right of dissent concerning each individualized education program, including the right to have the dissenting opinion recorded
o Individualized education programs should not be used as criteria for the evaluation of teachers
p Teachers, as mandated by law, must be appointed to state advisory bodies on special education
q Teachers must be allowed to take part in the U S Office of Special Education and Rehabilitative Services on-site visits to states Teachers should be invited to these meetings
r Incentives for teacher participation in in-service activities should as mandated by law be made available for teachers
s Local associations must be involved in monitoring school systems compliance with P L 94-142
t Student placement must be based on individual needs rather than space availability (78 80)

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