INVESTIGATIONS IN MATHEMATICS EDUCATION

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An editorial comment . . .

Mathematics Education Research: Expectations, Problems, and Directions

David Wheeler
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I think it was Hilbert who said that mathematicians work at mathematics all week and worry about the foundations of mathematics on Sundays. Mathematicians are fortunate indeed to have so much security embedded in their methods and traditions that they know, most of the time, that they may get on with the job and let the justification for their activity look after itself. Mathematics educators are not so lucky. They not only have to worry about the foundations of their activity but also about its effects. And the news is not good from either front (see Kilpatrick [1981] for a critical analysis of the present situation).

I comment briefly here on three aspects of the general research situation: on expectations regarding research, on particular difficulties that afflict mathematics education research, and on a neglected component.

1. Perhaps it is healthier to expect too much from mathematics education research than too little, but let's put the prospects into perspective. Mathematics can provide us with a useful comparison case. It has been developed over a very long period of time and it is undeniably one of humanity's success stories. Nevertheless, as any mathematician will admit (e.g., Hammer, 1969), mathematics, by its nature, can only handle a very small fraction of the questions that our world proposes to us, and the mathematics that has been developed so far is only a small fraction of the mathematics that we can envision. What appears from one point of view to be an outstanding achievement is, from another angle, quite minor progress. When we consider mathematics education and how far it has developed, perhaps we should remember its much shorter history and be consequently more realistic in our expectations of what it can achieve today - or even tommorrow. Our situation is not made more cheering by the fact that not much expertise is needed to <u>ask</u> very pointed questions pertinent to mathematics education. So everyone is as aware of the state of our ignorance as we are ourselves (whereas

only the mathematicians know how ignorant they are). We can be easily tempted into trying to prove, by overplaying our hand, that we are achieving something useful. But this only serves to reinforce the prevailing mismatch between expectations and achievements. Ignorance about the answers to very difficult questions is really nothing to be ashamed of, and it is better to use our energy on them rather than waste it on trying to fool others about what we have achieved.

2. There are certain difficulties intrinsic (or endemic) to research in mathematics education which we do well to recognise.

(a) Mathematics education is a field of study, not a unified discipline. Any question that is considers seems to belong, wholly or in part, to one or more other subject fields as well. It has no research paradigms or research methodologies s own and borrows from other fields and disciplines.

(b) The attea onsider mathematics education as a science appears to be thwarted by th t that to every general result it tries to claim, one can always find counter-examples. (This difficulty, however, it shares with all the human sciences.)

(c) Mathematics education is an applied field of study -- i.e., its results can be expected to hold implications for practice. But research in mathematics education (unlike research in medicine or engineering) takes place in a context where only a very small minority of people really believe that research is necessary, or that it can provide guidleines for practice. Given this climate it is not surprising that serious research is not received seriously -- and, conversely, that the lack of a serious reception tends to make research unserious too.

(d) There is little agreement among educators about the foundations of the central questions of mathematics education. Researchers therefore tend to have very little common ground, so there is little in the way on an accumulation of effort or result. There is almost no sense of progress in the field as a whole and much "reinventing of the wheel".

(e) Questions in mathematics education partake of two kinds of complexity. There is the "theoretical" complexity that arises from the lack of precise boundaries to the field of study. Any moderately large question in mathematics education often seems to be a mathematical question, a psychological question, a philosophical question, a sociological question ... (and so on)

... as well. There is also the "practical" complexity of mathematics education *in situ* -- i.e., the multiplicity of uncontrollable variables. The problem for the researcher is how to steer safely through these complexities -- not pretending that they do not exist and taking refuge in a form of reductionism, nor attempting to take on the full range of complexity and thereby making advances impossible to obtain.

Of course, these are not original observations and perhaps I should have introduced them by saying we do well to keep them in mind. They can be expressed in different language too. Simon remarks in the introduction to his book (Simon, 1981) that "Engineering, medicine, business, architecture and painting are concerned not with the necessary but the contingent -- not with how things are but how they might be -- in short, with design." (I wonder why education wasn't included in his list?) He asks, as by implication I ask too, whether a science of the contingent -- the "artificial" -- is possible. Simon says yes. I say I hope so.

3. In thinking about what research in mathematics education presently lacks, I find myself concluding that what it needs is not just more of the same but more of something different. Researchers rarely seem to study questions with an epistemological orientation, yet it seems to me that it is in some form of epistemology that the foundation of mathematics education must be laid.

A question like "What part does language play in the learning of mathematics?" is an example of what I mean by a research question with an epistemological orientation. It's a very large question, of course, something that could occupy a researcher for several lifetimes. But one of my points is that research in mathematics education needs the underpinning provided by sustained concentration on such fundamental issues.

In a reasonably clear sense I can say that this question is not one for a linguist, or a mathematician, or a psychologist, or a philosopher, however useful some of their professional insights might be. The locus of the questions seems to me to be firmly within the ballpark of mathematics education. And I think that (in principle) the educator has two significant advantages as an investigator of this question: first, he is a professional observer of mathematics learning and, second, he is not essentially tied, as a theoretician or an experimenter, to particular disciplinary modes of inquiry.

This kind of question, it seems to me, cries out for an examination from first principles, an examination without any preconceptions about possible answers and without any commitment to methodological tools borrowed from other sources.

We can say (again in principal) that because educators have learned mathematics themselves and have observed others learning it, they already know a great deal about the matter. The problem is that they do not yet know that they know so much, or in detail what it is that they know. The study is difficult because it requires bringing into consciousness all those things which were in consciousness when learning took place, but which have since disappeared from consciousness; as well as those which may never have passed through consciousness and are nevertheless aspects of the functioning of people who have learned mathematics. But if language does indeed play any part in the learning of mathematics, then the researcher <u>can</u> catch it in action, for nothing that we experience is absolutely and forever irrecoverable.

But how to catch it!

I have to be tentative because if I knew more clearly what the procedure was I would spend my time following it and not writing about it. It is not introspection, though introspection can provide helpful clues. It is a kind of integrative action in which the researcher holds together (i) his awareness of himself as a learner, (ii) his analysis of the content of his learning, and (iii) his identification with anyone else who is in the process of learning what he has already learned. This integrated perception of the problem, this very special way of looking at it, can be applied in quite specific cases -- learning to count, say, or to add; learning to solve linear equations; learning the way the exponential function behaves: whatever topic may serve the investigation.

The second of the three strands is closest to traditional epistemology -- i.e., studying what it means to know (a specific) something. But epistemology does not necessarily concern itself with the process of <u>coming to know</u> -- though Piaget might claim that his genetic epistemology does. However, it seems to me a lack in Piaget's work that it doesn't really study "coming to know" but only "the development of knowing", and that this is the reason why his work, though immensely powerful, is not particularly instructive when applied to the classroom. The third strand of my tentative model is almost entirely missing in Piagetian studies.

I use the word "identification" in my description of the third strand because I cannot think of a suitable alternative, but I do not mean that the researcher achieves, or tries to achieve, a personal identification with a particular learner he may be observing. The process is more generalized than that. Because the researcher knows how people go about learning, and knows precisely the content of the challenges that the learner is facing, and has sensitively observed the behavior of particular children confronted with these challenges, he can create (or recreate) in himself a direct knowledge of what steps a learner, at a certain stage in his life, must take if the challenges are to be conquered. The researcher knows, as if he were in that situation himself, what the learner will do because he must.

There are assumptions, but no theories, in this method. It is truly a study from first principles.

I recommend anyone who wants to see a realized example of this kind of epistemological approach to look at a short paper in which Gattegno outlines the steps in which children come to know counting, numerals, and the first ideas of numbers (Gattegno, 1970).

Notes and References

1. Some of the ideas in this editorial have been put into circulation at a meeting of the Groupe des Didacticiens de la Mathématique du Québec on November 27, 1981.

2. Kilpatrick, Jeremy, "The reasonable ineffectiveness of research in mathematics education". For the Learning of Mathematics, 2, 2 (November 1981): 10-16.

3. Hammer, Preston C. "The role and nature of mathematics". Mathematics Teacher 57: 514-521 (December 1965).

4. Simon, Herbert A. The sciences of the artificial. Second Edition. Cambridge, Mass.: MIT Press, 1981.

5. Gattegno, Caleb. "Notes on a new epistemology: teaching and education". Mathematics Teaching 50 (Summer 1970).

INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1982

Becker, Joanne Rossi. DIFFERENTIAL TREATMENT OF FEMALES AND MALES IN MATHEMATICS CLASSES. Journal for Research in Mathematics Education 12: 40-53; January 1981.

Abstract and comments prepared for I.M.E. by FRANK F. MATTHEWS, University of Houston.

1. Purpose

The purpose of the study is to investigate the relationship between student/teacher interactions and the sex of the student. This is intended to highlight some factors of the school milieu which might have an impact on female attrition rates.

2. Rationale

One of the most important issues in mathematics education today is the determination of the causes of high attrition rates of members of minority groups in secondary mathematics classes. This study investigates the area of teacher/student interactions as a potential factor causing such attrition of female students.

3. Research Design and Procedures

The study utilized a two-pronged approach to the question. In one part, frequency counts of teacher/student interactions were taken, classified according to the Brophy-Good Teacher Child Dyadic Interaction System. The second approach utilized participant observation techniques to develop a non-quantitative description of the classroom setting.

The subjects were one Geometry class each for ten teachers located in two school districts. The classes were each observed for one week early in the Fall and one week at least four weeks later. Within each observation, data collection was alternated between the two approaches. All observations were made by the author with 11 involving a second observer to measure inter-rater reliability on the Brophy-Good instrument.

A two-way ANOVA with repeated measures was performed on six categories of interaction. The two factors were sex of student vs. week of observation. Descriptive statistics were provided for the remaining data.

4. Findings

For the six categories of data treated with ANOVA, three showed significant sex differences. All of these were in favor of the male students. The significant categories were: 2) Teacher choice of a volunteer to an open question, 3) Higher-order teacher questions, and 5) Teacher choice of those calling out responses.

The descriptive data are reported in three tables and a narrative discussion. The tabular results show differences in favor of male students in most teacher-initiated interactions vs. female students in student-initiated interactions. In fact, although the number of instances of discouragement was somewhat limited, almost all occurred with female students. As might be expected from traditional stereotypes, the female students tended to be quieter and more passive. However, the author points to several indications that this is situationally induced. Surprisingly contrary to stereotypic expectations, the disciplinary contacts were balanced, refuting the explanation that behavior problems cause unbalanced interaction. Finally, several instances of sex stereotyping in both the setting and materials are cited.

5. Interpretations

The basic finding by the author was that differential treatment did occur based upon the sex of the student. In addition, these differences generally worked in a positive way for males. The author proposes a three-step mechanism for sex interaction patterns:

1) Teachers start from sex-based stereotypes.
2) Teachers treat students differentially based on sex, favoring males.
3) Students respond differentially, reinforcing sex stereotypes.

The author then comments on the potential interaction of these steps; in particular, that students arrive with well-developed sex-role socialization and differential behavior patterns. These can, in part, cause the differential treatment.

Abstractor's Comments

My concerns about this study are both methodological and philosophical. First, there are several references to 100 classes when referring to the tabular data and its commentary. This tends to exaggerate the scope of the study, since it is, rather, 10 repeated measures of 10 classes. This is especially true in the note on sex balance. The author did use n=10 in the ANOVA, however. The selection criteria for participants is also open to question. The ratio of male to female mathematics teachers in the schools is 2:1 and is cited as typical. Yet, there are 7 female and 3 male teachers in the study, reversing that ratio. In other studies of sex-differentiated treatment, the sex of the teacher has been an interesting variable and might be relevant in this context also. While other data suggest that male blacks and female whites take mathematics at comparable rates and there were significant black populations in the schools studied, the classes observed were nearly all white. The mechanism proposed for differential selection should address this earlier attrition of blacks in this study.

Finally, there is some question as to the appropriateness of the response categories used. In particular, two of the three significant categories -- "Open questions" and "Call-outs" -- seem to involve essentially the same type of interaction behavior. In both cases the teacher initiates a question, a group of students indicates a response to that question, and the teacher selects a student to respond. The only difference is in the mechanism by which a student indicates a response.

My philosophical objections are of a different nature. I do not disagree with the finding that female and male mathematics students are treated differently. Nor do I think that this doesn't affect their selection of mathematics courses. I do disagree, though, with the implied comment that sex is a primary factor in this selection. I think that in studying education we tend to overestimate the importance of the school environment. This problem is much more complex and more difficult to treat than this study would suggest. Also, while the geometry course studied is often the last taken before females cease to take mathematics, I think that this is because two years of mathematics is commonly required and that is their first chance to get out. Other studies of student attitude especially indicate that students' relations toward mathematics are developing at a much earlier age.

Campbell, Richard L. INTELLECTUAL DEVELOPMENT, ACHIEVEMENT AND SELF CONCEPT OF ELEMENTARY MINORITY SCHOOL CHILDREN. School Science and Mathematics 81: 200-204; March 1981.

Abstract and comments prepared for I.M.E. by NORMAN WEBB, Manager of Research, Wisconsin Educational Communication Board.

1. Purpose

The purpose of the study was to examine the relationship between self-concept and intellectual development of minority children. The two questions that were tested are:

- Can the relationship between self-concept and academic achievement be re-confirmed with minority school children?
- Is there a relationship between minority (black and hispanic) school children's self-concept and their intellectual development?

2. Rationale

This study was designed to link the findings from research in two areas. The findings of research in one area support the notion that self-concept and academic achievement are related. In the second area, research indicates a strong relationship between Piagetian intellectual development stages and academic achievement. It is believed that low self-concept among minority school children is a major reason for their lack of success in schools. If a link is established between self-concept and intellectual development, then stages of development can be taken into consideration with an emphasis on the "self" to enhance the success of minority children.

3. Research Design and Procedures

Three sets of tests were administered to 51 second-grade students enrolled in a predominantly bi-ethnic (black and hispanic) school. The Stanford Achievement Test was used to measure academic achievement. The Pier-Harris Self-Concept Scale was used to assess students' self-concepts. Seven Piaget-like tasks administered by trained interviewers were used to measure intellectual development. The tasks measured the students' attainment of the concrete operational stages IIA and IIB using conservation tasks including number, length, area, weight, and volume. For each task students were given one point for a correct response and one point for a correct explanation. The analysis consisted of computing Pearson correlation coefficients between the Stanford Achievement subtests and the Pier-Harris Self-Concept Scale and between the self-concept scale and the Piagetian tasks. Only the correlations significantly different from zero were reported.

4. Findings

Self-concept scores we.e significantly positively correlated with the Stanford Achievement subtests of reading-word (.60), reading-comprehension (.48), social studies (.40), science (.50), and listening comprehension (.38). Self-concept scores were significantly positively correlated with the Piagetian tasks of length-response (.40), length-explanation (.42), area-explanation (.36), weight-explanation (.29), and volume-explanation (.[illegible]0).

5. Interpretations

It was concluded that the results of the study supported the idea that intellectual development is a function of self-concept. A relationship between minority school children's self-concept and their intellectual development was partially supported by the positive corre..cions between self-concept and four Piagetian tasks. It was further concluded that self-concept enhancement is more a function of high-level thought than it is of low-level thought. Thus, having a high positive self-concept makes it easier to achieve higher levels of intellectual development as a minority child moves from one stage of development to another.

Abstractor's Comments

Very little can be concluded from the results of this study. The ex.stence of positive correlations does not indicate that one variable is a function of another but only that there is a relationship between the two variables. Relationships between intellectual development and achievement and between self-concept and achievement have been shown to exist in other studies. Without any partial correlations it is difficult to establish the unique relationship between self-concept and intellectual development after academic achievement has been partialed out. The use of the Pearson correlation coefficients to establish a relation between a continuous variable (self-concept) and a dichotomous variable (conservation tasks) is in question. Other measures of relationships are probably more appropriate under the given situation. Because of the above problems in the design, it is impossible to make the suggested implication from the study that having a high-positive self-concept makes it easier to achieve higher levels of intellectual development as a minority child moves from one stage of development to another.

Fraser, Barry J. and Koop, Anthony J. CHANGES IN AFFECTIVE AND COGNITIVE OUTCOMES AMONG STUDENTS USING A MATHEMATICAL PLAY. School Science and Mathematics 81: 55-60; January 1981.

Abstract and comments prepared for I.M.E. by JACK D. WILKINSON, University of Northern Iowa.

1. Purpose

The purpose of this study was to measure both cognitive and attitudinal growth of ninth-grade students who spent two or three class periods producing and viewing a play about the mathematician Thales and the topic of similar triangles.

2. Rationale

In response to numerous pleas that the history of mathematics be included in the school mathematics curriculum, the authors note that textbooks and mathematics teachers have generally neglected the history of mathematics in their classroom presentations. It was felt that the introduction of the historical content through the presentation of a play would be a source of motivation.

3. Research Design and Procedures

The sample consisted of 117 ninth-grade students from typical schools around Sydney, Australia. While the sample was not randomly assigned it was, according to the authors, "representative of the range of schools and pupils found around Sydney."

A play about the life of Thales and the properties of similar triangles was presented in mathematics classes in two or three class periods of 40 minutes each. Students read the play and acted out the parts. Both participants and spectators were included in the sample.

Classroom teachers administered a pretest on both affective and cognitive outcomes and after the play the posttest was administered. There was no "instruction" on the topic of similar triangles until after the posttest was administered.

The four dependent affective variables were: attitude toward learning about the lives of ancient mathematicians, attitude toward learning about the history of mathematics, attitude toward the practical applications of mathematics to daily life, and attitude toward using mathematical plays in mathematics lessons.

There were five dependent variables which dealt with cognitive items. They were: knowledge of the approximate time in which Thales lived, knowledge of Thales' birthplace, and three items which measured applications of similar triangles.

The major hypothesis dealt with measured changes in the whole sample during the time of using the play. Three subsidiary hypothesis dealt with "...whether differential changes in performance during the time of the play occurred for students varying in general ability, general mathematics attitude, and sex."

The major hypothesis for the attitudinal outcomes was tested using a t test. A z test was used for the cognitive outcomes.

Multiple regression techniques were used for the three subsidiary hypothesis.

4. Findings

Significant results ($p < .05$) were reported for two additional outcomes and two cognitive outcomes, namely, attitude toward learning history of mathematics, attitude toward using mathematical plays in mathematics lessons, knowledge of the approximate time when Thales lived, and knowledge of Thales' birthplace.

Results dealing with the three subsidiary hypothesis are also presented.

5. Interpretations

The authors feel that the use of the Thales play was more effective in the historical sense than in teaching about similar triangles. Getting significant results on four of nine variables is noteworthy because only two or three class periods were devoted to the play. Finally, the use of a play in a mathematics class is rather novel and might be a fruitful endeavor for other mathematics teachers.

Abstractor's Comments

The novel nature of this treatment seems of particular interest. I'm not at all sure about the availability of such plays, but if teachers can find the right play and arrange it with the right "touch", it could be "a fun experience" and have interesting consequences in both attitude and cognitive growth.

On an even more removed key, is it possible that students could research and write such plays or skits in conjunction with modern language and writing classes?

Malin, Jane T. INFORMATION-PROCESSING LOAD IN PROBLEM SOLVING BY NETWORK SEARCH. Journal of Experimental Psychology: Human Perception and Performance 5: 379-390; May 1979.

Abstract and comments prepared for I.M.E. by GERALD KULM, Purdue University.

1. Purpose

The purpose of the study was to examine the choice, efficiency, and effect of problem domain on strategies of solving means-end algebra problems.

2. Rationale

Research concerned with the efficiency of strategies has characterized problem solving as a search through a network of possible solution paths. Some strategies are efficient because they reduce the search by eliminating the paths to be tested. An example is the heuristic (strategy) of working backwards. On the other hand, search-reducing strategies may overload information processing and memory. The present study sought to determine the effects of both amount of search and amount of processing load. Two separate experiments were performed.

3. Research Design and Procedures

Experiment 1

Thirty-two paid subjects from a college freshman and sophomore pool were used. Eight males and eight females were randomly assigned to each of two groups. Subjects were tested individually. First, each S memorized five addition and muitiplication equations; then half were taught a forward-working strategy and half were taught a backward-working strategy. The forward strategy consisted of combining givens until the unknown was derived. The backward strategy consisted of substituting variables for the unknown until all substituted variables were on the givens list.

Each S solved 50 problems. In each problem, one of the variables from one of the five equations was the unknown and a list of givens or known variables was supplied. The problems varied according to the number of steps (1 to 5), number of subgoals (0 to 2), and number of blind alleys (0 to 3). Solutions to 20 of the problems were used for data collection. The data consisted of solution times to the nearest second.

Experiment 2

The 54 subjects were from the same population pool and the procedures were the same as experiment 1. In experiment 2, however, the five equations were either (1) grouped for easier chunking and memory or (2) given meaningful contexts, e.g., time worked, rate of pay, overtime pay. Each subject solved 29 problems and data were gathered on 14 problems which represented different types. Subjects could choose a forward or backward strategy and were asked to report their solution in a format that revealed which strategy they used.

4. Findings

Experiment 1

A significant interaction between strategy and problem type was found, indicating that the two groups did use different directions of search, as they were instructed to do. A comparison of strategies on equivalent problems revealed significant differences in solution times only on four- or five-step problems. The backward strategy was a little faster and less variable overall.

Experiment 2

Each subject was classified according to strategy used. Almost all subjects started with a backward step, then some changed direction. Subjects were classified as mixed if they changed directions in at least 7 of the 11 problem types.

The subjects in the Grouped and Context treatments preferred forward strategies, whereas the Ungrouped subjects preferred backward strategies. Subjects using the preferred strategy for their group solved the problems significantly faster than subjects using the nonpreferred strategy. There was a strategy-by-treatment interaction indicating an improvement from nonpreferred to preferred strategy in Grouped and Context treatments and a small reverse effect in the Ungrouped treatment.

5. Interpretations

The use of a backward strategy in Ungrouped equations is consistent with studies showing the use of backward strategies on difficult problems.

The strategy by treatment interaction suggests the possibility that aptitude by treatment interactions may be explained by strategy differences between treatment groups.

The results of both studies support the case for memory or information processing load differences between strategies. To test this assumption, information-processing models of the three strategies were constructed and tested. Using seven parameters, a formula was constructed for each problem type and strategy. Observed and predicted times were compared. The model fit the observed data very well, helping to explain the strategy preferences. The mixed strategy had three advantages: easy to discover a new step, easy to recover from memory failure, and possible to avoid difficult recall from memory.

Abstractor's Comments

This study provides interesting and significant results for mathematics educators from several perspectives. First, the use of well-defined yet reasonably realistic problem tasks provides an opportunity for a meaningful bridge between information-processing research and the mathematics classroom. The finding that carefully organized information can reduce memory load and influence efficient strategy selection by subjects has important instructional implications.

Although the method may appear forced or not properly motivated, the success demonstrated in teaching a specific solution strategy is encouraging. Working forward, backward, or "mixed" applies to broad classes of problems. The finding that subjects could, once taught each strategy, choose the one or combination that was most appropriate to a problem was even more exciting.

The significance of the results and the possible classroom implications must be tempered by the simplicity of the tasks and the trivial outcome measure (solution time). However, the strength of a simple model which accounts for the specific processes used in a detailed manner balances these criticisms.

Nelson, Doyal. STUDYING PROBLEM SOLVING BEHAVIOR IN EARLY CHILDHOOD. Alberta, Canada: University of Alberta Printing Service, 1980.

Abstract and comments prepared for I.M.E. by PAUL R. TRAFTON, National College of Education.

1. Purpose

The purpose of this series of studies was to observe the behavior of three- to nine-year-olds in solving problems constructed in accordance with specific criteria, including relationship to specific mathematical content and the active manipulation of materials and/or apparatus.

2. Rationale

There is a need for greater knowledge regarding the problem-solving behavior of children at a significant time in their development. This is particularly true for problems which can be represented in various physical contexts and which can be solved apart from explicit knowledge of the underlying context.

3. Research Design and Procedures

A set of twelve paired problems representing six areas of mathematical content were developed in accordance with a set of criteria. These guidelines are described in the Thirty-seventh Yearbook of the NCTM (Nelson and Kirkpatrick, 1975). The problems dealt with division, space and geometry, reflections in a plane, co-ordinates, factors, and sequences. Special apparatus was developed that placed the problems in real-world contexts.

In the first year of the study, 15 children at each age level from three to eight were administered six problems. Four content areas were represented in each set with students solving both the primary and second problem in two areas. Thus, the first problem was presented to 10 children at each age level with 5 children doing the equivalent problem. The following year 74 of the original students were administered six additional problems. The interviews with the children were videotaped and the tapes were analyzed by problem.

4. Findings

Findings were reported by problem, with results given in broad detail for four of the six problems. Space limitations preclude giving specific findings by problems. Interested readers should note that the division study results were abstracted for I.M.E. by Reys and Bestgen (Vol. 11, No. 4) and the space and geometry study results were abstracted by Lindquist (Vol. 12, No. 2).

5. Interpretations

The author made these observations based on the six exploratory studies:

1) Children were eager to do the manipulations and showed sponteneity in dealing with the problems. Older children also chose to manipulate, although in some cases they were capable of more symbolic approaches.
2) Irrelevant attributes in a situation were a major distractor, particularly for children less than seven years old.
3) Many children were reluctant to predict outcomes. Those who made predictions willingly were almost always correct, and asking children to predict seemed to make them reflect on the information they had.
4) The seven problem-solving criteria were useful in constructing problems and provided a good framework. However, problems which seemed equally to fit the criteria produced different responses from children. Also, the criteria alone did not produce problems that were equally accessible by children.
5) Caution should be observed in attempting to draw implications from these studies for curriculum work.

Abstractor's Comments

It is difficult to know how to react to this monograph which summarizes two years of clinical investigations. One is impressed by the imagination that led to interesting problems with ingenious apparatus; the thoroughness with which designing protocols, recording interviews, and coding responses were approached; the willingness to devote extensive time to nontraditional research; and the attempts to delineate and implement guidelines for "good" problems. In terms of results, certainly insights about how children approach tasks are always interesting and often useful.

Yet one is also left feeling disappointed. First, the study appears flawed by its dual focus. It was as much an attempt to create and implement guidelines for problems as it was to study problem-solving behaviors in any systematic way. This split focus led to the use of some problems which were not particularly appropriate for many children. Also there is little connection between the six sets of problems, which hinders collective interpretation of the results. Second, even exploratory studies need to be accompanied by careful piloting of approaches and procedures, and be guided by carefully framed, albeit tentative, questions so that there is a greater likelihood that results will provide better information and a better sense of direction for future work. In this study many problems that did occur could have been eliminated.

As it is, the study provides guidance for those who plan to do clinical investigations and interesting insights about how children approached these problems, but leaves us pondering what the findings mean and what their implications are, if any, for curriculum work.

O'Brien, David and Overton, Willis F. CONDITIONAL REASONING FOLLOWING CONTRADICTORY EVIDENCE: A DEVELOPMENTAL ANALYSIS. Journal of Experimental Child Psychology 30: 44-61; August 1980.

Abstract and comments prepared for I.M.E. by LIONEL PEREIRA-MENDOZA, Memorial University of Newfoundland.

1. Purpose

The purpose of the study was to investigate students' understanding of the conditional as it relates to hypothesis testing. Specifically, the study sought to determine the effect of contradictory training on the reasoning of students.

2. Rationale

In ordinary language, "If...then" statements are often interpreted as indicating temporal or causal relations, or as being equivalent to the biconditional, all of which are incorrect interpretations of the mathematical concept of the conditional. In light of the fact that the correct use of conditional testing is central to scientific reasoning, this is an area worthy of research. The researchers indicated that previous attempts to teach the application of conditional testing have not been productive. They further note that most research utilizing the contradictory training paradigm has only been attempted with young adults. Thus, this study attempts to extend previous research to a broader population and interpret the results within a cognitive structure, particularly Piaget's developmental theory.

3. Research Design and Procedures

The subjects consisted of 30 grade 3 students (16M, 14F), 20 grade 7 students (11M, 9F) and 20 college students (12M, 8F).

The inference task consisted of the rule "If a worker is __ years of age, or older, then the person will receive at least $350 each " (p. 48), followed by 12 numerical exemplars of the form 'There is a worker who is y years old and makes $z each week'. The task is to determine for each exemplar which of 3 choices about the age in the rule could be inferred from the exemplar, the choices being that the age is more than y (choice 1), at most y (choice 2) or nothing at all (choice 3). For example, following the exemplar that there is a worker 25 years old who makes $200 each week, choice 1 (that the age in the rule is more than 25) would be the correct inference regarding the age in the rule.

> Subjects tend to follow exemplars in which the monetary amount is less than that in the stated rule with the conditionally correct assertion. However, subjects usually assert that those exemplars in which the amount given is equal to or exceeds the amount stated in the rule, shows that the missing age in the rule is less than that in the exemplar. It is not recognized that someone could make more than $350 (q) without exceeding the age in the exemplar. (p. 48)

In the case of non-contradictory set of exemplars, all the situations were chosen such that if the age was less than 45 the salary was less than $350 and if the age was greater than or equal to 45, the salary was greater than or equal to $350. In the inference task with the contradictory data (contradictory treatment), only one exemplar, trial 6, was changed. The new exemplar stated that there was a 60-year-old who made $200 each week. Thus, this exemplar contradicts "the erroneous inference that was likely to have been made on the previous trials, and was directly contradictory to any error made in trial 5" (p. 48). The erroneous inference would be one made by a student interpreting the conditional as a biconditional. Trials 8, 9, and 11, which followed the alternative versions of trial 6, would result in different inferences depending on whether a subject used conditional or biconditional reasoning, and these trials were used to derive an inference task score.

The <u>selection task</u> consisted of 6 conditional statements each followed by 4 propositional types (p, q, $\bar{p}$, $\bar{q}$), and the task was to determine for each conditional statement what you needed to know to determine if the conditional statement was true or false.

The <u>evaluation task</u> consisted of the same conditional statements as the selection task followed by 4 pairs of propositional combinations ($p \cdot q$, $\bar{p} \cdot q$, $p \cdot \bar{q}$, $\bar{p} \cdot \bar{q}$) in random order. The task was to determine for each propositional combination whether or not it proved the rule true or false.

The <u>procedure</u> consisted of presenting each subject with one version of the inference task followed by either the selection or evaluation task, resulting in a level (grade 3, grade 7, college) by inference task (noncontradictory trial, contradictory trial) by task (selection, evaluation) matrix with 5 subjects per cell, accounting for a total of 60 subjects (20 per level). Each subject was interviewed individually. The subject read the inference task instructions while the tester read them aloud. If the subject indicated that he or she understood the instructions he or she proceeded to the first trial, and if not this trial was used by the instructor as an example. The subject proceeded through the 12 trials and after each had to indicate the inference for the age in the rule that could be drawn from that particular trial. The instructions for the selection and evaluation task were also read aloud and the subject responded to each conditional statement in order. All the materials were in booklet form.

The major analyses consisted of various ANOVAs designed to test the treatment effects (contradictory training), to examine the relative difficulty of the selection of choices given in the selection and evaluation tasks, and to examine the relative difficulty of the statements in the evaluation task.

The additional 10 grade 3 subjects received a slightly different version of the inference task, with an allowance of $3.50 being used to replace a salary of $350 since this new figure could be considered closer to their real-life experience.

4. Findings

A comparison between the performance of the grade 3 subjects on the two versions of the inference tasks resulted in no significant differences in performance, and consequently all further analyses were undertaken with the same task material.

The major findings were:

a) The comparisons of within-grade treatment effects indicated that the college students who received the contradictory treatment performed significantly better than the college students who received the non-contradictory treatment on the inference task, selection task, and evaluation task.

b) On the inference task, the college students performed significantly better than third and seventh graders only on the contradictory treatment.

c) On the selection task the college students performed significantly better than grade 3 or 7 students and there was no significant difference between the performance of students in grade in 3 and 7.

d) On the selection tasks it was significantly easier to assess p correctly than the other three choices ($\bar{p}$, q, $\bar{q}$) and significantly more difficult to assess q correctly.

e) On the selection task the introduction of contradictory evidence increased the occurrence of errors in selecting $\bar{p}$ by seventh graders.

f) On the evaluation task, third graders performed better than seventh graders in correctly evaluating p • q.

5. Interpretations

The authors note that the results with young adults replicate previous research. They feel that the success of students is more likely to be a function of the contradictory treatment alterting subjects to the asymmetry of conditional reasoning than to the single contradictory trial developing an understanding of such reasoning. They also conclude that evidence such as that presented in e) and f) of section 4, indicates that grade 7 students may be going through a transitional phase in the Piagetian sense. Finally, they conclude that the contradictory training paradigm is an appropriate method to improve young adults' conditional reasoning.

Abstractor's Comments

In scoring the selection task, the authors indicate that one point was given for each correctly selected propositional type, independent of the number of types selected. This would appear to provide a bias in favour of subjects who consistently select more responses. A similar comment can be made with respect to the scoring of the evaluation task.

The report did not indicate the length of time taken to administer the different tasks to the students. It would appear from the description that it could be a quite time-consuming task and if so, the length of time involved for the younger students might have been a factor contributing to their lower performance.

In the selection task, after each rule was presented the subject was asked: "Could you find out if the rule is true or false if you looked at ..." (p. 51) and was given a list of the four proportional types. The authors also note that the instructions were worded with an emphasis on falsification. A parallel procedure was followed for the evaluation task with the subject being asked: "Would you know if the rule was true or false ..." (pp. 51-52) and then given a list of propositional combinations. It would seem that the wording of the task might be complex for a grade 3 or 7 subject. This complexity could have accounted, in part, for the low number of totally correct responses. For example, after receiving the contradictory training only, one grade 7 student correctly chose just $p \cdot \bar{q}$ as the appropriate alternative. Thus, while the abstractor agrees with the conclusions regarding the effectiveness of the treatment for young adults, it is difficult to delineate the reasons for the low level of performance of grade 3 and grade 7 subjects. It might well be that a different wording or explanation of the selection and evaluation tasks could result in greater success for the younger students.

In spite of the concerns expressed in the previous paragraphs, the report raises some interesting questions. The contradictory training paradigm seems to be effective for "young adults". The question raised by the authors regarding college students' understanding of conditional reasoning after the contradictory training is worth pursuing, possibly through interview studies. The question of whether grade 7 students are in a transitional phase needs further investigation. Why do students find it easier to assess p than other propositions? Why was it harder to assess q? Finally, would the placement of the contradictory trials at a different place in the sequence or the use of further contradictory trials be more effective?

Peterson, Penelope L. and Janicki, Terence C. INDIVIDUAL CHARACTERISTICS AND CHILDREN'S LEARNING IN LARGE-GROUP AND SMALL-GROUP APPROACHES. Journal of Educational Psychology 71: 677-687, October 1979.

Abstracts and comments prepared for I.M.E. by DOUGLAS E. SCOTT, Amphitheater High School, Tucson, Arizona.

1. Purpose

Questions investigated by this study were:

(a) Do students learn better when they work essentially by themselves (with teacher supervision) in large-group settings, or when they work in small groups with much student-to-student interaction?

(b) Is there any aptitude-treatment interaction? Specifically, do high-ability and low-ability students differ in their learning accomplishments when working as individuals in a large group as compared to working as a member of a small (four-person), mixed-ability group?

2. Rationale

This investigation is an outgrowth of a study by N. M. Webb (1977) in which high school students worked either alone, in groups of four students of similar ability, or in groups of four students of mixed ability. The underlying assumptions of the Peterson and Janicki study seem to be (although nowhere explicitly stated) that:

(a) there is some factor in the psychological makeup of a particular student that makes him or her more receptive in a particular kind of classroom situation (i.e., small or large group), and

(b) this factor can be identified.

3. Research Design and Procedures

One hundred students in grades four through six who had not previously studied fractions were divided into four groups for a nine-day unit on that topic. Two groups were taught (by different teachers) in a "standard" format: the teacher would introduce the day's topic, explain and ask questions, work some sample problems, and make a workbook assignment. Students then worked individually, requesting teacher assistance as necessary.

The other two groups functioned in much the same way, except that after the workbook assignment had been made students worked in previously-assigned groups of four with instructions to seek help from others in the group before asking the teacher. Only if none of the group could assist would the teacher provide help.

The design was multivariate; previous mathematics achievement, mathematics anxiety, locus of control (internal/external), attitude toward mathematics, group size preference, and treatment (small group/large group) all were measured and treated in one way or another as independent variables. Dependent variables were: scores on two achievement tests, one given immediately upon completion of the nine-day unit and an identical test given two weeks later (with no feedback from the previous testing) to measure retention; scores on a mathematics attitude posttest; and scores on an attitude-toward-group-size posttest.

In addition, an observation instrument was constructed and used to verify that a difference in classroom procedure did in fact exist between the two groups. This instrument was used by an observer to check the activities (teacher explaining to whole group, student working in small group, etc.) being performed by students during 20-second blocks of time.

Statistical procedures used included a factor analysis on the various measures of student aptitudes and generalized regression analyses on achievement and attitude scores which were used to test for aptitude-treatment interaction.

4. Findings

Despite the fact that there was a real difference in observable behavior between the two groups (students in the large-group classes did not confer with each other or form small groups at all, insofar as measured by the observers), there was no significant difference between the groups on either the immediate achievement test or the retention test.

There was, however, a significant ATI: high-ability students did better in small groups, while low-ability students did better in the large-group approach. "Ability," one of the two factors that emerged from the factor analysis, had components reflecting mathematics computational ability and knowledge of mathematics concepts. A second factor, "Attitude/Anxiety," was composed of scores on measures of those characteristics. A relation between all of these factors appeared, with high-ability students having a more positive attitude, as well as higher scores, in the small-group setting and low-ability students having a more positive attitude in the large-group classes.

One possibly surprising finding was that student preference, expressed before the study, for small-group or large-group learning environment was in fact negatively correlated with the students' actual accomplishment: those who said they preferred a large-group approach actually did worse in that setting and better in the small-group classes.

		Attitude/Anxiety: Good	Attitude/Anxiety: Poor
Ability	High	SMALL	LARGE
Ability	Low	LARGE	SMALL

Figure 1. Group Size in Which Best Scores Resulted

Figure 1 summarizes the three-way relationship that was found to exist between ability, attitude/anxiety, and treatment. (In the Attitude/Anxiety columns, "Good" describes students whose scores reflected a positive attitude and low anxiety; "Poor", the opposite on both.)

5. Interpretations

In their discussion of the two findings of their investigation (no significant difference between treatments, and significant ATI), Peterson and Janicki suggest that apparently the more-able students, by serving as tutors in the small groups, did learn more (or learn better) in the small-group situation. The investigators found that receiving an explanation, however, did not seem to produce any effect on the student's scores; the less-able ones evidently were helped more by receiving explanations from the teacher in the large-group classes than by receiving explanations from other students in the small groups. It may also be surmised that these students did not know the material well enough to be able to explain it to others, and thus did not benefit appreciably from the interaction in the small groups.

The authors conclude their paper by suggesting that educators need to "provide for planning and implementing instruction suitable for the individual students" (p. 686).

Abstractor's Comments

When an investigation finds no significant difference between the results of two treatments, one has a responsibility to ask: Is the lack of difference (in this investigation, a difference between means of approximately 1.5 with sample SDs of 7.5 to 10) real, or is it a construct of the investigation? If it is real, what meaning does it have for the teacher in the classroom?

In the present study, the apparent lack of difference between the scores of students who studied in small groups and the scores of those who worked by themselves in a large group is the result of a significant aptitude-treatment interaction: high-ability students apparently did somewhat better than might bave been expected when they worked in small groups, and somewhat less well in the large-group approach. Low-ability students showed an opposite trend, and the overall result was the finding of no significant difference.

Peterson and Janicki identify this ATI effect with some clarity, but then proceed to obscure it with discussion of aptitudes (such as locus of control and attitude toward teaching) that their data show have little or no effect.

The authors also devote overmuch space to a comparison of their results with those of Webb, and one senses a feeling of apology because the results of the present study did not match those found by him. In fact, there are more than enough dissimilarities in procedure to account for the differences in results between the two studies: Webb used high school students, rather than elementary; individual study rather than large-group instruction; and both uniform-ability and mixed-ability in the small groups.

Peterson and Janicki, as well as Webb, suggest (although not in exactly these words) that it is the internalization of concepts by the more-able students as an adjunct to answering questions or providing help in the small groups that produces the extra improvement in their scores. On the other hand (and the authors do not address this aspect), in the small group the less-able student may be able to avoid thinking about the material and in general may act in a much more passive manner. In the large group, if the teacher is moving about the room, the less-able students may be forced to do more actual work than they would as relatively inactive members of a four-person group.

The question of the validity of the various test instruments is not discussed, although one comment seems to reflect favorably upon the validity of the final test: "Ability accounted for the largest portion of the variance (51.4%) in retention scores" (p. 683).

The measure of attitude seemed to be a stable one, since we are told that "students' attitudes toward math were best predicted by their initial attitudes toward math" (p. 684). There was an ATI effect here, too, because high-ability students had a more positive attitude in the small-group classes and low-ability students had a more positive attitude in the large groups.

The actual findings in this study seem to merit a more firmly-stated conclusion than the authors' trite and trivial "...students' achievements and attitudes can be improved by adapting instruction to the needs of the individual student" (p. 686). That statement could have been made without ever conducting the investigation at all, and yet the results of the investigation would fully justify a statement such as: "In the intermediate grades, high-ability students benefit from the opportunity to work in small, mixed-ability groups. Further research to refine and generalize this conclusion is needed."

Reference

Webb, N. M. Learning in Individual and Small Group Settings (Technical Report No. 7). Stanford, California: Aptitude Research Project, School of Education, Stanford University, 1977.

Rosier, Malcolm J. CHANGES IN SECONDARY SCHOOL MATHEMATICS IN AUSTRALIA 1964-1978. Victoria, Australia: Australian Council for Education Research Limited, May 1980. ERIC: ED 195 402.

Abstract and comments prepared for I.M.E. by THOMAS E. KIEREN, University of Alberta.

1. Purpose

The major purpose of this report was to document changes in achievement by students of secondary school mathematics in Australian schools as reflected in data collected in the 1964 and 1978 First and Second IEA studies. These achievement performances are explained in light of curriculum and other background factors. A secondary purpose was to describe changes in attitude towards mathematics and mathematics instruction between these two years.

2. Rationale

In this study curriculum, its central construct, is viewed in dynamic terms -- that is, in the period studied curriculum is likely to have changed. Curriculum is viewed as having three stages of implementation. The first of these stages in the initial design -- seen here to be in the charge of various state government agencies in Australia. The second stage is the translation of this syllabic curriculum into the practical curriculum of the classroom. The final stage in the sequence is reflected by the extent to which students have learnt the material in this curriculum.

This study uses common items from the IEA studies of 1964 and 1978 to document student performance in terms of the third stage above and with respect particularly to IEA populations 1 and 3 (13-year-olds and Year 12 students completing a secondary school mathematics program). This report is tied to the reports of the 1964 study by Husén and various Australian studies based on 1964 data.

Because of the curriculum sequence theory, student performance changes were explained in two ways. The first took into account the control of local instructional practice. Hence, state-by-state results rather than national results are compared. Within this constraint both the curriculum relevance (reflected in a Curriculum Content per Item variable) and the instructional (reflected in an Opportunity to Learn variables) were related to performance.

Individual achievement performance was assumed to be affected by a number of personal (e.g., socio-economic status) and instructional variables. This led to a comparison of causal models of individual performance in the two years studied.

3. Research Design and Procedures

A substantial portion (four chapters) of this report is devoted to a discussion of design matters. The first of these discusses changes in the system of mathematics education in Australia between 1964 and 1978. This reflects the concern for explaining performance and performance pattern change in terms of syllabic and instructional change. Some of the changes documented were institutional in nature and varied from state to state. For example, in 1964 in New South Wales there were seven primary levels (K-6) and five secondary levels, whereas in 1978 there was an added secondary level with concomitant changes in upper secondary course contents. There were substantial increases in the numbers of students at the 13-year-old level, substantial increases in the numbers and percentage of students taking Year 12 mathematics, decreases in time spent on mathematics at both levels, as well as changes in the average number of Year 8 13-year-olds. There were, of course, variances in these trends among states.

There were also general changes in mathematics education. Some were societally sponsored -- the change to decimal currency, metrication, and growth in use of electronic calculators. The "new mathematics" emphasis on structure and understanding was just starting to have an impact in 1964, whereas its impact pervaded curriculums by 1978. The use of concrete materials from the work of Dienes and Gattegno also were adopted in some states.

The principal instruments used in this study were the common items in the 1964 and 1978 IEA Population 1 and 3 tests. These item sets contained 64 and 69 items. The items were classified (by IEA) into subtests on the basis of various content and process level categories. These tests were used to generate various performance scores for 1964 and 1978.

The categorization of items allowed for state-by-state curricular analysis which generated a Curriculum Content per Item score for the tests and subtests. This was done by relating the type of objective and its relative emphasis in the curriculum to the items on the test.

Teachers also rated each test item on the basis of whether students had an opportunity to learn the related content. This opinionnaire was used on a state-by-state basis to generate an Opportunity to Learn score per items for the test and subtests. Thus, by design, there were scores which reflected the three elements in the assumed curriculum implementation sequence computed at the state level.

The relevant achievement tests were administered to a probability sample, with each student representing a certain number of students from a particular stratum (e.g., primary vs. secondary schools for the 13-year-old population). The sampling procedure took into account non-response rates as well as stratum size in order to allow for the development of state data. In 1964 only students from government schools in five Australian states took part in the IEA study. Thus, for presentation and analysis of results, information on two 1978 samples is presented. First, there is a general sample of students in government and non-government schools in seven states. Second, a restricted subsample meant to match the 1964 sample is considered. The teachers whose opinions were used to generate school information and Opportunity to Learn scores were teachers of students selected for the study and hence were not a random sample.

In order to complete other parts of the study, similar attitude questionnaires were administered in 1964 and 1978. The scales were intended to measure attitudes toward mathematics instruction, importance of mathematics, facility with mathematics, school enjoyment, and control of the environment. In addition, student, teacher, and school questionnaires were used to ascertain a variety of home and school background measures.

4. Findings

Mathematics test and subtest mean scores, Curriculum Content scores, and Opportunity to Learn scores were generated for the 1964, 1978, and 1978 restricted samples for both the 13-year-old and the Year 12 tests. The achievement tests and various subtests were standardized on the basis of relevant total scores for 1964 and 1978. These standardized scores were used in graphically presenting within-state and between-state performance change patterns for 1964, 1978, and 1978 (R) samples.

For the 13-year-old group, in four of five states there was a slight decline in total mathematics scores, with two of these differences being significant at the 95 percent probability level. In terms of content subtests, there were declines in all five states on Advanced Arithmetic and Geometry. Two states showed a significant gain on the New Mathematics subtest. In terms of process level, there was neither universal decline on Computation nor universal gain on Comprehension; in fact, on the latter there was universal decline across states. To give some indication of the level of performance, the 1964 mean score per item ranged from .36 to .47, while this score ranged from .37 to .46 across states in 1978.

In 1964 the Curriculum Content per Item score was compared with mean test scores per item (the Opportunity to Learn data for 1964 was lost in the original IEA study). All such correlations were negative. With one exception both Curriculum Content and Opportunity to Learn scores correlated positively with all test and subtest performance scores in 1978.

Performance of individual students as related to home and school variables was studied using a causal path model analysis. In 1964, median values of state model path coefficients indicated that the year (7, 8, or 9) of the 13-year-olds in school and their fathers' occupations were leading causal contributors. In 1978, the leading contributors were the same when Opportunity to Learn was excluded. The latter variable had the largest coefficient when it was included. In neither year was sex linked to performance in these models.

Similar analyses were carried out for the Year 12 population. In general, mean score analyzes indicated a general gain in test and subtest scores from 1964 to 1978. The only exceptions to this trend occurred on Geometry, Trigonometry, and Calculus subtests. There were positive changes (in general) on Computation, Translations, and Comprehension subtests. As with the 13-year-old group, there was considerable variation among states. Performance scores ranged from .30 to .46 across states in 1964 and .33 to .49 in 1978.

As suggested previously, more students enrolled in more diverse courses were represented in the 1978 Year 12 age cohort. Yield scores indicated that more students in Year 12 did better on mathematics performance.

Opportunity to Learn and Curriculum Content each correlated positively with test and subtest performance at Year 12 in both assessments.

For individual student performances, causal model analyses indicated that Class Time and Opportunity to Learn were strongly related to performance in 1964 and 1978. A separate model including calculator use was derived for 1978 data and such use figured in explaining individual performance. In Year 12 models, Father's Occupation did not play a role, while Sex of Student figured in a secondary way in the 1978 model, being related to Opportunity to Learn (females appeared to have more opportunity).

Five attitude scales were completed by students in 1964 and 1978. With the exception of the Facility with Mathematics scale (accessibility of mathematics to learners), attitudes were generally less positive in 1978 than 1964. There was inter-state consistency at the 13-year-old level on the Mathematics Teaching scale, indicating that this attitude did not explain the inter-state mathematics performance variations.

5. Intrepretations

13-Year-Old Populations. There was a slight general decline in performance from 1964 to 1978. Due to unstable curriculum circumstances in 1964 (reflected in low correlations with performance), such changes cannot be explained in terms of a changed curriculum. The decline in performance manifested itself mainly with an increase in low scores in 1978, indicating that the 1978 curriculum may be less effective for low-achieving students. Further, although more 13-year-olds were exposed to more higher level content in 1978 than 1964, this did not necessarily lead to increased performance.

Year 12 Populations. Because of an increased percentage of students in Year 12 Mathematics and the higher scores in 1978 than in 1964, it was concluded that the 1978 program resulted in a greater "yield" as compared with the 1964 Year 12 program. This was achieved at no expense to the percentage of high scorers, which also increased. Such charges were even to be linked to corresponding curriculum implementation changes. Such positive changes occurred in the face of less total time (especially homework time) spent on mathematics in 1978 than in 1964.

Calculator use, while having little impact at the 13-year-old level, varied across states at the Year 12 level. There was a tendency for states with high calculator use to show slightly higher mathematics performance scores.

Student Attitudes. There was no evidence of a deterioration of attitude towards mathematics from the 13-year-old to the Year 12 level in either assessment. Year 12 students exhibited less polarized reactions to liking mathematics. However, there seemed to be a general between-year decline in attitudes, especially with respect to liking school and the ability to control the environment. It was concluded, especially relating 13-year-old performance and its attitude decline, that a more utilitarian mathematics program would better balance cognitive and attitudinal components.

Abstractor's Comments

This report would be of interest to any mathematics educator in countries where various regions, provinces, or states set their own curriculums. The above abstract does not do justice to the amount or complexity of quantitative information developed for this study, nor does it reflect all facets which the author attempted to illuminate. It would seem that consideration of the total document would be worthwhile by anyone pursuing work of a similar nature.

As a person living in a country with provincial rather than national curriculums, I was interested that, although curriculums between states in 1978 were more similar than in 1964, the achievement or performance differences between states seem to be maintained. Thus, reducing curriculum differences in mathematics did not appear to alter dramatically performance differences. The author did not undertake an evaluation effort to explain such differences.

Rosier does remark that students' earlier experience in mathematics might explain later performance at a population level (p. 198). He backs this by saying that Queensland's strong emphasis on mathematics in primary school might explain the level of this state's students' performances scores. However, either the relative strength of the Queensland program does not continue or it is less important for mathematically inclined students, for Queensland's Year 12 results are in the middle of state performance scores.

It is important to note what this study was not. Because of its relationship to IEA and the nature of the IEA instruments, the study reported here was not an evaluation study. It did not attempt to evaluate totally or on a state-by-state basis what Australian school mathematics programs were trying to do or how well these goals were being accomplished. Yet the study did try to relate the elements in the triangle -- matter meant, matter taught, and matter learnt. The IEA mathematics tests were intended to provide students an opportunity to exhibit a variety of mathematical behaviors. For both populations in both years and across states, mean performances are in the 35 to 50 percent range. The author does not comment on this level of performance -- is it good, or adequate? When Opportunity to Learn scores or Curriculum Content scores are taken into account, relative performance appears to increase. However, even though such scores correlated positively with achievement in 1978, little is done to evaluate performance in relation to opportunity as has been done elsewhere. It would have been of interest to non-Australians to have the author comment more on the level of performance and then on aspects of the curriculum (or instruction) which relate to such performance.

The study reported here was quantitative in nature. Information was collected from relatively large and representative samples on a variety of attributes. These data were analyzed in many ways, some of which were unique, to try to relate performance and other curricular and personal attributes. Yet the results of such analysis seem to leave our picture of such relationships incomplete. Why is this so? Obviously the author was restricted by available variables. Thus, for instance, the causal models were perhaps either more simple or containing less powerful variables than the author might wish. Or it may be more qualitative analyses are necessary to explain mathematical performance and how it might best be effected by instruction.

Rosnick, P. and Clement, J. LEARNING WITHOUT UNDERSTANDING: THE EFFECT OF TUTORING STRATEGIES ON ALGEBRA MISCONCEPTIONS. Journal of Mathematical Behavior 3: 3-27: Autumn 1980.

Abstract and comments prepared for I.M.E. by JAMES FEY and THOMAS SONNABEND, University of Maryland.

1. Purpose

The studies described in this report explored causes of student misconceptions about algebraic expression of proportional relations in verbal problems and tested several strategies for correcting common errors.

2. Rationale

Translation of conditions in verbal problems to appropriate algebraic form is an essential skill in application of mathematics and, unfortunately, a source of great difficulty for many students. In earlier work, Clement and colleagues had found that freshman engineering students performed poorly on problems like:

1) There are six students (S) for every professor (P). How are S and P related? (63 percent correct)
2) At a restaurant, for every four people ordering cheesecake there are five ordering strudel. How are C and S related? (27 percent correct)

The authors sought explanations for the large number of errors on these apparently simple problems.

3. Research Design and Procedures

Following up on results from an earlier study indicating that the "students and professors" and "cheesecake" problems were surprisingly difficult for college students, the investigators conducted three series of clinical interviews to probe the causes of errors. The first videotaped interviews revealed a number of types of misunderstanding involving variable and equations. They suggested a list of potentially useful corrective teaching strategies.

Next, nine calculus students who had missed the "students and professors" problem were instructed through pilot tutoring interviews designed to correct their misunderstandings and improve performance on similar problems. These interviews showed that common error patterns were remarkably resistant to remediation, and a second series of standardized tutoring interviews was conducted to test the informal findings.

The six calculus students who were subjects in the standardized tutoring interviews had all missed the "students and professors" problem. They studied a brief teaching unit. They then tried five new problems, thinking out loud as they worked. The taped interviews were analyzed to determine effects of instruction.

4. Findings

The heart of this report focuses on description and analysis of the pilot and standardized tutoring interviews. Most of the nine students in the pilot interviews had resilient misconceptions about proportional relations word problems. None of the teaching strategies attempted effectively overcame these misconceptions. In the follow-up standardized interviews, six calculus students attempted to solve five word problems. Most of the six continued to make mistakes, but all were able to correct their mistakes after being reminded of the recommended strategy for checking solutions. Although all the students arrived at correct solutions to all problems, five of the six students failed to demonstrate conceptual understanding of each problem. These students either reversed the meaning of the original problem, said that the correct answer did not make sense, misunderstood the meaning of the correct equation, or misinterpreted the meaning of the variables in the equation.

5. Interpretations

The investigators concluded that student misconceptions about the algebraic word problems in the study could not be corrected by brief tutoring interviews or a short teaching unit. Although tutoring intervention enabled students to find correct solutions, interviews revealed that underlying misconceptions often remained unchanged.

The authors infer from these findings that educators need to distinquish more carefully "between performance and understanding as outcomes of instruction". The underlying concepts in these word problems -- equation and variable -- need to be taught more carefully. Teachers should present variables as representing number, not objects themselves, and equations as "active operations on variables". Suggesting that mathematics education currently stresses manipulative skill measured by test performance, the authors urge more attention on concept development.

Abstractor's Comments

This study addresses one of the central questions in mathematics education, the role of conceptual understanding in skilled performance. In earlier studies, the authors found that students who had successfully reached the study of calculus were unable to handle simple algebraic problems. In this study, they made effective use of clinical interviews to demonstrate convincingly that the errors are caused by misunderstanding of basic algebraic concepts and those misconceptions are remarkably resistant to remediation. The study relates directly to important issues of classroom practice, suggesting distinctions that teachers ought to make and offering instructional ideas that teachers might try.

There are, however, several features of the study that raise cautions about extrapolation of the findings. First, the two series of interviews involved only 15 students, with little information given about their abilities other than the college course from which they were drawn. The "pilot" interviews included only six students, yet they are given more attention than the "standardized" interviews and provide the most interesting information.

The study claims to show resilience of misconceptions despite corrective teaching. However, the instructional procedure for the "standardized" interviews merely suggested that students guess an equation to fit the problem and check given information in the equation. This method makes no attempt to correct misconceptions; it only helps students obtain solutions efficiently.

The mathematics problems studied involved only one type of algebra problem from a family of proportional thinking tasks that are well-known to be very difficult. To infer from this study that school algebra instruction has fatal flaws seems unjustified by the evidence. While shocking the reader with evidence of poor mathematical performance, the study makes no effort to relate these findings to the rich literature on meaningful learning in mathematics or proportional thinking and its development as studied by the Piagetians.

Classroom teachers and future research must decide how indicative these results are of student conceptual understanding in algebra. The results of such research may point to an overemphasis on standardized tests which encourage some teachers to use quick-fix methods like the procedure taught in the study. While the study is inconclusive about conceptual development, it makes an important distinction between correct solutions and conceptual understanding and suggests an appropriate research methodology based upon interviewing. This is a useful, provocative entry into an important research area, but only a beginning.

Shumway, Richard J.; White, Arthur L.; Wheatley, Grayson H.; Reys, Robert E.; Coburn, Terrence G.; and Schoen, Harold L. INITIAL EFFECT OF CALCULATORS IN ELEMENTARY SCHOOL MATHEMATICS. Journal for Research in Mathematics Education 12: 119-141; March 1981.

Abstract and comments prepared for I.M.E. by THOMAS E. ROWAN, Montgomery County Public Schools, Rockville, Maryland.

1. Purpose

The authors state as their purpose, "to determine the effect the availability of calculator-related curriculum resources, consultant resources, and in-service workshops for teachers had on the elementary school children's attitudes and achievement in mathematics, Grades 2-6" (p. 119).

2. Rationale

The issue of calculator use in mathematics classes is one that affects virtually every school and classroom nationwide. The public appears to be most concerned over whether calculators should be used in elementary schools, according to a study cited by the authors. Serious design and sampling problems severely limit the conclusions which can be drawn from previously reported studies. It was therefore seen as important and appropriate to carry out the investigation reported in this article.

3. Research Design and Procedures

Attitudes of students were measured using two six-item semantic differentials with five response options. One scale was used to measure calculator attitudes. Four basic facts tests, each made up of twenty randomly selected facts, were used as indication of the students' knowledge of basic facts. These tests were read to the students. Mathematics achievement was measured with the Mathematics Tests of the Stanrord Achievement Tests. Achievement in concepts, computation, and applications was measured by these tests. Two of the computation posttests were given with student access to a calculator. A twelve-item multiple-choice test was designed by the researchers and used to measure estimation skills. Two levels of "special topics" posttests were designed by the researchers. These tests had two parts, one taken without the use of calculators and the other taken with the use of calculators. The non-calculator portion of the test presented questions which would not likely benefit from calculator use. The calculator portion was more oriented toward problems requiring computations of some type. The final type of data collected was observational data. The major purpose of this was to document the degree and type of calculator use.

The sampling design considered site, grade level, and treatment. The sites included five states, the grade levels included were two through six, and the treatments were calculator and no-calculator. Fifty teachers and their classes were involved in the study, five from each of the five sites. Five classes at each site were in the no-calculator treatment and five in the calculator treatment. Twenty-five teachers and their classes were included in each treatment.

The differences between treatments were on availability of calculators, teacher workshops on use of calculators, availability of calculator-based instructional materials, and researchers' availability as consultants to teachers. The treatments began in October and ended in February, and had a duration of 67 school days. It is notable that teachers in the no-calculator group were specifically instructed to ask their students not to use the calculator for any mathematics work.

4. Findings

The following summary of "Results by Hypotheses" was offered by the authors on pages 136 and 137:

Hypothesis I. No evidence was found to support the hypothesis that use of calculators influences student attitudes toward mathematics.

Hypothesis II. Evidence was found to support the hypothesis that students have a more positive attitude toward calculators than toward mathematics. These effects were observed at grades 2-3 and grades 4-6 ($p \leq .001$) independent of calculator use.

Hypothesis III. Evidence was found to support the hypothesis that students with and without calculators show gains for basic facts (+, -, x, ÷) ($p \leq .001$). These gains were independent of calculator use.

Hypothesis IV. No evidence of effects of calculator use on student knowledge of basic facts or on student mathematics achievement was found.

Hypothesis V. No evidence of effects of calculator use on student achievement of estimation skills or special topics of mathematics was found.

Hypothesis VI. Evidence was found to support the hypothesis that the use of the calculator on computations tests would increase student computations test scores. This effect was observed at grades 2-4 ($p \leq .001$) and grades 5-6 ($p \leq .01$) independent of Treatment Group.

5. Interpretations

The authors concluded from their results that children have more positive attitudes toward calculators than toward mathematics; that children achieve better on basic fact and mathematics achievement tests taken without calculators regardless of whether they used calculators during instruction; "the use of calculators increases children's 'computational power' with little instruction"; and that they found no evidence of detrimental or positive effects on grades 2-6 arising from first-year use of calculators in mathematics classes.

In addition to the conclusions stated, the authors made recommendations for future research, suggested directions for school practices, and raised a basic philosophical question. The suggested need for research was focused on three potential advantages for calculator use:

"1. increased number and variety of examples of mathematical concepts and computations

2. facilitation of the introduction of new topics such as decimals, metric measure, negative integers, and number theory earlier in a student's mathematical training

3. improvement in student and teacher attitudes toward mathematics." (p. 140)

In suggesting school practices, the authors stressed that no evidence was found that student abilities were debilitated as a result of calculator use. They endorsed the use of calculators when accompanied by appropriate monitoring and planning. They decried two common errors in calculator use, the failure to use them when they would be most effective, and the tendency to encourage children to use calculators but then test them only without calculators.

Philosophically, the authors ask whether it is "possible to use calculators in such a way that algorithms such as the long division algorithm need not be taught and would not be learned" (p. 141). Investigation of such questions through surveys, philosophical research, and clinical research is recommended.

Abstractor's Comments

This was a very thorough and well-supported study of calculator use in elementary mathematics teaching. The questions which were investigated are important questions. There may be other equally important questions which were not considered, but perhaps no single study can be expected to treat all important dimensions of a complex issue. This study involved a large number of teachers and their students in what appeared to be a minimal training effort, supplied them with adequate numbers of calculators to use in their classrooms, provided teachers with activities which could be used with students, and made consultant support available to teachers while the research was in progress. Although the report did not say this specifically, it appeared that the emphasis in the project was to make the equipment and materials available and to provide adequate instruction to get teachers started with calculators. The emphasis appeared <u>not</u> to be on providing intensive guidance on the use of calculators or their integration into the classroom instructional scheme. The total teacher workshop time for the calculator group was three hours. Periodic consultant availability after the program was underway reinforced this workshop training. The effect of the calculator in such a minimal training setting would thus be the question at issue. In a sense, the full potential of the calculator for influencing instruction and learning is not really investigated; only a portion of that potential is treated. Given the nature of the real world as far as in-service training for teachers is concerned, this minimal training approach may be the most appropriate one to evaluate. It would be interesting, though, to see an investigation into the potential of the calculator which involved more intensive training and development of appropriate materials for making the calculator an integral part of the mathematics program. It seemed that in this research study the sampling design and statistical analysis were more sophisticated and powerful than the treatment itself. Again, this may be appropriate, given a goal of determining calculator effect with minimal training and the widely varying teaching conditions which exist in different school systems in this country.

This writer was somewhat puzzled at the way the hypotheses were handled in the research report. No mention was made of the hypotheses to be tested being formulated prior to the conduct of the study. In fact, the wording of the hypotheses as they are initially presented in the "Summary of Results by Hypotheses" section of the paper (pp. 136-137) leads one to speculate on whether they were written before or after the study.

This speculation would appear to be refuted by the following statement under the "Results" section of the paper, "The results include the analysis of pretest and post-test data relevant to the hypotheses stated earlier" (p. 128). The only problem is that the hypotheses cannot be found stated on any of the "earlier" pages, only in the location already cited. It is difficult for this writer to envision how Hypothesis III, "that students with and without calculators show gains for basic facts (+, −, x, ÷ : $p \leq .001$) and for mathematics achievement (Concepts, Computations, and Applications, $p \leq .001$)" (p. 136), would have been formulated prior to the review of the data. This same question carries over to Hypothesis VI, which spoke to increased student computations test scores when calculators were used, independent of the treatment group. It is unclear whether the independence from treatment group was a part of the hypothesis, or an added observation. It seems unlikely that it could have been part of a hypothesis forumulated prior to the data review.

It was of some concern to this writer that the findings stated in Hypothesis VI were not discussed at greater length. The fact that the no-calculator groups were able to apply the calculator in the testing situation as well as did the calculator-trained students is an interesting result. The possible implication is that extensive efforts to teach children to use calculators might be wasteful of time for both teachers and students. Of course, it must be kept in mind that the teachers, and presumably therefore the students, in this particular study did not have extensive training on how to use the calculator. It would have been interesting to know more of what the writers saw in this finding, given their more intensive knowledge of the project and how the classroom activities really differed for the two treatments.

The authors do a good job describing the limitations of their study. They saw these mainly in three dimensions, breadth (meaning limitations of the treatment and testing), duration, and power (referring to the power of the statistical procedures used). Whatever the limitations are, it seems to this writer that the primary contribution of this study is its support for the claim that calculator use does not have a detrimental effect on computational achievement. This support must be viewed in the context of the minimal training provided to teachers. Would teachers who had more extensive training with calculators have used them differently, or more frequently, and would there consequently have been an effect on mathematics computational achievement?

The "Future Directions" section of the report poses a number of interesting questions for further study. Some of these deal with ways of facilitating instruction in the traditional mathematics curriculum through use of calculators; others deal with testing and standards. These are important questions, but difficult to investigate in the current educational atmosphere, where everyone seems caught up with achieving minimal competencies. Variations from traditional roads to these competencies find little acceptance among the public, and among many teachers.

The philosophical question posed by the authors, whether some parts of the curriclum, such as long division, are still necessary, is even more difficult in today's society. The way that curriculum is determined in United States schools provides almost no vehicle for significant changes in relatively short periods of time. The closest we have come to such changes in recent years were the "new math" of the sixties and the metric changes of the seventies. Both of these changes made significant inroads because of relatively strong centralized leadership funded by the National Science Foundation and the Office of Education. Prospects for such leadership in the near future seem dim indeed.

Overall, this was an interesting research report dealing with an issue which may not be as important today as it was five to eight years ago, at least not in the minimal training form used in this project. Many teachers are probably quite willing to use calculators in whatever ways seem supportive of the content they are teaching. They need more guidance on appropriate places to use them in the curriculum and how to change other aspects of their teaching to accomodate and reflect their use. They also need training which will enable them to use calculators effectively in their own lives so that they can teach from an experience base.

Silver, Edward A. RECALL OF MATHEMATICAL PROBLEM INFORMATION: SOLVING RELATED PROBLEMS. Journal for Research in Mathematics Education 12: 54-64; January 1981.

Abstract and comments prepared for I.M.E. by ROLAND F. GRAY, University of British Columbia.

1. Purpose

The purpose of this study was threefold:

(a) To test previous findings which indicated that high-ability problem solvers tend to recall the mathematical structure of previously solved problems, while low-ability students tend to recall the context of the problems.

(b) To study the relationship of such recall in the solution of problems related to previously solved problems.

(c) To study information transfer and recall when discussion of a first problem is a mediating factor.

2. Rationale

This study was developed from a series of earlier studies, cited by the author, which indicated that successful and unsuccessful problem solvers differed in the quality of information recalled from earlier problems. Previous studies were done with university students. This present study was done with junior high students working typical word problems.

3. Research Design and Procedures

(a) Sample

The sample consisted of 67 grade seven mathematics students, all members of three sections of grade seven in a suburban New York school. The subjects' I.Q. scores were "well above" national norms (p. 55).

(b) Tasks

On a 16-item card sort task, subjects were asked to classify problems that were nathematically related and to explain why (CST). Four sets of problems were structurally related and four were related by context. A subtest of 12 items constituted a verbal problem solving test (VPST).

A week later, the students were asked to solve two "target problems" of different mathematical structure (p. 55). After work was done, subjects were asked to write down all they remembered. They were asked to do the same the next day. This was followed by a discussion of the problems. On the next day and four weeks later, the same recall task was performed. Altogether there were four recall tasks.

Following the third recall task, subjects completed a six-item related problems tests (RPT). For each of the two target problems, one problem was related by structure, one was related by context, and one was unrelated.

(c) Scoring and Analysis

Scoring procedures were developed such that the subjects were classified as good (N = 16), average (N = 37), and poor (N = 14) problem solvers. Except for two cases, this corresponded to teacher judgments.

4. Findings

The major findings are presented only in briefest summary.

(a) "Good problem solvers" tended to form groups of problems on the basis of common problem structure, while poor problem solvers grouped on the basis of common problem details.

(b) On related problem tests, pre-solution judgment was positively related to problem-solving success, with those finding a structural relationship being vastly more successful than those judging a relationship of detail (18 to 0 for target problem #1 and 26 to 0 for target problem #2).

(c) Recall of problem structure was less frequent than recall of other aspects except for the fourth recall.

(d) "Good problem solvers" who solved target problems successfully tended to have a better recall of structure than good problem solvers who were unsuccessful.

(e) Recall of structure improved after class discussion of structure.

(f) "Poor problem solvers recalled other aspects as well as (did) good...problem solvers. " (p. 61)

5. Interpretations

In a discussion section, the author drew the following conclusions and implications.

(a) The findings of this study supported earlier studies that tended to show that good and poor problem solvers differed with respect to their solution success and also in their recall of problem information. Good problem solvers tended to show a reasonably accurate recall of details, but so did poor problem solvers. Thus, difficulties of poor problem solvers are probably not due to any systematic memory defect, but may be due to their lack of ability to see structural relationships.

(b) The findings were related to studies of reading in which good and poor readers differed in their recall of structural relationships. Poor readers exhibited difficulties with comprehension.

(c) A significant transfer of information on target problems to related problems was postulated, as one related problem was more difficult than the target problem.

However, both good and poor problem solvers failed to report an influence of a first solution on a second.

(d) Contrary to the above (c), it might have been the case that for the good problem solvers the solution method was independent of the previously solved target problem.

Abstractor's Comments

The researcher has done excellent work in conceptualizing his study from previous work and from a limited body of emerging theory relating to problem solving in mathematics.

The author's series of recall tasks seemed well designed, but most of the evidence in his study, despite his view to the contrary, seems to support earlier findings of rapid forgetting. The recall of structural relationships, for example, showed a decline of total recall from 11 to 0 over four weeks for good problem solvers. (Poor problem solvers remembered none.) Further, the number 11 was a small proportion of the total of 67 subjects and of the 30 who solved target problem one and the 38 who solved target problem two.

I am left somewhat in doubt as to whether or not transfer of information from target problems to related problems did, in fact, occur. In one place the author suggests that "...a significant transfer of information occurred from the target problems to related problems" (p. 63). Further on he states "...Neither good nor poor problem solvers tended to acknowledge having used information from the target problems..." (p. 63). I did not find a satisfactory explication of this apparent inconsistency.

The major finding seems to be that successful problem solvers tended to see structural relationships between related problems. However, in this study, recognition of similar structure seemed to be evidenced by the ability to relate a particular form of equation to several similar problems. While this can be seen as a structural relationship, it may not be a profound one. I am not sure what enables some students to see such relationships and not others; however, I suspect this ability may relate to some form of little understood mental process. Noting that the 67 subjects had substantially higher than normal I.Q.s and that only 16 were classed as good problem solvers, there must be a potentially large number of students who cannot solve problems. This is briefly acknowledged by the author, but not dealt with at any great length. Ultimately, this must become a more engaging question than those examined in this study.

All of the foregoing comments may arise from the general complexity of the study and the difficulty I found following some of the written exposition. The article was undoubtedly an abstract of a larger paper for which it may have been difficult to write in a simple and direct manner. (Further abstraction was difficult indeed!) I think this was probably an important study, carefully designed and carefully carried out, but readers might be advised to go directly to the larger work for a more complete and fuller understanding.

Smith, Lyle R. and Cotten, Mary Linda. EFFECT OF LESSON VAGUENESS AND DISCONTINUITY ON STUDENT ACHIEVEMENT AND ATTITUDES. Journal of Educational Psychology 72: 670-675; October 1980.

tract and comments prepared for I.M.E. by THOMAS J. COONEY, ersity of Georgia.

1. Purpose

The purpose of the investigation was to study the effects of low-inference teacher clarity behaviors on student achievement and attitudes. Specifically the following question was addressed: What is the joint effect of lesson discontinuity and teacher vagueness terms on student achievement and student perception of lesson effectiveness?

2. Rationale

Previous research has established relationships between discontinuity and a lesson's clarity and the behavior of students. In the present study two forms of discontinuity were defined. One was defined as the teacher's interruption in the flow of the lesson with information which was irrelevant to the lesson. The second was defined as the teacher's interjection of relevant stimuli at inappropriate times in the lesson.

Various studies have established negative correlations between teacher use of vagueness terms and student achievement. Vagueness terms consist of phrases of approximation, indeterminate quantification, ambiguous designation, and other statements of imprecision. This study was designed to provide additional evidence on the joint effects of variables related to teacher verbal patterns (such as vagueness terms) and to transitions within lessons (such as lesson discontinuity) on student achievement and on student perception of lesson effectiveness.

3. Research Design and Procedures

Each of the 100 seventh-grade subjects was randomly assigned to one of four groups (n = 25 each) which were defined by the possible combinations of two continuity conditions (discontinuity, continuity) and two vagueness conditions (vagueness terms, no vagueness terms). The four groups listened to a 12- to 14-minute audiotaped mathematics lesson while they observed overhead projectors with corresponding content. The content consisted of three geometric theorems involving relationships between chords, secants, and tangents. The theorems are traditionally taught in high school geometry. None of the students had prior instruction concerning the theorems.

Student comprehension of the lessons was determined by administering a 20-item test immediately after each lesson was completed. The test problems required students to select the appropriate theorem to solve a given problem and then to perform correct computations. Immediately after students completed the test they were administered a six-item lesson evaluation.

Two of the lessons contained 120 vagueness terms and two lessons contained no vagueness terms. Also, two lessons contained 40 instances of discontinuity while the other two lessons contained no instances of discontinuity. Of the 40 instances of discontinuity, 14 involved the interjection of irrelevant stimuli and 26 involved the interjection of relevant stimuli at inappropriate times.

A 2 (discontinuity vs. continuity) x 2 (vagueness terms vs. no vagueness terms) analysis of variance was performed on the student achievement scores and on the lesson evaluation scores.

4. Findings

The main effect due to lesson discontinuity was significant ($p < .01$), as was the main effect due to vagueness terms ($p < .001$), with respect to studen achievement. The interaction between discontinuity and vagueness terms was also significant ($p < .02$). Neither the main effect due to lesson discontinuity nor the interaction between discontinuity and vagueness terms was significant for lesson evaluations. The vagueness terms main effect was significant ($p < .01$) for lesson evaluations.

5. Interpretations

Two cautions were noted. First, the lessons were very short, averaging only 13 minutes. Second, the content was very difficult for seventh graders as evidenced by a mean score of 8.86 out of a possible 20 points for the highest achieving group. Still, several conclusions were made. The study supports findings from previous research which suggest a high frequency of vagueness terms negatively influences achievement and causes pupils to perceive the teacher as disorganized and unprepared. The authors also stated that "the findings of this study indicate a cause-and-effect relation between discontinuity and achievement. In the light of prior research indicating that discontinuity inhibits teacher clarity, such results might be expected. Perhaps of greater interest is the interaction between vagueness terms and discontinuity. The combined effect of the two variables on achievement merits further study. The effect of discontinuity on student perception was somewhat unexpected." The authors suggest that "students are somewhat accustomed to teachers who digress, interject irrelevant anecdotes, and generally meander through lessons."

The investigators argued that further research on vagueness terms and discontinuities should focus on the threshold levels at which the variables influence achievement. The introduction of noise, if extensive enough, is sure to cause some disharmony with the instructional process. The question then becomes the identification of the threshold level at which the noise begins negatively to affect student achievement.

Abstractor's Comments

This study utilizes a simplistic design to answer rather specific questions. One of the study's strengths lies in the fact that it coordinates well with previous research and follows a rather consistent trend of research associated with vagueness terms and discontinuities specifically and clarity in general. The investigation provides further evidence of the effect of clarity on achievement.

The authors identified several limitations which indeed are quite severe. One involves the fact that the study is limited in scope. The fact that instruction is so short (13 minutes) and somewhat artificial (audiotapes plus overhead) raises serious questions as to the viability of the treatments. Be that as it may, one must also be concerned about the arbitrariness of the intensity levels of the vagueness terms and discontinuities. Perhaps there is something to be said for "turning the noise up loud enough" to see if interference eventuates. If so, then the question of the threshold of the noise can be investigated, as suggested by the authors. On the other hand, 120 vagueness terms for a 13-minute lesson is a rather hefty dose of vagueness, nearly 10 terms per minute or one every six seconds! That level of noise is unrealistic in terms of the teachers this writer has observed. The intense level of vagueness terms and discontinuities causes considerable concern over the "face validity" of the treatment and the applicability of the conclusions reached. The investigators also gave quite a lot of credit for doing computations correctly. It is hard to see why the ability to compute should play a role in determining the effectiveness of instruction. The lessons were not aimed at promoting computational expertise.

The authors state that "perhaps the single most relevant suggestion for teacher training and teacher evaluation is that trainers and evaluators focus on low-inference teacher behaviors that can be critiqued objectively." Certainly the authors have a point regarding the utility of low-inference behaviors. But it is also possible that the occurrence of vagueness terms and discontinuities may be a second-order phenomena, where the primary difficulty stems from feelings of insecurity, lack of knowledge of the content, or any other number of factors which quite possibly need to be addressed directly.

Nevertheless, the study does provide some useful information on effective instruction. Hopefully those interested in the phenomena of discontinuities and vagueness terms will turn their attention to questions involving threshold levels or will investigate the occurrence of such phenomena in actual classrooms.

An Announcement . . .

FIFTH INTERNATIONAL CONGRESS ON MATHEMATICS EDUCATION

Adelaide, Australia - Friday 24 August to Thursday 30 August 1984

The International Commission on Mathematical Instruction (ICMI) has accepted an invitation from the Australian Academy of Science to hold the Fifth International Congress on Mathematical Education at the University of Adelaide.

The Congress Program

Determination of the major emphasis in the Congress Program is the responsibility of the International Program Committee (IPC), appointed by ICMI. Dr. M. F. Newman of the Australian National University is the chairman of IPC.

It is expected that the program will span all levels of education and discuss problems of general interest while recognising different cultural perspectives.

A principal objective of the Congress will be to facilitate both professional and personal contact amongst its participants. In particular, the organisers seek to encourage existing working groups in mathematics education to meet at the Congress and to encourage overseas participants to visit Australian colleagues in their home educational institutions.

Languages

The official language of the Congress is English. There will be translations of important sessions into several languages. There will also be provision for translating abstracts or summaries of presented papers into several languages. The selection of languages will be dependent upon the needs of the participants and also on the availability of assistance in providing translations prior to the Congress. Languages considered currently for selection include Japanese, Chinese, Indonesian, French, German, Russian and Spanish.

Future Announcements

The ICME 5 Organising Committee in Australia expects to issue a first announcement by May 1982. This will contain general information of relevance to prospective participants. A second announcement is expected to be available by May 1983. It will contain details of the scientific program and ancillary activities and include a registration form. The second announcement will automatically be sent to all respondents to the first announcement.

Request for Comment

The Organising Committee requests comment from prospective participants which might assist it in planning Congress activities. Remarks on the weaknesses and strengths of previous Congresses and other relevant international meetings will be greatly appreciated. Responses received before July 1982 will be especially helpful.

Please write to

ICME 5,
Wattle Park Teachers' Centre,
424 Kensington Road,
WATTLE PARK, SOUTH AUSTRALIA 5066,
AUSTRALIA

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EJ 242 308. How the U.S. Compares with Other Countries. Educational Leadership, v38 n5, 368-70, February 1981.

EJ 242 824 Braine, Martin D. S.; Rumain, Barbara. Development of Comprehension of "Or": Evidence for a Sequence of Competencies. Journal of Experimental Child Psychology, v32 n1, 46-70, February 1981.

EJ 242 826 McLaughlin, Judith A. Development of Children's Ability to Judge Relative Numerosity. Journal of Experimental Child Psychology, v31 n1, 103-14, February 1981.

EJ 242 908 McKillip, William D. Computational Skill in Division: Results and Implications from National Assessment. Arithmetic Teacher, v28 n7, 34-37, March 1981.

EJ 242 909 Peck, Donald M.; Jencks, Stanley. Share and Cover. Arithmetic Teacher, v28 n7, 38-41, March 1981.

EJ 242 946 Galbraith, P. L. Aspects of Proving: A Clinical Investigation of Process. Educational Studies in Mathematics, v12 n1, 1-28, February 1981.

EJ 242 947 Booth, Lesley R. Child-Methods in Secondary Mathematics. Educational Studies in Mathematics, v12 n1, 29-41, February 1981.

EJ 242 948 Kemme, S. L. References of Speech Acts as Characteristics of Mathematical Classroom Conversation. Educational Studies in Mathematics, v12 n1, 43-58, February 1981.

EJ 242 949 Zepp, Raymond A. Relationships Between Mathematics Achievement and Various English Language Proficiencies. Educational Studies in Mathematics, v12 n1, 59-70, February 1981.

EJ 242 950 Hasemann, Klaus. On Difficulties with Fractions. Educational Studies in Mathematics, v12 n1, 71-87, February 1981.

EJ 242 951 Janvier, Claude. Use of Situations in Mathematics Education. Educational Studies in Mathematics, v12 n1, 113-22, February 1981.

EJ 242 952 Lang, Bryan; Ruane, Peter. Geometry in English Secondary Schools. Educational Studies in Mathematics, v12 n1, 123-32, February 1981.

EJ 242 953 Von Glasersfeld, Ernst. An Attentional Model for the Conceptual Construction of Units and Number. Journal for Research in Mathematics Education, v12 n2, 83-94, March 1981.

EJ 242 954 Houlihan, Dorothy M; Ginsburg, Herbert P. The Addition Methods of First- and Second-Grade Children. Journal for Research in Mathematics Education, v12 n2, 95-106, March 1981.

EJ 242 955 Wagner, Sigrid. Conservation of Equation and Function under Transformations of Variable. Journal for Research in Mathematics Education, v12 n2, 107-18, March 1981.

EJ 242 956 Shumway, Richard J.; And Others. Initial Effect of Calculators in Elementary School Mathematics. Journal for Research in Mathematics Education, v12 n2, 119-41, March 1981.

EJ 242 957 Zehavi, Nurit; Bruckheimer, Maxim. A Method of Analyzing Tests Using the Teacher's Predictions. Journal for Research in Mathematics Education, v12 n2, 142-51, March 1981.

EJ 242 958 Kraus, William H. Using a Computer Came to Reinforce Skills in Addition Basic Facts in Second Grade. Journal for Research in Mathematics Education, v12 n2, 152-55, March 1981.

EJ 243 023 Carpenter, Thomas P.; And Others. Decimals: Results and Implications from National Assessment. Arithmetic Teacher, v28 n2, 34-37, April 1981.

EJ 244 527 Smith, Lehi T.; Haley, J. M. Inservice Education: Teacher Response and Student Achievement. School Science and Mathematics, v81 n3, 189-94, March 1981.

EJ 244 528 House, Peggy A. One Small Step for the Mathematically Gifted. School Science and Mathematics, v81 n3, 195-99, March 1981.

EJ 244 533 Benson, Vada P. Chisanbop - Can Students Benefit from It? School Science and Mathematics, v81 n3, 236-38, March 1981.

EJ 244 588 Kleiman, Glenn; And Others. Microcomputers and Hyperactive Children. Creative Computing, v7 n3, 93-94, March 1981.

EJ 244 619 Eisenberg, Theodore. Remedial Mathematics and Open Admissions. School Science and Mathematics, v81 n4, 341-46, April 1981.

EJ 244 621 Hess, Charlotte M.; Shrigley, Robert L. A Study of the Effect of Three Modes of Teaching on Metric Knowledge and Attitude. Science Education, v65 n2, 121-38, April 1981.

EJ 244 631 Buccino, Alphonse; Evans, Paul L. Views from the Field on Needs in Precollege Science, Mathematics, and Social Studies Education. Science Education, v65 n2, 237-41, April 1981.

EJ 244 632 Tomizuka, C.; And Others. Mathematical Ability: Is Sex a Factor? Science, v212 n4491, 114-21, April 1981.

EJ 244 638 Clement, John; And Others. Translation Difficulties in Learning Mathematics. American Mathematical Monthly, v88 n4, 286-90, April 1981.

EJ 244 930 Skon, Linda; And Others. Cooperative Peer Interaction Versus Individual Competition and Individualistic Efforts: Effects on the Acquisition of Cognitive Reasoning Strategies. Journal of Educational Psychology, v73 n1, 83-92, February 1981.

EJ 246 032 DeLuca, Frederick P.; Scouten, Donald C. A Microcomputer System as a Tool for Research in Logical Thinking. Journal of Educational Technology Systems, v9 n1, 3-20, 1980-81.

EJ 246 180 Saxe, Geoffrey B. Body Parts as Numerals: A Developmental Analysis of Numeration Among the Oksapmin in Papua New Guinea. Child Development, v52 n1, 306-16, March 1981.

EJ 246 249 Olstad, Roger G.; Beal, Jack L. The Search for Teachers: Supply and Demand in Washington State. Science Teacher, v48 n4, 26-28, April 1981.

EJ 246 263 Nummedal, Susan G.; Collea, Francis P. Field Independence, Task Ambiguity, and Performance on a Proportional Reasoning Task. Journal of Research in Science Teaching, v18 n3, 255-60, May 1981.

EJ 246 272 Schoedler, James. A Comparison of the Use of Active Game Learning with a Conventional Teaching Approach in the Development of Concepts in Geometry and Measurement at the Second Grade Level. School Science and Mathematics, v81 n5, 365-70, May-June 1981.

EJ 246 276 Wiebe, James H. The Use of Manipulative Materials in First Grade Mathematics: A Preliminary Investigation. School Science and Mathematics, v81 n5, 388-90, May-June 1981.

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