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ABSTRACT

The aim of this project was to develop a curriculum for first-year algebra students to provide a wide range of applications of mathematics. This type of application would permit students to understand how algebraic symbolism develops out of natural needs. A pilot draft of the materials was field-tested during the 1974-75 school year, revised, and field-tested in three schools the following year. A basic element of the curriculum which evolved from the testing and revision was the development of a skill workbook which employed principles of mastery learning. Nationwide field testing was accomplished during the 1976-77 school year. The final product is a course similar to a standard algebra course in much of the content, but with more emphasis on probability, statistics, and applications of word problems. Results of field-testing in 20 schools indicated little differences in measures of achievement between project and non-project classes. Teacher attitudes were divided, almost evenly, into two groups--those who felt the course to be appropriate for first-year students, and those who felt it was too difficult or non-traditional. (Author)

# THE FIRST-YEAR ALGEBRA VIA APPLICATIONS DEVELOPMENT PROJECT

## Summary of Activities and Results[1]

In 1974, The University of Chicago was awarded $36,400 by the National Science Foundation for a 15-month period to develop a one-year algebra course in which, as stated in the grant proposal, "algebraic symbolism develops out of natural needs." There was never the expectation that such a course could be planned, written, and tested in so short a time. Further grants were awarded the next two years, bringing the total amount to $144,500. The report of the testing appeared in December, 1978 and the project officially terminated in January, 1979. I was project director. The purpose of this article is to summarize and make known the work of this project so that mathematics educators may take advantage of this work and so that future curriculum workers will neither repeat the mistakes nor waste time rediscovering what we have learned.

### PREPARATION PRIOR TO GRANT RECEPTION

In 1970-71, the project director had written the first draft of a second-year algebra course in which applications of mathematics were distinguished from the "contrived" word problems (e.g., digit, age, etc.) which are part of that curriculum.

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One lesson learned from this experience was that some students could not distinguish an application from a contrived situation. For example, students were asked to judge the reality of the following problem.

> Approximately 8% of the items produced by a company are found defective before shipping. How many items should be manufactured in order to ship 1000 non-defective items?

Many students responded that the problem was not real because too high a percentage of items were defective. (This was before the appearance of the early calculators, whose percentage of defects ranged around 10%.) One student responded that the problem was not real "because of the word 'approximately'."

In doing this work, a large number of applications of mathematics were found to be appropriate for the second-year algebra course. However, there was not time in that course for many applications because too many other things had to be taught. To insert applications into that course, something had to be taken out. This was taken as aximomatic in the first-year algebra course as well.

In 1972, Max Bell's Mathematical Uses and Models in Our Everyday World appeared. It gave many simpler applications. The same year we began offering summer workshops in the applications of mathematics to teachers and degree students at The University of Chicago. More applications were collected. Because many important applications were to statistics and probability, the next year we offered a workshop in the teaching of probability and statistics. This helped prepare us for the task which lay ahead.

# 1974-75 SCHOOL YEAR

The grant began in July, 1974. During the 1974-75 school year, the grant enabled the director to devote full-time to the writing of a first draft and the simultaneous teaching of that draft in a suburban high school (here called School A) carefully selected for its socio-economic closeness to some sort of national average. One other teacher at that high school watched this class and taught the same material later. A third teacher used the materials at an all-black all-girl middle-class parochial school (School B) in the city.

There are some who believe that you should test immediately to see how you are doing. In some circumstances, this is unwise. For instance, in our situation, both schools A and B were not using the materials under normal conditions. The director wrote the materials just a few days ahead of where he was in the teaching. As a consequence, no teacher could plan far ahead. At School A, the author was teaching. At School B, the <u>other</u> algebra classes were taught under an individualized approach. These characteristics made any sort of comparison testing unwise.

Yet even without testing we learned much, not only about this course but also about standard algebra courses. For example, in School A, when doing early work in solving equations of the forms $ax = b$ and $a + x = b$, we found that there were not enough practice exercises in the materials. The department chairman was asked if there were any materials around which could be duplicated and used as worksheets in the class. It turned out that there was an entire storeroom of practice sheets, arranged skill by skill, neatly piled, often with copies in the hundreds. So it was apparent that, for this school, commercial textbooks also didn't have anough problems. This had important implications for what we did the following year.

In each school, we did administer the Educational Testing Service Cooperative Algebra Tests at the end of the year to see if our students were in the ballpark. This test must be considered biased against students who have studied applications, because there are no questions which allow them to demonstrate skills they may have learned in this area. At School A, project classes had a mean 5 points lower than other algebra classes. At SChool B, the mean score was 11 (out of 40) for our class. It seemed that we had failed completely. However, 11 was also the mean score for the students in this school taught a standard course under the individualized approach.

How can end-of-year scores be so low? On this test, a score of 8 would be expected from random guessing! School B, like many private schools in the inner-city, attracts its students by being "college-prep" and so offers algebra to students who do not have enough arithmetic skills to cope with the generalizations which are fundamental to an understanding of algebra. The next year this school, for the first time, offered a class "below" algebra. The lesson for us was that a good knowledge of arithmetic was necessary for success not only in the new course but also for standard courses.

## 1975-76 SCHOOL YEAR

School A and two other schools (C and D) were the settings for the second pilot draft of the materials. The author taught in School C as he had done the previous year in School A, revising just ahead of the students.

The materials were overhauled, but the approach begun in the previous year was substantially unaltered. We describe this approach below, just prior to a discussion of the field testing.

From the first pilot year, a need for skill workbooks was apparent. Commercially available workbooks were found unsuitable to an applications approach, for they tended to contain even fewer hints at uses of mathematics than textbooks. Furthermore, we considered certain application ideas (e.g., calculations involving formulas for compound interest) as at least as important as traditional algebraic skills. Finally, research on our campus had suggested that a workbook which enabled employment of principles of mastery learning would be useful.

Mastery learning is based upon a simple principle: If a student "masters" prerequisite concepts, i.e., if a student answers questions on that concept with 80-85% correct (the percentage is called the criterion level), then the student will learn later concepts faster and more completely.

Consequently, under a mastery strategy, the first thing that the teacher does is teach a basic idea or skill, as would be done in any classroom. Then a quiz is given to ascertain whether the student has mastered the idea. This quiz, called a formative test, never counts for a grade. The results are made known to the student. Those who do not master the idea are given time to practice and take a second quiz. In theory, students take a quiz, receive feedback, practice, take another quiz, get feedback, and so on, repeating the process until mastery occurs. At the end of a chapter, which might have six to ten or more concepts to master, the student takes a test (called a summative test) for a grade.

Mastery strategies require some individualization, because there are always some students who master earlier than others. A common strategy is to have the early masterers help those who have not mastered. To keep a class together, it is important that early masterers not be accelerated; otherwise the differences between faster and slower students will increase.

The mastery learning skill workbook went through many stages of evolution during the 1975-76 school year. Ultimately, the following scheme was used for each notion to be mastered.

(1) There was a short 5-10 minute formative test in the workbook, taken by all students simultaneously.

(2) Answers to each question were immediately announced.

(3) Those who mastered were directed to a place in the workbook where they would read a newspaper clipping or other piece using the mathematical ideas of the quiz, or do a puzzle which used the mathematics. Those who did not master were directed to a very short explanation in the workbook, followed by prac-

tice arranged in a programmed instructional way: answers to one item were found beside the next item, often with explanations.

(4) A second quiz was available in the materials for students to take whenever they wished. Answers to these questions were individually checked by the teacher.

It took one chapter for students to trust the teacher. That is, the students did not really believe that the questions on the chapter test would be taken almost exclusively from ideas which they were expected to have mastered. After that time, we found that almost all of these students were mature enough (even though half were freshmen) to do the practice and grade themselves honestly. They tended to welcome the quizzes and practice because it let them know what would be on exams.

During the school year 1975-76, the National Science Foundation's Science Education area was in turmoil due to controversy involving materials from a social science curricular project, Man - A Course of Study (MACOS). This affected every curricular project in science and mathematics education and continued funding was not at all assured. (Prior to this, NSF had usually followed through on all of its curricular projects from inception to implementation.) We learned of our third year of funding only in the late spring of 1976, and were told that this would be the last funding year. This was one year less than we had planned at the time of the second proposal and meant that we never did carry the mastery learning materials and other teacher aids to their conclusion.

Otherwise, the project proceeded on schedule. From her experience with research and with National Assessment, Professor Jane Swafford of Northern Michigan University was asked to head the nationwide field testing of the materials planned for the 1976-77 school year. Professor Henry Kepner of the University of Wisconsin agreed to assist in this endeavor, due to his particular expertise in the construction of tests at the ninth grade level. At the NCTM and National Council of Supervisors of Mathematics annual meetings in Atlanta in April, 1976, solicitations were made for schools to be involved in the field testing.

The response to the solicitation was overwhelming. We distributed approximately 560 announcements, over 400 of these to a wide variety of people who attended one session. We received over 80 of these announcement forms back from schools or school districts wishing to be involved in the field test. A response rate of even 3% is considered fine for any solicitation; our rate of close to 15% was extraordinary and attests to the interest of the mathematics education community in doing something with applications in the first-year algebra course. Making this even more impressive, only a small handful of these schools and school districts had ever seen the materials; the announcements contained reduced copies of only 4 pages of the second draft.

## THE MATERIALS

The major goal of the project was to producc a text which would (in the words of the original proposal):

"(1) offer the student a picture of the wide range of applications of mathematics, from which algebraic symbolism develops out of natural needs;

(2) cover the standard skills associated with the first-year algebra course with perhaps two notable exceptions: contrived verbal problems (to be replaced by more realistic applications); and complicated factoring and fractional expression problems (unless they are needed for applications - some will arise from statistics and probability);

(3) to develop the algebraic properties associated with standard skills by means of applications and/or embodiments as often as is practical, as opposed to developments in which the properties come before applications and embodiments;

(4) to work arithmetic skills and concepts in the natural framework of the rest of the course, since many students are not yet comfortable with arithmetic processes.

(5) to be no more difficult (and hopefully easier) than standard algebra courses, and

(6) to be easily implementable in schools; that is, the course should not require much (if anything) in the way of teacher training and should not require the school to modify any other courses in its curriculum. (Such modification of other courses because of experience with this course, would, however, be a fortunate side effect.)"

For the field testing, the drafts of the materials were again revised and printed in two paperback volumes entitled Algebra Through Applications. These volumes are now available through NCTM.

Here is a brief description of the materials, particularly as they differ from standard first-year algebra courses. There are 16 chapters, 8 in each volume.

1: Some Uses of Numbers
2: Patterns and Variables
3: Addition and Subtraction
4: Multiplication
5: Models for Division
6: Sentence-Solving
7: Linear Expressions and Distributivity, Part I
8: Linear Expressions and Distributivity, Part II
9: Slopes and Lines
10: Powering
11: Operations with Powers
12: Squares and Square Roots
13: Sets and Events
14: Linear Systems
15: Quadratic Equations
16: Functions

Each chapter has 7-11 lessons. A lesson consists of reading and examples followed by problems entitled "Questions Covering the Reading," "Questions Testing Understanding of the Reading," "Questions Applying the Reading," or "Questions for Discussion."

From the earliest drafts, it seemed evident that a student must understand the applications of arithmetic in order to understand that algebra can be applied. That is, to translate the expression

$$3x + 5$$

into real terms, you must understand first what 3 and 5 might represent, what addition (the plus sign) and multiplication (in 3x) could stand for, and what it means to have the variable x standing for a number. The first five chapters were devoted to these underpinnings.

In the first chapter, extending an idea which appears in the book of Bell mentioned earlier, the uses of numbers are detailed: counting, measuring, identification or coding, comparison, scoring, locating, and ordering. Negative numbers appear whenever a situation has two directions. In the second chapter, the uses of variables are given. The third, fourth, and fifth chpaters examine the basic types of applications of each of the four fundamental operations. These basic types are called <u>models</u>.

Table 1 lists the models of the fundamental operations as they occur in the materials. There is not room to detail all of these models, so we choose to concentrate on multiplication.

Repeated Addition Model: If n is a positve integer then $na = \underbrace{a + a + a + \cdots + a}_{n\ \text{terms}}$.

Orderered Pair Model: If the first coordinate of an ordered pair can be any of of x elements and the second coordinate can be any of y elements, then xy ordered pairs are possible.

Area Model: If a rectangle has dimensions x and y, then its area is xy.

Size Change Model: Let x be a scale factor. Then xy is x times as far from the origin as y is, and xy and y are:

in the same direction if x is positve.

in opposite directions if x is negative.

TABLE 1

MODELS FOR OPERATIONS

| Operation/Use of Numbers | Counting | Measuring | Transformation Comparison | Repetition |
|---|---|---|---|---|
| Addition | Union | Joining | Slide | |
| Subtraction | Take-Away | Cutting Off | Directed Distance | |
| Multiplication | Ordered Pair | Area | Size Change | Repeated Addition |
| Division | Splitting Up | Rate | Ratio | Repeated Subtraction |
| Powering | Permutation | Growth | | Repeated Multiplication |

Our students were only familiar with the repeated addition model. Though they had studied areas of rectangles, they never had connected it with multiplication. Thus they did not realize that multiplication of fractions is quite reasonable in the area model while it is impossible in the repeated addition model. The ordered pair model is usually taught as a fundamental counting principle just prior to a study of permutations and combinations; it was unknown to our students. The size change model explains multiplication with positive and negative numbers.

By having four models we were able to explain properties of operations where repeated addition does not work. For example, commutativity of multiplication is only obvious in the ordered pair and area models. The size change model explains multiplication with positive and negative numbers and also literally shows why multiplying by a positive number less than one (as in calculating a discount) gives a smaller product, while multiplying by a positive number greater than one (as with compount interest) gives a larger product. The mult. tive identity is most obvious from the ordered pair on size change models. And only the size change and area models allow for multiplication by fractions or decimals.

Of course, each model is introduced through many arithmetic examples. For instance, here are two problems from the section on size change models.

> "Each situation leads naturally to a size change model of multiplication. Name the scale factor. He is 48 years old and his son is half as old."

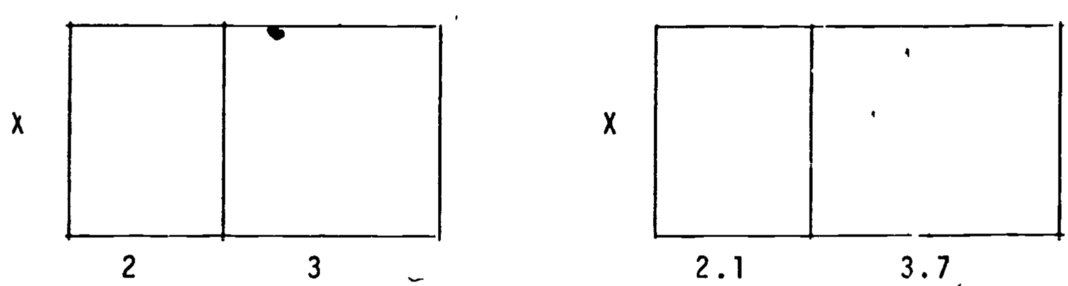
(Other similar statements follow.)

"Suppose a model plane is 1/100 actual size. A part of length x cm on the plane will correspond to a piece of length ____ on the model."

In Chapter 6, the models are applied to generate problems for linear sentence-solving.

"At an outdoor band concert, 60 people can be seated in each row. Suppose there are r rows. What is r if a seating capacity of 2500 is desired?" (Uses the area model.)

Chapters 7 and 8 combine the models for addition and multiplication. For example, the union (counting) model for addition and repeated addition explain 2x + 3x = 5x. So do length and area, which give the bonus as pictured below. While most algebra books utilize these models as mnemonic devices, in this approach these models are conceptualized to provide a fundamental curriculum in the applications of algebra.

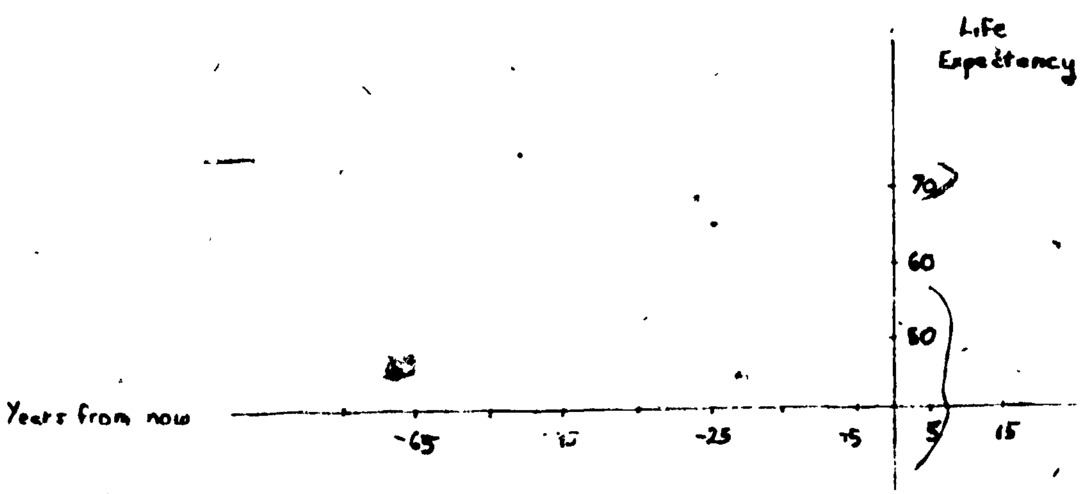



$2x + 3x = 5x$ $\qquad$ $2.1x + 3.7x = 5.8x$

Graphing of data is extremely important when discussing applications and occurs as early as Chapter 2. Here is a problem used to practice plotting of points.

"Life Expectancy. (a) Accurately graph the data given here. (Source: U.S. Dept. of Health, Education and Welfare, National Center for Health Statistics, 1971 Data) uses axes like those labelled. (b) The life expectancy is the average number of years you could expect to live if born at that time. What do you think the female life expectancy would be in 1980? Graph the point corresponding to your guess."

| No. of years ago (from 1975) | U.S. Female Life Expectancy |
|---|---|
| 75 | 48.3 |
| 65 | 51.8 |
| 55 | 54.6 |
| 45 | 61.6 |
| 35 | 65.2 |
| 25 | 71.1 |
| 15 | 73.1 |
| 5 | 74.6 |



Although all of the algebra students we taught had plotted points before ninth grade, they never had to consider different scales or the plotting of points where accuracy would help interpretation. The data for males is given as a companion question to the above problem. Graphing them both can give a nice picture of the difference between male and female life expectancy. The use of negatives is necessary if one is considering present time as time zero; otherwise one would get a graph which is a reflection image of the graph you would get if the x-axis represented "year."

In Chapter 9, after much graphing, the notion of slope is introduced. The first introduction is through questions involving rate of change.

> "Six years ago there were 1250 students in the school. Eleven years ago there were 800. Calculate the rate of change in school population per year from eleven years ago to six years ago."

Students can solve this problem without a formula. They think: "5 years elapsed and the population went down 450 students. So the population went down 90 students per year. Answer: -90 students/year." After a number of examples, the teacher notes that each of these problems involves two comparisons by subtraction (directed distance model) and a division (rate model). For the above problem, the answer is $\frac{1250 - 800}{-11 - 5}$.

$$\frac{(1250 - 800)}{(-11 - -5)} \frac{\text{people}}{\text{years}} = -90 \text{ people/year}$$

Then slopes are identified with lines and the slope-intercept equation of a line.

"Give an equation which describes the amount saved (y) in terms of the number of weeks (x).

(a) Begin with \$10.00, save \$2.00 a week.

(b) Begin with \$30.00 in debts, pay off \$2.00 a week.

etc."

Next studied is powering (the binary operation denoted by $a^b$), introduced through models which lead to the standard properties. The most unusual and useful of these is the growth model of powering.

Growth model of powering:

Suppose that a quantity is multiplied by a positive number B in every interval of unit length. Then in an interval of length t, the quantity is multiplied by $B^t$.

By going backwards in time, we get intervals of negative length. The undoing of a growth process can then be interpreted as $B^t \cdot B^{-t} = 1$. If the length of the interval is divided in half, then $B^{t/2} \cdot B^{t/2} = B^t$, from which fractional exponents can arise. Negative exponents are covered in detail. Fractional exponents are relegated to optional exercises.

There are a myriad of ways in which applications are used to generate other applications. The growth model above is an extension of the size change model of multiplication (B is a scale factor in the unit interval). The growth model can itself be extended to generate polynomials.

"Suppose a person pays $500 a year for life insurance. The insurance company can multiply the amount by m each year. How much will the insurance company have after:
1 year (include 2nd year's payment)
2 years
3 years
4 years
Evaluate how much the insurance company will have after 3 years if m is 1.10."

Two of the major unifying concepts in algebra, sets and functions, are not mentioned until late in the book, because of the author's view that a unifying concept is most effectively utilized when there are things to unify which have already been studied and just before there is payoff. The payoff for sets comes both with the probability of independent events and with systems. Those classes that are able to finish the book consist of better students; these students will see the payoff for functions in their later courses.

There is strong attention given to some basic concepts of statistics. These include displays of data, sampling, relative frequency, the false "law of averages," line-fitting, calculation of the mean and the mean absolute deviation (a nice application of absolute value). Subscripts, and — as the last lesson in the book — the calculation and interpretation of the Chi-square statistic. The probability ideas needed for an understanding of the statistics are present: calculation of simple probabilities, independent events, unusual events, and probability functions. Permutations and combinations are not needed for any of these discussions and not discussed, though permutations are present in conjunction with the ordered pair model for multiplication.

As indicated earlier, for every new idea put into the curriculum, some other idea must be deleted. Attention to quadratic factoring is minimal in these volumes, quadratic equations being solved by the formula. Complicated fractional expressions which require factoring for their simplification are non-existent. With the exception of mixture problems, the standard word problems involving age, digit, work, and $d = rt$ are not present, though distance-rate-time problems are available in many other contexts.

In summary, the materials are vastly similar to a standard algebra in much of the content, particularly as pertains to sentence-solving and graphing, but there are major differences. Factoring and the more complicated work with fractional expressions are replaced by the groundwork for the applications and work with probability and statistics. The standard word problems are replaced by applications to a wide variety of areas. If the approach taken in standard courses can be represented as:

properties → skill work → contrived word problems

then the approach in these materials might be represented as follows:

real world uses → models for operations ⇄ properties / skill work / more uses

## 1976-77 SCHOOL YEAR

If a single school is looking for materials to use, it is relatively easy for that school to set up a small experiment, look at the results, and make its decision. The relative ease of this task is due to the lack of desire for generalizing the results. The project was faced with a far more difficult task: to provide information which could be generalized to most any school district in the United States, and to do this with a rather small budget. (About $20,000 was allocated for the field test. This amount included all salaries and materials.)

The research is reported in detail elsewhere (Swafford & Kepner, 1978). Here we give a summary of the design and results.

Twenty public schools throughout the country were involved in the large-scale field testing of the materials. The schools were picked by geographic and socio-economic considerations from among volunteers and recommendations of supervisors. Four classes in each school were part of the study, giving a total of 80 classes and approximately 2400 students. Each school gave the project names of two teachers, each of whom would teach two classes. The project randomly chose the teacher who would teach from Algegra Through Applications. The other teacher was to use the school's typical algebra materials and could not utilize the project materials. Project texts for each student were supplied free to the schools; one copy of a revised version of the mastery workbook was supplied to each teacher.

Great care was taken to insure a fair test of the materials. Though several grade 10-12 high schools were involved, insuring slower students no accelerated classes were used. The author of the materials decided not to visit any of the school involved in the study at any time during the year. No help was given to teachers using the project materials other than a manual which suggested times of teaching, gave notes on each lesson, and included answers to all exercises. The goal was to make the year as much as possible like a normal year of teaching.

The data reported here are from only 17 of the original 20 schools. Two schools, both in large cities, did not provide enough test results to make inclusion of their scores reasonable. One school, in the state of Washington, dropped out of the study. These three schools had been suggested by supervisors and not become known to the researchers through the teachers themselves. Thus he attempt to secure representative non-volunteer schools (so as to increase generalizability of the results) carried with it the added risk of non-cooperation with the study.

The projected worked on a tight schedule. The first volume of the text reached schools at the very beginning of the school year. Some schools had already begun the initial testing without having seen the books. The second volume of the text reached school in December. The mastery workbook was mailed to each teacher in several sections, occasionally not in time to be used.

For these reasons. the use of mastery learning was never tested on a large scale. However, a smaller study using one chapter's worth of these materials was done in School C, a school which was not a part of the nation-wide study. In this study, it was concluded that the mastery materials increased student performance on a chapter test (Yildiran, 1978).

## RESULTS OF FIELD TEST

Fall testing: Each student was given the Mathematics Computation Subtest of the Stanford Achievement Test, Advanced Battery, Form A (1973); the Educational Testing Service (ETS) Cooperative Mathematics Test, Algebra I, Form A (1962), a 25-item Opinion Survey, and a 28-item Consumer Test. The data consistently verified that students in project and non-project classes were of comparable ability and attitudes. The variation in mean school scores was considerable, however. For example, on the ETS test, one school had a mean of 8.91, barely above random selection for this 40-item multiple choice test, while another school had a mean of 16.45. The second school's mean score for entering students was higher than the mean June score of students in the first school, whether they were taught by project or by standard materials.

In short, average students enter algebra classes in some parts of the country knowing more algebra than students in other parts know even after a year of algebra study. We had expected differences among schools, but not differences this great.

Achievement: It is a truism of research into different approaches that students learn what you teach them. Thus it was considered a certainty that, in June, students taught by standard materials would perform better on the ETS test, whereas students taught by the project materials would perform better on a test involving application concepts. The only questions concerned the degree and consistency of the differences.

A First-Year Algebra Test (FYAT) was developed as a measure of achievement on concepts unique to the project materials (12 items) or on concepts common to both yet not measured by the ETS Test (21 items). Wordings were carefully selected so that no question would be so strange to either group of students. It was planned that this be a companion test to the ETS test, biased towards the project to about the same degree that the ETS test is biased against.

Considering each school as a self-contained experimental unit, in 8 of the 17 schools the classes taught by standard materials scored significantly (in the statistical sense) higher on the ETS test than students taught by project materials. In the other 9 schools there was no statistical difference. And in 8 of the 17 schools (not the same 8) the classes taught by project materials scored significantly higher on the FYAT than students taught by standard materials. Again in the other 9 schools there was no statistical difference.

The project materials are substantially different than standard materials. Thus one is led to ask why there were not differences in <u>all</u> schools. Judging from the comments of teachers and of observers, there are two explanations. First, many of the teachers of the project materials skipped as much as they could of that which they had never taught before. Data collected from teachers indicated that this skipping took place as early as the third section of the first chapter, so it was not due to any lack of time. Second, many of the teachers of the project materials were under pressure from their schools--or felt pressured--to cover all standard skills. So they supplemented the materials with drill on factoring, fractional expressions, and polynomial manipulations. Thus the data does not reflect a "pure" study of the project materials.

Any differences between students in first-year algebra might tend to be dampened before entering the next algebra class. A large number of students need substantial review in their second algebra course. Still it is reasonable to assume that there would be retention of skills and ask what particular items on these tests showed differences that would be discernible to a teacher. The teacher would certainly notice that students using standard text were more skilled at multiplication and division of algebraic expressions and factoring. There might be a tendency to be better at dealing with roots. The students using the project materials would excel at graphing linear functions, relative frequency and probability, the metric system, and certain types of translation from verbal to algebraic expressions. There would tend to be no noticeable differences in solutions of linear equations and inequalities, substitution to evaluate expressions. A variety of topics of lesser importance would show no difference.

The item most differentiating the groups dealt with compound interest.

"If you invest \$100 at 6% yearly interest for 5 years, then how many dollars will you have at the end of that time?

A $100(1.30)$
B $100(.06)^5$
C $(106)^5$
D $100(1.06)^5$
E $100 + 5(.06)(100)$"

This question was answered correctly by 42% of students using project materials and only 8% of students using standard texts.

Because the project materials contain much of interest to consumers, it was felt that students using them would more easily cope with the mathematics needed by consumers. The project put much energy into developing a test of consumer skills. The test showed only a few differences between the groups. From this we may conclude that consumer skills develop at different rates only when specifically taught.

To summarize, the testing of skills seemed to demonstrate that the project materials were at the level of the average algebra student. Students learned content of project materials to about the same extent that their counterparts learned standard content. Thus it is a question of priority: Is factoring more important than compound interest? Is calculation of probability more important than polynomial manipulation? Is it worth taking less time on some standard topics in first-year algebra so as to allow time for these other skills, realizing that later, for those students who go on, some of the manipulative skills will have to be picked up?

Student attitudes: The Opinion Surveys, given in the Fall and Spring, were composed of 25 items designed to assess student feelings regarding enjoyment, value, and nature of mathematics, algebra, and their textbook. There was no difference between groups on any items dealing with enjoyment, a small difference favoring standard students on interest, and a small difference favoring project students on the value of algebra. The only sweeping trend concerned the textbook. On all items concerning the textbook, the project materials were consistently rated higher. Students found explanations easier to understand, the book more interesting, and read it more often.

Teacher attitudes: Fifteen of seventeen teachers completed a long textbook evaluation form. The teachers split into two camps of roughly equal size. Seven teachers thought the text was appropriate for the average first-year algebra student, easier to read and understand, and at about the same level of difficulty as other algebra books. Seven voiced no consensus but would not recommend the use of the text for an average first-year algebra class. These teachers felt either that the materials were too difficult for the students or too non-traditional for the brighter college-bound student. Only one teacher was neutral.

The teachers were asked to evaluate specific lessons and ideas in a variety of ways and to give suggestions concerning improvement. In almost all of these evaluations, those who were in general positive about the materials gave positive responses to the specifics; those who were generally negative provided almost all of the negative responses found on these forms.

Other schools, not a part of the formal study, used the materials and their teachers were asked for opinions on the text. These teachers tended to be volunteers rather than selected in the quasi-random fashion of the formal study. On 21 teachers using the materials, 12 returned the form. The split into camps was as sharp as in the formal study, but the ratio of favorable responses was much higher. Ten teachers responded very positively, two quite negatively.

Teacher attitudes vs. Student performance: When a school system adopts a textbook, it is ostensibly because it is believed that that book will help their students more than any other that has been evaluated. So it is helpful to ask whether teacher opinions were in any way related to student performance. The daca is most interesting in this regard. No significant relationship was found between comparative student achievement score in a school (i.e., whether project or standard students scored higher on ETS or FYAT tests) and teacher opinion of Algebra Through Applications.

However, data based upon this very small sample of teachers suggests that opinions towards the project materials were affected by the absolute level of student entering performance and the general ability of algebra students in the school. Tables 2 to 4 suggest that Algebra Through Applications would likely not be perceived as successful in schools where (1) mean September algebra student performance on the Stanford Achievement Test, Advanced Battery, Form A, is less than 10.00; (2) mean September beginning algebra student performance on the ETS Cooperative Algebra I Test, Form A, is less than 11.00; or (3) mean June first-y. algebra student performance on the ETS test is less than 21.00. Inversely, Algebra Through Applications would likely be perceived positively if the mean student performance on the three tests was greater than 30, 11, or 21, respectively, but here likelihood is not as great.

TABLE 2

TEACHER RECOMMENDATIONS OF ATA vs. SCHOOL MEAN ON STANFORD ACHIEVEMENT TEST

| | Mean ≤ 30 | 30 < Mean < 34 | 34 ≤ Mean |
|---|---|---|---|
| Would | 0 | 4 | 3 |
| Neutral | 1 | 0 | 0 |
| Would not | 3 | 3 | 1 |

TABLE 3

TEACHER RECOMMENDATIONS OF ATA vs. SCHOOL MEAN ON ETS COOPERATIVE ALGEBRA TEST (SEPTEMBER)

| | Mean ≤ 11 | 11 < Mean < 13 | 13 ≤ Mean |
|---|---|---|---|
| Would | 0 | 4 | 3 |
| Neutral | 1 | 0 | 0 |
| Would not | 4 | 1 | 2 |

TABLE 4

TEACHER ATTITUDES TOWARDS ATA vs. STANDARD COURSE MEAN ON ETS COOPERATIVE ALGEBRA TEST (JUNE)

| | Mean < 21 | 21 < Mean < 23 | 23 < Mean |
|---|---|---|---|
| Like | 1 | 3 | 3 |
| Neutral | 1 | 0 | 0 |
| Dislike | 5 | 1 | 1 |

## PUBLICATION

The project had hoped, from its inception, to develop materials which would be attractive enough so that there would be one or more commercial publishers interested in adapting the materials for commercial publishers. These adaptations could include the adding of color, professional artwork and editorial expertise, as well as ancillary materials (a teacher's edition, tests, etc.).

National Science Foundation rules are very strict regarding publication of materials developed with project monies. Opportunity must be given for all publishers who might be interested to see the materials and to present bids. National Science Foundation and The University of Chicago must approve any and all agreements. Copyright protection extends for only a short period of time, at most seven years.

A number of publishers expressed interest in the materials, but no commercial publisher both desired and was able to publish the materials as the project wished. Reasons given for the lack of interest never involved the quality of the materials, the ability of students to learn from them, nor the underlying goals. Cited were the lack of a second-year algebra text to go along with this one, the amount of reading in the materials, the lack of copyright protection, and the inability to sell alg ra teachers on the idea of replacing some of their favorite topics with applications. The project may have been handicapped because the director has commercial texts on the market; competing publishers may have been reluctant to consider the idea. A couple of interested publishers indicated that they would have bid but were involved in undertakings of their own which inhibited picking up this project.

Fortunately, the National Council of Teachers of Mathematics had expressed early interest in distributing the materials in the event that there was no agreement with a commercial publisher. Attesting to the difficulty of such negotiations is the length of time it took for the parties to agree, despite no significant stumbling block. The first call to publishers was made in March, 1977. By the beginning of 1978 all negotiations with commercial publishers had come to an end. National Council of Teachers of Mathematics approved publication in April, and drafted an agreement. After a number of modifications, NCTM and the University agreed on wording in August. National Science Foundation gave its approval and the final agreement was signed in March, 1979, two months after the end of funding for the project.

Under the agreement, NCTM is distributing the project materials for a period of at least three years, and possibly longer.

The materials have not been altered, but all known errors have been corrected. The agreement does not cover a teacher's edition nor the mastery workbook.

A number of commercial textbooks have already incorporated a few ideas from the project materials. The materials themselves have been used at the community college and junior high school levels, it seems quite successfully. No present publisher markets the same text materials for all these levels. National Council of teachers of Mathematics alone encompass this spectrum. Thus lack of commercial affiliation, which at one time seemed like a setback to the project, may in the long run help the work of the project in making the first-year algebra course more reflect what people do with mathematics today than the mathematics of a prior era.

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