DOCUMENT RESUME

ED 210 190 SE 035 943

AUTHOR	Rising, Gerald R.; And Others
TITLE	Using Calculators in Mathematics 12. Teacher Commentary.
INSTITUTION	State Univ. of New York, Buffalo. Dept. of Instruction.
SPONS AGENCY	National Inst. of Education (ED), Washington, D.C.
PUB DATE	80
CONTRACT	400-78-0013
NOTE	198p.; For related documents, see SE 035 940-942. Contains occasional light and broken type.
EDRS PRICE	MF01/PC08 Plus Postage.
DESCRIPTORS	*Answer Keys; *Calculators; Curriculum Development; Educational Technology; *Grade 12; Mathematical Concepts; Mathematics Education; *Mathematics Instruction; Secondary Education; *Secondary School Mathematics; Teaching Guides; Teaching Styles
IDENTIFIERS	*Programable Calculators

ABSTRACT

This teacher's guide is designed to aid in the incorporation of programable calculators in the school mathematics program for pupils in grade 12. Warnings are given, including the need for care in modifying the curriculum so that students are not punished in the process. The concept of "black boxing," of letting the computer or calculator take charge of education, is stated as a concern that pupils may lose conceptual understanding of computation and take for granted that these devices can carry out difficult computations easily and efficiently. However, the benefits are seen to present powerful arguments for calculator use in the instructional program. In addition to discussing the pros and cons of programable calculators, the brief introduction gives ideas on student access to calculators, rules and guidelines for calculator selection, approaches to classroom presentation, and hints on calculator-caused changes in classroom dynamics. The bulk of this document consists of answers to problems from the student textbook. (MP)

ED210190

Using Calculators in Mathematics 12

State University of New York at Buffalo

TEACHER COMMENTARY
1980

035 943

USING CALCULATORS IN MATHEMATICS 12

Gerald R. Rising

Betty J. Krist

Carl J. Roesch

Donald W. Stover

State University of New York at Buffalo
Department of Instruction

The work upon which this publication is based was performed pursuant to contract number NIE 400-78-0013 of the National Institute of Education. It does not, however necessarily reflect the views of that agency.

Using Calculators in Mathematics 12

TEACHER'S GUIDE

Introduction

Calculating tools have been utilized over the full span of civilization. The earliest records indicate that various forms of abacus-like apparatus were used still earlier. Today's hand held calculator provides only the latest step in the development of these labor saving devices. But these pocket-sized tools represent more than a difference in degree from their predecessors, such devices as slide rules; they represent a difference in kind. They triple or quadruple the number of digits accurately determined by a slide rule, thus multiplying accuracy by some ten million times! They carry out remarkably complex calculations that astound those of us who used paper, pencil, specialized tables, and much time to compute in the "old days" of just ten years ago! Thus we have in a few seconds of key punching:

$$\cos 32^\circ 14' 30'' = .84580542$$

and

$$\sqrt{3}^{\sqrt{5}} = 3.4153702$$

Consider computing those values to half this number of digits of accuracy before calculator access.

And now the programmables: the power of a half million dollar computer of twenty years ago shrunk into a $50 - $100 pocket-sized machine. Whole new vistas are opened to us. One of the earliest examples of practical use of a programmable communicated to me is one that will appeal to teachers.

> A school bargaining team was presented a modified salary proposal by a school board, a proposal the board negotiator said would require a postponement so that the full scale could be calculated. "No need," said the teacher representative, "We'll calculate that for you in ten minutes." And so they did,* providing not only the scale itself but also the cost of implementing that scale for current staff: all calculated on a programmable! Needless to say, the board was impressed.

---

* For example, a simple program like the following would generate a column in a 5% per step increase schedule:

HP - 25		TI - 58	
01	ENTER	01	LBL A
02	1	02	X
03	.	03	1
04	0	04	.
05	5	05	0
06	X	06	5
07	R/S**	07	=
08	GTO 01	08	R/S**
		09	GTO A

** The base salary is keyed into the calculator, R/S is pressed, and subsequent steps are read each time the calculator stops. To restart R/S is pressed again. For example, a scale starting at $9800 and incrementing in 5% steps would give

$9800.00
10290.00
10804.50
11344.72
11911.96, etc.

This simple but suggestive example only reaches the border of the wide range of programming applications.

The ready availability of programmable hand held calculators does not, of course, by itself imply that they should be used in the school mathematics program. Unless a useful role in the curriculum can be found for them, they belong there no more than does another recent invention, the hula hoop. Despite our facetious example, this is, we believe, an important issue. The school mathematics program is already a full one and we should always think carefully about tinkering with it. Curriculum workers have too often thought in terms of program additions rather than the more appropriate program replacements. When something new enters, something old must exit.

Our experience so far with programmables convinces us that there is an appropriate role for them in the grade eleven program as it is presently constituted. In fact we have convinced ourselves of the truth of the following postulates:

- The calculator is useful in a number of topics involving computation. Inversely, reasonable use of the calculator is restricted to those topics. Understanding this strict deliniation is important: the idea of a calculator in use every day of the school year is popular but wrong-headed.
- Gadget fascination sets traps as you address even appropriate curricular units. Playing with the calculator is fun and easily takes students and teachers away from mathematical

concerns.*

- There are activities that deserve either to be discarded or to be severely reduced in this calculator age, thus providing some of the curricular space for calculators.
- When using the calculator in the mathematics program, great care must be taken to avoid "black boxing" concepts.

In developing the textual materials for this program we have sought to respond to these postulates. Now consider their meaning and some of their implications.

Two years ago Professor Rising set as an assignment for in-service teachers in a graduate seminar the task of reviewing school texts in order to determine the fraction of the content appropriate for calculator enhancement. The results are striking and reinforce the purest of mathematicians: less than 10% at any grade level, elementary school through college, are amenable to calculator usage! More recently however, Professor Wallace Jewell of Edinboro State College in Pennsylvania carried out the same kind of page count for secondary school courses. His estimates came out in the range 20% - 50%, with the lower count geometry.

*We note here that care must be taken to separate the wheat from the chaff. Our salary scale example is not so trivial as it first appears. It has within it the basic elements of a geometric series and exponential growth.

Why the difference? The answer is instructive and should give better insight into our first postulate. Professor Jewell was studying calculators intensively at the time he made his survey, he had used them in his own instructional program, and he was sensitive to their application; the classroom teachers in the earlier group did not have these characteristics.

The message seems clear. As you start using claculators for classroom instruction, you will probably overvalue their application. But then, having reduced their use to those places where they enhance the program without question, you will begin to find more sophisticated use for them.

At some points in the curriculum calculator use is plainly signaled. They replace log and trig tables and in fact much computation by logarithms. Proofs on the other hand: never. But wait a minute: a [illegible]er substitute is: hardly ever. Motivating a theorem, for example, is an activity well supported by calculator. In this regard, consider maxima or minima for quadratic functions, $x \rightarrow ax^2 + bx + c$. A series of calculations for specific graphs can lead to the conjecture that $x^2 = -b/2a$ at the critical point. Now this result may be proved by standard means.

Gadget fascination. We should by now have learned from our experience with computers in the classroom how this operates. Computers, of course, just like calculators, have much to add to the mathematics program. Any examination of their use in school mathematics classrooms will suggest that their contribution to mathematics is not as great as might be expected and that, in fact, they often take classes of students away from mathematics into realms that are interesting but do not contribute to increasing mathematical sophistication. Across the country in thousands of mathematics classrooms students are working on such computer activities as sorting lists alphabetically, seeing that tables are printed in neat columns, and carrying out complex mathematical processes like inverting a matrix by a minimum of instructions. Such activities, and these are only examples, are good computer science but not good mathematics. We should learn our lesson from this. We as teachers should think very carefully about each place in the curriculum calculators are to apply. When they contribute to that curriculum they should be used, of course; but when they do not contribute to the curriculum and take us off on tangents, we would do better to stick with standard instructional techniques. We have attempted to follow this guideline in our development of the units of this program.

Replacement. This will continue to be a very serious and very difficult problem. For one thing, even though a topic becomes archaic it may still continue to appear on examinations that are important to our students' future programs in mathematics. A case in point: recently a member of the New York State Education Department Mathematics Office claimed that he had checked the eleventh year Regents examination and found that there were no questions that called for calculator usage. We looked at some recent examinations and found this comment to be inaccurate. For example, the following exercise appeared on an examination.

Find log 0.3145

Surely this problem is amenable to calculator computation. The solver need only key .3145 log to get the answer. He no longer needs to interpolate and to use care in determining the characteristic in writing his solution.

Note that the calculator solver gets a "different" answer from the solver who uses tables. The calculator solver's answer is

-0.5024

while the table solver's answer is

9.4976 - 10.

While mathematically equivalent these two answers differ remarkably in appearance. Many scorers would in fact fail to count the calculator answer correct. What has happened of course is that the negative characteristic has been combined with the mantissa to provide a single term result. This can be seen by carrying out the actual subtraction of ten from 9.4976. The result of this is that our old rules for characteristics no longer apply to numbers between 0 and 1.

The point we seek to make by our example should not be missed because of the details of that example. Yes, finding logarithms and tables is an archaic process, but the student who finds a multiple choice question on the SAT examination where no calculator answer is supplied finds himself in some difficulty. Thus we must be very careful as we modify curriculum not to punish our students in the process. This problem has long haunted curriculum developers and will continue to cause problems for them into the foreseeable future.

Having said that, we must still find ways to modify the curriculum significantly in order to do the new kinds of things that are important for contemporary and future use of mathematics. We cannot let our curriculum come to a dead halt because of problems like these.

Black boxing. Black boxing is letting the computer or calculator take charge. It is the first step into the science fiction robot-controlled world. As we look around us in modern society we see this more and more come into place. We see this, for example, in the supermarket where machines essentially replace most of the skills of the check-out personnel. The machines read the item and its price directly from a coded marking on the package, total the order, find the amount of change appropriate, and even provide feed-back to the store manager about inventory. This may very well be an appropriate course for modern engineering; it is inappropriate for the mathematics classroom.

It is important to understand that black boxing is not a new phenomenon, nor a necessarily inappropriate phenomenon. Consider again logarithmic and trigonometric tables. Where do they come from? They are, in fact, just as much a black box presentation of mathematics as is the calculator log key.

While such devices are occasionally appropriate because of the lack of sophistication of our students, we must exercise great care that we do not allow mathematical understandings that we wish to obtain to be lost in the black boxing process. We do not want our students to lose conceptual understanding of computation and what it involves just because the calculator can carry out these computations so quickly and efficiently. Here are two exercises that illustrate some of what we mean here:

Calculate 357895^3

Calculate π^π

Each of these exercises demands simple keying into a calculator for solution. In the case of the first, an answer like the following appears

4.5842736 16

If the student has no understanding of scientific notation, this answer is meaningless, and if the student does not understand something about rounding answers, the answer is inaccurate. In the case of the second calculation the answer comes up:

36.46215964

Here the student problems are more complex. What does it even mean to raise a number to an irrational, to say nothing of transcendental, power? Without conceptual underpinning the student has an answer to a process that is meaningless to him.

Value of Programmables

Having described all of these special concerns about teaching with programmable hand-held calculators, it will be well for us to turn now to some of the values of instruction with these devices.

Everyone knows the story of Mallory, who when asked why he would set out to climb Mount Everest replied, "It is there." Hand-held calculators are indeed there in modern society. One can get a sense of how wide is the distribution of small calculators by the fact that over the past several years calculator sales have outstripped circulation for the most popular magazine TV Guide. Of course programmables make up only a small fraction of total calculator sales, but they too are there. And today's high quality programmables cost less than standard "four banger" calculators of eight or ten years ago. Thus we have a readily available mathematical tool.

Availability is not enough. With the limited instructional time available to mathematics in the schools, we must make priority decisions on what we teach. All curricular decisions in mathematics must be made on a first-things-first basis. Programmable calculators, we believe, meet this stern test.

One of our basic roles in the schools is to prepare our students for modern society. The computer is a central feature of modern society. Work with programmable hand-held calculators provides students with insights into how computers operate at a very rudimentary level. Given this kind of understanding they may or may not go on to learn how to operate the larger, more complex machines, but even if they do not, they carry with them a general understanding of how these machines operate.

This kind of argument justifies programmable hand-held calculators in the school program, but not necessarily in the mathematics program. The textual materials that we have developed should demonstrate to you just as our experience with these materials in the classrooms with students demonstrates to us, that programmables have a definite contribution to make to school mathematics at the eleventh grade level. We have found, as you will too, that students gain insights into mathematical activities through use of these devices and that they refine their understanding of concepts gained earlier as well.

As a trivial example of what we mean by this last comment, consider an episode that occurred in one of our earlier classes when we were showing youngsters how to use the calculators. Professor Rising asked the tenth grade students in the experimental class to enter 4 in their calculators and then to press the reciprocal (1/x) key. The calculator display then showed 0.25. He then asked a student what multiplier would change the display to 1. The student could not answer. Professor Rising wrote on the chalkboard

$$\frac{1}{x} \cdot \underline{\qquad} = 1 .$$

The student readily suggested x as the number that should fill in the blank. But he still did not know what number to use as a multiplier in answer to the first question. He finally suggested 25. Here was a case in which this reasonably intelligent student was confused by the representation of a common fraction as a decimal to such an extent that he could not apply a basic concept that in other contexts he could use readily. Thus the calculator gave the opportunity to expose and respond to a student's weakness, in this way to refine his understanding of mathematical concepts.

The basic role of any calculator is to take over routine tasks of computation. As they do that, they free the user to concentrate on more serious problems: deciding how to respond to the problem, organizing the solution, determining the reasonableness and accuracy of the answer, thinking about related problems, and otherwise generalizing the solution. When you use calculators in your classroom you should keep this continually in mind. Performing a series of multiplication exercises by calculator is not a mathematical activity.

But we have found and the text pages should display a wide range of places in the standard curriculum for eleventh grade where the calculator contributes to student understanding. Consider, for example, a long standing problem having to do with graphing curves. Every teacher has had the experience of the broken line "graph" of a quadratic. The students plot a few points and connect the points by segments. More points; more segments. This problem is rather hard to address without calculators by any means other than a teacher edict. Why? Because the work required in calculating additional points is considerable, especially when fractions or decimal values are involved. But now suppose our function is something of the form

$$y = 2x^2 - 5x + 6.$$

Students can quickly program this function into their calculators and run successive x values to generate points on the curve. Now they can literally plot dozens of points until they really can see the shape of the curve. The reader should think about this example carefully. Notice how the calculator only takes over a computation role. It in no way substitutes for understanding of the procedure. In fact, the student had to know the procedure for calculating y values in order to prepare the program. What he did not have to do is carry out the complex computations to evaluate the function point by point. In fact there is more than this. The easy generation of additional points allows the student to focus his attention on areas where the concerns are critical. What is the minimum y value for this function? Before the student knows how to determine the turning point from the equation itself, he can locate that turning point by trial and error with his simple program. Thus he develops initial insights into a problem that he can later solve by algebraic technique.

Some teachers feel that by taking over this kind of work calculators will make students lazy. We do not fear this. Our observation of students at work with calculators is that they work harder. The difference is that their work is focussed on concepts, the calculator taking over routine.

Calculator Access

Certainly the best access to calculators is continuous access. We have found in our work at SUNY/Buffalo with simpler calculators that student ownership is the easiest policy. Few problems arise here, because the cost of the calculators is approximately that of school textbooks. This cost equivalence may solve the problem for providing inexpensive calculators in the schools as well. If a school has a textbook distribution (loan) system, calculators can be acquired and distributed within this same program. Student loss of a calculator then is no different from student loss of a textbook and would be treated the same.

Although the cost of programmable hand held calculators has come down markedly over the past several years, these costs are still high enough to make the programmable situation more complex. Best access is still continuous access; but teachers will have to use their best judgment in determining how near they can come to this preferred policy. Our experience in the experimental classes may be of interest and use here. At the outset we were extremely careful about calculator security. We even had some of our calculators secured in locking cradles. As time went on, however, it became clear to us that we had to relax our restrictions or students would not get full value from the experience. For that reason we have adopted a very open program, allowing the students to sign out calculators for overnight use. We have not yet lost a calculator by following this procedure. At the same time we note that we have lost one calculator from the facility in which they are stored at the university.

This still leaves the local school and often the individual classroom teacher to make procedural decisions. We would rank in order of preference the following four possibilities:

1. student ownership
2. long term assignment
3. overnight check out
4. use only in class and in special work rooms.

Which calculator?

Our development work within this project has given us an opportunity to try out and work with a rather wide range of programmable hand held calculators. As we have worked with these machines, we have each developed personal preferences. The key word here is "personal". When working with calculators we have found that you tend to like what you know.

This rule applies especially to machine language. A number of people have made strong cases for the "natural" language of algebraic order calculators, but a personal story may be in order here. Professor Rising's wife, a non-mathematician, has used for several years one of the earliest reverse Polish notation calculators. She has become skilled in the use of this machine. More to the point, she has considerable difficulty adapting to the algebraic order calculators. This suggests that the idea of "natural" order is something to be considered less seriously than we have been tempted to do in the past.

We are not in a position to recommend a particular calculator. For one thing, a recommendation at the date of this writing may very well be inappropriate a year or two years hence. One concern does seem clear to us and it represents a very serious problem. As costs come down, quality is reduced as well. The most distressing comment that has been made to us over the time of our work with programmables was the one made by a representative of a major calculator manufacturer that "The programmables are only being made to last through one year's operation." We have had some difficulties with calculator break-down, yes; but in general our experience with medium-priced ($80 - $100) programmables is that they will last for at least several years. Interestingly it appears that hard use, that is such things as dropping the calculator on the floor, does not seriously affect the calculator operation. The lesson in this is, we believe, that teachers should use caution in purchasing the least expensive available calculators.

Before you select calculators for your students you would do well to experiment with the models you are considering yourself. Some vendors are willing to let you take a calculator overnight to familiarize yourself with its operation. Others will spend considerable time with you in showing you how the machine works. We suggest the following as basic concerns that you should address in selecting calculators:

- complexity of operation
- number of program steps (merged steps save here)
- number of storage locations
- programming language
- instruction manuals

We have found fifty program steps and a half-dozen storage locations entirely satisfactory for high school use. Very rarely will more program steps be needed and only occasionally will more storage locations be necessary for complex programs.

For the simpler "four banger" calculators we recommend battery replacement. For programmables, which draw somewhat more electricity, it seems appropriate to utilize re-charging devices. Since virtually all programmables have plug-in rechargers, this should not be a matter of concern to selectors.

With the advent of liquid crystal display programmables such as the Casio 502, battery charging problems disappear. The batteries on such calculators need only be replaced about once each school year. Users would do well to examine such calculators.

If you will be using this text with microprocessor, most of what we have said will still apply but you will probably have different and often additional problems. Access to the equipment is probably the most difficult.

Classroom Presentation

Now you have your calculators and you are ready to go. The students are all excited about the new toys and they want to get to them just as quickly as possible. Don't be trapped by this situation into a complete departure from your mathematics goals to focus on this device. You must constantly keep in mind the fact that the calculator is another tool for teaching mathematics, not a device that is an end in itself. When it is appropriate to use it, do so. When it is inappropriate to use the calculator, have your students set them aside.

What we have done in preparation of these textual materials is to select units which may be enhanced partly by calculator use. You will notice that many other units we do not touch at all. Activities like factoring, solution techniques for linear equations, word problems, in fact, about half the course are not enhanced by use of the calculator. Even the topics that we have developed have sections where you will not wish to use the calculators. The basic rule: don't force the calculator into places in which it doesn't belong.

Another don't. Don't attempt to assign motivation to the calculator. That is a false hope. Your best bet for motivating your students is a serious approach to the teaching and learning of mathematics. The calculator by itself as a motivating device will last like all other such devices about ten minutes. But the calculator used effectively in your instructional program will enhance that program and add to the general motivation that good instruction can contribute.

It is not necessary for you to spend time teaching your students how to use the particular calculators that they have in hand before starting the units in this text. The first unit includes, along with the study of order of operations, sections devoted to introducing the students to their own calculators. These sections and in fact the entire book consider both algebraic order and reverse Polish order operations. We think that it is important for your students to learn both. You and we do not know what kind of calculator or computer access your students will have when they leave school. But clearly, you will wish to focus main attention on the kind of calculator that your students have. At appropriate points you may wish to supplement the instruction by use of, for example, some ideas from the instruction manual for the specific calculator the students have.

Classroom Dynamics

You will soon find as we did that classroom organization changes when you are using calculators. In fact, you will not be able to assign a particular teaching style to the use of calculators. Things are not that simple. There do seem to be two quite different formats for classroom instruction with calculators. We identify these for you so that you can prepare to adapt your instruction to them. Remarkably they are at opposite ends of the instructional spectrum.

The first is the technique that you will wish to use when you want to take your students through a series of keystrokes. This is the most lock-step, regimented kind of instruction. In fact if you depart at all from a step-by-step, "do this", "do this" kind of presentation you will find that your students will diverge frightfully from the pattern that you hope to accomplish. After a few false starts when you learn the lessons that we learned, we expect that you will find yourselves like us saying something like the following: "All right class, now all together turn your calculators off and on and get ready together to follow these keystrokes. First press the ......" In our instruction we found that we could make fun of this kind of ~~activity by~~ saying something like, "Now it's time for close order drill." The students reacted favorably to this. As this is only an occasional instructional activity, you will not find that your classroom is changed into a nineteenth century presentation by these occasional rigid structures.

The second instructional mode is almost exactly the opposite. You will wish to provide your students with opportunities for very open attack on problems. You will want to give them time to organize their own calculator procedures and to apply them to assigned exercises or larger tasks. While they are doing this you will wish to circulate among them to answer specific questions and to give assistance where it is needed. Here we urge you to keep the atmosphere as open as possible, and in particular to allow students to help each other. It will quickly become clear to you which students are leaning too heavily on their neighbor's assistance. In those cases you will wish to intervene. You may wish to give additional assistance to the student being helped in order to wean him from his reliance on his neighbor, or you may wish to comment to the tutor that he may be providing too much help and so preventing the other student from learning the material for himself.

We do not mean to suggest that these are the only teaching styles that will come up in your instructional program. Quite the contrary, you will find that you will use your entire range of instructional techniques. We have only stressed that these extremes are also included. Many of you who are accustomed to working with your class as a unit will find that the second kind of instruction, which opens up the classroom to individual activities, will make you somewhat uncomfortable at first. Recall in this regard that our main business is student learning, and that teacher's teaching sometimes gets in the way.

Calculators do not eliminate student errors. Far from it, they merely highlight these errors. Carelessness will continue to annoy you and to a lesser extent the students themselves as errors are made. But some students will be far worse than others. You will probably wish to give them additional careful instruction. For example, we found that one student constantly pressed two keys at once. We finally had to work with him to get him to use only one finger in that vertical position known to piano instructors and to make a fist of the rest of his hand. This reduced the number of errors by about 75%, bringing him down to just a little above the average of his classmates.

We cannot of course in this brief introductory section head off all the problems you will have as you introduce these devices into your classroom instruction period. Just as we did, you will find unique situations which arise and will need to be dealt with thoughtfully. Along with the individual section exercise answers we provide some suggestions about classroom presentation. You will wish to look at these and to look carefully at the textual materials themselves in preparing your classroom presentations. Here as elsewhere your thoughtful instruction is the key to student learning.

Exercise Set 1.1

1) 12 2) 18

3) 12 4) 24

5) 17 6) 45

7) 17 8) 24

9) $ab + cd$ 10) $ac + ad + bc + bd$

11) $\frac{a}{b} \div \frac{c}{d} \times \frac{e}{f} = \frac{ade}{bcf}$ 12) $\frac{a}{b} \div (\frac{c}{d} \times \frac{e}{f}) = \frac{adf}{bce}$

13) $a[b + c(d + e)] = ab + acd + ace$ 14) $a + b + c - d$

15) $(ab + c)\,d + e = abd + cd + e$ 16) $\frac{\sqrt{a + b}}{cd - e}$

17) (9) 26 (10) 54

(11) -7 (12) $-\frac{1}{7}$

Notice that (11) and (12) are reciprocals since $ad = bc$ when $a = 6$, $b = 3$, $c = 4$, $d = 2$.

(13) 234 (14) 11

(15) 51 (16) 3

18) 4 19) 4

20) 4 21) 1

22) 1 23) $\frac{1}{25}$

Exercise Set 1.2

1) [+], [-], [x], [÷] . These keys would not be used because they do not separate calculations. When the [=] is used the calculator automatically separates the calculations.

2) The equal step between 38 and [÷] may be eliminated. On some calculators, usually the more sophisticated models, an order of operations is already wired into the machine. Thus, on simple calculators the keystrokes [4] [+] [3] [÷] [7] = 1 because the order is left to right. On more advanced calculators [4] [+] [3] [÷] [7] = 4.428571 because the order of operations by hierarchy is designed into the wiring of the machine. If your calculator has this hierarchy of operations no step can be eliminated.

3) There are several answers to this question that not only represent different problem solving approaches but also reflect the individual characteristics of specific calculators. The following are some reasonable responses to the question.

(a) If your calculator has at least 2 storage registers you may solve the problem by storing the numerator in one register, the denominator in another register and recalling the registers at the appropriate times as follows:

[4] [9] [+] [3] [8] [=] [STO] [A] (STO means store)

[8] [5] [+] [9] [6] [=] [STO] [B]

[RCL] [A] [÷] [RCL] [B] [=]

Remember that the labeling of storage registers is dependent upon the particular calculator you are using.

(b) If your calculator has a key that switches the contents of two registers the problem may be solved as follows:

[4] [9] [+] [3] [8] [=] [STO] [A] (The display is 87 and 87 is stored in register A.)

[8] [5] [+] [9] [6] [=] (the display is 181)

[EXC] [A] (EXC means the display and the storage register contents are exchanged. At this point the display is 87 and 181 is in the register storage labeled A)

[÷] [RCL] [A] (the display is 181.)

[=] (the display is 0.4806629)

c) If your calculator has only a single storage register <u>or</u> no storage register the problem must be solved by writing down the intermediate results or reentering them into the calculator.

4) - 10) Some calculators, because of their wiring, can correctly solve problems by simply working left to right because they have a built-in order of operations where, for example, multiplication takes precedence over addition. On others it is necessary to reorganize the problem so that the operations are performed in the correct order.

(4)	10110.9 6	(5)	5235.47 1
(6)	5214.	(7)	-2297.52 93

(8) 21.952 Your calculator may have a $\boxed{y^x}$ key that would be appropriate to use here. Your calculator may have a constant multiplying key.

(9) .320118(1592) (10) 3.12384(6527)

(11) They are reciprocals (multiplicative inverses) of each other. If your calculator has one, you might wish to discuss the $\boxed{1/x}$ key at this time. The answer to (10) can be obtained by the following key strokes:

(answer to 9) $\boxed{1/x}$.

Notice that it is unnecessary to use $\boxed{=}$ in this case.

If you are dealing with calculators that have several storage registers, you could ask the students to calculate these exercises in more than one way, without using parenthesis. Have them write down the sequence of key strokes and consider which is a better method. At this point you may wish to consider efficiency of methods in terms of fewer key strokes.

(12) 0.02688(5465) (13) 37.1948(1878)

(14) -179907.(84) (15) -9.44695(6522)

(16) -5.47988(5646)

Exercise Set 1.3

Some calculators are wired for a hierarchy of operations. In those calculators even more parenthesis may be deleted without storing.

1) (a) 3 + 5 - 7

(b) 3 + 5 - 7

1

2) (a) 20 x 10 ÷ 5

(b) 20 x 10 ÷ 5

40

3) (a) $\frac{2 \times 7}{31 - 14}$

(b) $\frac{2 \times 7}{(31 - 14)}$

.823529(4118)

4) (a) 20 ÷ (10 x 5)

(b) 20 ÷ (10 x 5)

.4

5) (a) (8 + 7)(3 + 5)

(b) (8 + 7)(3 + 5)

120

6) (a) (27.3 + 41.7) 3.6

(b) (27.3 + 41.7)3.6

248.4

7) (a) 27.3 + 41.7 x 3.6

(b) 27.3 + (41.7 x 3.6)

177.42

8) (a) 41.7 x 3.6 + 27.3

(b) 41.7 x 3.6 + 27.3

177.42

9) (a) 41.7 x (3.6 + 27.3)

(b) 41.7 x (3.6 + 27.3)

1288.53

10) (a) $\frac{28 \times 3 + 8}{(26 + 7) \times 4}$

(b) $\frac{(28 \times 3) + 8)}{((26 + 7) \times 4)}$

$.696\overline{969}$

11) 40.068

12) 40.068

Look at where 11 and 12 are the same

37.8 + (.06 x 37.8) = (1 + .06)37.8 = 1.06 x 37.8

13) 162553.306 14) -422.4

15) In algebraic - memory 264 - 189 [STO]

327.84 ÷ [RCL] =

4.3712

In (algebraic) and in (AOS)

327.84 ÷ (264 - 189) =

4.3712

16) In algebraic - memory

48.3 + 27.9 [=] [STO]

79.4 - 43.7 [=] [x] [RCL] [=] [STO]

67.1 - 4 [=] [x] [RCL] [=]

171653.454

In (algebraic) or (AOS)

(48.3 + 27.9) [x] (79.4 - 43.7) [x] (67.1 - 4) [=]

171653.454

17) 1 + 1st day

(1 + 2) + 2nd day

(1 + 2 + 3) 3rd day

⋮

(1 + 2 + 3 + ... + 12) 12th day

= 12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6) + 6(7) + 5(8)

+ 4(9) + 3(10) + 2(11) + 1(12)

= 2 [12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6)]

on an (AOS)

2 [x] [(] [(] 12 [)] [+] [(] 11 [x] 2 [)] [+] [(] 10 [x]

3 [)] [+] [(] 9 [x] 4 [)] [+] [(] 8 [x] 5 [)] [+] [(]

7 [x] 6 [)] [)] [=]

364

The gifts will be returned on Christmas Eve of the following year (if it is not a leap year).

Exercise Set 1.4

1.

1		1
2	+	1
3		2
4		7

2.

1		1
2	÷	1
3		2
4		.75

3.

8		8
0	x	8
2		0
2		4

4.

1		2
2	ENT	3.
3		4
4		4

5.

1		1		2
2	5	2	or	3
3		3		4
4		45		5

6.

1		1
2	CLX	2
3		3
4		0

7.

1	f clear	0
2	STK	0
3		0
4		0

8.

1		1
1	CHS	1
1		1
1		-1

9.

1		1
2	-	1
3		2
4		-1

10.

1		1
2	÷	2
3		3
0		0
		ERROR

11.

0		0
0	ENT	0
0		35
35		35

12.

35		0
0	ENT	0
0		0
0		0

13.

5	0	ENT	0	3	0	+	0
	0		0		0		0
	0		5		5		0
	5		5		3		8

14.

5	0	ENT	0	X	0
	0		0		0
	0		5		0
	5		5		25

15.

5	0	X	0
	0		0
	0		0
	5		0

16.

3	0	ENT	0	ENT	0	+	0	X	0
	0		0		3		0		0
	0		3		3		3		0
	3		3		3		6		18

17.

2	3	0	ENT	0	5	0	÷	0
		0		0		0		0
		0		23		23		0
		23		23		5		4.6

18.

5	0	ENT	0	4	0	ENT	0	+	0	÷	0
	0		0		0		5		0		0
	0		5		5		4		5		0
	5		5		4		4		8		.625

19.
- (14) 5 x 5
- (15) 5 x 0
- (16) (3 + 3) 3
- (17) 23 ÷ 5 or $\frac{23}{5}$
- (18) $\frac{5}{4 + 4}$ or $\frac{5}{8}$

20. 2 ENT 3 + 4 X 20

21. 2 ENT 3 ÷ 4 + 4.66666$\overline{6}$

22. 4 ENT 2 ENT 3 ÷ + 4.6666$\overline{6}$

23. 2 ENT 3 + 4 ENT 5 + X 45

24. 2 ENT 3 ÷ 4 ENT 5 ÷ + 1.46$\overline{6}$

25. 2 ENT 3 + 4 ENT 5 + ÷ .5$\overline{5}$

26. 2 ENT 3 + 4 ENT 5 + 6 ENT 7 + X X 585

27. 5 ENT 9 ENT 1 3 X X

28. 6 ENT 7 ENT 8 ENT

29) (16)

6	3	7	ENT	8	ENT	8	+	8	X	8
7		8		8		3		8		8
8		8		3		3		3		8
8		3		3		3		6		18

(18)

6	5	7	ENT	8	4	8	ENT	5	+	5	÷	5
7		8		8		5		5		5		5
8		8		5		5		4		5		5
8		5		5		4		4		8		.625

30) [12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6)] 2

12 ENT 11 ENT 2 X + 10 ENT

3 X + 9 ENT 4 X + 8 ENT

5 X + 7 ENT 6 X + 2 X

Exercise Set 1.5

1) 25   2) 3   3) .25

4) 1000   5) -8   6) error message

7) 25

0
0
5
25

8) 30   9) 100

10) 49   11) .25   12) .25

13) 0   14) 1   15) 10

16) 100,000   17) 4

18) INT gives the largest integer less than or equal to the number.

19) FRACT gives the part of the number after the decimal point - the fractional part of the number.

20) ABS gives the absolute value of the number.

21) AOS [8] [y^x] [5] [=]

RPN [8] [ENTER] [5] [y^x]

22) AOS [1.23] [y^x] [3] [=]

RPN [1.23] [ENTER] [3] [y^x]

23) AOS [1] [÷] [16] [+] [1] [÷] [7] [=]

RPN [1] [ENTER] [16] [÷] [1] [ENTER] [7] [÷] [+]

24) AOS [16] [+] [7] [=] [$\frac{1}{x}$]

or [1] [÷] [(] [16] [+] [7] [)] [=]

RPN [1] [ENTER] [16] [ENTER] [7] [+] [÷]

25) AOS [10] [y^x] [5] - [5] [y^x] [7] [=]

or [10] [y^x] [5] - [(] [5] [y^x] [7] [)] [=]

RPN [10] [ENTER] [5] [y^x] [5] [ENTER] [7] [y^x] [-]

26) AOS [35] [√x] [X] [45] [SIN] [÷] [34] [y^x] [3] [=]

RPN [35] [√] [45] [SIN] [X] [34] [ENTER] [3] [y^x] [÷]

27) AOS [10] [y^x] [60] [TAN] [=] [INT]

RPN [10] [ENTER] [60] [TAN] [y^x] [INT]

28) AOS [3.7] [√x] [+] [10] [COS] [=] [÷] [(] [.13] [y^x] [3] [-] [27] [$\frac{1}{x}$] [)] [=]

RPN [3.7] [√x] [10] [COS] [+] [.13] [ENTER] [3] [y^x] [1] [ENTER] [27] [÷] [-] [÷]

29) AOS [b] [CHS] [illegible] [a] [=] or [b] [-] [a] [=] [CHS]

RPN [b] [CHS] [ENTER] [a] [+] or [b] [ENTER] [a] [-] [CH

30) AOS [b] [÷] [a] [=] [$\frac{1}{x}$]

RPN [b] [ENTER] [a] [÷] [$\frac{1}{x}$]

or [b] [ENTER] [a] [x ≷ y] [÷]

Exercise Set 1.6

1) 2009.811741

2) 502.4529

3) 502.4529

4) 0

5) 212

6) 0

7) 20

8) 37

9) F = C at -40

10) let C = F

$$C = \frac{5}{9}(C - 32)$$

$$9C = 5C - 160$$

$$4C = -160$$

$$C = -40.$$

11) 3.8302

12) 34.6410

13) 6.6418

14) 4349.8139

15) 5.48095031

16) 26.61950336

17) .501019369

18) 2.97190930

19) $t = \sqrt{\frac{2h}{9.8}}$

$h = \frac{t^2(9.8)}{2}$ when t = 10, h = 490 trick on HP 33 and

X [ENTER] y [g] [→p] change rectangular to polar coords.

on TI-57 use X [X ≷ t] y [INV] [2nd] [P→R] [X ≷ t]

20) 6.4031

21) 18.8213

22) 15.5878

23) 19.0394

24) 151.29

25) 151.29

26) 87.65

27) 1860.867

28) $(x + y)^2 = x^2 + 2xy + y^2$

29) $(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^2 \neq x^2 + y^2$

30) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

Exercise Set 1.7

1) 1.749635531

2) 4.517539515

3) 14.28571429

4) 45.17539515

5) 42.47448214

6) 18.12090911

7)

RPN - HP-33	Algebraic - TI-57
ENTER	LRN
32	-
-	32
	=
5	X
ENTER	5
9	÷
÷	9
X	=
	R/S
	RST
	LRN
	RST

8) $-17.7\overline{7}$

9) $32.\overline{2}$

10) 10

11) -40

12) 320F = 160C

13)

RPN - HP-33	Algebraic - TI-57
ENTER	LRN
.07	X
X	.07
	=
	R/S
	RST
	LRN
	RST

14) $35

15) $3.17 (24)

16) $20.99 (65) = $21.

17) $.19(53) = $.20

18) When rounded to two decimal places any answer between $14.22 and $14.35, inclusive, is correct.

Exercise Set 1.8

1) 265

2) 51

3) 339

4) 25

5) 28, 53

6) 60, 75

7) 108, 117

8) 200, 205

9) 1012, 1013

10) $2.45; $37.40

11) $.12; $1.79

12) $209.65; $3204.65

13) $44.28; $676.78

14) $7.00; $106.95

15) $7.00; $107.

16) In HP33 program after step 4, key 8 you are done.

17) p x .07 x 107 ÷ 7 =

18)

	tax	cost
suit	8.28	146.23
overcoat	6.76	91.26
shoes	2.20	33.65
hat	1.11	19.61
totals	18.35	290.75

Solutions to Exercise set 1.9

1) The hypotenuse of a right triangle, given legs a and b.

2) Celsius temperature given Fahrenheit temperature.

3) The 7% tax of an item and the total cost.

4) $(a + bi)^2 = c + di$

5) stop

6) Replace x by 2x

7) Replace x by 1/x

8) Replace x by 5

9) Replace x by x - 1

10) The information is not displayed
(1) enter x
(2) $x \leftarrow x + 1$
(3) display x
(4) stop

11) The variables are not initialized.
(1) enter a and b
(2) $c \leftarrow a + b$, display c
(3) stop

12) The process has no way to stop.
(1) enter x, y
(2) $z \leftarrow x + y$, display c
(3) stop

13) (1) enter ℓ and w
(2) $a \leftarrow \ell \times w$, display a
(3) stop

14) (1) Remember s
(2) $p \leftarrow 3s$, display p
(3) $a \leftarrow \dfrac{s^2\sqrt{3}}{4}$, display a
(4) stop

15) (1) Remember a, b, c, d.
(2) $x = b - d$
(3) $y = c - a$
(4) $m = x \div y$, display m
(5) stop

16) (1) Remember a, b, c, d
(2) $x \leftarrow ad + bc$
(3) $y \leftarrow bd$
(4) $s \leftarrow x/y$, display s
(5) stop

17) (1) Remember a, b, c, d.
(2) $x \leftarrow (a - c)^2$
(3) $y \leftarrow (b - d)^2$
(4) $z \leftarrow \sqrt{x + y}$, display z
(5) stop

18) (1) Remember x, y.
(2) $a \leftarrow \frac{x + y}{2}$, display a
(3) $g \leftarrow \sqrt{xy}$, display g
(4) stop

19) (1) Remember a, b, c, d.

TI 58		HP 33E	TRS 80
STO 00	($R_0 = a$)	STO 0	INPUT A, B, C, D
R/S		R/S	
STO 01	($R_1 = b$)	STO 1	
R/S		R/S	
STO 02	($R_2 = c$)	STO 2	
R/S		R/S	
STO 03	($R_3 = d$)	STO 3	

(2) $x \leftarrow (a - c)^2$

TI 58		HP 33E	TRS 80
RCL 00		RCL 0	X = (A - C) ↑ 2
-		RCL 2	
RCL 02		-	
=		gx^2	
x^2			
STO 04	($R_4 = x$)		

(3) $y \longleftarrow (b - d)^2$

RCL 01	RCL 1	Y = (B - D) ↑ 2
-	RCL 3	
RCL 03	-	
=	gx^2	
x^2		

(4) $z \longleftarrow \sqrt{x + y}$, display z

+	+	Z = SQR (x + y)
RCL 04	f $\sqrt{x}$	PRINT Z
=		
$\sqrt{x}$		
R/S		

(5) stop

last command in 4	calculator automatically resets to 00 and stops	END

20) (1) Remember x, y

STO 00 $(R_0=x)$ R/S STO 01 $(R_1=y)$	STO 0 R/S STO 1	INPUT X, Y

(2) $a \longleftarrow \frac{x + y}{2}$, display a

+ RCL 00 = ÷ 2 = R/S	+ 2 ÷ R/S	A = (x + y)/2

(3) $g \longleftarrow \sqrt{xy}$

RCL 00 x RCL 01 = $\sqrt{x}$ R/S	RCL 0 RCL 1 X f $\sqrt{x}$	G = SQR (x*Y)

(4)	stop last command in 3	calculator automatically resets to 00 and stops	END

In general $g \leq a$ that is $\sqrt{xy} \leq \frac{x + y}{2}$. The geometric mean of two numbers is less than or equal to the arithmetic mean of those two numbers.

Solutions to Exercise Set 10.1

1)

a	b	n	k	c	d	e	f
1	-1	3	2	1	0	1	-1
			1	1	-1	0	-2
			0	0	-2	-2	-2

2) $(1 - i)^3 = -2 - 2i$

3) $(1 - i)^3 = 1^3 + 3(1)^2(-i) + 3(1)(-i)^2 + (-i)^3$

$= 1 - 3i + 3i^2 - i^3$

$= 1 - 3i - 3 + i$

$= -2 - 2i$

4)

a	b	n	k	c	d	e	f
0	2	4	3	1	0	0	2
			2	0	2	-4	0
			1	-4	0	0	-8
			0	0	-8	16	0

$(2i)^4 = 16$

5)

a	b	n	k	c	d	e	f
-1	0	3	2	1	0	-1	0
			1	-1	0	1	0
			0	1	0	-1	0

$(-1)^3 = -1$

6) completed in text

7) n is stored in R_2 and k ← n-1

HP 33E: 1, STO 3, 0, STO 4, RCL 2, 1, -, STO 5
(R_3 = c, R_4 = d, R_5 = k)

TI 58: 1, STO 03, 0, STO 04, RCL 02, -, 1, =, STO 05
(R_3 = c, R_4 = d, R_5 = k)

TRS-80: C = 1, D = 0, K = N-1

8) HP 33E: RCL 0, RCL 3, X, RCL 1, RCL 4, X, -, STO 6
(R_6 = e)

TI 58: LBL A, RCL 00, X, RCL 03, =, -, RCL 01, X, RCL 04,
=, STO 06 (R_6 = e)

TRS-80: E = AC - BD

9) HP 33E: RCL 0, RCL 4, X, RCL 1, RCL 3, X, +, STO 7
(R_7 = f)

TI 58: RCL 00, X, RCL 04, =, +, RCL 01, X, RCL 03, =,
STO 07 (R_7 = f)

TRS-80: F = AD + BC

10) HP 33E: RCL 6, STO 3, RCL 7, STO 4, RCL 5, 1, -, STO 5

TI 58: RCL 06, STO 03, RCL 07, STO 04, RCL 05, -, 1, =, STO 05

TRS-80: C = E, D = F, K = K-1

11) HP 33E: R/S, GTO 14 (see below)

TI 58: R/S, GTO A (see below)

TRS-80: PRINT E, F GO TO 30

GTO statements for HP 33E's are followed by program step numbers.

GTO statements for TI 58's are followed by labels.

Solutions to Exercise Set 1.11

1)	4	2)	.2	3)	.5
4)	1	5)	.01	6)	error
7)	.25	8)	.2	9)	.5
10)	1	11)	.01	12)	error
13)	-1	14)	-5	15)	-2
16)	0	17)	-100.	18)	0
19)	+1	20)	-5	21)	-2
22)	0	23)	-100	24)	0

25) Do you have any empty pockets left?

26) It counts the number of pockets that you have.

Solutions to Exercise Set 1.12

(A) For HP 33E

1) programmable calculator
2) RPN
3) on/off switch
4) yes
5) write it out on paper
6) no
7) 8, named 0, 1, 2, 3, 4, 5, 6, 7
8) the 5 is lost and the 7 is stored in that register
9) (a) 3
STO + 5

(b) 2
STO - 5

(c) RCL 1
RCL 2
X
STO 2

(d) RCL 4
RCL 3
÷
STO 3
STO 4

(e) 0
STO X5

(or)

0
STO 5

10) Change to program mode by using program/run switch.
11) Change to run mode by using program/run switch.
12) No, but more than one program can be carried at a time by carefully going to a specified step number that begins a program and ends in R/S. e.g. 00 ⋮ 09 R/S } one program; 10 ⋮ 25 R/S } another program.

To get to the second program key STO 10 in RUN mode and press R/S.

13) Either g RTN or GTO 00
14) For unconditional branching: GTO program step number.
For conditional branching: $x \neq y$, $x = y$, $x > y$, $x \leq y$, $x \neq 0$, $x = 0$, $x > 0$, $x < 0$.
15) program step
16) SST, BST, MANT
17) SST
18) Yes, write over the steps to be replaced.
19) Yes, write NOP over these program steps to be deleted.
20) Yes, if you use a subroutine.

(B) For TI 58

1) programmable calculator
2) AH
3) on/off switch
4) yes
5) write it out on paper
6) no
7) 31, 00 through 29 and t
8) the new number replaces the old number and the old number is lost

9) (a)
```
3
SUM
05
```
(b)
```
2
INV SUM
05
```
(c)
```
RCL 01
X
RCL 02
=
STO 02
```
(d)
```
RCL 04
÷
RCL 03
=
STO 03
STO 04
```
(e)
```
0          0
PRD   or   STO
05         05
```

10) Press LRN in run mode.
11) Press LRN in program mode.
12) Yes, A through E, A' through E' and most keys like C05 etc.
13) RST
14) For unconditional looping: GTO or RST. For conditional looping: $x \geq t$, $x < t$, $x = t$, $x \neq t$, DSZ, INV 2nd DSZ
15) a label
16) PRM, LRN, SST, BST
17) SST, BST
18) Yes, write over the steps to be replaced
19) Yes, use Del or Nop.
20) Yes, use INS.

(C) For TRS-80

1) microprocessor
2) AH
3) on/off
4) YES
5) store on tape or disc
6) YES, LPRINT
7) 48000, coded hexadecimally
8) REPLACES
9) REGISTER ARITHMETIC
10) turning to ON

11) Cannot be done without writing a special program.
12) No, but programs may be saved on tape or disc for future use.
13) TYPE LOAD followed by "name of program". This loads the program from a disc.

CLOAD "Name" - loads program from cassette
14) GO TO STEP NO.
OR FOR TO STATEMENT
AND NEXT STATEMENT
15) STEP
16) NOT APPLICABLE
17) TYPE LIST
18) YES, EDIT STEP NO., ENTER
19) YES, DELETE STEP NO., ENTER
20) YES, if there has been space left between STATEMENT NUMBERS.

Solutions to Exercise Set 1.13

The following are possible solutions.

1) HP 33E: STO 0, R/S, STO 1, X, R/S, RCL 0, 2, X, RCL 1, 2, X, +

TI 58: STO, 00, R/S, X, STO, 01, =, R/S, RCL, 00, X, 2, =, +, RCL, 01, X, 2, =, R/S, RST

TRS-80:
```
10 INPUT, L, w
20 A = L*w
30 PRINT A
40 P = 2*L + 2 * w
50 PRINT P
60 END
```

2) HP 33E: STO 0, x^2, 3, $\sqrt{x}$, X, 4, ÷, R/S, RCL 0, 3, X

TI 58: STO, 00, x^2, X, 3, $\sqrt{x}$, =, ÷, 4, =, R/S, RCL, 00, X, 3, =, R/S, RST

TRS-80:
```
10  INPUT S
20  A = S ↑ 2 * SQR(3)/4
30  PRINT A
40  P = 3 * S
50  PRINT P
60  END
```

3) HP 33E: STO 0, R/S, STO 1, R/S, STO 2, R/S, STO 3, RCL 1, -, STO 4, RCL 2, RCL 0, -, RCL 4, x ≶ y, ÷

TI 58: STO, 00, R/S, STO, 01, R/S, STO, 02, R/S, STO, 03, -, RCL, 01, =, STO, 04, RCL, 02, -, RCL, 00, =, STO, 05, RCL, 04, ÷, RCL, 05, =, R/S, RST

TRS-80:
```
10  INPUT A, B, C, D
20  E = D - B
30  F = C - A
40  G = E/F
50  PRINT  G
60  END
```

4) HP 33E: STO 0, R/S, STO 1, R/S, STO 2, R/S, STO 3, RCL 0, X, RCL 1, ENTER, RCL 2, X, +, RCL 1, RCL 3, X, ÷

TI 58: STO, 00, R/S, STO, 01, R/S, STO, 02, R/S, STO, 03, X, RCL, 00, =, STO, 04, RCL, 02, X, RCL, 03, =, ÷, RCL, 04, 1/x, R/S, RST

TRS-80:

```
10  INPUT A, B, C, D
20  N = (A*D) + (B*C)
30  D = B*D
40  X = N/D
50  PRINT X
60  END
```

5) HP 33E: STO 0, R/S, STO 1, 9, 0, x ≷ y, -, R/S (other acute angle), STO 2, COS, RCL 0, X, R/S, (one leg) STO 3, RCL 2, SIN, RCL 0, X, R/S (other leg), STO 4, RCL 3, +, RCL 0, +, R/S (perimeter), RCL 4, RCL 3, X, 2, ÷ (area)

TI 58: STO, 00, R/S, STO, 01, +/-, +, 9, 0, =, R/S (other acute angle), STO, 02, COS, X, RCL, 00, =, R/S (other leg), STO, 03, RCL, 02, SIN, X, RCL, 00, =, R/S, (other leg), STO 04, + RCL, 03, + RCL, 00, =, R/S (perimeter), RCL, 04, X, RCL, 03, ÷, 2, =, R/S (area), RST

TRS-80:

```
10  INPUT H(hypotenuse), A(acute angle)
20  B = 90-A, PRINT B (other acute angle)
30  L1 = COS A * H, PRINT L1 (one leg)
40  L2 = SIN A * H, PRINT L2 (other leg)
50  P = H + L1 + L2, PRINT P (perimeter)
60  A = L1 * L2/2, PRINT A (area)
```

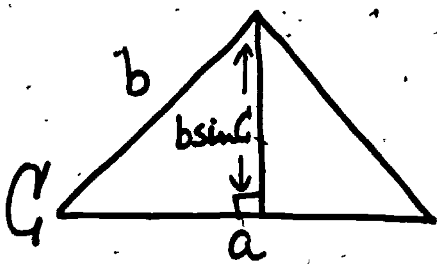
6) HP 33E: STO 0, R/S, STO 1, R/S, STO 2, RCL, 1, CHS, 2, ENTER, RCL, 0, X, ÷, R/S (abscissa of vertex), STO 3, RCL 0, X, RCL 1, +, RCL 3, X, RCL 2, +, R/S (ordinate of vertex), RCL 3, R/S, (k in X = k, axis of symmetry), RCL 1, RCL 0, ÷, CHS, R/S, (sum of the roots), RCL 2, RCL 0, ÷ (product of the roots)

TI 58: STO, 00, R/S, STO, 01, R/S, STO, 02, RCL, 01, +/-, ÷ 2, =, ÷ RCL, 00, R/S (abscissa of vertex), STO, 03, X, RCL, 00, +, RCL, 01, =, X, RCL, 03, =, RCL, 02, =, R/S (ordinate of vertex), RCL, 03, R/S (k in X = k, axis of symmetry), RCL, 01, ÷, RCL, 00, +/-, R/S (sum of the roots), RCL, 02, ÷, RCL, 00, =, R/S (product of the roots), RST

TRS-80:

```
10   INPUT, A, B, C
20   X = -B/(2*A)
30   Y = A*X ↑ 2 + B*X + C
40   PRINT  X, Y (vertex)
50   PRINT "X = " X (equation of axis of symmetry)
60   S = -B/A, PRINT S (sum of roots)
70   P = C/A, PRINT P (product of roots)
```

7)



HP 33E: STO 0 (a), R/S, STO 1 (b), R/S, STO 2 (c), SIN, RCL 1, X, RCL 0, X, 2, ÷

TI 58: STO, 00 (a), R/S, STO, 01 (b), R/S, STO, 02 (c), SIN, X, RCL, 01, X, RCL, 00, ÷, 2, =, R/S (area) RST

TRS-80:

```
10   INPUT A, B, C
20   AREA = B*SIN C*A/2
30   PRINT AREA
40   END
```

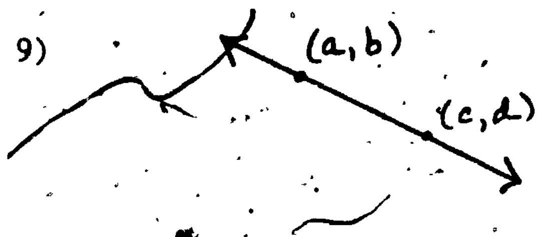
8) area = $\sqrt{s(s - a)(s - b)(s - c)}$ where $s = \frac{1}{2}(a + b + c)$

(Hero's formula)

HP 33E: STO 0 (a), R/S, STO 1 (b), R/S, STO 2 (c), +, +, 2, ÷, STO 3 (s), ENTER, RCL 0, -, X, RCL 3, RCL 1, -, X, RCL 3, RCL 2, -, X, √x

TI 58: STO, 00 (a), R/S, +, STO, 01 (b), R/S, +, STO, 02 (c), =, STO, 03 (s), -, RCL, 00, =, X, RCL, 03, =, X, (, RCL, 03, -, RCL, 01,), X, (, RCL, 03, -, RCL, 02,), =, √x, R/S (area), RST

TRS-80:
```
10   INPUT A, B, C
20   S = (A + B + C)/2
30   AREA = SQR(S*(S - A)*(S-B)* (S-C))
40   PRINT AREA
```

9) 

$$m = \frac{d-b}{c-a}$$

$$x - a = m(y-b)$$

$$x + (-m)y = a - mb$$

This program will give the coefficient of y (-m) and the constant (a - mb).

HP 33E: STO 0 (a) R/S, STO 1 (b), R/S, STO 2 (c), R/S, STO 3 (d), RCL 1, -, RCL 2, RCL 0, -, ÷, CHS, R/S (coefficient of y), RCL 1, X, RCL, 0, + (constant)

TI 58: STO, 00 (a), R/S, STO, 01 (b), R/S, STO, 02 (c), R/S, STO, 03 (d), -, RCL, 01, =, ÷, (, RCL, 02, -, RCL, 00,) =, +/-, R/S (coefficient of y), X, RCL, 01, +, RCL, 00, =, R/S, (constant), RST.

<u>TRS-80</u>:

```
10   INPUT A, B, C, D
20   M = (D - B)/(C - A)
30   PRINT "X +" - M"y=" A - M*B
40   END
```

10) <u>HP 33E</u>:

01	2	11	RCL 1	21	2
02	R/S	12	÷	22	STO + 1
03	3	13	f int	23	GTO 10
04	R/S	14	f last x	24	RCL 0
05	STO 0	15	f x=y	25	R/S
06	STO 1	16	GTO 26	26	2
07	RCL 0	17	RCL 2	27	STO + 0
08	f √x	18	RCL 1	28	3
09	STO 2	19	f x>y	29	STO 1
10	RCL 0	20	GTO 24	30	GTO 07

g RTN, keep pressing R/S until you get to whatever is the largest prime you want. A prime list may be started at any odd number, X, by: $R_0 = X$, $R_1 = 3$, start program at step 07 and continue to press R/S.

<u>TI 58</u>:

00	2	21	RCL	42	01
01	R/S	22	01	43	B
02	Clr	23	=	44	2nd Lbl
03	3	24	STO	45	C
04	R/S	25	03	46	RCL
05	STO	26	x ≷ t	47	00
06	00	27	RCL	48	R/S
07	STO	28	03	49	2nd Lbl
08	01	29	2nd INT	50	D
09	2nd Lbl	30	2nd x=t?	51	2
10	A	31	D	52	SUM
11	RCL	32	RCL	53	00
12	00	33	01	54	3
13	√x	34	x ≷ t	55	STO
14	STO	35	RCL	56	01
15	02	36	02	57	GTO
16	2nd Lbl	37	2nd INV	58	A
17	B	38	x ≥ t?		
18	RCL	39	C		
19	00	40	2		
20	÷	41	SUM		

TRS-80:

```
10   A = 2, PRINT A,
20   A = 3, PRINT A,
30   A = 5
40   B = SQR(A)
50   D = 3
60   If INT (A/D) = A/D  THEN 100
70   D = D + 2
80   If D > B  THEN 130
90   GO TO 60
100  A = A + 2
110  If A < 100 THEN 40 ELSE END
120  END
130  PRINT A,
140  GO TO 100
```

11) Each of these programs will find the sum of the squares of consecutive integers from N to M inclusive (M ≥ N).

HP 33E:

```
01  STO 1 (N)      10  -          19  f x = y?
02  R/S            11  1          20  GTO 25
03  STO 2 (M)      12  +          21  1
04  1              13  STO 4      22  STO + 3
05  STO 3          14  RCL 1      23  STO + 1
06  0              15  x2         24  GTO 14
07  STO 5          16  STO + 5    25  RCL 5
08  RCL 2          17  RCL 4
09  RCL 1          18  RCL 3
```

TI 58:

```
00  STO            17  1            34  B
01  01 (N)         18  =            35  1
02  R/S            19  STO          36  SUM
03  STO            20  04           37  03
04  02 (M)         21  2nd Lbl      38  SUM
05  1              22  A            39  01
06  STO            23  RCL          40  GTO A
07  03             24  01           41  2nd Lbl
08  0              25  x2           42  B
09  STO            26  SUM          43  RCL
10  05             27  05           44  05
11  RCL            28  RCL          45  R/S
12  02             29  04           46  RST
13  -              30  x><t
14  RCL            31  RCL
15  01             32  03
16  +              33  2nd x=t?
```

<u>TRS-80</u>:

```
10   INPUT N, M
20   A = 1
30   D = 0
40   G = M - N + 1
50   S = N ↑ 2
60   D = D + 2
70   If C = A  THEN 110
80   N = N + 1
90   A = A + 1
100  GO TO 50
110  PRINT D
120  END
```

12) (a) <u>HP 33E</u>:

01	STO 0	13	STO 2	25	RCL 0
02	√x	14	RCL 0	26	f clear reg.
03	STO 1	15	RCL 2	27	RTN
04	RCL 0	16	÷	28	RCL 2
05	2	17	ENTER	29	f clear reg.
06	STO 2	18	INT	30	RTN
07	÷	19	f(x=y)	31	2
08	ENTER	20	GTO 28	32	STO + 2
09	INT	21	RCL 1	33	GTO 14
10	f(x=y)	22	RCL 2	34	RTN
11	GTO 28	23	f x ≤ y		
12	3	24	GTO 31		

(b) Press R (number divided by prime factor displayed) g RTN R/S to get next prime factor. Continue until all factors are found.

TI 58:

```
00  2nd Lbl        24  3              48  2nd x≥t?
01  A              25  STO            49  D
02  STO            26  02             50  RCL
03  00             27  2nd Lbl        51  00
04  √x             28  C              52  2nd CMS
05  STO            29  RCL            53  RST
06  01             30  00             54  2nd Lbl
07  2              31  ÷              55  B
08  STO            32  RCL            56  RCL
09  02             33  02             57  02
10  x><t           34  =              58  R/S
11  RCL            35  STO            59  2nd CMS
12  00             36  03             60  RST
13  ÷              37  x><t           61  2nd Lbl
14  x><t           38  RCL            62  D
15  =              39  03             63  2
16  STO            40  2nd INT        64  SUM
17  03             41  2nd x=t?       65  02
18  x><t           42  B              66  GTO
19  RCL            43  RCL            67  B
20  03             44  02             68  RST
21  2nd INT        45  x><t
22  2nd x=t?       46  RCL
23  B              47  01
```

(b) RCL 03 (number divided by prime factor displayed) RST R/S to get next prime factor. Continue until all factors are found.

TRS-80:

(a)

```
10   INPUT N
20   A = SQR(N)
30   B = 2
40   If INT (N/B) = N/B
       THEN 100
50   B = 3
60   If INT (N/B) = N/B
       THEN 100
70   If B > A  THEN 120
80   B = B + 2
90   GO TO 60
100  PRINT B
110  END
120  PRINT N
130  END
```

(b)

```
10 - 100   same as A
110        N = N/B
120        GO TO 20
130        PRINT N
140        END
```

Solutions to 1.14 - Chapter I test

(1 - 4) Answers may vary.

HP33E

1) 5, ENTER, 6 ENTER, 7, ÷, +

2) 3, ENTER, 4, +, 5, ENTER, 7, +, X

3) 37, f SIN, 6, X, 3, g 1/x, f y^x

4) 2, ENTER, 3, +, 5, ENTER, 7, +, ÷

TI 58

1) 5, +, 6, ÷, 7, =

2) 3, +, 4, =, X, (5, +, 7,), =

3) 37, 2nd SIN, X 6, =, y^x, 3, 1/x, =

4) 2, +, 3, =, ÷, (, 5, +, 7,), =

5) 10 6) 35.4

7) 2 8) 1.1412 (13562)

9) 0 10) .36

11) 0 12) 1

13) 1.91

14) 9.18

15) 13.59

16) (a) remember C

F ← 9/5 C + 32

display F

Stop

(b) HP-33E

01	9	05	3
02	X	06	2
03	5	07	+
04	-		

TI-58

00	X	06	3
01	9	07	2
02	-	08	=
03	5	09	R/S
04	=	10	RST
05	+		

TRS-80

```
10    INPUT C
20    F = 9/5 * C + 32
30    PRINT F
40    STOP
```

(c) $28^{\circ}C = 82^{\circ}F$; $16^{\circ}C = 61^{\circ}F$

17) (a) $\frac{1}{a + bi} = \frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2} i$

(b)
1. remember a, b
2. $c \leftarrow a^2 + b^2$, remember c
3. $x \leftarrow a \div c$
4. $y \leftarrow -b \div c$
5. display x, y
6. stop

(c) HP 33E

01	STO 0	07	+	13	RCL 1
02	R/S	08	STO 2	14	CHS
03	STO 1	09	RCL 0	15	RCL 2
04	x^2	10	x ≷ y	16	÷ (y)
05	RCL 0	11	÷		
06	x^2	12	R/S (x)		

TI 58

00	STO	07	RCL	14	00	21	01
01	00	08	00	15	÷	22	+/-
02	R/S	09	x^2	16	RCL	23	÷
03	STO	10	=	17	02	24	RCL
04	01	11	STO	18	=	25	02
05	x^2	12	02	19	R/S(x)	26	=
06	+	13	RCL	20	RCL	27	R/S(y)
						28	RST

TRS-80

```
10  INPUT A, B
20  C = A ↑ 2 + B ↑ 2
30  X = A/C
40  Y = -B/C
50  PRINT X, Y
60  STOP
```

(d) .1538(46154) - .2307(69231) i

(e) -.5 - .5i

(f) 0 - i

(g) .0675(67568) + .0945(94595)i

18) (a) 1. Remember a, b, c

2. $S \leftarrow \frac{1}{2}(a + b + c)$

3. $x \leftarrow S - a$, remember x

4. y ← s-b, remember y

5. z ← s-c, remember z

6. w ← s · x · y · z

7. Q = $\sqrt{w}$

8. display Q.

9. stop

(b). HP 33E

01	f FIX 0	08	+	15	RCL 3	22	RCL 2
02	STO 0	09	2	16	X	23	
03	R/S	10	÷	17	RCL 3	24	X
04	STO 1	11	STO 3	18	RCL 1	25	f $\sqrt{x}$
05	R/S	12	ENTER	19	-		
06	STO 2	13	RCL 0	20	X		
07	+	14	-	21	RCL 3		

TI 58

00	2nd FIX	12	01	25	03	38	X
01	0	13	+	26	-	39	(
02	STO	14	RCL	27	RCL	40	RCL
03	00	15	00	28	00	41	03
04	R/S	16	=	29	)	42	-
05	STO	17	÷	30	X	43	RCL
06	01	18	2	31	(	44	02
07	R/S	19	=	32	RCL	45	)
08	STO	20	STO	33	03	46	=
09	02	21	03	34	-	47	$\sqrt{x}$
10	+	22	X	35	RCL	48	R/S
11	RCL	23	(	36	01	49	RST
		24	RCL	37	)		

TRS 80

```
10    INPUT A, B, C
20    S = (A + B + C)/2
30    AREA = SQR (S* (S-A) * (S-B) * (S - C))
40    PRINT AREA
50    STOP
```

(c) 3152

(d) 0

(e) In any triangle, the sum of the lengths of two sides must be greater than the third side. If the sides are 2, 3, 5 it cannot be a triangle.

Solutions to Exercise Set 2.1

1) 1, 3, 5, 7, 9, 11

2) 0, 2, 6, 12, 20, 30

3) 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$

4) 2, 4, 8, 16, 32, 64

5) 1, 4, 27, 256, 3125, 46,656

6) 1, 1.4142, 1.4422, 1.4142, 1,3797, 1.3480

7) -1, 1, -1, 1, -1, 1

8) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$

9) .3, .03, .003, .0003, .00003, .000003

10) 79

11) 9900

12) .02

13) 1,048,576

14) 1.2089×10^{64}

15) 1.1374

16) 1

17) .99

18) 3×10^{-40}

(19 - 22)

HP 33E

01	1	07	RCL 0	13	R/S (S_n)
02	STO 0	08	÷	14	RCL 0
03	5	09	STO 1	15	1
04	×	10	RCL 0	16	+
05	1	11	R/S (n)	17	GTO 02
06	+	12	RCL 1		

TI 58

00	1	10	÷	20	01
01	2nd Lbl	11	RCL	21	R/S (S_n)
02	A	12	00	22	RCL
03	STO	13	=	23	00
04	00	14	STO	24	+
05	×	15	01	25	1
06	5	16	RCL	26	=
07	+	17	00	27	GTO
08	1	18	R/S (n)	28	A
09	=	19	RCL		

TRS 80 (this program will print the first 20 terms of the sequence)

```
10   N = 1
20   S = (5*N+1)/N
30   DISPLAY N, S
40   N = N + 1
50   IF N < 21  THEN 20
60   END
```

19) 6, 5.5, $5.\bar{3}$, 5.25, 5.2, $5.1\bar{6}$

20) 5

21) yes

22) If n is large, $\frac{1}{n}$ is very close to zero.

(23 - 26)

HP 33E

01	1	07	RCL_x 0	13	RCL 1
02	STO 0	08	f y^x	14	R/S (S_n)
03	3	09	÷	15	RCL 0
04	f y^x	10	STO 1	16	1
05	2	11	RCL 0	17	+
06	ENTER	12	R/S (n)	18	GTO 02

TI 58

00	1	08	÷	16	RCL	24	+
01	2nd Lbl	09	2	17	00	25	1
02	A	10	y^x	18	R/S (n)	26	=
03	STO	11	RCL	19	RCL	27	GTO
04	00	12	00	20	01	28	A
05	y^x	13	=	21	R/S (S_n)		
06	3	14	STO	22	RCL		
07	=	15	01	23	00		

TRS 80 (this program prints the first 20 terms of this sequence)

```
10   N = 1
20   S = N ↑ 3/ 2 ↑ N
30   DISPLAY  N, S
40   N = N + 1
50   IF N < 21  THEN 20
60   END
```

23) .5, 2, 3.375, 4, 3.9063, 3.3750

24) $n = 4$, $S_n = 4$

25) 0

26) The denominator is getting larger much faster than the numerator.

Solutions to Exercise Set 2.2

1) yes, $d = -2$

2) no

3) yes, $d = 3$

4) yes, $d = 0$

5) no

6) yes, $d = -.9$

7) 3, 7, 11, 15, 19

8) 5, -2, -9, -16, -23

9) p-2q, p-q, p, p+q, p+2q

10) 2, 4, 6, 8, 10

11) 41

12) -32

13) 20

14) 70

15) The difference between any 2 consecutive terms is d. $S_7 - S_6 = d$, $S_8 - S_7 = d$, $S_9 - S_8 = d$, so the difference between S_6 and S_9 is 3d.

16) 4d

17) 15d

19) 17.5

20) $\frac{a + b}{2}$

21) The average of two numbers is the sum of the numbers divided by 2. This is the same as finding the arithmetic mean between the two numbers.

22) $d = 2$; 5, 7, 9, 11, 13

23) $d = -3$; 37, 34, 31, 28, 25, 22, 19

24) $S_{26} = 1950 + (+25)(-50) = \$700$

25) $S_w = 1950 + (w-1)(-50)$

26) $S_{17} = -16$ $d = -4$, $S_1 = 48$

27) m + b

28) $d = 2m + b - (m + b) = m$

29) $m + b = 5$

$m = +2, \quad b = 3$

$S_n = 2n + 3$ or $(2n + 3)$

30) $1 + \frac{1}{4} = \frac{3}{4} = m$

$m + b = .25$

$b = -.5$

$Sn = .75n - .5$ or $(.75n - .5)$

Solutions to Exercise Set 2.3

1) yes, $r = 5$

2) yes, $r = \frac{1}{4}$

3) no, $\frac{18}{6} = 3$, but $\frac{72}{18} = 4$

4) yes, $r = 5/6$

5) yes or no

6) yes, $r = -3$

7) 3, 6, 12, 24, 48

8) 1, .6, .36, .216, .1296

9) a, ab, ab^2, ab^3, ab^4

10) $a/b^2, a/b, a, ab, ab^2$.

11) .5

12) .98304

13) $r = 1/3$; $S_8 = 8/248 = .032921811$

14) $r = 1.5$, $S_2 = 12$

15) 8

16) 8

17) a^2

18) $\sqrt{ab}$

19) $\sqrt{ab}$

20) they are the same

21) $r = 3$; 21, 63, 189

22) $r = 2/3$; 378, 252

23) 1.6, 1.28, 1.024, .8192, .65536 $r = .8$

24) side $s_1 = s$ $r = \frac{\sqrt{2}}{2}$

side $s_2 = \frac{s}{2}\sqrt{2}$ each new side is hypotenuse

side $s_3 = s/2$ of an isosceles right triangle

side $s_4 = \frac{s}{4}\sqrt{2}$ whose sides is half the side of the square before

area $s_1 = s^2$ $r = \frac{1}{2}$

area $s_2 = s^2/2$ since the sides are a

area $s_3 = s^2/4$ geometric sequence the

area $s_4 = s^2/8$ areas are too. $S_7 = 1.5$

Solutions to Exercise Set 2.4

1) 1275

2) 2,097,150

3) arithmetic; $\$_{20} = 400$

4) arithmetic; $\$_n = n^2$

5) geometric; $\$_{10} = 15.984375$

6) geometric; $\$_5 = 968.75$

7) arithmetic; $\$_{20} = -530$

8) geometric; $\$_7 = 152.\overline{518}$

9) other; $\$_5 = 55$

10) arithmetic; $\$_{21} = 6,300$

11) 1¢, 2¢, 4¢, 8¢, is geometric sequence; r = 2

$\$_{30}$ = 1,073,741,823¢ = $10,737,418.23 which is much larger than $1,000,000.

12) HP33E

01	1	07	x	13	RCL 1	19	RCL 0
02	STO 0	08	1	14	R/S (Sn)	20	1
03	0	09	-	15	RCL 2	21	+
04	STO 2	10	STO 1	16	+	22	STO 0
05	RCL 0	11	RCL 0	17	R/S ($\$_n$)	23	GTO 06
06	2	12	R/S(n)	18	STO 2		

TI 58

00	1	11	2	22	R/S (Sn)	33	1
01	STO	12	-	23	+	34	=
02	00	13	1	24	RCL	35	STO
03	0	14	=	25	02	36	00
04	STO	15	STO	26	=	37	GTO
05	02	16	01	27	R/S ($n)	38	A
06	RCL	17	RCL	28	STO		
07	00	18	00	29	02		
08	2nd Lbl	19	R/S(n)	30	RCL		
09	A	20	RCL	31	00		
10	X	21	01	32	+		

TRS 80

```
10   N = 1,  SUM = 0
20   S = 2*N - 1
30   PRINT  N, S
40   SUM = SUM + S
50   PRINT   SUM
60   IFN = 20    THEN 90
70   N = N + 1
80   GO TO 20
90   END
```

13)
1. Set $n \leftarrow 1$, $\$ \leftarrow 0$
2. Remember S_1, d
3. $S_n \leftarrow S_1$
4. Display n, S_n
5. $\$ \leftarrow \$ + S_n$
6. Display $
7. If n is large enough, stop
8. $n \leftarrow n + 1$, $S_n = S_n + d$
9. Go to step 4

14)
1. Set $n \leftarrow 1$, $\$ \leftarrow 0$
2. Remember S_1, r
3. $S_n \leftarrow S_1$
4. Display n, S_n
5. $\$ \leftarrow \$ + S_n$
6. Display $
7. If n is large enough, stop
8. $n \leftarrow n + 1$, $S_n = S_n \cdot r$
9. Go to step 4.

15)

$$
\begin{array}{r|l}
 & 1 \quad + r \quad + r^2 \quad + r^3 \\
\hline
1 - r & 1 + 0r + 0r^2 + 0r^3 - r^4 \\
 & \underline{1 - r} \\
 & \quad + r + 0r^2 \\
 & \quad \underline{r - r^2} \\
 & \qquad + r^2 + 0r^3 \\
 & \qquad \underline{+ r^2 - r^3} \\
 & \qquad\quad + r^3 - r^4 \\
 & \qquad\quad \underline{+ r^3 - r^4}
\end{array}
$$

$$
\begin{array}{l}
1 + r + r^2 + r^3 \\
\underline{1 - r} \\
1 + r + r^2 + r^3 \\
\underline{\quad - r - r^2 - r^3 - r^4} \\
1 \qquad\qquad\quad - r^4
\end{array}
$$

16) formula (13) : $S_n = \frac{5 - 1^n(5)}{1 - 1} = \frac{0}{0}$

formula (16) : $S_n = \frac{5 - 1(5)}{1 - 1} = \frac{0}{0}$

The formulas are not appropriate, but $S_n = n \cdot S_1$

17) $S_n = \frac{n}{2}\left[2S_1 + (n-1)(0)\right]$

$= \frac{n}{2}\left[2\ S_1\right] = n \cdot S_1$

Solutions to Exercise Set 2.5

1) $.99^5 = .950990$

$.99^{10} = .904382$

$.99^{100} = .366032$

$.99^{1000} = .000043$

2) $(-.99)^5 = -.590990$

$(-.99)^{10} = .904382$

$(-.99)^{100} = .366032$

3) $(1.1)^5 = 1.610510$

$(1.1)^{10} = 2.593742$

$(1.1)^{100} = 13,780.61234$

4) $(-1.01)^5 = -1.051010$

$(-1.01)^{10} = 1.104622$

$(-1.01)^{100} = 2.704814$

5) $1^5 = 1$

$1^{10} = 1$

$1^{100} = 1$

6) $(-1)^5 = -1$

$(-1)^{10} = +1$

$(-1)^{100} = +1$

7) Yes, because if $|r| \geq 1$ the numbers are getting larger in magnitude.

8) Yes, $0^n = 0$.

9) Answers vary. 5, 0, 0, 0, 0, ...

The formula works. $S_n = \frac{5}{1-0} = 5$.

10) The sequence always keeps getting bigger or smaller.

11) Answers vary

S_n has a limit 0, 0, 0, 0, 0, ... $\rightarrow 0$

S_n has no limit 5, 5, 5, 5, 5, ... $\rightarrow \infty$

12) -2.5

13) $$\frac{\frac{3n^2}{n^3} - \frac{10n^3}{n^3} - \frac{n}{n^3}}{\frac{7n}{n^3} + \frac{4n^3}{n^3} - \frac{n^2}{n^3}} = \frac{\frac{3}{n} - 10 - \frac{1}{n^2}}{\frac{7}{n} + 4 - \frac{1}{n}}$$

14) (a) $S_1 = -.6063$ $S_{15} = -.0001$

$S_2 = -.3666$ $S_{19} = -1.1084 \times 10^{-8}$

$S_5 = -.0778$ $S_{20} = 0$

$S_{10} = -.0041$

S_n appears to be approaching zero.

(b) $S_{100} = 0.1317$

$S_{1,000} = 0.8187$

$S_{1,000,000} = 0.9998$

S_n appears to be approaching one.

15) $$\left(\frac{\frac{n - 20}{n}}{\frac{n + 20}{n}}\right)^5 = \left(\frac{1 - \frac{20}{n}}{1 + \frac{20}{n}}\right)^5$$

16) 10 8 6.4

$r = .8$ $S_n = \frac{10}{1 - .8} = \frac{10}{.2} = 50$ meters

17) 32, 16, 8 $r = \frac{1}{2}$ $S = \frac{32}{1 - .5} = 64$

18) 64, 16, 4, 1 ... $r = \frac{1}{4}$, $S = \frac{64}{1-.25} = 85\frac{1}{3}$

19) $1 - (\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots)$

$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots \frac{1}{4^n} + \ldots = \frac{.25}{1-.25} = \frac{.25}{.75} = \frac{1}{3}$

So the remaining piece approaches 2/3 of the original square.

Solutions to Exercise Set 2.6

1) $\$_n = S_1 + S_2 + S_3 + \ldots + S_n$

$\$_{n+1} = S_1 + \ldots + S_n + S_{n+1} = \$_n + S_{n+1}$

$S_{n+1} + S_{n \cdot r}$

$\$_{n+1} = \$_n + S_n \cdot r$

2) $S \leftarrow S \cdot r$

$\$ \leftarrow \$ + S$

3) $\frac{n+1}{n}$

4) $\frac{2n+1}{2n-1}$

5) 2

6) 3

7) $\frac{n}{n+1}$

8) $\frac{(n+1)(n+1)}{(2n+2)(2n+1)}$

9) in text

10) $S \leftarrow S \cdot \frac{n+1}{n}$

$\$ \leftarrow \$ + S$

11) $S \leftarrow S \cdot 3$

$\$ \leftarrow \$ + S$

12) $S \leftarrow S \cdot \frac{n}{n+1}$

$\$ \leftarrow \$ + S$

13) $S \leftarrow \frac{S(n+1)(n+1)}{(2n+2)(2n+1)}$

$\$ \leftarrow \$ + S$

14) HP 33E

01	0	08	1	15	RCL 0	22	1
02	STO 0	09	+	16	1	23	+
03	STO 1	10	÷	17	+	24	STO 0
04	1	11	STO 2	18	R/S(n)	25	GTO 06
05	STO 2	12	RCL 1	19	RCL 1		
06	RCL 2	13	+	20	R/S (\$)		
07	RCL 0	14	STO 1	21	RCL 0		

TI 58:

00	0	17	1	34	RCL
01	STO	18	)	35	01
02	00	19	=	36	R/S($)
03	STO	20	STO	37	RCL
04	01	21	02	38	00
05	1	22	+	39	+
06	STO	23	RCL	40	1
07	02	24	01	41	=
08	2nd Lbl	25	=	42	STO
09	A	26	STO	43	00
10	RCL	27	01	44	GTO
11	02	28	RCL	45	A
12	÷	29	00		
13	(	30	+		
14	RCL	31	1		
15	00	32	=		
16	+	33	R/S (n)		

TRS-80:

```
10    N = 0, SUM = 0, S = 1
20    S = S/(N + 1)
30    SUM = SUM + S
40    PRINT  N + 1, SUM
50    IF  N + 1 = 12   THEN 80
60    N = N + 1
70    GO TO 20
80    END
```

HP 33E: $\$_{12} = 1.718281830$; $e^1 - 1 = 1.718281828$

TI 58: $\$_{12} = 1.718281828$; $e^1 - 1 = 1.718281828$

e' is determined by the keystroke sequence 1,

[INV], [ℓn x]

15) 1, 1, 2, 3, 5, 8, 13, 21

16) <u>HP 33E</u>:

01	1	08	R/S	15	RCL 2	22	1
02	R/S	09	3	16	+	23	STO + 0
03	1	10	STO 0	17	STO 3	24	RCL 1
04	R/S	11	1	18	RCL 0	25	STO 2
05	2	12	STO 1	19	R/S(n)	26	RCL 3
06	R/S	13	STO 2	20	RCL 3	27	STO 1
07	1	14	RCL 1	21	R/S(S_n)	28	GTO 14

<u>TI 58</u>:

00	1	17	01	34	03
01	R/S	18	STO	35	R/S(S_n)
02	CLR	19	02	36	1
03	1	20	2nd Lbl	37	SUM
04	R/S	21	A	38	00
05	CLR	22	RCL	39	RCL
06	2	23	01	40	01
07	R/S	24	+	41	STO
08	CLR	25	RCL	42	02
09	1	26	02	43	RCL
10	R/S	27	=	44	03
11	CLR	28	STO	45	STO
12	3	29	03	46	01
13	STO	30	RCL	47	GTO
14	00	31	00	48	A
15	1	32	R/S(n)		
16	STO	33	RCL		

<u>TRS-80</u>:

```
10   N = 1, S = 1
20   PRINT  N, S
30   N = 2, S = 1
40   PRINT  N, S
50   N = 3, A = 1, B = 1
60   S = A + B
70   PRINT  N, S
80   IF N = 12  THEN 110
90   N = N + 1, B = A, A = S
100  GO TO 60
110  END
```

n	1	2	3	4	5	6	7	8	9	10	11	12
S_n	1	1	2	3	5	8	13	21	34	55	89	144

17) HP 33E:

01	1	08	÷	15	÷	22	R/S(n)
02	STO 0	09	STO 1	16	.	23	RCL 2
03	5	10	RCL 1	17	5	24	R/S(S_n)
04	f $\sqrt{x}$	11	RCL 0	18	+	25	1
05	1	12	f y^x	19	g INT	26	STO + 0
06	+	13	5	20	STO 2	27	GTO 10
07	2	14	f $\sqrt{x}$	21	RCL 0		

TI 58:

00	1	14	A	28	2nd INT
01	STO	15	RCL	29	STO
02	00	16	01	30	02
03	5	17	y^x	31	RCL
04	$\sqrt{x}$	18	RCL	32	00
05	+	19	00	33	R/S (n)
06	1	20	÷	34	RCL
07	=	21	5	35	02
08	÷	22	$\sqrt{x}$	36	R/S (S_n)
09	2	23	=	37	1
10	=	24	+	38	SUM
11	STO	25	.	39	00
12	01	26	5	40	GTO
13	2nd Lbl	27	=	41	A

TRS-80:

```
10   N = 1
20   A = (1 + SQR 5)/2
30   B = A ↑ N/SQR 5 + .5
40   S = INT B
50   PRINT  N, S
60   IF  N = 12  THEN 90
70   N = N + 1
80   GO TO 30
90   END
```

18) $S_0 = -1$; $S_{-1} = .3$; $S_{-2} = -5$

$S_{-n} = -S_{n+1}$

19) $S_0 = 1.5$; $S_{-1} = .75$; $S_{-2} = .375$

$S_{-n} = S_n/4^n$

20) $S_0 = 0$; $S_{-1} = -1$; $S_{-2} = 2$

$S_{-n} = S_n$ if n is odd

$S_{-n} = -S_n$ if n is even

Solutions to Exercise Set 2.7

1) Thirty terms are convincing.

HP 33E:

```
01   1
02   STO 0
03   0
04   STO 2
05   RCL 0
06   g x^2
07   2
08   RCL 0
09   f y^x
10   ÷
11   STO 1
12   RCL 0
13   R/S (n)
14   RCL 1
15   RCL 2
16   +
17   STO 2
18   R/S ($_n)
19   1
20   STO + 0
21   GTO 05
```

TI 58:

```
00   1
01   STO
02   00
03   0
04   STO
05   02
06   2nd Lbl
07   A
08   RCL
09   00
10   x^2
11   ÷
12   (
13   2
14   y^x
15   RCL
16   00
17   )
18   =
19   STO
20   01
21   RCL
22   00
23   R/S (n)
24   RCL
25   01
26   +
27   RCL
28   02
29   =
30   STO
31   02
32   R/S ($_n)
33   1
34   SUM
35   00
36   GTO
37   A
```

TRS-80:

```
10   N = 1, S = 0
20   A = N ↑ 2/2 ↑ N
30   S = S + A
40   PRINT N, S
50   IF N = 30 GO TO 80
60   N = N + 1
70   GO TO 20
80   END
```

2) Answers vary - student trials

3) HP 33E:

alter steps 6 $\left\langle \begin{array}{l} 3 \\ f\ y^x \end{array} \right\}$ 22 steps in program

TI 58:

alter steps 10 $\langle$ y^x / 3 } 138 steps in program

TRS-80:

alter 20 A = N ↑ 3/ 2 ↑ N

The fair cost is $26.

4) Student trials - answers vary.

5) $(1 + \frac{.06}{4})^4 - 1 = 6.14\%$ 6) $(1 + \frac{.1}{12})^{12} - 1 = 10.47\%$

7) $(1 + \frac{.085}{360})^{365} - 1 = 9\%$ 8) $(1 + \frac{.0575}{360})^{365} - 1 = 6\%$

9) $(1 + \frac{.015}{12})^{12} - 1 = 1.51\%$

$(1 + \frac{.02}{12})^{12} - 1 = 2.02\%$

If you take 1 year to pay for an item

(a) at 1½% you are paying 1½ times the cost

(b) at 2% you are payimg more than twice the cost

10) $100

11) $100 + .10(1000) = $200 $100 + .10(400) = $140

$100 + .10(900) = $190 $100 + .10(300) = $130

$100 + .10(800) = $180 $100 + .10(200) = $120

$100 + .10(700) = $170 $100 + .10(100) = $110

$100 + .10(600) = $160 $100 + .10(0) = $100

$100 + .10(500) = $150

12) $1650 13) $650

14) $1000(1 + .1)^{10}$ = $2593.74; interest is $1593.74

15) Average yearly payment is $165.

16) the borrower

17) 1979 - 1626 = 353 years

$\$24.(1 + .07)^{353} \doteq \$565,828,429,700$

18) <u>HP 33E</u>:

01	1	06	STO 1	11	RCL 1
02	6	07	1	12	f y^x
03	2	08	.	13	2
04	6	09	0	14	4
05	-	10	7	15	x

<u>TI 58</u>:

00	-	09	.	18	4
01	1	10	0	19	=
02	6	11	7	20	R/S
03	2	12	y^x	21	RST
04	6	13	RCL		
05	=	14	01		
06	STO	15	=		
07	01	16	X		
08	1	17	2		

<u>TRS-80</u>:

```
10   INPUT  N
20   A = N - 1626
30   V = 24 × (1.7) ↑ A
40   PRINT V
50   END
```

1700 ⟶ $3586.
1776 ⟶ $613,448.20
1800 ⟶ $3,111,634.36
1864 ⟶ $236,347,128.8
1900 ⟶ $2,700,015,936.
1918 ⟶ $9,125,871,007
1970 ⟶ $307,773,175,400.

19) 2

20) 4

21) 8

22) 2^n, n is the number of generations

23) 68

24) 2.9515×10^{20}

25) $(2040 + 480)/30 = 84$; $2^{84} = 1.9343 \times 10^{25}$

26) The world's population at that time was less than this number.

27) Our calculations assumed that the current family relationships have always existed.

Solutions to 2.8 - Chapter 2 test

1) b - geometric

2) b - geometric

3) c - neither

4) a - arithmetic

5) any nonzero constant sequence

6) -.0234375

7) -3/128

8) $\text{ctn}^{28}\theta$

9) .8336

10) -76

11) 400

12) 996.09375

13) $\frac{n(n+1)}{2}$

14) 2

15) $\frac{3}{2}$

16) $\frac{811}{333}$

17) $-\frac{1}{3}$

18) e

19) 87,178,291,200

20) d = -1

Teacher Manual Materials   Section 3.1

Notes. If you wish to develop another example in class that parallels Example 3.1 - 1 of the text, many choices are available, but only a few give integral results. Here is the general formula

$$x = \sqrt{y + \sqrt{y + \sqrt{y + \dots}}}$$

$$x^2 = y + \sqrt{y + \sqrt{y + \dots}} = y + x$$

$$x^2 - x - y = 0$$

$$x = \frac{1 + \sqrt{1 + 4y}}{2}$$ (The negative value of the radical is extraneous here.)

To make this expression an integer, $1 + 4y$ must be a square. Here are some $(y, 1 + 4y)$ pairs: (1, 5), (2,9)*, (3, 13), (4, 17), (5, 21), (6, <u>25</u>)*, (7, 29), (8, 33), (9, 37), (10, 41), (11, 45), (12, <u>49</u>). This last would be a good class example.

When $x = \sqrt{12 + \sqrt{12 + \sqrt{12 + \dots}}}$, $x = \frac{1}{2} + \frac{7}{2} = 4$.

*(2,9) is used in Example 3.1 - 1, (b, 25) in Exercise 3.1 - 1.

Solutions to Exercises 3.1

1) 3

2) $x_{n+1} = \sqrt{6 + x_n}$

3) $x = \sqrt{6 + x}$, $x^2 = 6 + x$, $x^2 - x - 6 = 0$ $(x-3)(x+2) = 0$, $x = 3$.

4) $2,\ 2 + \frac{1}{2},\ 2 + \frac{1}{2+\frac{1}{2}},\ 2 + \frac{1}{2 + \frac{1}{2+\frac{1}{2}}}$

5-6) $2,\ 2 + \frac{1}{2},\ 2 + \frac{1}{2+\frac{1}{2}},\ 2 + \frac{1}{2 + \frac{1}{2+\frac{1}{2}}}$.

7) $x_3 + 2 + \frac{1}{x_2}$, $x_4 = 2 + \frac{1}{x_3}$

8) $x_{n+1} = 2 + 1/x_n$

9) AH: RCL 0, 1/x, +, 2, =, STO 0, R/S, RST

RPN: RCL 0, 1/x, 2, +, STO 0

BASIC:

```
10 LET x = 2
20 FOR N = 1 to 20, PRINT X,
30 X = 2 + 1/X
40 NEXT N
50 STOP
```

10)

n	x_n	n	x_n	n	x_n
1	2	6	2.4142^+	11	2.4142135^+
2	2.5	7	2.414[illegible]$^+$	12	2.414213564
3	2.4	8	2.41421^+	13	2.414213562
4	2.416^+	9	2.414213^+		same
5	2.413^+	10	2.414213^+		

11) $x = 2 + 1/x$, $x^2 = 2x + 1$, $x^2 - 2x - 1 = 0$,

$$x = \frac{2 + \sqrt{4 + 4}}{2} = 1 + \sqrt{2}.$$ Same.

12) AH: RCL 0, ÷, RCL 1, +, RCL 1, =, ÷, 2, =, R/S, STO 0, RST

RPN: RCL 0, RCL 1, ÷, RCL 1, +, 2, ÷, R/S, STO 0

BASIC:
```
10 PRINT "SQUARE ROOT OF N: WHAT VALUE OF N";
20 INPUT N
30 LET X = 1
40 FOR I = 1 TO 10
50 PRINT I, X
60 X = (X + N/X)/2
70 NEXT I
```

13)

n	x_n
1	4
2	4.375
3	4.358928572
4	4.358898944

14) $4 < \sqrt{19} < 5$

15)

n	x_n	n	x_n
1	1	5	4.36^+
2	10	6	4.3589^+
3	5.95^+	7	4.358898944
4	4.57^+		

16)

n	x_n	n	x_n	n	x_n
1	1000^+	6	31^+	11	4.36^+
2	500^+	7	16^+	12	4.358899^+
3	250^+	8	8^+	13	4.358898944
4	125^+	9	5^+		
5	62^+	10	4.4^+		

17)

n	x_n	n	x_n
1	-10^+	4	-4.36^+
2	-5^+	5	-4.3589^+
3	-4.5^+	6	-4.358898944

18) $x = \sqrt[3]{N}$, $x^3 = N$, $x = N/x^2$, $3x = 2x + N/x^2$, $x = (2x + N/x^2)/3$

19)

n	x_n	n	x_n	n	x_n
1	1	5	6^+	9	3.9148^+
2	20^+	6	4^+	10	3.91486764
3	13^+	7	4^+		
4	9^+	8	3.9^+		

20) $x = \sqrt[5]{N}$, $x^5 = N$, $x = N/x^4$, $5x = 4x + N/x^4$, $x = (4x + N/x^4)/5$

n	x_n	n	x_n	n	x_n
1	1	9	42^+	17	7^+
2	200.8	10	33^+	18	5^+
3	160^+	11	26^+	19	4^+
4	128^+	12	21^+	20	4^+
5	102^+	13	17^+	21	4^+
6	82^+	14	13^+	22	3.981^+
7	65^+	15	11^+	23	3.9810717^+
8	52^+	16	8^+	24	3.981071706

Solutions to Exercise Set 3.2

1)

x	y
5	4
1	2
3	3
2	2.5
2.5	2.75
2.25	2.625
2.375	2.6875
2.3125	2.65625
2.34375	2.671875
2.328^+	2.664^+
2.335^+	2.667^+
2.332^+	2.666^+
2.333^+	2.667^-
2.333^+	2.667^-

3) $\begin{cases} x + y = 5 & (\ell) \\ y = 2x - 3 & (m) \end{cases}$

Substituting (m) into (ℓ)

$$x + (2x-3) = 5$$

$$3x = 8$$

$$\begin{cases} x = 2\frac{2}{3} \\ y = 2\frac{1}{3} \end{cases}$$

4) Algorithm:

1. Set a = 0 (This is x_0
2. Let $a \leftarrow 2^{-a}$, display a (This is y_n.)
3. Let $a \leftarrow a^2$, display a (This is x_{n+1})
4. Stop when accuracy is achieved.
5. Go back to step 2.

x	y
0	.5
.25	$.84^+$
$.71^-$	$.61^+$
$.38^-$	$.77^+$
$.59^+$	$.66^+$
$.44^-$	$.74^-$
$.54^+$	$.69^-$
$.47^+$	$.72^+$
$.52^+$	$.70^-$
$.49^-$	$.71^+$
$.51^+$	$.70^+$
$.49^+$	$.71^+$
$.50^+$	$.70^+$
$.50^-$	$.71^-$
$.50^+$	$.71^-$

5) $x \to \frac{1}{2}$ $y = 2^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} = \sqrt{2}/2 = 1.414/2 = .707$

6) Same

7) Yes $\sqrt{2}/2 = 2^{-\frac{1}{2}} = \sqrt{2}/2$

8) $x = \frac{3y+3}{4}$

9) $y = \frac{2x+3}{3}$

10) Algorithm:

(1) Set a = 0 (This is x_1)

(2) Let a ← (2a+3)/3, display a. (This is y_n.)

(3) Let a ← (3a+3)/4, display a. (This is x_n.)

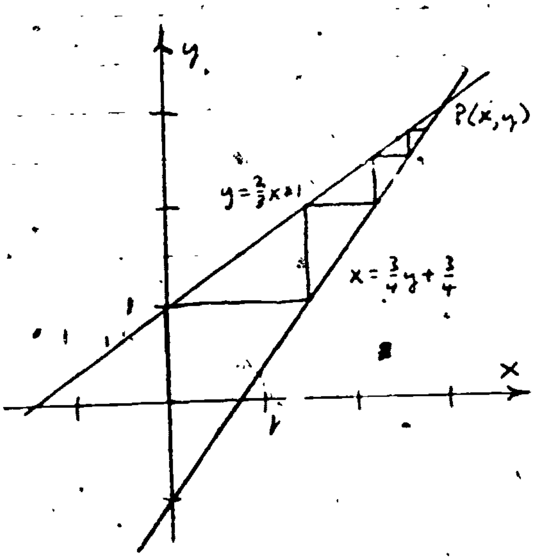
(4) Stop when accuracy is achieved.

(5) Go back to step 2.

11) x = 3, y = 3

12) Same.

13) See graph



14) $3y - 2x = 3$

$\underline{3y - 4x = -3}$

Subtracting: $2x = 6$ and $x = 3$, $3y - 2(3) = 3$ yields $y = 3$

15)

x_n	y_n
0	-3
8	13
-8	-19
24	45

16) Diverging means going away from or branching off, as opposed to converging which means going toward a target.

17) Resolve the two equations to reverse the roles of x and y.

$x = f(y)$ convert to $y = F(x)$

$y = g(x)$ convert to $x = G(y)$

Solve the new (equivalent) system.

18)

x	y
0	4
3	-2
-3	10
9	-14

Since this system is diverging we convert

$$\begin{cases} y = 4 - 2x \\ x = y - 1 \end{cases} \text{ to } \begin{cases} x = \frac{4-y}{2} \\ y = x + 1 \end{cases}$$

x	y
0	2
3	.5
1.5	1.25
2.25	.875
1.875	1.0625
2.0625	$.97^-$
1.97^-	1.02^-
2.02^-	$.99^+$
1.99^+	1.00^+
2.00^+	1.00^-
2.00^-	

$x = 2$, $y = 1$

19) $\begin{cases} 2x + y = 3 \\ 2x - y = 1 \end{cases} \equiv \begin{cases} y = 3 - 2x \\ x = \frac{y+1}{2} \end{cases}$

x	y
0	3
2	-1
0	3
2	-1
⋮	

This cycle repeats:

$x_{n+2} = x_n$ and $y_{n+2} = y_n$

20)

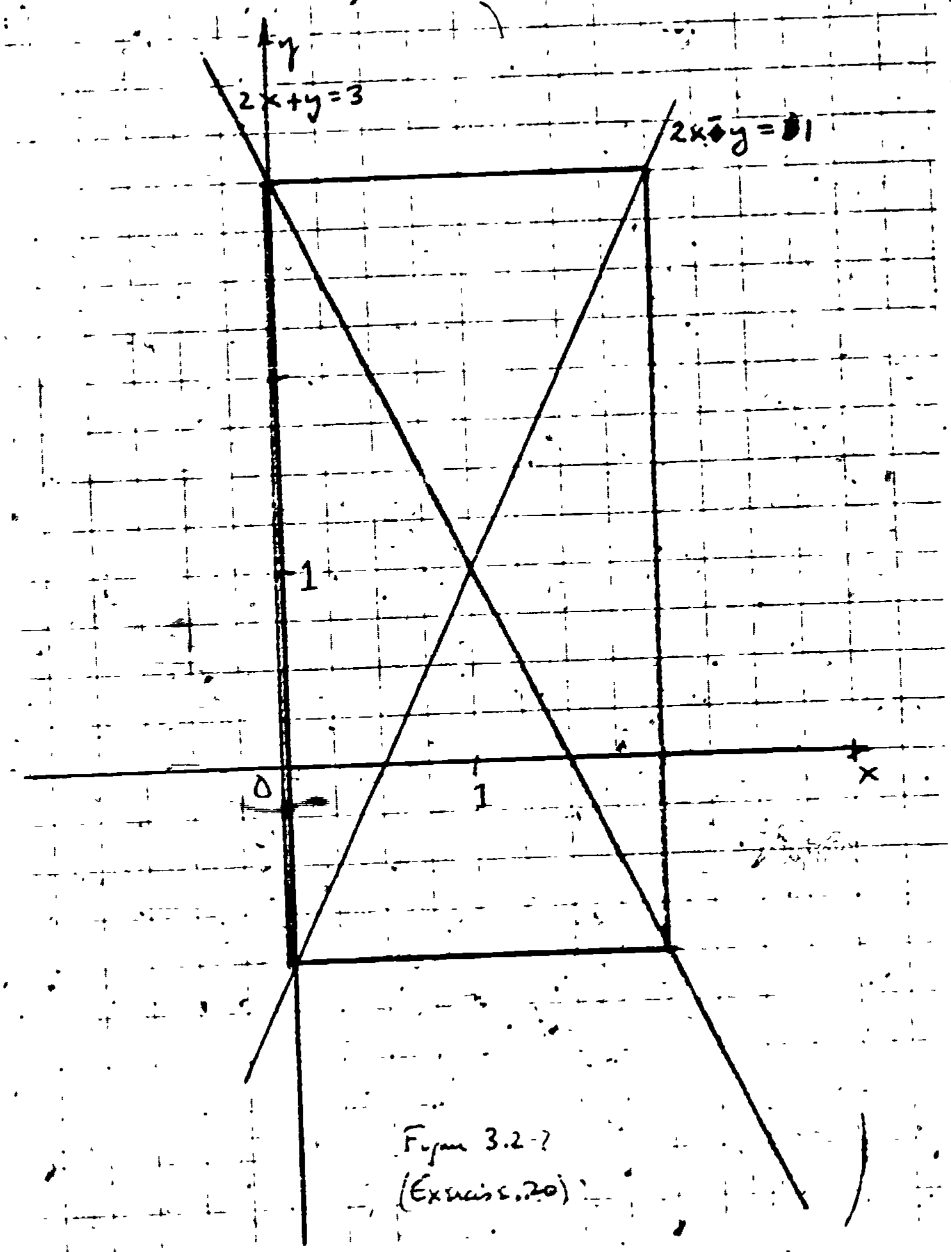


Figure 3.2-? (Exercise 20)

21) No. The route would be reversed.

22) $m(1) = -2$, $m(2) = 2$

23)

x	y
.5	-.75
$.31^+$	$-.90^+$
$.27^+$	$-.92^+$
$.27^-$	$-.93^-$
$.27^-$	$-.93^-$
.	.
.	.
.267949192	-.92810323

24) $$x = \frac{(x^2 - 1) + 2}{4} = \frac{x^2 + 1}{4}$$

25) $$x_{n+1} = (x_n^{\ 2} + 1)/4$$

26) It gives only the x-column of the table in exercise (23).

27) $x^2 - 4x + 1 = 0$

28) $(.267949192)^2 - 4(.267949192) + 1 \doteq 0$

Solutions to Exercise Set 3.3

1)

n	$t_{1,n}$	$t_{2,n}$	$t_{3,n}$	$t_{4,n}$	$t_{5,n}$	$t_{6,n}$
1	50	50	50	50	50	50
2	50	37.5	46.875	50	34.375	45.3125
3	46.89	32.03	44.34	45.31	30.66	43.75
4	44.34	29.83	43.40	43.75	29.33	43.18

2) $t_{1,n} = t_{3,n-1}$ $t_{4,n} = t_{6,n-1}$

They check by symmetry.

Note that t_2 and t_5 are getting closer. They should also move toward equality because of symmetry.

3) $t = \frac{100 + 0 + T + t}{4}$

$T = \frac{0 + t + T + t}{4}$

n	t	T
1	50	50
2	50	37.5
3	46.875	32.8125
8	42.99	28.70
12	42.8743	28.58

$t \doteq 43$

$T \doteq 29$

4) Notice that $t_1 = t_3$, $t_4 = t_6$ by symmetry. Thus we can work with t_1, t_2, t_4, and t_5

Recursion equations: $t_1 = \frac{100 + t_4 + t_2}{4}$

$$t_2 = \frac{2t_1 + t_5}{4}$$

$$t_4 = \frac{200 + t_1 + t_5}{4}$$

$$t_5 = \frac{100 + t_2 + 2t_4}{4}$$

Arbitrarily starting with t's = 50

n	$t_{1,n}$	$t_{2,n}$	$t_{4,n}$	$t_{5,n}$
1	50	50	50	50
2	50	37.5	75	71.875
3	53.125	44.53	81.25	76.76
8	58.35	49.04	84.45	79.49
12	58.38	49.07	84.47	79.50

5) Recursion equations:

$$t_1 = \frac{100 + t_2}{4}$$

$$t_2 = \frac{t_1 + t_3}{4}$$

$$t_3 = \frac{t_2}{4}$$

n	t_1	t_2	t_3
1	50	50	50
2	37.5	21.875	5.46875
3	30.46875	8.9844^-	2.2461^-
4	27.2461^-	7.3730^+	1.8433^-
5	26.8433^-	7.1716^+	1.7929^+
10	26.7857^+	7.1429^-	1.7857^+
20	26.7857^+	7.1429^-	1.7857^+

6) $p_2 = (0 + p_1 + p_5 + p_3)/4$

$p_3 = (1 + 1 + p_2 + p_6)/4$

$p_4 = (0 + p_1 + p_5 + p_7)/4$

$p_5 = (p_2 + p_4 + p_6 + p_8)/4$

$p_6 = (p_3 + p_5 + p_9 + 0)/4$

$p_7 = (0 + 0 + p_4 + p_8)/4$

$p_8 = (p_5 + p_7 + p_9 + 0)/4$

$p_9 = (p_6 + p_8 + 0 + 0)/4$

7)

n	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9
1	.5	.5	.5	.5	.5	.5	.5	.5	.5
2	.25	.313	.703	.313	.406	.402	.203	.277	.170
3	.156	.316	.680	.191	.297	.287	.117	.146	.108
⋮									
8	.068	.202	.601	.059	.130	.199	.029	.056	.064
⋮									
11	.063	.197	.599	.054	.126	.197	.027	.054	.063

8) BEST C_3 WORST C_7 (Note symmetries also.)

9) $p_1 = (1 + 1 + p_2 + 0)/4$

$p_2 = (1 + p_1 + 0 + p_3)/4$

$p_3 = (p_2 + 0 + p_4 + p_6)/4$

$p_4 = (p_2 + 0 + p_5 + p_7)/4$

$p_5 = (0 + 1 + p_4 + p_8)/4$

$p_6 = (1 + p_3 + p_7 + p_9)/4$

$p_7 = (p_4 + p_6 + p_8 + p_{10})/4$

$p_8 = (1 + p_5 + p_7 + p_{11})/4$

$p_9 = (1 + 1 + p_6 + p_{10})/4$

$p_{10} = (1 + p_7 + p_9 + p_{11})/4$

$p_{11} = (1 + 1 + p_8 + p_{10})/4$

n	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}
1	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5
2	.63	.53	.38	.38	.47	.60	.49	.62	.77	.69	.83
3	.63	.50	.37	.37	.50	.66	.58	.73	.84	.81	.88
10	.63	.52	.43	.45	.56	.77	.73	.80	.92	.89	.92
15	.63	.52	.43	.45	.56	.77	.73	.80	.92	.89	.92

10) various answers

11) C ← .75C + .2M

M ← .6M + 200

12)

n	G	C	M
1	300	300	400
2	255	305	440
3	220	317	464
10	117	384	499
20	101	399	500

13) a)

n	G	C	M
1	0	0	0
2	0	0	200
3	0	40	320
10	44	327	495
20	90	396	500

13) continued

b)

n	G	C	M
1	2000	0	0
2	1600	0	200
3	1280	40	320
10	312	327	495
20	119	396	500

14) (1) $G = .8G + .05C$

(2) $C = .75C + .2M$

(3) $M = .6M + 200$

from (3) $.4M = 200$ and $\underline{M = 500}$

substituting in (2): $C = .75C + 100$ and $.25C = 100$, $\underline{C = 400}$

substituting in (1): $G = .8G + 20$ and $.2G = 20$, $\underline{G = 100}$.

Answers in (12) and (13) are converging to these values.

Solutions to Exercise Set 3.4

Commentary: If you choose to have your students challenge each other with functions as suggested on page 3.1 - 2, you probably should place some restrictions on the functions allowed. In order not to avoid complex functions you may only wish to restrict the number of program steps to, say, eight. Students can still come up with functions very difficult to guess in this many steps.

This process is designed to show what the challenge of data structuring is. It is quite different from the more restricted challenge of exercise (25).

1) 2 2) 1 3) 3 4) 2

5) $f(n) = 2n^2 - 3n + 4$ 6) $g(n) = 3n + 1$

7) $h(n) = n^3 + n^2 - 3n + 3$ 8) $j(n) = -n^2 + 10n + 5$

9) It identifies the constant term immediately as $f(0)$, thus reducing the number of equations to process.

10) $f(n) = 3n^2 - 15n$ 11) $f(n) = -2n + 10$

(12 - 13)

n	f(n)	1	2
0	c		
		> a + b	
1	a+b+c		> 2a
		> 3a+b	
2	4a+2b+c		> 2a
		> 5a+b	
3	9a+3b+c		> 2a
		> 7a+b	
4	16a + 4b + c		

14) $\Delta^2 = 2a$ Since Δ^2 is constant, degree is 2.

15)

n	$f(n) = an^3 + bn^2 + cn + d$	Δ^1	Δ^2	Δ^3
0	d			
		a+b+c		
1	a+b+c+d		6a+2b	
		7a+3b+c		6a
2	8a+4b+2c+d		12a+2b	
		19a+5b+c		6a
3	27a+9b+3c+d		18a+2b	
		37a+7b+c		6a
4	64a+16b+5c+d		24a+2b	
		61a+9b+c		
5	125a+25b+5c+d			

16)

n	f(n)	Δ^1	Δ^2	Δ^3
0	1	1		
1	2		1	
		2		2
2	4	4	2	
3	8			
		8	4	4
4	16			

Differences are the same each time, that is $f(n) = \Delta^1 = \Delta^2 = \Delta^3 = \ldots$ and will never be constant.

(17-18)

n	log f(n)	1
0	0	
		.301
1	.301	
		.301
2	.602	
		.301
3	.903	
		.301
4	1.204	
		.301
5	1.505	

degree 1 - linear: $\log f(n) = an + b$

19) $y = an + b$

$0 = a \cdot 0 + b \Rightarrow b = 0$

$.301 = a \cdot 1 + b \Rightarrow a = .301$

$\therefore y = .301n$

20) $\log f(n) = .301n$

$10^{\log f(n)} = 10^{.301n}$

$f(n) = (10^{.301})^n = 2^n$

$f(n) = 2^n$

21) 3 moves

22) 8 moves

(23 - 24)

n	f(n)	Δ	Δ^2
1	3		
		5	
2	8		2
		7	
3	15		2
		9	
4	24		

$f(n) = an^2 + bn + c$

$3 = a+b+c$
$8 = 4a+2b+c$
$15 = 9a+3b+c$

$5 = 3a+b$
$7 = 5a+b$

$2 = 2a \Rightarrow a=1$

$a=1, b=2, c=0$

$f(n) = n^2 + 2n$

(23 - 24 continued)

① ② ③ ④ ⑤ ⑥ ⑦

3 coins each 7 spaces

3 → 4	1 → 2	4 → 6
5 → 3	3 → 1	2 → 4
6 → 5	5 → 3	3 → 2
4 → 6	7 → 5	5 → 3
2 → 4	6 → 7	4 → 5

The answer is 15 moves.

4 coins 9 spaces

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨

4 → 5	4 → 2	3 → 5	7 → 8
6 → 4	6 → 4	1 → 3	5 → 7
7 → 6	8 → 6	2 → 1	3 → 5
5 → 7	9 → 8	4 → 2	4 → 3
3 → 5	7 → 9	6 → 4	6 → 4
2 → 3	5 → 7	8 → 6	5 → 6

The answer is 24 moves.

Teacher's Manual 3.5

Induction requires much practice and many worked out examples for students to follow the form of the proof. It is especially difficult for them to understand that they are given S_k in PFI Part (2). We have worked out some additional examples for your use in class demonstrations or as additional exercises.

Exercise Set Solutions 3.5

1) False. Fails for n = 2: $2(2)^2-1 = 9 = 3 \cdot 3$.

2) False. Fails for n = 2: $2^2 \not> 2^3$

3) True. (Proof by PFI: Part (1): 3 divides $2 \cdot 4^1 + 1 = 9$.
Part (2) Given that 3 divides $2 \cdot 4^k + 1$, we must show that 3 divides $2 \cdot 4^{k+1} + 1$. First examine $2 \cdot 4^{k+1}$: it equals $2 \cdot 4^k \cdot 4 = 2 \cdot 4^k (3+1) = 2 \cdot 4^k \cdot 3 + 2 \cdot 4^k$, the first term of which is divisible by 3 since it has 3 as a factor. Our proof then goes as follows:

3 divides $2 \cdot 4^k + 1$ — given

3 divides $3 \cdot 2 \cdot 4^k$ — since 3 is a factor

3 divides $\underline{3 \cdot 2 \cdot 4^k} + \underline{2 \cdot 4^k + 1}$ — since 3 divides the underscored parts

3 divides $2 \cdot 4^k (3+1) + 1$ — factoring

3 divides $2 \cdot 4^{k+1} + 1$ — $4^k \cdot 4 = 4^{k+1}$

4) False. For n = 40: $40^2 + 40 + 41 = 40(40+1) + 41 = 40 \cdot 41 + 41 = 41^2 = 1681$

5) True. (Proved in exercises 8 - 12).

6) True. (Proved in exercises 13 - 16).

7) $n = 10$. True

8) True

9) n^2

10) $1 = 1^2$

11)

$1 + 3 + 5 + \ldots + (2k-1) = k^2$	given
$2(k+1) - 1 = 2(k+1) = 1$	identity
$1 + 3 + 5 + \ldots + (2k-1) + 2(k+1)-1) = k^2 + 2(k+1) - 1$	adding
$= k^2 + 2k + 1$	
$= (k+1)^2$	

12) Yes. By PFI.

13) When $n = 1$ there are no chords, and $0(0-1)/2 = 0$.

14) $(k+1)\left[(k+1) -1\right]/2$ or $k(k+1)/2$

15) k

16)

$S_k = k(k-1)/2$	given
$S_{k+1} = k(k-1)/2 + k$	by exercise (15)

$$= \frac{k^2 - k}{2} + \frac{2k}{2}$$

$$= \frac{k^2 + k}{2}$$

$$= k(k+1)/2$$

17) PFI Part (1) $1 = 2^1 - 1$

PFI Part (2):

$$1 + 2 + 2^2 + \ldots + 2^{k-1} = 2^k - 1$$

$$2^k = 2^k$$

$$1 + 2 + 2^2 + \ldots + 2^{k-1} + 2^{(k+1)-1} = 2^k + 2^k - 1$$

$$= 2 \cdot 2^k - 1$$

$$= 2^{k+1} - 1$$

18) Data

n	t_n	Δ^1	Δ^2
1	1		
		2	
2	3		1
		3	
3	6		1
		4	
4	10		1
		5	
5	15		

$$t_n = a n^2 + bn + c$$

$$1 = a + b + c$$
$$3 = 4a + 2b + c$$
$$6 = 9a + 3b + c$$

$$2 = 3a + b$$
$$3 = 5a + b$$
$$1 = 2a$$

$$a = \tfrac{1}{2},\ b = \tfrac{1}{2},\ c = 0$$

$$t_n = \tfrac{1}{2}n^2 + \tfrac{1}{2}n = n(n+1)/2$$

19) PFI Part (1): $t_1 = 1 = 1(1+1)/2$

PFI Part (2):

$$t_k = k(k+1)/2 \qquad \text{given}$$

$$t_{k+1} = k(k+1)/2 + k+1 \qquad \text{k+1 balls added to form the } (k+1)^{st} \text{ row.}$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} = (k+1)\left[(k+1)+1\right]/2$$

20) Various forms of scientific induction.

21) The argument from k to k+1 fails for going from k=1 to k=2 when there is no overlap.

An additional problem you might like to work out with your students is: 16 divides $5^n - 4n - 1$.

Solutions to Exercise Set 3.6

1)

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1

2) $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

3) $a^{10} + 10a^9b + 45a^8b^2 + 120a^7b^3 + 210a^6b^4 + 252a^5b^5 + 210c^4b^6$
$+ 120a^3b^7 + 45a^2b^8 + 10ab^9 + b^{10}$

4) $64x^6 + 576x^5y + 2160x^4y^2 + 4320x^3y^3 + 4860x^2y^4 + 2916xy^5 + 729y^6$

5) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$

6) $256x^8 + 512x^7 + 448x^6 + 224x^5 + 70x^4 + 14x^3 + \frac{7}{4}x^2 + \frac{1}{8}x + \frac{1}{256}$

7) $84\, a^6b^3$

8) $21a^2b^5$

9) a^{100}

10) $500a^{499}b$

11) $15360\, x^9y$

12) $-105x^4y^3$

13) $-30x^{29}y$

14) $-30xy^{29}$

15) 8615125

16) $200^3 + 3\cdot 200^2\cdot 5 + 3\cdot 200\cdot 5^2 + 5^3 = 8\,000\,000 + 600\,000 + 15\,000$
$+ 125 = 8\,615\,125$

17) $1^6 + 6\cdot 1^5(.04) + 15\cdot 1^4(.04)^2 - 1 + .24 + .0240$

18) $1 + 8(.002) = 1.016$

19) $20^4 + 4 \cdot 20^3(.03) = 160\ 000 + 960 = 160\ 960$

20) $160\ 960 + 6 \cdot 20^2(.03)^2 + 4 \cdot 20(.03)^4 = 160960 + 2.16 + .00216$
$+ .00000081 = 160\ 962.16210081$ Error is 2.16210081

Solutions to Chapter 3 TEST

1)

n	x_n	y_n
1	0	1
2	1	.5
3	.25	.8409
4	.7071	.6125
5	.3752	.7710
10	.5211	.6968
15	.4967	.7087
20	.5005	.7068
25	.4999	.7071

2)

$$p_1 = \frac{1 + 1 + p_2 + p_3}{4}$$

$$p_2 = \frac{p_1 + 1 + p_4}{3}$$

$$p_3 = \frac{p_1 + p_4}{3}$$

$$p_4 = \frac{p_3 + p_2 + p_5}{4}$$

$$p_5 = \frac{p_4}{2}$$

$p_1 = .76$

$p_2 = .69$

$p_3 = .35$

$p_4 = .30$

$p_5 = .15$

3) $R_1 \leftarrow M$

$R_2 \leftarrow C$

$R_3 \leftarrow G$

$R_4 \leftarrow N$

N	M	C	G
0	800	400	200
1	680	436	182
2	608	449	168
3	565	449	157
4	539	445	148
5	523	438	140
10	502	412	116
20	500	401	102
29	500	400	100
30	500	400	100

4) $f(n) = n^3 - 2n^2 - 3n + 4$

5) for n = 1 $\quad 1 = \frac{1((3 \cdot 1) - 1)}{2}$

$1 = 1$

Given: $1 + 4 + 7 + \dots + (3k - 2) = \frac{k(3k-1)}{2}$

$1 + 4 + 7 + \dots + (3k-2) + 3(k+1) - 2 =$

$\frac{k(3k-1)}{2} + \frac{3k + 1}{1} =$

$\frac{3k^2 - k + 6k + 2}{2} =$

$\frac{3k^2 + 5k + 2}{2} =$

$\frac{(k+1)(3k+2)}{2} =$

thus $1 + 4 + 7 + \dots + (3k-2) + 3(k+1) - 2 =$

$\frac{(k+1)(3k+2)}{2}$

6) $81 x^8 - 54 x^6 y + \frac{27}{2} x^4 y^2 - \frac{3}{2} x^2 y^3 + \frac{1}{16} y^4$

7) $-448 x^3 y^{10}$

Solutions to Exercise Set 4.1

1) Yes

2) 12 · 12 = 144

3) 72

4) 20

Solutions to Exercise Set 4.2

1) $\frac{1000}{17,576,000} \doteq .0001$

2) $\frac{1}{181,440} \doteq .0000055$

3) $\frac{1}{5040} \doteq .0002$

4) $\frac{1}{8} = .125$

5) $\frac{8}{27} \doteq .2963$

6) a) $\frac{20}{52} \doteq .3846$

b) $\frac{32}{52} \doteq .6154$

c) If p = probability of event occurring

and p = probability of event not occurring

Conculsions p = 1 - p

10) a) odd, even ⟷ heads, tails

b) pairs of digits across a row using only the digits 1 through 6 inclusive.

12) Depending on book size. First three or four digits in a column for page number. Next digit for column or page. Next three digits for number of names down the column.

13) Each eligible individual in a selective service district is assigned a four digit number 0001 ⟷ 9999. Numbers are then selected from a random digit table.

14) 23

Solutions to Exercise Set 4.3

1) $N \to R_1$

$R \to R_2$

$P = 1 \to R_3$

01)	RCL 2	10)	GTO 13
02)	CHS	11)	RCL 3
03)	+	12)	R/S
04)	RCL 1	13)	STO X 3
05)	+	14)	1
06)	STO 4	15)	STO + 4
07)	RCL 1	16)	RCL 4
08)	x >< y	17)	GTO 07
09)	x ≤ y ?		

2) a) $P(8,8) = 40,320$

b) $P(8,5) = 6720$

c) $P(7,7) = 5040$

d) $7 \cdot P(7,7) = 35280$

e) $P(9,9) = 362,880$

3) a) $P(10,3) \cdot 10^4 = 7,200,000$

b) $\frac{1}{5\cdot4\cdot3\cdot5^4} = \frac{1}{37500} \doteq .0052$

4) a) $P(n,n) = \frac{n!}{(n-n)!} = \frac{n!}{0!}$

b) $0! = 1$

5) $$\frac{1}{P(n,1)} + \frac{1}{P(n,2)} = \frac{1}{\frac{n!}{(n-1)!}} + \frac{1}{\frac{n!}{(n-2)!}}$$

$$= \frac{1}{\frac{n(n-1)!}{(n-1)!}} + \frac{1}{\frac{n(n-1)(n-2)!}{(n-2)!}}$$

$$= \frac{1}{n} + \frac{1}{n(n-1)}$$

$$= \frac{n-1}{n(n-1)} + \frac{1}{n(n-1)}$$

5) continued:

$$= \frac{n}{n(n-1)}$$

$$= \frac{1}{n-1} \quad \text{Q.E.D.}$$

6) a) $P(9,9) = 362,880$

b) Consider hy as one letter $P(8,8) = 80,640$. Divide by 2 since half of these words have the y before the h. 40,320

c) $362,880 - 80,640 = 282,240$

7) $\frac{1}{17576} \doteq .0001$

8) a) $\frac{P(8,7)}{2!\cdot 2!\cdot 2!\cdot 2!} = \frac{40320}{16} = 2520$

b) $\frac{4 \cdot P(7,6)}{16} = 1260$

c) .5

d) $\frac{P(6,5)}{2!\cdot 2!\cdot 2!} = \frac{720}{8} = 90$

$\frac{90}{2520} \doteq .0357$

9) $P(n, r+1) = \frac{n!}{(n-r-1)!} = \frac{n!(n-r)}{(n-r-1)!(n-r)} = \frac{n!(n-r)}{(n-r)!}$

$$= \frac{n!}{(n-r)!} \cdot (n-r)$$

$$= P(n,r) \cdot (n-r)$$

11)

r	P(n,r)
0	1
1	9
2	72
3	504
4	3024
5	15120

r	P(n,r)
6	60480
7	181440
8	362880
9	362880

Solutions to Exercise Set 4.4

1) $N \rightarrow R_1$

$R \rightarrow R_2$

01	1	09	STO X 1
02	-	10	x >< y
03	x = 0 ?	11	GTO 01
04	GTO 12	12	RCL 1
05	STO X 2	13	RCL 2
06	x >< y	14	÷
07	1		
08	-		

2) 161,700

3) 161,700

4) 2,598,960

5) 2,598,960

6) 3003

7) 3003

8) 7

9) 1

10) 6

11) 12

12) 8

13) n - x

14) C(23,3) = 1771

15) C(A,3) = 364

16) C(9,3) = 84

17) C(7,3); P(7,3); 7! + 3!; 10!; $\frac{20!}{2}$

18) C(13,1) · C(4,4) · C(48,1) = 624 $\frac{624}{2598960}$ = .0002

19) $C(13,1) \cdot C(4,3) \cdot C(12,1) \cdot C(4,2) = 3744 \quad \frac{3744}{2598960} \doteq .0014$

20) $C(4,1) \cdot C(13,5) = 5148$

$5148 - 40 = 5108 \quad \frac{5108}{2598960} \doteq .0020$

21) "4 of a kind" - lowest probability

"full house"

"flush" - highest probability

Solutions to Exercise Set 4.5

fix 0

1) N

01	f Reg	09	R/S
02	1	10	Σ +
03	STO 0	11	STO ÷ 0
04	+	12	-
05	ENTER	13	STO X 0
06	ENTER	14	CLX
07	ENTER	15	GTO 08
08	RCL0		

2) $(\frac{a}{2})^8 + 8(\frac{a}{2})^7 + 28(\frac{a}{2})^6 + 56(\frac{a}{2})^5 + 70(\frac{a}{2})^4 + 56(\frac{a}{2})^3 + 28(\frac{a}{2})^2 + 8(\frac{a}{2}) + 1$

$\frac{a^8}{256} + \frac{a^7}{16} + \frac{7a^6}{16} + \frac{7}{4}a^5 + \frac{35a^4}{8} + 7a^3 + 7a^2 + 4a + 1$

3) $1 + 8(\frac{x^2}{2}) + 28(\frac{x^2}{2})^2 + 56(\frac{x^2}{2})^3 + 70(\frac{x^2}{2})^4 + 56(\frac{x^2}{2})^5 + 28(\frac{x^2}{2})^6$

$+ 8(\frac{x^2}{2})^7 + (\frac{x^2}{2})^8$

$1 + 4x^2 + 7x^4 + \frac{35}{8}x^8 + \frac{7}{4}x^{10} + \frac{7x^{12}}{16} + \frac{x^{14}}{16} + \frac{x^{16}}{256}$

4) $(x^2)^9 + 9(x^2)^8(-x^3) + 36(x^2)^7(-x^3)^2 + 84(x^2)^6(-x^3)^3 + 126(x^2)^5(-x^3)^4$

$+ 126(x^2)^4(-x^3)^5 + 84(x^2)^3(-x^3)^6 + 36(x^2)^2(-x^3)^7 + 9(x^2)(-x^3)^8$

$+ (-x^3)^9$

$x^{18} - 9x^{19} + 36x^{20} - 84x^{21} + 126x^{22} - 126x^{23} + 84x^{24} - 36x^{25} +$

$+ 9x^{26} - x^{27}$

5) $a^9 + 9(a)^8(-ax) + 36(a)^7(-ax)^2 + 84(a)^6(-ax)^3 + 126(a)^5(-ax)^4 +$

$126(a)^4(-ax)^5 + 84(a)^3(-ax)^6 + 36(a)^2(-ax)^7 + 9(a)(-ax)^8 +$

$+(-ax)^9$

$a^9 - 9a^9x + 36a^9x^2 - 84a^9x^3 + 126a^9x^4 - 126a^9x^4 - 126a^9x^5 +$

$+ 84a^9x^6 - 36a^9x^7 + 9a^9x^8 - a^9x^9$

6) $(3c)^7 + 7(3c)^6(6) + 21(3c)^4(6)^3 + 35(3c)^3(6)^4 + 21(3c)^2(6)^5 +$
$+ 7(3c)(6)^6 + 6^7$

$2187c^7 + 30618c^6 + 183,708c^5 + 612,360c^4 + 1,224,720c^3 +$

$1,469,664c^2 + 979776c + 279936$

7) $(2)^7 + 7(2)^6(4m) + 21(2)^5(4m)^2 + 35(2)^4(4m)^3 + 35(2)^3(4m)^4$
$+ 21(2)^2(4m)^5 + 7(2)(4m)^6 + (4m)^7$

$128 + 1792m + 10752m^2 + 35840m^3 + 71680m^4 + 86016m^5 +$
$57344m^6 + 16384m^7$

8) $$C(n,r-1) + C(n,r) = \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!(r)!}$$

$$= \frac{r \cdot n!}{r(n-r+1)!(r-1)!} + \frac{(n-r+1)(n!)}{(n-r+1)(n-r)!(r)!}$$

$$= \frac{(r+n-r+1)n!}{(n-r+1)!\ r!}$$

$$= \frac{n!\ (n+1)}{(n+1-r)!\ r!}$$

$$= \frac{(n+1)!}{(n+1-r)!r!}$$

$$= C(n+1,r)$$ Q.E.D.

Solutions to Exercise Set 4.6

1) $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} \doteq .8333$

2) $3^0 + 3^2 + 3^4 + 3^6 + 3^8 = 7381$

3) $1^1 + 2^2 + 3^3 + 4^4 + 5^5 = 3395$

4) $\frac{2^0}{1} + \frac{2^1}{2} + \frac{2^2}{3} + \frac{2^3}{4} + \frac{2^4}{5} + \frac{2^5}{6} + \frac{2^6}{7} + \frac{2^7}{8} + \frac{2^8}{9} + \frac{2^9}{10} + \frac{2^{10}}{11} \doteq 212.7449$

5) 500

6) $\frac{3}{1} + \frac{3}{2} + \frac{3}{3} + \frac{3}{4} + \frac{3}{5} + \frac{3}{6} + \frac{3}{7} + \frac{3}{8} + \frac{3}{9} + \frac{3}{10} \doteq 8.7869$

7) $(12 - 10 + 1) + (27-15+1)+(48-20+1)+(75-25+1)+(108-30+1) = 175$

8) $2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10} = 2047$

9) $>$

10) $>$

11) $>$

12) $<$

13) $\sum_{k=1}^{5} (\frac{1}{2})^k$

14) $\sum_{k=1}^{4} (\frac{3}{5})^k$

15) $\sum_{k=0}^{7} (1+2k)$

16) $\sum_{k=1}^{4} \frac{k}{1+5k}$

Solutions to Chapter 4 Test

1) 1,073,741,824

2) .00000013

3) 4989600

4) 670870.5882

5) 2598960

6) $x = n-y$

7) $\frac{1}{r!}$

8) $\frac{1}{n-1}$

9) $1689600y^2b^{10}$

10) 4

11) 10

12) 5.3750

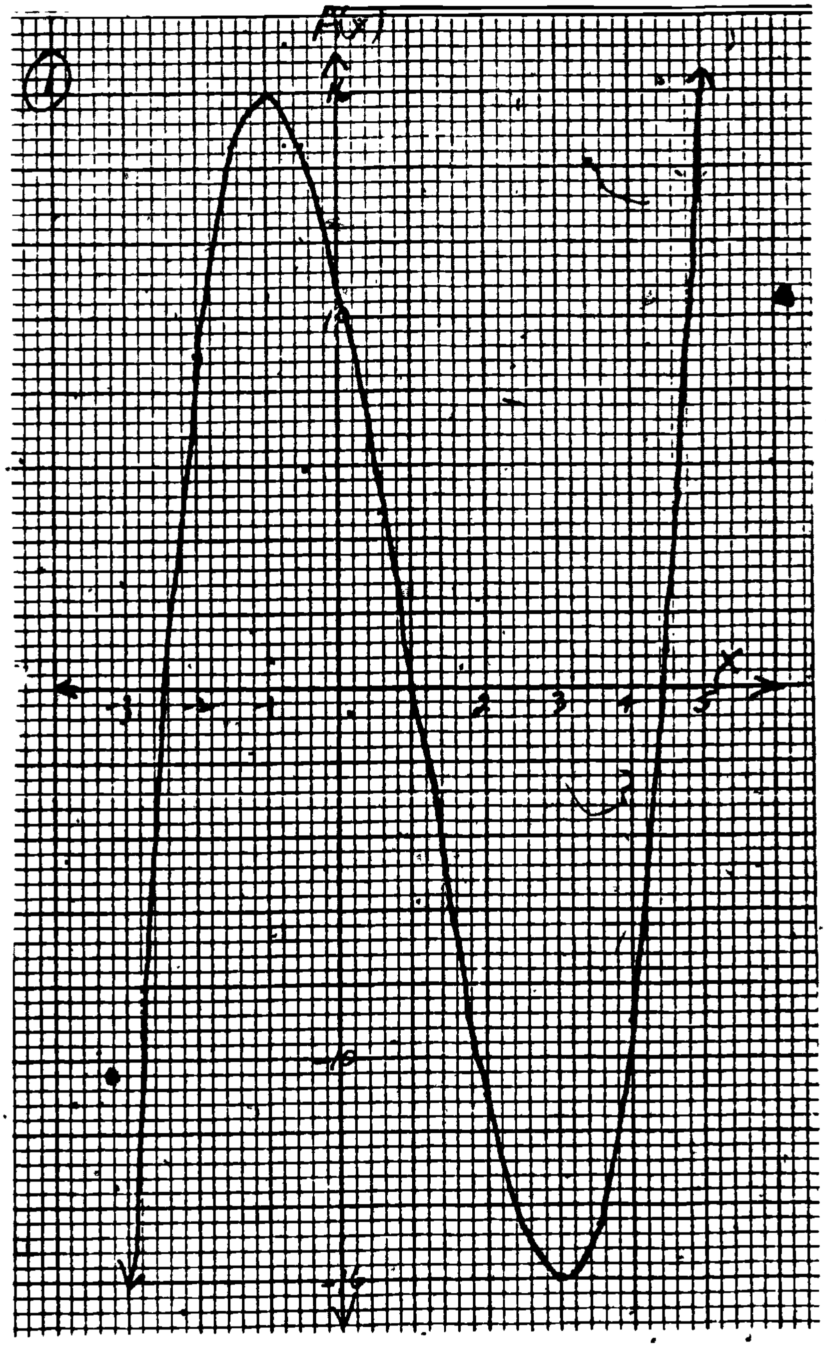
13) $\sum_{k=0}^{n} C(n,k)a^{n-k}b^{k}$

14) $\frac{11}{12} \cdot \frac{10}{12} \doteq .7639$

Exercise Set 5.1

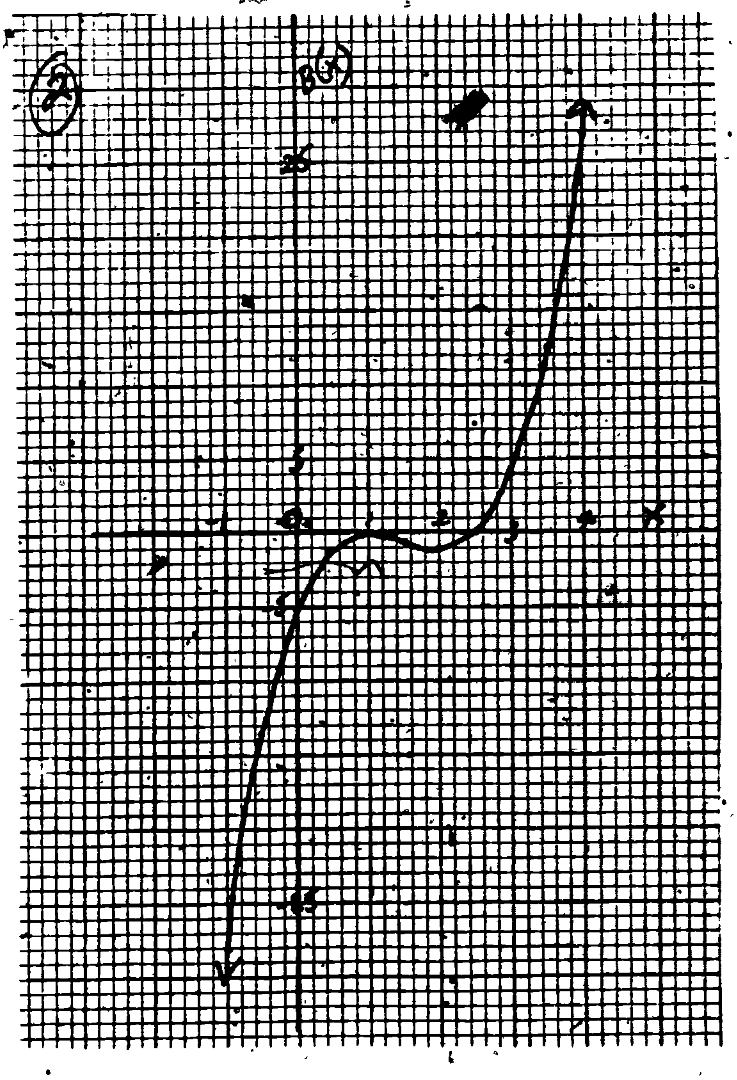
1) $A(x) = x[x(x-3) - 9] + 11$

x	A(x)
-3	-16
-2.5	-.9
-2	9
-1.5	14.4
-1	16
-.5	14.6
0	11
.5	5.9
1	0
1.5	-5.9
2	-11
2.5	-14.6
3	-16
3.5	-14.4
4	-9
4.5	.9
5	16



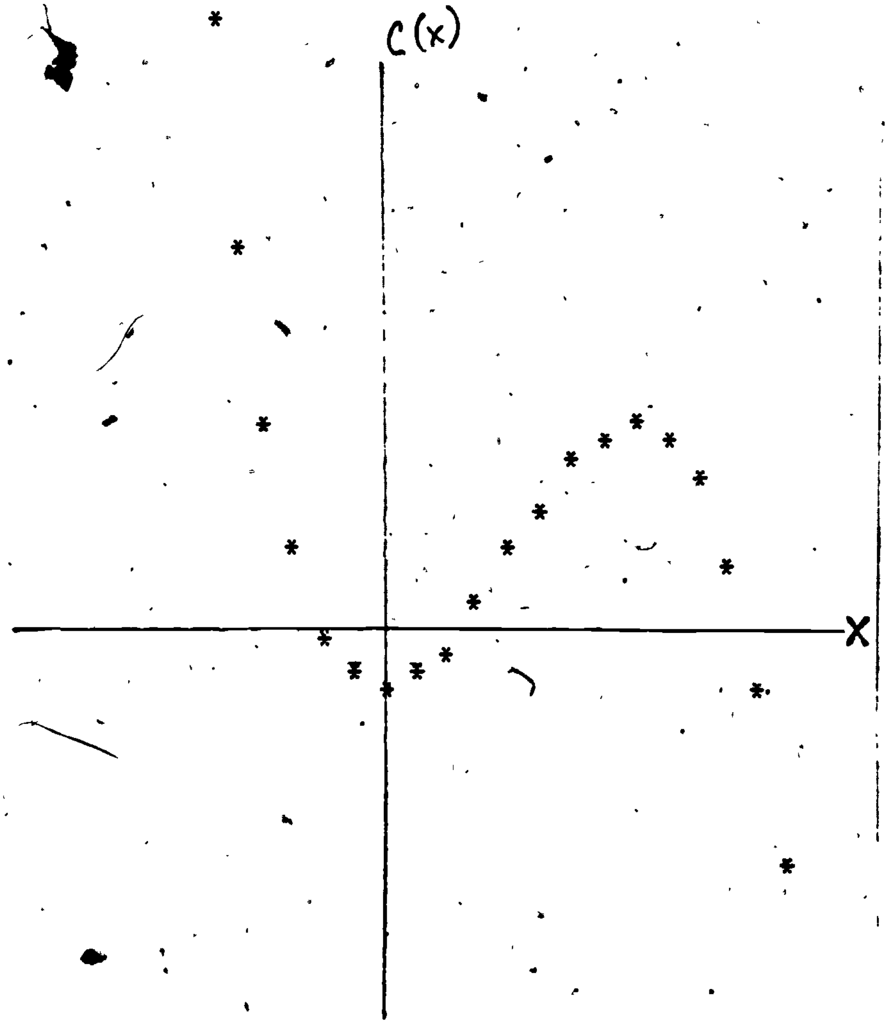
2) $B(x) = x\,[x\,(2x-9) + 12]\ -5$

x	B(x)
-1	-28
-5	-13.5
0	-5
.5	-1
1	0
1.5	-.5
2	-1
2.5	0
3	4
3.5	12.5
4	27



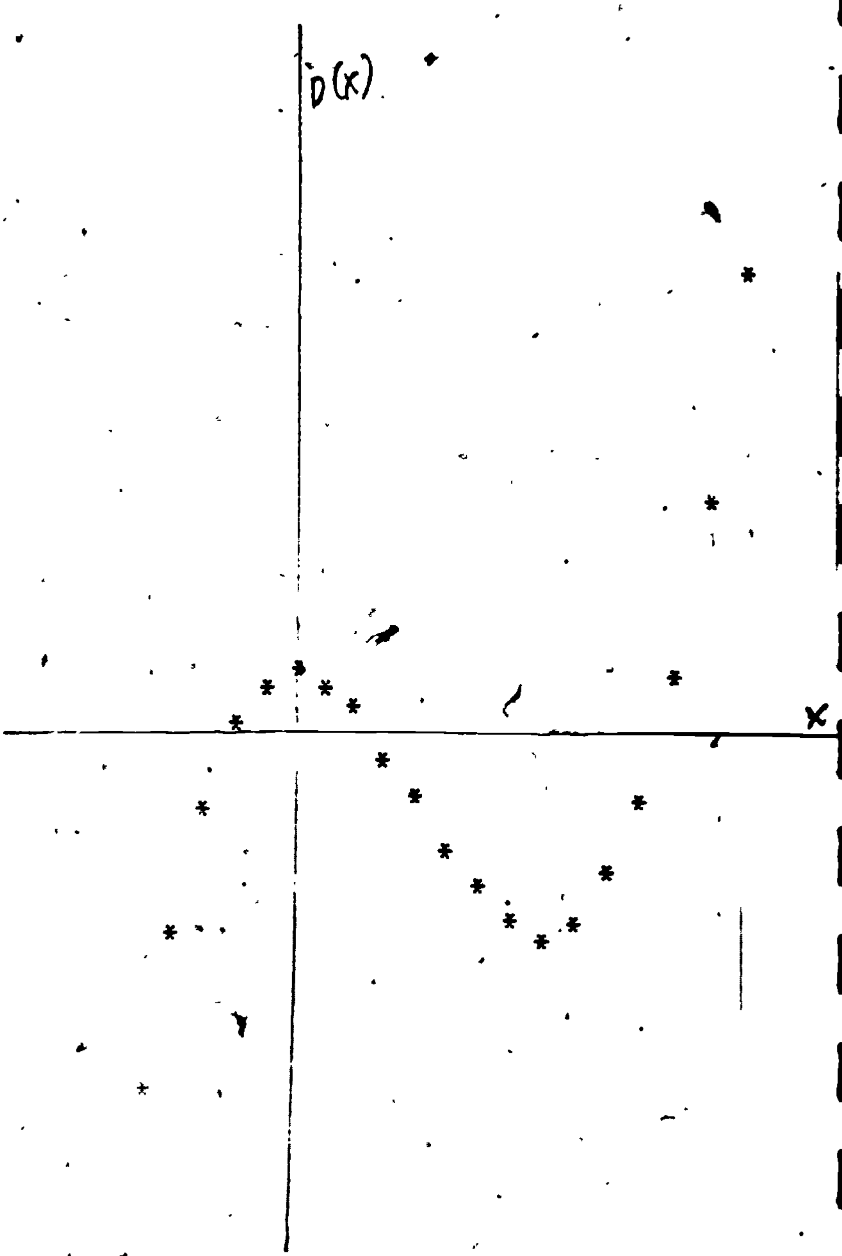
3) $C(x) = x[x(-x+3)] - 1$

x	C(x)
-1.5	9.1
-1.25	5.6
-1	3
-.75	1.1
-.5	-.1
-.25	-.8
.0	-1
.25	-.8
.5	-.4
.75	.3
1	1
1.25	1.7
1.5	2.4
1.75	2.8
2	3
2.25	2.8
2.5	2.1
2.75	.9
3	-1
3.25	-3.6
3.5	-7.1



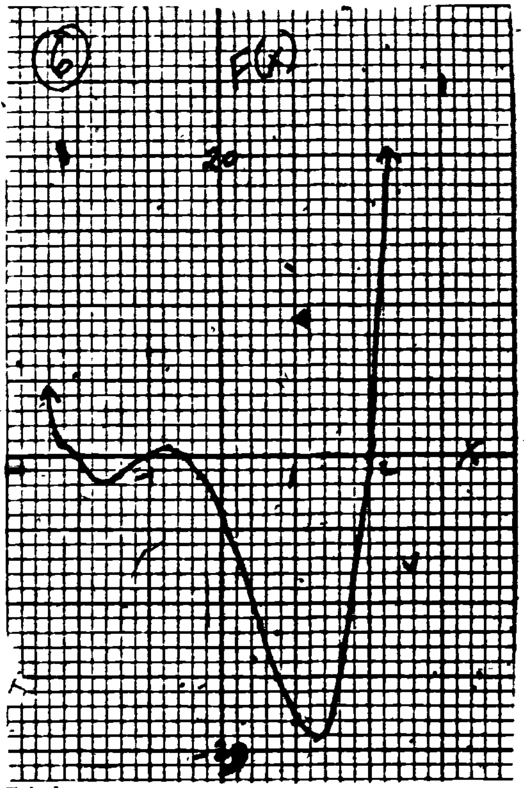
4) $D(x) = x\,[x\,(x-3)] + 1$

x	D(x)
-1.5	-9.1
-1.25	-5.6
-1	-3
-.75	-1.1
-.5	.1
-.25	.8
0	1
.25	.8
.5	.4
.75	-.3
1	-1
1.25	-1.7
1.5	-2.4
1.75	-2.8
2	-3
2.25	-2.8
2.5	-2.1
2.75	-.9
3	1
3.25	3.6
3.5	7.1



3) $E(x) = x\,[x(x(4x + 2) - 19) - 11] + 6$

x	D(x)
-2.25	14.3
-2	0
-1.75	-6.1
-1.5	-6.8
-1.25	[illegible] 1
-1	0
-.75	4.0
-.5	6.8
-.25	7.5
0	6
.25	2.1
.5	-3.8
.75	-10.8
1	-18
1.25	-23.8
1.5	-26.3
1.75	-23.2
2	-12
2.25	10.4



6) F(x) = x (x (x (2x + 3) -7) -12) -4

x	F(x)
-2.25	1
-2	0
-1.75	-1.8
-1.5	-1.7
-1.25	-.9
-1	0
-.75	.4
-.5	0
-.25	-1.5
0	-4

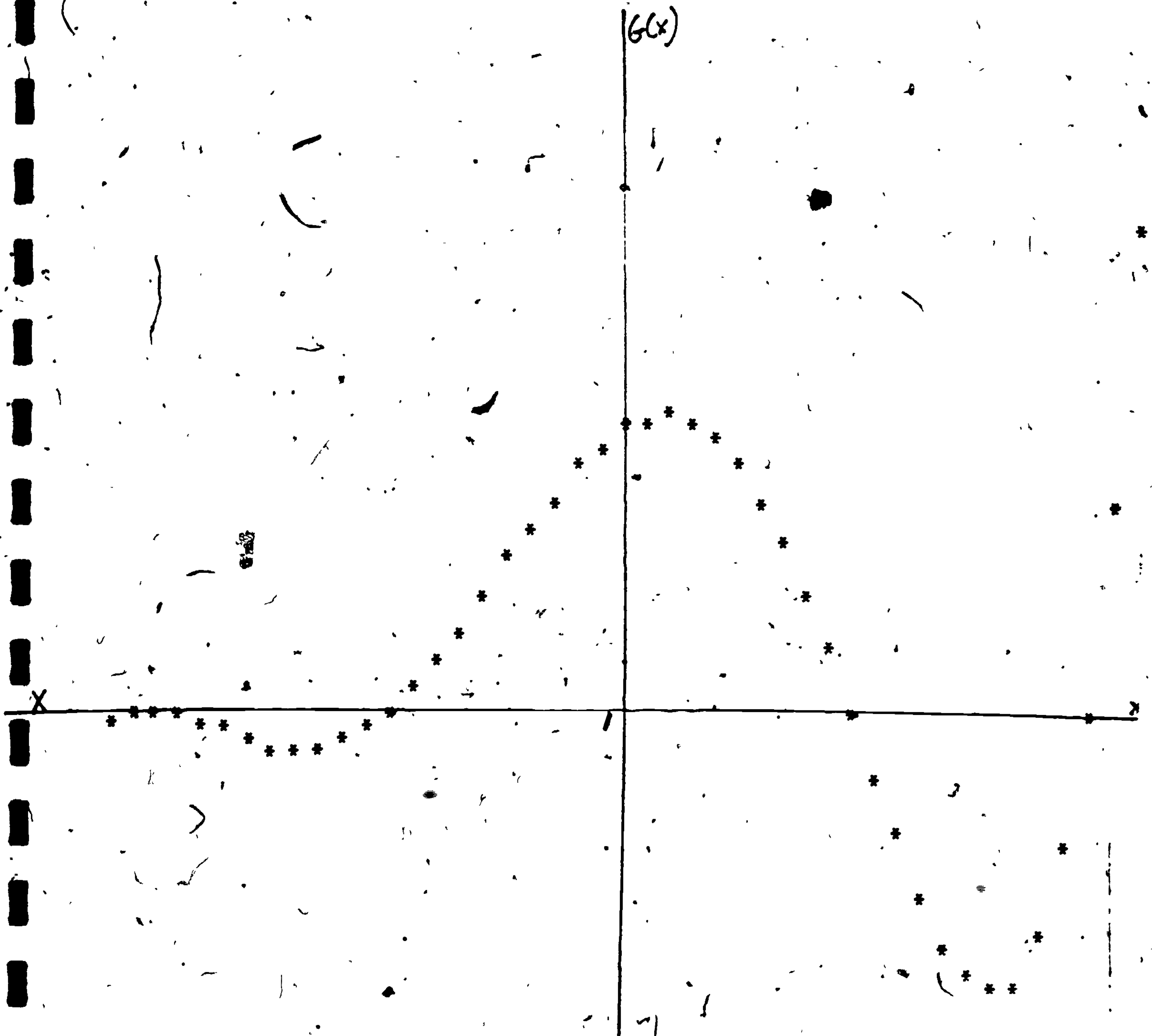
x	F(x)
.25	-7.4
.5	-11.3
.75	-15.0
1	-18
1.25	-19.2
1.5	-17.5
1.75	-11.6
2	0
2.25	19.0

7) G(x) = x (x (x (x (x + 2) -5) -10) +4) + 8

x	G(x)
-2.2	-.6
-2.1	-.1
-2	0
-1.9	-.1
-1.8	-.3
-1.7	-.6
-1.6	-.9
-1.5	-1.1
-1.4	-1.2
-1.3	-1.1
-1.2	-.9
-1.1	-.5
-1.0	0
-.9	.7
-.8	1.5
-.7	2.3
-.6	3.3
-.5	4.2
-.4	5.2
-.3	6.0
-.2	6.8
-.1	7.5
0	8

x	G(x)
.1	8.3
.2	8.4
.3	8.2
.4	7.7
.5	7.0
.6	6.1
.7	4.8
.8	3.4
.9	1.8
1	0
1.1	-1.8
1.2	-3.6
1.3	-5.3
1.4	-6.7
1.5	-7.7
1.6	-8.1
1.7	-7.8
1.8	-6.5
1.9	-4
2	0
2.1	5.7
2.2	13.5

7)



Solutions to Exercise Set 5.2

1) a) (-1, 0)

   b) -4/3 or $-1.\overline{3}$

2) a) (2, 0)

   b) (0, -4)

3) a) Relative maximum (-1, 16)
   Relative minimum (3, -16)

   b) 1, 4.5, -2.5

4) a) Relative maximum (1, 0)
   Relative minimum (2, -1)

   b) 1, 1, 2.5

5) a) maximum (2, 3) minimum (0, -1) for C(x)

   b) maximum (0, 1) minimum (2, -3) for D(x)

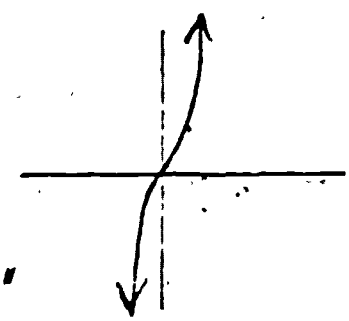
   c) If two functions are opposites such as f(x) and g(x), f(x) = -g(x). If f(a) = maximum when x = a, then g(a) = minimum when x = a. Similarly if f(b) = minimum when x = b, then g(b) = maximum.

6) -2.0, -1.0, .3, 2.2

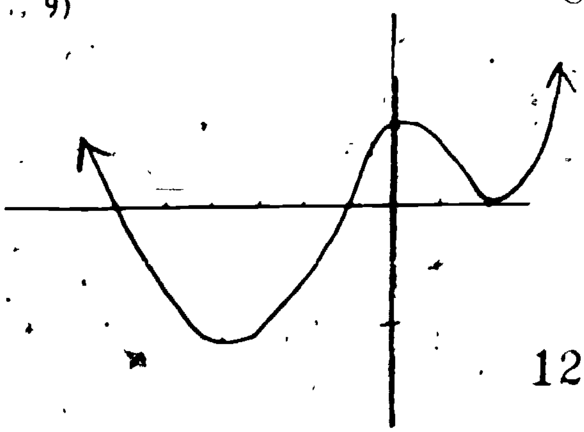
   $F(x) = 2(x + \frac{1}{2})(x + 1)(x + 2)(x - 2)$

7) $G(x) = (x + 2)(x + 1)(x - 1)(x - 2)^2$

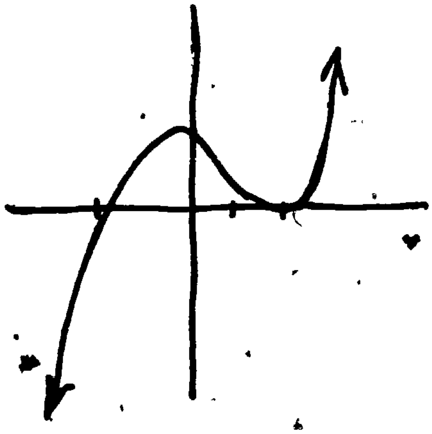
8) $y_1 = 2x(x^2 + 9) = 2x(x + 3i)(x - 3i)$



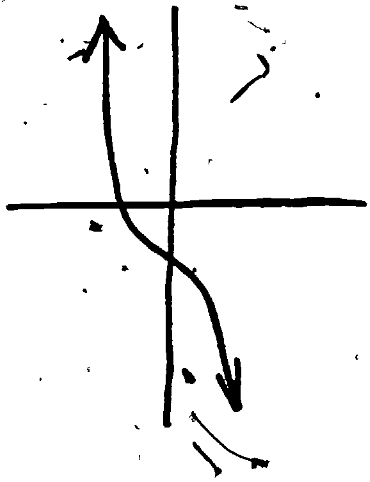
9)



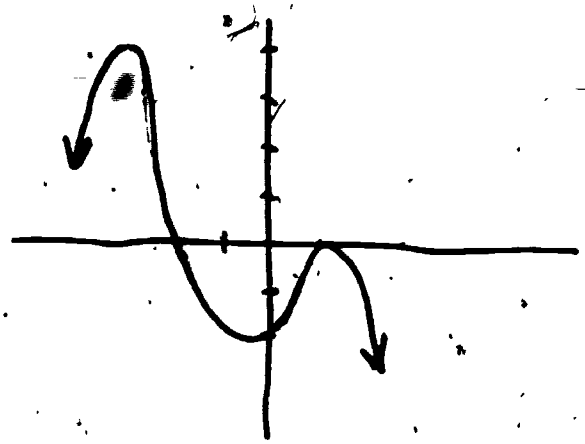
10)



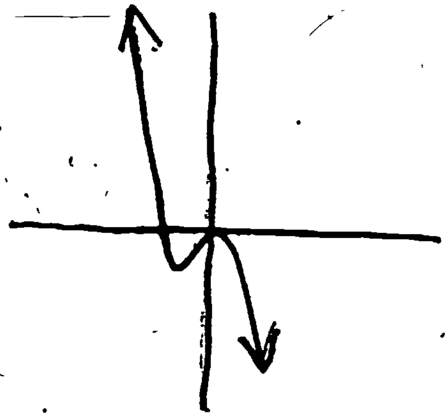
13)



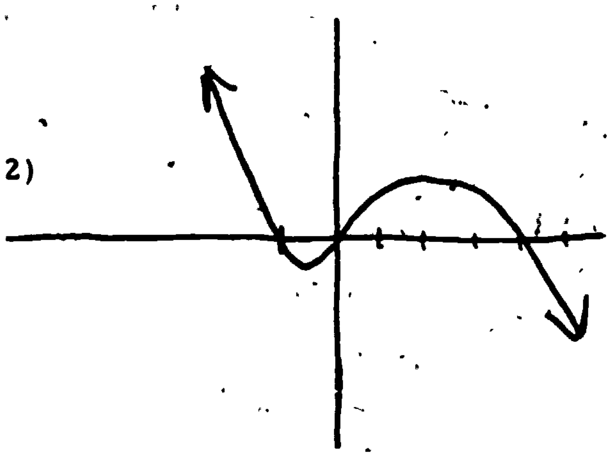
11)



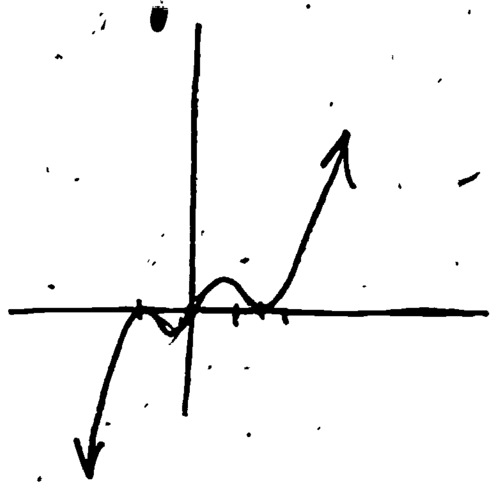
14)



12)



15)



Solutions to Exercise Set 5.3

1)

1	1	1.5	-6	-2
		1.0	2.5	-3.5
	1	2.5	-3.5	-5.5
		1	3.5	
	1	3.5	0	

2) (2,16)(1,17)

3) (1,0), (-3, 32)

4) (-2, 54), (3, -71)

5) (-1, 11), (3, -21)

6) (0, 3), (1, -2), (-2, -29)

7) (1, -2), (-1, 6)

8) a) (1.2, 2.1)

b) (-.5, -.6)

9) y = -7x - 7

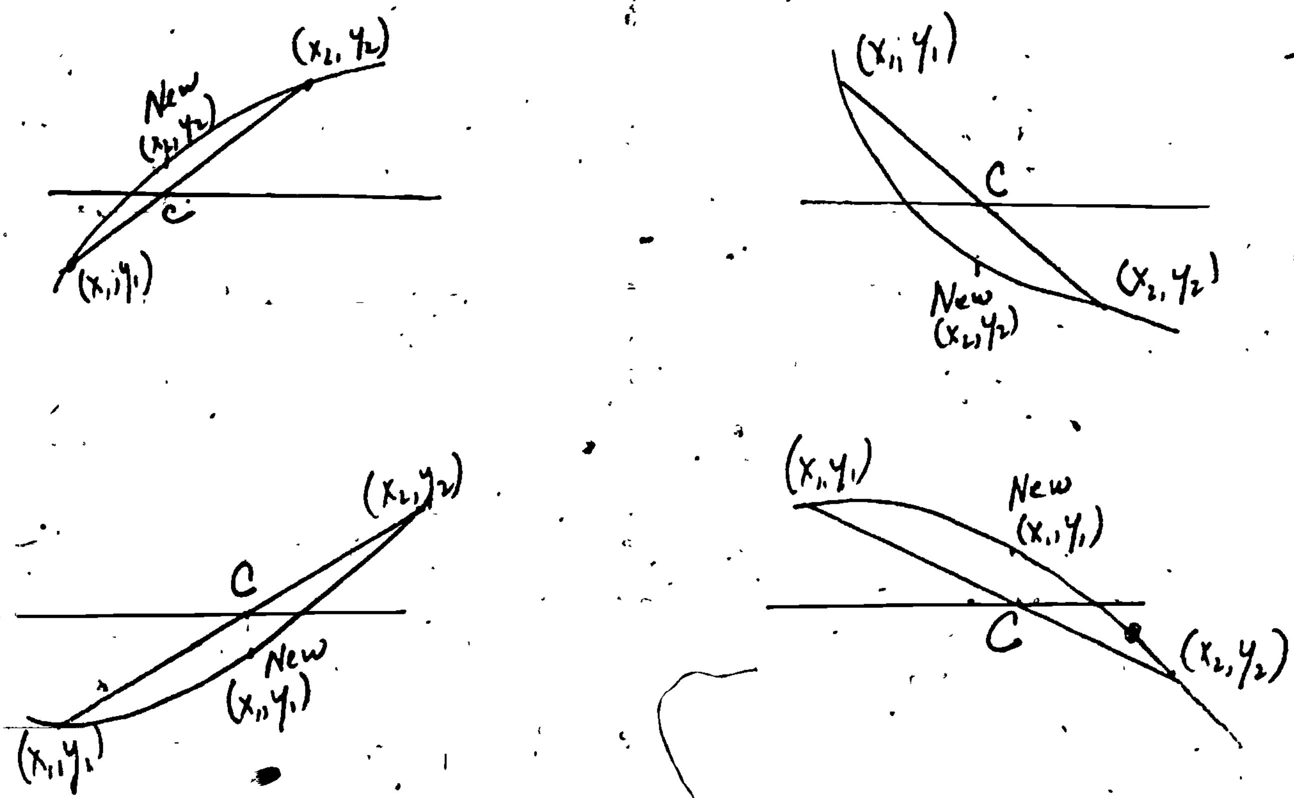
10) y = -5x + 1

11) 22x + 4y + 37 = 0

12) 26x - 4y -25 = 0

Solutions to Exercise Set 5.4

1)



2) HP 33E Program

x_2 ENTER x_1

if display shows 0,
RCL 4 for root.

01	F REG	15	÷	29	RCL 2	43	X
02	STO 0	16	R/S	30	GTO 01	44	3
03	R↓	17	STO 2	31	STO 4	45	+
04	STO 1	18	GSB 31	32	2	46	g X = 0
05	GSB 31	19	RCL 0	33	-	47	R/S
06	RCL 0	20	X	34	RCL 4	48	g RTN
07	Σ +	21	g X > 0	35	X		
08	RCL 0	22	GTO 28	36	4		
09	GSB 31	23	RCL 2	37	-		
10	STO 0	24	RCL 3	38	RCL 4		
11	RCL 1	25	RCL 1	39	X		
12	f Σ -	26	+	40	6		
13	RCL 7	27	GTO 01	41	+		
14	RCL 5	28	RCL 1	42	RCL 4		

2.4142

7 iteration.

3) -1.7321, -.4142, 1.7321

4) -1.8794, .3473, 1.5321

5) .1127, .5, .8873

6) -1, -2, 6.6056, -.6056

7) 1, 2, .5826, -8.5826

8) -2.4265

9) -3.8737, 0.1497, 1.7240

Solutions to Exercise Set 5.5

1) $A(x) = (x + 1)(3x - 2)(2x + 3)$

2) $B(x) = (x - 3)(x + 2)(2x - 3)$

3) $C(x) = (x + 3)(x + 3)(x + 1)(2x - 3)$

4) $D(x) = (x - 1)(x - 2)(x - 2)(2x - 1) =$

5) $E(x) = (x - 2)(x - 2i)(x + 2i)$

6) $F(x) = (3x - 5)(x + i)(x - i)$

7) $G(x) = (x - 2)(x + 2)(3x^2 - x + 5) =$

$= (x - 2)(x + 2)(x - \frac{1}{6} - \frac{\sqrt{59}}{6} i)(x - \frac{1}{6} + \frac{\sqrt{59}}{6} i)$

8) $H(x) = (x - 3)(x + 3)(2x^2 - x + 3)$

$(x - 3)(x + 3)(x - \frac{1}{4} - \frac{\sqrt{23}}{4} i)(x - \frac{1}{4} + \frac{\sqrt{23}}{4} i)$

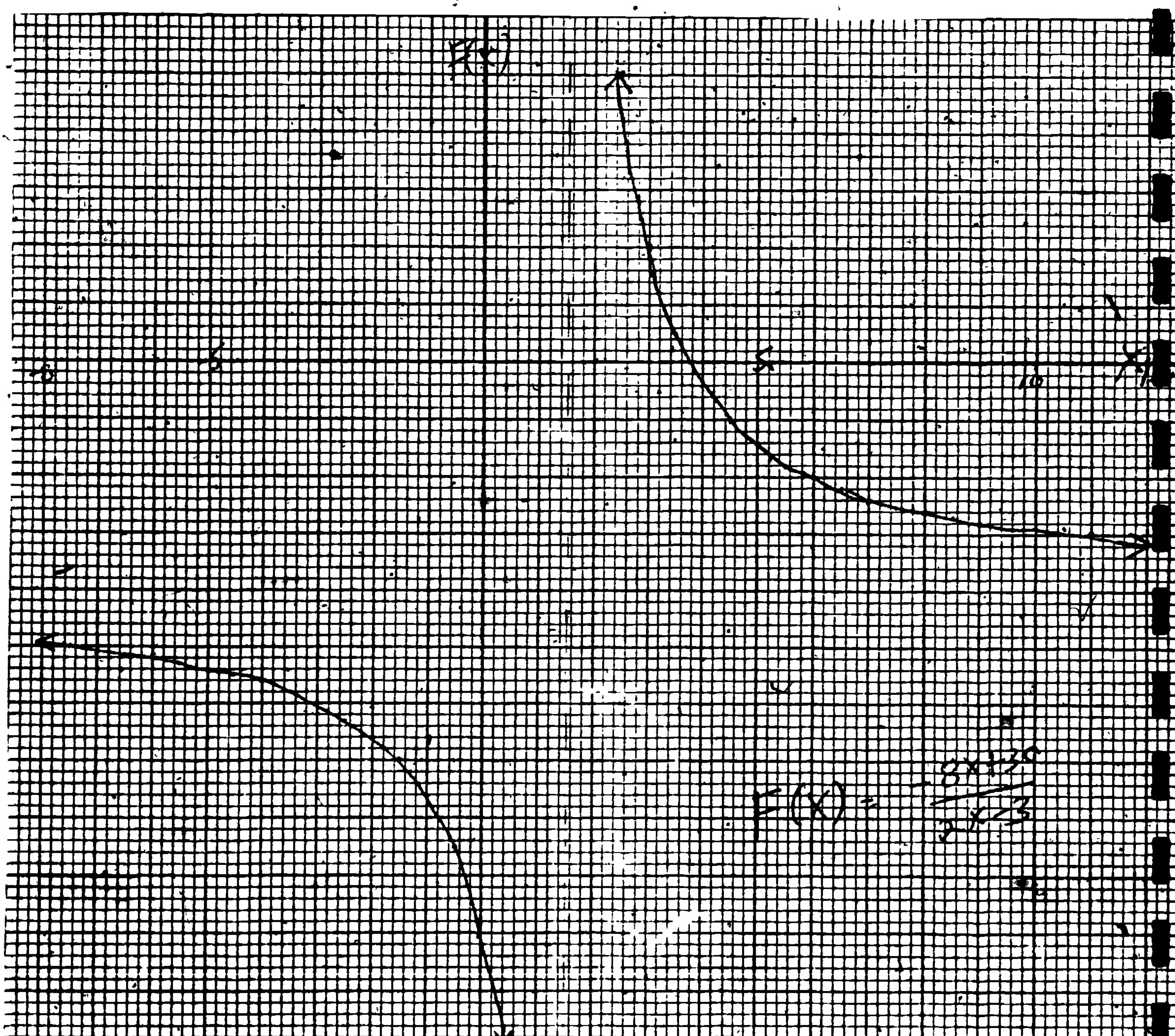
9) $J(x) = x(2x - 3)(2x - 3)(2x - 3)$

10) $K(x) = x(3x - 2)(3x - 2)(5x + 2)$

Solutions to Exercise Set 5.6

1) The value x = 1.5 is substituted in f(x)

x = 1.5 is a vertical asymptote. Error signal will be displayed. Division by zero. GTO adifferent step in your program. F(x) = -4 is a horizontal asymptote.



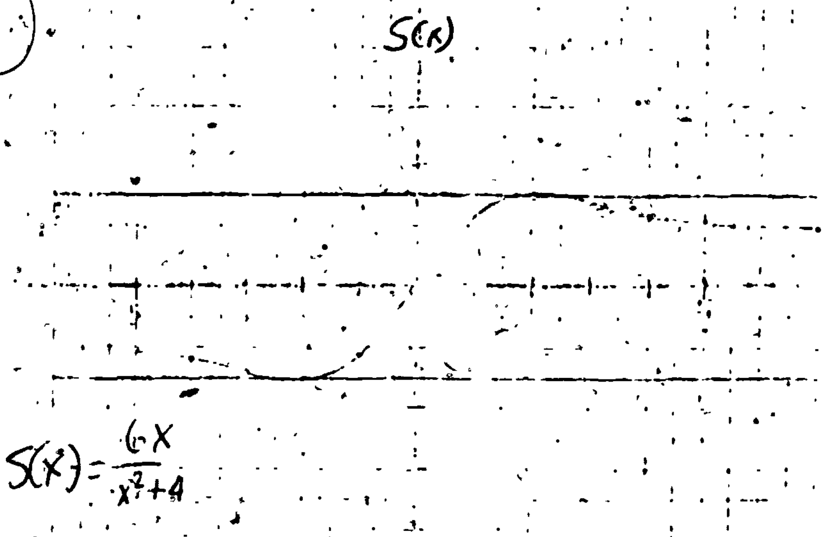
2) Serpentine

Domain $(-\infty, \infty)$

Range $[-1.5, 1.5]$

$S(x) = 0$

horizontal asymptote



3) Pilaster

Domain

$(-\infty, -2) \cup (-2,2) \cup (2, \infty)$

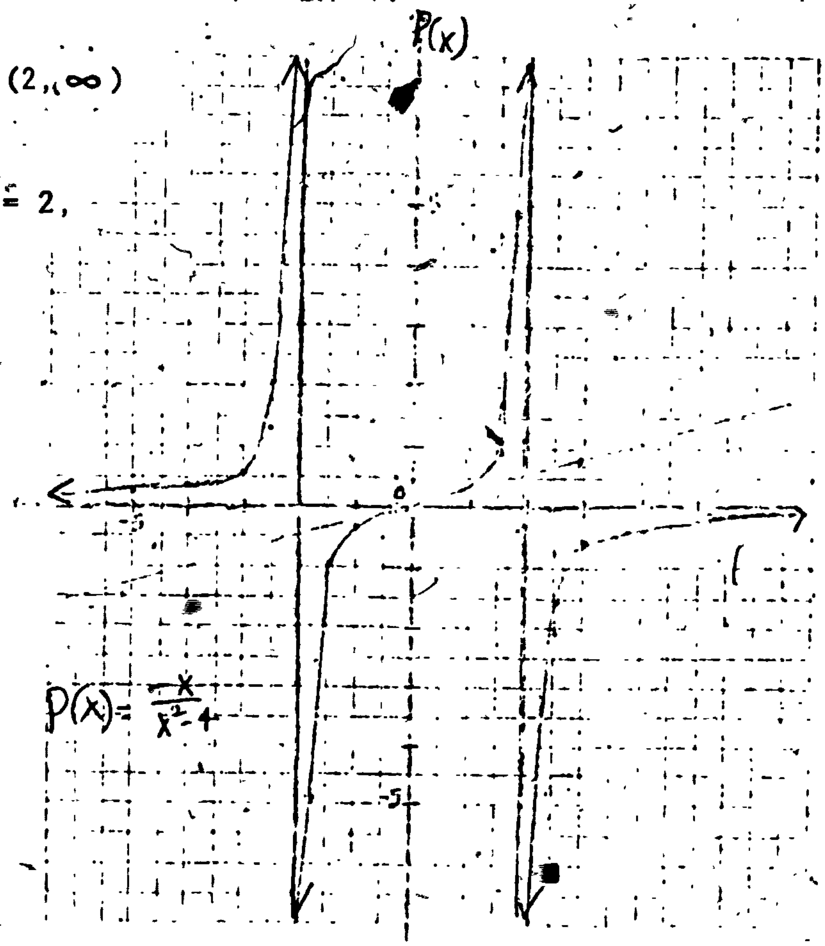
Range $(-\infty, \infty)$

Vertical asymptotes $x = 2$,

$x = -2$

Horizontal asymptote

$P(x) = 0$



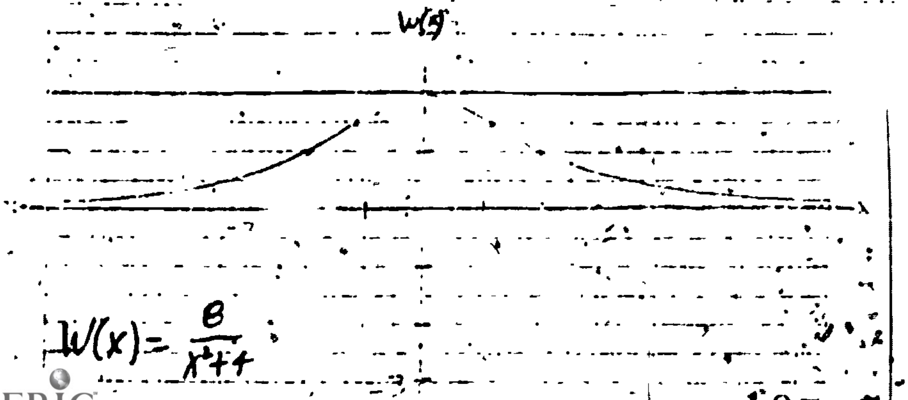
4) Witch

Domain $(-\infty, \infty)$

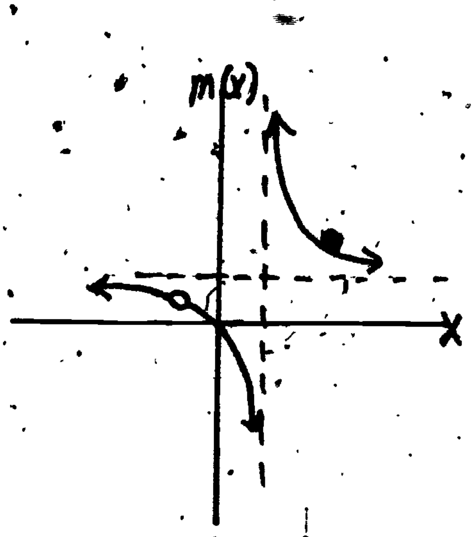
Range $(0, 2)$

Horizontal Asymptote

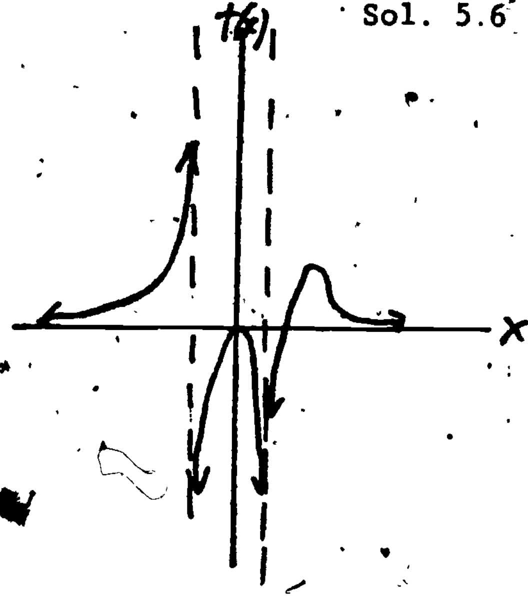
$W(x) = 0$



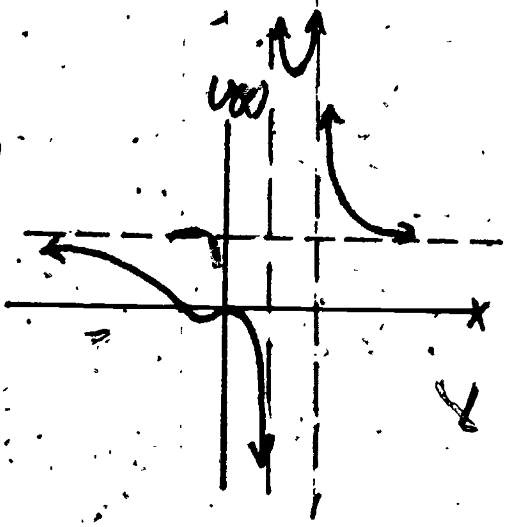
5)



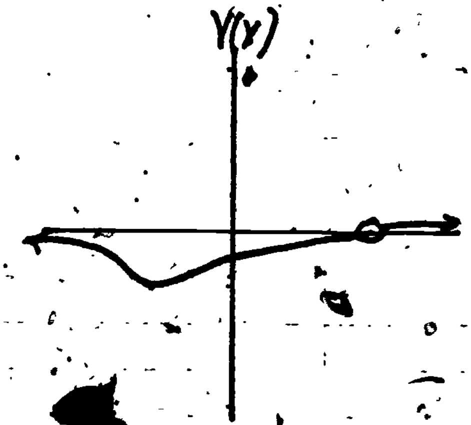
6)



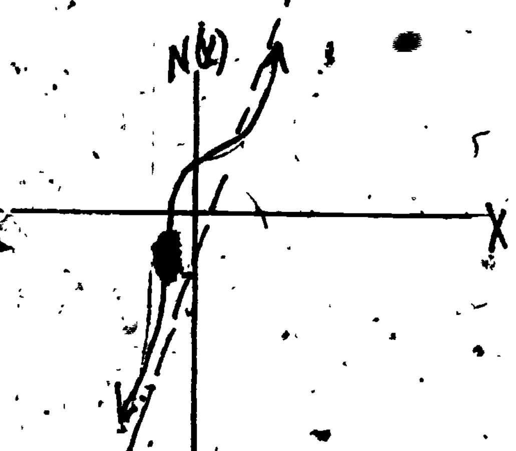
7)



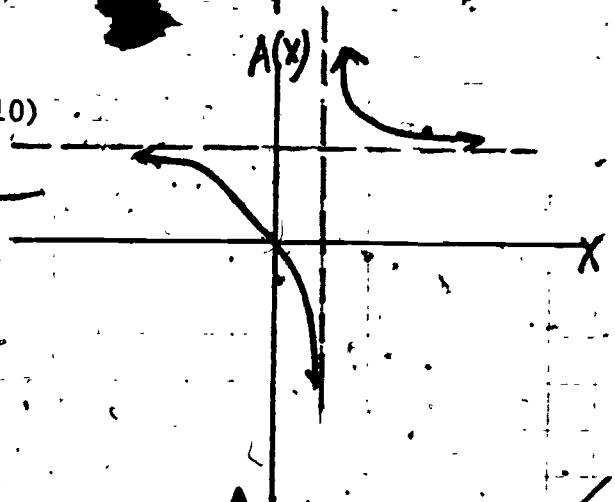
8)



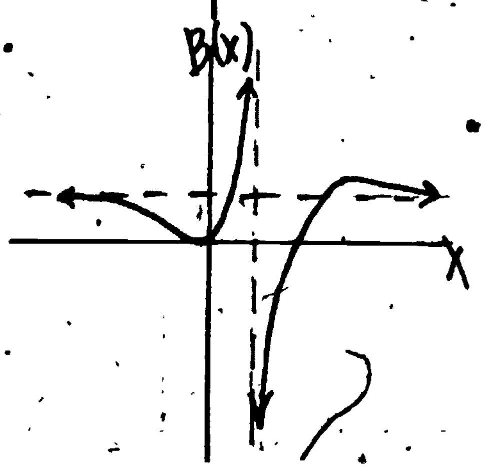
9)



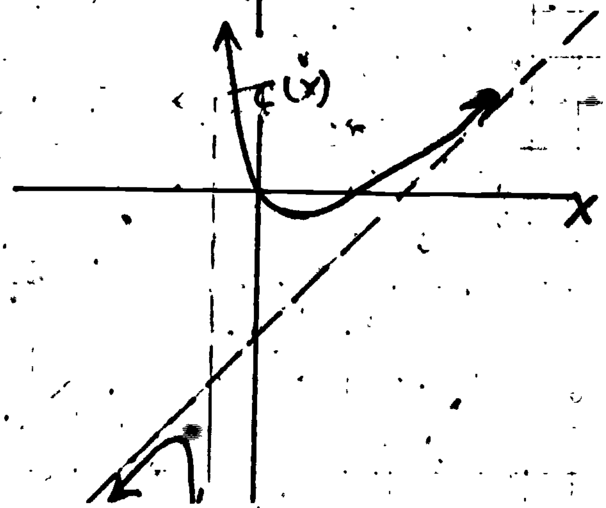
10)



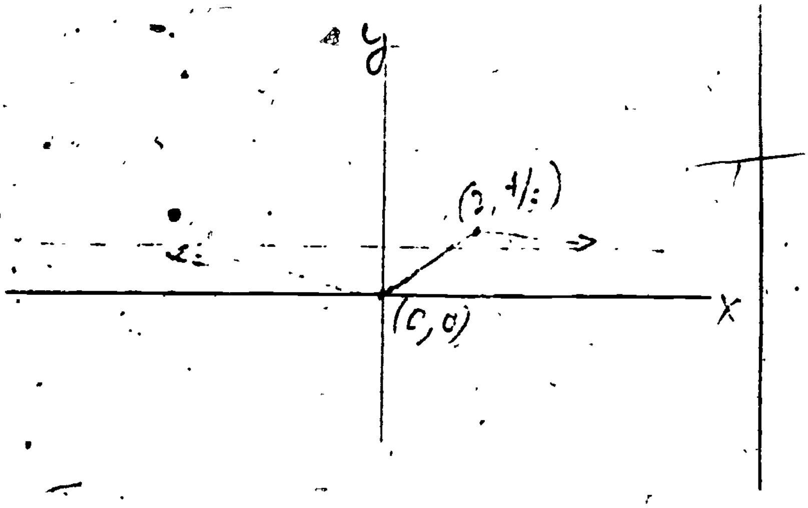
11)



12)



13)



$(L, H) \longleftarrow (0, 4/3)$

Range $[0, 4/3]$

14) Solve for x in terms of y

$$yx^2 - yx + y = x^2$$

$$(y - 1)x^2 - yx + y = 0$$

Using the quadratic formula

$$x = \frac{y \pm \sqrt{y^2 - 4(y - 1)(y)}}{2(y - 1)}$$

$$x = \frac{y \pm \sqrt{-3y^2 + 4y}}{2y - 2}$$

Fox x to be real $b^2 - 4ac \geq 0$

$$-3y^2 + 4y \geq 0$$

$$y(4 - 3y) \geq 0$$

$y \geq 0$ and $4 - 3y \geq 0$ OR $y \leq 0$ and $4 - 3y \leq 0$

$y \geq 0$ and $4 \geq 3y$ OR $y \leq 0$ and $4 \leq 3y$

$y \geq 0$ and $y \leq 4/3$ $y \leq 0$ and $y \geq 4/3$

$[0, 4/3]$ $\emptyset$

Solutions to Exercise Set 5.7

1) 64

2) 8

3) 76

4) 3.75

5) $4/21 \doteq .1905$

6) 4392

7) $71/48 \doteq 1.4792$

8) Since the $f(x) > y(x)$ for all x in the interval $[a, b]$, find the function $d(x) = f(x) - g(x)$. Using the function $d(x)$ the required area may be found.

9) $\frac{343}{48} \doteq 7.1458$

$N = 100$, area is 7.1451

10) $\frac{504}{5} = 100.8$

$N = 100$, area is 100.

Solutions to 5.8 - Chapter 5 Test

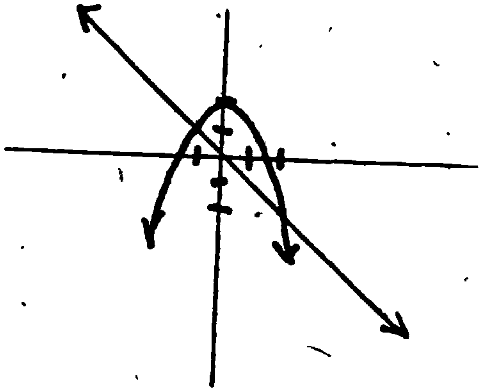
1) (3) (2) (2)

3) (4) (4) (3)

5) (1) (6) (2)

7) a) -2, 1, -.5, -.5

7(b)



c) $f(x) = 4x^4 + 8x^3 - 3x^2 - 7x - 2$

d)

-1	4	8	-3	-7	-2
		-4	-4	7	0
	4	4	-7	0	-2
		-4	0	7	
	4	0	-7	7 = m	

e) $(-2, -\frac{1}{2})$

or

$(-\frac{1}{2}, \ )$

$y - y_1 = m(x - x_1)$

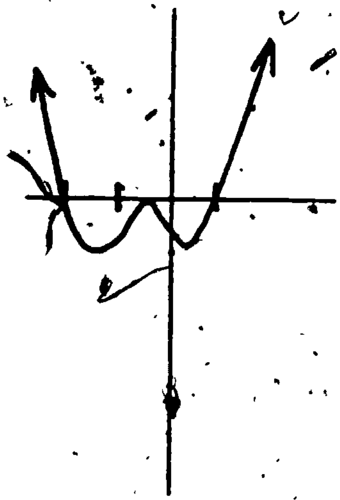
$y - (-2) = 7(x - (-2))$

$y = 7x + 9$

8) a) $1, \frac{+1}{-3}, \frac{+1}{-9}, \frac{+5}{-3}, \frac{+5}{-9}$

b) $-1, \frac{1}{3}, \frac{1}{3}$

9) a)



b) (2, -1) (-1, 1)

c) .25

Solutions to Exercise Set 6.1

1) (a) $\sqrt[3]{8(4096)(-1)}$

$\sqrt[3]{-32768}$

-32

(b) $(8(-2)^{12}(-1)^{9})^{\frac{1}{3}}$

$2(-2)^{4}(-1)^{3}$

$2(16)(-1) = -32$

2) (a) $\sqrt[4]{81(.0000009536743164)}$

$\sqrt[4]{.00007724761963}$

$.09375$

(b) $(81(.0625)^{5})^{\frac{1}{4}}$

$3(.5)^{5}$

$3(.03125) = .09375$

3) (a) $\frac{2.6879(.09)}{.09}$

2.6879

(b) $\frac{3^{.9}(.3)^{2}}{(.3)^{2}}$

$3^{.9} = 2.6879$

4) (a) $\frac{2.9240 + 2.9240}{2.9240}$

2

(b) $\frac{5^{4/3}(1 + 1)}{5^{4/3}}$

$1 + 1 = 2$

5) (a) $(2.6651)^{\sqrt{2}}$

4

(b) $2^{\sqrt{2}\cdot\sqrt{2}}$

$2^{2} = 4$

6) (a) $(2.1746)^{\sqrt{2}}$

3

(b) $\sqrt{3}^{\sqrt{2}\cdot\sqrt{2}}$

$\sqrt{3}^{2} = 3$

7) (a) $\frac{.1790}{.09}$

1.9889

(b) $\frac{1}{2.7^{\sqrt{3}}(.3)^{2}}$

$\frac{1}{5.5866(.09)} = \frac{1}{.5028}$

1.9889

8(a) $(.2887)^{-8}$

$20{,}735.99\overline{9}$

(b) $\left(\dfrac{6\sqrt{2}}{\sqrt{6}}\right)^{+8}$

$\left(6\sqrt{\dfrac{1}{3}}\right)^{+8}$

$6^{+8}\left(\dfrac{1}{3}\right)^{+4} = \dfrac{6^8}{3^4}$

$= \dfrac{3^8 \cdot 2^8}{3^4}$

$3^4 \cdot 2^8$

$20{,}736$

9) $8.6023i$

10) $1.7725i$

11) $13.1149i$

12) $-4.9619i$

13) -1.5518

14) -1.7333

15) $8^2 \doteq 8^1 \cdot 8^{1/2} \cdot 8^{1/4} \cdot 8^{1/8} \cdot 8^{1/16} \cdot 8^{1/32} \cdot 8^{1/64} \cdot 8^{1/128} \cdot 8^{1/256} \cdot 8^{1/512} \cdot$

$8^{1/1024} \cdot 8^{1/2048} \cdot 8^{1/4096} \cdot 8^{1/8192} \cdot 8^{1/16384}$

$= 8(2.824)(1.6818)(1.2968)(1.1388)(1.0671)(1.0330)$

$(1.0164)(1.0082)(1.0031)(1.0010)(1.0010)(1.0005)$

$(1.0003)(1.0001)$

$= 63.9919$

$8^2 = 64$ by direct calculation

16) $9^{3/2} = 9^1 \cdot 9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \cdot 9^{1/81} \cdot 9^{1/243} \cdot 9^{1/729} \cdot 9^{1/2187} \cdot 9^{1/6561} \cdot 9^{1/19683}$

$= 9(2.0801)(1.2765)(1.0848)(1.0275)(1.0091)(1.0030)(1.0010)(1.0003)(1.0001)$

$= 26.9985$

$9^{3/2} = 27$ by direct calculation

17) $27^{1/3} = 27^{.3} \cdot 27^{.03} \cdot 27^{.003} \cdot 27^{.0003} \cdot 27^{.00003}$

$= (2.6879)(1.1039)(1.0099)(1.0010)(1.0001)$

$= 3$

$27^{1/3} = 3$ by direct calculation

18) $5^{4/7} = 5^{.5} \cdot 5^{.07} \cdot 5^{.001} \cdot 5^{.0004} \cdot 5^{.00002}$

$= (2.2361)(1.1193)(1.0016)(1.0006)(1.0000)$

$= 2.5084$

$5^{4/7} = 2.5085$ by direct calculation

19) $6.1^{\sqrt{2}} = 6^1 \cdot 6^{.4} \cdot 6^{.01} \cdot 6^{.004} \cdot 6^{.0002} \cdot 6^{.00001}$

$= 6(2.0477)(1.0181)(1.0072)(1.0004)(1.0000)$

$= 12.9009$

$6.1^{\sqrt{2}} = 12.9010$ by direct calculation

20) $8.6^{\sqrt{5}} = 8.6^2 \cdot 8.6^{.2} \cdot 8.6^{.03} \cdot 8.6^{.006} \cdot 8.6^{.00007}$

$= (73.96)(1.5378)(1.0667)(1.0130)(1.0002)$

$= 122.9147$

$8.6^{\sqrt{5}} = 122.9142$ by direct calculation

21) $\pi^{\pi} = \pi^{3} \cdot \pi^{.1} \cdot \pi^{.04} \cdot \pi^{.001} \cdot \pi^{.0005} \cdot \pi^{.00009}$

$= (31.0063)(1.1213)(1.0469)(1.0012)(1.0006)(1.0001)$

$= 36.4620$

$\pi^{\pi} = 36.4622$ by direct calculation

22) $\sqrt{2}^{\sqrt{3}} = \sqrt{2}^{1} \cdot \sqrt{2}^{.7} \cdot \sqrt{2}^{.03} \cdot \sqrt{2}^{.002} \cdot \sqrt{2}^{.00005}$

$= (1.4142)(1.2746)(1.0105)(1.0007)(1.0000)$

$= 1.8226$

$\sqrt{2}^{\sqrt{3}} = 1.8226$ by direct calculation

23) $(1734826)^2 = [17348 \times 10^2 + 26]^2$

$= (17348 \times 10^2)^2 + 2(17348 \times 10^2)(26) + 26^2$

$= 300,953,104 \times 10^4 + 902,096 \times 10^2 + 676$

$= 3,009,531,040,000$
$+ \quad 90,210,276$
$\quad 3,009,621,250,276$

24) $(12345)^3 = (123 \times 10^2 + 45)^3$

$= (123 \times 10^2)^3 + 3(123 \times 10^2)^2(45) + 3(123 \times 10^2)(45)^2 +$

$= (1860867 \times 10^6) + (2042415 \times 10^4) + (747225 \times 10^2) + 91,$

$= 1,860,867,000,000$
$\quad 20,424,150,000$
$+ \quad 74,813,625$
$= 1,881,365,963,625$

25) $(27)^8 = 387,420,489 \times 27^2 =$

$387,420,489\ [729] = 387,420,489\ [700 + 20 + 9]$

$= 271,194,342,300 + 7,74[illegible],409,780 + 3,486,784,901$

$= 282,429,536,481$

26) $2^{40} = 20^{30} \cdot 2^{10} = 1{,}073{,}741{,}824\ [1024]$

$= 1{,}073{,}741{,}824\ [1000 + 20 + 4]$

$= 1{,}073{,}741{,}824{,}000$

$\quad 21{,}474{,}836{,}480$

$\quad \underline{4{,}294{,}967{,}296}$

$\quad 1{,}099{,}511{,}627{,}776$

27) $(\sqrt{2})^{\sqrt{2}^{\sqrt{2}}} \doteq 1.7608, \quad (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = 2$

so $(\sqrt{2})^{\sqrt{2}^{\sqrt{2}}} < (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}}$

Notice that in general, $y^x \neq x^y$.

Solutions to Exercise Set 6.2

1) $\log_3\sqrt{6} = \frac{1}{2}\log_3 6$

$= \frac{1}{2}(1.6310) = .8155$

$3^{.8155} = 2.4496$

$\sqrt{6} = 2.4495$ by direct calculation

2) $\log_3\sqrt{8} = \frac{1}{2}\log_3 8$

$= \frac{1}{2}(1.8930) = .9465$

$3^{.9465} = 2.8288$

$\sqrt{8} = 2.8284$ by direct calculation

3) $\log_3(8\times 7) = \log_3 8 + \log_3 7$

$= 1.8930 + 1.7712 = 3.6642$

$3^{3.6642} = 56.0103$

$8\times 7 = 56$ by direct calculation

4) $\log_3(9\times 6) = \log_3 9 + \log_3 6$

$= 2 + 1.6310 = 3.6310$

$3^{3.6310} = 54.0042$

$9\times 6 = 54$ by direct calculation

5) $\log_3\sqrt[5]{6} = \frac{1}{5}\log 6$

$= \frac{1}{5}(1.6310) = .3262$

$3^{.3262} = 1.4310$

$\sqrt[5]{6} = 1.4310$

6) $\log_3\sqrt[6]{5} = \frac{1}{6}\log_3 5$

$= \frac{1}{6}(1.4650) = .2442$

$3^{.2442} = 1.3077$

$\sqrt[6]{5} = 1.3077$ by direct calculation

7) $\log_3 \frac{1}{4} = \log_3 1 - \log_3 4$

$= 0 - 1.2620 = -1.2620$

$3^{-1.2620} = .25$

$\frac{1}{4} = .25$ by direct calculation

8) $\log_3 \frac{1}{5} = \log_3 1 - \log_3 5$

$= 0 - 1.4650 = -1.4650$

$3^{-1.4650} = .2$

$\frac{1}{5} = .2$ by direct calculation

9) $\log_3 (\frac{6\sqrt{5}}{8}) = \log_3 6 + \frac{1}{2}\log_3 5 - \log_3 8$

$= 1.6310 + \frac{1}{2}(1.4650) - 1.8930$

$= .4705$

$3^{.4705} = 1.6768$

$\frac{6\sqrt{5}}{8} = 1.6771$ by direct calculation

10) $\log_3 \left(\frac{5\sqrt[3]{7}}{4}\right) = \log_3 5 + \frac{1}{3}\log_3 7 - \log_3 4$

$= 1.4650 + \frac{1}{3}(1.7712) - 1.2620$

$= .7934$

$3^{.7934} = 2.3908$

$\frac{5\sqrt[3]{7}}{4} = 2.3912$ by direct calculation

11) $-\log_3\left(\frac{5}{3\sqrt[3]{9}}\right) = \log_3 5 - (\log_3 3 + \frac{1}{3}\log_3 9)$

$= 1.4650 - (1 + \frac{1}{3}(2))$

$= -.2017$

$3^{-.2017} = .8013$

$\frac{5}{3\sqrt[3]{9}} = .8012$ by direct calculation

12) $\log_3\left(\frac{2}{5\sqrt{7}}\right) = \log_3 2 - (\log_3 5 + \frac{1}{2}\log_3 7)$

$= .6310 - (1.4650 + \frac{1}{2}(1.7712))$

$= -1.7196$

$3^{-1.7196} = .1512$

$\frac{2}{5\sqrt{7}} = .1512$ by direct calculation

(13 - 18)

n	11	12	13	14	15
log n	2.1827	2.2620	2.3347	2.4022	2.4650

to compute $\log_3 11$

$3^2 = 9$ small

$3^3 = 27$ big

$3^{2.5} = 15.5885$ big

$3^{2.125} = 10.3248$ small

$3^{2.3125} = 12.6868$ big

$3^{2.2188} = 11.4449$ big

$3^{2.1719} = 10.8708$ small

$3^{2.1954} = 11.1544$ big

$3^{2.1837} = 11.0120$ big

$3^{2.1778} = 10.9414$ small

$3^{2.1808} = 10.9770$ small

$3^{2.1823} = 10.9951$ small

$3^{2.1830} = 11.0041$ big

$3^{2.1827} = 10.9999$ small

$\log_3 12 = \log_3 3 + \log_3 4 = 1 + 1.2620 = 2.2620$

to compute $\log_3 13$ from previous work

$3^{2.5} = 15.5885$ big	$3^{2.3302} = 12.9349$ small
$3^{2.3125} = 12.6868$ small	$3^{2.3331} = 12.9769$ small
$3^{2.4063} = 14.0628$ big	$3^{2.3346} = 12.9976$ small
$3^{2.3594} = 13.3573$ big	$3^{2.3353} = 13.0083$ big
$3^{2.3360} = 13.0176$ big	$3^{2.3350} = 13.0033$ big
$3^{2.3243} = 12.8514$ small	$3^{2.3348} = 13.0012$ big
	$3^{2.3347} = 12.9997$

$\log_3 14 = \log_3 2 + \log_3 7 = .6310 + 1.7712 = 2.4022$

$\log_3 15 = \log_3 3 + \log_3 5 = 1 + 1.4650 = 2.4650$

13) $\log_3 1.2 = \log_3(\frac{12}{10}) = \log_3 12 - \log_3 10 = 2.2620 - 2.0960$

$= .1660$

$3^{.1660} = 1.2001$ by direct calculation

14) $\log_3 1.4 = \log_3(\frac{14}{10}) = \log_3 14 - \log_3 10 = 2.4022 - 2.0960$

$= .3062$

$3^{.3062} = 1.3999$ by direct calculation

15) $\log_3 .13 = \log_3(\frac{13}{10^2}) = \log_3 13 - 2 \log_3 10$

$= 2.3347 - 2(2.0960)$

$= -1.8573$

$3^{-1.8573} = .1300$ by direct calculation

16) $\log_3 .15 = \log_3(\frac{15}{10^2}) = \log_3 15 - 2 \log 10$

$= 2.4650 - 2(2.0960)$

$= -1.7270$

$3^{-1.7270} = .1500$ by direct calculation

17) $\log_3 .0015 = \log_3(\frac{15}{10^4}) = \log_3 15 - 4 \log 10$

$= 2.4650 - 4(2.0960)$

$= -5.9190$

$3^{-5.9190} = .0015$ by direct calculation

18) $\log_3 .00012 = \log_3(\frac{12}{10^5}) = \log_3 12 - 5 \log 10$

$= 2.2620 - 5(2.0960)$

$= -8.2180$

$3^{-8.2180} = .00012$ by direct calculation (to five decimal places)

19) $\log n < 0$ when $n < 1$.

20) $0^n = 0$ for all $n \neq 0$.

21) $1^n = 1$ for all n.

22) $(-b)^x$ would be positive or negative so that signs would be an inconvenient problem in computation.

23) No. Negative numbers do not have logarithms.

24) b = N.

25)

a	b	c	d	e	f	g	b^g
2	3	1	3	0	1	.5	1.732
				.5	.75	.75	2.2795
				.625	.6875	.625	1.9870
				.6290	.6563	.6875	2.1282
					.6407	.6563	2.0564
					.6329	.6407	2.0215
						.6329	2.0042
						.6290	1.9957
						.6310	2.0000

27) If $\{n_1, n_2, n_3, \ldots\}$ is a geometric sequence it can be rewritten as $\{n_1, n_1r, n_1r^2, n_1r^3, \ldots\}$. Thus $\{\log_b n_1, \log_b n_2, \log_b n_3, \ldots\}$ becomes

$\{\log_b n_1, \log_b n_1 r, \log_b n_1 r^2, \log_b n_1 r^3, \ldots\} =$

$\{\log_b n_1, \log_b r + \log_b n_1, 2 \log_b r + \log_b n_1, 3 \log_b r + \log_b n_1, \ldots\}$

which is arithmetic with $d = \log_b r$. $b > 0, b \neq 1$ by the normal conversion of bases for logs.

Solutions to Exercise Set 6.3

1)	$\log_2 54 = 5.7549$	2)	$\log_3 150 = 4.5609$
3)	$\log_{11} 0.009 = -1.9644$	4)	$\log_4 0.416 = -0.6327$
5)	$\log_{\sqrt{2}} 6 = 5.1699$	6)	$\log_{\sqrt{3}} 7 = 3.5425$
7)	$\log_{\pi} 8 = 1.8165$	8)	$\log_{\pi} 1 = 0$
9)	$\log_{.03} 10 = -0.6567$	10)	$\log_{.7} 9 = -6.1603$
11)	$\log_{\sqrt{2}} \sqrt{2} = 1$	12)	$\log_{\pi} \pi = 1$
13)	$\log_a \sqrt{a} = \frac{1}{2}$	14)	$\log_x x^2 = 2$

15) $4^{20} = 10^x$

$20 \log 4 = x$

$12.0412 = x$

4^{20} has 13 digits

16) $5^{18} = 10^x$

$18 \log 5 = x$

$12.5815 = x$

5^{18} has 13 digits

17) $127^{19} = 10^x$

$19 \log 127 = x$

$39.9723 = x$

127^{19} has 40 digits

18) $253^{12} = 10^x$

$12 \log 253 = x$

$28.8374 = x$

253^{12} has 29 digits

19) $57^{90} = 10^x$

$90 \log 57 = x$

$158.0287 = x$

57^{90} has 159 digits

20) $63^{85} = 10^x$

$85 \log 63 = x$

$152.9439 = x$

63^{85} has 153 digits

21) $\frac{4 + 6}{4} = 2.5$

22) $\frac{4 + 0}{2 + 6} = .5$

23) $\frac{2 + 0}{5 \cdot (.5)} = .\overline{8}$

24) $\frac{-2 \div 5/2}{\frac{1}{4} - 5/2} = 3.\overline{5}$

25) $5 + 0 - 2 + 1 + 0 = 4$

26) $6 + 2 + 9 - 1 = 16$

27)

$\log_2 3$	$\log_3 2$	$\log_5 7$	$\log_7 5$	$\log_{10} 2$	$\log_2 10$
1.5850	0.6309	1.2091	0.8271	.3010	3.3219

28) various answers

29) $\log_a b\ (\log_b a) = 1$

30) let $\log_a b = x$ $\qquad \log_b a = y$

$a^x = b \qquad b^y = a$

$$(b^y)^x = b$$

$$xy = 1$$

31) Verbal algorithm to compute $\log_b a$

1. Remember a, b
2. c ← log a
3. d ← log b
4. Compute ℓ = c ÷ d
5. Display ℓ

32)

HP 33E	TI 58	TRS 80
PRGM	LRN	Input A, B
f log	2nd log	C = LOG(A)
STO 1	STO 01	D = LOG(B)
R/S	R/S	L = C/D
f log	2nd log	PRINT L
RCL 1	STO 02	
x >< y	RCL 01	
÷	÷	
RUN	RCL 02	
g RTN	=	
	R/S	
	RST	
	LRN	
	RST	

Solutions to 6.4

EXAMPLE 6.4.1

n	a_n	n	a_n
1	2	6	2.5216
2	2.25	7	2.5465
3	2.3704	8	2.5658
4	2.4414	9	2.5812
5	2.4883	10	2.5937

EXAMPLE 6.4.2

n	s_n	n	s_n
1	1	9	2.71827877
2	2	10	2.71828153
3	2.5	11	2.71828180
4	2.66666667	12	2.71828183
5	2.70833333	13	2.71828183
6	2.71666667	14	2.71828183
7	2.71805556	15	2.71828183
8	2.71825397	16	2.71828183

Solutions to Exercise Set 6.4

1) HP 33E: [1] [e^x]

   TI 58: [1] [INV] [ℓn x]

   TRS-80: EXP (1)

2) 2.715568521

3) 2.718010050

4) 2.718281827

5) by calculation 1

6) Because of rounding the calculator adds

$$1 + 10^{-10} = 1.$$

7) The sequence never quits so the number that it represents never stops.

8) About 1 minute and 5 seconds

9) About 2 minutes and 40 seconds

10) Based on 6 minutes to do 100 terms, 5×10^7 minutes $\doteq$ 833,333 hours $\doteq$ 34,722 days $\doteq$ 95 years.

11) Based on 5 minutes to do 100 terms, 5×10^8 minutes $\doteq$ 950 years.

12) 2.718281836

13) 2.718281834

14) No, because $\sum_{n=0}^{30} \frac{1}{n!} - \sum_{n=0}^{20} \frac{1}{n!} = \sum_{n=21}^{30} \frac{1}{n!} \neq 0$

15) The series never quits so the number that it represents never stops.

16) $\ln 10 \, (\log_{10} e) = 1$

17) Let $x = \ln 10$ then $e^x = 10$

$y = \log_{10} e$ then $10^y = e$

$(10^y)^x = 10$

$xy = 1$

18) false. $\ln 1 = 0$, $\log 1 = 0$

19) true. This is a special case of $\log_b(\frac{x}{y}) = \log_b x - \log_b y$

20) false. $\ln x^p = p \ln x$

21) false. $\ln e = 1$ and $e^1 = e$

22) true.

23) 1.4118

24) 1.4141

25) 2.7216

26) 2.7183

27) $\sqrt{e^{i\pi}}$ does not exist. $f(x) = \sqrt{x}$ is a function only for real numbers x.

Solutions to Exercise Set 6.5

1) a) $N = 200\ e^{.27(30)} \doteq 658893.6 \times 150$ bacteria

b) $N = 200\ e^{.27(120)} = 2.355977884 \times 10^{16}$ bacteria

c) $15,000 = 200\ e^{.27t}$

$75 = e^{.27t}$

$\ln 75 = .27t \ln e$

$\frac{\ln 75}{.27} = t = 7.4626$ It will take 15.9907 minutes

2) a) $7.5 = 15\ e^{k(5)}$

$.5 = e^{5k}$

$\ln .5 = 5k \ln e$

$\frac{\ln .5}{5} = k = -0.1386$

$y = 15\ e^{-0.1386(12)}$

$= 2.8420$

12.1580 grams will disappear
2.8420 grams will remain

b) $9 = 15\ e^{-0.1386t}$

$.6 = e^{-0.1386t}$

$\ln .6 = -0.1386t$

$3.6856 = t$ It will take 3.6856 days.

3) $y = c\ e^{kt}$

$.04 = 1\ e^{k(2)}$

$\ln .04 = 2k$

$-1.6094 = k$

$.5 = 1\ e^{-1.6094t}$

$\ln .5 = -1.6094t$

$.4307 = t$

The half-life is .4307 hours.

4)

$$y = c\,e^{kt} \qquad 2000 = 1000\,e^{.1946t}$$
$$7000 = 1000\,e^{k\cdot 10} \qquad 2 = e^{.1946t}$$
$$7 = e^{10k} \qquad \ln 2 = .1946t$$
$$\ln 7 = 10k \qquad 3.5619 = t$$
$$.1946 = k$$

It takes 3.5619 hours.

5) $A = P\,e^{rt}$

$$A = 5000\,e^{.05(7)} = \$7095.34$$

6) Sidney: $A = 5000\,e^{.05(5)} = \$6420.13$

Susie: $A = 5000\left(1 + \frac{.055}{12}\right)^{5(12)} = \$6578.52$

Susie made the better investment.

7) $462,768 = 532,759\,e^{k(10)}$

$$\ln 462,768 = \ln 532,759 + 10k$$

$$\frac{\ln 462,768 - \ln 532,759}{10} = k = -.0141$$

$$y = 532,759\,e^{-.0141(20)} = 401,846.$$

The population will be 401,846.

8)

$$497,000 = 487,000\,e^{k(10)} \qquad 1,000,000 = 487,000\,e^{.002t}$$
$$497 = 487\,e^{10k} \qquad 1000 = 487\,e^{.002t}$$
$$\ln 497 = \ln 487 + 10k \qquad \ln 1000 = \ln 487 + .002t$$
$$\frac{\ln 497 - \ln 487}{10} = k = .0020 \qquad \frac{\ln 1000 - \ln 487}{.002} = t$$
$$359.7456 = t$$

In 1960 + 360 = 2320 the population of Atlanta will be 1 million.

9) $2 = 1e^{k \cdot 33}$ $y = 3 \times 10^9 e^{.0210(30)}$

$\ln 2 = 33k$ $y = 5,633,585,463$

$.0210 = k$

10) a) $y = c\, e^{kt}$

c is original amount of sugar

y is the amount of sugar at time t

k is to be determined

b) $11 = 30e^{k(4)}$

$\ln 11 = \ln 30 + 4k$

$\frac{\ln 11 - \ln 30}{4} = k = -.2508$

$20\% \,(30) = 30\, e^{-.2508t}$

$6 = 30e^{-.2508t}$

$.2 = e^{-.2508t}$

$\ln .2 = -.2508t$

$6.4116 = t$

It will take 6.4116 hours.

11) a) $P = 50e^{-365/250}$

$= 11.6118$ watts at the end of one year

b) $25 = 50e^{-t/250}$

$.5 = e^{-t/250}$

$\ln .5 = \frac{-t}{250}$

$-250\,(\ln .5) = t$

173.2868 days = t, the half-life of the power supply

c) $$10 = 50e^{-t/250}$$

$$.2 = e^{-t/250}$$

$$(-250)(\ln .2) = t = 402.3595$$

The operational life of the satellite is 402 days.

12) a) $$P = 14.7(.5)^{\frac{h}{3.25}}$$

$h \leq 50$, h in miles

P is pressure in $lbs/in.^2$

b) $$P = 14.7\ (.5)^{\frac{20}{3.25}}$$

$$= .2065 \text{ pounds per square inch}$$

c) $$.25(14.7) = 14.7\ (.5)^{\frac{h}{3.25}}$$

$$.25 = (.5)^{h/3.25}$$

$$\frac{3.25\ (\ln .25)}{\ln .5} = h$$

$$6.5 = h$$

The altitude is 6.5 miles.

d) $$.01\ (14.7) = 14.7\ (.5)^{\frac{h}{3.25}}$$

$$\frac{3.25\ (\ln .01)}{\ln .5} = h$$

$$21.5925 = h$$

21.6 miles is above 99% of the atmosphere.

Solutions to Exercise Set 6.6

1) $\ln l = \ln a + (n-1)\ln r$

$\ln l - \ln a = n \ln r - \ln r$

$\dfrac{\ln l - \ln a + \ln r}{\ln r} = n$

2) $\ln x = \ln a + cn \ln b$

$\dfrac{\ln x - \ln a}{c \cdot \ln b} = n$

3) $3^x = n$

4) $x^2 - 3 = \ln n$

$e^{x^2-3} = n$

5) $7x \ln 5 = (5x+1) \ln 7$

$7x \ln 5 = 5x \ln 7 + \ln 7$

$x(7 \ln 5 - 5 \ln 7) = \ln 7$

$x = \dfrac{\ln 7}{7 \ln 5 - 5 \ln 7}$

$x = 1.2664$

ck:

$5^{7(1.2664)}$	$7^{5(1.2664)+1}$
1571192.118	1571298.978

6) $(x-2) \ln 3 = (x-2) \ln 5$

$x \ln 3 + 2 \ln 3 = x \ln 5 - 2 \ln 5$

$x \ln 3 - x \ln 5 = -2 \ln 5 - 2 \ln 3$

$x = \dfrac{+2 \ln 5 + 2 \ln 3}{-\ln 3 + \ln 5}$

$x = 10.6026$

ck:

$3^{10.6026+2}$	$5^{10.6026-2}$
1030310.973	1030289.697

7) $\log x + 1 = 10^{-.2}$

$x + 1 = (10)^{10^{-.2}}$

$x = 3.2752$

ck:

~~log~~ log ~~(3.2752~~ + 1)	-.2
-.2	

8) $\ln (x-3) = e^{-1.2}$

$x-3 = (e)^{e^{-1.2}}$

$x = 4.3515$

ck: $\ln \ln (4.3515-3) \mid -1.2$

-1.1999

9) $\log_5(x+1) + \log_5 (x+2) = 1$

$(x+1)(x+2) = 5$

$x^2 + 3x + 2 = 5$

$x^2 + 3x - 3 = 0$

$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(-3)}}{2}$

$x = \frac{-3 \pm \sqrt{21}}{2}$

$x = .7913 \quad x = -3.7913$

reject $x \geq 0$

ck: $\log_5(.7913+1)+\log_5(.7913+2)$

$.3622 + .6378$

1

10) $\log_3(x+3) - \log_3 x = 2$

$\frac{x+3}{x} = 9$

$x + 3 = 9x$

$3 = 8x$

$.375 = x$

ck: $\log_3(.375+3) - \log_3 .375 \mid 2$

$1.1072 - (-.8928)$

2

11) $\frac{\ln(7x-12)}{\ln x} = 2$

$\log_x(7x-12) = 2$

$x^2 = 7x - 12$

$x^2 - 7x + 12 = 0$

$(x-4)(x-3) = 0$

$x = 4,\ x = 3$

ck: $\frac{\ln(7(4)-12)}{\ln 4}$ | 2

$\frac{\ln 16}{\ln 4}$

2

$\frac{\ln(7(3)-12)}{\ln 3}$ | 2

$\frac{\ln 9}{\ln 3}$

2

12) $\frac{\log(5x-6)}{\log x} = 2$

$\log_x(5x-6) = 2$

$x^2 = 5x-6$

$x^2-5x+6 = 0$

$(x-3)(x-2) = 0$

$x = 3,\ x = 2$

ck: $\frac{\log(5(3)-6)}{\log 3}$ | 2

$\frac{\log 9}{\log 3}$

$\frac{\log(5(2)-6)}{\log 2}$ | 2

$\frac{\log 4}{\log 2}$

13) $\ln x \cdot \ln x = \ln 1.5$

$(\ln x)^2 = \ln 1.5$

$\ln x = \sqrt{\ln 1.5}$

$x = 1.8903$

ck: $1.8903^{\ln 1.8903}$ | 1.5

1.5

14) $\log x \cdot \log x = \log 2$

$(\log x)^2 = \log 2$

$\log x = \sqrt{\log 2}$

$x = 3.5372$

ck: $3.5372^{\log 3.5372}$ | 2

2.0000

15) $(e^{-x}-2)(e^{-x}-1) = 0$

$e^{-x} = 2 \qquad e^{-x} = 1$

$-x = \ln 2 \quad -x = \ln 1$

$x = -.6931 \quad x = 0$

ck: $e^{-2(-.6931)} - 3e^{+.6931} + 2$ | 0

$4 \quad -6 \quad +2$

0

$e^0 - 3e^0 + 2$ | 0

$1 - 3 + 2$

0

16) $(e^x-3)(e^x+2) = 0$

$e^x = 3 \qquad e^x = -2$

$x = \ln 3, \; x = \ln(-2)$

$x = 1.0986$, reject

ck: $e^{2(1.0986)} - e^{1.0986}$ | 6

$8.9998 - 3$

5.9998

17) $e^{\ln x} = e^{1.2}$

$x = (e)^{e^{1.2}}$

$x = 27.6636$

ck: $\ln(\ln 27.6636)$ | 1.2

1.2

18) $10^{\log x} = 10^{-.3}$

$x = (10)^{10^{-.3}}$

$x = 3.1709$

ck: $\log(\log 3.1709)$ | -.3

$-.3$

19) $x = \sqrt{3 + x}$

$x^2 = 3 + x$

$x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1 + 4(3)}}{2}$

$x = \frac{1 + \sqrt{13}}{2}, \quad x = \frac{1 - \sqrt{13}}{2}$

$x = 2.3028 \qquad$ reject, $x > 0$

20) $x = \sqrt{5 + x}$

$x^2 = 5 + x$

$x^2 - x - 5 = 0$

$x = \frac{1 \pm \sqrt{1 + 4(5)}}{2}$

$x = \frac{1 + \sqrt{21}}{2}, \quad x = \frac{1 - \sqrt{21}}{2}$

$x = 2.7913 \qquad$ reject, $x > 0$

21) $x - 2 = \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \cdots}}}}$

$$x = 2 + \frac{1}{4 + x - 2}$$

$$x = 2 + \frac{1}{x + 2}$$

$$x(x+2) = 2(x+2) + 1$$

$$x^2 + 2x = 2x + 4 + 1$$

$$x^2 = 5$$

$$x = \sqrt{5} \doteq 2.2361$$

22) $x - 3 = \cfrac{1}{6 + \cfrac{1}{6 + \cfrac{1}{6 + \cdots}}}$

$$x = 3 + \frac{1}{6 + x - 3}$$

$$x = 3 + \frac{1}{x+3}$$

$$x(x-3) = 3(x+3) + 1$$

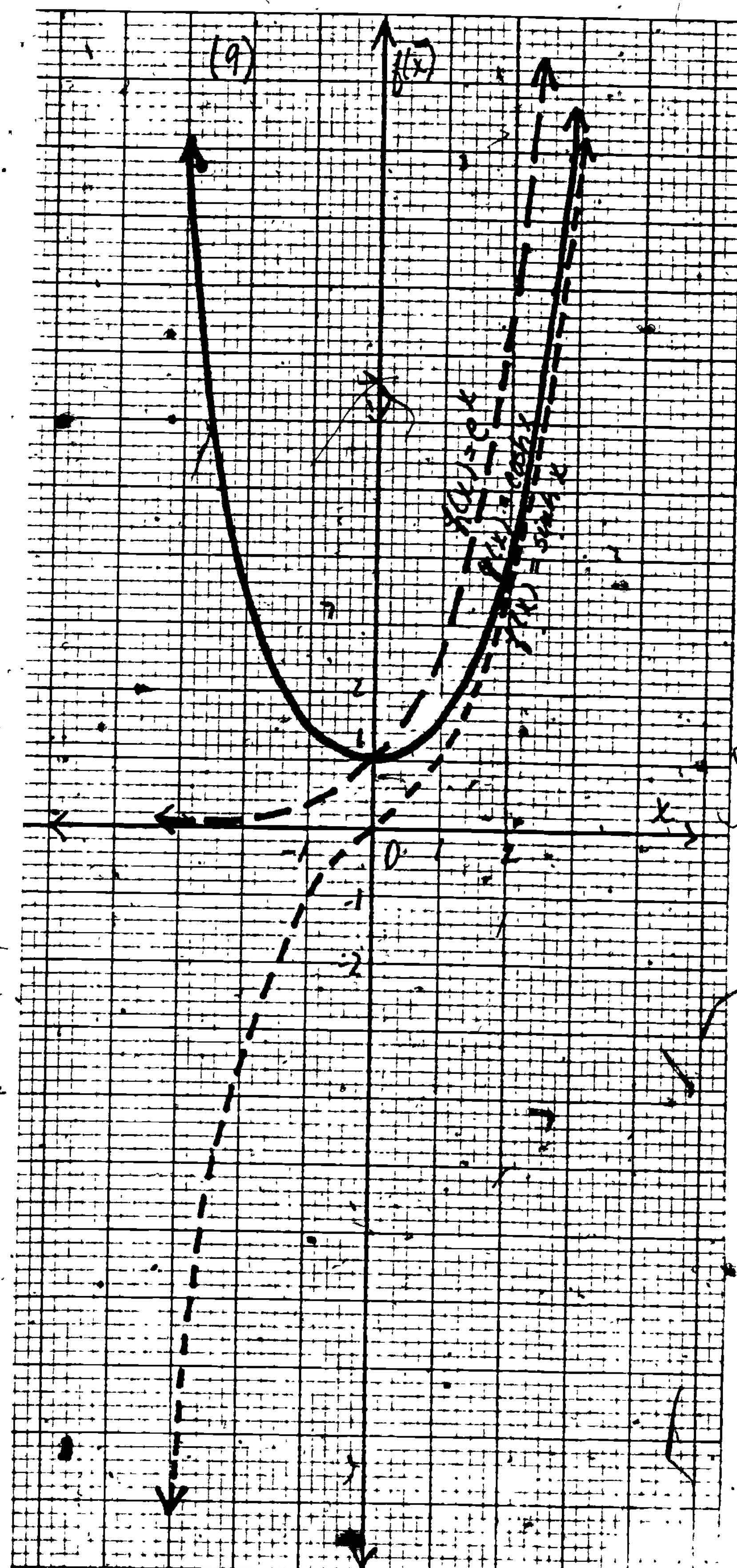
$$x^2 - 3x = 3x + 9 + 1$$

$$x^2 = 10$$

$$x = \sqrt{10} \doteq 3.1623$$

Solutions to Exercise Set 6.7

1) <
2) >
3) >
4) <
5) <
6) =
7) =
8) >
9) sinh x < cosh x < e^x



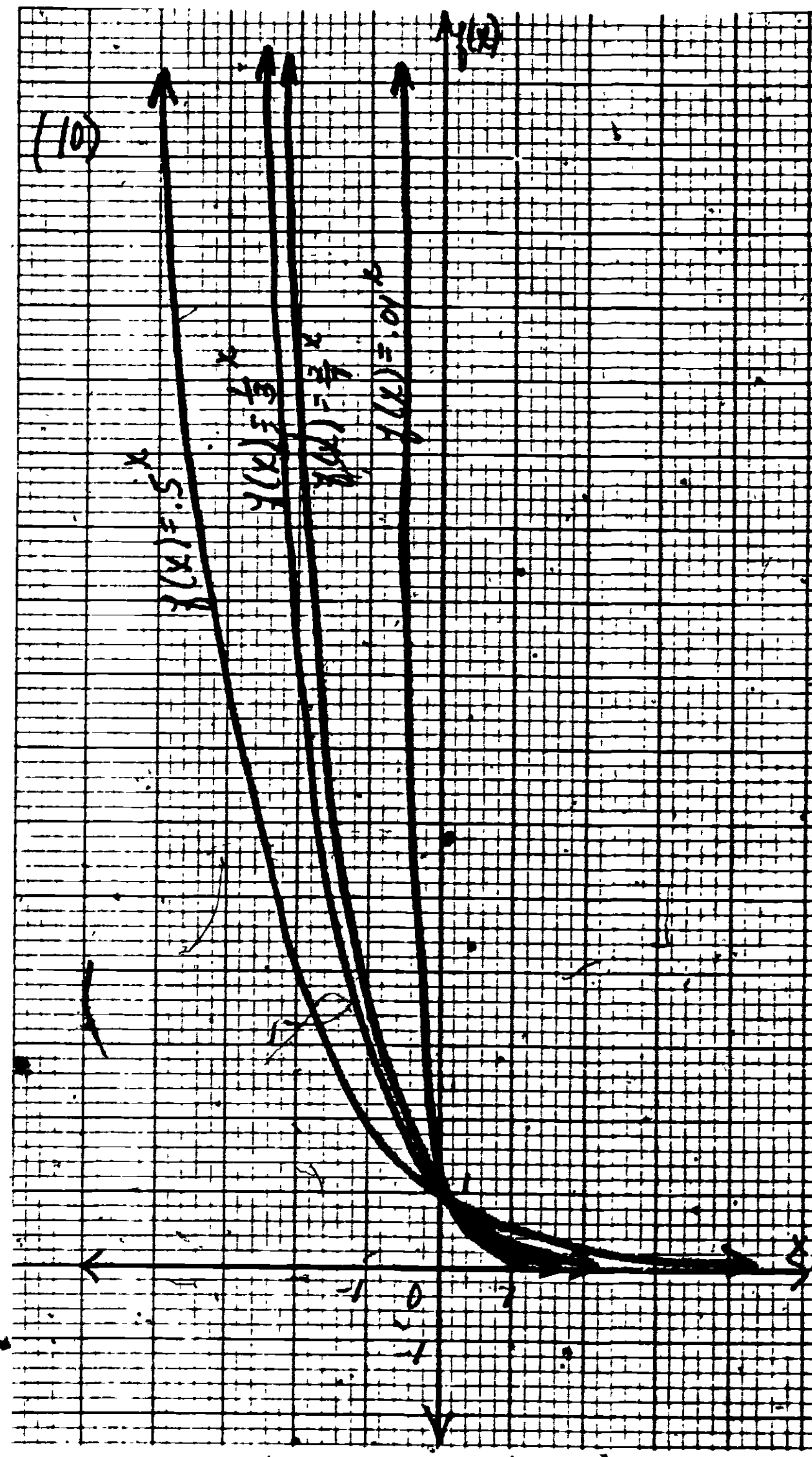
10) see graph

$.5^x$

$.01^x$

$\frac{1}{3}^x$

$\frac{2}{7}^x$



a) all reals
all reals
all reals
all reals

b) $x > 0$
$x > 0$
$x > 0$
$x > 0$

c) 1
1
1
1

d) .5
.01
$\frac{1}{3}$

e) $f(x) \rightarrow +\infty$
$f(x) \rightarrow +\infty$
$f(x) \rightarrow +\infty$
$f(x) \rightarrow +\infty$

f) $f(x) \rightarrow 0$
$f(x) \rightarrow 0$
$f(x) \rightarrow 0$
$f(x) \rightarrow 0$

g) cont. cont. cont. cont.

h) $\log_{.5} x$ $\log_{.01} x$ $\log_{\frac{1}{3}} x$ $\log_{\frac{2}{7}} x$

i) monotonic decreasing

11) The domain is all real numbers.

The range is all positive real numbers.

$f(0) = 1$

$f(1) = b$

$\lim_{x \to -\infty} f(x) = +\infty$

$\lim_{x \to +\infty} f(x) = 0$

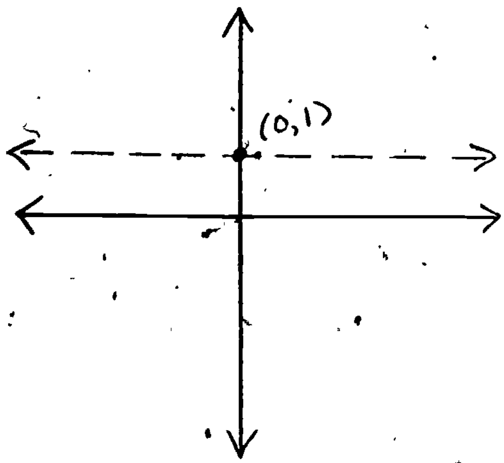
The functions are continuous.

The inverse is $f(x) = \log_b x$.

The functions are monotonic decreasing.

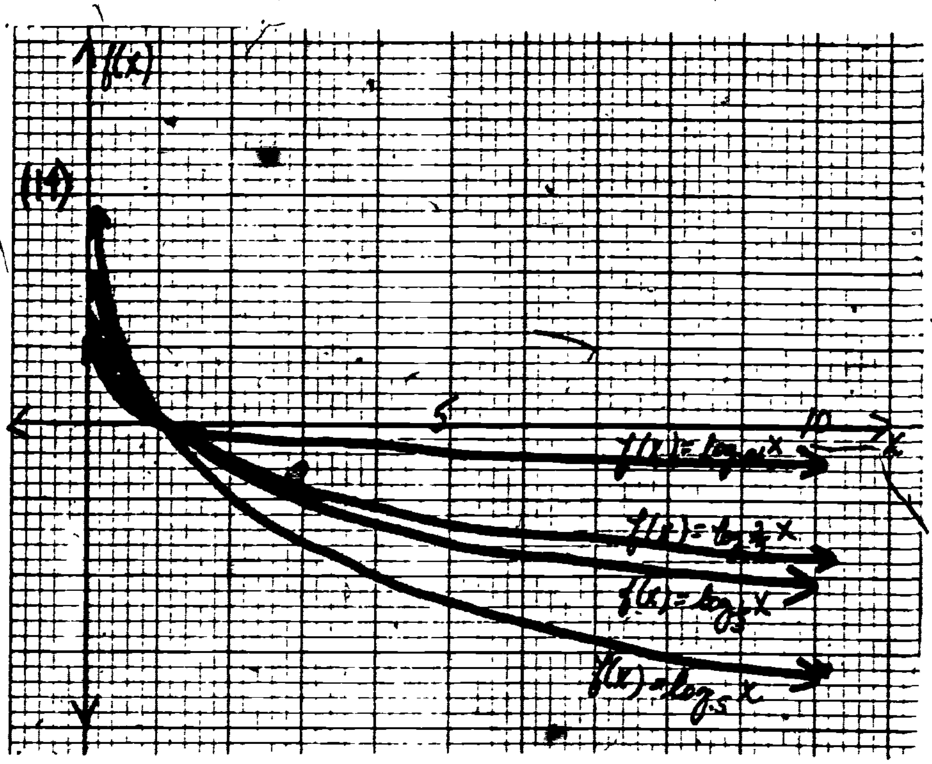
12) Generalizations about $f(x) = b^x$ when $b > 0$, $b \neq 1$.

1. The domain is all real numbers.
2. The range is the positive real numbers.
3. $f(0) = 1$
4. $f(1) = b$
5. The functions are continuous.
6. The functions are monotonic.
7) $f^{-1}(x) = \log_b x$.

13) 

$f(x) = 1^x$ fits all generalizations except (2) and (7). It is not considered an exponential function because $f(x) \equiv 1$ for all x.

14)



	$\log_{.5} x$	$\log_{.01} x$	$\log_{\frac{1}{3}} x$	$\log_{\frac{2}{7}} x$
a	$x > 0$	$x > 0$	$x > 0$	$x > 0$
b	all reals	all reals	all reals	all reals
c	1	1	1	1
d	.5	.01	$\frac{1}{3}$	$\frac{2}{7}$
e	$f(x) \to +\infty$	$f(x) \to +\infty$	$f(x) \to +\infty$	$f(x) \to +\infty$
f	$f(x) \to -\infty$	$f(x) \to -\infty$	$f(x) \to -\infty$	$f(x) \to -\infty$
g	cont.	cont.	cont.	cont.
h	$.5^x$	$.01^x$	$\frac{1}{3}^x$	$\frac{2}{7}^x$
i	monotonic decreasing →			

15) The domain is all positive real numbers

The range is all real numbers

$f^{-1}(0) = 1$

$f^{-1}(1) = b$

$\lim_{x \to 0} f(x) = +\infty$

$\lim_{x \to \infty} f(x) = -\infty$

The functions are continuous

$f^{-1}(x) = b^x$

The functions are monotonic decreasing.

16) The domain is all positive real numbers

The range is all real numbers

$f^{-1}(0) = 1$

$f^{-1}(1) = b$

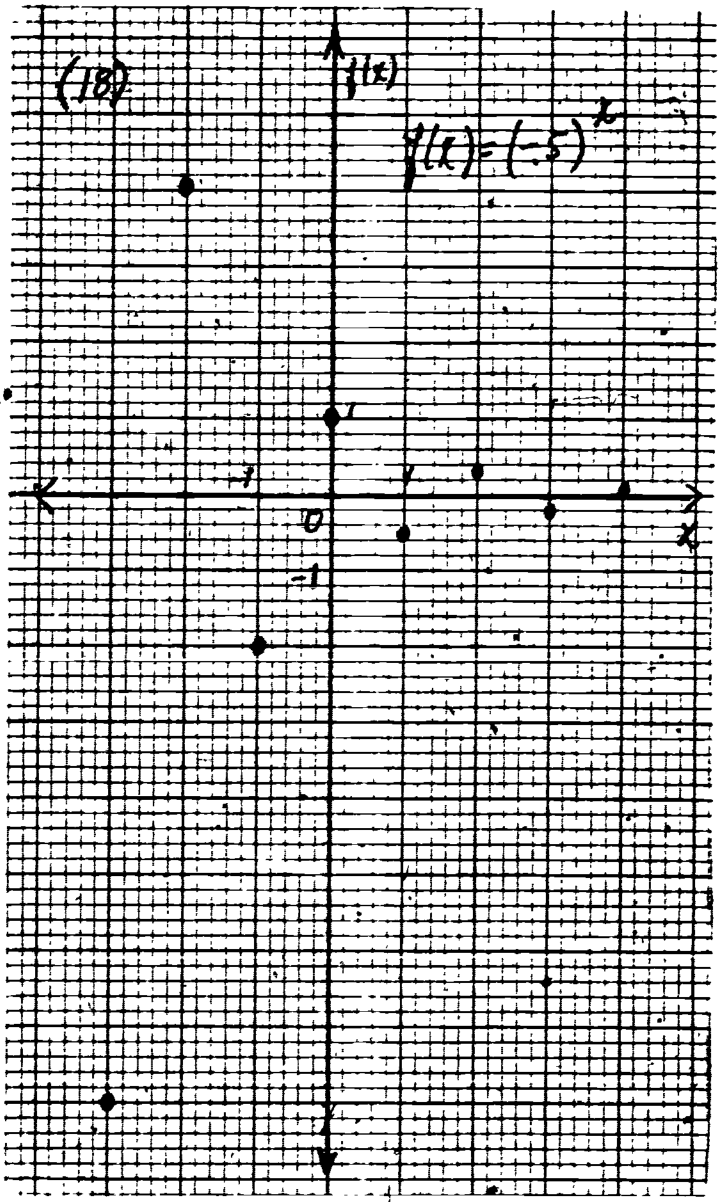
The functions are continuous

$f^{-1}(x) = b^x$

The functions are monotonic.

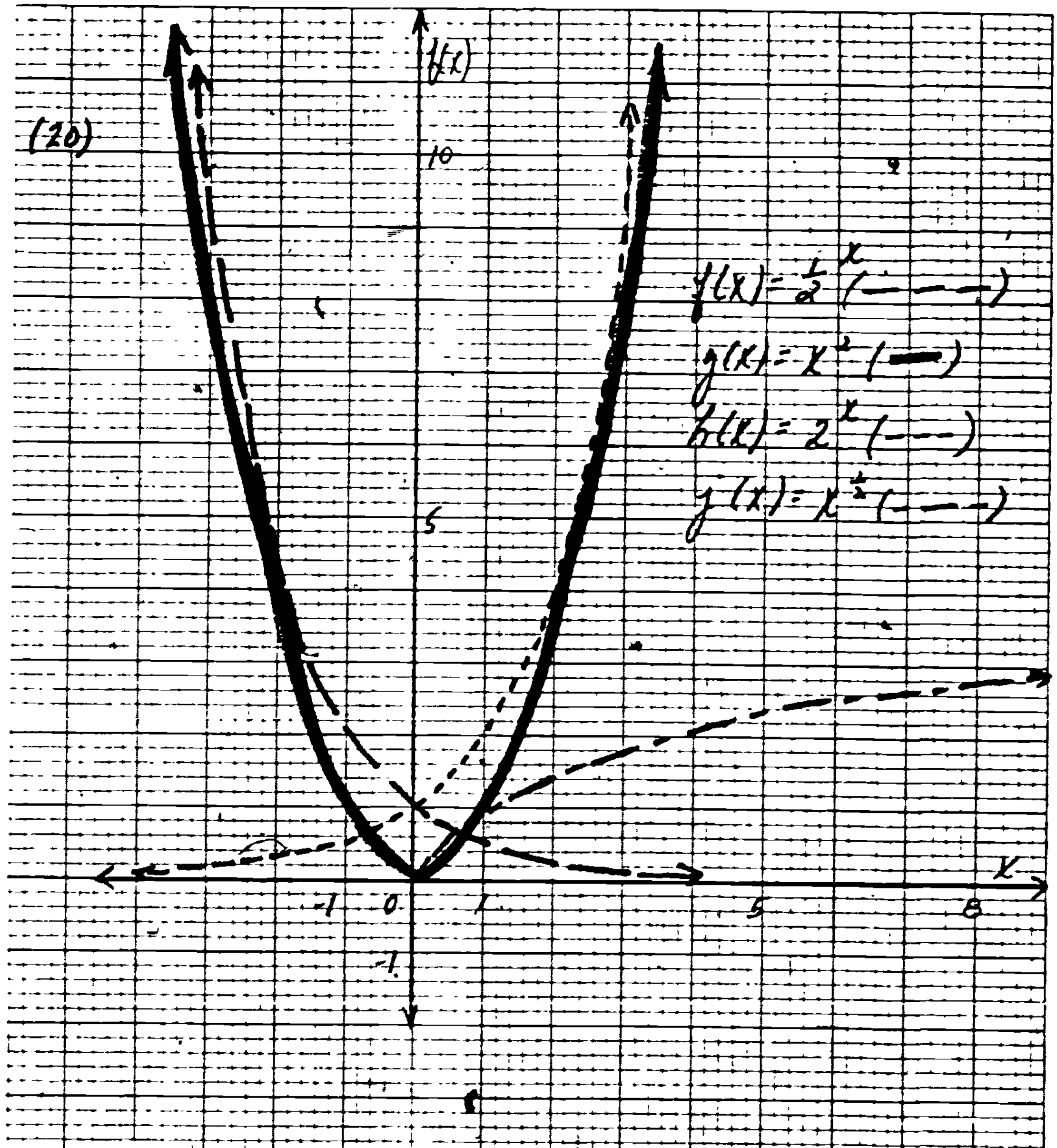
17) $f(x) = \log_b x$ when $b = 1$ cannot be graphed because 1 cannot be the base for logs. 1 is not a base for logs because $1^x = 1$ for all x.

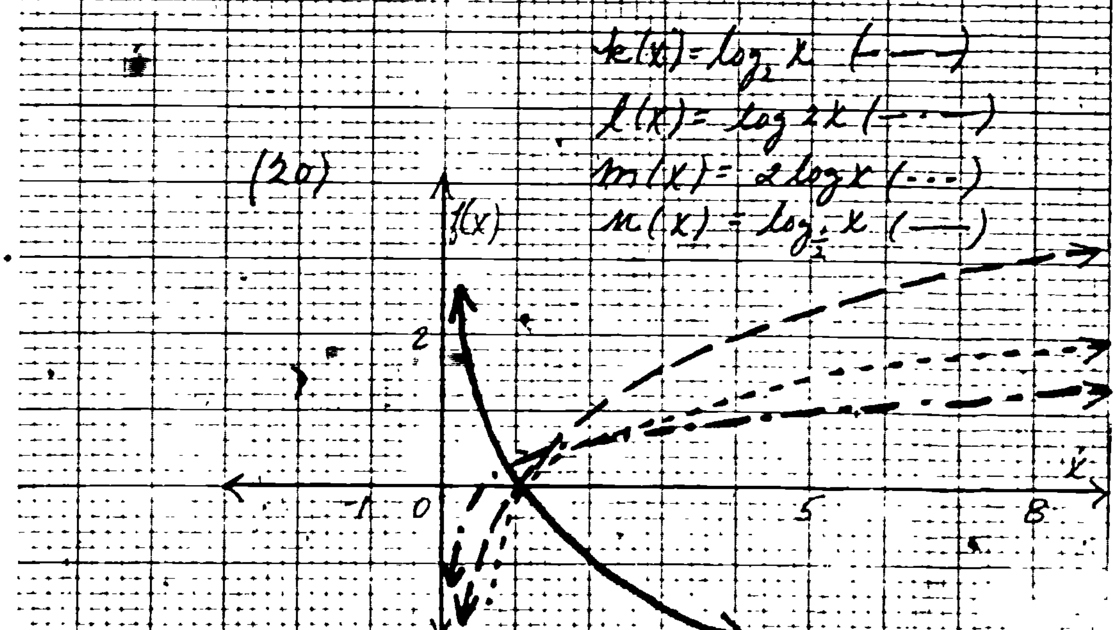
18)



19) This function is everywhere discontinuous.

20)


(20)
f(x)
10
5
-1
0
1
5
8
x
f(x) = 1/2^x (— — —)
g(x) = x² (▬▬)
h(x) = 2^x (- - -)
j(x) = x^½ (— — —)

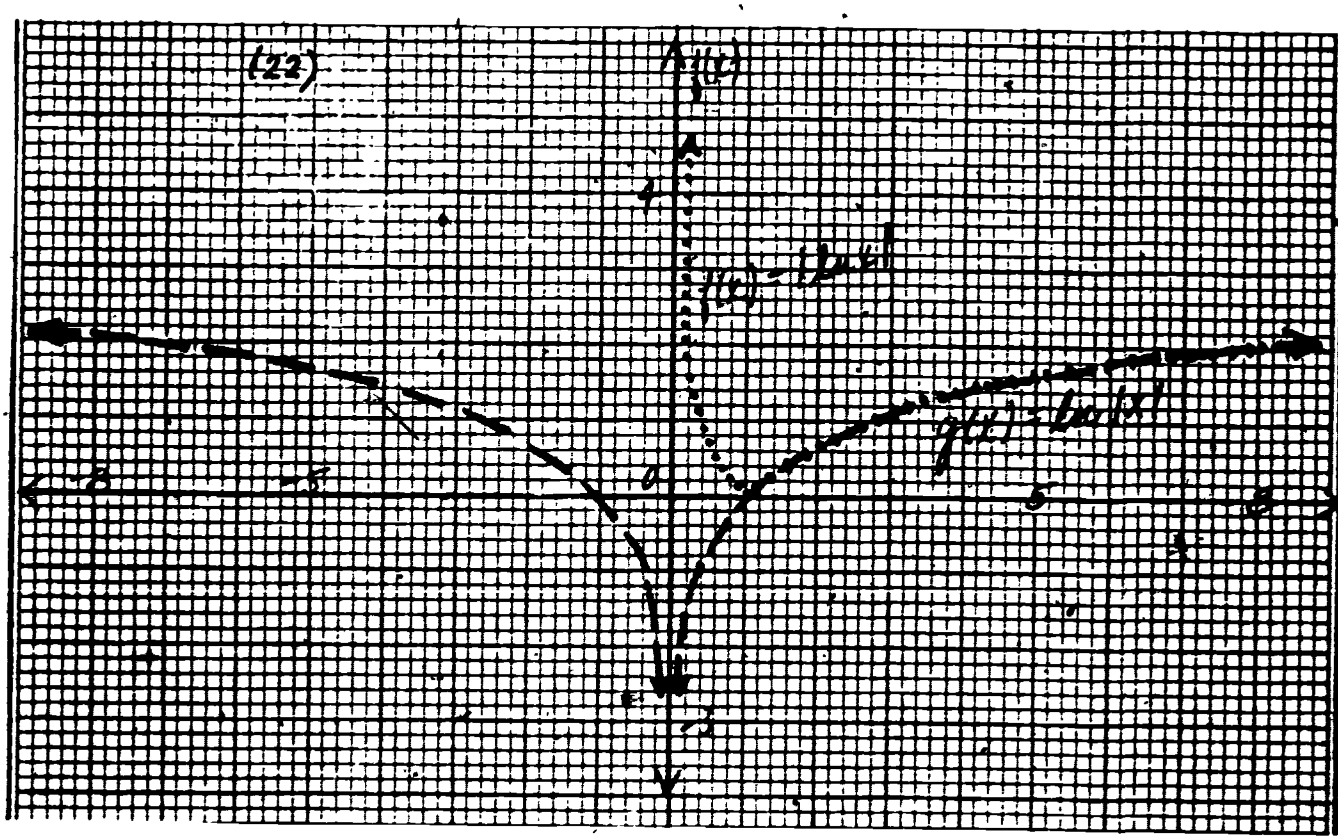

k(x) = log₂ x (——)
l(x) = log 2x (— · —)
m(x) = 2 log x (· · ·)
n(x) = log½ x (——)
(20)
f(x)
2
-1
0
5
8
x

21) $f(x) = \frac{1}{2}^{x}$ and $n(x) = \log_{\frac{1}{2}} x$ are inverses.

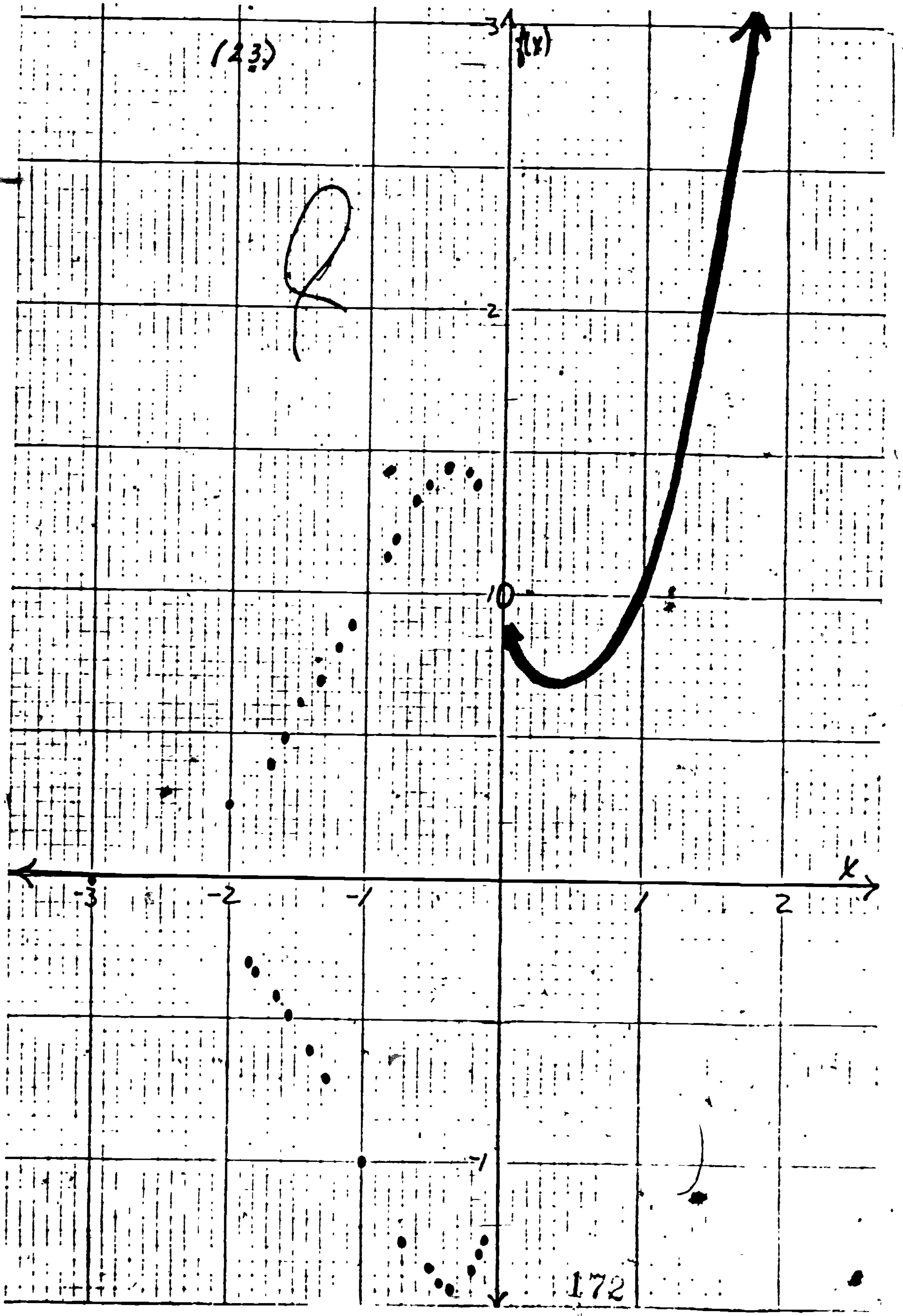
$g(x) = x^2$ and $j(x) = x^{\frac{1}{2}}$ are inverses for $x > 0$

$h(x) = 2^x$ and $k(x) = \log_2 x$ are inverses.

22) $|\ln x| \geq \ln |x|$ when $x > 0$



23) 0^0 is undefined.

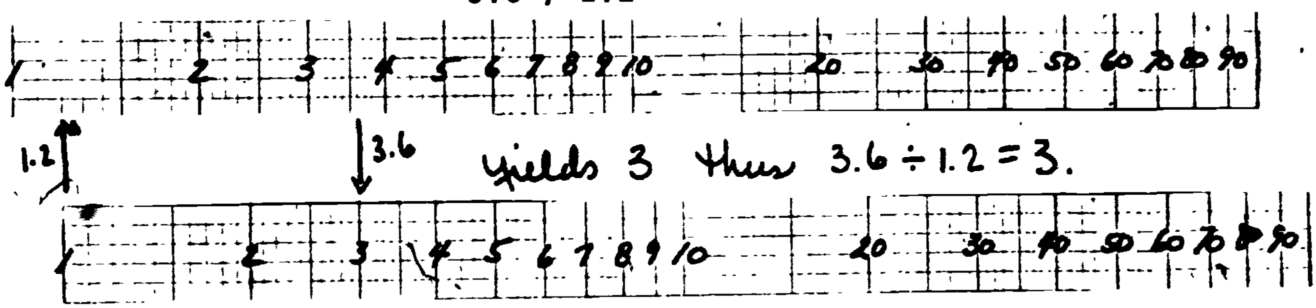


Solutions to Exercise Set 6.8

(1 - 10) Build slide rules and physically perform calculations.

(11 - 14) Divisions are performed on slide rules by the following operation:

$3.6 \div 1.2$



(15 - 18) Tables below:

15) $xy = 3$

x	5	10	50	100	500	1000
y	.6	.3	.06	.03	.006	.0003

16) $xy^3 = 4$

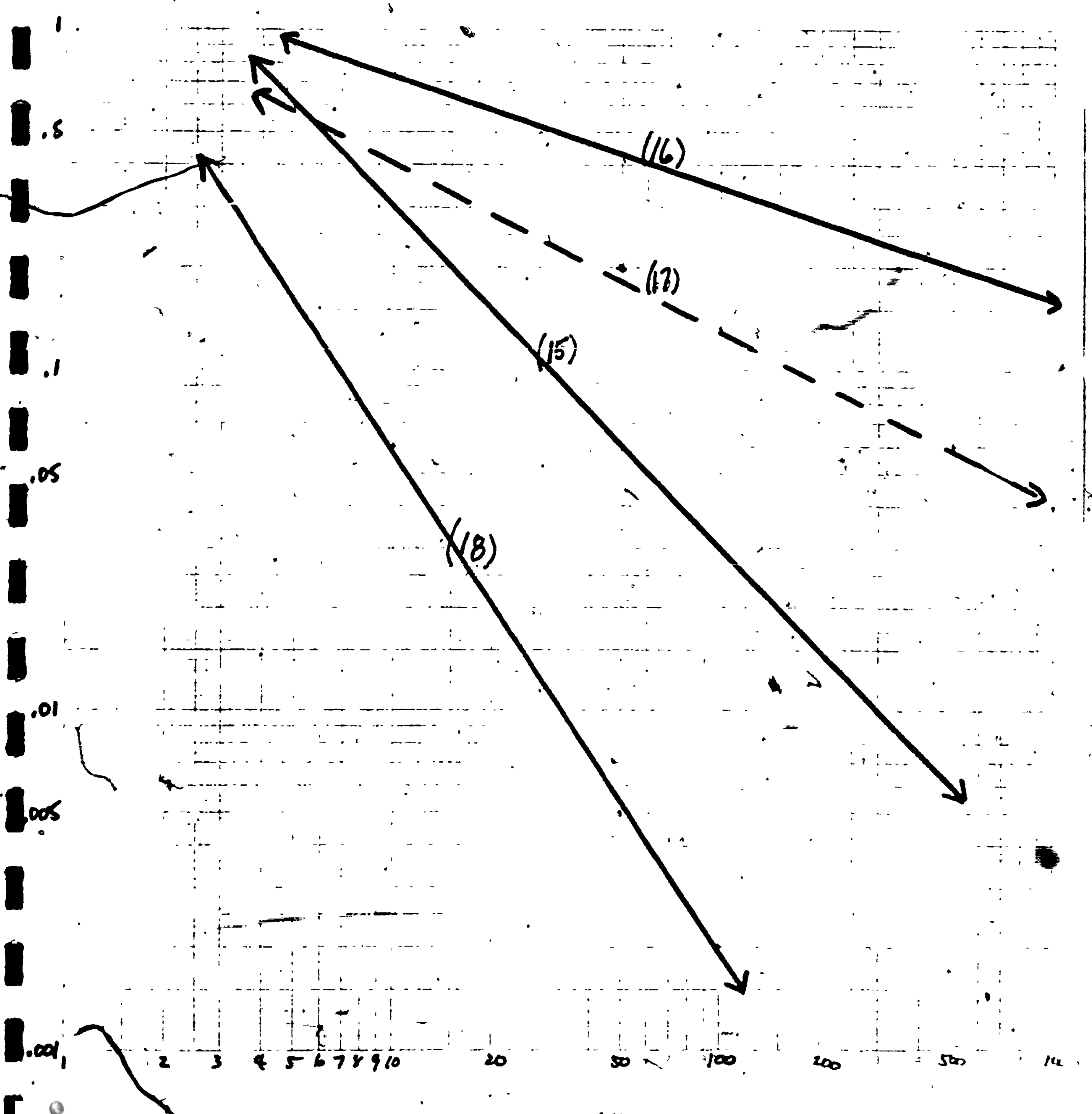
x	5	10	50	100	500	1000
y	.93	.74	.43	.34	.20	.16

17) $x y^2 = 1.7$

x	5	10	50	100	500	1000
y	.58	.41	.18	.13	.06	.04

18) $x^3y^2 = 3.1$

x	5	10	50	100	500	1000
y	.16	.06	.005	.002	.0002	.0001



(19 - 22) tables appear below

19) $y = 1.7^x$

x	0	1	2	3	4	5	6
y	1	1.7	2.89	4.91	8.35	14.2	24.14

20) $y = 2.5^x$

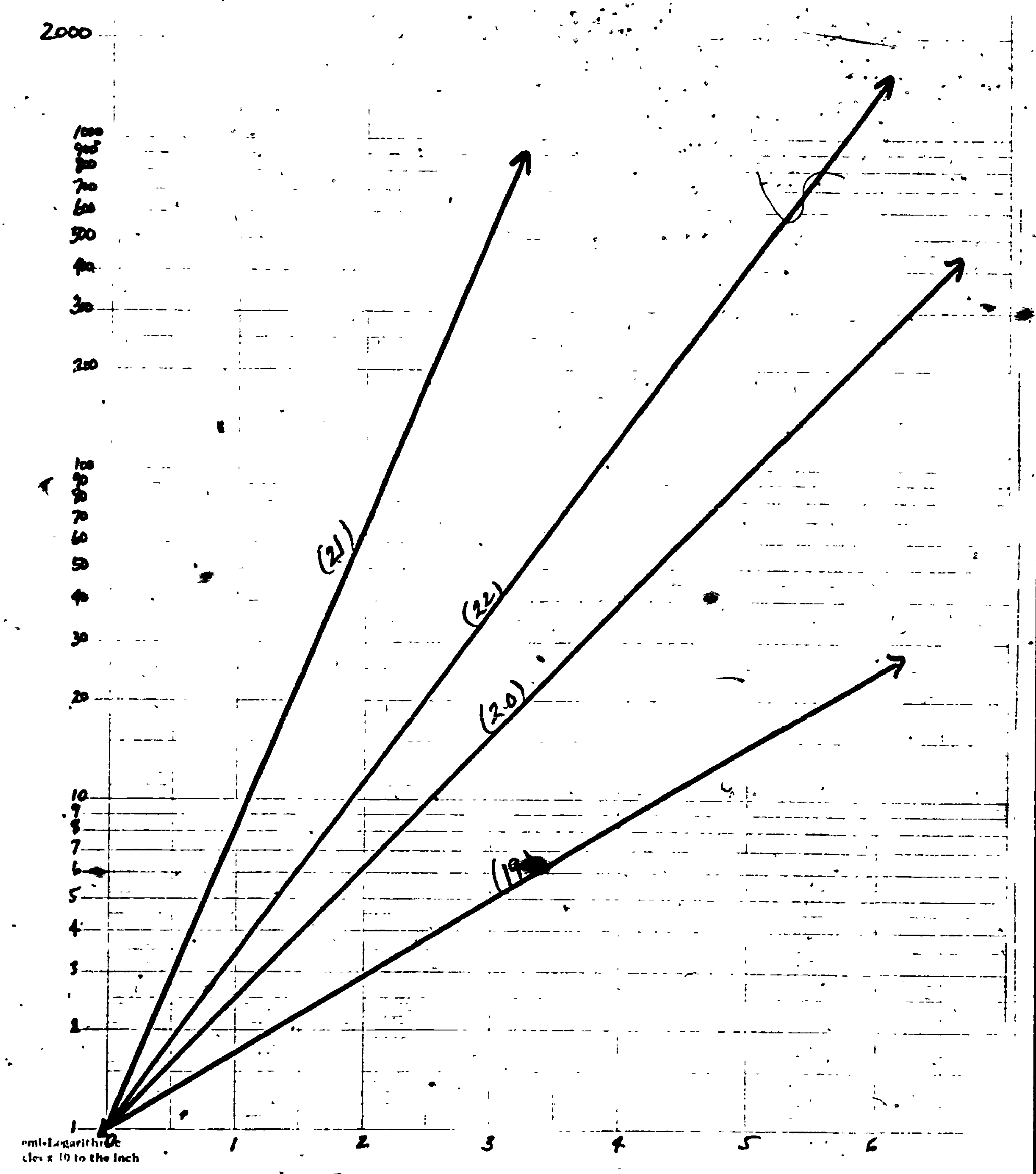
x	0	1	2	3	4	5	6
y	1	2.5	6.25	15.63	39.06	97.66	244.14

21) $y = 2^{3x}$

x	0	1	2	3	4	5	6
y	1	8	64	512	4096	32,768	262,144

22) $y = 3^{1.1x}$

x	0	1	2	3	4	5	6
y	1	3.35	11.21	37.54	125.70	420.89	1409.29


2000
1000
900
800
700
600
500
400
300
200
100
90
80
70
60
50
40
30
20
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
(21)
(22)
(20)
(19)
emi-Logarithmic
cles x 10 to the inch

23) $\log y = 3 \log x$ full log

24) $2 \log y = 3 \log x$ full log

25) $3 \log x = 1.2 \log y$ full log

26) $\log y = \log 5 + 3 \log x$ full log

27) $\log y = 2x \log 7$ semilog

28) $\log y = x \log 2$ semilog

Solutions to Chapter 6 Test

1) 1.47581

2) 1.4650

3) 852,891,037,441

4) $\frac{2 + .5}{2 - 2.5} = \frac{4}{-.5} = -8$

5) e

6) log S = log a + (n-1) log r

log X - log a = n log r - log r

$\frac{\log S - \log a + \log r}{\log r} = n$

$\frac{\log (\frac{Sr}{a})}{\log r} = n$

7) (x+2) log 5 = (x-2) log 3

x log 5 + 2 log 5 = x log 3 - 2 log 3

x log 5 - x log 3 = -2 log 5 - 2 log 3

$x = \frac{-2 \log 5 - 2 \log 3}{\log 5 - \log 3} \doteq -10.6026$

8) $\ln (x-3) = 10^{-.2}$

$x-3 = e^{10^{-.2}} \doteq 1.8794$

$x \doteq 4.8794$

9) full log

10) semilog

11) a) $N = 1000e^{.25(30)} \doteq 1,808,042$

b) $50,000 = 1000e^{.25t}$

$50 = e^{.25t}$

$\ln 50 = .25t \ln e$

$\frac{\ln 50}{.25} = t \doteq 15.6481$ minutes

12) positive real numbers

13) b

14) b^x

15) 0

16) false

17) true

18) false

19) true

20) false

Solutions to Exercise Set 7.1

(1 - 6) Answers may vary. General forms are listed below. In each case k is an integer.

1) $C(7 + k\,2\pi)$ 2) $D(5 + k\,2\pi)$

3) $C(5\pi + k\,2\pi)$ 4) $C(3\pi + k\,2\pi)$

5) $C(-.3573 + k\,2\pi)$ 6) $C(-1.5826 + k\,2\pi)$

(7-12) Answers may vary. General forms are listed below. In each case k is an integer.

7) $C^*(1.2 + 4k)$ 8) $C^*(-3.7 + 4k)$

9) $C^*(-\pi/6 + 4k)$ 10) $C^*(\sqrt{2} + 4k)$

11) $C^*(e + 4k)$ 12) $C^*(-e/\pi + 4k)$

(13-20)

13) (0,1) 14) (0,-1)

15) (-.8660, -.5) 16) (-.7071, -.7071)

17) (.2837, -.9589) 18) (-.8391, -.5440)

19) (.1559, .9878) 20) (-.9117, .4108)

21) $.7854 + k\,2\pi$ radians
$45 + k\,360$ degrees

22) $1.0472 + k\,2\pi$ radians
$60 + k\cdot 360$ degrees

23) $5.5535 + k\,2\pi$ radians
$318.1897 + k\,360$ degrees

24) $.6155 + k\,2\pi$ radians
$35.2644 + k\,360$ degrees

25) $4.0689 + k\,2\pi$ radians

$233.1301 + 360$ degrees

26) $4.3009 + k\,2\pi$ radians

$246.4215 + 360$ degrees

27) a) $(5,0)$

b) 10π

28) a) $(3,0)$

b) 6π

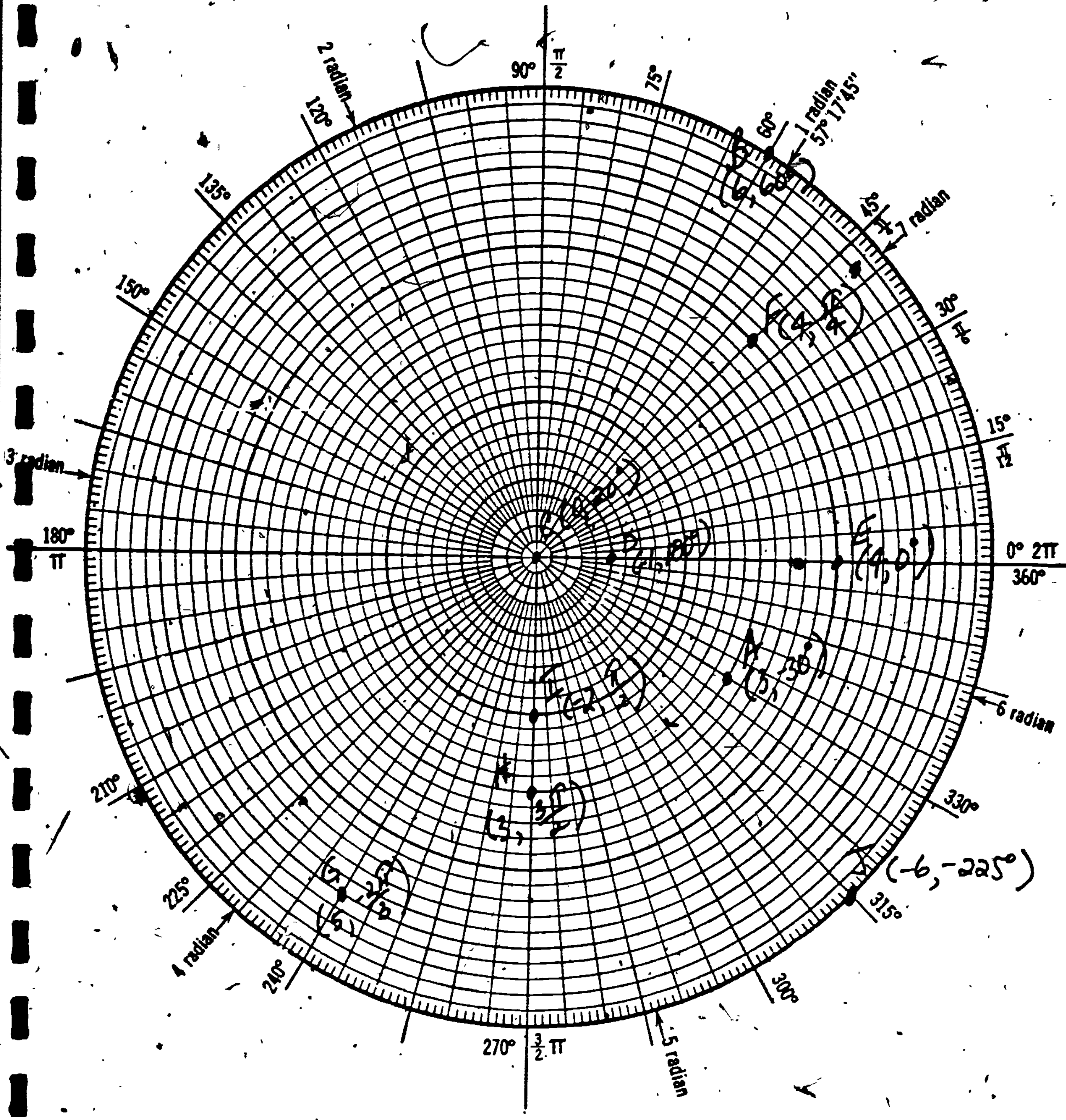
29) a) $(\pi, 0)$

b) $2\pi^2$

30) a) $(5.8310, 0)$

b) 36.6370

(1-10) Solutions to Exercise Set 7.2



11) A circle whose center is the pole and whose radius is 2.

12) A circle whose center is the pole and whose radius is 1.

13) A line through the pole which makes a 45° angle with the axis.

14) A horizontal line.

15)

HP33E	TI-58	TRS-80
(key θ)	(key ρ)	INPUT R: T*
ENTER	$x \gtrless t$	X = R*COS(T*.0174533)
(key ρ)	(key θ)	Y = R*SIN(T*.0174533)
f → R	2nd P → R	PRINT "("X","Y")"
(display is x)	(display is y)	
R ↓	$x \gtrless t$	
(display is y)	(display is x)	

16) $(2, 90^\circ)$

17) $(1, 180^\circ)$

18) $(2, 135^\circ)$

19) $(4, 330^\circ)$

20) $(5, 216.8699^\circ)$

21) $(13, 247.3801^\circ)$

22) $(0, -6)$

23) a) $(4\sqrt{3}, -4)$

b) $(6.9282, -4)$

24) $(-3, 3)$

25) a) $(\sqrt{3}, -1)$

b) $(1.7321, -1)$

26) $(10, .5)$

27) a) $(-\sqrt{3}, -1)$

b) $(-1.7321, -1)$

28) Circular functions behave like polar coordinates having radius = $\frac{\text{period}}{2\pi}$.

29) $C(5) \equiv (1, 5 \text{ radians}) = (1, 286.4789^\circ)$
$= (.2837, -.9589$ rectangular

*R represents ρ, T represents θ

30) $(.5, \sqrt{3}/2)_{rectangular} = (1, 60^{o})_{polar} = (1, 1.0472 \text{ radians})_{polar}$
$t = 1.0472$

Solutions to Exercise Set 7.3

1) $\rho \cos \theta = 4$

2) $\rho \sin \theta = -3$

3) $2\rho \cos \theta - 3\rho \sin \theta = 7$

4) $\rho \cos \theta = \rho \sin \theta + 4$

5) $\rho^2 = 9$

6) $\rho^3 \sin^2 \theta \cos \theta = 10$

7) $\rho^2 \sin \theta \cos \theta = 7$

8) $\rho^2 - 3\rho \cos \theta + 2\rho \sin \theta = 0$

9) $\rho^2 \cos^2 \theta - \rho^2 \sin^2 \theta = 10$

10) $\rho \sin^2 \theta = 3 \cos \theta$

11) $x^2 + y^2 = 9$

12) $\sqrt{3}x - 3y = 0$

13) $x^2 + y^2 - 3x - 3y = 0$

14) $x^2 + y^2 + 2x - 3y = 0$

15) $x = 8$

16) $x^2 y + y^3 = 9$

17) $8x^2 - y^2 - 12x + 4 = 0$

18) $2x + y = 3$

19) $x = 1$

20) $x^4 = x^2 y^2 - y^2 = 0$

21) limaçon - inner loop

22) cardiod

23) 4 leafed rose

24) 3 leafed rose

25) lemniscate w/o origin

26) line

27) ellipse

28) parabola

29) cardioid

30) limacon - no loop

31) Spiral of Archimedes

32) reciprocal spiral

33) logarithmic spiral

34) line

Solutions to Exercise Set 7.4

1) $3.6056 (\cos 303.6901^\circ + i \sin 303.6901^\circ)$

2) $5.8310 (\cos 149.0362^\circ + i \sin 149.0362^\circ)$

3) $6 (\cos 330^\circ + i \sin 330^\circ)$

4) $2 (\cos 60^\circ + i \sin 60^\circ)$

5) $1 (\cos 180^\circ + i \sin 180^\circ)$

6) $6 (\cos 0^\circ + i \sin 0^\circ)$

7) $5 (\cos 90^\circ + i \sin 90^\circ)$

8) $3 (\cos 270^\circ + i \sin 270^\circ)$

9) $4.3301 + 2.5i$

10) $-2.5981 - 1.5i$

11) $-3i$

12) 1

13) $-.3427 + .6446i$

14) $-1.5526 - 3.0472i$

15) $.1510 - 1.7255i$

16) $-1.9662 + 1.7704i$

17) $15 (\cos 116^\circ + i \sin 116^\circ)$

18) $14 (\cos 348^\circ + i \sin 348^\circ)$

19) $3 (\cos 51^\circ + i \sin 51^\circ)$

20) $\sqrt{2}/2 (\cos 135^\circ + i \sin 135^\circ)$

21) $1/3 (\cos 150^\circ + i \sin 150^\circ)$

22) $2.5 (\cos 161^\circ + i \sin 161^\circ)$

23) $9 (\cos 36^\circ + i \sin 36^\circ)$

24) $25 (\cos 140^\circ + i \sin 140^\circ)$

25) $2 + 3i\sqrt{3} \doteq 5.5678 (\cos 68.9483^\circ + i \sin 68.9483^\circ)$

$2 - i\sqrt{3} \doteq 2.6458 (\cos 319.1066^\circ + i \sin 319.1066^\circ)$

product $\doteq 14.7313 (\cos 28.0549^\circ + i \sin 28.0549^\circ) \doteq 13.0003 + 6.93$

26) $\sqrt{3} - i = 2\ (\cos 330^\circ + i \sin 330^\circ)$

$1 + i\sqrt{3} = 2\ (\cos 60^\circ + i \sin 60^\circ)$

quotient $= 1\ (\cos 270^\circ + i \sin 270^\circ) = -i$

27) $5 - 2i \doteq 5.3852\ (\cos 338.1986^\circ + i \sin 338.1986^\circ)$

$6 + .5i \doteq 6.0208\ (\cos 4.7636^\circ + i \sin 4.7636^\circ)$

quotient $\doteq -.6356\ (\cos 333.4224^\circ + i \sin 333.4224^\circ)$

$\doteq -.5684 + .2844i$

28) $(1 + 3i) \doteq 3.1623\ (\cos 71.5651 + i \sin 71.5651)$

power $\doteq 100.0028\ (\cos 286.2604 + i \sin 286.2604)$

$\doteq 28.0011 - 96.0026i$

$= 28 - 96i$

29) $d = \sqrt{(a-c)^2 + (b-d)^2}$

30) $r_1 = r_2$ because the radii must be the same

$\theta_1 = \theta_2 + k.360^\circ$ because the angles must put the point in the same position in the plane and $x = x + k360$ for all angles x.

Solutions to Exercise Set 7.5

1) $[3.1623 (\cos 161.5651^\circ + i \sin 161.5651^\circ)]^4$

$100 (\cos 286.2628^\circ + i \sin 286.2628^\circ)$

$28 - 96i$

2) $[3.6056 (\cos 303.6901^\circ + i \sin 303.6901^\circ)]^5$

$609.3793 (\cos 78.4505^\circ + i \sin 78.4505^\circ)$

$122 + 597i$

3) $243 (\cos 140^\circ + i \sin 140^\circ) \doteq -186.1488 + 156.1974i$

4) $128 (\cos 29^\circ + i \sin 29^\circ) \doteq 111.9513 + 62.0556i$

5) $[1.7321 (\cos 305.2644^\circ + i \sin 305.2644)]^{10}$

$243 (\cos 172.6440^\circ + i \sin 172.6440^\circ) = -241 + 31.1123i$

6) $[2 (\cos 330^\circ + i \sin 330^\circ)]^7 =$

$128 (\cos 150^\circ + i \sin 150^\circ) = -110.8513 + 64i$

7) $[1.4142 (\cos 225 + i \sin 225)]^{-8} =$

$.0625 (\cos 0^\circ + i \sin 0^\circ) = .0625$

8) $[1.4142 (\cos 135 + i \sin 135)]^{-6} =$

$.1250 (\cos 270 + i \sin 270) = .1250\, i$

9) $2 - 2i\sqrt{3} = 4 (\cos 300 + i \sin 300)$

$1.4142 (\cos 75^\circ + i \sin 75^\circ) = .3660 + 1.3660i$

$1.4142 (\cos 165^\circ + i \sin 165^\circ) = -1.3660 + .3660i$

$1.4142 (\cos 255^\circ + i \sin 255^\circ) \doteq -.3660 - 1.3660i$

$1.4142 (\cos 345^\circ + i \sin 345^\circ) = 1.3660 - .3660i$

10) $(-5 + 3i) = 5.8310\ (\cos 149.0362^\circ + i \sin 149.0362^\circ)$

$1.7999\ (\cos 49.6787^\circ + i \sin 49.6787^\circ) = 1.1647 + 1.3723i$

$1.7999\ (\cos 169.6787^\circ + i \sin 169.6787^\circ) = -1.7708 + .3225i$

$1.7999\ (\cos 289.6787^\circ + i \sin 289.6787^\circ) = .6061 - 1.6948i$

11) $1 = (\cos 0^\circ + i \sin 0^\circ)$

$1\ (\cos 0^\circ + i \sin 0^\circ) = 1$

$1\ (\cos 120^\circ + i \sin 120^\circ) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$1\ (\cos 240^\circ + i \sin 240^\circ) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$

12) $i = 1\ (\cos 90^\circ + i \sin 90^\circ)$

$1\ (\cos 22.5^\circ + i \sin 22.5^\circ) = .9239 + .3827i$

$1\ (\cos 112.5^\circ + i \sin 112.5^\circ) = -.3827i + .9239i$

$1\ (\cos 202.5^\circ + i \sin 202.5^\circ) = -.9239 - .3827i$

$1\ (\cos 292.5^\circ + i \sin 292.5^\circ) = .3827 - .9239i$

13) $x = 1^{\frac{1}{4}} = \left[1\ (\cos 0^\circ + i \sin 0^\circ)\right]^{\frac{1}{4}}$

$1\ (\cos 0^\circ + i \sin 0^\circ) = 1$

$1\ (\cos 90 + i \sin 90^\circ) = i$

$1\ (\cos 180 + i \sin 180^\circ = -1$

$1\ (\cos 270 + i \sin 270^\circ = -i$

14) $x = 32^{\frac{1}{5}} = \left[32\ (\cos 0^\circ + i \sin 0^\circ)\right]^{\frac{1}{5}}$

$2\ (\cos 0^\circ + i \sin 0^\circ) = 2$

$2\ (\cos 72^\circ + i \sin 72^\circ) = .6180 + 1.9021i$

$2\ (\cos 144^\circ + i \sin 144^\circ) = -1/6180 + 1.1756i$

$2\ (\cos 216^\circ + i \sin 216^\circ) = -1.6180 - 1.756i$

$2\ (\cos 288^\circ + i \sin 288^\circ) = .6180 - 1.9021i$

15) $x = (-27i)^{\frac{1}{3}} = \left[-27 (\cos 90^\circ + i \sin 90^\circ)\right]^{\frac{1}{3}}$

$-3 (\cos 30^\circ + i \sin 30^\circ) = -2.5981 - 1.5i$

$-3 (\cos 150^\circ + i \sin 150^\circ) = 2.5981 - 1.5i$

$-3 (\cos 270^\circ + i \sin 270^\circ) = 3i$

16) $x = (-1 + i\sqrt{2})^{\frac{1}{2}} = \left[1.732 (\cos 125.2644^\circ + i \sin 125.2644^\circ)\right]^{\frac{1}{2}}$

$1.3161 (\cos 31.3161^\circ + i \sin 31.3161^\circ) = 1.1244 + .6841i$

$1.3161 (\cos 211.3161^\circ + i \sin 211.3161^\circ) = -1.1244 - .6841i$

17) $\left[r (\cos \theta + i \sin \theta)\right]^n = \left[r e^{i\theta}\right]^n = r^n (e^{i\theta})^n =$

$r^n (e^{i n\theta}) = r^n \left[\cos(n\theta) + i \sin (n\theta)\right]$.

18)
$$\begin{array}{l} \cos\theta + i \sin\theta = e^{+i\theta} \\ \underline{-\cos\theta + i \sin\theta = -e^{-i\theta}} \end{array}$$

$$\frac{2i \sin\theta}{2i} = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

19)
$$\begin{array}{l} \cos\theta + i \sin\theta = e^{i\theta} \\ \underline{\cos\theta - i \sin\theta = +e^{-i\theta}} \end{array}$$

$$\frac{2 \cos\theta}{2} = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

20) $2^{\sqrt{2}}$ has infinitely many values, because $360 \sqrt{2}\, n$ is not an integral multiple of 360 so there will never be two coterminal angles. $2^{\sqrt{2}}$ is always imaginary since $\sqrt{2}\,(90 + 360n)^\circ$ will never be a number on the real axis.

Solutions to Exercise Set 7.6

1) $e^2 \doteq 1 + 2 + \frac{2^2}{2} + \frac{2^3}{6} + \frac{2^4}{24} + \frac{2^5}{120} + \frac{2^6}{720} \doteq 7.3556$

$e^2 \doteq 7.3891$ by calculating device

2) $e^3 \doteq 1 + 3 + \frac{3^2}{2} + \frac{3^3}{6} + \frac{3^4}{24} + \frac{3^5}{120} + \frac{3^6}{720} \doteq 19.4125$

$e^3 \doteq 20.0855$ by calculating device

3) $e^{.3} \doteq 1 + .3 + \frac{.3^2}{2} + \frac{.3^3}{6} + \frac{.3^4}{24} + \frac{.3^5}{120} + \frac{.3^6}{720} \doteq 1.3499$

$e^{.3} \doteq 1.3499$ by calculating device

4) $e^{.6} \doteq 1 + .6 + \frac{.6^2}{2} + \frac{.6^3}{6} + \frac{.6^4}{24} + \frac{.6^5}{120} + \frac{.6^6}{720} \doteq 1.8221$

$e^{.6} \doteq 1.8221$ by calculating device

5) $e^{-2} \doteq 1 - 2 + \frac{2^2}{2} - \frac{2^3}{6} + \frac{2^4}{24} - \frac{2^5}{120} + \frac{2^6}{720} \doteq$ ~~.1556~~

$e^{-2} \doteq .1353$ by calculating device

6) $e^{-1} \doteq 1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720} \doteq .3681$

$e^{-1} \doteq .3679$ by calculating device

7) $\sin \pi \doteq \pi - \frac{\pi^3}{6} + \frac{\pi^5}{120} - \frac{\pi^7}{5040} + \frac{\pi^9}{362880} \doteq .0069$

$\sin \pi = -.0000000004$ by calculating device

8) $\cos \pi \doteq 1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \frac{\pi^8}{40320} \doteq -.9760$

$\cos \pi = -1$ by calculating device

9) $\cos \frac{\pi}{6} \doteq 1 - \frac{\left(\frac{\pi}{6}\right)^2}{2} + \frac{\left(\frac{\pi}{6}\right)^4}{24} - \frac{\left(\frac{\pi}{6}\right)^6}{720} + \frac{\left(\frac{\pi}{6}\right)^8}{40320} \doteq .8660$

$\cos \frac{\pi}{6} = .8660$ by calculating device

10) $\sin \frac{\pi}{2} \doteq \frac{\pi}{2} - \frac{\left(\frac{\pi}{2}\right)^3}{6} + \frac{\left(\frac{\pi}{2}\right)^5}{120} - \frac{\left(\frac{\pi}{2}\right)^7}{5040} + \frac{\left(\frac{\pi}{2}\right)^9}{362880} \doteq 1.0000$

$\sin \frac{\pi}{2} = 1$ by calculating device

11) $\cos(-\pi) \doteq 1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \frac{\pi^8}{40320} \doteq -.9760$

$\cos(-\pi) = -1$ by calculating device

12) $\sin(-\pi) \doteq -\pi + \frac{\pi^3}{6} - \frac{\pi^5}{120} + \frac{\pi^7}{5040} - \frac{\pi^9}{362880} \doteq -.0069$

$\sin(-\pi) = -.0000000004$ by calculating device

13) $\sin 27^\circ \doteq \sin(.4712) = .4712 - \frac{(.4712)^3}{6} + \frac{(.4712)^5}{120}$

$- \frac{(.4712)^7}{5040} + \frac{(.4712)^9}{362880} \doteq .4540$

$\sin 27^\circ \doteq .4510$ by calculating device

14) $\cos 123^\circ = \cos(2.1468) \doteq 1 - \frac{(2.1468)^2}{2} + \frac{(2.1468)^4}{24}$

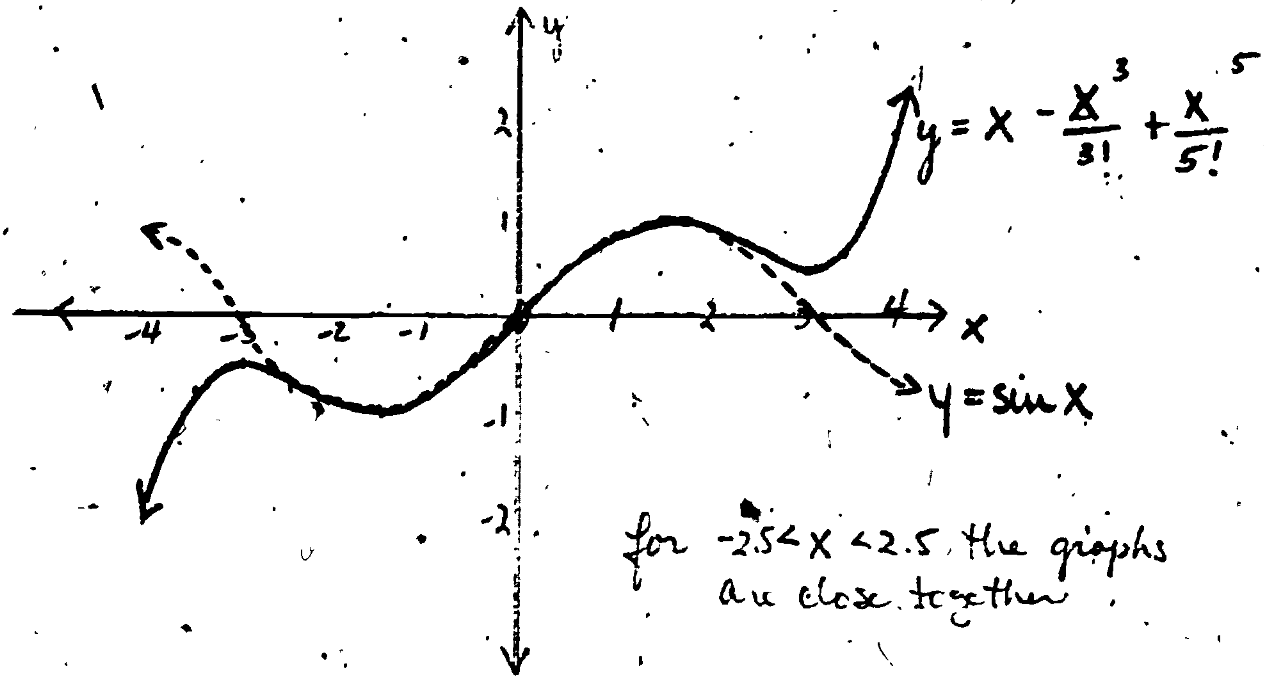
$- \frac{(2.1468)^6}{720} + \frac{(2.1468)^8}{40320} \doteq -.5441$

$\cos 123^\circ = -.5446$ by calculating device

15) One decimal place because $\frac{2^6}{720} \doteq .0889$

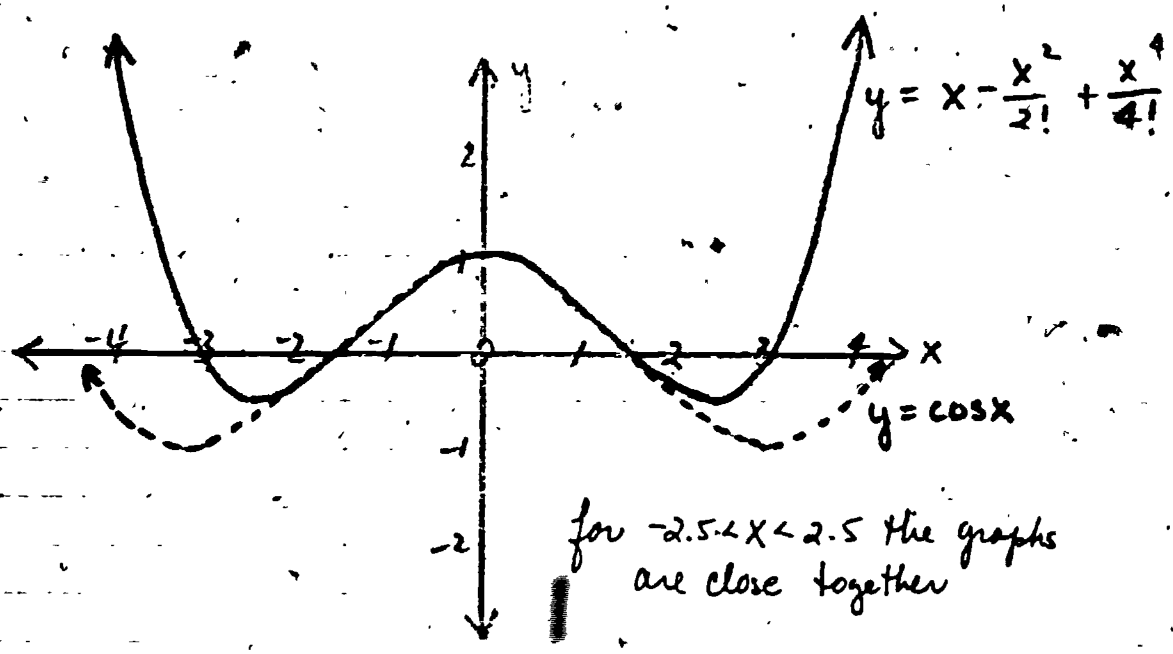
16) Six decimal places because $\frac{\left(\frac{\pi}{6}\right)^8}{40320} \doteq .000000140$

17.)



for $-2.5 < x < 2.5$ the graphs are close together

18.)



for $-2.5 < x < 2.5$ the graphs are close together

19) .8660 (25404)

20) -1

21) -1

22) -.5446 (39036)

Solutions to Chapter 7 TEST

1) (3,3)

2) $(-4, 330^\circ)$

3) $\rho\cos\theta + \rho\sin\theta = 6$

4) $x^2 + y^2 + 3x = 4$

5) $-1.7321 - i$

6) $\rho^2 = 2\rho\cos\theta + 15$

7) $1.4142(\cos 315^\circ + i\sin 315^\circ)$

8) $3.4641 + 2i$

9) $2(\cos 330^\circ + i\sin 330^\circ)$

10) $12 + 0i$

11) 2

12) $7.2111(\cos 123.6901^\circ + i\sin 123.6901^\circ)$

13) $8 + 8\sqrt{3}i$

14) $2(\cos 45^\circ + i\sin 45^\circ)$

15) $1.1402(\cos 127.875^\circ + i\sin 127.875^\circ)$

16) a) $32(\cos 225^\circ + i\sin 225^\circ)$

b) $-8 + 0i$

17) a) $.5215 + 1.0235i$
$-.8123 + .8123i$
$-1.0235 - .5215i$
$.1797 - 1.1346i$
$1.1346 - .1797i$

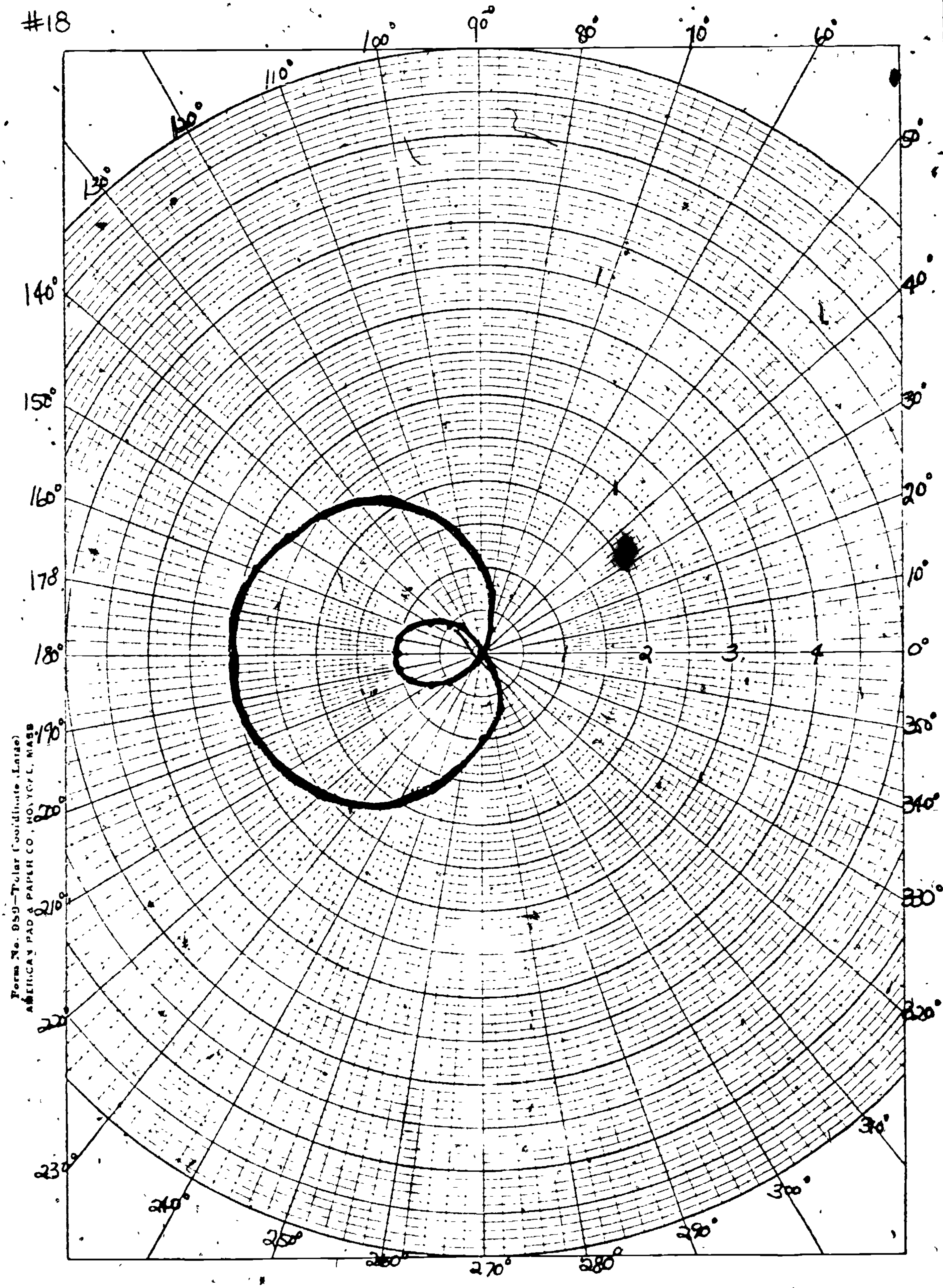
b) $1.1487(\cos 63^\circ + i\sin 63^\circ)$
$1.1487(\cos 135^\circ + i\sin 135^\circ)$
$1.1487(\cos 207^\circ + i\sin 207^\circ)$
$1.1487(\cos 279^\circ + i\sin 279^\circ)$
$1.1487(\cos 351^\circ + i\sin 351^\circ)$

18) $r = 1 - 2\cos\theta$

θ	r
10	-1.0
20	-.9
30	-.7
40	-.5
50	-.3
60	0
70	.3
80	.7
90	1.0
100	1.3
110	1.7
120	2.0
130	2.3
140	2.5
150	2.7
160	2.9
170	3.0
180	3.0

θ	r
190	3.0
200	2.9
210	2.7
220	2.5
230	2.3
240	2.0
250	1.7
260	1.3
270	1.0
280	.7
290	.3
300	0
310	-.3
320	-.5
330	-.7
340	-.9
350	-1.0
360	-1.0

#18


90°
100°
80°
70°
60°
110°
120°
50°
130°
140°
40°
150°
30°
160°
20°
170°
10°
180°
0°
2
3
4
190°
350°
200°
340°
210°
330°
220°
320°
230°
310°
240°
300°
250°
290°
260°
270°
280°