DOCUMENT RESUME

ED 211 369 SE 036 050

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TITLE Illinois Inventory of Educational Progress: 1979 Mathematics Results, 4th Grade.
INSTITUTION Illinois State Board of Education, Springfield.
PUB DATE [80]
NOTE 35p.; For related documents, see SE 036 051-052.

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Achievement Tests; Cognitive Objectives; *Educational Assessment; Educational Objectives; Educational Research; Elementary Education; *Elementary School Mathematics; Grade 4; Mathematical Concepts; *Mathematics Achievement; Mathematics Curriculum; Mathematics Education; Mathematics Instruction
IDENTIFIERS Illinois; *Illinois Inventory of Educational Progress; *Mathematics Education Research

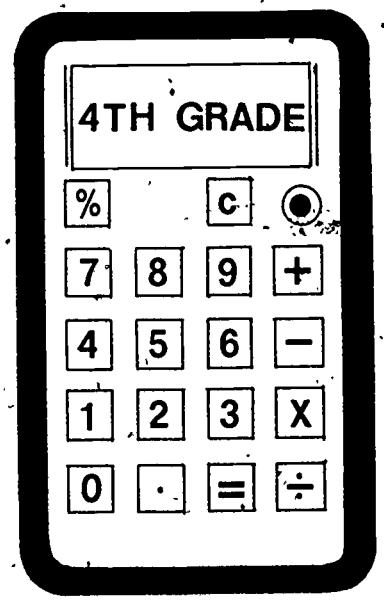
ABSTRACT

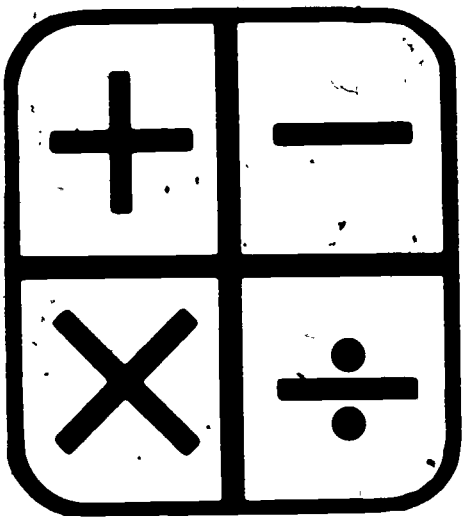
Presented is an overview of the 1979 Illinois Inventory of Educational Progress (IIEP) for fourth-grade mathematics. The IIEP is a systematic effort by the Illinois State Board of Education to collect information on the educational achievement of Illinois students in certain areas and to make that information available to education decision-makers. The IIEP employs an objective-referenced approach, with desired student performance expressed in terms of objectives. Students to be tested are selected in a two-stage random sampling method. Since the IIEP is geared towards determining how groups of Illinois students perform on given tasks, no individual student, teacher, or school district is identified. As part of the study, teachers of participating students were asked to estimate the percentage of students who would obtain correct answers to individual test items. Of the eight measured objectives, it was found that teacher estimates were significantly higher than student performance for only one objective. (MP)

1979
Mathematics
Results

Illinois
State Board of
Education

Illinois Inventory of Educational Progress





FOREWORD

What follows is designed to provide an overview of the 1979 Illinois Inventory of Educational Progress (IIEP) in fourth grade mathematics. The test has been administered by the Illinois State Board of Education since 1976; however, this analytical report is in a new and more usable format.

Development of the IIEP is discussed, and results and analyses of the test administered to fourth grade students are presented. Results and analyses of eighth and eleventh grade tests can be found in separate reports. It is hoped that the information contained here will enhance instruction in Illinois schools.

While many state staff members contributed to the preparation of this report, I would like to especially acknowledge the efforts of Dr. Mervin M. Brennan as the main writer. Any questions concerning this report may be addressed to Dr. Brennan or Dr. Thomas Kerins, Manager of the Program Evaluation and Assessment Section of the Department of Planning, Research and Evaluation of the Illinois State Board of Education.

Donald G. Gill
State Superintendent of Education

PREFACE

Purpose

The Illinois Inventory of Educational Progress (IIEP) is a systematic effort by the Illinois State Board of Education to collect information on the educational achievement of Illinois students in certain areas and to make that information available to educational decision makers.

The three goals of the IIEP are:

1) to make available relevant, reliable, and valid data on the educational attainments of Illinois students;

2) to identify any trends (growth, stability, or decline) in educational attainments which occur over time; and

3) to publish results of the research conducted in connection with the IIEP.

Student Selection

A random sample with two sampling stages is used to select those students attending Illinois public schools who will participate.

First, schools throughout the state are chosen randomly. A sample of fourth, eighth, and eleventh graders is then randomly selected from lists of eligible students submitted by schools for participation. These grade levels are selected to correspond roughly with the end of the primary, elementary, and secondary levels of education.

Since the IIEP is geared toward determining how groups of Illinois students perform on given tasks, no individual student, teacher, school, or district is identified in any reports of the results.

Type of Test

The IIEP employs an objective-referenced approach. An objective-referenced assessment instrument assesses student performance. Desired student performance is expressed in terms of objectives. An objective is a statement of desired student performance, for example: "Fourth grade students should be able to recognize geometric shapes such as circles, etc." Student performance is measured by test items designed to determine whether or not certain groups of students are able to do what the objectives state they should be able to do.

Subject Areas

The IIEP has been in existence since 1976. A number of subject areas have been assessed, for example, reading, mathematics, science, citizenship, energy and nutrition, as well as student attitudes about themselves and education in general.

Base line data is collected during the first year that any subject area is assessed. For each succeeding year that a subject area is reassessed, comparisons can be made concerning student performance on specific objectives, and any growth or decline in achievement can be noted.

TABLE OF CONTENTS

CHAPTER 1

Illinois Inventory of Educational Progress - Mathematics

Development of the 1979 Mathematics-IIEP

In the spring of 1978, a panel of six mathematics educators with elementary junior high, high school, and college teaching and administrative experience was convened to assist State Board staff in formulating the 1979 mathematics IIEP (a roster of panel members appears in Appendix D). Charged with redrafting the objectives which had been developed for the 1976 IIEP, the group met over a two-month period. The results of their work are discussed later in this report.

Additionally, results of a teacher survey that was administered with the previous year's IIEP (1978) were used in developing the 1979 mathematics IIEP. Produced by State Board staff, the survey sought to: (1) validate the test; (2) supply an additional perspective on the results; and (3) provide a standard of performance, based upon teacher estimates, with which student results could be compared.

Toward that end, one mathematics teacher from each school which participated in the IIEP was asked to do three things for each test item. Teachers were asked to determine (1) whether students had been exposed to the material and (2) whether the item was of an appropriate level of difficulty. Teachers were also asked (3) to estimate the percentage of students that could be expected to answer each item correctly. A sample of the teacher survey is contained in Appendix C. Results of the teacher survey are discussed in Chapter 2.

The Test

The test was a domain and objective-referenced test, which means simply that the items tested the general domain of mathematics and that items are derived from or keyed to a set of curricular objectives.

Mathematics objectives for the 1979 IIEP were developed by the aforementioned panel of educators. The following mathematics topics and abilities reflect those objectives. A list of topics precedes a summary description of abilities. Some of the topics are self-explanatory; a brief definition is provided for those which are less common. The abilities are a bit more detailed; essentially, they are the skills required for success in mathematics. Each mathematics objective describes a particular ability with reference to a specific topic.

Mathematics Topics

I. NUMERATION CONCEPTS. This topic refers to the concepts of numeration and place value, and the processes of naming numerals, approximating numbers, and rounding off numbers.

II. PROPERTIES OF NUMBERS AND OPERATIONS. This topic also includes characteristics of numbers and operations and comparisons among numbers.

III. NUMBERS.

A. WHOLE NUMBERS. Whole numbers are the numbers used by children to count. Whole numbers include 0, 1, 2, 3, etc.

B. FRACTIONS.

C. DECIMALS.

D. PERCENT.

E. INTEGERS. Integers are positive and negative whole numbers and zero as distinguished from fractions. The numbers -3, -2, -1, 0, +1, +2, +3, etc., are integers.

F. RATIONALS. Rationals is an all-inclusive term for topics A through E, both positive and negative. Examples are +2, +1/2, +.50, +50%, -2, -1/2, -.50, and -50%.

G. REALS. Reals is an all-inclusive term for topics A through F and numbers such as π, $\sqrt{2}$, etc.

IV. MEASUREMENT.

V. ALGEBRA.

VI. GEOMETRY.

VII. PROBABILITY AND STATISTICS.

VIII. PERSONAL AND CONSUMER MATHEMATICS.

Mathematics Abilities

1. Ability to recall and recognize facts, definitions, and symbols quickly. Perception is the primary mental act used.

2. Ability to perform computations, procedures, and complex counting where the operations are indicated.

3. Ability to understand concepts, facts, and processes. The mental operations of analysis and synthesis are used to make comparisons and evaluative judgments.

4. Ability to solve complex word problems. Several of the following operations must be involved: interpretation of the question, identification of the relevant data from the given information, decisions about which operations need to be performed on the data, correct performance on the operations and interpretations of the results.

Each mathematics item tested a student ability with respect to one of the mathematics topics. The matrix of mathematics topics and abilities (Table 1) shows the conceptual model of the IIEP mathematics tests. Each cell of the matrix is a specific mathematics objective.

The test contained items on six topics and four abilities. There were items related to 13 objectives within the topics and abilities. A topic, ability, or objective was considered to be measured if there were three or more items testing it. By that standard, the test measured six topics and four abilities, and eight objectives within them. The test is described more fully in subsequent chapters of this report.

TABLE 1
MATRIX OF MATHEMATICS OBJECTIVES

MATHEMATICS CATEGORIES BY ABILITIES

Mathematics Topics	Mathematics Abilities 1		2		3		4	
	Ability to recall and recognize facts, definitions, and symbols quickly		Ability to perform computations, procedures, and complex counting where the operations are indicated		Ability to understand concepts, facts, and processes		Ability to solve complex word problems	
I. NUMERATION CONCEPTS	1	*4	2		3	8	4	
II. PROPERTIES OF NUMBERS AND OPERATIONS	5	4, 8	6	8	7	4	8	8
III. NUMBERS								
A. WHOLE NUMBERS	9	4	10	4, 8, 11	11	8	12	4
B. FRACTIONS	13	8	14	11	15	11	16	
C. DECIMALS	17		18	8	19		20	
D. PERCENT	21		22	8	23		24	
E. INTEGERS	25		26	4, 8	27		28	8, 11
F. RATIONALS	29	8	30	11	31	11	32	8, 11
G. REALS	33		34		35		36	
IV. MEASUREMENT	37	4, 8 11	38	8, 11	39	8	40	4, 8 11
V. ALGEBRA	41		42	8, 11	43		44	
VI. GEOMETRY	45	8	46		47	8, 11	48	11
VII. PROBABILITY AND STATISTICS	49		50	11	51		52	
VIII. PERSONAL AND CONSUMER MATHEMATICS	53	4	54	4, 8	55	4	56	4, 8, 11

* The numerals (4, 8, 11) indicate the grade level(s) at which these items were tested in the 1979 IIEP.

Chapter 2

ITEM RESULTS

As mentioned in Chapter 1, teachers of participating students were asked to estimate the percentage of students who would obtain the correct answers to the items. The hypothesis was that the teacher estimates would be higher than the student scores. Chapter 4 shows the statistical results.

It was anticipated that there would be some discrepancies between teacher estimates and student scores which could not be submitted to statistical tests or would not reach significance levels, but would lend themselves to suggestions for future research. After statistical analysis of the data, experienced Illinois mathematics educators were asked to comment on the results.

The following descriptions were used for discrepancies between teacher estimates and student scores:

- approximating for discrepancies of ten or less percentage points,
- higher than/lower than for discrepancies of 11 to 20 points, and
- considerably higher than/lower than for discrepancies of more than 20 points.

These discrepancy guidelines were established because consultants suggested the use of consistent standards. Ten percentage points was used since standard deviations for previously calculated data were usually near .10.

The panel of mathematics educators were asked to analyze and interpret the test results using the test data and the teacher survey data. They reflected upon the data for each curricular topic and each objective within the topics. This chapter gives the data and the panel's comments. Correct answers are underlined. Teacher estimates are abbreviated as teach. est., Student scores are abbreviated as stu. score.

The comments are solely those of the experts and are not to be taken as the official position of the State Superintendent of Education or the Illinois State Board of Education.

Topic I: Numeration Concepts

The test contained four items on this topic. Table 2 shows the results.

Table 2

Objective 1: Recognition of Numeration Concepts

Item 30. What digit is in the tens place in 4,263?

a.	2	teach. est.:	83%
b.	3	stu. score:	78%
c.	4		
d.	6		

Item 34. Which of the following is true?

a.	653,804 < 653,084	teach. est.:	69%
b.	653,804 > 653,084	stu. score:	61%
c.	653,804 = 653,084		

Item 31. 1029 is written as

a.	ten hundred twenty-nine.	teach. est.:	74%
b.	ten thousand twenty-nine.	stu. score:	40%
c.	one thousand two hundred nine.		

Item 41. Four and two-fifths is written as

a.	42/5	teach. est.:	49%
b.	4/25	stu. score:	73%
c.	4 2/5		
d.	None of these		

Panel Comments: Student scores approximated teacher estimates for items 30 and 34. Scores were considerably lower than the teacher estimate for item 31. In regard to item 31, the panel noted that fourth graders are usually taught to read the numeral 1029 as "one thousand twenty-nine." Many students may not have known that "ten hundred twenty-nine is also correct." Student scores were considerably higher than teacher estimates for item 41, which related to understanding numeration concepts.

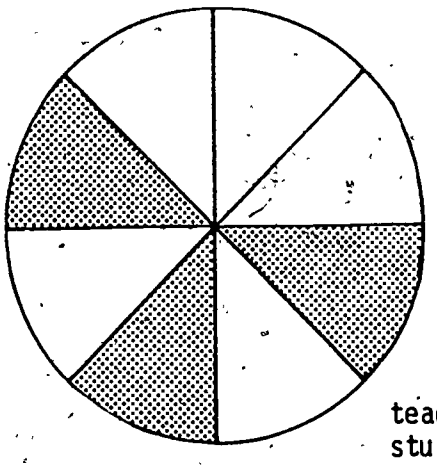
Topic II: Properties of Numbers and Operations

There were six items on this topic. Table 3 shows the results.

Table 3

Objective 5: Recognition of Properties of Numbers and Operations

Item 42. The figure below is divided into equal parts. What fractional part is shaded?



a. $\frac{1}{2}$

b. $\frac{3}{6}$

c. $\frac{3}{8}$

d. $\frac{3}{10}$

teach. est.: 57%
stu. score: 74%

Item 29. What is the next larger odd number after 5?

a. 6
b. 7
c. 8
d. 9

teach. est.: 77%
stu. score: 70%

Item 33. Which of the following is a prime number?

a. 12
b. 13
c. 15
d. 18

teach. est.: 32%
stu. score: 24%

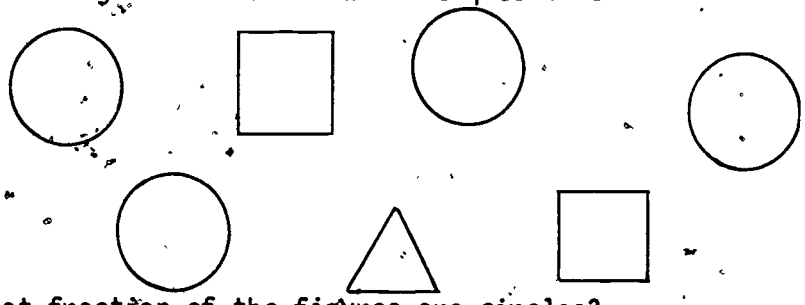
Item 32. Which of the following is a prime number?

a. 6
b. 7
c. 9
d. 15

teach. est.: 33%
stu. score: 18%

Objective 7: Understanding of Properties of Numbers and Operations

Look at the drawings below and answer the questions that follow.



Item 39. What fraction of the figures are circles?

a. $\frac{4}{4}$

b. $\frac{3}{7}$

c. $\frac{2}{7}$

d. $\frac{4}{7}$

teach. est.: 51%
stu. score: 47%

Item 40: What fraction of the figures are geometric shapes?

a. $\frac{3}{7}$ — teach. est.: 44%, stu. score: 29%

b. $\frac{7}{7}$

c. $\frac{4}{7}$

d. $\frac{2}{7}$

Panel Comments: ~~The~~ student average score (47%) approximated the teacher estimate (50%) for objective 5. The range of scores was large, from 77% to 18%. Sixty-two percent of the teachers responded in the IIEP teacher survey that their students had received no exposure to the material tested by items 32 and 33. They said those items were too difficult. Fifty-three percent of the teachers reported little or no student exposure to the material tested by item 42. The student score (74%) was higher than the teacher estimate (57%) for that item. Sixty-two percent of the teachers also reported little or no exposure to the material tested by the items related to objective 7. The student average score for the objective was 38%; the teacher estimate was 48%.

Topic III: Numbers

The test contained 11 items on this topic. Eight items were related to whole numbers; three items tested integers.

Whole Numbers

Whole numbers were tested by one item on objective 9, four items on objective ten, and three items on objective 12. Table 4 shows the results.

Table 4

Objective 9: Recognition of Whole Number Facts

Item 26. Divide: $9 \div 3 =$

a. 3 — teach. est.: 85%, stu. score: 86%

b. 6

c. 12

d. 27

Objective 10: Computation of Whole Numbers

Item 25. Multiply: 38
X 9

a. 342 — teach. est.: 77%
b. 272 — stu. score: 83%
c. 2772
d. 372

Item 28. The number that is 200 less than 800 is

a. 1000 — teach. est.: 81%
b. 400 — stu. score: 78%
c. 600

Item 23. Find the difference: 2043
- 317

a. 1,736 — teach. est.: 76%
b. 1,627 — stu. score: 78%
c. 1,726
d. 1,636

Item 27. What number is 3 more than 999?

a. 2,997 — teach. est.: 79%
b. 996 — stu. score: 73%
c. 333
d. 1,002

Objective 12: Problem Solving Using Whole Numbers

Item 47. An astronaut is to orbit the earth in a space capsule for seven days. If he drinks three pints of water each day, how many pints of drinking water will be needed for the trip?

a. 4 pints — teach. est.: 69%
b. 7 pints — stu. score: 73%
c. 10 pints
d. 21 pints

Item 45. John has 385 stamps in his stamp collection. Greg has 230, Pete has 310 and Bob has 175. The number of stamps the boys have all together is:

a.	900 stamps	teach. est.: 77%
b.	1,000 stamps	stu. score: 68%
c.	1,100 stamps	
d.	1,200 stamps	

Item 49. A sports car owner says that the car gets 22 miles per gallon of gasoline. How many miles could the car go on seven gallons of gasoline?

a.	144 miles	teach. est.: 65%
b.	154 miles	stu. score: 65%
c.	164 miles	
d.	174 miles	

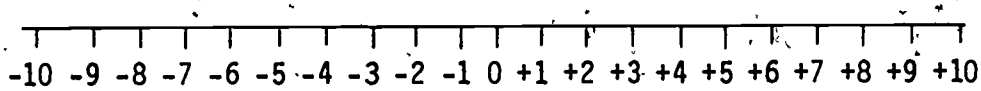
Panel Comments: The student scores on whole numbers approximated the teacher estimates for the topic as a whole, for each objective, and for the individual items. For the topic, the student score was 76%, which was identical to the teacher estimate. For objective 9, the student score was 86%; the teacher estimate was 85%. For objective 10, the student score was 78%, compared to the teacher estimate of 78%. For objective 12, the student score was 69%, compared to the teacher estimate of 70%. Student scores ranged from 86% to 65% for whole numbers. The highest score was on item 26, which was on recognition of whole number facts. The lowest score was for item 49, which tested problem-solving ability. As shown in table 5, post hoc analysis indicated that students scored significantly higher on computation (objective 10) than in problem solving (objective 12).

Integers

Three items measured objective 26. Table 5 shows the results.

Table 5

Objective 26: Computation with Integers



Using the number line above, solve the following problems.

Item 53. What is the difference between +2 and +4?

a. 1
b. 2
c. 3
d. 4

teach. est.: 54%
stu. score: 58%

Item 51. What is the difference between +2 and +5?

a. 2
b. 3
c. 7
d. 10

teach. est.: 54%
stu. score: 50%

Item 52. What is the difference between -5 and +2?

a. 2
b. 3
c. 5
d. 7

teach. est.: 32%
stu. score: 40%

Panel Comments: The student scores on the items approximated teacher estimates. The average score was 49% for the objective. The teacher estimate was 47%. The scores appeared to be low, but it was hypothesized that students were not heavily exposed to the materials tested. That hypothesis was supported by the IIEP teacher survey data. Forty-three percent of the teachers reported that their students had received no exposure to the material tested by items 51 and 53. Seventy-one percent reported no exposure to the material tested by item 52.

Topic IV: Measurement

Seven items tested objective 37. There was one item on objective 40. Table 6 shows the results.

Table 6

Objective 37: Recognition of Measurement Facts

Item 35. How many inches are there in one foot?

a.	12 inches	teach. est.: 76%
b.	24 inches	stu. score: 80%
c.	36 inches	
d.	48 inches	

Item 38. How many quarts are in one gallon?

a.	1 quart	teach. est.: 60%
b.	2 quarts	stu. score: 62%
c.	3 quarts	
d.	4 quarts	

Item 57. Which is the CLOSEST to the size of one square centimeter?

a.	A tennis court	teach. est.: 44%
b.	Your thumbnail	stu. score: 53%
c.	A slice of bread	
d.	The cover of a record album	

Item 36. How many ounces are there in one pound?

a.	8 ounces	teach. est.: 62%
b.	16 ounces	stu. score: 52%
c.	20 ounces	
d.	32 ounces	

Item 56. In the United States, we usually buy gasoline by the gallon. In France, where the metric system is used, people buy gasoline by the.

a.	meter.	teach. est.: 42%
b.	liter.	stu. score: 42%
c.	quart.	
d.	gram.	

Item 37. How many pints are there in one quart?

a.	1 pint	teach. est.: 59%
b.	2 pints	stu. score: 41%
c.	3 pints	
d.	4 pints	

Item 58. Box B holds one liter. How many milliliters does it hold?

a.	1	teach. est.: 34%
b.	10	stu. score: 31%
c.	100	
d.	1,000	

Objective 40: Problem Solving in Measurement

Item 50. Mary earned $1.00 raking leaves. Candy bars cost 15 cents. How many candy bars can she buy with her money?

a.	3	teach. est.: 52%
b.	4	stu. score: 50%
c.	6	
d.	7	

Panel Comments: The student scores were not significantly different from teacher estimates for objective 37. Despite the large range in scores (from 80% to 31%), student scores approximated teacher estimates for almost all of the items. Interpretations should take into account the amount of student exposure. Eighty-one percent (81%) of the teachers reported that students had been exposed to the content of items 35, 36, 37, and 38, and 51% of them

described the exposure as adequate to heavy. Only 56% of the teachers reported that students had been exposed to the content of items 56, 57, and 58, and only 33% described the exposure as adequate to heavy. The student score also approximated the teacher estimate for the item related to objective 40. Seventy-three percent (73%) of the teachers reported that students had been exposed to the content of the item. Fifty-three percent described the exposure as adequate to heavy.

Topic V: Algebra
Topic VI: Geometry
and
Topic VII: Probability/Statistics

The mathematics panel noted during the test development phase that Algebra, Geometry and Probability/Statistics are not included in mathematics curricula until after fourth grade. Those topics were not tested.

Topic VIII: Personal and Consumer Mathematics

The test contained nine items on this topic. All four abilities were tested with respect to the topic. Table 7 shows the results.

Table 7

Objective 53: Recognition of Personal and Consumer Mathematics Facts

Item 46. John has 13 cents. He wants to buy a 25 cent toy. How much more money does he need?

a. 12 + 13 = 25
b. 25 - 12 = 13
c. 25 - 13 = 12

teach. est.: 77%
stu. score: 59%

Item 44. How many apples did you have at the start if you gave away 9 apples and have 6 apples left?

a. 9 - 6 = 3
b. 9 + 6 = 15
c. 6 + 3 = 9

teach. est.: 73%
stu. score: 56%

Item 43. Jane and Sue each had 10 cents, Mary had 9 cents. How much money did the girls have all together?

a. 10 + 9 + 10 = 29
b. 20 - 9 = 11
c. 9 + 10 = 19

teach. est.: 80%
stu. score: 51%

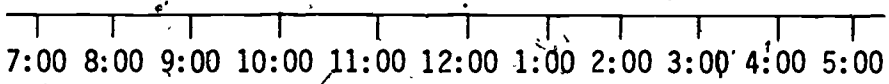
Objective 54: Computation in Personal and Consumer Mathematics

Item 24. Find the difference: $4.21
- 2.17

a. $214
b. $204
c. $2.04
d. $2.14

teach. est.: 77%
stu. score: 52%

Objective 55: Understanding of Personal and Consumer Mathematics Concepts



The line above shows the hours from 7:00 a.m. to 5:00 p.m. Use the line to solve the following problems.

Item 54. Peter left home at 2:00 p.m. and arrived back home at 5:00 p.m. How many hours was he away from home?

a. 1 hour
b. 2 hours
c. 3 hours
d. 4 hours

teach. est.: 64%
stu. score: 70%

Item 55. Janet worked from 8:00 a.m. to 5:00 p.m. She had an hour for lunch. How many hours did she work?

a. 3 hours
b. 5 hours
c. 8 hours
d. 10 hours

teach. est.: 58%
stu. score: 58%

Objective 56: Problem Solving in Personal and Consumer Mathematics

Item 48. It takes Fred five minutes to wash one window. How many minutes will it take him to wash ten windows?

a. 25 minutes
b. 15 minutes
c. 50 minutes
d. 60 minutes

teach. est.: 70%
stu. score: 75%

Item 49. A sports car owner says that the car gets 22 miles per gallon of gasoline. How many miles could the car go on seven gallons of gasoline?

a. 144 miles
b. 154 miles
c. 164 miles
d. 174 miles

teach. est.: 65%
stu. score: 65%

Item 50. Mary earned $1.00 raking leaves. Candy bars cost 15 cents. How many candy bars can she buy with her money?

a. 3
b. 4
c. 6
d. 7

teach. est.: 52%
stu. score: 50%

Panel comments: For objective 53, the average student score (55%) was significantly lower than the teacher estimate (77%). The teacher survey indicated that students were exposed to the material and that the items were of an appropriate difficulty level. However, panel discussion of the results led to a reappraisal of the items themselves. The panel's judgment was that the items needed revision. The correct responses are not straightforward for any of the three items. Additionally, all the answer choices for item 46 are mathematically correct, which makes it a bad item.

The student score (52%) was considerably lower than the teacher estimate (77%) for item 24 (objective 54). When analyzing the results for that item, panelists noted that 39% of the students chose response "b" (an incorrect response). Testing experts caution that speculation about choice of incorrect responses should be done only in exceptional cases. Item 24 was judged to be an exceptional case, and the panel commented that students who subtracted the digits 217 correctly from the digits 421 would obtain 204 (the digits of incorrect response choice "b"). They expressed concern that the 39% who chose response "b" may have attended only to mechanics, paid insufficient attention to the decimal point, and not used enough common sense to note that one who has four dollars could not end up with two hundred dollars by giving away two dollars.

The student score (64%) approximated the teacher estimate (61%) for objective 55. The same was true for objective 56 (student score: 63%; teacher estimate: 62%).

Chapter 3

Discussion of the Results

The mathematics panel was asked to discuss the results of the IIEP data and give their reflections. The comments were based primarily on 1) the performance of 4th, 8th, and 11th grade Illinois students on the 1976, 1978, and 1979 IIEP tests, 2) the results of the 1979 IIEP teacher surveys, and 3) the findings of relevant mathematical and educational research.

The panel noted that two factors emerged from the 1979 fourth grade test (See Chapter 4) and suggested that further research be done to replicate that finding. They indicated surprise that the factors appeared to be ability factors rather than topic factors. The nature of the factor structure has important implications for curriculum, teaching, and learning.

The panel noted that the IIEP has been a census-type test until now. It has tested a representative sample of all 4th grade mathematics content and provided a snapshot of general student performance. The panel suggested that future tests take a new direction.

One suggestion was that comparisons be made across years and across grade levels. The IIEP has charted trends of general mathematics performance since 1978 (See the 1979 IIEP Annual Report[1] for the trends from 1976 to 1979). The panel suggested that trends should be charted for specific objectives, topics, and abilities where sufficient IIEP data exists. Comparisons could also be made across grade levels where there is comparable data.

A second suggestion was that a long-range IIEP research program should be developed. Future IIEP tests should focus on specific objectives, topics, and abilities. A research design should be utilized to test specific hypotheses. The aim should be to move from description toward explanation. The development of focused tests would provide sound tests which school districts and teachers could use to good advantage. The results of the hypothesis testing could indicate curricular strengths and weaknesses and point out needs for educational emphasis.

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[1] 1979 Annual Report of the Illinois Inventory of Educational Progress. Springfield, Illinois 62777: Illinois State Board of Education, 1981.

Chapter 4

Factor Analysis Results

The IIEP was first administered in 1976. Results from the test gave base line data regarding mathematics achievement. In 1978, the objectives were revised in terms that were more easily understood and more amenable to research on learning processes as they occur in students. Results were subjected to factor analysis, a statistical procedure which helps identify student abilities and the strategies which they use in learning.

Factor analysis is a highly technical mathematical and statistical procedure which cannot be fully explained here. However, an intuitive understanding of factors and their derivation is possible. Fred Kerlinger, in his book Foundations of Behavioral Research (1973) wrote:

> Factor analysis is a method for determining the number and nature of the underlying variables among large numbers of measures.
>
> Generally speaking, if two tests measure the same thing, the scores obtained from them can be added together. If, on the other hand, the two tests do not measure the same thing, their scores cannot be added together. Factor analysis tells us, in effect, what tests or measures can be added and studied together rather than separately. It thus limits the variables with which the scientist must cope. It also (hopefully) helps the scientist to locate and identify unities or fundamental properties underlying tests and measures.
>
> A factor is a construct, a hypothetical entity that is assumed to underlie tests and test performance. A number of factors have been found to underlie intelligence, for example: verbal ability, numerical ability, abstract reasoning, spatial reasoning, and memory.

A HYPOTHETICAL EXAMPLE

Suppose we administer six tests to a large number of seventh grade pupils. We suspect that the six tests are not measuring six, but some smaller number of variables. The tests are: vocabulary, reading, synonyms, numbers, arithmetic (standardized tests), and arithmetic (teacher-made tests). The names of these tests indicate their nature. We label them respectively, V, R, S, N, AS, and AT. (The last two tests, though both arithmetic, have different contents and reliabilities. We assume a good reason for including them both in a test battery.) After the tests are administered and scored, coefficients of correlation are computed between each test and every other test. We lay out the r's in a correlation matrix (usually called R matrix). The matrix is given in Table 37.1 (Table 8).

Table 8

TABLE 37.1 R MATRIX: COEFFICIENTS OF CORRELATION AMONG SIX TESTS

		V	R	S	N	AS	AT
Cluster I	V		.72	.63	.09	.09	.09
	R	.72		.57	.15	.16	.09
	S	.63	.57		.14	.15	.09
	N	.09	.15	.14		.57	.63
	AS	.09	.16	.15	.57		.72
	AT	.00	.09	.09	.63	.72	

Cluster II

...How many underlying variables or factors are there?...The factors are presumed to be underlying unities between the test performances. They are reflected in the correlation coefficients. If two or more tests are substantially correlated, then the tests share variance. They have common factor variance. They are measuring something in common.

...There are two factors. This is indicated by the clusters of r's circled and labeled I and II in Table 37.1. Note that V correlates with R,.72; V with S,.63; and R with S,.57. V, R, and S appear to be measuring something in common. It is important to note, however, that the tests in Cluster I, though themselves intercorrelated, are not to any great extent correlated with the tests in Cluster II. Likewise, N, AS, and AT, though themselves intercorrelated, are not substantially correlated with the tests V, R, and S. What is measured in common by the tests in Cluster I is evidently not the same as what is measured in common by the tests in Cluster II. There appear to be two clusters or factors in the matrix. (pp. 659-661).[2]

For further discussion of factor analysis, see Kerlinger (1973) pp. 659-692 and cited references.

Results for Factors and Hypotheses

Hypothesis 1 was that four ability factors would be indicated. The hypothesized factors were: 1) recognition of mathematical facts, 2) computational skills, 3) an understanding of concepts, and 4) problem-solving ability. The data showed two factors. Factor I was composed of

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[2] Fred N. Kerlinger. Foundations of Behavioral Research (Second Edition). New York: Holt Rinehart, and Winston, Inc., 1973.

items related to problem-solving ability. Factor II was composed of items related to abilities 1, 2, and 3 and was labeled learned mathematical facts, computations, and concepts.

Hypothesis 2 was that there would be five topic factors, one factor for each topic which was measured. Hypothesis 2 was not supported. No topic factor was indicated.

Hypothesis 3 was that teacher estimates would be significantly higher than the student scores for the items loading on the factors. Hypothesis 3 was not supported. Tables 9 and 10 show the results with respect to hypotheses 1, 2, and 3. The identical means and similar standard deviations indicate that there is no significant difference.

Table 9

Factor I: Problem Solving

Objective	Item	Teacher Estimate	Student Performance
40	50	52%	50%
56	48	70%	75%
	49	65%	65%
	50	52%	50%
Mean		.60	.60
Standard Deviation		.09	.12
N		4	4

$t = .026^{+}$ $\quad df = 6$

$^{+}$ N.S.

Table 10

Factor II: Learned Material for Student Abilities in Recognition, Computation, and Understanding

Objective	Item	Teacher Estimate	Student Performance
1	30	83%	78%
	31	74%	40%
	34	69%	62%
5	29	77%	70%
	32	33%	18%
	33	32%	24%
	42	57%	74%
7	39	51%	47%
	40	44%	29%
9	26	85%	86%
10	23	76%	78%
	25	77%	83%
	27	79%	73%
	28	81%	78%
26	51	54%	50%
	52	32%	40%
	53	54%	58%
37	35	76%	80%
	36	62%	52%
	37	59%	41%
	38	60%	62%
	56	42%	42%
	57	44%	53%
	58	34%	31%
53	43	80%	51%
	44	73%	56%
	46	77%	59%
55	54	64%	70%
	55	58%	58%
Mean		.62	.57
Standard Deviation		.17	.19
N		29	29

$t = 1.03^{+}$ df = 56 $r^2 = .001$

+ N.S.

Hypothesis 4 was that the teacher estimates would be higher than the student scores for the items of the eight measured objectives. Hypothesis 4 was supported only for objective 53 (recognition of consumer mathematics facts) where the r^2 indicates the degree to which student performance differed from teacher estimates. Table 11 shows the results.

Table 11

T-test Results for Factors and Objectives
Teacher Estimates vs. Student Scores

Label	Teacher Estimates M_1	S.D.	Student Scores M_2	S.D.	n	t	df	r^2
Factor I	.60	.09	.60	.12	4	-.03+	6	--
Obj. 40	--				1			
Obj. 56	.62	.09	.63	.13	3	-.12+	4	--
Factor II	.62	.17	.57	.19	29	1.03+	56	.001
Obj. 1	.68	.14	.63	.17	4	1.18+	6	.06
Obj. 5	.50	.22	.47	.30	4	.19+	6	--
Obj. 7	--				2			
Obj. 9	--				1			
Obj. 10	.78	.02	.78	.04	4	.13+	6	--
Obj. 26	.47	.13	.49	.09	3	-.29+	4	--
Obj. 37	.54	.15	.52	.16	7	.28+	12	--
Obj. 53	.77	.04	.55	.04	3	7.08**	4	.89
Obj. 55	--				2			
NON-FACTOR								
12	.703	.06	.69	.040	3	.39+	4	--

T-test results are for one tailed test

** significant $p < .01$

+ N.S.

Additional Analyses

In order to identify hypotheses for future research, post hoc analyses of the data were done. Correlations were computed between teacher estimates and student scores to explore the relationship between the two. This was done for each factor as a whole and for each measured objective. Significant correlations were found for Factors I and II and for objectives 5, 26, 37, and 56 as indicated by the degree to which student performance was accounted for by the teacher estimates (r^2). Table 12 shows the results.

Table 12

Teacher Estimates Correlated with Student Scores by Factor and Objective for the 1979 Fourth Grade IIEP.

Label	r	df	r^2
Factor I	.99***	6	.98
Factor II	.81***	56	.66
Objective 1	-.14	6	.02
Objective 5	.90**	6	.81
Objective 10	$-.37^{+}$	6	.14
Objective 12	$.19^{+}$	4	.04
Objective 26	.90*	4	.81
Objective 37	.83***	12	.69
Objective 53	$-.552^{+}$	4	.30
Objective 56	.99***	4	.98

* significant $p < .05$

** significant $p < .01$

*** significant $p < .001$

\+ N.S.

In another post hoc analysis, T-tests were computed comparing student scores among objectives. Six T-tests were significant. In particular, student performance was higher for objectives 10 and 12 than for objectives 26 and 53 as indicated by the r^2s. Twenty-two of the twenty-eight tests were not significant. Table 13 shows the results.

Table 13

T-test Results between Objectives

Obj.	$Mean_1$	$S.D._1$	n_1	Obj.	$Mean_2$	$S.D._2$	n_2	t	df	r^2
10	.78	.04	4	12	.69	.04	3	2.99*	5	.53
10	.78	.04	4	53	.55	.04	3	7.30***	5	.88
10	.78	.04	4	37	.52	.16	7	3.16*	9	.50
10	.78	.04	4	26	.49	.09	3	5.76**	5	.82
12	.69	.04	3	53	.55	.04	3	4.10*	4	.72
12	.69	.04	3	26	.49	.09	3	3.41*	4	.64

T-test results are for two-tailed tests

* significant $p < .05$

** significant $p < .01$

*** significant $p < .001$

APPENDIX A

INDEX OF MATHEMATICS OBJECTIVES FOR THE 1979 FOURTH GRADE IIEP

Objective	Item	Page(s)
1	30	6
1	31	6
1	34	6
1	41	6
5	29	8
5	32	8
5	33	8
5	42	7
7	39	8
7	40	9
9	26	9
10	23	10
10	25	10
10	27	10
10	28	10
12	45	11
12	47	10
12	49	11
26	51	12
26	52	12
26	53	12
37	35	13
37	36	14
37	37	13
37	38	13
37	56	14
37	57	13
37	58	14
40	50	14
53	43	16
53	44	15
53	46	15
54	24	16
55	54	16
55	55	17
56	48	17
56	49	17
56	50	17

APPENDIX B

INDEX OF MATHEMATICS ITEMS FOR THE 1979 FOURTH GRADE MATHEMATICS IIEP

Item	Objective	Page(s)
23	10	10
24	54	16
25	10	10
26	9	9
27	10	10
28	10	10
29	5	8
30	1	6
31	1	6
32	5	8
33	5	8
34	1	6
35	37	13
36	37	13
37	37	14
38	37	13
39	7	8
40	7	9
41	1	6
42	5	7
43	53	16
44	53	15
45	12	11
46	53	15
47	12	10
48	56	17
49	12,56	11,17
50	40,56	14,17
51	26	12
52	26	12
53	26	12
54	55	16
55	55	17
56	37	14
57	37	13
58	37	14

APPENDIX C

STATE BOARD OF EDUCATION
ILLINOIS OFFICE OF EDUCATION
Program Evaluation and Assessment Section
100 North First Street
Springfield, Illinois 62777

(1) (2) (3) (4)

4th GRADE MATH ATTENDANCE CENTER TEACHER SURVEY

INSTRUCTIONS Starting with Column 8, indicate your response by placing a number corresponding to your opinion in the appropriate box. Return the form to your building principal when completed.

ITEM NUMBER (5-7)	WHEN WERE STUDENTS EXPOSED TO THE SUBJECT MATTER? 1 Exposed prior to this grade level 2 Exposed this year 3 Have not been exposed (8)	TO WHAT DEGREE HAVE STUDENTS BEEN EXPOSED TO THE SUBJECT MATTER? 1 Heavily 2 Adequately 3 Minimally (9)	HOW IMPORTANT IS MASTERY OF THIS SKILL? 1 Very Important 2 Important 3 Not Important (10)	DOES THE EXERCISE MEASURE SUBJECT MATTER? 1 Yes 2 No (11)	EXERCISE DIFFICULTY INDEX 1 Too Easy 2 Appropriate Level of Difficulty 3 Too Difficult (12)	WHAT PERCENTAGE OF STUDENTS WILL ANSWER THIS ITEM CORRECTLY? (13-15)
23						
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46						

ITEM NUMBER (5-7)	WHEN WERE STUDENTS EXPOSED TO THE SUBJECT MATTER? 1 Exposed prior to this grade level 2 Exposed this year 3 Have not been exposed (8)	TO WHAT DEGREE HAVE STUDENTS BEEN EXPOSED TO THE SUBJECT MATTER? 1 Heavily 2 Adequately 3 Minimally (9)	HOW IMPORTANT IS MASTERY OF THIS SKILL? 1 Very Important 2 Important 3 Not Important (10)	DOES THE EXERCISE MEASURE SUBJECT MATTER? 1 Yes 2 No (11)	EXERCISE DIFFICULTY INDEX 1 Too Easy 2 Appropriate Level of Difficulty 3 Too Difficult (12)	WHAT PERCENTAGE OF STUDENTS WILL ANSWER THIS ITEM CORRECTLY? (13-15)
47						
48						
49						
50						
51						
52						
53						
54						
55						
56						
57						
58						

APPENDIX D

LIST OF MATHEMATICS PANEL

BUSINESS ADDRESS

Mr. Willie D. Anderson
Carbondale Community High School
Carbondale, Illinois

Mrs. Janet Barnard
Parkside Jr. High School
Normal, Illinois

Mrs. Marie Jernigan
Bureau of Mathematics
Chicago Board of Education
Chicago, Illinois

Mrs. Betty F. Schuerman
Springfield District 186
Springfield, Illinois

Dr. Margariete Montague Wheeler
Northern Illinois University
Department of Mathematical Science
DeKalb, Illinois

Dr. Aurum I. Weinzweig
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Mr. Wendell Meeks
Educational Consultant
Program, Planning, and
Development Section
Illinois State Board
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Dr. Mervin M. Brennan
Department of Planning,
Research, and Evaluation
Illinois State Board
of Education

APPENDIX E

LIST OF PUBLICATIONS DESCRIBING THE RESULTS OF THE 1979 IIEP

1979 Illinois Inventory of Educational Progress Annual Report

Fourth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Eighth Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Eleventh Grade Mathematics Results of the 1979 Illinois Inventory of Educational Progress

Energy Results of the Fourth, Eighth, and Eleventh Grade Illinois Inventory of Educational Progress

GAO/17[illegible]2f