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ABSTRACT

This teacher's guide is designed to aid in the incorporation of programable calculators in the school mathematics program for pupils in grade 11. Warnings include the need for care in modifying the curriculum so that students are not punished in the process. The concept of "black boxing," of letting the computer or calculator take charge of education, is stated as a concern that pupils may lose conceptual understanding of computation and take for granted that these devices can carry out difficult computations easily and efficiently. However, the benefits are seen to present powerful arguments for calculator use in the instructional program. In addition to discussing the pros and cons of programable calculators, the brief introduction gives ideas on student access to calculators, rules and guidelines for calculator selection, approaches to classroom presentation, and hints on calculator-caused changes in classroom dynamics. The bulk of this document consists of answers to problems from the student textbook. (MP)

USING CALCULATORS IN MATHEMATICS

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USING CALCULATORS IN MATHEMATICS 11

TEACHER'S GUIDE

Introduction

Calculating tools have been utilized over the full span of civilization. The earliest records indicate that various forms of abacus-like apparatus were used still earlier. Today's hand held calculator provides only the latest step in the development of these labor saving devices. But these pocket-sized tools represent more than a difference in degree from their predecessors, such devices as slide rules; they represent a difference in kind. They triple or quadruple the number of digits accurately determined by a slide rule, thus multiplying accuracy by some ten million times! They carry out remarkably complex calculations that astound those of us who used paper, pencil, specialized tables, and much time to compute in the "old days" of just ten years ago! Thus we have in a few seconds of key punching:

$$\cos 32^\circ\ 14'\ 30'' = .84580542$$

and

$$\sqrt{3}^{\sqrt{5}} = 3.4153702$$

Consider computing those values to half this number of digits of accuracy before calculator access.

And now the programmables: the power of a half million dollar computer of twenty years ago shrunk into a \$50 - \$100 pocket-sized machine. Whole new vistas are opened to us. One of the earliest examples of practical use of a programmable communicated to me is one that will appeal to teachers.

> A school bargaining team was presented a modified salary proposal by a school board, a proposal the board negotiator said would require a postponement so that the full scale could be calculated. "No need," said the teacher representative, "We'll calculate that for you in ten minutes." And so they did,* providing not only the scale itself but also the cost of implementing that scale for current staff: all calculated on a programmable. Needless to say, the board was impressed.

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*For example, a simple program like the following would generate a column in a 5% per step increase schedule:

HP - 25		TI - 58	
01	ENTER	01	LBL A
02	1	02	X
03	.	03	1
04	0	04	.
05	5	05	0
06	X	06	5
07	R/S**	07	=
08	GTO 01	08	R/S**
		09	GTO A

**The base salary is keyed into the calculator, R/S is pressed, and subsequent steps are read each time the calculator stops. To restart R/S is pressed again. For example, a scale starting at $9800 and incrementing in 5% steps would give

$9800.00
10290.00
10804.50
11344,72
11911.96, etc.

This simple but suggestive example only reaches the border of the wide range of programming applications.

The ready availability of programmable hand held calculators does not, of course, by itself imply that they should be used in the school mathematics program. Unless a useful role in the curriculum can be found for them, they belong there no more than does another recent invention, the hula hoop. Despite our facetious example, this is, we believe, an important issue. The school mathematics program is already a full one and we should always think carefully about tinkering with it. Curriculum workers have too often thought in terms of program additions rather than the more appropriate program replacements. When something new enters, something old must exit.

Our experience so far with programmables convinces us that there is an appropriate role for them in the grade eleven program as it is presently constituted. In fact we have convinced ourselves of the truth of the following postulates:

- The calculator is useful in a number of topics involving computation. Inversely, reasonable use of the calculator is restricted to those topics. Understanding this strict deliniation is important: the idea of a calculator in use every day of the school year is popular but wrong-headed.
- Gadget fascination sets traps as you address even appropriate curricular units. Playing with the calculator is fun and easily takes students and teachers away from mathematical

concerns.*

- There are activities that deserve either to be discarded or to be severely reduced in this calculator age, thus providing some of the curricular space for calculators.
- When using the calculator in the mathematics program, great care must be taken to avoid "black boxing" concepts.

In developing the textual materials for this program we have sought to respond to these postulates. Now consider their meaning and some of their implications.

Two years ago Professor Rising set as an assignment for in-service teachers in a graduate seminar the task of reviewing school texts in order to determine the fraction of the content appropriate for calculator enhancement. The results are striking and reinforce the purest of mathematicians: <u>less than 10% at any grade level</u>, elementary school through college, are amenable to calculator usage! More recently however, Professor Wallace Jewell of Edinboro State College in Pennsylvania carried out the same kind of page count for secondary school courses. His estimates came out in the range 20% - 50%, with the lower count geometry.

*We note here that care must be taken to separate the wheat from the chaff. Our salary scale example is not so trivial as it first appears. It has within it the basic elements of a geometric series and exponential growth.

Why the difference? The answer is instructive and should give better insight into our first postulate. Professor Jewell was studying calculators intensively at the time he made his survey, he had used them in his own instructional program, and he was sensitive to their application; the classroom teachers in the earlier group did not have these characteristics.

The message seems clear. As you start using claculators for classroom instruction, you will probably overvalue their application. But then, having reduced their use to those places where they enhance the program without question, you will begin to find more sophisticated use for them.

At some points in the curriculum calculator use is plainly signaled. They replace log and trig tables and in fact much computation by logarithms. Proofs on the other hand: never. But wait a minute: a better substitute is: hardly ever. Motivating a theorem, for example, is an activity well supported by calculator. In this regard, consider maxima or minima for quadratic functions, $x \mapsto ax^2 + bx + c$. A series of calculations for specific graphs can lead to the conjecture that $x^2 = -b/2a$ at the critical point. Now this result may be proved by standard means.

Gadget fascination. We should by now have learned from our experience with computers in the classroom how this operates. Computers, of course, just like calculators, have much to add to the mathematics program. Any examination of their use in school mathematics classrooms will suggest that their contribution to mathematics is not as great as might be expected and that, in fact, they often take classes of students away from mathematics into realms that are interesting but do not contribute to increasing mathematical sophistication. Across the country in thousands of mathematics classrooms students are working on such computer activities as sorting lists alphabetically, seeing that tables are printed in neat columns, and carrying out complex mathematical processes like inverting a matrix by a minimum of instructions. Such activities, and these are only examples, are good computer science but not good mathematics. We should learn our lesson from this. We as teachers should think very carefully about each place in the curriculum calculators are to apply. When they contribute to that curriculum they should be used, of course; but when they do not contribute to the curriculum and take us off on tangents, we would do better to stick with standard instructional techniques. We have attempted to follow this guideline in our development of the units of this program.

Replacement. This will continue to be a very serious and very difficult problem. For one thing, even though a topic becomes archaic it may still continue to appear on examinations that are important to our students' future programs in mathematics. A case in point: recently a member of the New York State Education Department Mathematics Office claimed that he had checked the eleventh year Regents examination and found that there were no questions that called for calculator usage. We looked at some recent examinations and found this comment to be inaccurate. For example, the following exercise appeared on an examination.

Find log 0.3145

Surely this problem is amenable to calculator computation. The solver need only key .3145 log to get the answer. He no longer needs to interpolate and to use care in determining the characteristic in writing his solution.

Note that the calculator solver gets a "different" answer from the solver who uses tables. The calculator solver's answer is

-0.5024

while the table solver's answer is

9.4976 - 10.

While mathematically equivalent these two answers differ remarkably in appearance. Many scorers would in fact fail to count the calculator answer correct. What has happened of course is that the negative characteristic has been combined with the mantissa to provide a single term result. This can be seen by carrying out the actual subtraction of ten from 9.4976. The result of this is that our old rules for characteristics no longer apply to numbers between 0 and 1.

The point we seek to make by our example should not be missed because of the details of that example. Yes, finding logarithms and tables is an archaic process, but the student who finds a multiple choice question on the SAT examination where no calculator answer is supplied finds himself in some difficulty. Thus we must be very careful as we modify curriculum not to punish our students in the process. This problem has long haunted curriculum developers and will continue to cause problems for them into the foreseeable future.

Having said that, we must still find ways to modify the curriculum significantly in order to do the new kinds of things that are important for contemporary and future use of mathematics. We cannot let our curriculum come to a dead halt because of problems like these.

Black boxing. Black boxing is letting the computer or calculator take charge. It is the first step into the science fiction robot-controlled world. As we look around us in modern society we see this more and more come into place. We see this, for example, in the supermarket where machines essentially replace most of the skills of the check-out personnel. The machines read the item and its price directly from a coded marking on the package, total the order, find the amount of change appropriate, and even provide feed-back to the store manager about inventory. This may very well be an appropriate course for modern engineering; it is inappropriate for the mathematics classroom.

It is important to understand that black boxing is not a new phenomenon, nor a necessarily inappropriate phenomenon. Consider again logarithmic and trigonometric tables. Where do they come from? They are, in fact, just as much a black box presentation of mathematics as is the calculator log key.

While such devices are occasionally appropriate because of the lack of sophistication of our students, we must exercise great care that we do not allow mathematical understandings that we wish to obtain to be lost in the black boxing process. We do not want our students to lose conceptual understanding of computation and what it involves just because the calculator can carry out these computations so quickly and efficiently. Here are two exercises that illustrate some of what we mean here.

Calculate 357895^3

Calculate π^{π}

Each of these exercises demands simple keying into a calculator for solution. In the case of the first, an answer like the following appears

4.5842736 16

If the student has no understanding of scientific notation, this answer is meaningless, and if the student does not understand something about rounding answers, the answer is inaccurate. In the case of the second calculation the answer comes up:

36.46215964

Here the student problems are more complex. What does it even mean to raise a number to an irrational, to say nothing of transcendental, power? Without conceptual underpinning the student has an answer to a process that is meaningless to him.

Value of Programmables

Having described all of these special concerns about teaching with programmable hand-held calculators, it will be well for us to turn now to some of the values of instruction with these devices.

Everyone knows the story of Mallory who when asked why he would set out to climb Mount Everest replied, "It is there." Hand-held calculators are indeed there in modern society. One can get a sense of how wide is the distribution of small calculators by the fact that over the past several years calculator sales have outstripped circulation for the most popular magazine TV Guide. Of course programmables make up only a small fraction of total calculator sales, but they too are there. And today's high quality programmables cost less than standard "four banger" calculators of eight or ten years ago. Thus we have a readily available mathematical tool.

Availability is not enough. With the limited instructional time available to mathematics in the schools, we must make priority decisions on what we teach. All curricular decisions in mathematics must be made on a first-things-first basis. Programmable calculators, we believe, meet this stern test.

One of our basic roles in the schools is to prepare our students for modern society. The computer is a central feature of modern society. Work with programmable hand-held calculators provides students with insights into how computers operate at a very rudimentary level. Given this kind of understanding they may or may not go on to learn how to operate the larger, more complex machines, but even if they do not, they carry with them a general understanding of how these machines operate.

This kind of argument justifies programmable hand-held calculators in the school program, but not necessarily in the mathematics program. The textual materials that we have developed should demonstrate to you just as our experience with these materials in the classrooms with students demonstrates to us, that programmables have a definite contribution to make to school mathematics at the eleventh grade level. We have found, as you will too, that students gain insights into mathematical activities through use of these devices and that they refine their understanding of concepts gained earlier as well.

As a trivial example of what we mean by this last comment, consider an episode that occurred in one of our earlier classes when we were showing youngsters how to use the calculators. Professor Rising asked the tenth grade students in the experimental class to enter 4 in their calculators and then to press the reciprocal (1/x) key. The calculator display then showed 0.25. He then asked a student what multiplier would change the display to 1. The student could not answer. Professor Rising wrote on the chalkboard

$$\frac{1}{x} \cdot \underline{\qquad} = 1 .$$

The student readily suggested x as the number that should fill in the blank. But he still did not know what number to use as a multiplier in answer to the first question. He finally suggested 25. Here was a case in which this reasonably intelligent student was confused by the representation of a common fraction as a decimal to such an extent that he could not apply a basic concept that in other contexts he could use readily. Thus the calculator gave the opportunity to expose and respond to a student's weakness, in this way to refine his understanding of mathematical concepts.[3]

The basic role of any calculator is to take over routine tasks of computation. As they do that, they free the user to concentrate on more serious problems: deciding how to respond to the problem, organizing the solution, determining the reasonableness and accuracy of the answer, thinking about related problems, and otherwise generalizing the solution. When you use calculators in your classroom you should keep this continually in mind. Performing a series of multiplication exercises by calculator is not a mathematical activity.

But we have found and the text pages should display a wide range of places in the standard curriculum for eleventh grade where the calculator contributes to student understanding. Consider, for example, a long standing problem having to do with graphing curves. Every teacher has had the experience of the broken line "graph" of a quadratic. The students plot a few points and connect the points by segments. More points: more segments. This problem is rather hard to address without calculators by any means other than a teacher edict. Why? Because the work required in calculating additional points is considerable, especially when fractions or decimal values are involved. But now suppose our function is something of the form

$$y = 2x^2 - 5x + 6.$$

Students can quickly program this function into their calculators and run successive x values to generate points on the curve. Now they can literally plot dozens of points until they really can see the shape of the curve. The reader should think about this example carefully. Notice how the calculator only takes over a computation role. It in no way substitutes for understanding of the procedure. In fact, the student had to know the procedure for calculating y values in order to prepare the program. What he did not have to do is carry out the complex computations to evaluate the function point by point. In fact there is more than this. The easy generation of additional points allows the student to focus his attention on areas where the concerns are critical. What is the minimum y value for this function? Before the student knows how to determine the turning point from the equation itself, he can locate that turning point by trial and error with his simple program. Thus he develops initial insights into a problem that he can later solve by algebraic technique.

Some teachers feel that by taking over this kind of work calculators will make students lazy. We do not fear this. Our observation of students at work with calculators is that they work harder. The difference is that their work is focussed on concepts, the calculator taking over routine.

Calculator Access

Certainly the best access to calculators is continuous access. We have found in our work at SUNY/Buffalo with simpler calculators that student ownership is the easiest policy. Few problems arise here, because the cost of the calculators is approximately that of school textbooks. This cost equivalence may solve the problem for providing inexpensive calculators in the schools as well. If a school has a textbook distribution (loan) system, calculators can be acquired and distributed within this same program. Student loss of a calculator then is no different from student loss of a textbook and would be treated the same.

Although the cost of programmable hand held calculators has come down markedly over the past several years, these costs are still high enough to make the programmable situation more complex. Best access is still continuous access, but teachers will have to use their best judgment in determining how near they can come to this preferred policy. Our experience in the experimental classes may be of interest and use here. At the outset we were extremely careful about calculator security. We even had some of our calculators secured in locking cradles. As time went on, however, it became clear to us that we had to relax our restrictions or students would not get full value from the experience. For that reason we have adopted a very open program, allowing the students to sign out calculators for overnight use. We have not yet lost a calculator by following this procedure. At the same time we note that we have lost one calculator from the facility in which they are stored at the university.

This still leaves the local school and often the individual classroom teacher to make procedural decisions. We would rank in order of preference the following four possibilities:

1. student ownership
2. long term assignment
3. overnight check out
4. use only in class and in special work rooms.

Which calculator?

Our development work within this project has given us an opportunity to try out and work with a rather wide range of programmable hand held calculators. As we have worked with these machines, we have each developed personal preferences. The key word here is "personal". When working with calculators we have found that you tend to like what you know.

This rule applies especially to machine language. A number of people have made strong cases for the "natural" language of algebraic order calculators, but a personal story may be in order here. Professor Rising's wife, a non-mathematician, has used for several years one of the earliest reverse Polish notation calculators. She has become skilled in the use of this machine. More to the point, she has considerable difficulty adapting to the algebraic order calculators. This suggests that the idea of "natural" order is something to be considered less seriously than we have been tempted to do in the past.

We are not in a position to recommend a particular calculator. For one thing, a recommendation at the date of this writing may very well be inappropriate a year or two years hence. One concern does seem clear to us and it represents a very serious problem. As costs come down, quality is reduced as well. The most distressing comment that has been made to us over the time of our work with programmables was the one made by a representative of a major calculator manufacturer that "The programmables are only being made to last through one year's operation." We have had some difficulties with calculator break-down, yes; but in general our experience with medium-priced ($80 - $100) programmables is that they will last for at least several years. Interestingly it appears that hard use, that is such things as dropping the calculator on the floor, does not seriously affect the calculator operation. The lesson in this is, we believe, that teachers should use caution in purchasing the least expensive available calculators.

Before you select calculators for your students you would do well to experiment with the models you are considering yourself. Some vendors are willing to let you take a calculator overnight to familiarize yourself with its operation. Others will spend considerable time with you in showing you how the machine works. We suggest the following as basic concerns that you should address in selecting calculators:

- complexity of operation
- number of program steps (merged steps save here)
- number of storage locations
- programming language
- instruction manuals

We have found fifty program steps and a half-dozen storage locations entirely satisfactory for high school use. Very rarely will more program steps be needed and only occasionally will more storage locations be necessary for complex programs.

For the simpler "four banger" calculators we recommend battery replacement. For programmables, which draw somewhat more electricity, it seems appropriate to utilize re-charging devices. Since virtually all programmables have plug-in rechargers, this should not be a matter of concern to selectors.

With the advent of liquid crystal display programmables such as the Casio 502, battery charging problems disappear. The batteries on such calculators need only be replaced about once each school year. Users would do well to examine such calculators.

If you will be using this text with microprocessor, most of what we have said will still apply but you will probably have different and often additional problems. Access to the equipment is probably the most difficult

Classroom Presentation

Now you have your calculators and you are ready to go. The students are all excited about the new toys and they want to get to them just as quickly as possible. Don't be trapped by this situation into a complete departure from your mathematics goals to focus on this device. You must constantly keep in mind the fact that the calculator is another tool for teaching mathematics, not a device that is an end in itself. When it is appropriate to use it, do so. When it is inappropriate to use the calculator, have your students set them aside.

What we have done in preparation of these textual materials is to select units which may be enhanced partly by calculator use. You will notice that many other units we do not touch at all. Activities like factoring, solution techniques for linear equations, word problems, in fact, about half the course are not enhanced by use of the calculator. Even the topics that we have developed have sections where you will not wish to use the calculators. The basic rule: don't force the calculator into places in which it doesn't belong.

Another don't. Don't attempt to assign motivation to the calculator. That is a false hope. Your best bet for motivating your students is a serious approach to the teaching and learning of mathematics. The calculator by itself as a motivating device will last like all other such devices about ten minutes. But the calculator used effectively in your instructional program will enhance that program and add to the general motivation that good instruction can contribute.

It is not necessary for you to spend time teaching your students how to use the particular calculators that they have in hand before starting the units in this text. The first unit includes, along with the study of order of operations, sections devoted to introducing the students to their own calculators. These sections and in fact the entire book consider both algebraic order and reverse Polish order operations. We think that it is important for your students to learn both. You and we do not know what kind of calculator or computer access your students will have when they leave school. But clearly, you will wish to focus main attention on the kind of calculator that your students have. At appropriate points you may wish to supplement the instruction by use of, for example, some ideas from the instruction manual for the specific calculator the students have.

Classroom Dynamics

You will soon find as we did that classroom organization changes when you are using calculators. In fact, you will not be able to assign a particular teaching style to the use of calculators. Things are not that simple. There do seem to be two quite different formats for classroom instruction with calculators. We identify these for you so that you can prepare to adapt your instruction to them. Remarkably they are at opposite ends of the instructional spectrum.

The first is the technique that you will wish to use when you want to take your students through a series of keystrokes. This is the most lock-step, regimented kind of instruction. In fact if you depart at all from a step-by-step, "do this", "do this" kind of presentation you will find that your students will diverge frightfully from the pattern that you hope to accomplish. After a few false starts when you learn the lessons that we learned, we expect that you will find yourselves like us saying something like the following: "All right class, now all together turn your calculators off and on and get ready together to follow these keystrokes. First press the ......" In our instruction we found that we could make fun of this kind of activity by saying something like, "Now it's time for close order drill." The students reacted favorably to this. As this is only an occasional instructional activity, you will not find that your classroom is changed into a nineteenth century presentation by these occasional rigid structures.

The second instructional mode is almost exactly the opposite. You will wish to provide your students with opportunities for very open attack on problems. You will want to give them time to organize their own calculator procedures and to apply them to assigned exercises or larger tasks. While they are doing this you will wish to circulate among them to answer specific questions and to give assistance where it is needed. Here we urge you to keep the atmosphere as open as possible, and in particular to allow students to help each other. It will quickly become clear to you which students are leaning too heavily on their neighbor's assistance. In those cases you will wish to intervene. You may wish to give additional assistance to the student being helped in order to wean him from his reliance on his neighbor, or you may wish to comment to the tutor that he may be providing too much help and so preventing the other student from learning the material for himself.

We do not mean to suggest that these are the only teaching styles that will come up in your instructional program. Quite the contrary, you will find that you will use your entire range of instructional techniques. We have only stressed that these extremes are also included. Many of you who are accustomed to working with your class as a unit will find that the second kind of instruction, which opens up the classroom to individual activities, will make you somewhat uncomfortable at first. Recall in this regard that our main business is student learning, and that teacher's teaching sometimes gets in the way.

Calculators do not eliminate student errors. Far from it, they merely highlight these errors. Carelessness will continue to annoy you and to a lesser extent the students themselves as errors are made. But some students will be far worse than others. You will probably wish to give them additional careful instruction. For example, we found that one student constantly pressed two keys at once. We finally had to work with him to get him to use only one finger in that vertical position known to piano instructors and to make a fist of the rest of his hand. This reduced the number of errors by about 75%, bringing him down to just a little above the average of his classmates.

We cannot of course in this brief introductory section head off all the problems you will have as you introduce these devices into your classroom instruction period. Just as we did, you will find unique situations which arise and will need to be dealt with thoughtfully. Along with the individual section exercise answers we provide some suggestions about classroom presentation. You will wish to look at these and to look carefully at the textual materials themselves in preparing your classroom presentations. Here as elsewhere your thoughtful instruction is the key to student learning.

Exercise Set 1.1

1) 12 | 2) 18

3) 12 | 4) 24

5) 17 | 6) 45

7) 17 | 8) 24

9) $ab + cd$ | 10) $ac + ad + bc + bd$

11) $\frac{a}{b} \div \frac{c}{d} \times \frac{e}{f} = \frac{ade}{bcf}$ | 12) $\frac{a}{b} \div \left(\frac{c}{d} \times \frac{e}{f}\right) = \frac{adf}{bce}$

13) $a[b + c(d + e)] = ab + acd + ace$ | 14) $a + b + c - d$

15) $(ab + c)\,d + e = abd + cd + e$ | 16) $\frac{\sqrt{a + b}}{cd - e}$

17) (9) 26 (10) 54

(11) -7 (12) $-\frac{1}{7}$

Notice that (11) and (12) are reciprocals since $ad = bc$ when $a = 6$, $b = 3$, $c = 4$, $d = 2$.

(13) 234 (14) 11

(15) 51 (16) 3

18) 4 | 19) 4

20) 4 | 21) 1

22) 1 | 23) $\frac{1}{25}$

Exercise Set 1.2

1) [+], [-], [x], [÷] . These keys would not be used because they do not separate calculations. When the [=] is used the calculator automatically separates the calculations.

2) The equal step between 38 and [÷] may be eliminated. On some calculators, usually the more sophisticated models, an order of operations is already wired into the machine. Thus, on simple calculators the keystrokes [4] [+] [3] [÷] [7] = 1 because the order is left to right. On more advanced calculators [4] [+] [3] [÷] [7] = 4.428571 because the order of operations by hierarchy is designed into the wiring of the machine. If your calculator has this hierarchy of operations no step can be eliminated.

3) There are several answers to this question that not only represent different problem solving approaches but also reflect the individual characteristics of specific calculators. The following are some reasonable responses to the question.

(a) If your calculator has at least 2 storage registers you may solve the problem by storing the numerator in one register, the denominator in another register and recalling the registers at the appropriate times as follows:

[4] [9] [+] [3] [8] [=] [STO] [A] (STO means store)

[8] [5] [+] [9] [6] [=] [STO] [B]

[RCL] [A] ÷ [RCL] [B] [=]

Remember that the labeling of storage registers is dependent upon the particular calculator you are using.

(b) If your calculator has a key that switches the contents of two registers the problem may be solved as follows:

[4] [9] [+] [3] [8] [=] [STO] [A] (The display is 87 and 87 is stored in register A.)

[8] [5] [+] [9] [6] [=] (the display is 181)

[EXC] [A] (EXC means the display and the storage register contents are exchanged. At this point the display is 87 and 181 is in the register storage labeled A)

[÷] [RCL] [A] (the display is 181.)

[=] (the display is 0.4806629)

c) If your calculator has only a single storage register <u>or</u> no storage register the problem must be solved by writing down the intermediate results or reentering them into the calculator.

4) - 10) Some calculators, because of their wiring, can correctly solve problems by simply working left to right because they have a built-in order of operations where, for example, multiplication takes precedence over addition. On others it is necessary to reorganize the problem so that the operations are performed in the correct order.

(4) 10110.9 6 (5) 5235.47 1

(6) 5214. (7) -2297.52 93

(8) 21.952 Your calculator may have a $\boxed{y^x}$ key that would be appropriate to use here. Your calculator may have a constant multiplying key.

(9) .320118(1592) (10) 3.12384(6527)

(11) They are reciprocals (multiplicative inverses) of each other. If your calculator has one, you might wish to discuss the $\boxed{1/x}$ key at this time. The answer to (10) can be obtained by the following key strokes:

(answer to 9) $\boxed{1/x}$.

Notice that it is unnecessary to use $\boxed{=}$ in this case.

If you are dealing with calculators that have several storage registers, you could ask the students to calculate these exercises in more than one way, without using parenthesis. Have them write down the sequence of key strokes and consider which is a better method. At this point you may wish to consider efficiency of methods in terms of fewer key strokes.

(12) 0.02688(5465) (13) 37.1948(1878)

(14) -179907.(84) (15) -9.44695(6522)

(16) -5.47988(5646)

Exercise Set 1.3

Some calculators are wired for a hierarchy of operations. In those calculators even more parenthesis may be deleted without storing.

1) (a) $3 + 5 - 7$

(b) $3 + 5 - 7$

1

2) (a) $20 \times 10 \div 5$

(b) $20 \times 10 \div 5$

40

3) (a) $\frac{2 \times 7}{31 - 14}$

(b) $\frac{2 \times 7}{(31 - 14)}$

.823529(4118)

4) (a) $20 \div (10 \times 5)$

(b) $20 \div (10 \times 5)$

.4

5) (a) $(8 + 7)(3 + 5)$

(b) $(8 + 7)(3 + 5)$

120

6) (a) $(27.3 + 41.7)\ 3.6$

(b) $(27.3 + 41.7)3.6$

248.4

7) (a) $27.3 + 41.7 \times 3.6$

(b) $27.3 + (41.7 \times 3.6)$

177.42

8) (a) $41.7 \times 3.6 + 27.3$

(b) $41.7 \times 3.6 + 27.3$

177.42

9) (a) $41.7 \times (3.6 + 27.3)$

(b) $41.7 \times (3.6 + 27.3)$

1288.53

10) (a) $\frac{28 \times 3 + 8}{(26 + 7) \times 4}$

(b) $\frac{(28 \times 3) + 8)}{((26 + 7) \times 4)}$

$.696\overline{69}$

11). 40.068

12) 40.068

Look at where 11 and 12 are the same

$$37.8 + (.06 \times 37.8) = (1 + .06)37.8 = 1.06 \times 37.8$$

13) 162553.306 14) -422.4

15) In algebraic - memory 264 - 189 $\boxed{\text{STO}}$

327.84 ÷ $\boxed{\text{RCL}}$ =

4.3712

In (algebraic) and in (AOS)

327.84 ÷ (264 - 189) =

4.3712

16) In algebraic - memory

48.3 + 27.9 $\boxed{=}$ $\boxed{\text{STO}}$

79.4 - 43.7 $\boxed{=}$ $\boxed{\times}$ $\boxed{\text{RCL}}$ $\boxed{=}$ $\boxed{\text{STO}}$

67.1 - 4 $\boxed{=}$ $\boxed{\times}$ $\boxed{\text{RCL}}$ $\boxed{=}$

171653.454

In (algebraic) or (AOS)

(48.3 + 27.9) $\boxed{\times}$ (79.4 - 43.7) $\boxed{\times}$ (67.1 - 4) $\boxed{=}$

171653.454

17) 1 + 1st day

(1 + 2) + 2nd day

(1 + 2 + 3) 3rd day

⋮

(1 + 2 + 3 + ... + 12) 12th day

= 12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6) + 6(7) + 5(8)

+ 4(9) + 3(10) + 2(11) + 1(12)

= 2 [12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6)]

on an (AOS)

2 $\boxed{\times}$ $\boxed{(}$ $\boxed{(}$ 12 $\boxed{)}$ $\boxed{+}$ $\boxed{(}$ 11 $\boxed{\times}$ 2 $\boxed{)}$ $\boxed{+}$ $\boxed{(}$ 10 $\boxed{\times}$
3 $\boxed{)}$ $\boxed{+}$ $\boxed{(}$ 9 $\boxed{\times}$ 4 $\boxed{)}$ $\boxed{+}$ $\boxed{(}$ 8 $\boxed{\times}$ 5 $\boxed{)}$ $\boxed{+}$ $\boxed{(}$
7 $\boxed{\times}$ 6 $\boxed{)}$ $\boxed{)}$ $\boxed{=}$

364

The gifts will be returned on Christmas Eve of the following year (if it

Exercise Set 1.4

1.

1		1
2	+	1
3		2
4		7

2.

1		1
2	÷	1
3		2
4		.75

3.

8		8
0	x	8
2		0
2		4

4.

1		2
2	ENT	3
3		4
4		4

5.

1		1		2
2	5	2	or	3
3		3		4
4		45		5

6.

1		1
2	CLX	2
3		3
4		0

7.

1	f clear	0
2	STK	0
3		0
4		0

8.

1		1
1	CHS	1
1		1
1		-1

9.

1		1
2	-	1
3		2
4		-1

10.

1		1
2	÷	2
3		3
0		0
		ERROR

11.

0	ENT	0
0		0
0		35
35		35

12.

35	ENT	0
0		0
0		0
0		0

13.

5	0	ENT	0	3	0	+	0
	0		0		0		0
	0		5		5		0
	5		5		3		8

14.

5	0	ENT	0	X	0
	0		0		0
	0		5		0
	5		5		25

15.

5	0	X	0
	0		0
	0		0
	5		0

16.

3	0	ENT	0	ENT	0	+	0	X	0
	0		0		3		0		0
	0		3		3		3		0
	3		3		3		6		18

17.

2	3	0	ENT	0	5	0	÷	0
		0		0		0		0
		0		23		23		0
		23		23		5		4.6

18.

5	0	ENT	0	4	0	ENT	0	+	0	÷	0
	0		0		0		5		0		0
	0		5		5		4		5		0
	5		5		4		4		8		.625

19. (14) 5 x 5

(15) 5 x 0

(16) (3 + 3) 3

(17) 23 ÷ 5 or $\frac{23}{5}$

(18) $\frac{5}{4 + 4}$ or $\frac{5}{8}$

20. 2 ENT 3 + 4 X 20

21. 2 ENT 3 ÷ 4 + 4.666666

22. 4 ENT 2 ENT 3 ÷ + 4.66666

23. 2 ENT 3 + 4 ENT 5 + X 45

24. 2 ENT 3 ÷ 4 ENT 5 ÷ + 1.466

25. 2 ENT 3 + 4 ENT 5 + ÷ .55

26. 2 ENT 3 + 4 ENT 5 + 6 ENT 7 + X X 585

27. 5 ENT 9 ENT 1 3 X X

28. 6 ENT 7 ENT 8 ENT

29) (16)

6	3	7	ENT	8	ENT	8	+	8	X	8
7		8		8		3		8		8
8		8		3		.3		3		8
8		3		3		3		6		18

(18)

6	5	7	ENT	8	4	8	ENT	5	+	5	÷	5
7		8		8		5		5		5		5
8		8		5		5		4		5		5
8		5		5		4		4		8		.625

30) $[12(1) + 11(2) + 10(3) + 9(4) + 8(5) + 7(6)]\ 2$

12 ENT 11 ENT 2 X + 10 ENT

3 X + 9 ENT 4 X + 8 ENT

5 X + 7 ENT 6 X + 2 X

Exercise Set 1.5

1) 25   2) 3   3) .25

4) 1000   5) -8   6) error message

7) 25 $\boxed{0}$ $\boxed{0}$ $\boxed{5}$ $\boxed{25}$ 8) 30   9) 100

10) 49   11) .25   12) .25

13) 0   14) 1   15) 10

16) 100,000   17) 4

18) INT gives the largest integer less than or equal to the number.

19) FRACT gives the part of the number after the decimal point - the fractional part of the number.

20) ABS gives the absolute value of the number.

21) AOS $\boxed{8}$ $\boxed{y^x}$ $\boxed{5}$ $\boxed{=}$

RPN $\boxed{8}$ $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{y^x}$

22) AOS $\boxed{1.23}$ $\boxed{y^x}$ $\boxed{3}$ $\boxed{=}$

RPN $\boxed{1.23}$ $\boxed{\text{ENTER}}$ $\boxed{3}$ $\boxed{y^x}$

23) AOS $\boxed{1}$ $\boxed{\div}$ $\boxed{16}$ $\boxed{+}$ $\boxed{1}$ $\boxed{\div}$ $\boxed{7}$ $\boxed{=}$

RPN $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{16}$ $\boxed{\div}$ $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{7}$ $\boxed{\div}$ $\boxed{+}$

24) AOS $\boxed{16}$ $\boxed{+}$ $\boxed{7}$ $\boxed{=}$ $\boxed{\frac{1}{x}}$

or $\boxed{1}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{16}$ $\boxed{+}$ $\boxed{7}$ $\boxed{)}$ $\boxed{=}$

RPN $\boxed{1}$ $\boxed{\text{ENTER}}$ $\boxed{16}$ $\boxed{\text{ENTER}}$ $\boxed{7}$ $\boxed{+}$ $\boxed{\div}$

25) AOS $\boxed{10}$ $\boxed{y^x}$ $\boxed{5}$ - $\boxed{5}$ $\boxed{y^x}$ $\boxed{7}$ $\boxed{=}$

or $\boxed{10}$ $\boxed{y^x}$ $\boxed{5}$ - $\boxed{(}$ $\boxed{5}$ $\boxed{y^x}$ $\boxed{7}$ $\boxed{)}$ $\boxed{=}$

RPN $\boxed{10}$ $\boxed{\text{ENTER}}$ $\boxed{5}$ $\boxed{y^x}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ $\boxed{7}$ $\boxed{y^x}$ $\boxed{-}$

26) AOS $\boxed{35}$ $\boxed{\sqrt{x}}$ $\boxed{X}$ $\boxed{45}$ $\boxed{SIN}$ $\boxed{-}$ $\boxed{34}$ $\boxed{y^x}$ $\boxed{3}$ $\boxed{=}$

RPN $\boxed{35}$ $\boxed{\sqrt{x}}$ $\boxed{45}$ $\boxed{SIN}$ $\boxed{X}$ $\boxed{34}$ $\boxed{ENTER}$ $\boxed{3}$ $\boxed{y^x}$ $\boxed{-}$

27) AOS $\boxed{10}$ $\boxed{y^x}$ $\boxed{60}$ $\boxed{TAN}$ $\boxed{=}$ $\boxed{INT}$

RPN $\boxed{10}$ $\boxed{ENTER}$ $\boxed{60}$ $\boxed{TAN}$ $\boxed{y^x}$ $\boxed{INT}$

28) AOS $\boxed{3.7}$ $\boxed{\sqrt{x}}$ $\boxed{+}$ $\boxed{10}$ $\boxed{COS}$ $\boxed{=}$ $\boxed{-}$ $\boxed{(}$ $\boxed{.13}$ $\boxed{y^x}$ $\boxed{3}$ $\boxed{-}$ $\boxed{27}$ $\boxed{\frac{1}{x}}$ $\boxed{)}$ $\boxed{=}$

RPN $\boxed{3.7}$ $\boxed{\sqrt{x}}$ $\boxed{10}$ $\boxed{COS}$ $\boxed{+}$ $\boxed{.13}$ $\boxed{ENTER}$ $\boxed{3}$ $\boxed{y^x}$ $\boxed{1}$ $\boxed{ENTER}$ $\boxed{27}$ $\boxed{\div}$ $\boxed{-}$ $\boxed{-}$

29) AOS $\boxed{b}$ $\boxed{CHS}$ $\boxed{+}$ $\boxed{a}$ $\boxed{=}$ or $\boxed{b}$ $\boxed{-}$ $\boxed{a}$ $\boxed{=}$ $\boxed{CHS}$

RPN $\boxed{b}$ $\boxed{CHS}$ $\boxed{ENTER}$ $\boxed{a}$ $\boxed{+}$ or $\boxed{b}$ $\boxed{ENTER}$ $\boxed{a}$ $\boxed{-}$ $\boxed{CHS}$

30) AOS $\boxed{b}$ $\boxed{\div}$ $\boxed{a}$ $\boxed{=}$ $\boxed{\frac{1}{x}}$

RPN $\boxed{b}$ $\boxed{ENTER}$ $\boxed{a}$ $\boxed{\div}$ $\boxed{\frac{1}{x}}$

or $\boxed{b}$ $\boxed{ENTER}$ $\boxed{a}$ $\boxed{x \gtrless y}$ $\boxed{\div}$

Exercise Set 1.6

1) 2009.811741

2) 502.4529

3) 502.4529

4) 0

5) 212

6) 0

7) 20

8) 37

9) F = C at -40

10) let C = F $\quad C = \frac{5}{9}(C - 32)$

$$9C = 5C - 160$$

$$4C = -160$$

$$C = -40$$

11) 3.8302

12) 34.6410

13) 6.6418

14) 4349.8139

15) 5.48095031

16) 26.61950336

17) .501019369

18) 2.97190930

19) $t = \sqrt{\frac{2h}{9.8}}$

$h = \frac{t^2(9.8)}{2}$ when t = 10, h = 490 trick on HP 33 and

X $\boxed{\text{ENTER}}$ y $\boxed{\text{g}}$ $\boxed{\to\text{p}}$ change rectangular to

polar coords.

on TI-57 use X $\boxed{\text{X} \gtrless \text{t}}$ y $\boxed{\text{INV}}$ $\boxed{\text{2nd}}$ $\boxed{\text{P}\to\text{R}}$ $\boxed{\text{X} \gtrless \text{t}}$

20) 6.4031

21) 18.8213

22) 15.5878

23) 19.0394

24) 151.29

25) 151.29

26) 87.65

27) 1860.867

28) $(x + y)^2 = x^2 + 2xy + y^2$

29) $(x + y)^2 = x^2 + 2xy + y^2$

$(x + y)^2 \neq x^2 + y^2$

30) $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

Exercise Set 1.7

1) 1.749635531

2) 4.517539515

3) 14.28571429

4) 45.17539515

5) 42.47448214

6) 18.12090911

7)

RPN – HP-33	Algebraic – TI-57
ENTER	LRN
32	-
-	32
5	=
ENTER	X
9	5
÷	÷
X	9
	=
	R/S
	RST
	LRN
	RST

8) $-17.77\overline{7}$

9) $32.\overline{2}$

10) 10

11) -40

12) 320°F = 160°C

13)

RPN – HP-33	Algebraic – TI-57
ENTER	LRN
.07	X
X	.07
	=
	R/S
	RST
	LRN
	RST

14) $35

15) $3.17 (24)

16) $20.99 (65) = $21.

17) $.19(53) = $.20

18) When rounded to two decimal places any answer between $14.22 and $14.35, inclusive, is correct.

Exercise Set 1.8

1) 265

2) 51

3) 339

4) 25

5) 28, 53

6) 60, 75

7) 108; 117

8) 200, 205

9) 1012, 1013

10) $2.45; $37.40

11) $.12; $1.79

12) $209.65; $3204.65

13) $44.28; $676.78

14) $7.00; $106.95

15) $7.00; $107.

16) In HP33 program after step 4, key 8 you are done.

17) p × .07 × 107 ÷ 7 =

18)

	tax	cost
suit	8.28	146.23
overcoat	6.76	91.26
shoes	2.20	33.65
hat	1.11	19.61
totals	18.35	290.75

Solutions to Chapter 1 test

1) 1/4

2) no solution ~~or error~~

3) 2

4) 100

(5 - 8) possible RPN solutions

5) 5 $\boxed{\text{ENTER}}$ 6 $\boxed{\text{ENTER}}$ 7 $\boxed{\div}$ $\boxed{+}$

6) 2 $\boxed{\text{ENTER}}$ 3 $\boxed{+}$ 4 $\boxed{\text{ENTER}}$ 7 $\boxed{+}$ $\boxed{\text{X}}$

7) 37 $\boxed{\text{f}}$ $\boxed{\text{sin}}$ 6 $\boxed{\text{X}}$ $\boxed{\text{f}}$ $\boxed{\sqrt{x}}$

8) 2 $\boxed{\text{ENTER}}$ 3 $\boxed{+}$ 4 $\boxed{\text{ENTER}}$ 7 $\boxed{\text{X}}$ $\boxed{-}$

(5 - 8) possible Algebraic solutions - (AOS) logic

5) 5 $\boxed{+}$ 6 $\boxed{\div}$ 7 $\boxed{=}$

6) $\boxed{(}$ 2 $\boxed{+}$ 3 $\boxed{)}$ $\boxed{\text{X}}$ $\boxed{(}$ 4 $\boxed{+}$ 7 $\boxed{)}$ $\boxed{=}$

7) 37 $\boxed{\text{2nd}}$ $\boxed{\text{sin}}$ $\boxed{\text{X}}$ 6 $\boxed{=}$ $\boxed{\sqrt{x}}$

8) $\boxed{(}$ 2 $\boxed{+}$ 3 $\boxed{)}$ $\boxed{-}$ 4 $\boxed{\div}$ 7 $\boxed{=}$

9) 1

10) 13

11) 142.5206

12) -2.4429

13) 1.84

14) 346.27

15) 1588.7

16) (a) np

(b) $n(110 - 2n) = 110n - 2n^2$

(c) $np - (600 + 10n + n^2) = -600 + 100n - 3n^2$

(d) $-600 + 100(8) - 3(8)^2 = 8$

(e) see next page

HP-33 Solution		TI-57 Solution	
01	ENTER	00	STO 1
02	STO 1	01	x^2
03	g x^2	02	X
04	3	03	3
05	X	04	+/-
06	CHS	05	+
07	RCL 1	06	RCL 1
08	1	07	X
09	0	08	1
10	0	09	0
11	X	10	0
12	+	11	-
13	6	12	6
14	0	13	0
15	0	14	0
16	-	15	=
		16	R/S
		17	RST

(f) 17

17) (a) $x = \sqrt{z^2 - y^2}$

HP-33 Solution		TI-57 Solution	
01	g x^2	00	x^2
02	R/S	01	-
03	g x^2	02	R/S
04	-	03	x^2
05	f $\sqrt{x}$	04	=
		05	$\sqrt{x}$
		06	R/S

(b)

HP-33 Solution	TI-57 Solution
RUN	LRN
SST	RST
R/S	z
z	R/S
R/S	y
y	R/S
R/S	

17) (c) (i) 22.81
(ii) 15.38

18) (a)

HP-33 Solution

01	STO 1
02	R/S
03	STO 2
04	R/S
05	STO 3
06	+
07	+
08	2
09	÷
10	STO 4
11	ENTER
12	ENTER
13	RCL 1
14	-
15	X
16	RCL 4
17	RCL 2
18	-
19	X
20	RCL 4
21	RCL 3
22	-
23	X
24	f √x

TI-57 Solution

00	STO 1
01	+
02	R/S
03	STO 2
04	+
05	R/S
06	STO 3
07	=
08	÷
09	2
10	=
11	STO 4
12	X
13	(
14	RCL 4
15	-
16	RCL 1
17	)
18	X
19	(
20	RCL 4
21	-
22	RCL 2
23	)
24	X
25	(
26	RCL 4
27	-
28	RCL 3
29	)
30	=
31	√x
32	R/S

(b) (i) .3152

(ii) 0

(iii) The sum of two sides of a triangle must be greater than the third side. 2, 3, 5 are not sides of a triangle.

Exercise Set 2.1

1) x^{12}

2) $6a^2b^3$

3) $2w^4$

4) $6a^6$

5) c^{15}

6) $8w^9$

7) $3a^3$

8) 25

9) x^8y^{12}

10) $64c^{18}d^6$

11) $\frac{8}{27}$

12) $\frac{x^8y^{20}}{z^{12}}$

13) $a^{20}b^4$

14) $7x^2$

15) $-32x^5 \div 4x^2 =$
$-8x^3$

16) $\sin^5 x$

17) $11 \sin^2 x \cos^2 x$

18) e^{2x+1}

19) πr

20) $2^{3y} = 8^y$

21) $25^2 = 625$

22) $8^2 = 64$

23) a^3b^2

24) $16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

25) $x^6y^3 \div y^2 = x^6y$

26) $\frac{1}{8} t^{21}$

27) $34.328125 = 34 \frac{21}{64}$

28) $5^{3x+3} = (125)^{x+1}$

29) $(x + 7)^3 = x^3 + 21x^2$
$+ 147x + 343$

30) $(a + 3)^6 = a^6 + 18a^5 + 135a^4$
$+ 540a^3 + 1215a^2 + 1458a + 729$

31) $\cos^3 x$

32) $-r^2$

33) $16y^4$

34) x^{5a}

35) $2b^4$

36) x^2

37) $9 \tan^6 x$

38) 3^{xa-2a}

39) x^4

40) $2^{a+2} x^{3a+6}$

41) $(x^2y^3)^2 = x^4y^6$
$3^4(-1)^6 = 81$

42) $5x^3 + 4y^2 \longrightarrow 5(-1)^3 + 4(3)^2 = -5 + 36 = 31$

43) $(-2ab^2)^3 = -8a^3b^6$

$$-8(5)^3(2)^6 = -8(125)(64) = -64000$$

44) $3a^2 - (5b)^3 = 3a^2 - 125b^3$

$$3(2)^2 - 125(-2)^3 = 12 + 125(8)$$
$$= 12 + 1000 = 1012$$

45) $\sqrt[3]{8a^{12}b^9} = 2a^4b^3$

$$2(-2)^4(-1)^3 = 2(16)(-1) = -32$$

46) $2^c \cdot 2^{2c} = 2^{3c}$

$$2^{3(2)} = 2^6 = 64$$

47) $(x^{4a})(x^{2a+3})^2 = (x^{4a})(x^{4a+6}) = (-1)^{\text{even}}\ (-1)^{\text{even}}$

$$= (1)(1) = 1$$

48) $5^{2x-1} \div 5^3 = 5^{2x-4}$

$$5^{2(3)-4} = 5^2 = 25$$

49) $x^{105} \div x^{97} = x^8$

$$(-1)^8 = 1$$

50) 0

51) When raising a power, multiply exponents. The exponent of a = 1. The correct answer is $25a^2x^6$.

52) A radical without an index has an index of 2.

$(\sqrt{x^4})^3 = (x^2)^3 = x^6$

53) To simplify exponential expressions the bases or the exponents must be the same. $(a^2)(b^3) = a^2b^3$.

54) When multiplying exponential expressions the base remains the same. $3^2 \cdot 3^4 = 3^6$.

55) When dividing exponential expressions the exponents should be subtracted. $18y^6 \div 9y^2 = 2y^4$.

56)

```
       ME
      /  \
     M    D
    / \  / \
   M   D M  D
```

2 1st generation

$4 = 2^2$ 2nd generation

⋮

$1024 = 2^{10}$ 10th generation

57) $4^4 = 256$

58) $x^a \cdot x^b = \underbrace{x \cdot x \cdot x \cdots x}_{a \text{ factors}} \cdot \underbrace{x \cdot x \cdot x \cdots x}_{b \text{ factors}}$ definition

$= \underbrace{x \cdot x \cdots x}_{a + b \text{ factors}}$ counting

$= x^{a+b}$ definition

59) $\left(\frac{x}{y}\right)^a = \underbrace{\frac{x}{y} \cdot \frac{x}{y} \cdots \frac{x}{y}}_{a \text{ factors}}$

$$= \frac{\overbrace{x \cdot x \cdots x}^{a \text{ factors}}}{\underbrace{y \cdot y \cdot y \cdots y}_{a \text{ factors}}}$$

$$= \frac{x^a}{y^a}$$

60) $x^{10} \div x^{12} = \dfrac{x^{10}}{x^{12}} = \dfrac{\overbrace{x \cdot x \cdots x}^{10 \text{ factors}}}{\underbrace{x \cdot x \cdots x}_{12 \text{ factors}}} = \dfrac{1}{x^2}$

$x^{15} \div x^{20} = \dfrac{1}{x^4}$

$x^3 \div x^7 = \dfrac{1}{x^4}$

$x^a \div x^b = \dfrac{1}{x^{b-a}}$ when $b > a$

Exercise Set 2.2

(1 - 6) are guesses

1) 5   2) 20

3) 12   4) 6

5) 10   6) 9

7) 1   8) 0

9) 1   10) -1

11) positive   12) negative

13) guess, less than

14) guess, greater than

15) $23^2 \cdot 23^2 = 279841$

16) $23^3 \cdot 23^1 =$

17) $(23^2)^2 =$

18) $(.023)^4 = \left(\frac{23}{1000}\right)^4 = \left(\frac{279841}{10^3}\right)^4 =$

$$\frac{279841}{10^{12}} = \frac{279841}{1,000,000,000,000} = .000000279841$$

19) (a) $6^6 \cdot 6^2 = 46656\ (30 + 6) = 46656 \times 30 + 46656 \times 6$

$46656(30) = 46656 \times 3 = 139968 \quad \times 10 = 1399680$

$+\ 46656(6) = 279936 \quad = \underline{279936}$

1679616

(b) $6^{11} = 6^9 \cdot 6^2 = 10077696\ (30 + 6) =$

$10077696\ (3) \times 10 = 30233088 \times 10 = 302330880$

$+\ 10077696\ (6) = \quad = \underline{60466176}$

362797056

(c) $6^{13} = 6^{11} \cdot 6^2 = 36279056\ (30 + 6) =$

$362797056\ (3) \times 10 = 1088391168 \times 10 = 10883911680$

$362797056\ (6) = \qquad = \underline{2176782336}$

13060694016

20) (a) $12^6 = 12^4 \cdot 12^2 = 20736\ (144) =$

$20736\ (100) = 2073600$

$20736\ (40) = 829440$

$20736\ (4) = \underline{82944}$

2985984

(b) $12^8 = 12^6 \cdot 12^2 = 2985984\ (144) =$

$2985984\ (100) = 298598400$

$2985984\ (40) = 119439360$

$2985984\ (4) = \underline{11943936}$

429981696

(c) $12^{10} = 12^8 \cdot 12^2 = 429981696\ (144) =$

$429981696\ (100) = 42998169600$

$429981696\ (40) = 17199267840$

$429981696\ (4) = \underline{1719926784}$

61917364224

Notice that in each of the above there is only one calculator computation (i.e., multiplying by 4).

21) $2^{17} = 131072$

22) $2^{34} = 2^{17} \cdot 2^{17} = 131072\ (131000 + 72) =$

$131072\ (131000) = 17170432000$

$131072\ (72) = \underline{9437184}$

$17,179,869,184$

On some calculators this calculation can be done entirely on the calculator but 2^{34} goes into scientific notation.

23) $5^{15} = 5^{10} \cdot 5^5 = 9765625 \times 3125$

$$9765625 \times 3100 = 30273437500$$

$$9765625 \times 25 = \underline{244140625}$$

$$30{,}517{,}578{,}125$$

24) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(5^{15}) = (5^5)^3 = (3125)^3 = (3100 + 25)^3 =$

$$3100^3 + 3 \times 3100^2 \times 25 + 3 \times 3100 \times 25^2 + 25^3$$

$3100^3 =$	29791000000
$3 \times 3100^2 \times 25 =$	720750000
$3 \times 3100 \times 25^2 =$	5812500
$25^3 =$	15625
	30,517,578,125

Exercise Set 2.3

1) 8,370,000

2) 5630

3) 29,000

4) 28.47

5) 627.3

6) 3333.8

7) 31,500,000

8) 953.7

9) 13,200,000,000,000,000,000,000,000

10) 30,000,000,000

11) 5,000,000,000

12) 6.9530×10^4

13) 8.34732×10^5

14) 1.46×10^2

15) 1.0×10^5

16) 1.47324×10^5

17) 5.328×10^2

18) 1.8435×10^2

19) 2.3764×10^3

20) 2.5×10^{13}

21) 5.878×10^{12}

22) 6×10^{23}

23) 656100 or 6.561×10^5

6.24 05 = 624000

+ 3.21 04 = + 32100

656100

24) 1.2×10^4 = 12000

$\sqrt{1.44 \quad 08}$ = $\sqrt{144000000}$

= 12000

25) $1.1872 \times 10^7 = 11,872,000$

$5.6 \times 10^4 = 56000$

$2.12 \times 10^2 = 212$

$$\begin{array}{r} 112000 \\ 56000 \\ 112000 \\ \hline 11,872,000 \end{array}$$

26) $6.25 \times 10^{10} = 62,500,000,000$

$(2.5 \quad 05)^2 = 250,000$

$$\begin{array}{r} 250,000 \\ \underline{250,000} \\ 12500000000 \\ \underline{500000} \\ 62,500,000,000 \end{array}$$

27) $3.75 \times 10^3 = 3750$

$4 \quad 03 = 4000$

$2.5 \quad 02 = -250$

$$\begin{array}{r} 4000 \\ -\ 250 \\ \hline 3750 \end{array}$$

28) $2 \times 10^4 = 20,000$

$(5 \quad 06) \div (2.5 \quad 02) = \frac{5,000,000}{2} = 20,000$

29) $3.72 \times 10^5 \qquad \frac{(9,300,000)\ (500)}{12,500} = 372000$

30) 6.5536×10^9

$$\frac{(400)^3\ (80000)^2}{(62,500,000)} = \frac{(64,000,000)\ (6,400,000,000)}{62,500,000}$$

$$= \frac{409,600,000,000,000,000}{62,500,000} = 6,553,600,000.$$

Exercise Set 2.4

1) $\sqrt[3]{y}$

2) $2\sqrt{x}$

3) $-\sqrt[4]{5}$

4) $\sqrt[y]{e^x}$

5) $\dfrac{a}{\sqrt[3]{b^2}}$

6) $\dfrac{1}{\sqrt[6]{2a}}$

7) $\dfrac{1}{\sqrt{y}}$

8) $\dfrac{3}{\sqrt[5]{a^3}}$

9) $\dfrac{1}{6} \doteq .1667$

10) $\dfrac{1}{8} = .125$

11) 3

12) $\dfrac{27}{8} = 3.375$

13) 21

14) $\dfrac{2}{81} \doteq .0247$

15) $\dfrac{1}{9} \doteq .1111$

16) $\dfrac{1}{32} = .03125$

17) $\dfrac{6}{5} = 1.2$

18) $\dfrac{1}{10^3} = .001$

19) -2

20) .2

21) 1

22) $\dfrac{243}{32} = 7\dfrac{19}{32} \doteq 7.59375$

23) a

24) $z^{\frac{1}{5}}$

25) y^8

26) $\sqrt[9]{x^4}$

27) $a^{\frac{1}{4}}$

28) x^3y^{-7}

29) $d^{-\frac{3}{4}}$.

30) $x^{-\frac{3}{2}}$

31) $\dfrac{32}{3} = 10\frac{2}{3} \doteq 10.6667$

32) $1\frac{1}{2} = 1.5$

33) $\dfrac{x^{2a}}{y^{4b}}$

34) $\dfrac{1}{9} - \dfrac{1}{3}(5)$

$\dfrac{1}{9} - \dfrac{5}{3} = \dfrac{-14}{9} \doteq -1.5556$

35) $0.166\overline{6} = \frac{1}{6}$

36) $0.1250 = \frac{1}{8}$

37) $3 = 3$

38) $3.375 = \frac{27}{8}$

39) $21 = 21$

40) $0.0247 = \frac{2}{81}$

41) $.111\overline{1} = \frac{1}{9}$

42) $0.0313 = \frac{1}{32}$

43) $1.2 = \frac{6}{5}$

44) $.001 = \frac{1}{1000}$

45) $-2 = -2$

46) $.2 = .2$

47) $1 = 1$

48) $7.5938 = \frac{243}{32}$

49) $32^{\frac{3}{5}} = 8$; $32^{.6} = 8$

50) $7^{-.25} \div 2^{-.25} = .7311$; $(\frac{7}{2})^{-.25} = (3.5)^{-.25} = .7311$

51) $8^{.5} \cdot 8^{.25} = 8^{.75} = 4.7568$; $8^{\frac{1}{2}} \cdot 8^{\frac{1}{4}} = (2.8284)(1.6818) = 4.7568$

52) $(81^{\frac{3}{4}})^{-2} = 81^{-\frac{3}{2}} = .0014$; $(\sqrt[4]{531441})^{-2} = 27^{-2} = .0014$

53) $(\frac{1}{100})^{-.5} \cdot 100^{.5} = .10 \cdot 10 = 1$; $\sqrt{\frac{1}{100}} \cdot \sqrt{100} = \sqrt{\frac{1}{100} \cdot 100} = \sqrt{1} = 1$

54) $.4796(.6127) = .2938$; $.23^{\frac{1}{2}+\frac{1}{3}} = .23^{\frac{5}{6}} = .2938$

55) $(\sqrt[5]{9})^{4} = (1.5518)^{4} = 5.7995$; $(3^{\frac{2}{5}})^{4} = 3^{\frac{8}{5}} = 5.7995$

56) $(.25) \div (.25)^{3} = 16$; $(.25) \div (.25)^{3} = .25^{-2} = 16$

57) $9 \cdot 140.2961 = 1262.6650$; $27^{\frac{3}{2}} \cdot 27^{\frac{2}{3}} = 27^{\frac{13}{6}} = 1262.6650$

58) $7 \div (\frac{1}{49})^{-.5} = 1$; $7 \div \sqrt{49} = 1$

59) $(4^{\frac{2}{3}})^{-\frac{3}{2}} = 4^{-1} = \frac{1}{4} = .25$; $(4^{.6666})^{-1.5} = 4^{-0.999} = .25$

60) $1.4142 \cdot 1.4142 = 2$; $2^{\frac{1}{2}} \cdot 4^{\frac{1}{4}} = 2^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} = 2^{1} = 2$

61) $\sqrt[4]{2401}$ and $\left(\frac{1}{49}\right)^{-\frac{1}{2}}$ are multiplicative inverses

62) $\sqrt[4]{4}$ is the same as $\sqrt{2}$.

Exercise Set 2.5

1) 4.12×10^{-7}

2) 2.578×10^{-3}

3) 1.37×10^{-1}

4) 1.247503×10^{6}

5) 2.372×10^{-2}

6) 2.301×10^{0}

7) 1.0026×10^{1}

8) 7.85×10^{-10}

9) 9.80665×10^{2}

10) 8.64×10^{5}

11) .000147

12) 2,563,000

13) 57,000,000

14) .0000103

15) .000003

16) .0000000682

17) 26,900,000,000

18) .0457

19) $\underbrace{.00 \ldots}_{\text{27 zeros}} 091066$

20) 29,977,600,000

21) 0.00 00

22) 0.00 any number

23) (a) 4 significant digits

(b) correct to the nearest ten

(c) range of error 13235 to 13244.

24) (a) 2 significant digits

(b) correct to the nearest thousandth

(c) range of error .0265 to .0274

25) (a) 2 significant digits

(b) correct to the nearest thousandth

(c) range of error .0605 to .0614

26) (a) 3 significant digits

(b) correct to the nearest ten thousandth

(c) range of error .06095 to .06104

27) (a) 4 significant digits

(b) correct to the nearest hundred

(c) range of error 326650. to 326740.

28) 1.234×10^{-3}

29) 47.32×10^{-3}

30) 10×10^{-9}

31) 1.23456×10^{3}

32) 1.237×10^{3} meters

33) 8.37×10^{-9} seconds

34) 6.32×10^{6} tons

35) 2.04×10^{-3} liters

Exercise Set 2.6

1)	$x = 49$	2)	$x = 125$
2)	$x = 16$	4)	$a = \pm 16\sqrt{2} = \pm\sqrt{512}$
5)	$y = \pm\frac{1}{5}$	6)	$c = \pm 2$
7)	$x = 343$	8)	$w = \frac{1}{9}$
9)	$m = \pm 27$	10)	$m = \pm 8$
11)	$x = 4$	12)	$x = 4$
13)	$x = 0$	14)	$x = -3$
15)	$y = \frac{1}{2}$	16)	$x = 5$
17)	$2^8 = 2^{6t-4}$ $12 = 6t$ $2 = t$	18)	$2^2 \cdot 2^4 = 2^{3a}$ $a = 2$
19)	$2y + 3 = -1$ $2y = -4$ $y = -2$	20)	$3x + 1 = 2x - 2$ $x = -3$
21)	$3(x-2) = 6(2x + 2)$ $3x - 6 = 12x + 12$ $-18 = 9x$ $-2 = x$	22)	$2(x - 2) = x + 1$ $2x - 4 = x + 1$ $x = 5$
23)	$3(x + 2) = -1(2 - x)$ $3x + 6 = x - 2$ $2x = -8$ $x = -$ [illegible]	24)	$2(x + 2) = 4(x - 1)$ $2x + 4 = 4x - 4$ $8 = 2x$ $4 = x$

25) $(4x - 3)(-2) = 3(-x-8)$

$-8x + 6 = -3x - 24$

$-5x = -30$

$x = 6$

(26) $3(2x - 6) = -2(3 - x)$

$6x - 18 = -6 + 2x$

$4x = 12$

$x = 3$

27) $3 - x + 2(2x - 1) = 4$

$3 - x + 4x - 2 = 4$

$3x + 1 = 4$

$3x = 3$

$x = 1$

28) $2x + 4x = -3$

$6x = -3$

$x = -\frac{1}{2}$

29) $2x + 2 = 3(x - 1)$

$2x + 2 = 3x - 3$

$5 = x$

30) $3(3^{x+2}) + 3^{x+2} = 12$

$3^{x+2}(3 + 1) = 12$

$3^{x+2} = 3$

$x + 2 = 1$

$x = -1$

31) false $x = \pm 5$

32) false x is any real number

33) true

34) true

35) false, $x = -2$

36) 4, 19

37) 1, 70

38) 2, 70

39) 1, 70

Exercise Set 2.7

n	4n
1.6	9.1896
1.7	10.5561
1.8	12.1257
1.9	13.9288
2.0	16.
2.1	18.3792
2.2	21.1121
2.3	24.2515

n	4n
2.4	27.8576
2.5	32.0000
2.6	36.7583
2.7	42.2243
2.8	48.5029
2.9	55.7152
3.0	64.

1) .8 + 2.1 = 2.9 answer is 55.7152

2) .2 + 2.5 = 2.7 answer is 42.2243

3) 1.6 + 1.2 = 2.8 answer is 48.5029

4) 1.3 + 1.3 = 2.6 answer is 36.7583

5) 1.4 + 1.6 = 3.0 answer is 64

6) 1.9 + .9 = 2.8 answer is 48.5029

7) .1 + 2.7 = 2.8 answer is 48.5029

8) 1 + 2.2 = 3.2 answer is 84.4485

9) 1 - 2

10) 1.1 - 1.2

11) 1.15 - 4.92
1.16 - 4.99
1.17 - 5.06 to nearest hundredth 1.16
1.18 - 5.13

12) 2 - 3

13) 2.1 - 2.2

14) 2.12 2.15 - 19.7
2.14 - 19.43
2.13 - 19.16
2.12 - 18.90

15)

n	4^n
1.16	5
2.12	19

16) $5 \times 19 = 4^{1.16 + 2.12} = 4^{3.28} = 94.3532$

the actual value is 95.

17) because of our approximations

18) 2

19) subtract

20) $2.1 - .8 = 1.3$ answer is 6.0629

21) $1.9 - .9 = 1$ answer is 4

22) $2.7 - 1.3 = 1.4$ answer is 6.9644

23) $2.2 - .7 = 1.5$ answer is 8

24) $2.8 - 1.4 = 1.4$ answer is 6.9644

25) $2.9 - 1.1 = 1.8$ answer is 12.1257

26) $2.12 - 1.16 = 0.96$ answer is 3.7842

27)

n	3^n	n	3^n	n	3^n	n	3^n
0	0	.4	1.5518	.7	12.1577	1	3
.1	.1161	.5	1.7321	.8	2.4082		
.2	1.2457	.6	1.9332	.9	2.6879		
.3	1.3904						

27) $.3 + .4 = .7$ answer is 2.1577

28) $.2 + .7 + .9$ answer is 2.6879

29) $.1 + .9 = 1$ answer is 3

30) $.3 + .3 = .6$ answer is 1.9332

Exercise Set 2.8

1)

n	10^n
1.1	12.5893
1.2	15.8489
1.3	19.9526
1.4	25.1189
1.5	31.6228

n	10^n
1.6	39.8107
1.7	50.1187
1.8	63.0957
1.9	79.4328
2.0	100.000

2) $10^{.1} \times 10^{.6} = 10^{.7} = 5.0119$

3) $10^{.3} \times 10^{.9} = 10^{1.2} = 15.8489$

4) $10^{1} \div 10^{.4} = 10^{.6} = 3.9811$

5) $10^{.8} \times 10^{.4} \div 10^{.7} = 10^{.5} = 3.1623$

6) $\sqrt{10^{1.4}} = 10^{.7} = 5.0119$

7) $(10^{.4})^{5} = 10^{2} = 100$

8) $(10^{.9})^{\frac{4}{3}} = 10^{1.2} = 15.8489$

9) $\sqrt[3]{10^{1.5}} = 10^{.5} = 3.1623$

10) $\dfrac{10^{.6} \times (10^{.7})^{2}}{10^{1.5}} = \dfrac{10^{2}}{10^{1.5}} = 10^{.5} = 3.1623$

11) Both contain the sequence of digits 12589254 but have the decimal points in different places.

12) The sequence of digits is the same but the placement of the decimal is different.

13) 2.1 - 125.8925
3.1 - 1258.925

14) Multiplying by 10 moves the decimal point one place to the right.

15) $10^{1} \cdot 10^{2.1} = 10^{3.1}$

16)

n	10^n
-.1	.7943
-.2	.6310
-.3	.5012
-.4	.3981
-.5	.3162

n	10^n
-.6	.2512
-.7	.1995
-.8	.1585
-.9	.1259
-1.0	.1000

17) They have the same sequence of digits as table 3 but in the reverse order. For example, .9 - 1 = -.1.

18) $10^1 \times 10^{-.1} = 10^{.9} = 7.9433$

19) $\sqrt[4]{10^{-.8}} = 10^{-.2} = .6310$

20) $10^{1.4} \div 10^{-.9} = 10^{2.3} = 199.5262$

21) $(10^{-.3})^3 = 10^{-.9} = .1259$

22) $\sqrt{\dfrac{10^{1.3} \times 10^{.3}}{10^{-.8}}} = \sqrt{10^{2.4}} = 10^{1.2} = 15.8489$

23) $(10^{-.6})^{\frac{2}{3}} = 10^{-.4} = .3981$

24) $\dfrac{10^{-.9} \times 10^{-.3}}{10^{-.4}} = \dfrac{10^{-1.2}}{10^{-.4}} = 10^{-.8} = .1585$

25) $\dfrac{10^{1.5} \times 10^{1.3}}{\sqrt[3]{10^{-.9}}} = \dfrac{10^{2.8}}{10^{-.3}} = 10^{3.1} = 1258.9254$

Exercise Set 2.9

1)	$12 = 10^{\log 12}$	2)	$.07 = 10^{\log .07}$
3)	$13 = 10^{\log 13}$	4)	$2846 = 10^{\log 2846}$
5)	$\frac{1}{2} = 10^{\log \frac{1}{2}}$	6)	$\pi = 10^{\log \pi}$
7)	$x = \log_3 27$	8)	$x = \log_2 16$
9)	$x = \log_{487} 1$	10)	$x = \log_{32} 2$
11)	$x = \log_{10} 1000$	12)	$x = \log_{10} .1$
13)	$x = \log_{10} 2$	14)	$x = \log_{10} 387$
15)	$x = \log_{10} 10$		

16)

(7)	$x = 3$	(8)	$x = 4$
(9)	$x = 0$	(10)	$x = 1/5 = .2$
(11)	$x = 3$	(12)	$x = -1$
(13)	$x = .3010$	(14)	$x = 2.5877$
(15)	$x = 1$		

17)	$\log_{10} 1000 = 3$	18)	$\log_{10} .1 = -1$
19)	$\log_2 32 = 5$	20)	$\log_3 9 = 2$
21)	$\log_{3.7} 26.33 = 2.5$	22)	$\log_{10} 2 = .3010$
23)	$\log_{10} 3 = .4771$	24)	$\log_{10} 6 = .7781$
25)	$\log_{10} \frac{1}{3} = -.4771$	26)	$\log_{10} .01 = -2$
27)	$\log_{10} 100,000 = 5$	28)	$\ln 7.39 = 2$

Exercise Set 2.10

1) Proof of II: $(\log (\frac{x}{y}) = \log x - \log y)$

(a) $\frac{x}{y} = 10^{\log \frac{x}{y}}$

(b) $x = 10^{\log x}$, $y = 10^{\log y}$

(c) $\frac{x}{y} = \frac{10^{\log x}}{10^{\log y}} = 10^{\log x - \log y}$

Since (a) = (c)

$10^{\log \frac{x}{y}} = 10^{\log x - \log y}$

2) $\log (74.1 \times 1.64) = \log 74.1 + \log 1.64 = 1.8698 + .2148$

$= 2.0846$

$10^{2.0846} = 121.5066$

3) $\log (.163 \div 2.18) = \log .163 - \log 2.18 = -0.7878 - .3385$

$= -1.1263$

$10^{-1.1263} = .0748$

4) $\log (82.7^{1.4}) = 1.4 (\log 82.7) = 1.4(1.9175)$

$= 2.6845$

$10^{2.6845} = 483.6239$

5) $\log \sqrt[3]{34} = \frac{1}{3} \log 34$

$= \frac{1}{3} (1.5315)$

$= 0.5105$

$10^{0.5105} = 3.2397$

6) $\log\left(\frac{38.5 \times 62.4}{71.8}\right) = \log 38.5 + \log 62.4 - \log 71.8$

$= 1.5855 + 1.7952 - 1.8561$

$= 1.5246$

$10^{1.5246} = 33.4657$

7) $\log\left(\frac{143.6}{71.2 \times 84.7}\right) = \log 143.6 - \log 71.2 - \log 84.7$

$= 2.1572 - 1.8525 - 1.9279$

$= -1.6232$

$10^{-1.6232} = 0.0238$

8) $\log(23.7 \times 41.3^2) = \log 23.7 + 2 \log 41.3$

$= 1.3747 + 2(1.6160)$

$= 4.6067$

$10^{4.6067} = 40429.6517$

9) $\log(\sqrt[5]{64.5} \times 81.2) = \frac{1}{5} \log 64.5 + \log 81.2$

$= \frac{1}{5}(1.8096) + 1.9096$

$= 2.2715$

$10^{2.2715} = 186.8530$

10) $\log(61.2 \div (43.6)^{1.3}) = \log 61.2 - 1.3 \log 43.6$

$= 1.7868 - 1.3(1.6395)$

$= -0.3446$

$10^{-0.3446} = .4523$

11) $\log(45^{.6} \times 34^{.02}) = .6 \log 45 + .02 \log 34$

$= .6(1.6532) + .02(1.5315)$

$= 1.0225$

$10^{1.0225} = 10.5317$

12) $\log(\pi^{\pi}) = \pi(\log \pi)$

$= 3.1416\,(.4972)$

$= 1.5618$

$10^{1.5618} = 36.4586$

Exercise Set 2.11

1) $\log x = \log 35 + \log 23 - \log 267$

2) $\log x = 2 \log 23 + \frac{1}{2} \log 35$

3) $\log x = \log 6720 - \log 7.6 - \log 14$

4) $\log x = 3 \log 41 + \frac{2}{3} \log 23 - \frac{1}{2} \log 17$

5) $\log a = 2 \log b + \log c$

6) $\log a = \log b - \log c - \frac{1}{2} \log d$

7) $\log xy = \log x + \log y = a + b$

8) $\log (\frac{x}{y}) = \log x - \log y = a - b$

9) $\log x^2 = 2 \log x = 2a$

10) $\log \frac{xy}{z^2} = \log x + \log y - 2 \log z = a + b - 2c$

11) $\log 1000z = \log 1000 + \log z = 3 + c$

12) $\log (.01\ y) = \log .01 + \log y = -2 + b$

13) $\log\ x\sqrt{y} = \log x + \frac{1}{2} \log y = a + \frac{b}{2}$

14) $\log \sqrt{xy} = \frac{1}{2}(\log x + y) = \frac{a + b}{2}$

15) $x = \frac{3}{5}$

16) $x = 3(5) = 15$

17) $x = 5^2 = 25$

18) $x = 100(5) = 500$

19) $x = \sqrt{36} = 6$

20) $x = y^2 z^3$

21) $\log\left(\frac{x}{100}\right) = 3$

$$10^3 = \frac{x}{100}$$

$$100,000 = x$$

22) $\log \frac{100}{x} = \log 5$

$$\frac{100}{x} = 5$$

$$x = 20$$

23) $\log x = a \Longrightarrow 10^a = x$

$$10^{2a} = (10^a)^2 = x^2$$

So antilog $2a = x^2$

24) $\log x = a \Longrightarrow 10^a = x$

$$10^{a+2} = 10^a \cdot 10^2 = 100 \cdot 10^a$$

So antilog $a + 2 = 100x$

25) $\log x = a \Longrightarrow 10^a = x$

$\log y = b \Longrightarrow 10^b = y$

$10^{2a} = x^2, \quad 10^{3b} = y^3$

$10^{2a-3b} = \dfrac{x^2}{y^3}$ So antilog $(2a - 3b) = \dfrac{x^2}{y^3}$

26) $3^x = 30$

$x \log 3 = \log 30$

$$x = \frac{\log 30}{\log 3} = \frac{1.4771}{.4771} = 3.096$$

27) $2^x = 10$

$x \log 2 = \log 10$

$$x = \frac{\log 10}{\log 2} = \frac{1}{.3010} = 3.3219$$

28) $5^x = .5$

$x \log 5 = \log .5$

$$x = \frac{\log .5}{\log 5} = \frac{-0.3010}{.6990} = -0.4307$$

29) $4^x = 21$

$x \log 4 = \log 21$

$$x = \frac{\log 21}{\log 4} = \frac{1.3222}{.6021} = 2.1962$$

30) $3.14^x = 5.12$

$x \log 3.14 = \log 5.12$

$$x = \frac{\log 5.12}{\log 3.14} = \frac{.7093}{.4969} = 1.4273$$

Exercise Set 2.12

1) $2^{.5} = 1.4$, (.5, 1.4) on graph

2) $2^{1.5} = 2.8$, (1.5, 2.8) on graph

3) $2^{2.5} = 5.6$, (2.5, 5.6) on graph

4) $2^{-.5} = .7$, (-.5, .7) on graph

5) $2^{-1.5} = .4$, (-1.5, .4) on graph

6) - 10) $y = \log_2 x \Longrightarrow 2^y = x$

$$y \log 2 = \log x$$
$$y = \frac{\log x}{\log 2}$$

6) (3,1.6)

7) (5, 2.3)

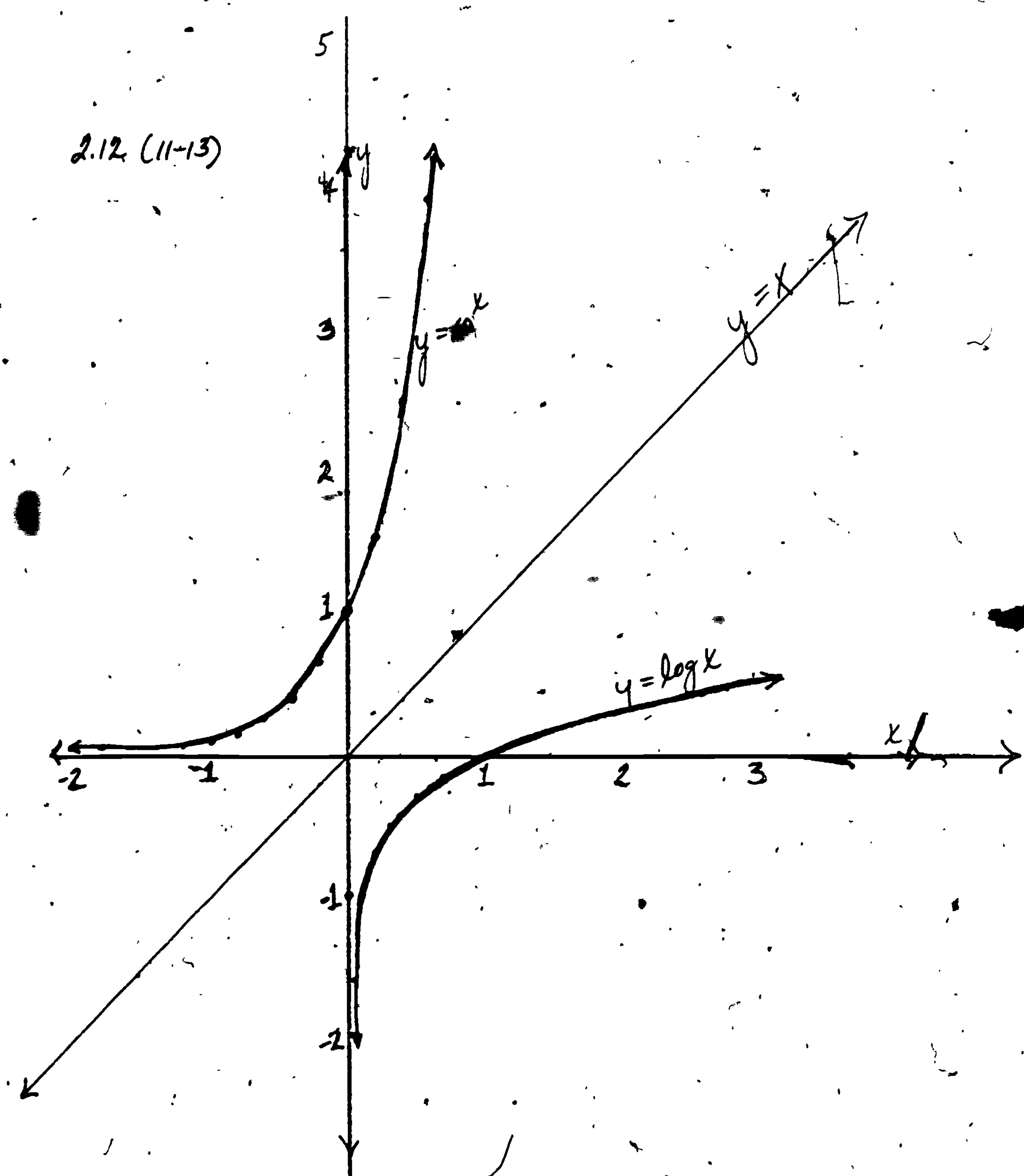
8) (6, 2.6)

9) (7, 2.8)

10) (.7, -.5)

11) - 13) see graph below

2.12 (11-13)

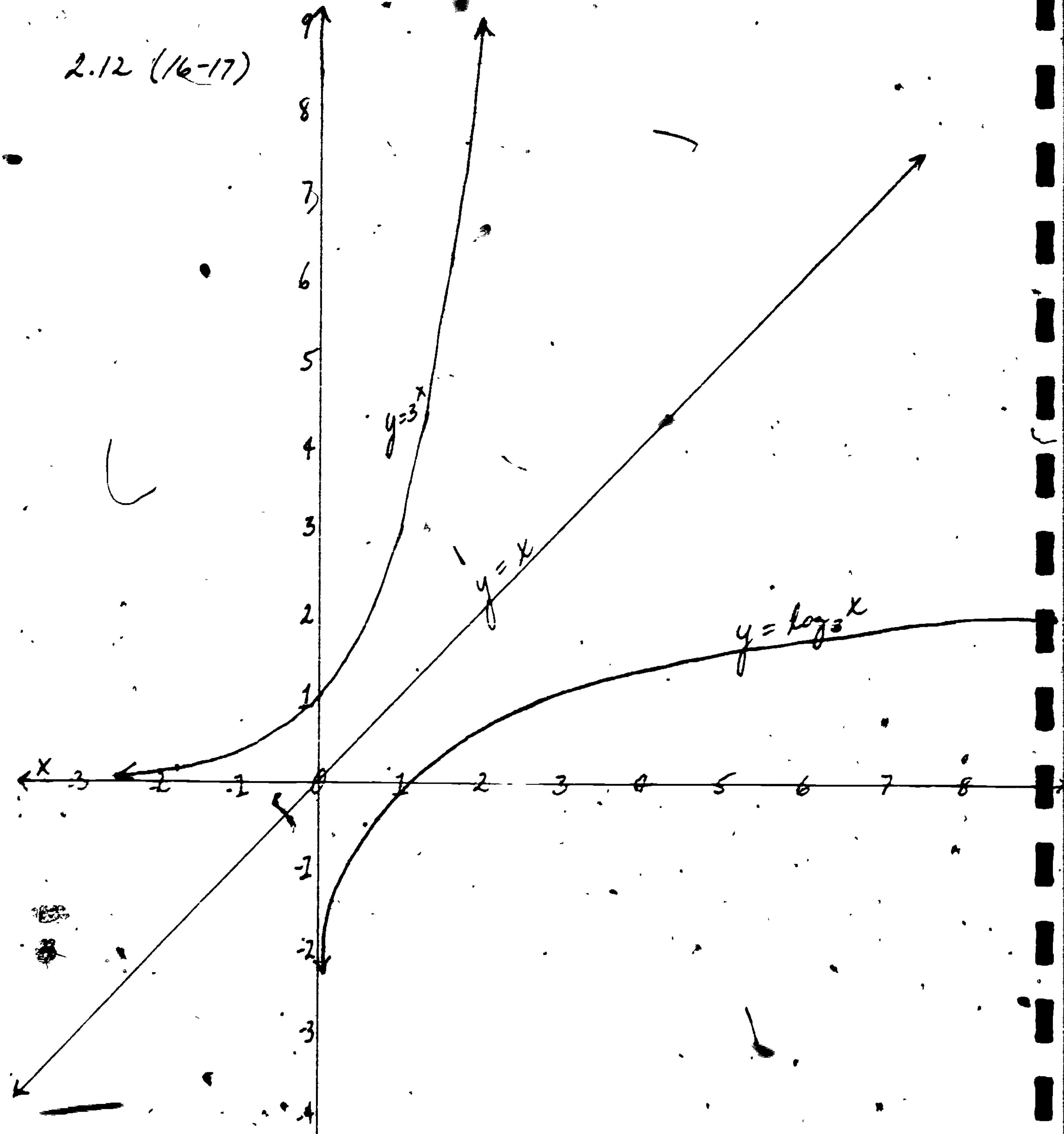


14) They fit exactly on each other.

15) They are symmetric about $y = x$ because these functions are inverses of each other.

16) - 17) see graph below

2.12 (16-17)



Solutions to Chapter 2 TEST

1)
$$\log n = 1 + \log 2$$
$$\log n = \log 10 + \log 2$$
$$\log n = \log (10 \cdot 2)$$
$$n = 20$$

2)
$$3^x = 9^y$$
$$3^x = (3^2)^y$$
$$x = 2y$$

3)
$$\log_b 81 = \frac{4}{3}$$
$$b^{\frac{4}{3}} = 81$$
$$b = 81^{3/4}$$
$$b = 27$$

4)
$$\left(\frac{16}{15}\right)^x > 10$$
$$x(\log 16 - \log 15) > \log 10$$
$$x > \frac{1}{\log 16 - \log 15}$$
$$x > 35.6\overline{7}$$
$$x = 36$$

5)
$$\log_a 54 = \log_a 3^3 \cdot 2$$
$$= 3 \log_a 3 + \log 2$$
$$= 3c + b$$

6)
$$y = \log 5$$
$$10^y = 5$$
$$10^{2y} = 5^2 = 25$$

7)
$$3^{2y+3} = \frac{1}{3}$$
$$3^{2y+3} = 3^{-1}$$
$$2y + 3 = -1$$
$$2y = -4$$
$$y = -2$$

8)
$$x^{-5/2} = 32$$
$$x = \frac{1}{\sqrt[5]{32^2}} = \frac{1}{4}$$

9) $\{x \mid x > 0\}$

10) 3.472×10^{-4}

11) $3^0(3^{-1} \div 3^{-4}) = 1(3^3) = 27$ (B)

12) $(2.7)(10^{-1}) = .27$ (D)

13)
$$2^{x+.3} = 8$$
$$2^{x+.3} = 2^3$$
$$x + .3 = 3$$
$$x = 2.7$$
(C).

14) $[(.3)^{-6}]^{-1/2} = .3^3 = .027$ (E)

15) $x^{-2/3} = 9$

$x = \pm 9^{-3/2} = \frac{\pm 1}{3^3} = \frac{\pm 1}{27}$ (A)

16) $27\sqrt{3} \div 3^{-1}\sqrt{3^3}$

$3^3 \cdot 3^{1/2} \div 3^{-1} \cdot 3^{3/2}$

$3^{7/2} \div 3^{1/2}$

$3^{6/2} = 27$ (B)

17) $3^{3/2} = \sqrt{3^3} = \sqrt{27}$ (F)

18) $(-27)^{-2/3} = \frac{-1}{\sqrt[3]{27^2}} = \frac{-1}{9}$ (G)

19) $(0.000027)(10^6) = 27$ (B)

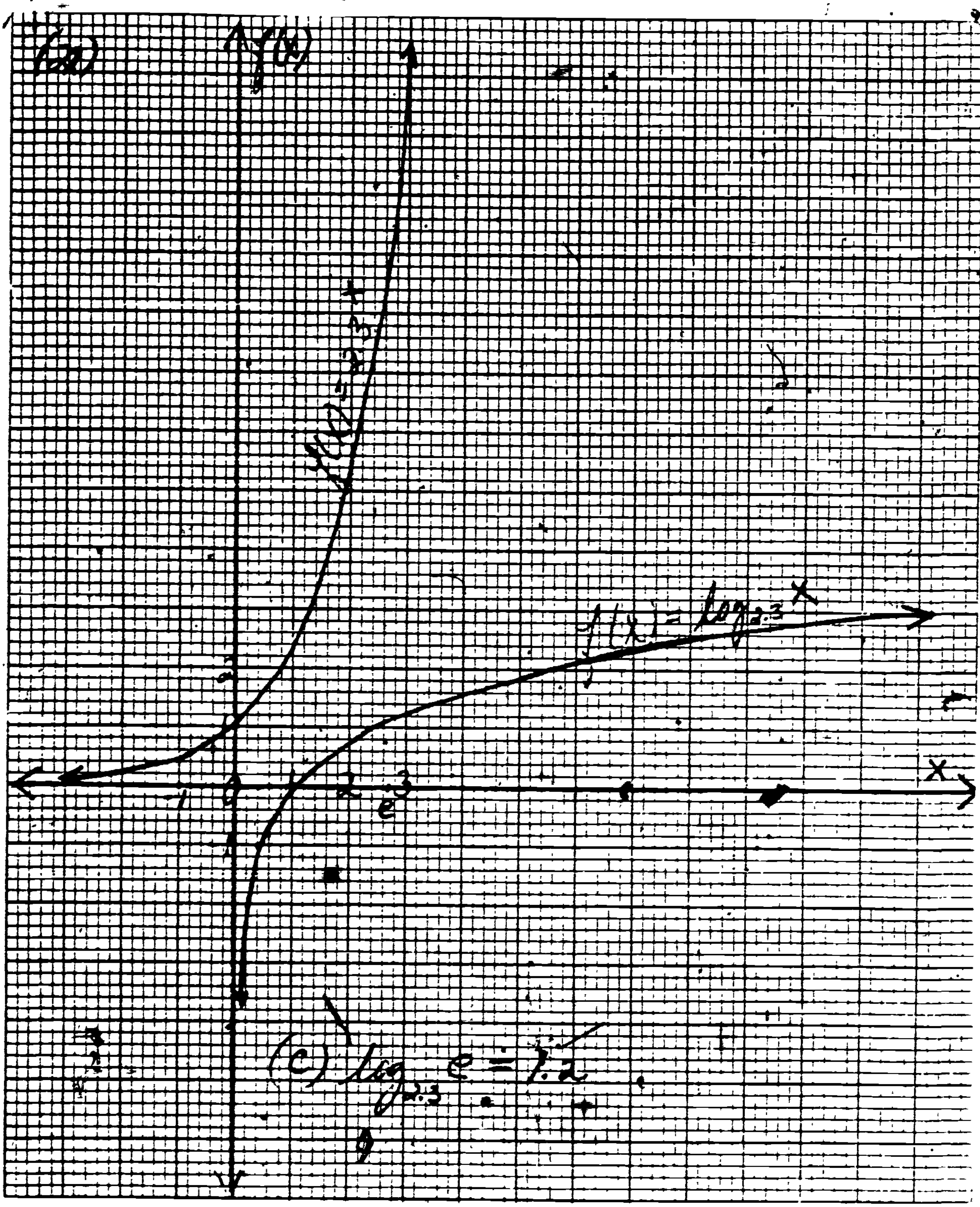
20) $\frac{27}{27^{1/2}} \cdot 27^0 \cdot 3^{-3}$

$27^{1/2} \cdot \frac{1}{3^3} = \frac{\sqrt{27}}{27}$ (G)

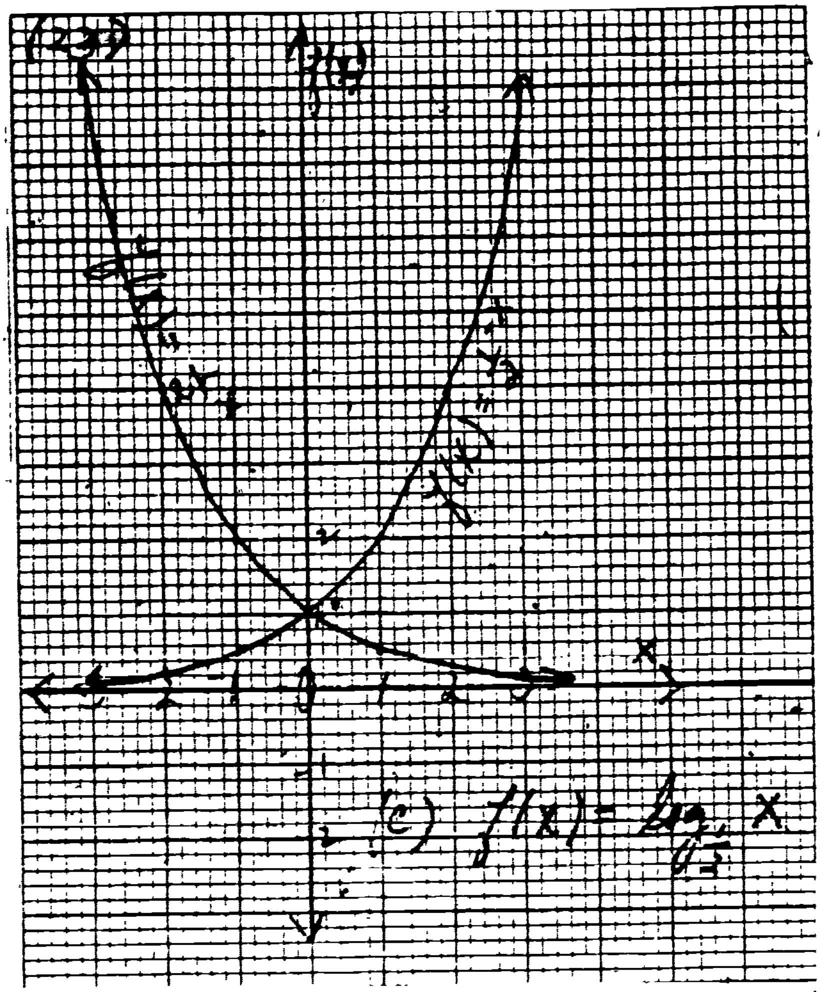
21) $(345621)^2 = (345600 + 21)^2 = 345600^2 + 2(345600)(21) + 21^2$

$$
\begin{aligned}
345600^2 &= 119439360000 \\
2(345600)(21) &= 14515200 \\
21^2 &= 441 \\
\hline
&\ 119453875641
\end{aligned}
$$

22)



23)



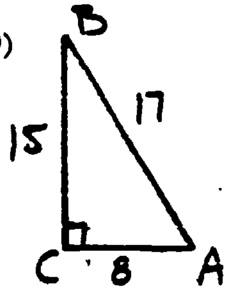
<u>Exercise Set 3.1</u>

1) $\frac{ZY}{XY}$ 2) $\frac{ZY}{XY}$

3) $\frac{ZY}{XZ}$ 4) $\frac{ZY}{XZ}$

5) $\frac{XY}{XZ}$ 6) $\frac{XY}{ZY}$

7) $\frac{XZ}{XY}$ 8) $\frac{XY}{ZY}$

9) 

$BC^2 + AC^2 = BA^2$

$BC^2 + 8^2 = 17^2$

$BC^2 + 64 = 289$

$BC^2 = 225$

$BC = 15$

$\sin \measuredangle A = \frac{15}{17}$

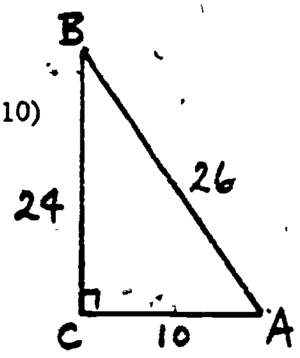
$\cos \measuredangle A = \frac{8}{17}$

$\tan \measuredangle A = \frac{15}{8}$

$\cot \measuredangle A = \frac{8}{15}$

$\sec \measuredangle A = \frac{17}{8}$

$\csc \measuredangle A = \frac{17}{15}$

10) 

$AC^2 + BC^2 = AB^2$

$10^2 + 24^2 = AB^2$

$100 + 576 = AB^2$

$676 = AB^2$

$26 = AB$

$\sin \measuredangle A = \frac{24}{26}$

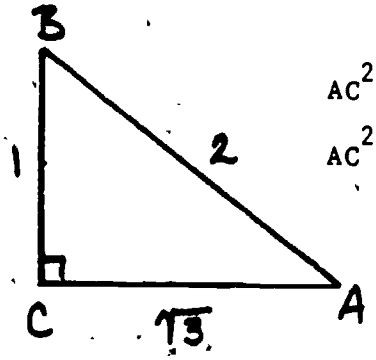
$\cos \measuredangle A = \frac{10}{26}$

$\tan \measuredangle A = \frac{24}{10}$

$\cot \measuredangle A = \frac{10}{24}$

$\sec \measuredangle A = \frac{26}{10}$

$\csc \measuredangle A = \frac{26}{24}$

11) 

$AC^2 + BC^2 = AB^2$

$AC^2 + 1^2 = 2^2$

$AC^2 = 3$

$AC = \sqrt{3}$

$\sin \measuredangle A = \frac{1}{2}$

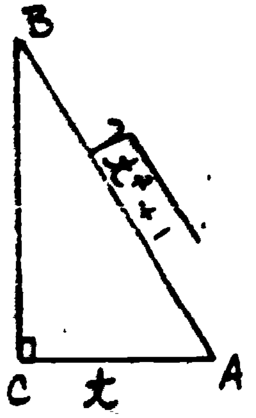
$\cos \measuredangle A = \frac{\sqrt{3}}{2}$

$\tan \measuredangle A = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\cot \measuredangle A = \frac{\sqrt{3}}{1}$

$\sec \measuredangle A = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\csc \measuredangle A = \frac{2}{1}$

12) 

$$AC^2 + BC^2 = AB^2$$
$$t^2 + 1^2 = AB^2$$
$$\sqrt{t^2+1} = AB$$

$$\sin \measuredangle A = \frac{1}{\sqrt{t^2+1}}$$
$$\cos \measuredangle A = \frac{t}{\sqrt{t^2+1}}$$
$$\tan \measuredangle A = \frac{1}{t}$$
$$\cot \measuredangle A = \frac{t}{1}$$
$$\sec \measuredangle A = \frac{\sqrt{t^2+1}}{t}$$
$$\csc \measuredangle A = \frac{\sqrt{t^2+1}}{1}$$

13) Because 2 angles of one triangle are congruent to 2 angles of another triangle.

14) $\sin \measuredangle B = .7193$ they are the same
$\sin \measuredangle E = .7193$ because the angles are the same.

15) $\frac{9.66}{10} = \frac{EF}{3}$

$EF = 2.90$

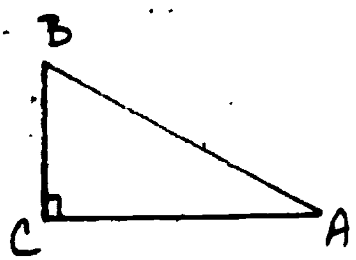
16) $\cos 44 = \frac{10}{BA}$ $\cos 44 = \frac{3}{DE}$

$\frac{10}{\cos 44} = BA$ $\frac{3}{\cos 44} = DE$

$13.90 = BA$ $4.17 = DE$

17) $10^2 + 9.6^2 = AB^2$ $2.90^2 + 3^2 = DE^2$

$13.90 = AB$ $4.17 = DE$

18) 

$\csc \measuredangle A = \frac{BA}{BC} = \sec \measuredangle B$

$\csc \measuredangle B = \frac{BA}{CA} = \sec \measuredangle A$

Thus $\csc x = \sec (90 - x)$ or
$\sec x = \csc (90-x)$

19) $\tan \angle A = \frac{BC}{CA} = \cot \angle B$

$\tan \angle B = \frac{CA}{BC} = \cot \angle A$, $\tan x = \cot (90-x)$

or $\cot x = \tan (90-x)$

20) $\frac{\sin 0^\circ}{\cos 0^\circ} = 0$

$\frac{\sin 1^\circ}{\cos 1^\circ} = .0175$

$\frac{\sin 15^\circ}{\cos 15^\circ} = .2679$

$\frac{\sin 30}{\cos 30} = .5774$

$\frac{\sin 75}{\sin 75} = 3.7321$

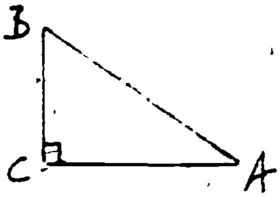
$\frac{\sin \angle A}{\cos \angle A} = \tan \angle A$

$\sin \angle A = \frac{BC}{BA}$

$\cos \angle A = \frac{CA}{BA}$

$\frac{\sin \angle A}{\cos \angle A} = \frac{\frac{BC}{BA}}{\frac{CA}{BA}} = \frac{BC}{BA} = \tan \angle A$

21) $\sec \angle A = \frac{BA}{CA}$



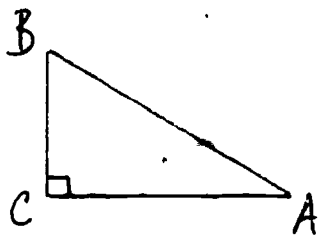
$\csc \angle A = \frac{BA}{BC}$

$\frac{\csc \angle A}{\sec \angle A} = \frac{\frac{BA}{BC}}{\frac{BA}{CA}} = \frac{\frac{1}{BC}}{\frac{1}{CA}} = \frac{CA}{BC} = \cot \angle A$

22) $\csc \angle A = \frac{BA}{BC}$, $\frac{BA}{BC} > 1$ because $BC < AB$

23) $\sec \angle A = \frac{BA}{CA}$, $\frac{BA}{CA} > 1$ because $CA < BA$

24) $\tan \angle A = \frac{BC}{CA}$, $\frac{BC}{CA} > 1$ when $BC > CA$



25) As the measure of an angle increases the length of the opposite side increases while the length of the hypotenuse remains constant.

26) As the measure of an angle increases the length of the adjacent side decreases, while the length of the hypotenuse remains constant.

27) $\tan \measuredangle A = \frac{BC}{CA}$ $\sin \measuredangle A = \frac{BC}{BA}$

since $BA > CA$

$$\frac{BC}{CA} < \frac{BC}{BA}$$

28) A triangle cannot have 2 right angles; division by zero is undefined.

29) If $\sin 0^\circ = 0$ the opposite side would have length = 0.

If $\cos 0^\circ = 1$ the adjacent side and the hypotenuse would be the same length.

30) true

31) false

32) true

33) true

34) true

35) false

Exercise Set 3.2

1) $\frac{AC}{BA}$

2) $\frac{AE}{DA}$

3) $\frac{AG}{FA}$

4) $\frac{AI}{HA}$

5) AI < AG < AE < AC

6) One, because the length of the adjacent side and the hypotenuse are nearly the same.

7) Zero, because the length of the adjacent side is very near zero.

8) Because the length of the adjacent side decreases and the length of the hypotenuse remains the same.

9) $\frac{1}{2} < \frac{3}{4}$ and $\frac{2}{1} > \frac{4}{3}$

10) $2 < 5$ and $\frac{1}{2} > \frac{1}{5}$

11) $\frac{5}{7} > \frac{2}{3}$ and $\frac{7}{5} < \frac{3}{2}$

12) $.256 < .583$ and $\frac{1}{.256} > \frac{1}{.583}$

13) $\frac{y}{x} > \frac{a}{b}$ because when the reciprocal of each side of an order relation is taken the order is reversed.

14) sin x

15) cos x

16) tan x

17) >

18) >

19) >

20) decreases

21) decreases

22) decreases

23) decreases

24) decreases

25) decreases

26)

function	value near 0°	behavior 0° to 90°	value near 90°
sine	near 0	increases	near 1
cosine	near 1	decreases	near 0
tangent	near 0	increases	very large
cosecant	very large	decreases	near 1
secant	near 1	increases	very large
cotangent	very large	decreases	near 0

27) The sine of an acute angle is always greater than <u>0</u> and less than <u>1</u>.

28) The cosine of an acute angle is always greater than <u>0</u> and less than <u>1</u>.

29) The tangent and cotangent of an acute angle is always greater than <u>0</u>.

30) The secant and cosecant of an acute angle is always greater than <u>1</u>.

Exercise Set 3.3

1) $\sin 30 + \cos 45$

(A) $\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{1+\sqrt{2}}{2}$

(B) $.5000 + .7071 = 1.2071$

(C) $\frac{1+\sqrt{2}}{2} \doteq \frac{1 + 1.4142}{2} = \frac{2.4142}{2} = 1.2071$

2) $\sin 30 \cos 60 + \cos 30 \sin 60$

(A) $\frac{1}{2}\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4} = 1$

(B) $.5\,(.5) + .87\,(.87) = .25 + .75 = 1$

(C) $1 = 1$

3) $1 + \tan 45$

(A) $1 + 1 = 2$

(B) $1 + 1 = 2$

(C) $2 = 2$

4) $2 \cos 30 + 3 \csc 30$

(A) $2\left(\frac{\sqrt{3}}{2}\right) + 3\left(\frac{2}{1}\right) = \sqrt{3} + 6$

(B) $2(.8660) + 3(2) = 7.7321$

(C) $\sqrt{3} + 6 = 1.7321 + 6 = 7.7321$

5) $\cos^2 30 + \sin^2 30$

(A) $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$

(B) $(.8660)^2 + (.5)^2 = .7500 + .2500 = 1$

(C) $1 = 1$

6) $2 \cos 45 - 3 \cot 60$

(A) $2(\frac{1}{\sqrt{2}}) - 3(\frac{1}{\sqrt{3}}) = \frac{2}{\sqrt{2}} - \frac{3}{\sqrt{3}} = \sqrt{2} - \sqrt{3}$

(B) $2(.7071) - 3(.5774) = 1.4142 - 1.7321 = -0.3178$

(C) $\sqrt{2} - \sqrt{3} = 1.4142 - 1.7321 = -0.3178$

7) $\sec 45 - 2 \cos 60$

(A) $\sqrt{2} - 2(\frac{1}{2}) = \sqrt{2} - 1$

(B) $1.4142 - 2(.5) = .4142$

(C) $\sqrt{2} - 1 = 1.4142 - 1 = .4142$

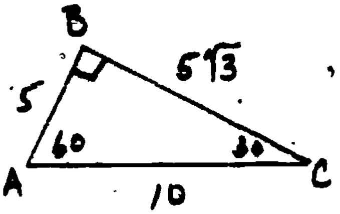
8) $\sec 30 + \csc 30$

(A) $\frac{2}{\sqrt{3}} + 2 = \frac{2\sqrt{3}}{3} + 2$

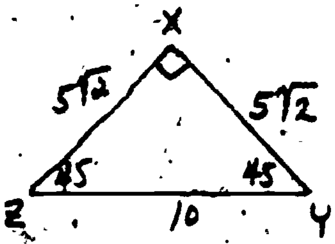
(B) $1.1547 + 2 = 3.1547$

(C) $\frac{2\sqrt{3}}{3} + 2 = 1.1547 + 2 = 3.1547$

9)



10)



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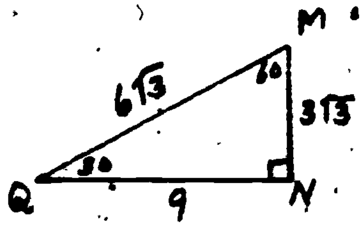

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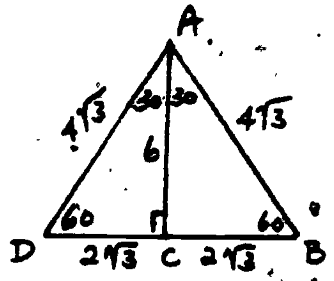

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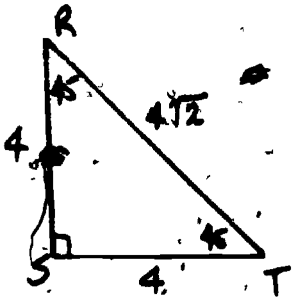
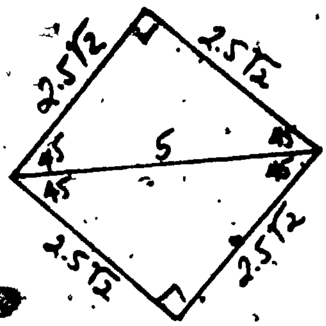


14)



Exercise Set 3.4

1) (a) $45^\circ\ 7'\ 12''$

(b) $\dfrac{120}{1000} = \dfrac{x}{3600}$

$1000x = 720$

$x = 432$ seconds

$432'' = 7'\ 12''$

2) (a) $39^\circ\ 45'\ 18''$

(b) $\dfrac{755}{1000} = \dfrac{x}{3600}$

$1000x = 2718000$

$x = 2718$

$2718'' = 45'\ 18''$

3) (a) $87^\circ\ 12'\ 54''$

(b) $\dfrac{215}{1000} = \dfrac{x}{3600}$

$1000x = 774000$

$x = 774$

$774'' = 12'\ 54''$

4) $51'\ 2'\ 15''$

(b) $\dfrac{375}{10000} = \dfrac{x}{3600}$

$10000x = 1350000$

$x = 135$

$135'' = 2'\ 15''$

5) 50.5 grads $= 45.45^\circ = 45^\circ\ 27'$

(b) $\dfrac{450}{1000} = \dfrac{x}{3600}$

$1000x = 162000$

$x = 1620$

$162'' = 27'$

6) (a) 13.5 grads = 12.15° = $12^\circ\ 9'$ (b) $\frac{150}{1000} = \frac{x}{3600}$

$1000x = 540000$

$x = 540$

$540'' = 9'$

7) (a) 14.175° (b) $10' = 600''$

$10'\ 30'' = 630''$

$\frac{630}{3600} = \frac{x}{1000}$

$x = 175$

8) (a) 68.3875° (b) $23' = 1380''$

$23'\ 15'' = 1395''$

$\frac{1395}{3600} = \frac{x}{1000}$

$x = 387.5$

$\frac{387.5}{1000} = .3875$

9) (a) $82.08\overline{33}^\circ$ (b) $5' = 300''$

$\frac{300}{3600} = \frac{x}{1000}$

$36x = 3000$

$x = 83.3\overline{3}$

$\frac{83.3\overline{3}}{1000} = .083\overline{3}$

10) (a) 70.505° (b) $30' = 1800''$

$30'\ 18'' = 1818''$

$\frac{1818}{3600} = \frac{x}{1000}$

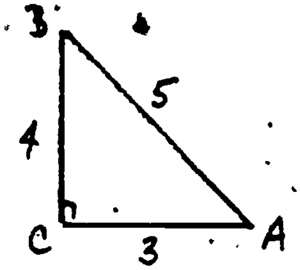
$3600x = 1818000$

$x = 505$

11) 81.225° (a) and (b) both obtained by multiplying by .9

12) 43.5° (a) and (b) both obtained by multiplying by .9

13)

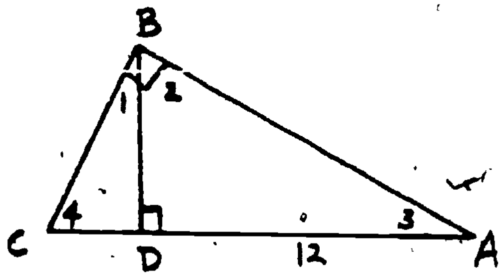


$\sin \measuredangle A = \frac{4}{5}$

$\sin^{-1} \measuredangle A = 53.1301^\circ = 53^\circ 7' 48''$

$m \measuredangle B = 90 - 53.1301^\circ = 36.8699^\circ = 36^\circ 52' 12''$

14)



$\tan \measuredangle 3 = \frac{5}{12}$

$m \measuredangle 3 = 22.6199^\circ$

$m \measuredangle 2 = 90 - 22.6199^\circ = 67.3801^\circ$

$m \measuredangle 1 = 22.6199^\circ = 22^\circ 37' 12''$

$m \measuredangle 4 = 67.3801^\circ = 67^\circ 22' 48''$

15) .9940

16) 78.9905° or $78^\circ 59' 26''$

17) error ($\csc x \geq 1$)

18) error ($\sec x \geq 1$)

19) $x = 26.1^\circ$

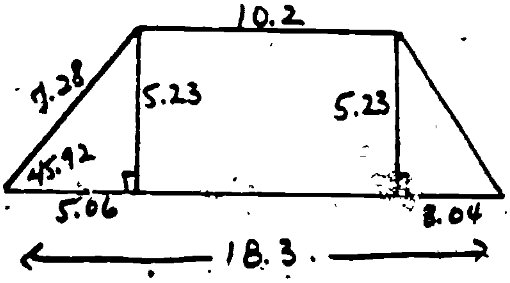
$y = 63.9^\circ$

20) $x = 42.3^\circ$

$y = 47.7^\circ$

21) $x = 45.9^\circ$

$y = 59.8^\circ$



22) $x = y = 39.3^\circ$

23) $\sin x$, $\sin^{-1} x$

$\cos x$, $\cos^{-1} x$

$\rightarrow$ H. MS, $\rightarrow$ H

ln, e^x

log, 10^x

$\sqrt{x}$, x^2

x, $\div$

$\frac{1}{x}$, $\frac{1}{x}$

+, -

$\rightarrow$R, $\rightarrow$P

Exercise Set 3.5

1) $x = 7.4$ (tan)

2) $46^\circ\ 14' = 46.23\overline{3}^\circ$

$x = 57.9$ (csc)

3) $x = 87.4$ (tan)

4) $43^\circ\ 46' = 43.76\overline{6}^\circ$

$x = 46.6$ (sin)

5) $x = 11.5$ (sin)

6) $2x^2 = 83.2$

$x = 6.4$

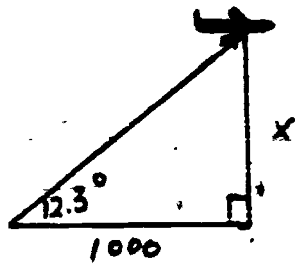
7) $x = 49.4^\circ$ (cos)

8) $x = 44.9$ (sin)

9) $x = 3.0$ (sin)

10) $x = 40.7^\circ$ (sin)

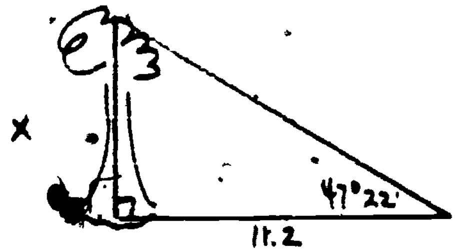
11)



$x = 218.(04)$ (tan)

The plane's altitude is 218 meters.

12)

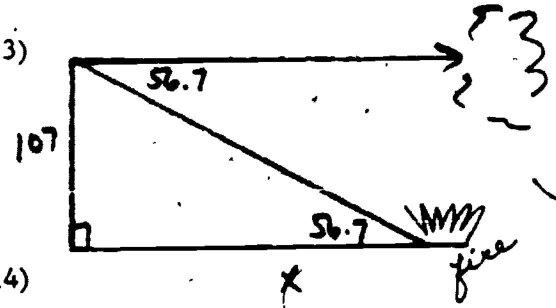


$47^\circ 22' = 47.37^\circ$

$x = 12.(17)$ (tan)

The tree is 12 meters high.

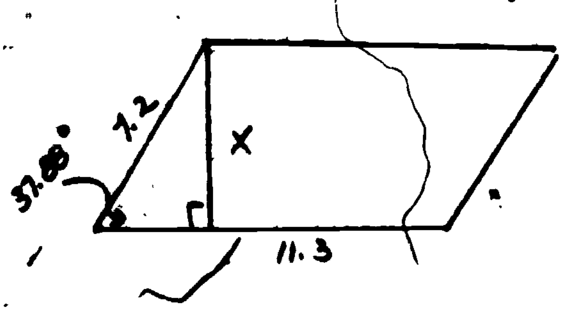
13)



$x = 70.2859$ (tan)

The fire is 70 meters away.

14)

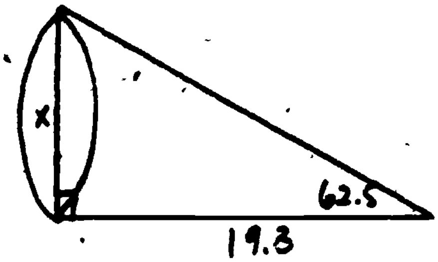


$37^\circ\ 53' = 37.88^\circ$

$= 4.42$ (sin)

The altitude to the longer side is 4.4 cm.

15)



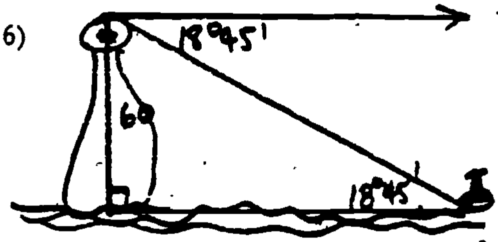
x = 37.08 (tan)

37.08 m = 370.8 dm.

1 meter = 10 decimeters

The spire is 371 dm. high.

16)

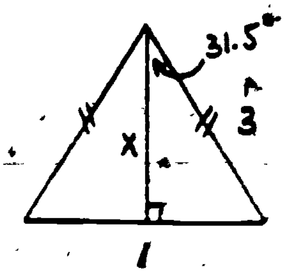


$18^\circ\ 45' = 18.75^\circ$

x = 176.75 (cot)

The buoy is 177 m. away.

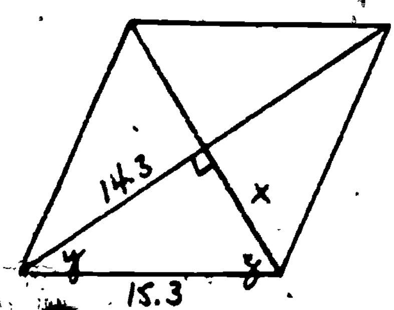
17)



x = 2.56 (cos)

The altitude is 2.6 inches.

18)



$x = \sqrt{15.3^2 - 14.3^2} = 5.4$

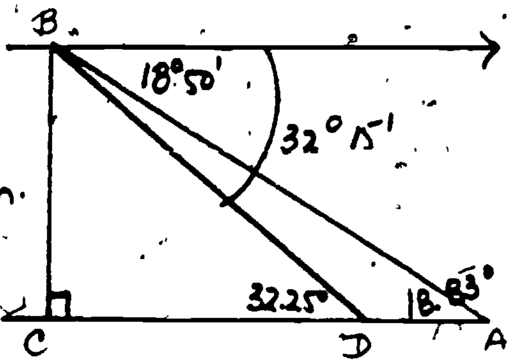
The other diagonal is 10.9.

y = 20.8 (cos)

z = 69.2

The angles are 41.6° and 138.4°.

19)



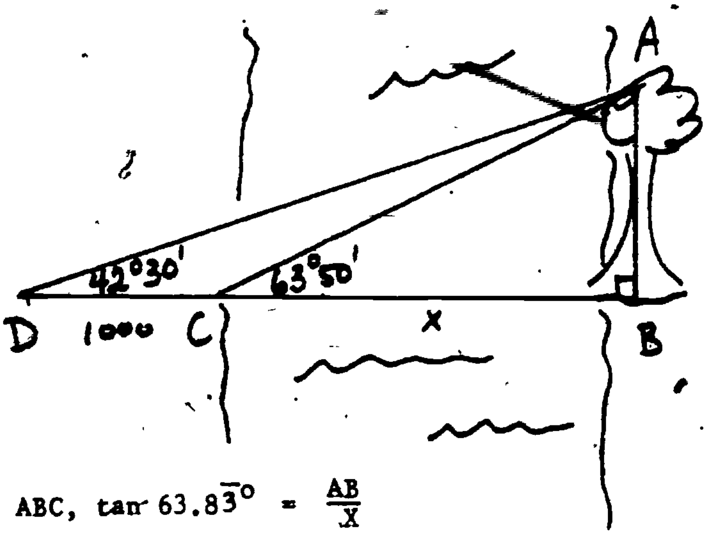
CD = 554.7 (cot)

CA = 1026.2 (cot)

DA = 471.5

The ships are 472 meters apart.

20)



In $\triangle$ ABC, $\tan 63.8\overline{3}^\circ = \frac{AB}{X}$

$2.0353X = AB$

In $\triangle$ ABD, $\tan 42.5^\circ = \frac{AB}{X + 1000} = \frac{2.0353X}{X + 1000}$

$.9163X + 916.3 = 2.0353X$

$916.3 = 1.1190X$

$818.85 = X$

The river is 819 meters wide.

Solutions to Chapter 3 test

1) true

2) false

3) true

4) true

5) false

6) false

7) false

8) true

9) false ($15^\circ 03' = 15.05^\circ$)

10) true

11 a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}+\sqrt{3}}{2}$

b) 1.5731

12 a) $(\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2 = \frac{2}{4} + \frac{2}{4} = 1$

b) 1

13 a) $\frac{\sqrt{3}}{3} + \frac{3}{\sqrt{3}} = \frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$

b) 2.3094

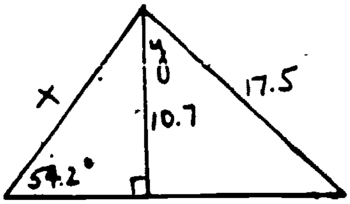
14 a) $\frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$

b) 2.3094

15) $37.25\overline{5}^\circ$

16) 78.525°

17)



$\sin 54.2^\circ = \frac{10.7}{x}$

$\csc 54.2^\circ = \frac{x}{10.7}$

$10.7 \csc 54.2^\circ = x$

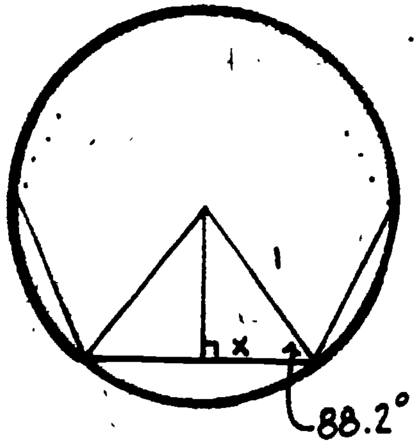
$13.2 = x$

$\cos y = \frac{10.7}{17.5}$

$y = \cos^{-1}(\frac{10.7}{17.5})$

$y = 52.3^\circ$

18 a)



$$\cos 88.2^\circ = \frac{x}{1}$$

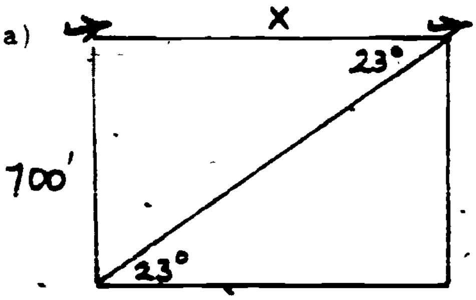
$$0.0314 = x$$

side of polygon = .0628

perimeter of polygon = 6.2822

b) The perimeter of the polygon is very nearly the circumference of the circle.

19 a)



$$\tan 23^\circ = \frac{700}{x}$$

$$\cot 23^\circ = \frac{x}{700}$$

$$1649.0967 = x$$

The plane's speed is 550 ft/sec.

b) 375 mph

Exercise Set 4.1

1) y_3 2) x_3

3) $\frac{y_3}{x_3}$ 4) $\frac{x_3}{y_3}$

5) $\frac{1}{x_3}$ 6) $\frac{1}{y_3}$

7) $x_3 < 0$, $y_3 < 0$ and the quotient of two negative numbers is positive.

8) $\sin \theta_3$, $\cos \theta_3$, $\sec \theta_3$ and $\csc \theta_3$ are negative because the quotient of a positive and a negative number is negative. Tan θ_3 and $\cot \theta_3$ are positive.

9) y_4 10) x_4

11) $\frac{y_4}{x_4}$ 12) $\frac{x_4}{y_4}$

13) $\frac{1}{x_4}$ 14) $\frac{1}{y_4}$

15) $\sin \theta_4 < 0$ because $y_4 < 0$. Cos $\theta_4 > 0$ because $x_4 > 0$.

16) $\sin \theta_4$, $\tan \theta_4$, $\cot \theta_4$ and $\csc \theta_4$ are negative because the quotient of a positive number and a negative number is negative. Cos θ_4 and $\sec \theta_4$ are positive because $x_4 > 0$ and $\frac{1}{x_4} > 0$.

17) $\sin 0^\circ = \frac{\text{ordinate of A}}{\text{distance to origin}} = \frac{0}{1} = 0$; $A = (1,0)$

18) $\cos 0^\circ = \dfrac{\text{abscissa of A}}{\text{distance to origin}} = \dfrac{1}{1} = 1; \quad A = (1,0)$

19) $\sin 90^\circ = \dfrac{\text{ordinate of A}}{\text{distance to origin}} = \dfrac{1}{1} = 1; \quad A = (0,1)$

20) $\cos 90^\circ = \dfrac{\text{abscissa of A}}{\text{distance to origin}} = \dfrac{0}{1} = 0; \quad A = (0,1)$

21) $\sin 180^\circ = \dfrac{\text{ordinate of A}}{\text{distance to origin}} = \dfrac{0}{1} = 0; \quad A = (-1,0)$

22) $\cos 180^\circ = \dfrac{\text{abscissa of A}}{\text{distance to origin}} = \dfrac{-1}{1} = -1; \quad A = (-1,0)$

23) $\sin 270^\circ = \dfrac{\text{ordinate of A}}{\text{distance to origin}} = \dfrac{-1}{1} = -1; \quad A = (0, -1)$

24) $\cos 270^\circ = \dfrac{\text{abscissa of A}}{\text{distance to origin}} = \dfrac{0}{1} = 0; \quad A = (0, -1)$

25) $\sin 360^\circ = \dfrac{\text{ordinate of A}}{\text{distance to origin}} = \dfrac{0}{1} = 0; \quad A = (1,0)$

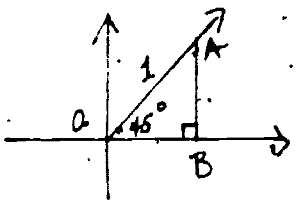
26) $\cos 360^\circ = \dfrac{\text{abscissa of A}}{\text{distance to origin}} = \dfrac{1}{1} = 1; \quad A = (1,0)$

27) $\tan 0^\circ = \dfrac{\text{ordinate of A}}{\text{abscissa of A}} = \dfrac{0}{1} = 0; \quad A = (1,0)$

28) $\tan 180^\circ = \dfrac{\text{ordinate of A}}{\text{abscissa of A}} = \dfrac{0}{-1} = 0; \quad A = (-1,0)$

29) $\tan 270^\circ = \dfrac{\text{ordinate of A}}{\text{abscissa of A}} = \dfrac{-1}{0}$ is undefined; $A = (0,-1)$

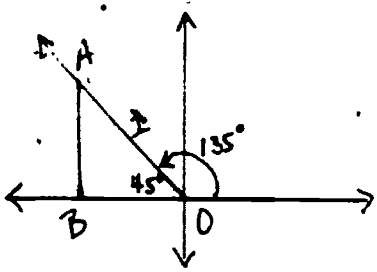
30) $\tan 360^\circ = \dfrac{\text{ordinate of A}}{\text{abscissa of A}} = \dfrac{0}{1} = 0; \quad A = (1,0)$

31) 

$OA = 1 \qquad OB = AB = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$A = \left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$

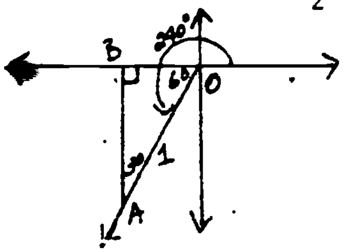
$\sin 45^\circ = \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2}}{2} = \dfrac{1.414}{2} = 0.707$

32) 

$OA = 1, \quad OB = AB = \frac{1}{\sqrt{2}}$

$A = \left(\frac{-\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

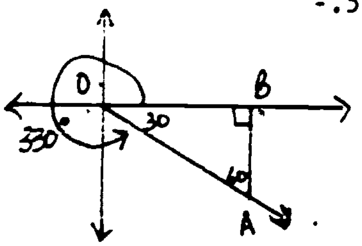
$\cos 135^\circ = \frac{-\sqrt{2}}{2} = \frac{-\sqrt{2}}{2} = \frac{-1.4142}{2} = -0.7071$

33) 

$OA = 1, \quad OB = .5, \quad AB = .5\sqrt{3}$

$A = (-.5, -.5\sqrt{3})$

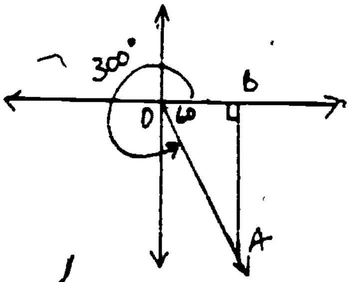
$\tan 240^\circ = \frac{-.5\sqrt{3}}{-.5} = \sqrt{3} = 1.7321$

34) 

$OA = 1, \ AB = .5, \quad OB = .5\sqrt{3}$

$A = (.5\sqrt{3}, -.5)$

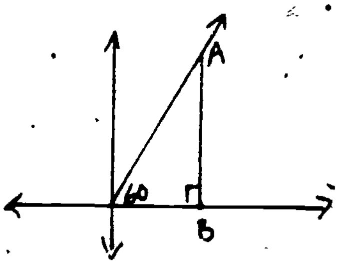
34 $\cot 330^\circ = \frac{.5\sqrt{3}}{-.5} = -\sqrt{3} = -1.732$

35) 

$OA = 1, \ OB = .5, \quad AB = .5\sqrt{3}$

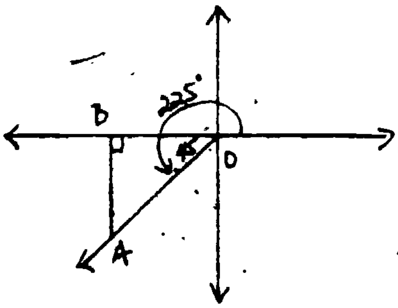
$A = (.5, -.5\sqrt{3})$

$\sec 300^\circ = \frac{1}{.5} = 2$

36) 

$OA = 1, \quad OB = .5, \quad AB = .5\sqrt{3}$

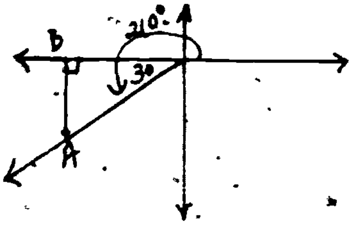
$\sin 60^\circ = .5\sqrt{3} = .5(1.732) = .8660$

37) 

OA = 1, AB = $\frac{\sqrt{2}}{2}$, BA = $\frac{\sqrt{2}}{2}$

A = $\left(\frac{-\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

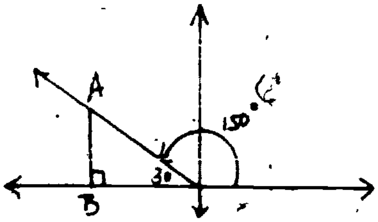
$\cos 225° = -\frac{\sqrt{2}}{2} = -.707$

38) 

OA = 1, AB = .5, OB = $.5\sqrt{3}$

A = $(-.5\sqrt{3}, -.5)$

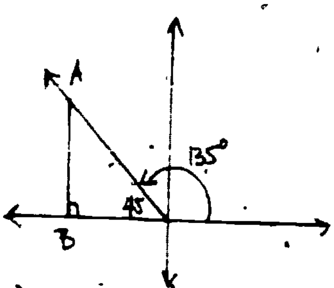
$\csc 210° = \frac{1}{-.5} = -2$

39) 

OA = 1, AB = .5, OB = $.5\sqrt{3}$

A = $(-.5\sqrt{3}, .5)$

$\tan 150° = \frac{.5}{-.5\sqrt{3}} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3} = \frac{-1.732}{3} = -.577$

40) 

OA = 1, BO = $\frac{\sqrt{2}}{2}$, AB = $\frac{\sqrt{2}}{2}$

A = $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\tan 135° = \frac{+\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$

Exercise Set 4.2

1)

θ	A	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
acute	1st quad.	pos.	pos.	pos.	pos.	pos.	pos.
obtuse	2nd quad.	pos.	neg.	neg.	neg.	neg.	pos.
reflex < 270	3rd quad.	neg.	neg.	pos.	pos.	neg.	neg.
$270 <$ reflex < 360	4th quad.	neg.	pos.	neg.	neg.	pos.	neg.

2) A secant is a line that intersects a circle in exactly two points.

3) 1

4) $\sec^2\theta$

5) $\csc^2\theta$

6) negative; sin in 4th quad. is negative

7) positive; cos in 1st quad. is positive

8) positive; cot in 3rd quad. is positive

9) negative; csc 4th quad. is negative.

10) positive; all functions are positive in the 1st quad.

11) negative; tan in 2nd quad. is negative

12) $\cot 725^\circ = 11.4301$; 725° has its terminal side in the 1st quadrant.

$\cot 725^\circ = \cot 5^\circ$

13) $\tan 1020^\circ = -1.7321$; $\tan 1020^\circ = \tan 300^\circ$

14) $\cos 512^\circ = -.8829$; $\cos 512^\circ = \cos 152^\circ$

15) $\sin 1432^\circ = -0.1392$; $\sin 1432^\circ = \sin 352^\circ$

16) $\sin(-115^\circ) = -0.9063$; $\sin(-115^\circ) = \sin 245^\circ$

17) $\cos(-90^\circ) = 0$; $\cos(-90^\circ) = \cos 270^\circ$

18) $\tan(-200^\circ) = -0.3640$; $\tan(-200^\circ) = \tan 160^\circ$

19) $\csc(-290^\circ) = 1.0642$; $\csc(-290^\circ) = \csc 70^\circ$

20) $90^\circ = \frac{1}{4}(360)$

$\frac{1}{4}(2\pi) = \frac{1}{2}\pi = 1.6$ units

21) $\frac{135^\circ}{360} = .3750 = \frac{3}{8};\quad 135 = .375(360)$

$.3750(2\pi) = \frac{3}{8}(2\pi) = \frac{3}{4}\pi = 2.4$ units

22) $180^\circ = \frac{1}{2}(360)$

$\frac{1}{2}(2\pi) = \pi = 3.1$ units

23) $225^\circ = .625(360) = \frac{5}{8}(360)$

$\frac{5}{8}(2\pi) = .6250(2\pi) = 3.9$ units

24) $330^\circ = .92(360) = \frac{11}{12}(360)$

$\frac{11}{12}(2\pi) = .9167(2\pi) = 5.8$ units

25) $120^\circ = \frac{1}{3}(360) = .3\overline{3}(360)$

$\frac{1}{3}(2\pi) = .3333(2\pi) = 2.1$ units

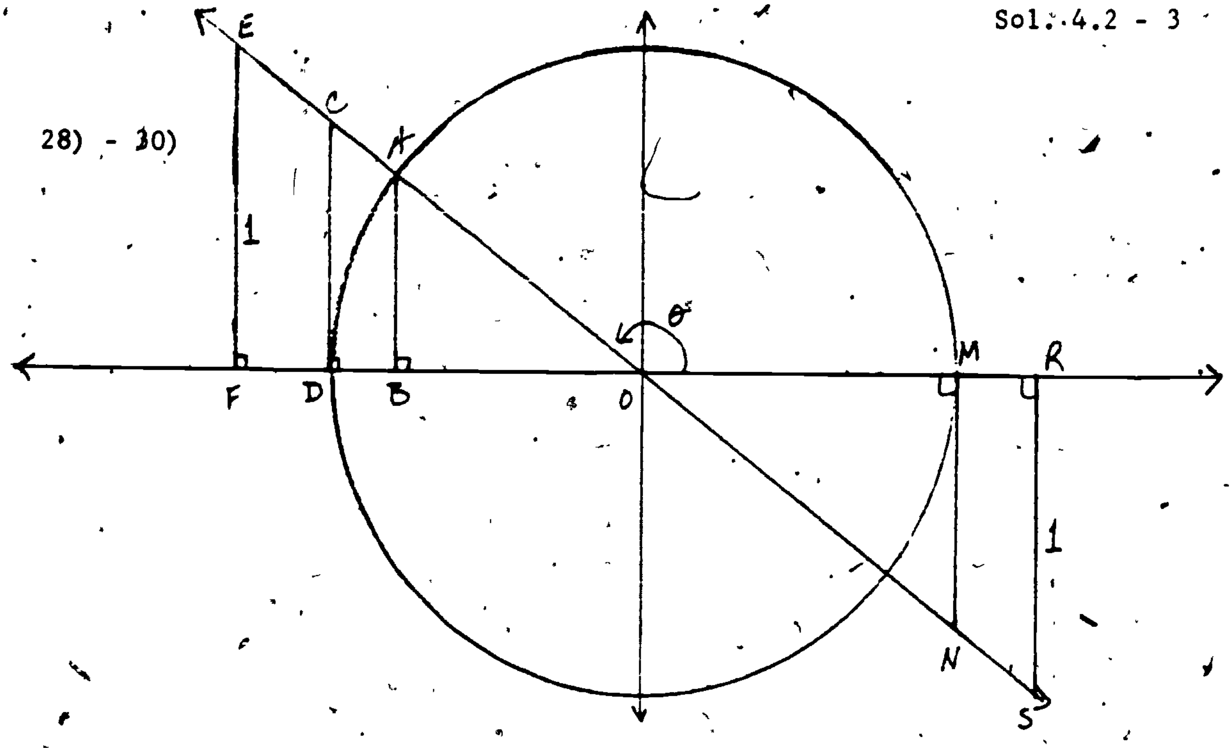
26) $210^\circ = .58(360) = \frac{7}{12}(360)$

$\frac{7}{12}(2\pi) = 3.7$ units

27) $345^\circ = .96(360) = \frac{23}{24}(360)$

$.96(2\pi) = 6.0$ units

28) - 30)

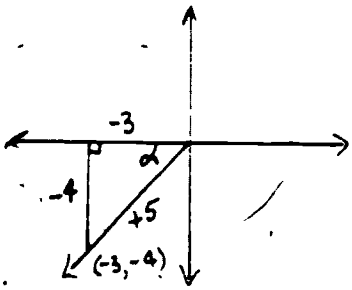


OR = cot θ

ON = sec θ

OE = csc θ

31)

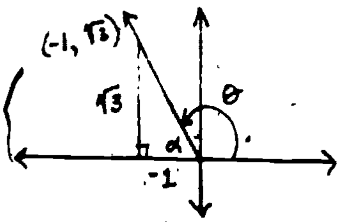


$\cos\alpha = \frac{3}{5} = .6$

$\alpha = 53.1$

$\theta = 180 + 53.1 = 233.1^\circ$

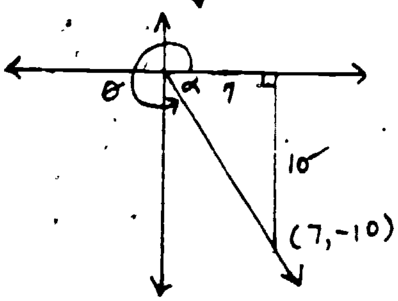
32)



$\tan\alpha = +\sqrt{3} = 60^\circ$

$\theta = 180 - 60 = 120^\circ$

33)

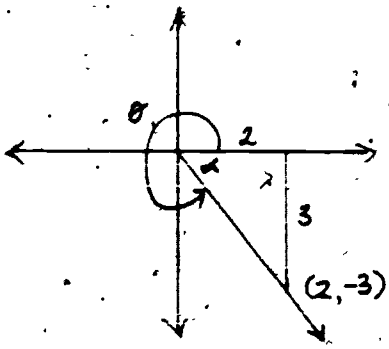


$\tan\alpha = \frac{10}{7}$

$\alpha = 55.01^\circ$

$\theta = 360 - 55.0 = 305^\circ$

34)

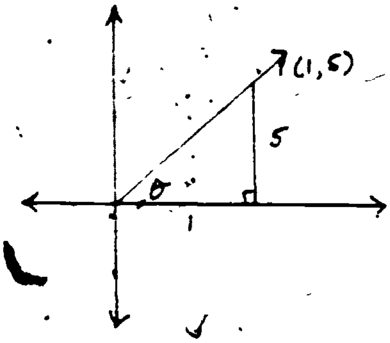


$\tan \alpha = \frac{3}{2}$

$\alpha = 56.3$

$\theta = 303.7^\circ$

35)



$\tan \theta = \frac{5}{1}$

$\theta = 78.7^\circ$

Exercise Set 4.3

1) 2958°

2) .0175 radians

3) π radians = 180°

$(\frac{\pi}{180})$ degrees = x radians

4) π radians = 180°

$\frac{180}{\pi}$ radians = x degrees

5) $\frac{3\pi}{4}$ radians = $\frac{180}{\pi}(\frac{3\pi}{4}) = 135^\circ$

6) 70.4738°

7) 2700°

8) -120°

9) 150°

10) -8594.3669°

A TI 57 programs to convert radian measure to decimal degrees is:

```
     LRN
00       x
01       1
02       8
03       0
04       ÷
05       2nd π
06       =
07       R/S
08       RST
     LRN
     RST
```

11) 4.1888 radians or $\frac{4}{3}\pi$ radians

12) 2.2253 radians or 0.7083π radians

13) .7854 radians or $\frac{\pi}{4}$ radians

14) 4.2542 radians or 1.3542π radians

15) 1.0472 radians or $\frac{\pi}{3}$ radians

16) -0.2732 radians or $-.0870\pi$ radians

17) .5236 radians or $(\frac{1}{6}\pi$ radians)

18) 9.4771 radians or 3.0167π radians

19) 5.7596 radians or $(\frac{11}{6}\pi$ radians)

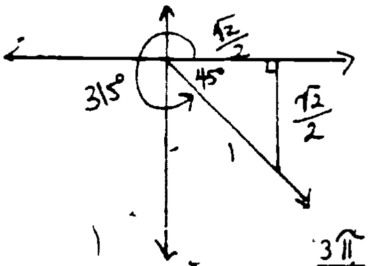
20) $397^\circ 15' = 397.25^\circ$

6.9333 radians or 2.2069π radians

A TI 57 program to convert degree angle measures to radians.

```
     LRN
00        x
01        2nd π
02        ÷
03        1
04        8
05        0
06        =
07        R/S  (gives radian measure)
08        ÷
09        2nd π
10.       =
11        R/S   (gives radian measure in terms of π )
12        RST
      LRN
      RST
```

21) -.71 ,



$-\frac{\sqrt{2}}{2} = -.71$

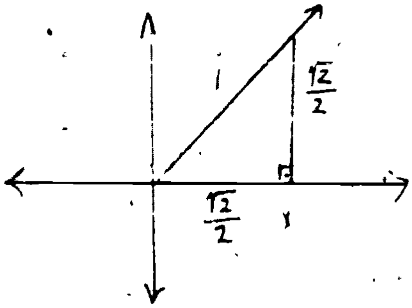
22) 0, $\frac{3\pi}{2}$ radians = 270°

23) .14, tan 548° = tan 188 which is positive

24) .98, 58.3 radians is close to 90°

25) -1.24, csc - 2.3π radians = csc - .3 radians
csc in 4th quad. is negative

26) 1.41, sec $\frac{\pi}{4}$ = sec 45°



sec 45 = $\frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}}$

27) $-4.1640390 \times 10^{-09} \approx 0$

sin 6π = sin 0°

28) tan - 93°35' = tan -93.58° = 15.97

tan -93°35' is positive becau it is in the 3rd quadrant.

29) 0.41, cot $\frac{3\pi}{8}$ = cot 67.5°

30) 1 , cos 2π = cos 0°

31) Any value of θ, $-\sin(+\theta)^\circ = \sin(360 - \theta)^\circ$

32) $\theta = 270, 90, 450$, $-\cos(+\theta)^\circ = \cos(180 + \theta)^\circ$

33) Any value of θ, $\tan(360 - \theta)^\circ = -\tan(\theta)^\circ$

34) Any value of θ, $\cot(360 - \theta)^\circ = -\cot(\theta)^\circ$

35) 0°, 180°, 360°, 540°, 720°

36) 90°, 270°, 450°

Exercise Set 4.4

Checks often reveal small rounding and approximation errors.

1) $3 \cot \theta = 3\sqrt{3}$

$\theta = 30, 210$

check: $5.1962 = 5.1962$

2) $\cos \theta + \frac{\sqrt{3}}{2} = 0$

$\theta = 150^\circ, 210^\circ$

check: $2 \times 10^{-10} \doteq 0$

3) $2 \cos \theta + 7 = 0$

$\cos \theta = \frac{-7}{2}$

the solution set is $\{\ \}$.
$\cos \theta \geq 0$ for all θ

4) $\sqrt{5}(\sin \theta + 1) = 4$

$\theta = 52.08^\circ, 127.92^\circ$

check $4 = 4$

5) $8.6 \sin \theta = 1 - \sin \theta$

$9.6 \sin \theta = 1$

$\theta = 5.98^\circ, 174.02^\circ$

check: $.8960 \doteq .8958$

6) $5 \cos \theta + 6 = 7$

$\theta = 78.46^\circ, 281.54^\circ$

check $7 = 7$

7) $4 - (2 \cot \theta - .9) - 3 \cot \theta = 1$

$4 - 2 \cot \theta + .9 - 3 \cot \theta = 1$

$-5 \cot \theta = -3.9$

$\theta = 52.05, 232.05^\circ$

check $1 = 1$

8) $\frac{\tan \theta + 2}{8.1} = \frac{\tan \theta - 2}{3.5}$

$3.5(\tan \theta + 2) = 8.1(\tan \theta - 2)$

$3.5 \tan \theta + 7 = 8.1 \tan \theta - 16.2$

$14.6 \tan \theta = 23.2$

$\theta = 78.79^\circ, 258.79^\circ$

check: $.87 = .87$

9) $3(\tan \theta - 5.6) = \tan \theta$

$3 \tan \theta - 16.8 = \tan \theta$

$2 \tan \theta = 16.8$

$\theta = 83.21^\circ, 263.21^\circ$

check $8.4 = 8.4$

10) $\frac{1 - \cos \theta}{7.1} = \cos \theta$

$1 - \cos \theta = 7.1 \cos \theta$

$1 = 8.1 \cos \theta$

$82.91^\circ, 277.09^\circ = \theta$

11) $2 \tan \phi + .57 = 1.23$

$\phi = .32, 3.46$ radians

check $1.23 = 1.23$

12) $\sin \phi + 1.8 = \sqrt{5}$

$\phi = .45, 2.69$ radians

check $2.23 \doteq 2.24$

13) $\sin \phi = \cos(\frac{\pi}{2} - \phi)$

$0 \leq \phi \leq 2\pi$

14) $\cos \phi - 1.8 = \sqrt{7}$

the solution set is $\{ \ \}$.

$\cos \phi \leq 1$ for all ϕ

15) $\sqrt{\tan x + 3} - 1 = 0$

$\tan x + 3 = 1$

$x = -63.42^{\circ} = 296.57^{\circ}, 116.58^{\circ}$

check: $6.52 \times 10^{-4} \doteq 0$

16) $\sqrt[3]{2 - 2 \cos x} - 3 = 0$

$2 - 2 \cos x = 27$

$-2 \cos x = 25$

$\cos x = -12.5$

the solution set is $\{ \ \}$.

$\cos x \geq 0$ for all x

17) $\sqrt{1 + \cos x} = 3\sqrt{\cos x}$

$1 + \cos x = 9 \cos x$

$1 = 8 \cos x$

$82.82^{\circ}, 277.18^{\circ} = x$

check: $1.06 = 1.06$

18) $\sqrt{1 - \sin x} = \frac{1}{\sqrt{5}}$

$1 - \sin x = .2$

$x = 53.13^{\circ}, 126.87^{\circ}$

check: $.45 = .45$

19) $\sqrt[3]{\tan x + 5} = 1$

$\tan x = -4$

$x = 284.04^{\circ}, 104.04^{\circ}$

check: $1.0004 \doteq 1$

20) $\sin x = .9149, \quad \sin x = -.9149$

$x = 66.19^{\circ}, 113.81^{\circ}, \quad 293.82^{\circ}, 246.19^{\circ}$

check: $.837 = .837$

21) $1 - \tan^2 x = \sqrt{3}$

$-\tan^2 x = \sqrt{3} - 1$

$\tan^2 x = 1 - \sqrt{3}$

the solution set is $\{\ \}$

since $1 - \sqrt{3} < 0$ and $\tan^2 x \geq 0$.

22) $2.47 = 3\cos^2 x$

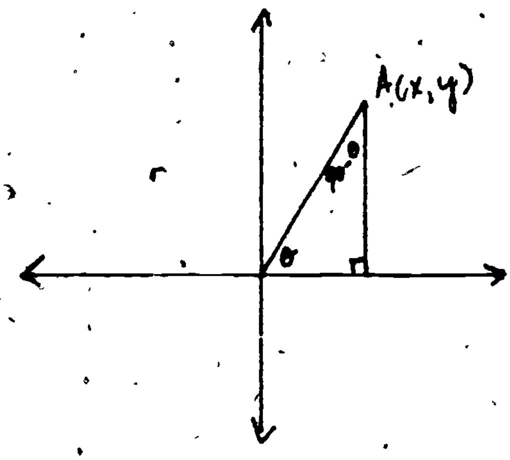
$.83 = \cos^2 x$

$.91, -.91 = \cos x$

$x = 24.85^\circ, 335.15^\circ$

$x = 155.15^\circ, 204.85^\circ$

check: $2.47 = 2.47$



$\sin\theta^\circ = y$ $\quad$ $\sin(90-\theta)^\circ = x = \cos\theta^\circ$

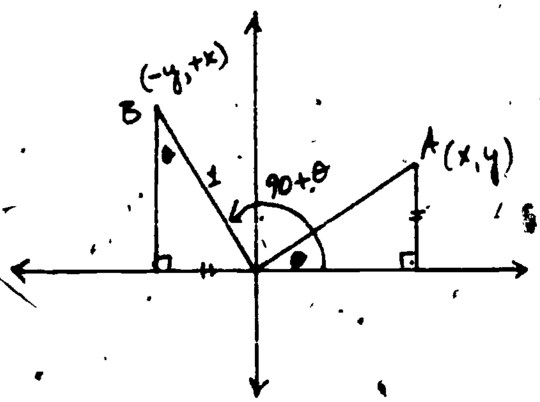
$\cos\theta^\circ = x$ $\quad$ $\cos(90-\theta)^\circ = y = \sin\theta^\circ$

$\tan\theta^\circ = \frac{y}{x}$ $\quad$ $\tan(90-\theta)^\circ = \frac{x}{y} = \cot\theta^\circ$

$\cot\theta^\circ = \frac{x}{y}$ $\quad$ $\cot(90-\theta)^\circ = \frac{y}{x} = \tan\theta^\circ$

$\sec\theta^\circ = \frac{1}{x}$ $\quad$ $\sec(90-\theta)^\circ = \frac{1}{y} = \csc\theta^\circ$

$\csc\theta^\circ = \frac{1}{y}$ $\quad$ $\csc(90-\theta)^\circ = \frac{1}{x} = \sec\theta^\circ$



$\sin(90+\theta)^\circ = x = \cos\theta^\circ$

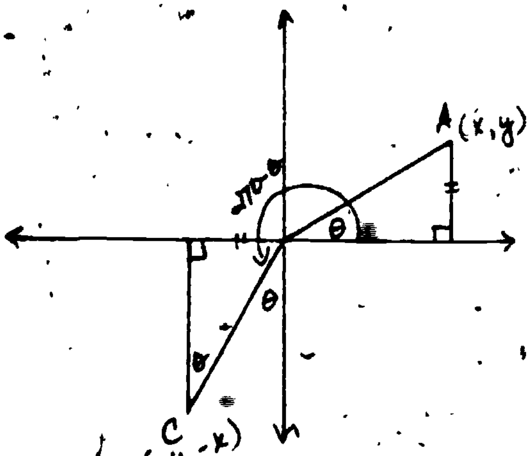
$\cos(90+\theta)^\circ = -y = \sin\theta^\circ$

$\tan(90+\theta)^\circ = \frac{-x}{-y} = -\tan\theta^\circ$

$\cot(90+\theta)^\circ = \frac{-y}{x} = -\cot\theta^\circ$

$\sec(90+\theta)^\circ = \frac{1}{-y} = -\csc\theta^\circ$

$\csc(90+\theta)^\circ = \frac{1}{x} = \sec\theta^\circ$



$\sin(270-\theta)^\circ = -x = -\cos\theta^\circ$

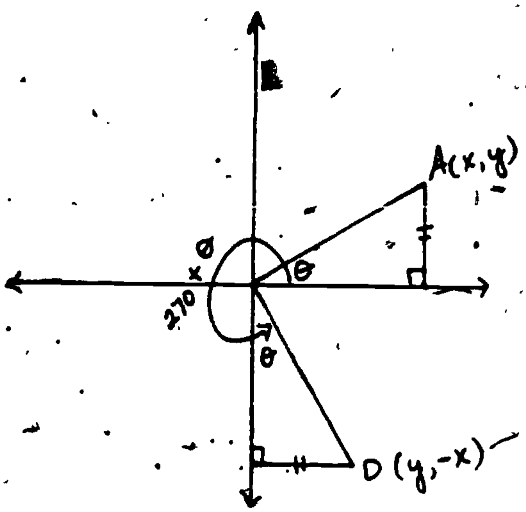
$\cos(270-\theta)^\circ = -y = -\sin\theta^\circ$

$\tan(270-\theta)^\circ = \frac{-x}{-y} = \cot\theta^\circ$

$\cot(270-\theta)^\circ = \frac{-y}{-x} = \tan\theta^\circ$

$\sec(270-\theta)^\circ = \frac{1}{-y} = -\csc\theta^\circ$

$\csc(270-\theta)^\circ = \frac{1}{-x} = -\sec\theta^\circ$



$\sin (270 + \theta)^\circ = -x = -\cos \theta^\circ$

$\cos (270 + \theta)^\circ = y = \sin \theta^\circ$

$\tan (270 + \theta)^\circ = \frac{-x}{y} = -\cot \theta^\circ$

$\cot (270 + \theta)^\circ = \frac{y}{-x} = -\tan \theta^\circ$

$\sec (270 + \theta)^\circ = \frac{1}{y} = \csc \theta^\circ$

$\csc (270 + \theta)^\circ = \frac{1}{-x} = -\sec \theta^\circ$

23) $f(90 - \theta)^\circ =$ cofunction $(\theta)^\circ$, $0^\circ \leq \theta < 90^\circ$

where f is a trigonometric function and sine and cosine, etc. are cofunctions.

24) $|f(90 + \theta)^\circ| =$ cofunction $(\theta)^\circ$, $0^\circ < \theta < 90^\circ$

where f is a trigonometric function, sine and cosine, etc. are cofunctions and the sign of the cofunction is determined by the sign of the function in the 2nd quadrant.

25) $|f(270 - \theta)^\circ| =$ cofunction $(\theta)^\circ$, $0^\circ < \theta < 90^\circ$

where f is a trigonometric function, sine and cosine, etc. are cofunctions and the sign of the cofunction is determined by the sign of the function in the 3rd quadrant.

26) $|f(270 + \theta)^\circ| =$ cofunction $(\theta)^\circ$, $0^\circ < \theta < 90^\circ$

where f is a trigonometric function, sine and cosine, etc. are cofunctions and the sign of the cofunction is determined by the sign of the function in the 4th quadrant.

Solutions - Chapter 4 TEST

1) $-\cos 70^\circ$

2) 2π

3) $-\sin x$

4) 45°

5) $-\frac{\pi}{2}$

6) 210°

7) 135°

8) .8 or 4/5

9) $4\pi/5$

10) -24/25 or -.96

11) (3)

12) (2)

13) (1)

14) (2)

15) (2)

16 a) $\overline{PD}$

b) $\overline{OD}$

c) $\overline{AC}$

d) $\overline{OP}$

e) $\overline{AB}$

Exercise Set 5.1

(1 - 3)

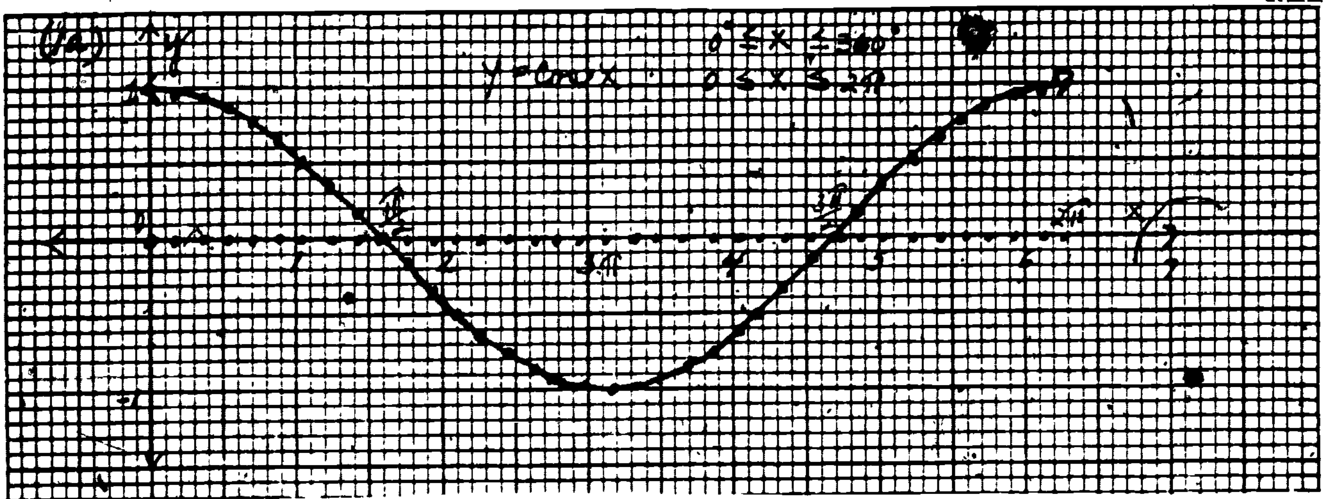
(a)

x	0	$\frac{\pi}{18}$	$\frac{\pi}{9}$	$\frac{\pi}{6}$	$\frac{2\pi}{9}$	$\frac{5\pi}{18}$	$\frac{\pi}{3}$	$\frac{7\pi}{18}$	$\frac{4\pi}{9}$	$\frac{\pi}{2}$	$\frac{5\pi}{9}$
	0	0.17	0.35	0.52	0.70	0.87	1.05	1.22	1.40	1.57	1.75
cos x	1	.98	.94	.87	[illegible]	.64	.50	.34	.17	0	-.17

x	$\frac{11\pi}{18}$	$\frac{2\pi}{3}$	$\frac{13\pi}{18}$	[illegible]	$\frac{5\pi}{6}$	$\frac{8\pi}{9}$	$\frac{17\pi}{18}$	π	$\frac{19\pi}{18}$	$\frac{10\pi}{9}$
	1.92	2.09	2.27	2.44	2.62	2.79	2.97	3.14	3.32	3.49
cos x	-.34	-.5	-.64	-.77	-.87	-.94	-.98	-1	-.98	-.94

x	$\frac{7\pi}{6}$	$\frac{11\pi}{9}$	$\frac{23\pi}{18}$	$\frac{4\pi}{3}$	$\frac{25\pi}{18}$	$\frac{13\pi}{9}$	$\frac{3\pi}{2}$	$\frac{14\pi}{9}$	$\frac{29\pi}{18}$	$\frac{5\pi}{3}$
	3.67	3.84	4.01	4.19	4.36	4.54	4.71	4.89	5.06	5.24
cos x	-.87	-.77	-.64	-.5	-.34	-.17	0	.17	.34	.50

x	$\frac{31\pi}{18}$	$\frac{16\pi}{9}$	$\frac{11\pi}{16}$	$\frac{17\pi}{9}$	$\frac{35\pi}{18}$	2π
	5.41	5.59	5.76	5.93	6.11	6.28
cos x	.64	.77	.87	.94	.98	1

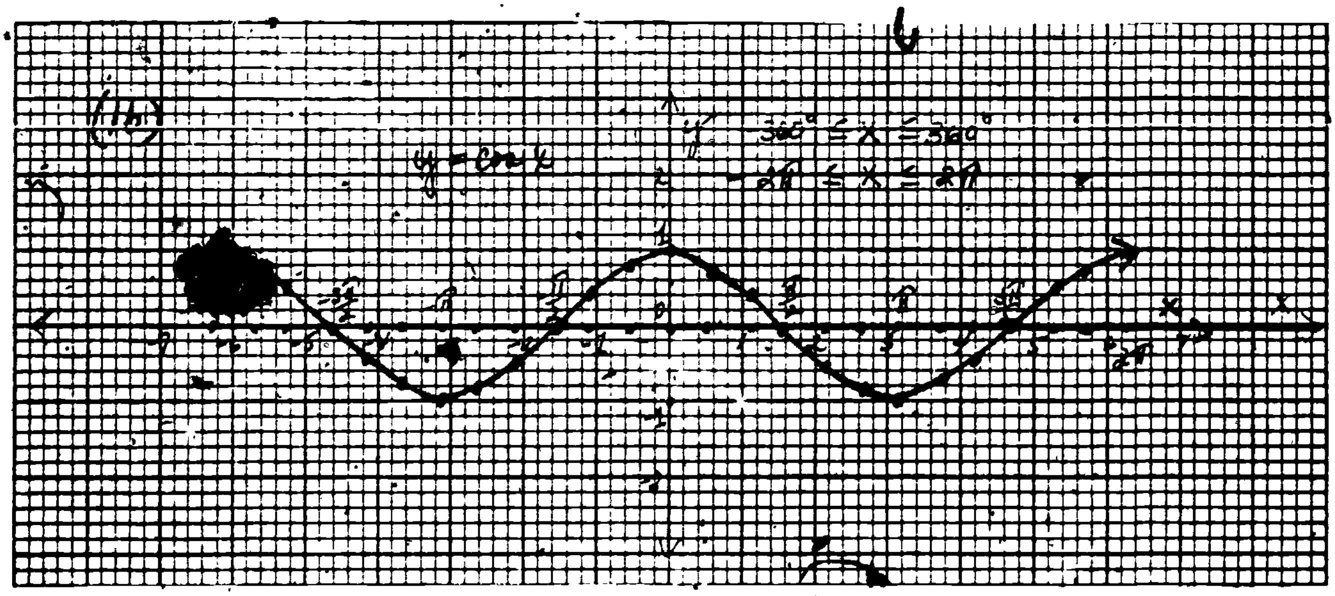


(b)

x	$\frac{-2\pi}{}$	$\frac{-11\pi}{6}$	$\frac{-5\pi}{3}$	$\frac{-3\pi}{2}$	$\frac{-4\pi}{3}$	$\frac{-7\pi}{6}$	$-\pi$	$\frac{-5\pi}{6}$	$\frac{-2\pi}{3}$
	-6.28	-5.76	-5.24	-4.71	-4.19	-3.67	-3.14	-2.62	-2.09
cos x	1	.87	.5	0	-.5	-.87	-1	-.87	-.5

x	$\frac{-\pi}{2}$	$\frac{-\pi}{3}$	$\frac{-\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
	-1.57	-1.05	-.52	0	.52	1.05	1.57	2.09	2.62	3.14
cos x	0	.5	.87	1	.87	.5	0	-.5	-.87	-1

x	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
	3.67	4.19	4.71	5.24	5.76	6.28
cos x	-.87	-.5	0	.5	.87	1



The HP-33 and TI-57 programs on p. 5.1 - 4 of the text can be easily adapted for each of these tables.

1) $\cos x = y$ decreases in quadrants I and II

increases in quadrants III and IV

is positive in quadrants I and IV

is negative in quadrants II and III

2) the amplitude of $y = \cos x$ is 1.

3) the period of $y = \cos x$ is 2π or 360°.

4) 2nd quadrant

5) 4th quadrant

6) 2nd quadrant

7) 3rd quadrant

8) $\frac{5\pi}{4}$ (225°)

9) $\frac{3\pi}{2}$ (270°)

10) $\frac{\pi}{4}$ (45°)

11) they have the same amplitude and the same period.

12) they do not have the same variation

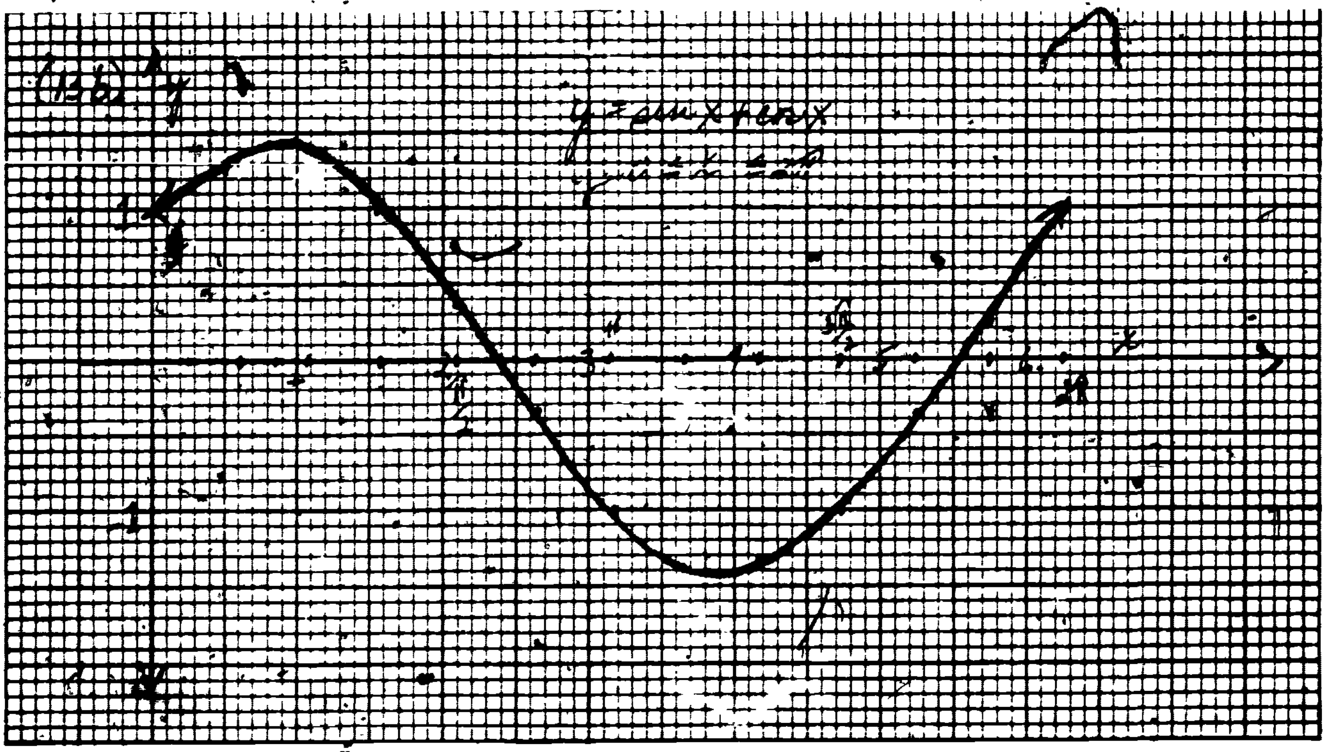
13 - 14)

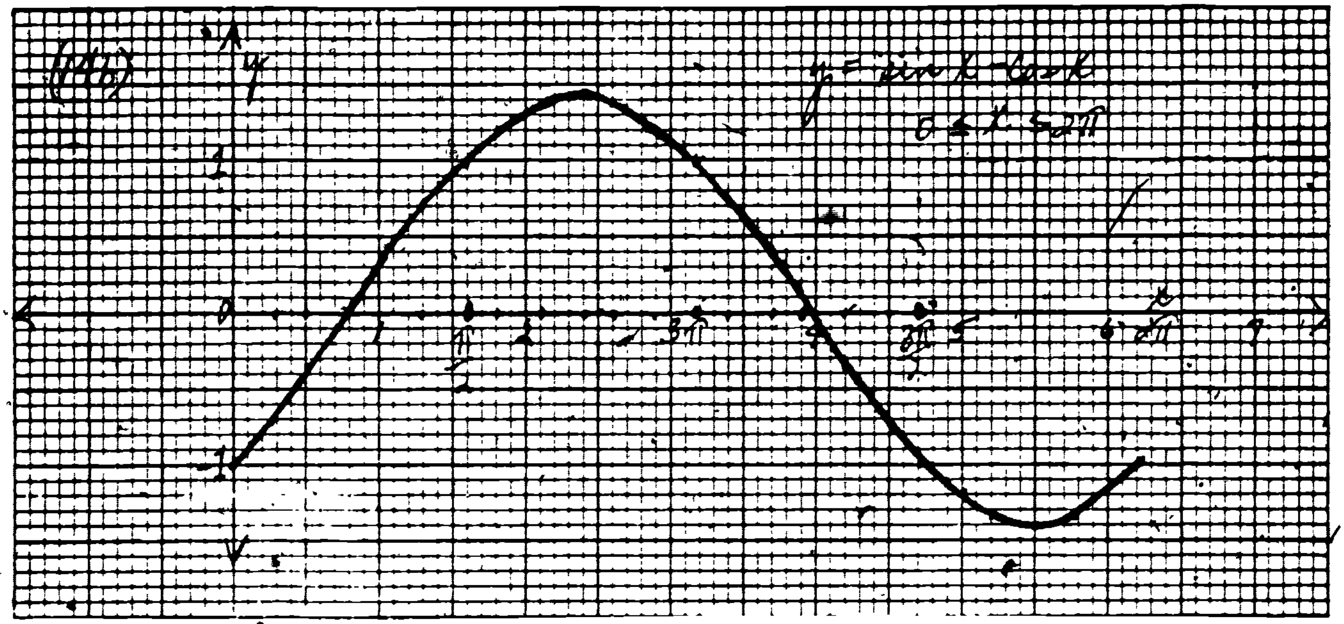
x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$
	0	.26	.52	.79	1.05	1.31	1.57	1.83
$\sin x + \cos x$	1	1.22	1.37	1.41	1.37	1.22	1	.71
$\sin x - \cos x$	-1	-.71	-.37	0	.37	.71	1	1.22

x	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π	$\frac{13\pi}{12}$	$\frac{7\pi}{6}$
	2.09	2.36	2.62	2.88	3.14	3.40	3.67
$\sin x + \cos x$	.37	0	-.37	-.71	-1	-1.22	-1.37
$\sin x - \cos x$	1.37	1.41	1.37	1.22	1	.71	.37

x	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	[illegible]	$\frac{3\pi}{2}$	$\frac{19\pi}{12}$	$\frac{5\pi}{3}$
	3.93	4.19	4.45	4.71	4.97	5.24
sin x + cos x	-1.41	-1.37	-1.22	-1	-.71	-.37
sin x - cos x	0	-.37	-.71	-1	-1.22	-1.37

x	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	$\frac{23\pi}{12}$	2π
	5.50	5.76	6.02	6.28
sin x + cos x	0	.37	.71	1
sin x - cos x	-1.41	-1.37	-1.22	-1





An HP- 33 program for this table is

```
01    ENTER
02    STO 1
03    f  SIN
04    Rcl  1
05    f  COS
06    +         (-)
07    R/S
08    RCL  1
09    3
10    0
11    +
12    STO 1
13    f  SIN
14    RCL  1
15    f  COS
16    +         (-)
17    R/S
18    GTO  08
```

Exercise Set 5.2

1) $\tan(-50^\circ) \doteq -1.2$

$x \doteq -50 - 180 = -230^\circ \doteq -4.01$ radians

$x \doteq -50 + 180 = 130^\circ \doteq 2.27$ radians

$x \doteq 130 + 180 = 310^\circ \doteq 5.41$ radians

2) $x \doteq -5.8$ radians $\doteq -332^\circ$

$\doteq -2.7$ radians $\doteq -152^\circ$

$\doteq .5$ radians $\doteq 28^\circ$

$\doteq 3.6$ radians $\doteq 208^\circ$

3) $x \doteq -4.25$ radians $\doteq -243^\circ$

$\doteq -1.11$ radians $\doteq -63^\circ$

$\doteq 2.03$ radians $\doteq 116^\circ$

$\doteq 5.18$ radians $\doteq 296^\circ$

4) 0°; 257.45°, -257.45°

0, 4.49341, -4.49341 radians

$\tan 0 = 0$, $\tan 4.49341 = 4.49342$

$\tan(-4.49341) \equiv 4.49342$

5) The graph of $y = \tan x$ exactly repeats itself every 180°.

6) Examples of functions with asympotes are hyperbolas

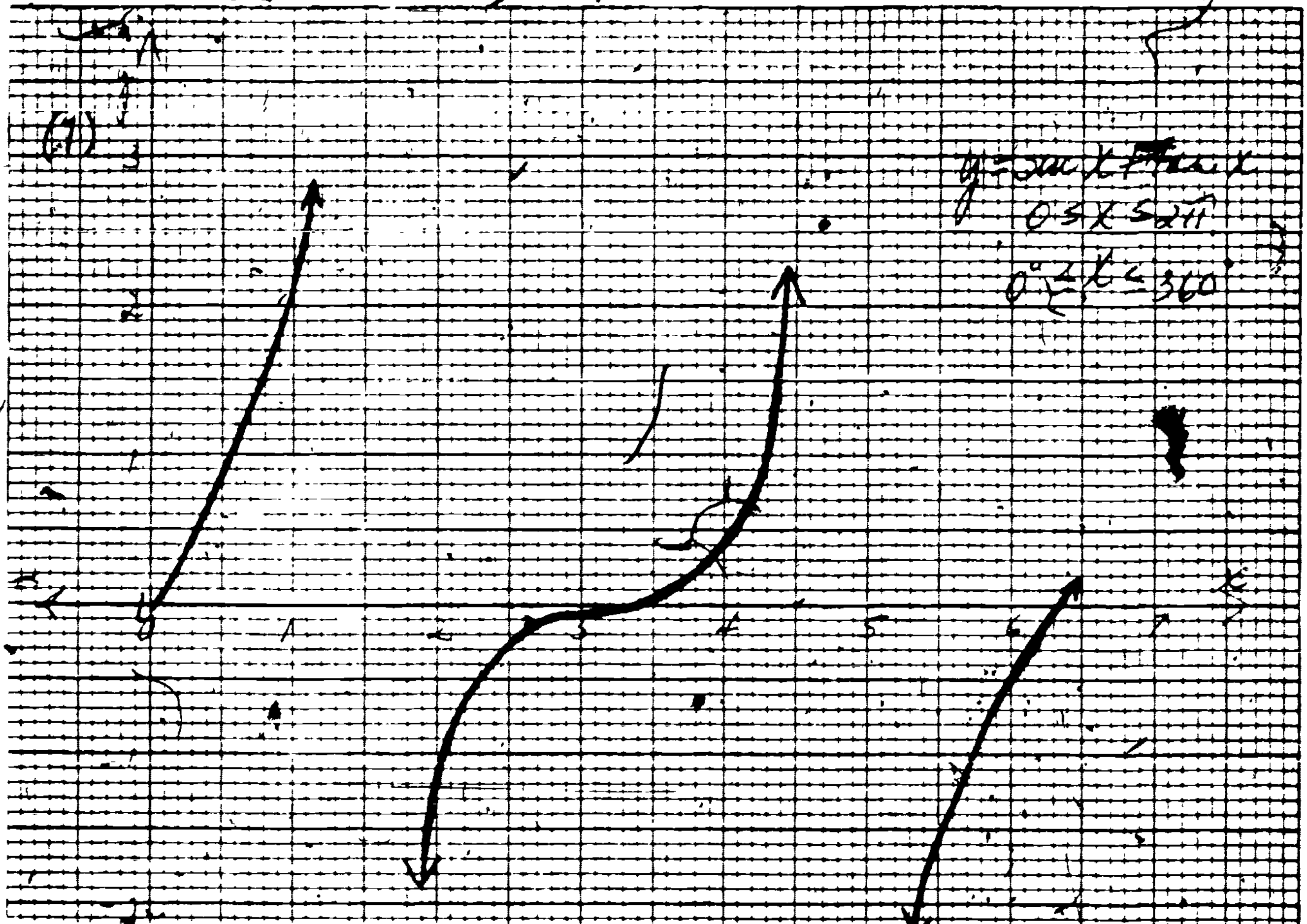
($xy = k$; $ax^2 - by^2 = c^2$) and exponential functions.

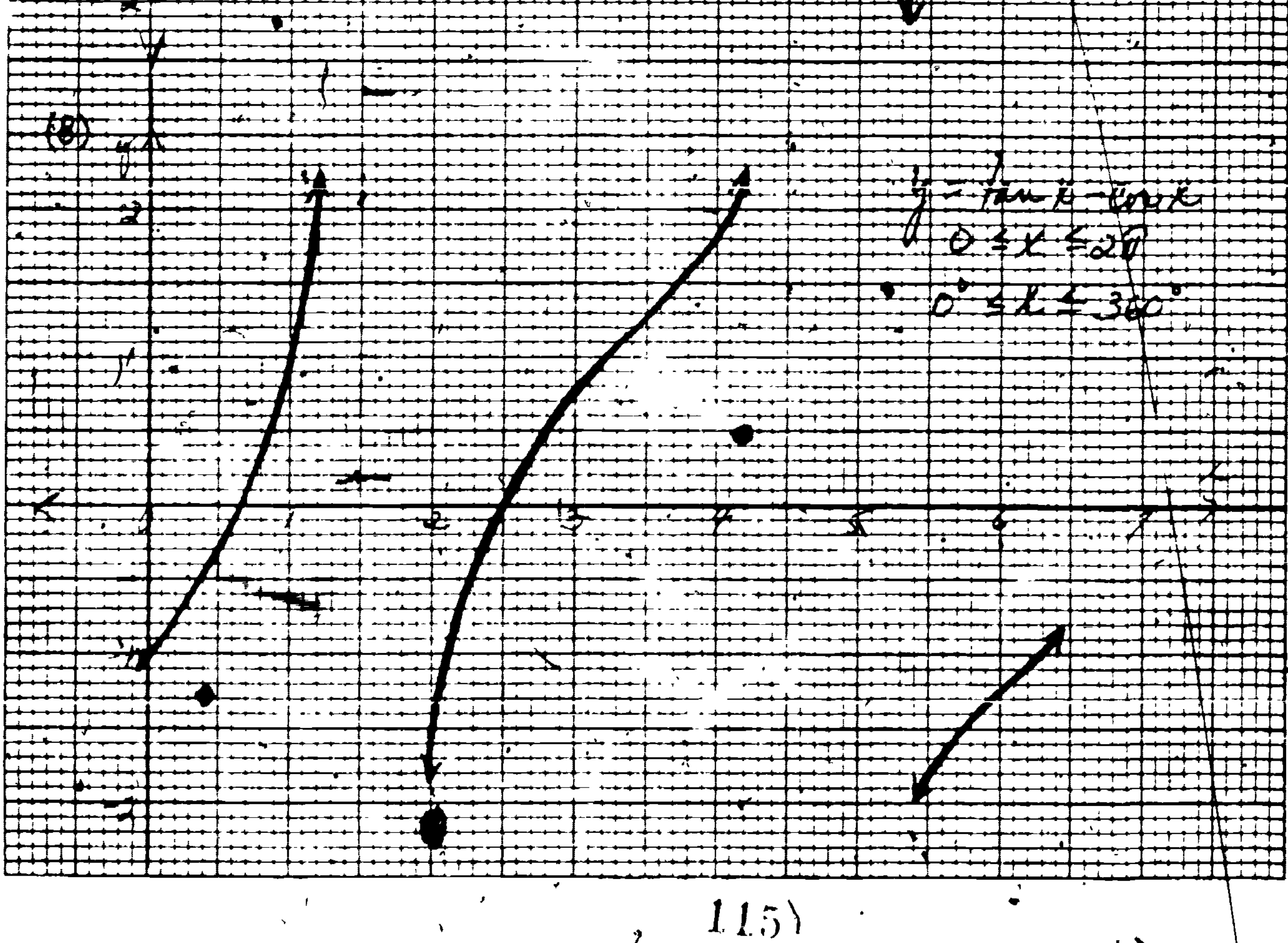
7 -8)

x	sin x + tan x	tan x - cos x
0	0	-1
.2	.4	-7.8
.4	.81	-.5
.6	1.25	-.14
.8	1.75	.33
1.0	2.4	1.02

7-8) continued

x	sin x + tan x	tan x - cos x
1.2	3.5	2.21
1.4	6.78	5.63
1.6	-33.23	-34.20
1.8	-3.31	-4.06
2.0	-1.28	-1.77
2.2	-.57	-.79
2.4	-.24	-.18
2.6	-.09	.26
2.8	-.02	.59
3.0	-.0014	.85
3.2	.00009	1.06
3.4	.008	1.23
3.6	.05	1.39
3.8	.16	1.56
4.	.40	1.81
4.2	.90	2.27
4.4	2.14	3.40
4.6	7.87	8.97
4.8	-12.38	-11.47
5.0	-4.34	-3.66
5.2	-2.77	-2.35
5.4	-1.99	-1.85
5.6	-1.45	-1.59
5.8	-.99	-1.41
6.0	-.57	-1.25
6.2	-.17	-1.08
6.4	.23	-.88


(7)
y = sec x + tan x
0 ≤ x ≤ 2π
0° ≤ x ≤ 360°


(8)
y = tan x − cos x
0 ≤ x ≤ 2π
0° ≤ x ≤ 360°

An HP-33 program that can be used to determine values for this table is

01	g RAD	
02	ENTER	
03	STO 1	
04	f SIN	f TAN
05	RCL 1	
06	f TAN	f COS
07	+	-
08	R/S	
09	RCL 1	
10	.	
11	2	
12	+	
13	STO 1	
14	f SIN	f TAN
15	RCL 1	
16	f TAN	f COS
17	+	-
18	R/S	
19	GTO 09	

Exercise Set 5.3

(1 - 3)

x	cot x	sec x	csc x
-2π	error	1	error
$\frac{-11\pi}{6}$	1.73	1.15	2
$\frac{-5\pi}{3}$	.58	2.	1.15
$\frac{-3\pi}{2}$	large -	large -	1
$\frac{-4\pi}{3}$	-.58	-2	1.15
$\frac{-7\pi}{6}$	-1.73	-1.15	2
$-\pi$	large +	-1	large -
$\frac{-5\pi}{6}$	1.73	-1.15	-2
$\frac{-2\pi}{3}$	.58	-2	-1.15
$\frac{-\pi}{2}$	large -	large +	-1
$\frac{-\pi}{3}$	-.58	2	-1.15
$\frac{-\pi}{6}$	-1.73	1.15	-2
0	large +	1	large +

x	cot x	sec x	csc x
$\frac{\pi}{6}$	1.73	1.15	2
$\frac{\pi}{3}$	.58	2	1.15
$\frac{\pi}{2}$	small -	large -	1
$\frac{2\pi}{3}$	-.58	-2	1.15
$\frac{5\pi}{6}$	-1.73	-1.15	2
π	large +	-1	large -
$\frac{7\pi}{6}$	1.73	-1.15	-2
$\frac{4\pi}{3}$	.58	-2.	-1.15
$\frac{3\pi}{2}$	small -	large +	-1
$\frac{5\pi}{3}$	-.58	2	-1.15
$\frac{11\pi}{6}$	-1.73	1.15	-2
2π	large +	1	large +

The graphs of the functions appear on p. 5.3 - 1 of the text.

An HP-33 program that can be used to determine values for this table is:

	cot x	sec x	csc x
01	g RAD		
02	STO 1		
03	f TAN	f COS	f SIN
04	g 1/x		
05	R/S		
06	RCL 1		
07	g π		
08	6		
09	÷		
10	+		
11	STO 1		
12	f TAN	f COS	f SIN
13	g 1/x		
14	R/S		
15	GTO 06		

(4 - 9)

	cot x	sec x	csc x
period	π	2π	2π
domain	all reals	all reals	all reals
range	all reals	$x \geq 1$ and $x \leq -1$	$x \geq 1$ and $x \leq -1$
discontinuities	at multiples of π	at odd multiples of $\pi/2$	at multiples of π
amplitude	unbounded	unbounded	unbounded

variation of cot x: always decreasing as x increases; cot x is positive for the intervals $\left\{\ldots\ (-2\pi, -\frac{3\pi}{2}), (-\pi, \frac{-\pi}{2}), (0, \frac{\pi}{2}), (\pi, \frac{3\pi}{2}) \ldots\right\}$ cot x is negative for the intervals $\left\{ (-\frac{3\pi}{2}, -\pi), (-\frac{\pi}{2}, 0), (\frac{\pi}{2}, \pi), (\frac{3\pi}{2}, 2\pi) \ldots\right\}$

variation of sec x: increasing for intervals $\{\ldots [-2\pi, -\pi], [0, \pi], [2\pi, 3\pi], \ldots\}$ as x increases.

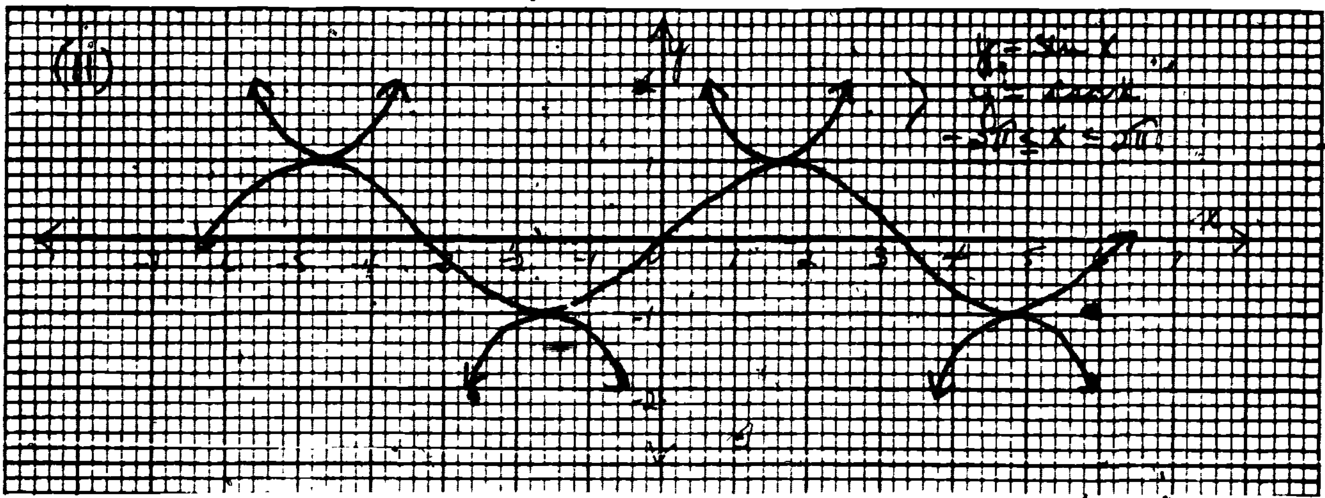
positive for intervals, $\{\ldots, (-\frac{5\pi}{2}, -\frac{3\pi}{2}), (-\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{5\pi}{2}), \ldots\}$

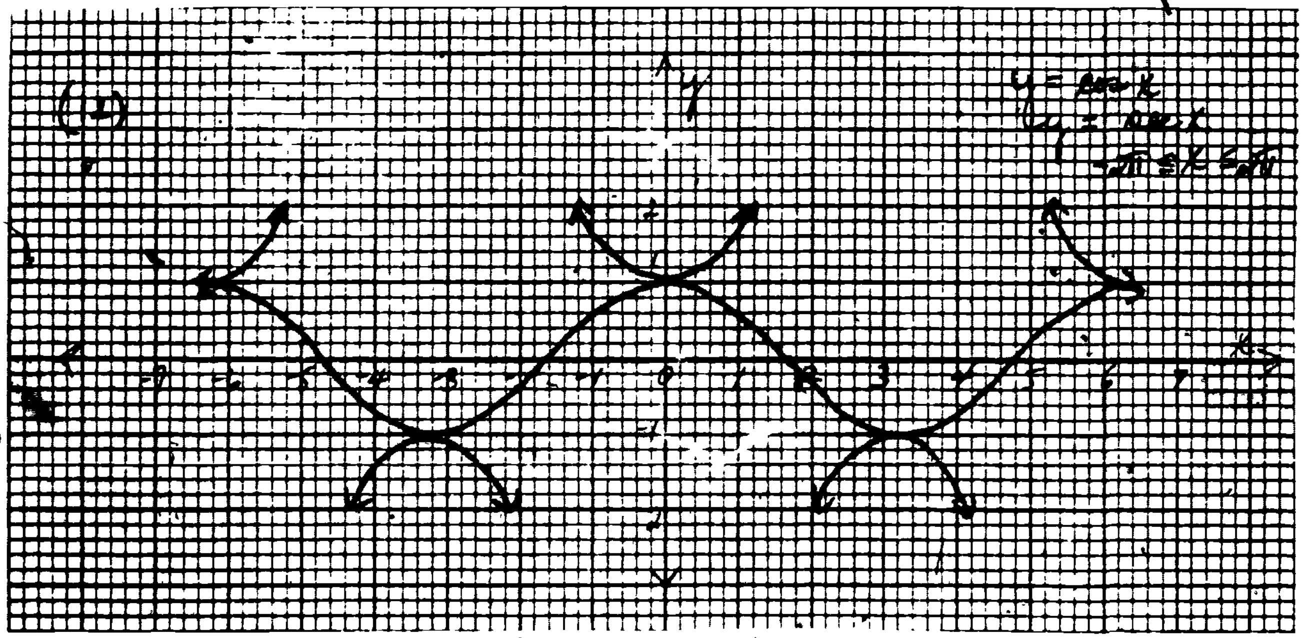
negative for intervals, $\{\ldots, (-\frac{3\pi}{2}, -\frac{\pi}{2}), (\frac{\pi}{2}, \frac{3\pi}{2}), \ldots\}$

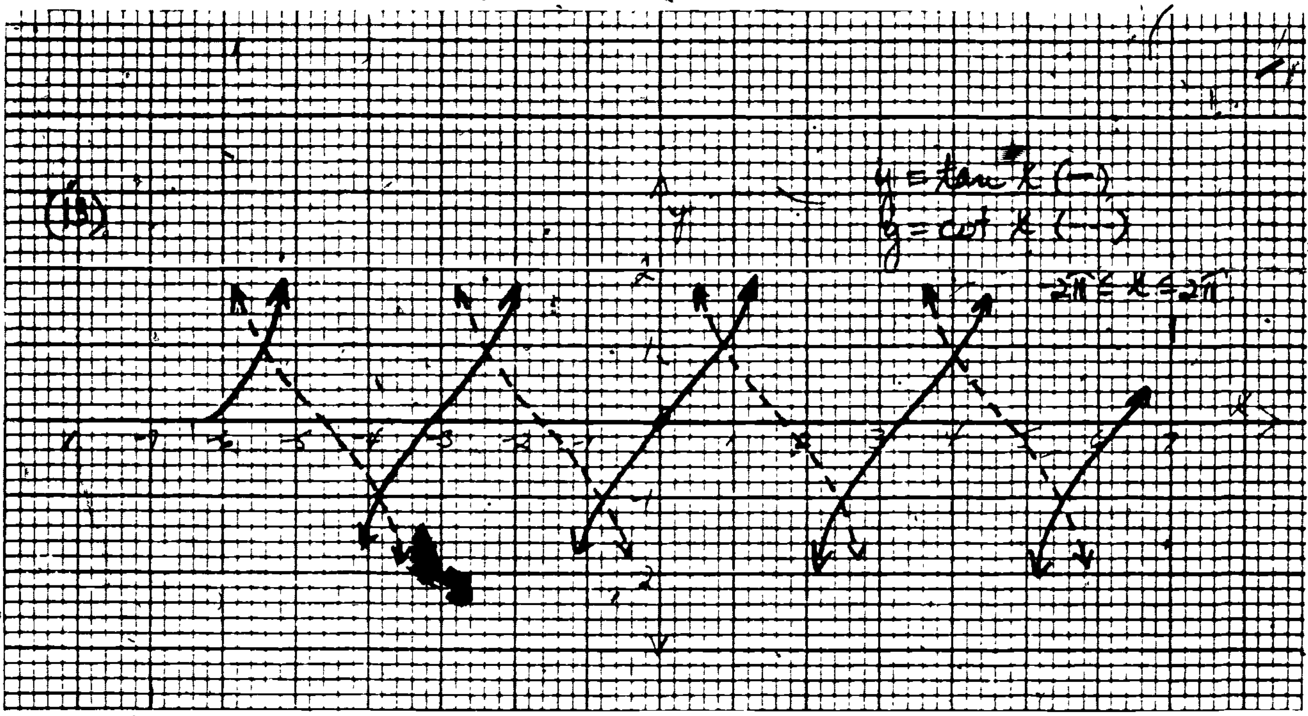
variation of csc x: increasing for intervals $\{\ldots, [-\frac{3\pi}{2}, -\frac{\pi}{2}], [\frac{\pi}{2}, \frac{3\pi}{2}], \ldots\}$ as x increases.

positive for intervals, $\{\ldots, (-2\pi, -\pi), (0, \pi), \ldots\}$

negative for intervals, $\{\ldots, (-\pi, 0), (\pi, 2\pi), \ldots\}$







14) Various answers, some examples:

the graphs of (11) and (12) are identical except that they are $\pi/2$ out of phase.

the graphs of y = tan x and y = cot x are mirror reflections of one another at the points $\frac{\pi}{2}$, $\frac{3\pi}{2}$, ...

Exercise Set 5.4

1) amplitude: 1

period: 2π radians or 360°

frequency: 1 cycle per 2π

2) amplitude: 2

period: 120° or $\frac{2\pi}{3}$ radians

frequency: 3 cycles per 2π

3) amplitude: $\frac{1}{2}$

period: 120° or $\frac{2\pi}{3}$

frequency: 3 cycles per 2π

4) amplitude: 4

period: 2π or 360°

frequency: 1 cycle per 360°

5) amplitude: $\frac{1}{3}$

period: 90° or $\frac{\pi}{2}$

frequency: 4 cycles per 2π

6) amplitude: .7

period: 10π or 1800°

frequency: $\frac{1}{5}$ cycles per 2π

7) amplitude: 1

period: 72° or $\frac{2\pi}{5}$

frequency: 5 cycles per 2π

8) amplitude: 5

period: 4π or 720°

frequency: $\frac{1}{2}$ cycle per 2π

9) amplitude: unbounded

period: 120° or $\frac{2\pi}{3}$

frequency: 3 cycles per 2π

10) amplitude: unbounded

period: 60° or $\frac{\pi}{3}$

frequency: 6 cycles per 2π

11) $y = 2 \sin 2x$

12) $y = \frac{1}{3} \sin 2x$

13) $y = .53 \sin 2x$

14) $y = 6.2 \sin 2x$

15) $y = 4 \cos x$

16) $y = 4 \cos 4x$

17) $y = 4 \cos \frac{2}{3} x$

18) $y = 4 \cos 5x$

19) max = 2

min = -2

20) max = 2

min = -2

21) max = .5

min = -.5

22) max = 12

min = -12

23)

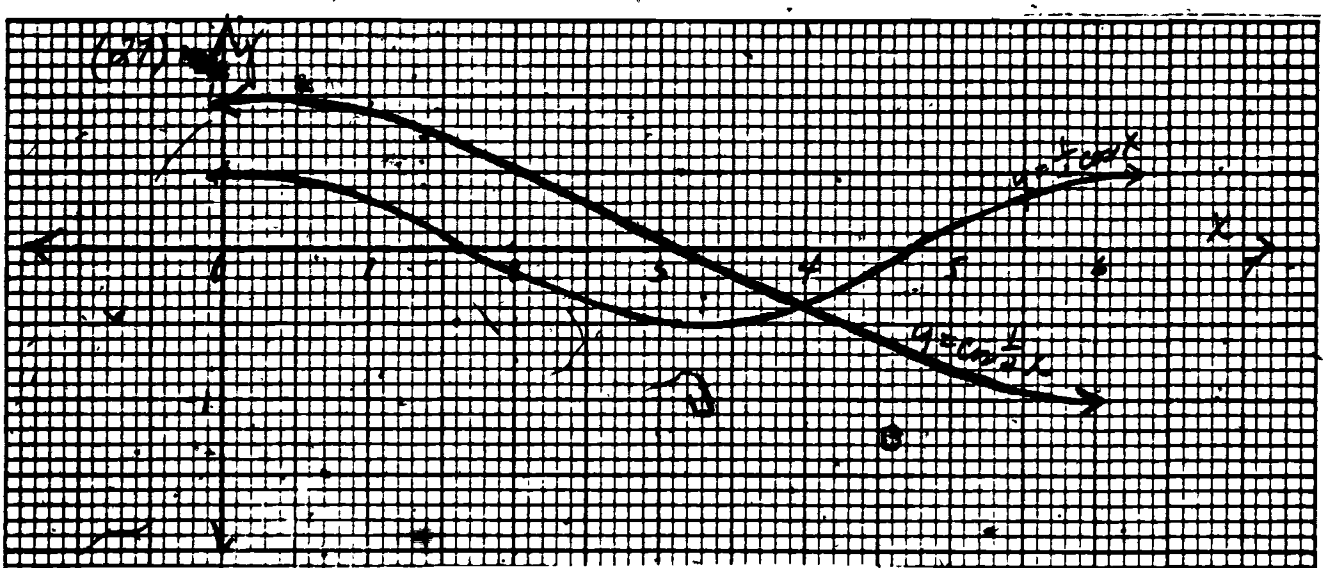
function	period	amplitude	frequency
$y = \sin x$	2π or 360°	1	1 per 2π
$y = 5 \sin x$	2π or 360°	5	1 per 2π
$y = \sin 5x$	$\frac{2\pi}{5}$ or 72°	1	5 per 2π
$y = 5 \sin 5x$	$\frac{2\pi}{5}$ or 72°	5	5 per 2π
$y = \cos x$	2π or 360°	1	1 per 2π
$y = 5 \cos x$	2π or 360°	5	1 per 2π
$y = 5 \cos 5x$	$\frac{2\pi}{5}$ or 72°	5	5 per 2π

24) The period of $y = \sin 2x$ is π while the period of $y = \sin x$ is 2π.

25) The period is how long it takes to complete one cycle. The frequency is how many cycles are completed in 2π.

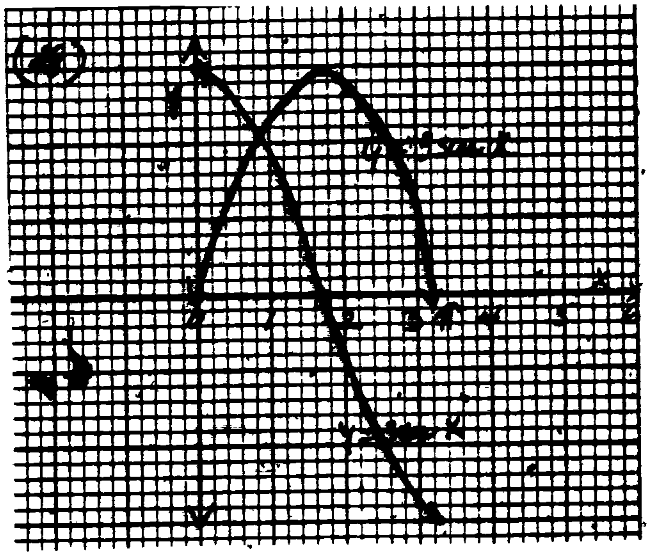
26) The amplitude of the function $y = \sin 3x$ is 1.

27) See graph.



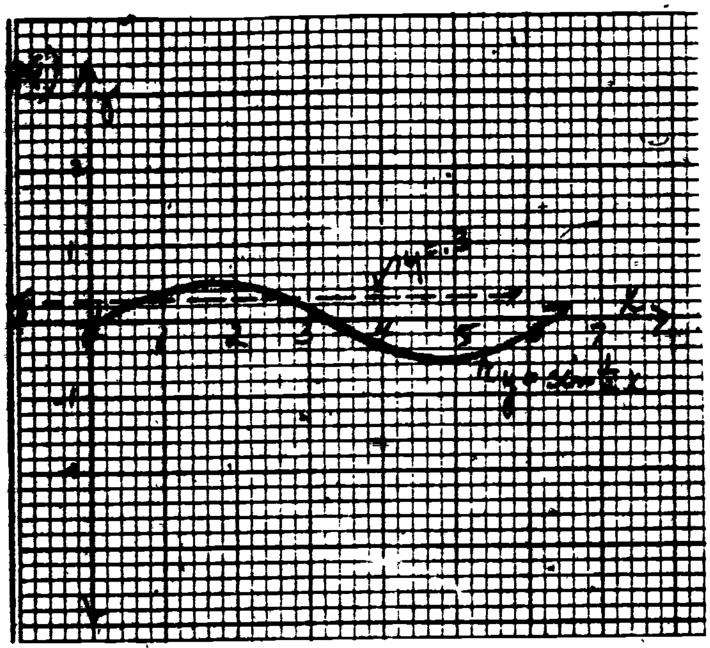
(b) There is one point of intersection.

28)



(b) There is one point between 0 and π where $3 \sin x = 3 \cos x$.

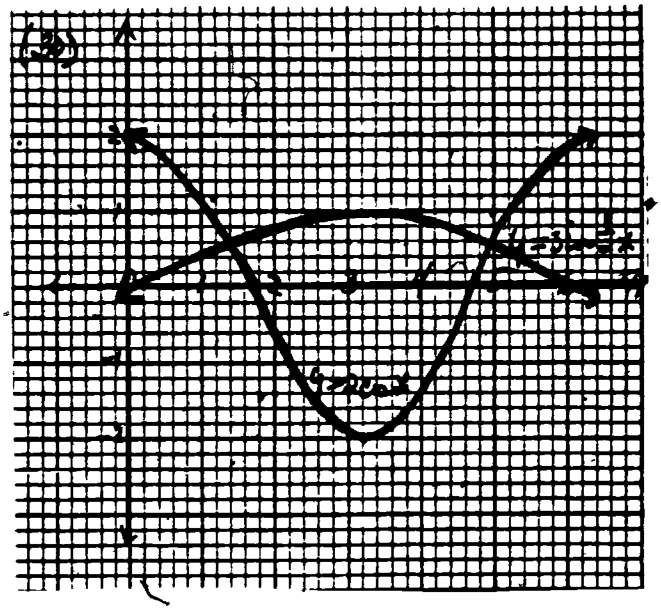
29)



(b) $x \doteq .6$ (34°)

$x \doteq 2.5$ (143°)

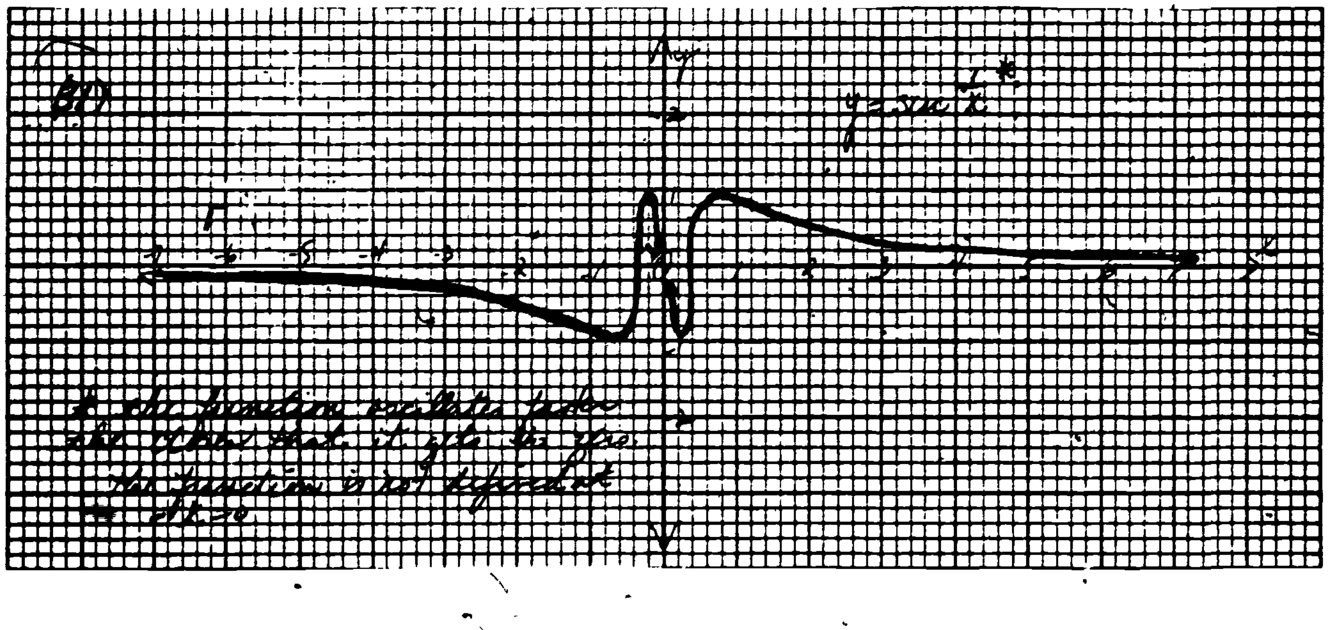
30)



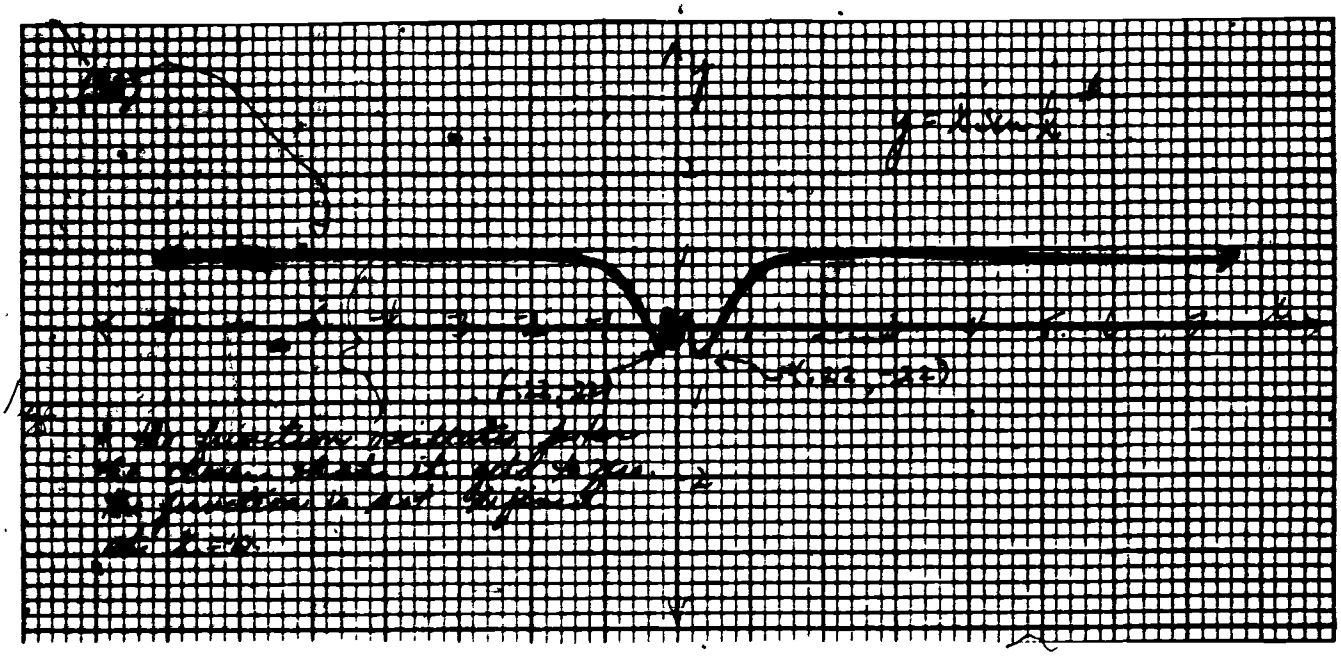
(b) $x \doteq 1.3 \quad (74^\circ)$

$x \doteq 5 \quad (286^\circ)$

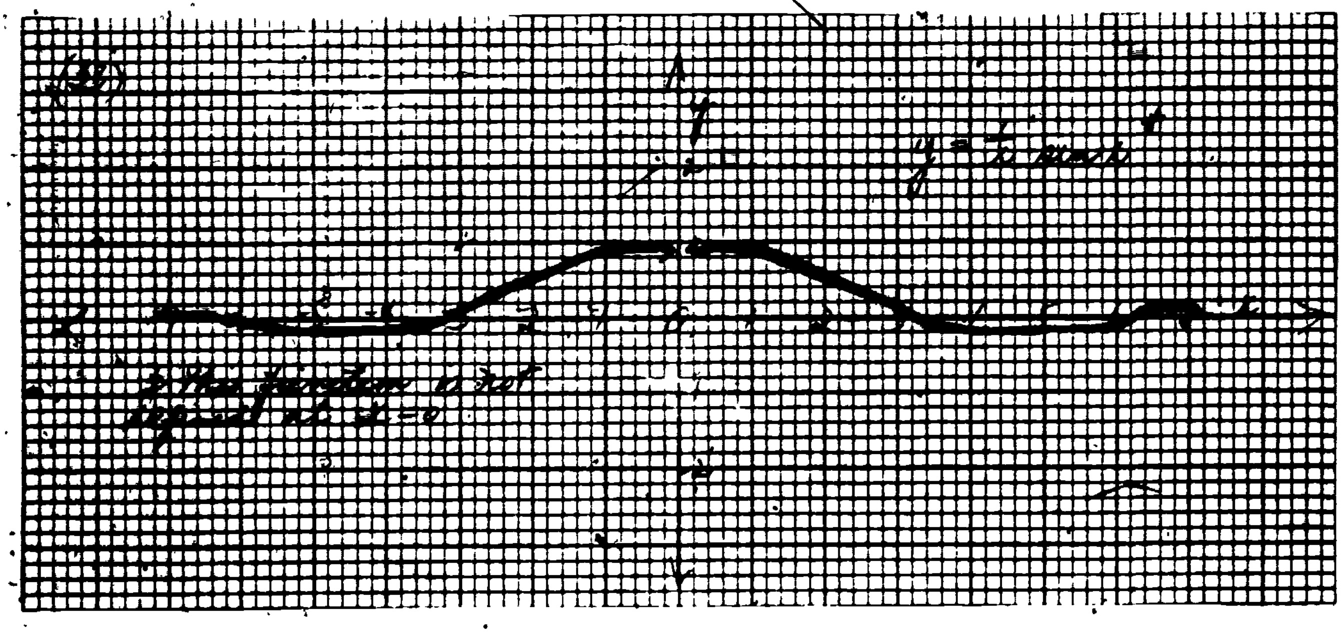
31)



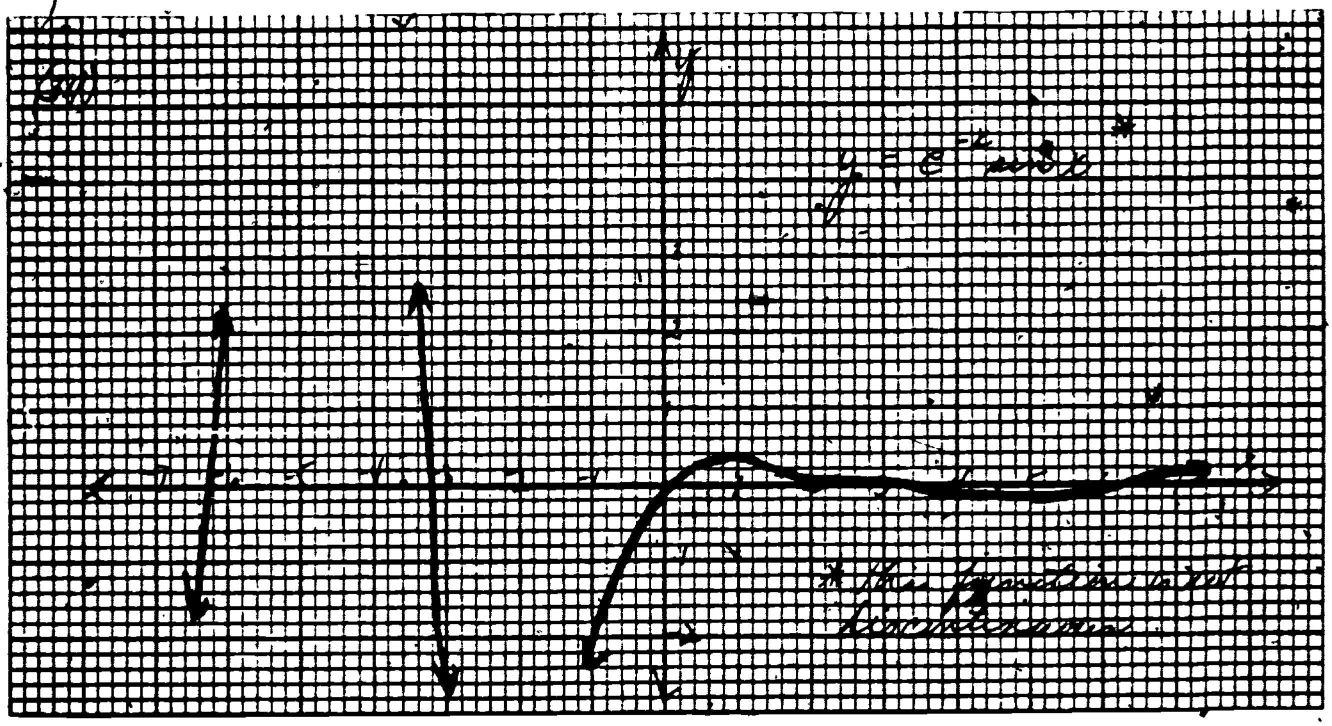
32)



33)



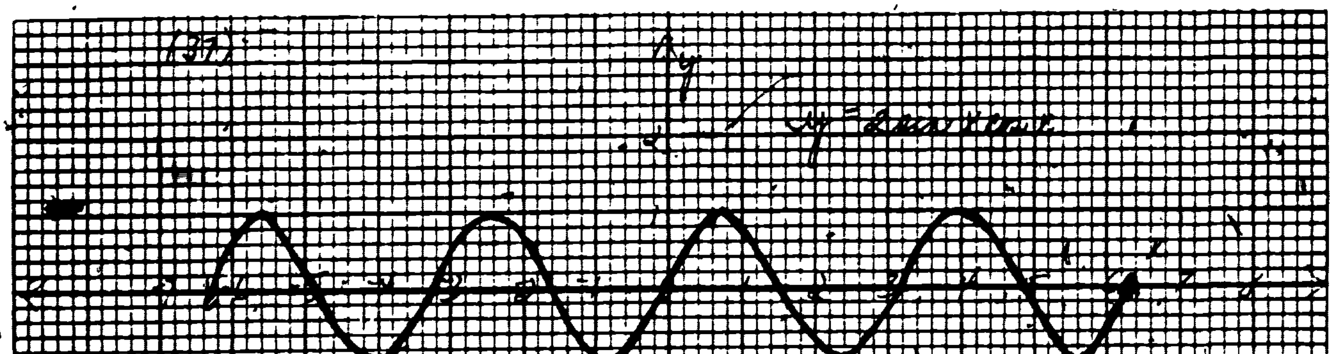
34)



35) None of these apply to these functions.

36) None apply because these functions are not of the form $y = a\, f\,(p \cdot x)$ where f is a trigonometric function and a and p are real numbers.

37)

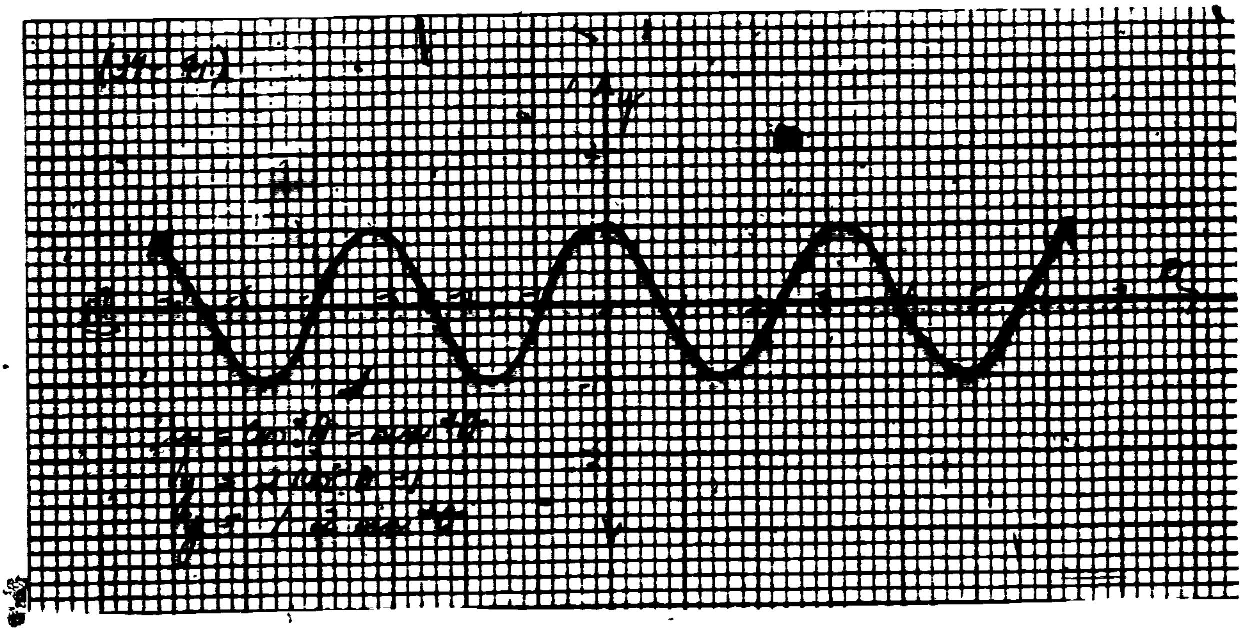


38) $2 \sin x \cos x = \sin 2x$

$\sin (x + y) = \sin x \cos y + \cos x \sin y$

$\sin 2x = \sin (x + x) = \sin x \cos x + \cos x \sin x = 2 \sin x \cos x$

(39 - 41)



42) Each of them is identical to $\cos 2\theta$.

$\cos 2\theta = \cos(\theta + \theta) = \cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta$

$= \cos^2\theta - \sin^2\theta$

but $\cos^2\theta = 1 - \sin^2\theta$ so

$\cos^2\theta = 1 - \sin^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

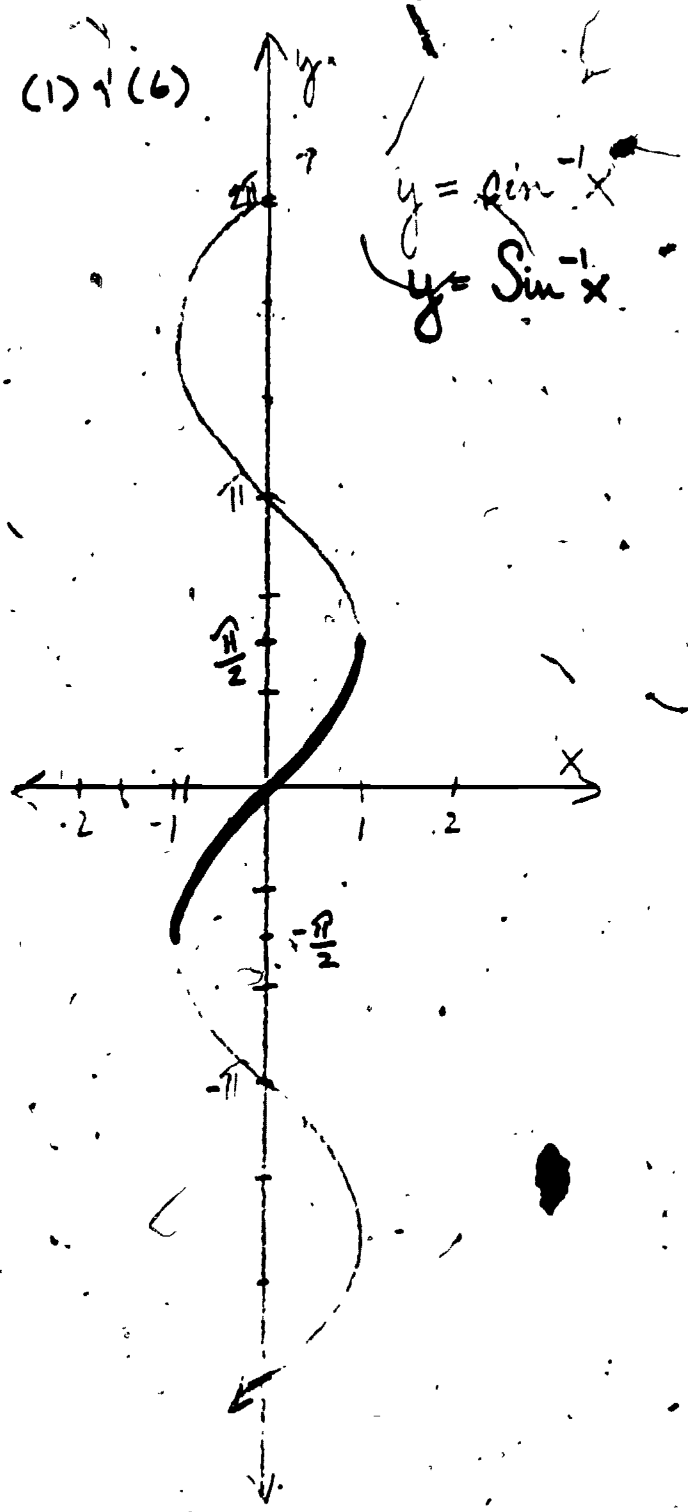
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

but $\sin^2\theta = 1 - \cos^2\theta$ so

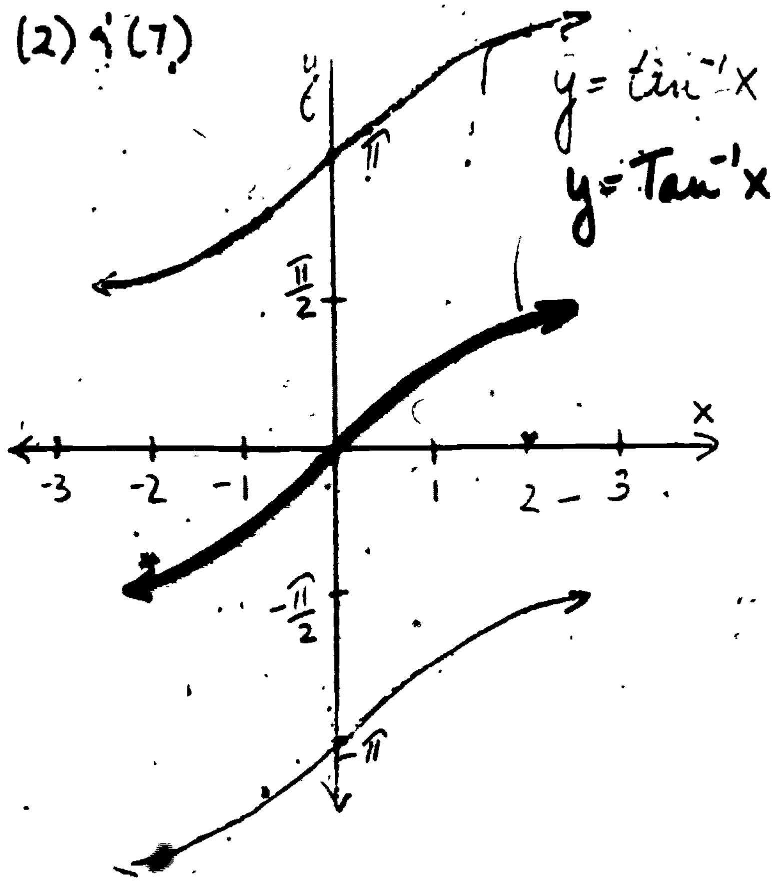
$\cos 2\theta = \cos^2\theta - 1 + \cos^2\theta = 2\cos^2\theta - 1$

Exercise set 5.5

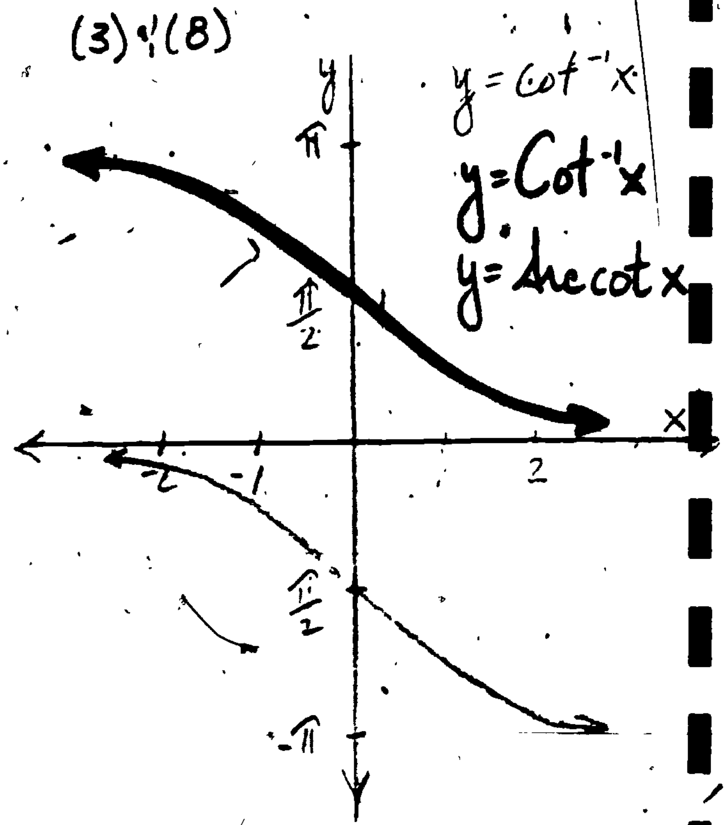
(1 - 10) See sketches



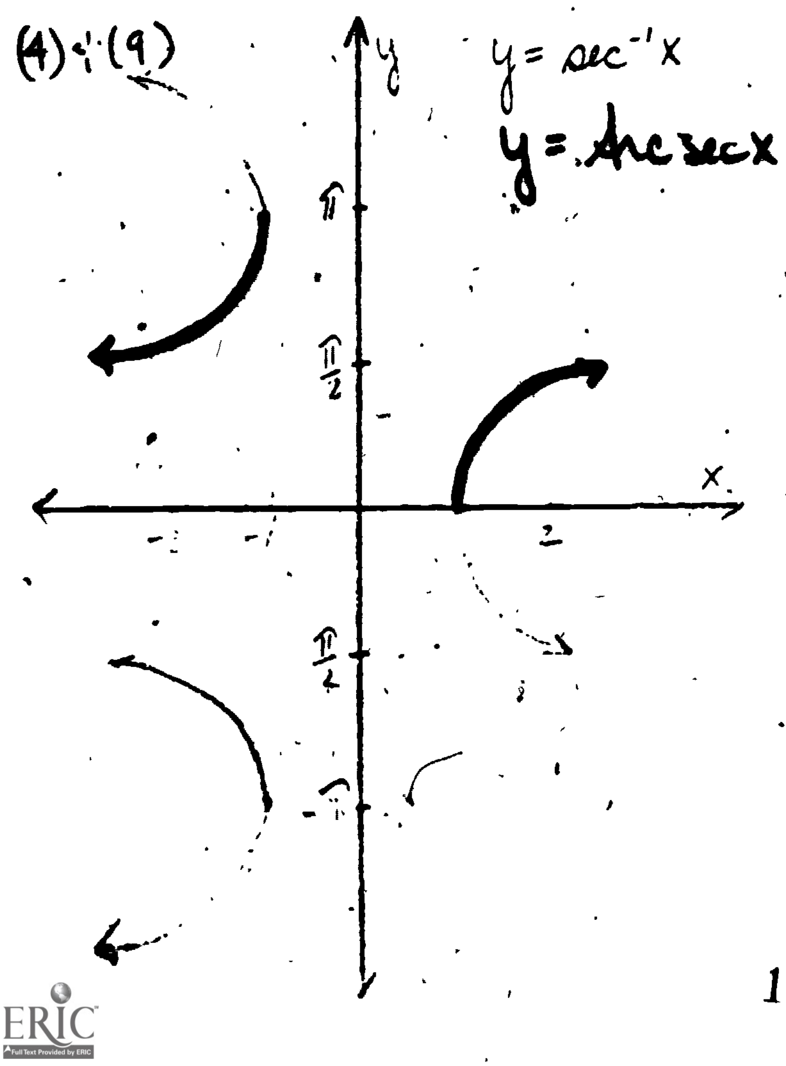
(2) & (7)


y = tan⁻¹x
y = Tan⁻¹x
π
π/2
-π/2
-π
x
y
-3 -2 -1 1 2 3

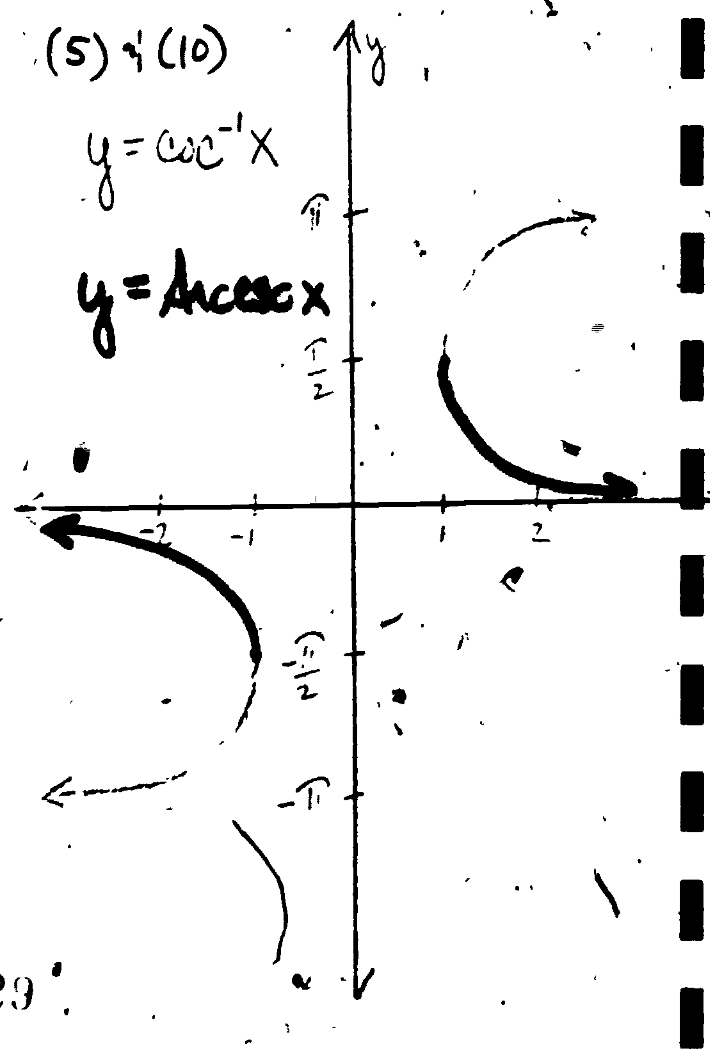
(3) & (8)


y = cot⁻¹x
y = Cot⁻¹x
y = Arccot x
π
π/2
-π/2
-π
x
y
-2 -1 2

(4) & (9)


y = sec⁻¹x
y = Arcsec x
π
π/2
-π/2
-π
x
y
2

(5) & (10)

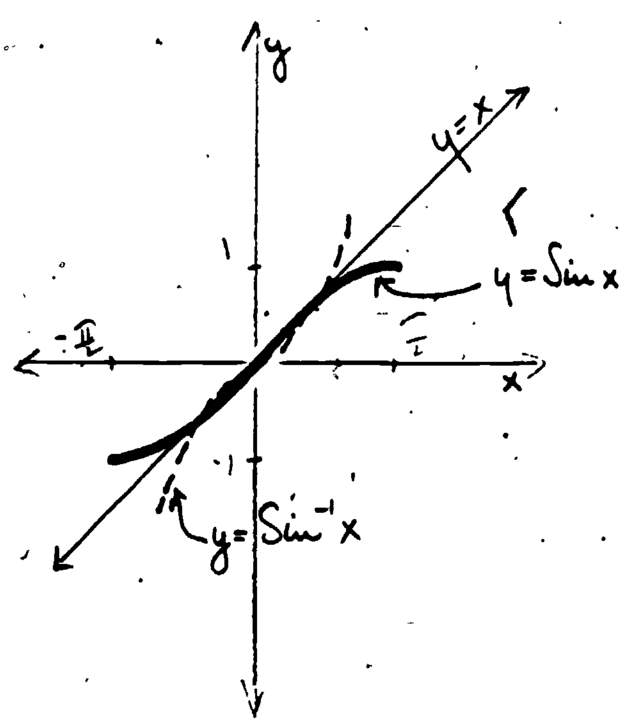

y = csc⁻¹x
y = Arccsc x
π
π/2
-π/2
-π
y
-2 -1 1 2

11)

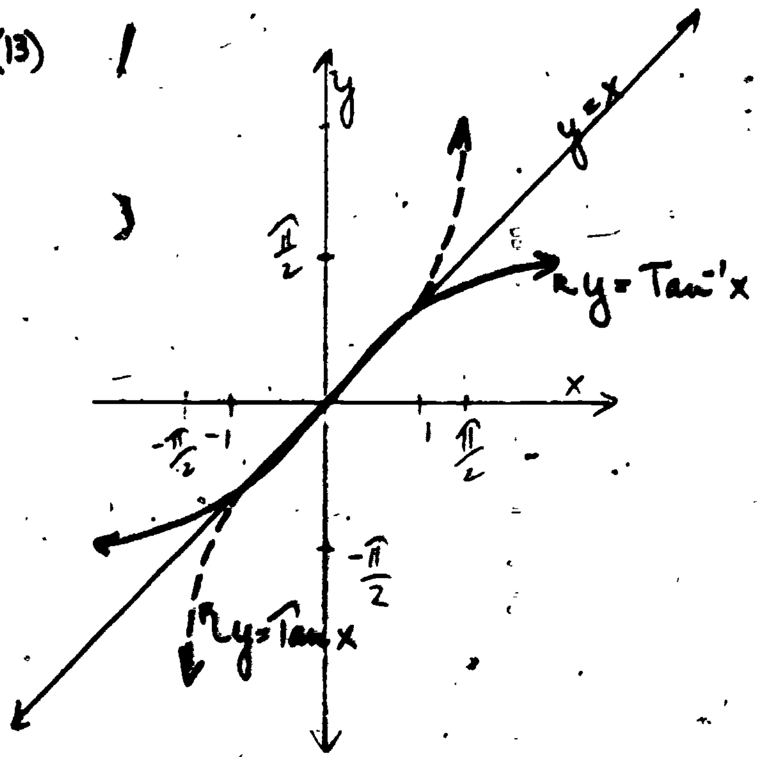
Function and Inverse	Domain	Range
$f : y = \cot x$	all reals except multiples of π	all reals
$f^{-1} : y = \cot^{-1} x$	all reals	all reals except multiples of π
$f : y = \sec x$	$0 \le x \le \pi,\ x \ne \frac{\pi}{2}$	$(-\infty, -1] \cup [1, +\infty)$
$f^{-1} : y = \sec^{-1} x$	$(-\infty, -1] \cup [1, +\infty)$	$0 \le y \le \pi,\ y \ne \frac{\pi}{2}$
$f : y = \csc x$	$-\frac{\pi}{2} \le x \le \frac{\pi}{2},\ x \ne 0$	$(-\infty, -1] \cup [1, +\infty)$
$f^{-1} : y = \csc^{-1} x$	$(-\infty, -1] \cup [1, +\infty)$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $y \ne 0$

(12 - 16) see graphs

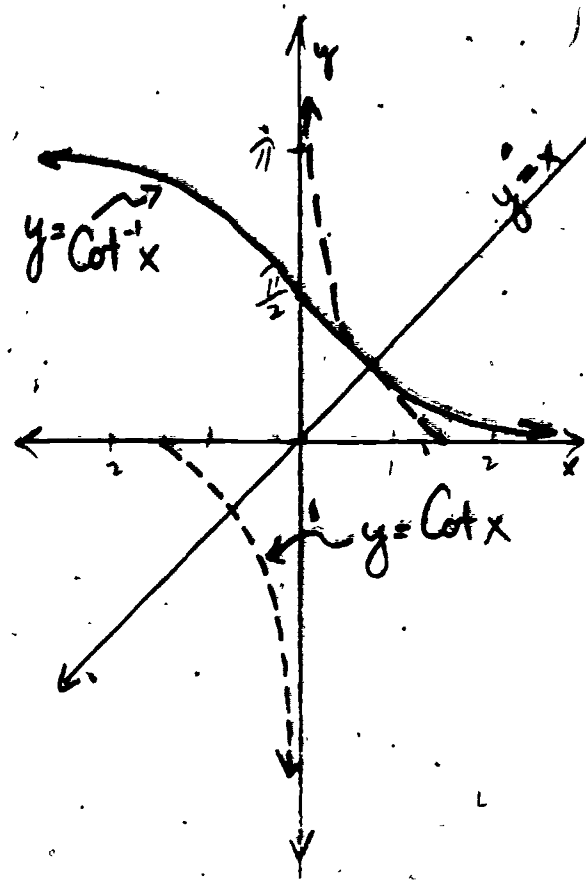
(12)



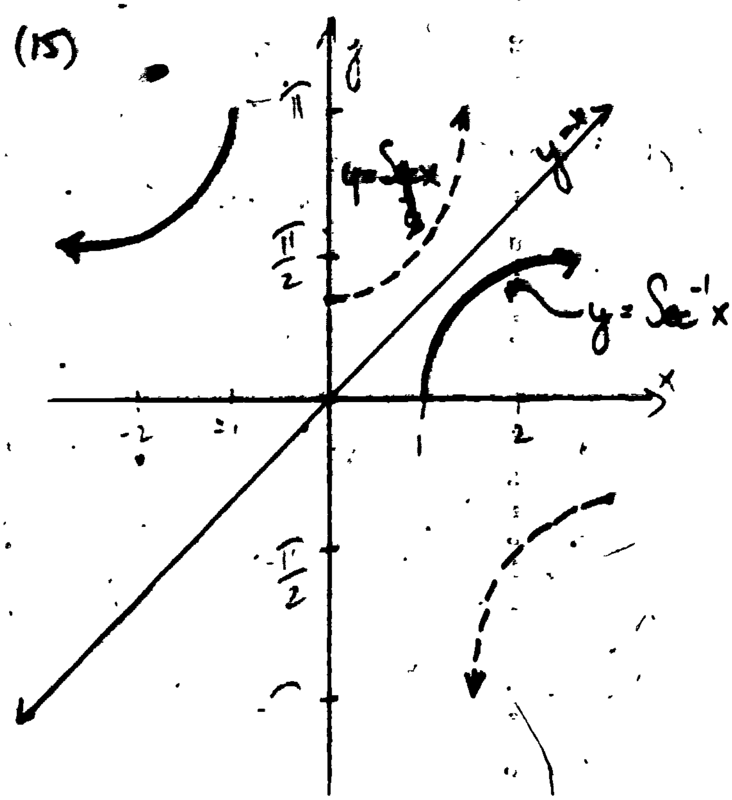
(13)


y
y=x
$\frac{\pi}{2}$
y = Tan⁻¹x
x
$-\frac{\pi}{2}$ -1
1 $\frac{\pi}{2}$
$-\frac{\pi}{2}$
y=Tan x

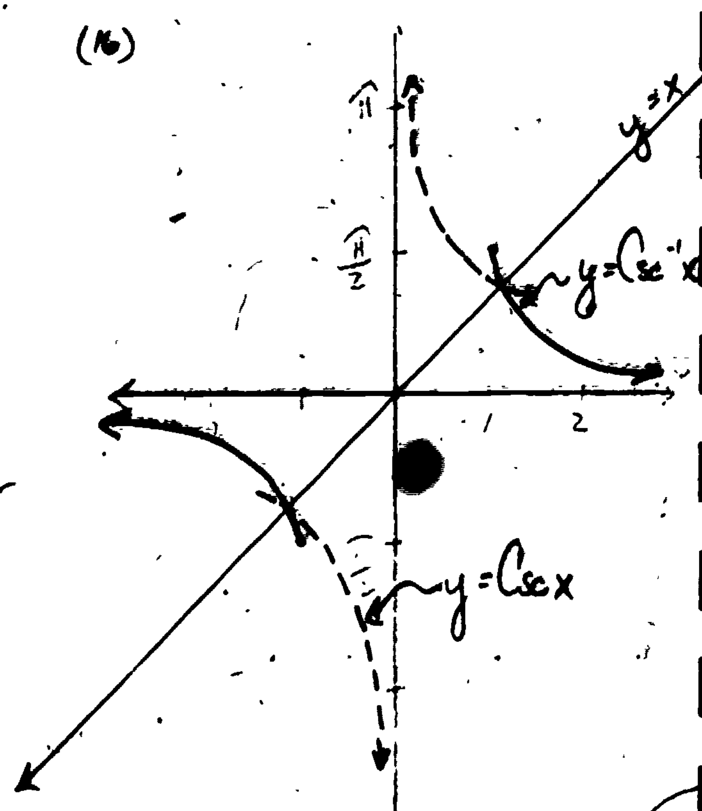
(14)


y
π
y=x
y=Cot⁻¹x
$\frac{\pi}{2}$
2
1 2 x
y=Cot x

(15)


y
π
y=Sec x
y=x
$\frac{\pi}{2}$
y = Sec⁻¹x
x
-2 -1
1 2
$-\frac{\pi}{2}$

(16)


π
y=x
$\frac{\pi}{2}$
y=Csc⁻¹x
1 2
y=Csc x

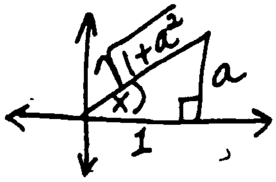
(17 - 18) Various graphs e.g., $f(x) = e^x$, $f(x) = \ln x$; $f(x) = x^2, x \geq 0$, $f(x) = \sqrt{x}$. A function and its inverse are always symmetric about the line $y = x$.

19) $\tan x = \dfrac{\sin x}{\cos x}$. Given: $\tan x = a$

Find x

$\cos x = \dfrac{1}{\sqrt{1 + a^2}}$

so $x = \text{Arc}\cos\left(\dfrac{1}{\sqrt{1 + a^2}}\right)$



20) (a) 45678 $\boxed{g}$ $\boxed{\tan^{-1}}$

(b) 89.9987° because the principal value of an angle whose tangent is a very large positive number is close to 90°.

21) (a) 45678 $\boxed{g}$ $\boxed{\sin^{-1}}$

(b) error because $-1 \leq \sin x \leq 1$ for all x.

22) (a) 45678 $\boxed{g}$ $\boxed{\cos^{-1}}$

(b) error because $-1 \leq \cos x \leq 1$ for all x.

23) (a) 45678 $\boxed{g}$ $\boxed{\frac{1}{x}}$ $\boxed{g}$ $\boxed{\tan^{-1}}$

(b) .0013 because the principal value of an angle whose cotangent is a very large positive number is near zero.

24) (a) 0 g $\boxed{\frac{1}{x}}$ $\boxed{g}$ $\boxed{\sin^{-1}}$

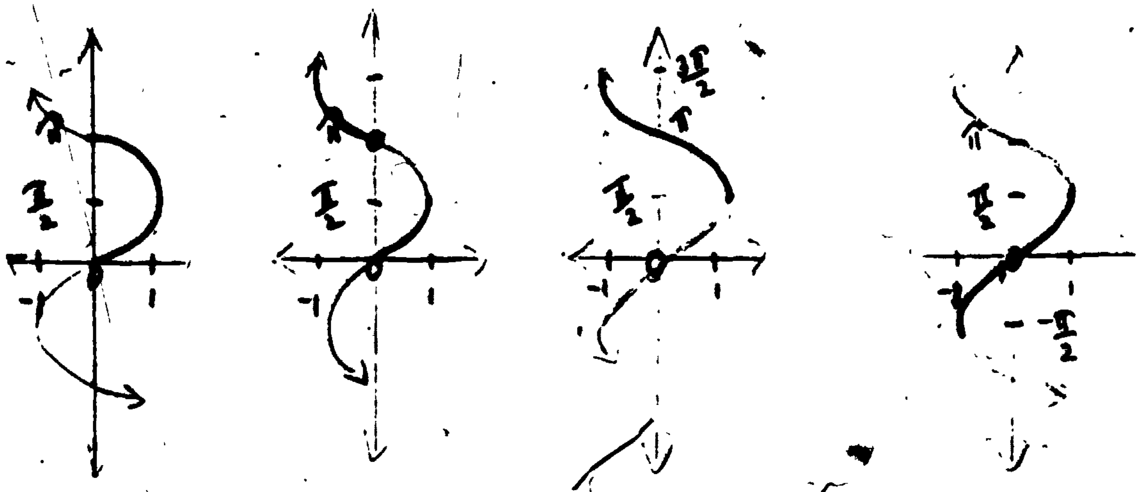
(b) error because division by zero is undefined and 0 is not in the domain of $f(x) = \sin^{-1} x$.

25) (a) $\boxed{g}$ $\boxed{\text{rad}}$ $\boxed{g}$ $\boxed{\pi}$ $\boxed{2}$ $\boxed{\div}$ $\boxed{g}$ $\boxed{\frac{1}{x}}$ g $\boxed{\cos}$

(b) .8807 because the principal value of $\sec^{-1} \frac{\pi}{2}$ is .8807 and $\sec .8807 \doteq \frac{\pi}{2}$. (.8807 radians $\doteq$ 50.4598°)

26) If a function is one-to-one then every element in the range corresponds to exactly one element in the domain so its inverse is also a function.

27) Other reasonable restricted domains of y = sin x could be

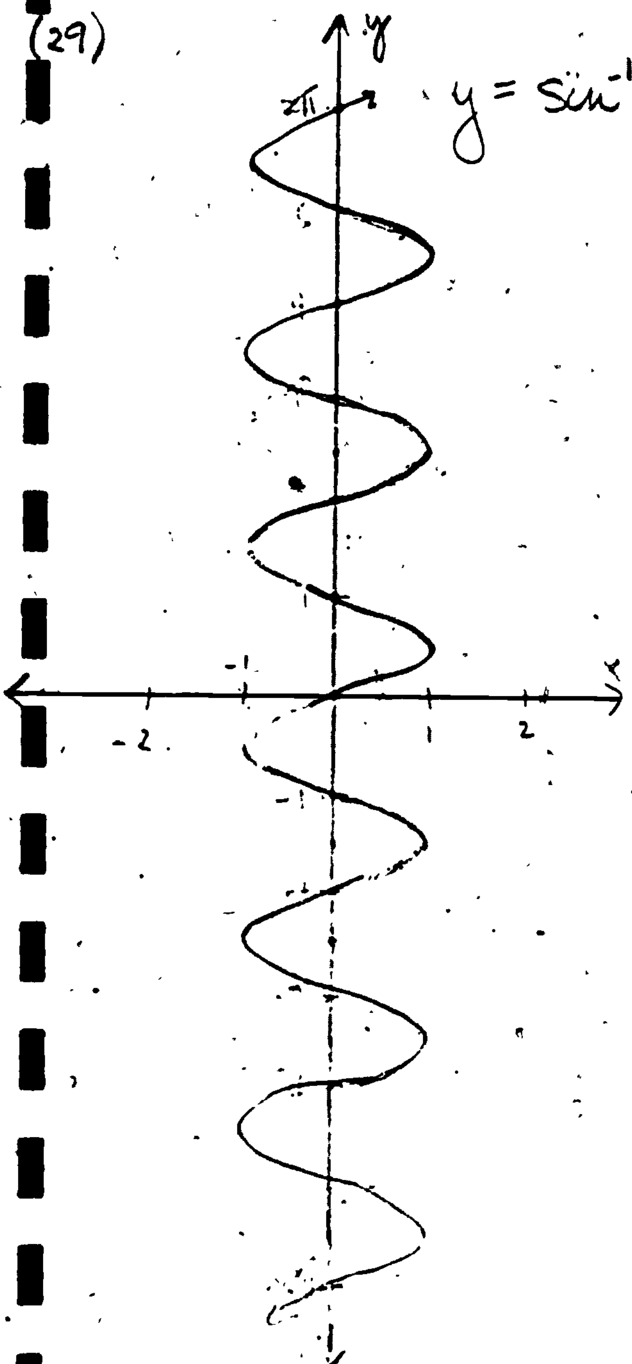


not a function | not continuous | not centered | just right

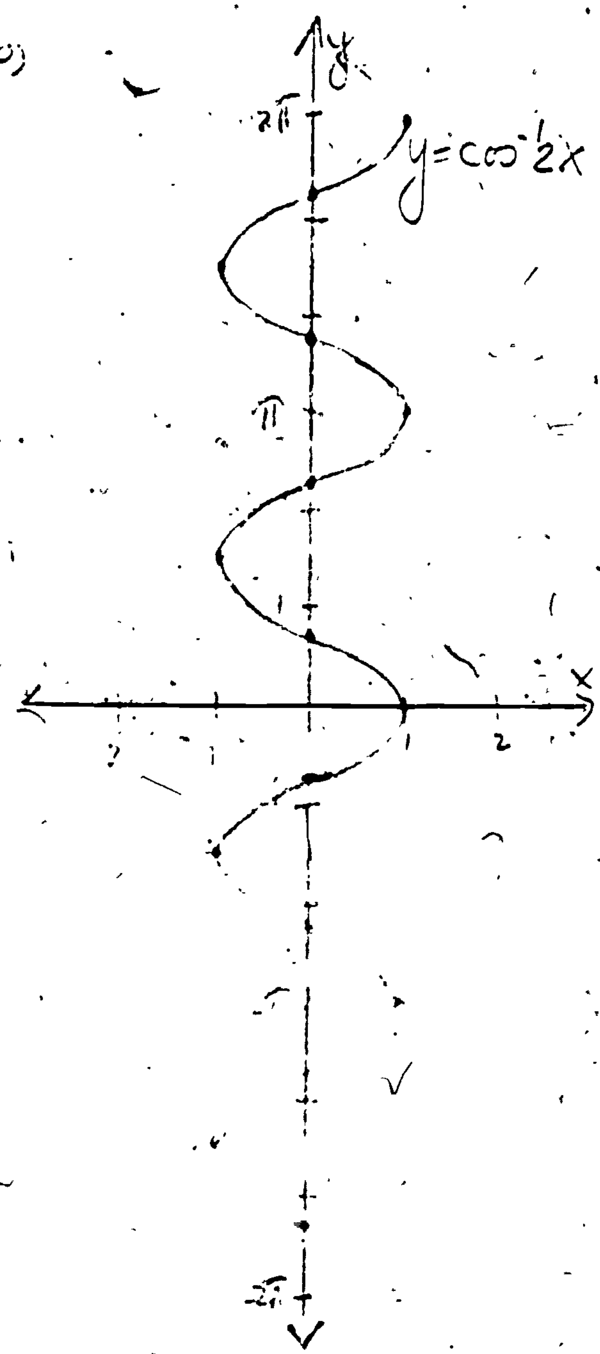
28) They are commutative with respect to composition.

(29 - 32) see sketches.

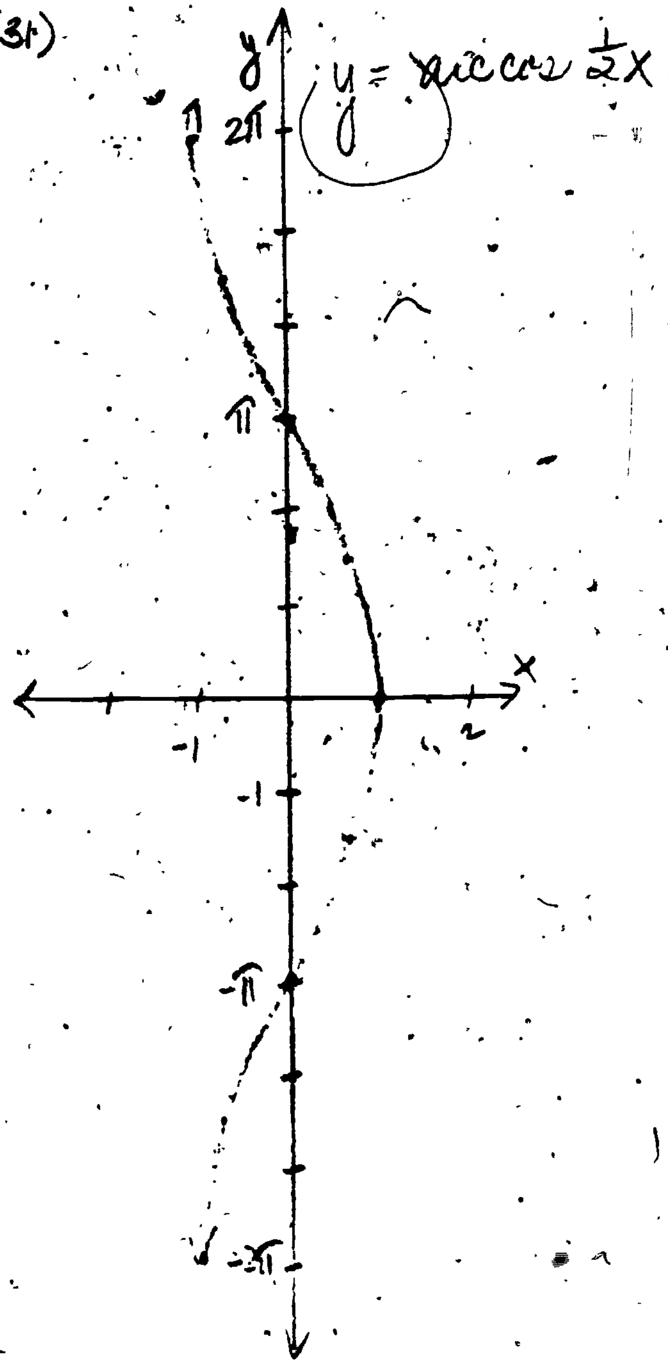
(29)



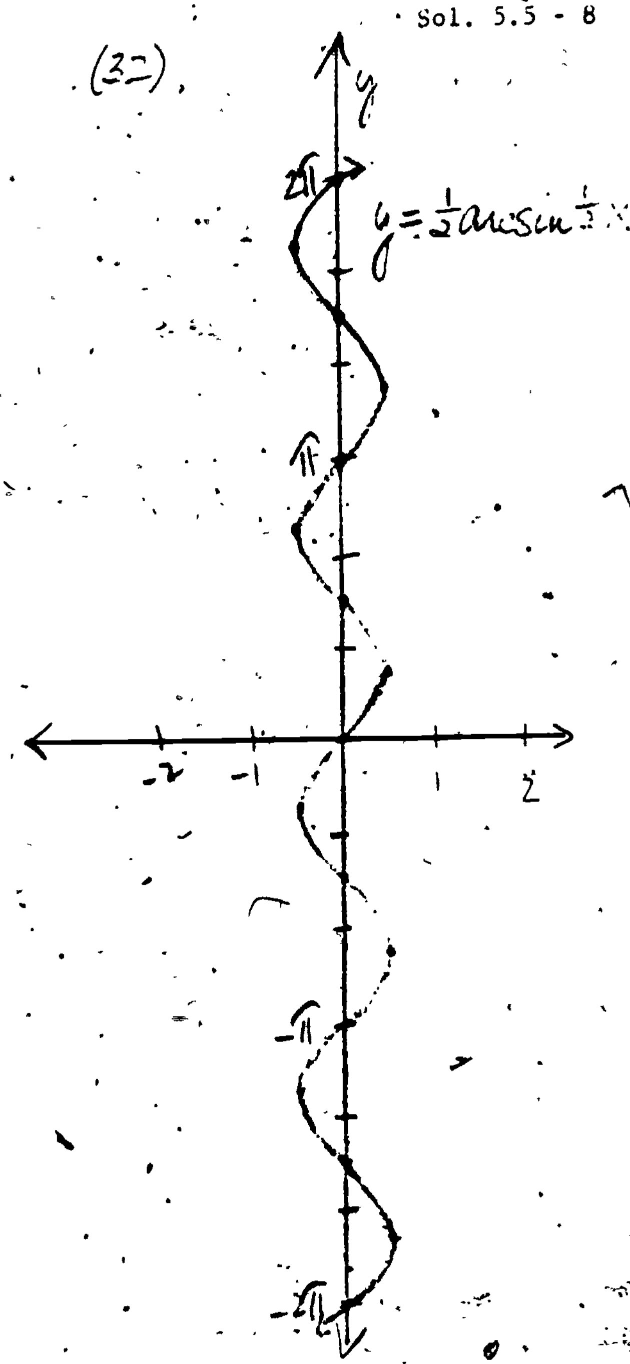
30)



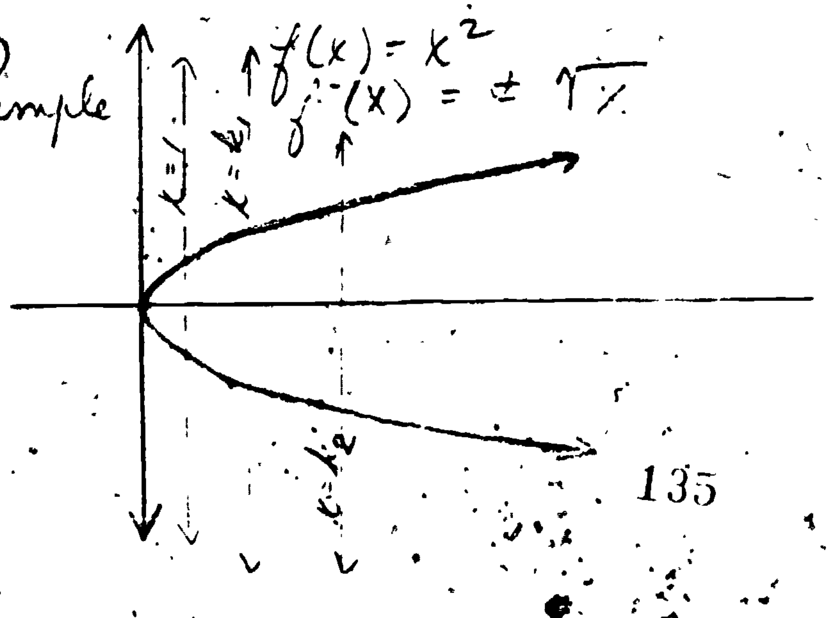
(31)



(32)



(33)
example



33) (example) $f(x) = x^2$, $f^{-1}(x) = \pm\sqrt{x}$

In this example when $k > 0$ the line $x = k$ intersects the graph in more than one place. This could not happen if $f^{-1}(x)$ where a function because then every element in the domain of $f^{-1}(x)$ would correspond to exactly one element in the range of $f^{-1}(x)$.

34 a) $\frac{\sqrt{3}}{3}$ b) .5774

35 a) $\frac{5}{13}$ b) .3846

36 a) 2 b) 2.0000

37 a) $\frac{1}{-\sqrt{3}}$ or $-\frac{\sqrt{3}}{3}$ b) -.5774

38 a) $\pm 1/2$ b) $\pm$ .5000

39 a) $\pm \frac{24}{25}$ b) $\pm$ .96

40 a) $-\frac{1}{\sqrt{3}}$ b) -.5774

Solutions to Chapter 5 TEST

1 a) 1.7321

b) $\sqrt{3}$

2 a) .3846

b) 5/13

3 a) -2.1250

b) -17/8

4) 1

5) undefined

6) 0

7) 90^{o} or $\frac{\pi}{2}$ or 1.5708

8) $\pi/4$, $3\pi/4$

9) increases

10) 1

11) D

12) C

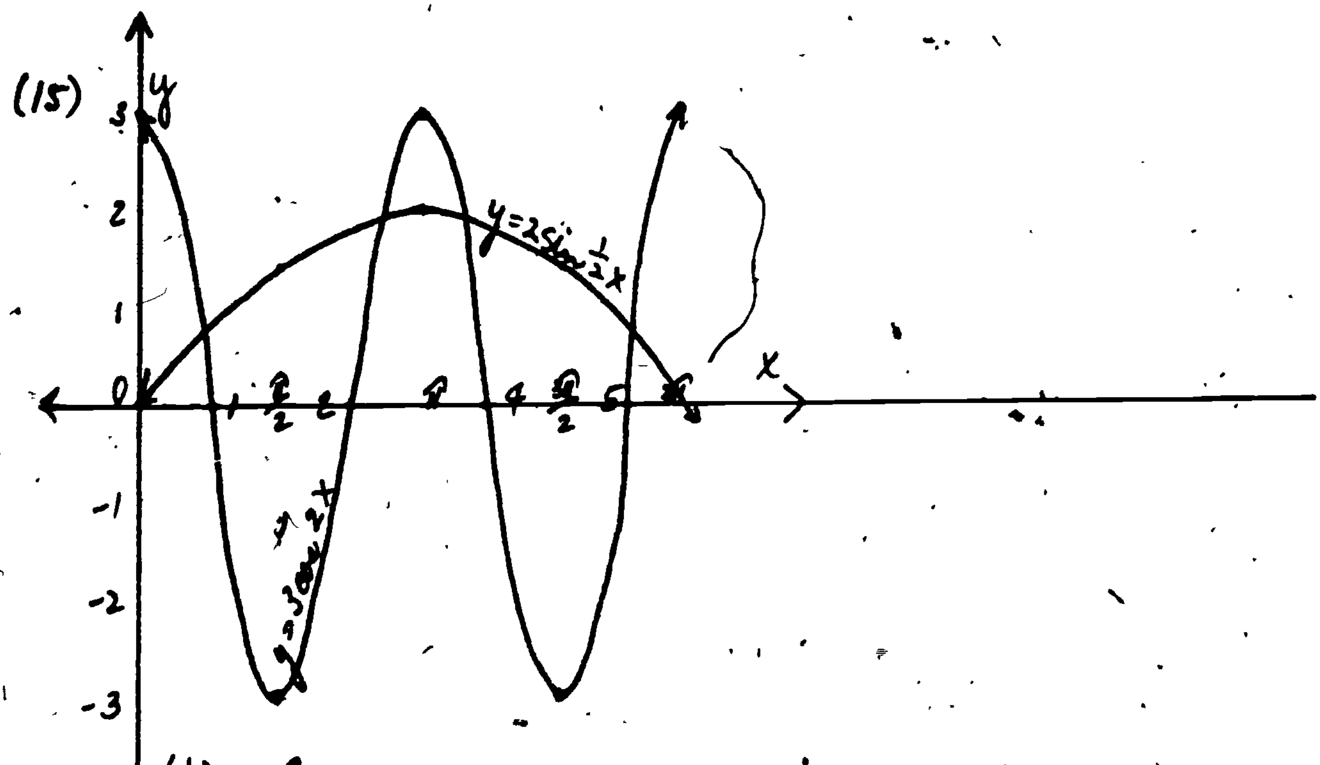
13) D

14) C

15) see graph

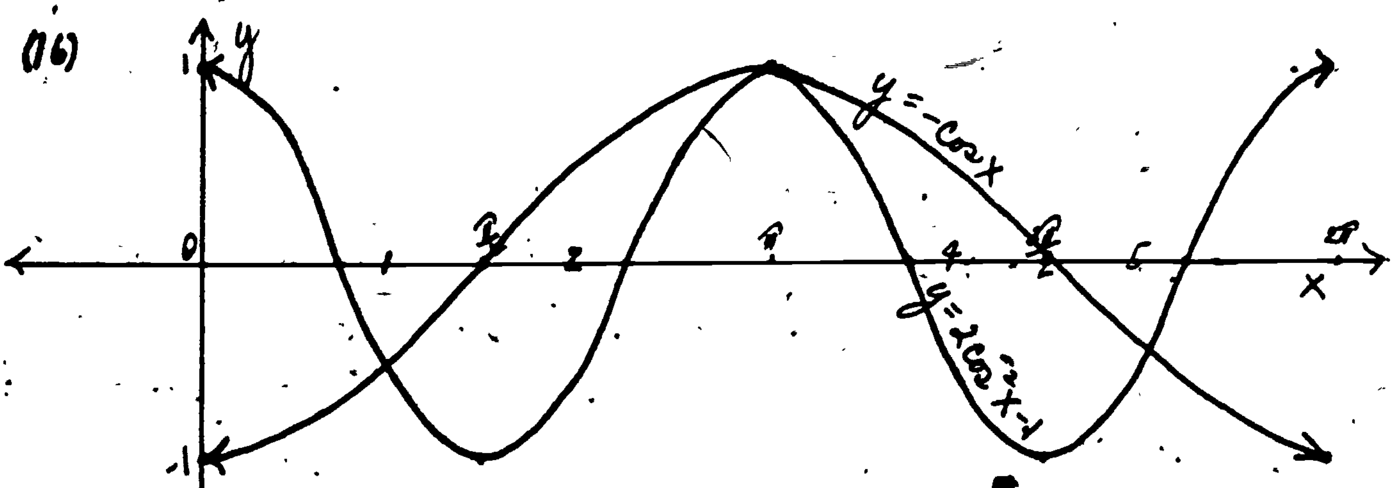
16) see graph

15)



(b) 2

16)



(b) π

Exercise Set 6.1

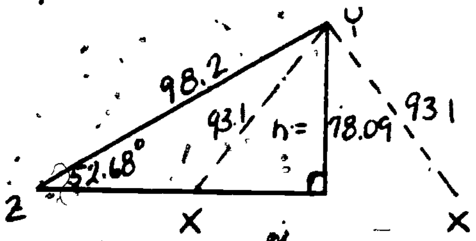
1) In any triangle, the largest angle is opposite the longest side.

2) If $\measuredangle X$ is a right angle then y is an altitude and x is the hypotenuse and x must be greater than y. Only one triangle is possible. This is really a special case of SAS, since z is determined uniquely by the Pythagorean Relation.

3) $\measuredangle X$ is acute, $x > z$ so one triangle is possible.

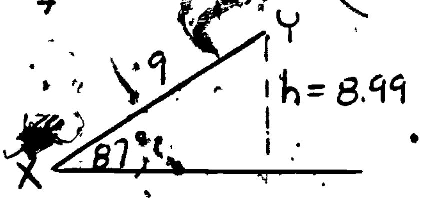
4) $\measuredangle Y$ is obtuse, $y < x$ so no triangle is possible.

5) $\measuredangle Z$ is acute, $z < x$



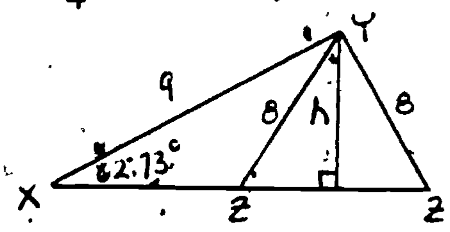
h, the altitude from Y to $\overline{XZ}$ is $78.09 < 93.1$ so 2 triangles are possible

6) $\measuredangle X$ is acute, $x < z$



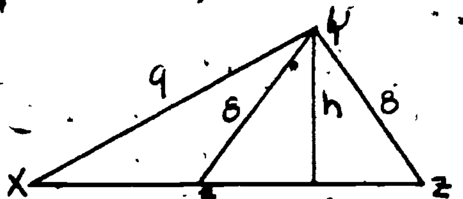
h, the altitude from Y to $\overline{XZ}$ is $8.99 > 8$ so no triangle is possible.

7) $\measuredangle X$ is acute, $x < z$



h, the altitude from Y to $\overline{XZ}$ is $7.9997 <$ [illegible] so two triangles are possible.

8) $\measuredangle X$ is acute, $x < z$



h, the altitude from Y to $\overline{XZ}$ is 1.0968 < 8 so two triangles are possible.

9) Let X be an angle then x is the side opposite that angle. Let y be a side adjacent to $\measuredangle X$.

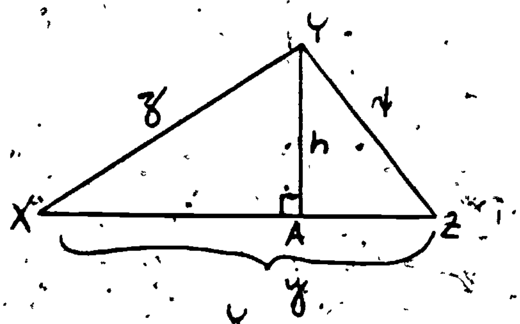
If $x = y \cdot \sin X$ then there is one triangle

$x < y \cdot \sin X$ then no triangle is possible

$x > y \cdot \sin X$ then two triangles are possible

Exercise Set 6.2

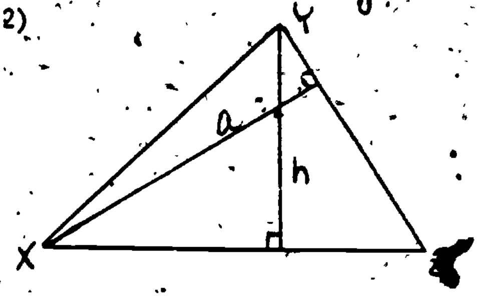
1)



In $\triangle XYA$, $h = z \sin X$

In $\triangle YZA$, $h = x \sin Z$

2)



Area $\triangle XYZ = \frac{1}{2} h \cdot y$ by exercise (1)

$\frac{1}{2} hy = \frac{1}{2} zy \sin X$

$\frac{1}{2} hy = \frac{1}{2} xy \sin Z$

also Area $\triangle XYZ = \frac{1}{2} a \cdot x$

$a = z \cdot \sin Y$. so

$\frac{1}{2} ax = \frac{1}{2} xz \sin Y$

Since each of these expressions represents the area of $\triangle XYZ$ they are equivalent so $\frac{1}{2} xy \sin Z = \frac{1}{2} xz \sin Y = \frac{1}{2} yz \sin X$.

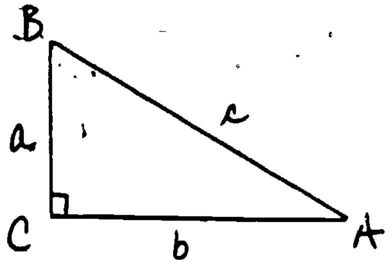
3) $\frac{1}{2} xy \sin Z = \frac{1}{2} xz \sin Y = \frac{1}{2} yz \sin X$

divide each expression by $\frac{1}{2} xyz$ thus

$$\frac{\sin Z}{z} = \frac{\sin Y}{y} = \frac{\sin X}{x}$$

4) In any proportion if $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$.

5)



$\sin A = \frac{a}{c}$

$\sin B = \frac{b}{c}$

$\sin C = 1$

$$\frac{a}{\sin A} = \frac{a}{\frac{a}{c}} = \frac{ac}{a} = c$$

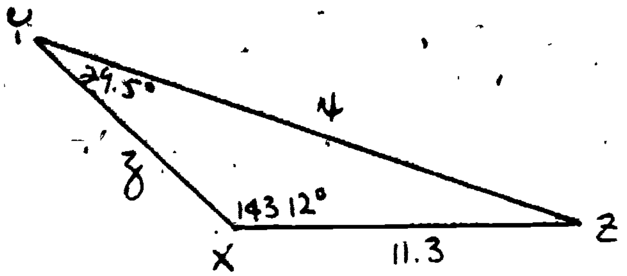
$$\frac{b}{\sin B} = \frac{b}{\frac{b}{c}} = \frac{bc}{b} = c$$

$$\frac{c}{\sin C} = \frac{c}{1} = c$$

So $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

6) a is the side opposite angle A

$\frac{\sin A}{a}$ is the ratio of the sine of an angle to the length of the side opposite it and since $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ this ratio is the same for any angle of the triangle so it is constant.

7) 

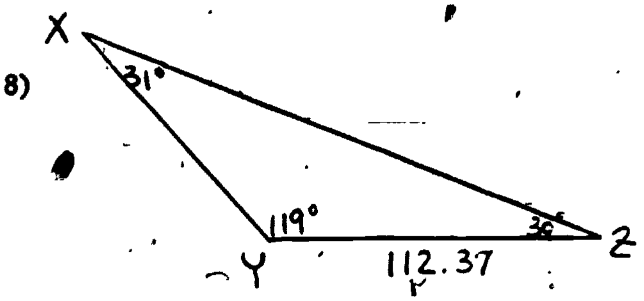
$m \angle Z = 7.3^\circ$

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 143.2^\circ} = \frac{11.3}{\sin 29.5^\circ} = \frac{z}{\sin 7.3^\circ} = 22.9$$

$x \doteq 13.8$, $z \doteq 2.9$

8)



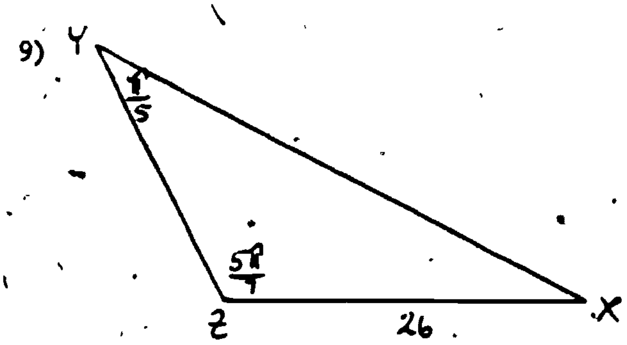
$m \measuredangle X = 31^\circ$

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{112.37}{\sin 31^\circ} = \frac{y}{\sin 119^\circ} = \frac{z}{\sin 30^\circ} \doteq 218.18$$

$y \doteq 190.82, \quad z \doteq 109.09$

9)



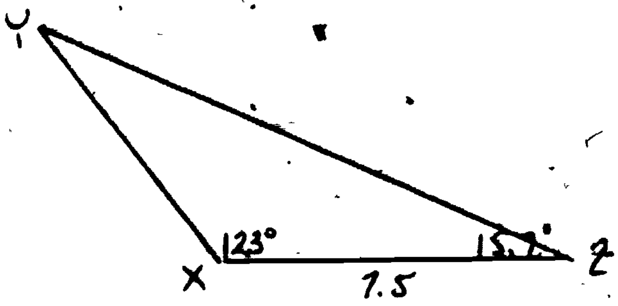
$m \measuredangle X = \frac{3\pi}{35}$

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin \frac{3\pi}{35}} = \frac{26}{\sin \frac{\pi}{5}} = \frac{z}{\sin \frac{5\pi}{7}} \doteq 44.23$$

$x \doteq 11.77 \doteq 12; \quad z \doteq 34.58 \doteq 35$

10)



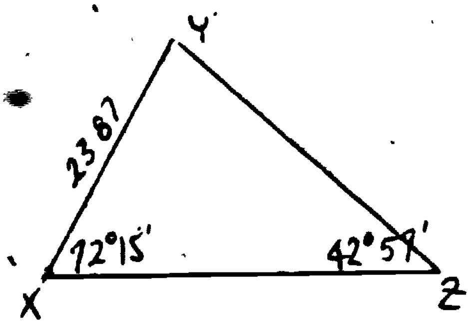
m∡ Y = 41.3°

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 123^\circ} = \frac{7.5}{\sin 41.3^\circ} = \frac{z}{\sin 15.7^\circ} \doteq 11.3636$$

$x \doteq 9.5;\quad z \doteq 3.1$

11)



m∡ X = 72° 15' = 72.25°

m∡ Z = 42° 57' = 42.95°

m∡ Y = 64.8°

$$\frac{x}{\sin X} = \frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{x}{\sin 72.25^\circ} = \frac{y}{\sin 64.8^\circ} = \frac{23.87}{\sin 42.95^\circ} \doteq 35.0329$$

$x \doteq 33.37,\quad y \doteq 31.70$

12) No triangle is possible because $m\angle X + m\angle Z > 180$.

13) Area ($\triangle ABC$) = $\frac{1}{2}(11)(7)(\sin 28^\circ) \doteq 18$

14) Area ($\triangle ABC$) = $\frac{1}{2}(22.7)(17.5)(\sin 148^\circ) \doteq 105.3$

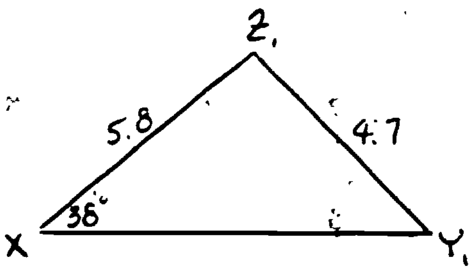
15) Area ($\triangle ABC$) = $\frac{1}{2}(12.26)(15.73)(\sin 38^\circ) \doteq 59.37$

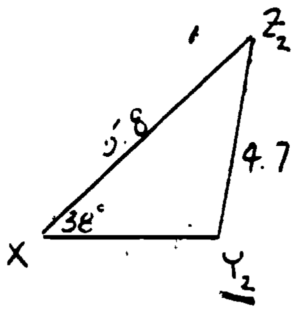
16) Area ($\triangle ABC$) = $\frac{1}{2}(7.25)(8.25)(\sin 109^\circ) \doteq 28.28$

17) $312.8 = \frac{1}{2}(146.2)(87.7)\sin C$

$2.8^\circ \doteq m\angle C$

18)





In $\triangle XY_1Z_1$

$$\frac{x}{\sin X} = \frac{y}{\sin Y_1} = \frac{z_1}{\sin Z_1}$$

$$\frac{4.7}{\sin 38^\circ} = \frac{5.8}{\sin Y_1}$$

$m\angle Y_1 \doteq 49^\circ$

$m\angle Z_1 \doteq 93^\circ$

$z_1 \doteq 7.6$

In $\triangle XY_2Z_2$

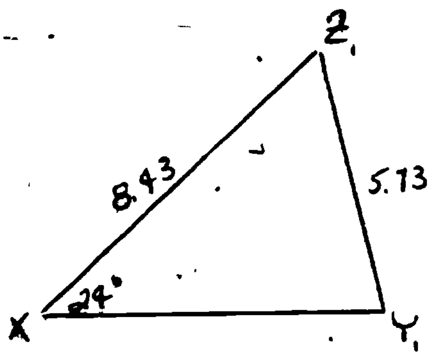
$m\angle Y_2 \doteq 180 - 49.44$

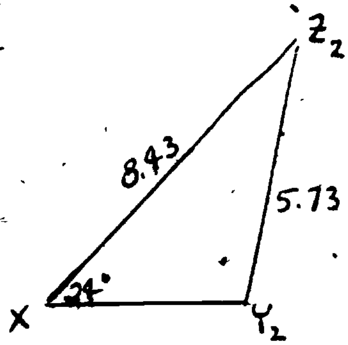
$m\angle Y_2 \doteq 130.56 \doteq 131^\circ$

$m\angle Z_2 \doteq 11^\circ$

$z_2 \doteq 1.5$

19)





In $\triangle XY_1Z_1$

$$\frac{x}{\sin X} = \frac{y_1}{\sin Y_1} = \frac{z_1}{\sin Z_1}$$

$$\frac{5.73}{\sin 24} = \frac{8.43}{\sin Y_1}$$

$$m\angle Y_1 \doteq 36.75^\circ \doteq 37^\circ$$

$$m\angle Z_1 \doteq 119.25 \doteq 119^\circ$$

$$z_1 \doteq 12.29$$

In $\triangle XY_2Z_2$

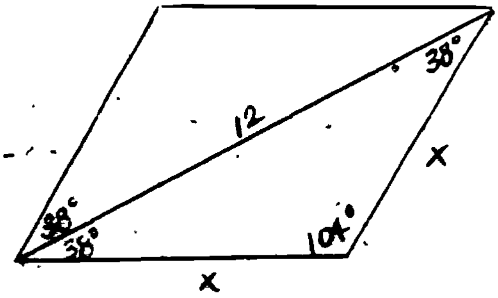
$$m\angle Y_2 \doteq 180 - 36.75$$

$$m\angle Y_2 \doteq 143.^\circ$$

$$m\angle Z_2 \doteq 13^\circ$$

$$z_2 \doteq 3.11$$

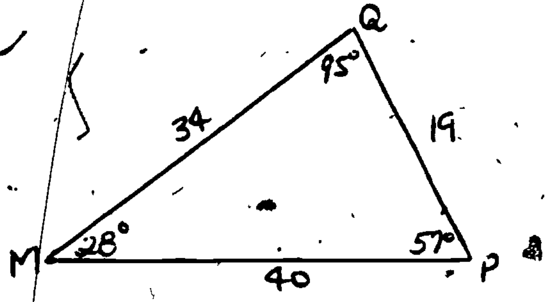
20)

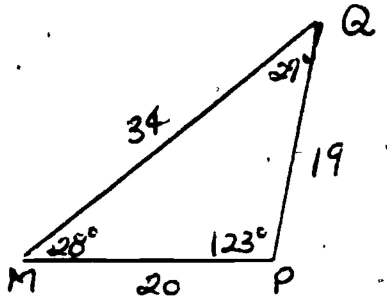


$$\frac{12}{\sin 104} = \frac{X}{\sin 38} \doteq 12.37$$

$$X \doteq 7.61 \doteq 8$$

21)





$$\frac{19}{\sin 28} = \frac{34}{\sin P}$$

$m\measuredangle P = 57$

$m\measuredangle Q = 95$

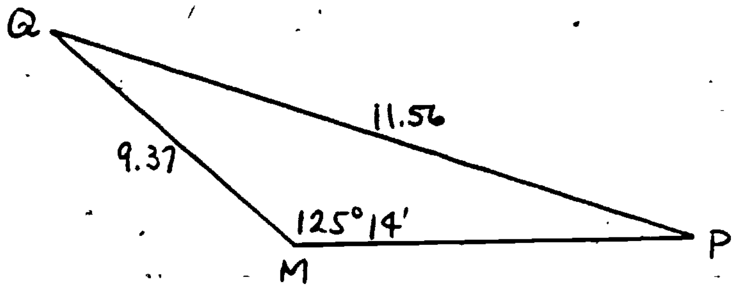
$q = 40$

$m\measuredangle P = 180 - 57 = 123$

$m\measuredangle Q = 29$

$q = 20$

22)



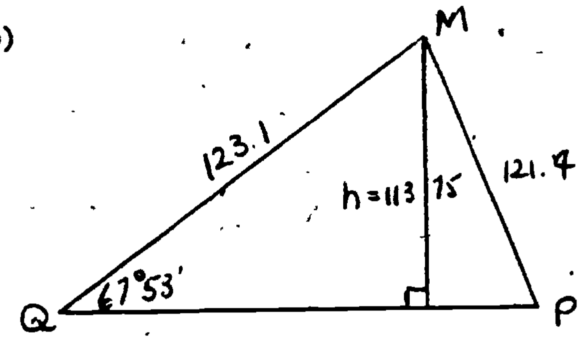
$$\frac{11.56}{\sin 125^\circ 14'} = \frac{9.37}{\sin \measuredangle P}$$

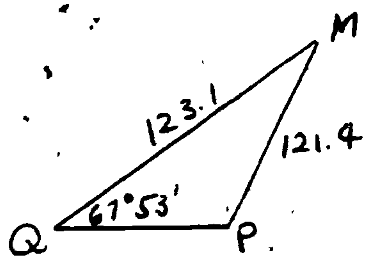
$41^\circ 27' = 41.46^\circ = m\measuredangle P$

$13^\circ 18' = 13.31^\circ = m\measuredangle Q$

$3.26 = q$

23)





$$\frac{121.4}{\sin 67^\circ 53'} = \frac{123.1}{\sin \measuredangle P}$$

$69^\circ 57' = 69.95^\circ \doteq m\measuredangle P$

$42^\circ 10' = 42.17^\circ \doteq m\measuredangle M$

$88.0 \doteq m$

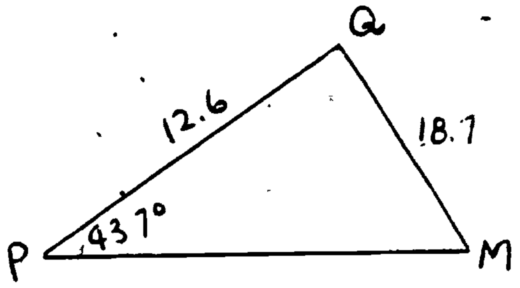
$m\measuredangle P \doteq 180^\circ - 69.95^\circ$

$m\measuredangle P \doteq 110.1^\circ$

$m\measuredangle M \doteq 2^\circ$

$m \doteq 4.6$

24)



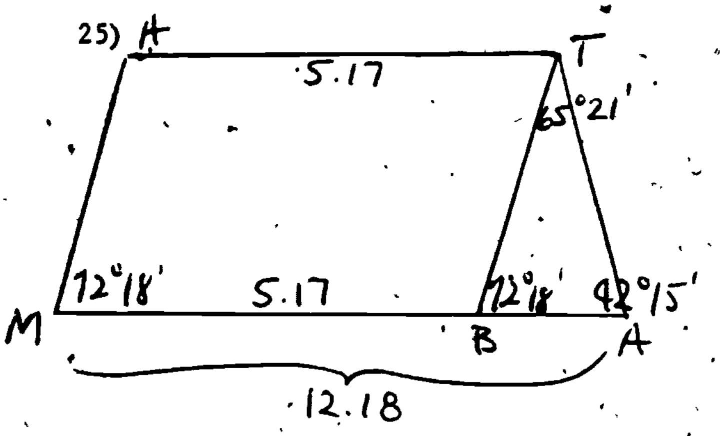
$$\frac{18.7}{\sin 43.7} = \frac{12.6}{\sin \measuredangle M}$$

$m\measuredangle M \doteq 27.7^\circ$

$m\measuredangle Q \doteq 108.6^\circ$

$q \doteq 25.7$

25)



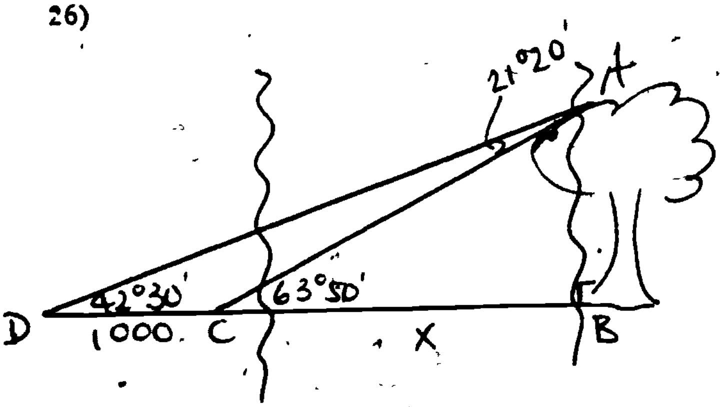
In $\triangle$ TBA

$$7.71 \doteq \frac{7.01}{\sin 65^\circ 27'} = \frac{a}{\sin 42^\circ 15'} = \frac{b}{\sin 72^\circ 18'}$$

$a \doteq 5.18 \doteq$ MH

$b \doteq 7.34 \doteq$ TA

26)



$m\angle$ ACD $= 116^\circ 10'$

$m\angle$ DAC $= 21^\circ 20'$

In $\triangle$DCA: $\dfrac{1000}{\sin 21^\circ 20'} = \dfrac{CA}{\sin 42^\circ 30'} = 2748.81$

$CA = 1857.07$

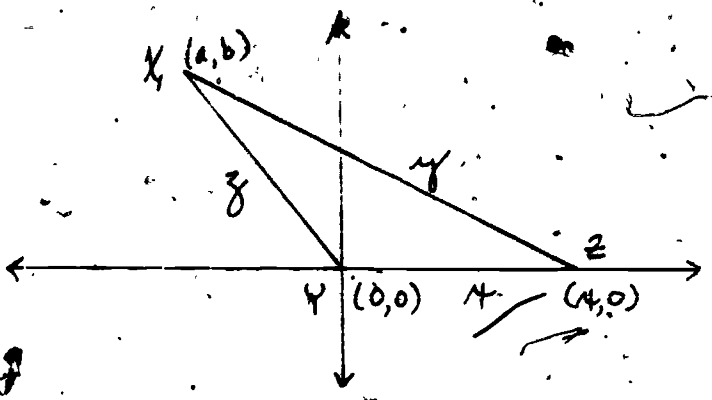
In $\triangle$ ABC $\quad \cos 63^\circ 50' = \dfrac{x}{CA}$

$818.94 = x$

The river is 819 meters wide.

Exercise Set 6.3

1)



$a = z \cos Y, \quad b = z \sin Y$

$$y = \sqrt{(z \cos Y - x)^2 + (z \sin Y)^2}$$

$$y^2 = (z \cos Y - x)^2 - (z \sin Y)^2$$

$$y^2 = z^2 \cos^2 Y - 2z \cdot x \cdot \cos Y + x^2 + z^2 \sin^2 Y$$

$$y^2 = z^2 (\cos^2 Y + \sin^2 Y) - 2z \cdot x \cdot \cos Y + x^2$$

$$y^2 = x^2 + z^2 - 2z \cdot x \cdot \cos \hat{Y}$$

2) If $\triangle ABC$ is a right triangle having a right angle at C then $\cos C = 0$ and for $c^2 = a^2 + b^2 - 2ab \cdot \cos C$

$$c^2 = a^2 + b^2 - 2ab\ (0) = a^2 + b^2$$

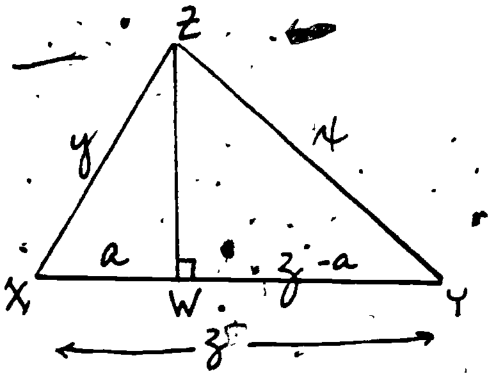
3) If $x^2 + y^2 = z^2$ then

$$x^2 + y^2 = x^2 + y^2 - 2xy \cos Z$$

$x \neq 0,\ y \neq 0$ so $\cos Z = 0$ and $m \angle Z = 90.$

4) The Pythagorean theorem is not a corollary to the Law of Cosines because the Pythagorean theorem is used to establish the distance formula which we used to prove the Law of Cosines.

5)



$a = y \cdot \cos X, \quad z - a = x \cdot \cos Y$

$$z = x \cdot \cos Y - y \cdot \cos X$$

$$y \cdot \cos X = x \cdot \cos Y - z$$

$$y^2 \cos^2 X = x^2 \cos^2 Y - 2x \cdot z \cdot \cos Y + z^2$$

by the Law of Sines

$$\frac{x}{\sin X} = \frac{y}{\sin Y}$$

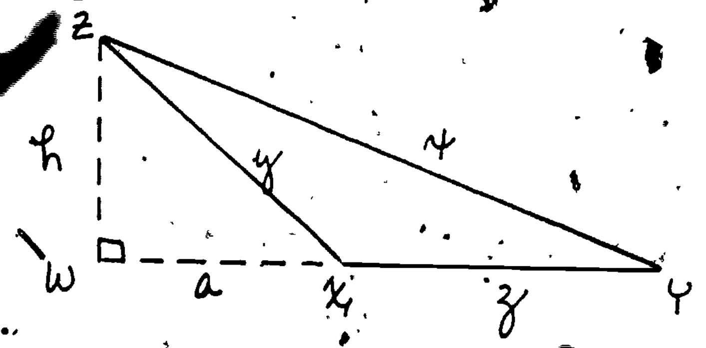
$$x^2 \sin^2 Y = y^2 \sin^2 X$$

$$y^2 \cos^2 X + y^2 \sin^2 X = x^2 \cos^2 Y + x^2 \sin^2 Y - 2x \cdot z \cdot \cos Y$$

$$y^2 (\cos^2 X + \sin^2 X) = x^2 (\cos^2 Y + \sin^2 Y) - 2x \cdot z \cdot \cos Y$$

$$y^2 = x^2 + z^2 - 2x \cdot z \cdot \cos Y$$

6)



In rt $\triangle$ XZW: $h^2 + a^2 = y^2$

$$h^2 = y^2 - a^2$$

In rt $\triangle$ WYZ: $x^2 = h^2 + (a + z)^2$

$$= y^2 - a^2 + a^2 + 2az + z^2$$

(1) $= y^2 + z^2 + 2az$

In rt $\triangle$ XZW: $\frac{a}{y} = \cos\,(180 - m\measuredangle ZXY)$

$= -\cos \measuredangle ZXY$

(2) $a = -y \cos \measuredangle ZXY$

using (1) and (2)

$$x^2 = y^2 + z^2 + 2z\,(-y \cos \measuredangle ZXY)$$

$$= y^2 + z^2 = 2yz \cdot \cos X$$

(7 - 12) The following program can be used to solve each triangle having sides x and y and angle Z. Key x in register 1, y in register 2 and Z (in decimal degrees) in register 3. When the program is run the first display is side z, the second display is the measure of $\measuredangle$ X in decimal degrees and the third display is the measure of $\measuredangle$ Y ~~in~~ decimal degrees. It is necessary to make x the smaller of x and y.

```
01  RCL 1              17  RCL 1
02  g x^2              18  RCL 3
03  RCL 2              19  f SIN
04  g x^2              20  x
05  +                  21  x ⇄ y
06  RCL 1              22  ÷
07  RCL 2              23  g SIN^-1
08  x                  24  R/S    (m ∡ X)
09  2                  25  RCL 3
10  x                  26  +
11  RCL 3              27  1
12  f COS              28  8
13  x                  29  0
14  -                  30  x ⇄ y
15  f √x               31  -      (m ∡ Y)
16  R/S   (z)
```

x	< y
R_1	x
R_2	y
R_3	Z

The formulas for this program are:

$$z^2 = x^2 + y^2 - 2\,xy \cdot \cos Z$$

$$X = \sin^{-1}\left(\frac{x \cdot \sin Z}{z}\right)$$

$$Y = 180^\circ - (X + Z)$$

7) $r = 12.8$, $m\angle S = 48^\circ$, $m\angle T = 75^\circ$

8) $s = 30.95$ $m\angle R = 21.2^\circ$, $m\angle T = 35.6^\circ$

* 9) $t = 15.3$, $m\angle S = 23.7^\circ$, $m\angle R = 124.0^\circ$

*10) $r = 20.36$, $m\angle S = 38.71^\circ$, $m\angle T = 26.73^\circ$

*11) $s = 5.00$, $M\angle R = 28.07^\circ$, $m\angle T = 17.93^\circ$

12) $t = 16.5$, $m\angle R = 54.8^\circ$, $m\angle S = 108.0^\circ$

(13 - 18) The following program can be used to solve each triangle having sides x, y and z. Key x in register 1, y in register 2 and z in register 3. When the program is run the first display is $m\angle Z$. The second display is $m\angle Y$ and the third display is $m\angle X$. All angles are measured in decimal degrees. It is necessary to make z the longest side.

```
01  RCL 1                 17  STO 0
02  g x^2                 18  f SIN
03  RCL 2                 19  RCL 2
04  g x^2                 20  x
05  +                     21  RCL 3
06  RCL 3                 22  ÷
07  g x^2                 23  g SIN^-1
08  -                     24  R/S  (m∠Y)
09  2                     25  RCL 0
10  ÷                     26  +
11  RCL 1                 27  1
12  ÷                     28  8
13  RCL 2                 29  0
14  ÷                     30  x >< y
15  g COS^-1              31  -      (m∠X)
16  R/S   (m∠Z)
```

z >	x
z >	y
R_1	x
R_2	y
R_3	z

13) $m\angle P = 46.2^\circ$, $m\angle Q = 53.1^\circ$, $m\angle R = 80.7^\circ$

14) $m\angle P = 72.5^\circ$, $m\angle Q = 72.5^\circ$, $m\angle R = 34.9^\circ$

**15) $m\angle R = 34.2^\circ$, $m\angle Q = 108.5^\circ$, $m\angle P = 37.3^\circ$

16) $m\angle P = 14.9^\circ$, $m\angle Q = 46.0^\circ$, $m\angle R = 119.1^\circ$

17) $m\angle P = 134^\circ$, $m\angle Q = 32^\circ$, $m\angle R = 14^\circ$

18) $m\angle P = 19.0^\circ$, $m\angle Q = 47.0^\circ$, $m\angle R = 114.0^\circ$

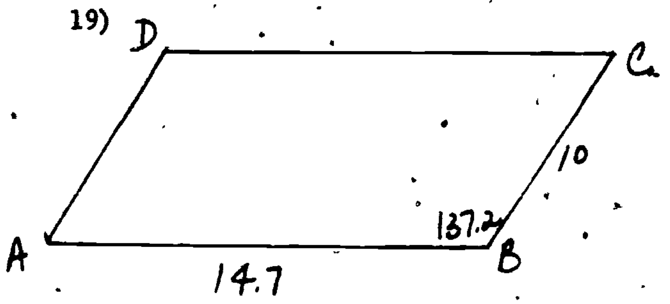
* Reletter to make x < y if using the accompanying program.

** Remember to reletter to make z the longest side if using the accompanying program.

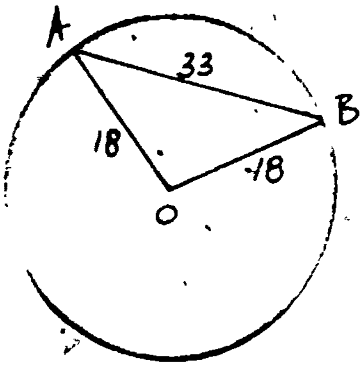
The formulas for this program are:

$$Z = \cos^{-1}\left(\frac{x^2 + y^2 - z^2}{2 \cdot x \cdot y}\right)$$

$$Y = \sin^{-1}\left(\frac{y \sin Z}{z}\right), \quad X = 180 - (Z + Y)$$

19) 

AC = 23.1
m ∡ A = 42.8
DB = 10.0

20) 

m ∡ AOB = 133°

Exercise Set 6.4

1) If the measures of the legs of an isosceles triangle are known then $x - y = 0$ and $m\angle X - m\angle Y = 0$ so no information is given by the formula.

2) If the measures of the legs of a right triangle are known then the hypotenuse of the triangle can more easily be found by the Pythagorean theorem and the acute angles can more easily be found by using the trigonometry of a right triangle. The Law of Tangents will however yield a solution.

(3 - 8) The following program can be used to solve each triangle having sides x and y and included angle Z. Key x in Register 1, y in Register 2 and Z (in decimal degrees) in Register 3. When the program is run the first display is $m\angle Y$, the second display is $m\angle X$ and the third display is the length of side z. It is necessary that $x < y$.

01	RCL 1	21	÷
02	RCL 2	22	g TAN^{-1}
03	+	23	2
04	STO 6	24	X
05	RCL 2	25	RCL 4
06	RCL 1	26	+
07	-	27	2
08	STO 5	28	÷
09	1	29	R/S ($m\angle Y$)
10	8	30	RCL 4
11	0	31	x ⇄ y
12	RCL 3	32	-
13	-	33	R/S ($m\angle X$)
14	STO 4	34	f SIN
15	2	35	RCL 3
16	÷	36	f SIN
17	f TAN	37	RCL 1
18	RCL 5	38	X
19	X	39	x ⇄ y
20	RCL 6	40	÷ (z)

x <	y
R_1	x
R_2	y
R_3	Z
R_4	180-Z
R_5	y - x
R_6	x + y

3) $m\angle Y = 97°.17'$, $m\angle X = 47°.46'$, $z = 4.84$

4) $m\angle Y = 97.2°$, $m\angle Z = 40.1°$, $x = 10.071$

5) $m\angle Z = 33.6°$, $m\angle X = 28.4°$, $y = 178.8$

6) $m\angle Y = 24.6°$, $m\angle Z = 19.2°$, $x = 25.9$

7) $m\angle X = 93.1°$, $m\angle Z = 4.2°$, $y = 0.308$

8) $m\angle X = 128°$, $m\angle Y = $ [illegible]$3°$, $z = 2.2$

(9 - 16)

9) aa ($\angle AGE \cong \angle OGB$, $\angle GBO \cong \angle GEA$)

10) Corresponding angles of similar triangles are congruent.

11) $\sin\phi = \frac{EC}{OE}$ so $EC = OE \sin\phi$

$\cos\phi = \frac{OC}{OE}$ so $OC = OE\cos\phi$

12) $\sin\phi = \frac{FE}{AE}$ so $FE = AE\sin\phi$

$\cos\phi = \frac{AF}{AE}$ so $AF = AE\cos\phi$

13) $\sin\theta = \frac{AE}{1}$ so $\sin\theta = AE$

$\cos\theta = \frac{OE}{1}$ so $OE = \cos\theta$

14) $\sin(\phi + \theta) = AB = AF + FG + GB = AF + EC$

15) $\sin(\phi + \theta) = AE\cos\phi + OE\sin\phi$

$= \sin\theta\cos\phi + \cos\theta\sin\phi$

(16 - 19)

16) $\sin(\phi - \theta) = \sin[\phi + (-\theta)]$

17) $\sin(\phi - \theta) = \sin\phi \cdot \cos(-\theta) + \cos\phi \cdot \sin(-\theta)$

18) $\cos(-\theta) = \cos\theta$; $\sin(-\theta) = -\sin\theta$

19) $\sin(\phi - \theta) = \sin\phi \cdot \cos\theta + \cos\phi(-\sin\theta)$

$= \sin\phi \cdot \cos\theta - \cos\phi \cdot \sin\theta$

(20 - 24)

20) $2A = \phi + \theta$, $2B = \phi - \theta$

21) $A = \dfrac{\phi+\theta}{2}$, $B = \dfrac{\phi-\theta}{2}$

22) $\sin(A + B) + \sin(A - B) = \sin A \cos B + \cos A \sin B$

$+ \sin A \cos B - \cos A \sin B$

$= 2 \sin A \cos B$

23) $\sin(A + b) - \sin(A - B) = \sin A \cos B + \cos A \sin B$

$- \sin A \cos B + \cos A \sin B$

$= 2 \cos A \sin B$

24) $\sin\phi + \sin\theta = \sin(A + B) + \sin(A - B)$

$= 2 \sin A \cos B$

$= 2 \sin\left(\dfrac{\phi+\theta}{2}\right) \cdot \cos\left(\dfrac{\phi-\theta}{2}\right)$

$\sin\phi - \sin\theta = \sin(A + B) - \sin(A - B)$

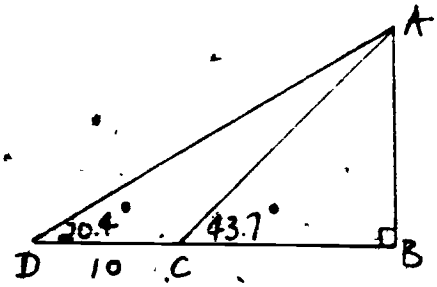
$= 2 \cos A \sin B$

$= 2 \cos\left(\dfrac{\phi+\theta}{2}\right) \cdot \sin\left(\dfrac{\phi-\theta}{2}\right)$

Exercise Set 6.5

Other methods of solution are possible. These solutions are samples.

1)



$m\angle ACD = 136.3^\circ$

$m\angle DAC = 23.3^\circ$

Using the Law of Sines in $\triangle DAC$

$$\frac{DA}{\sin 136.3^\circ} = \frac{10}{\sin 23.3^\circ}$$

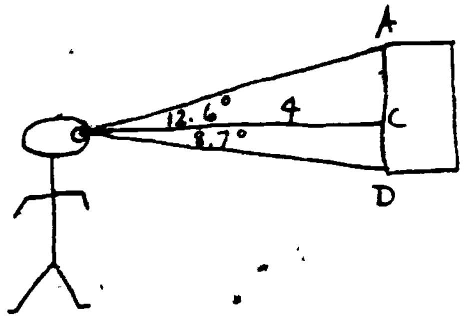
$$DA = 17.47 \text{ meters}$$

In $\triangle DAB$ $\sin 20.4^\circ = \frac{AB}{17.47}$

$$6.09 = AB$$

The tree is 6 meters high.

2)



In $\triangle ABC$, $\tan 12.6 = \frac{AC}{4}$

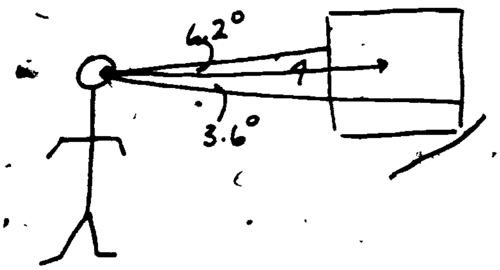
$.89 = AC$

In $\triangle BCD$, $\tan 8.7 = \frac{CD}{4}$

$.61 = CD$

$AD = AC + CD = 1.51$ meters

The painting is 1.5 meters high.

3) 

In $\triangle ABC$, $\tan 6.2 = \frac{AC}{4}$

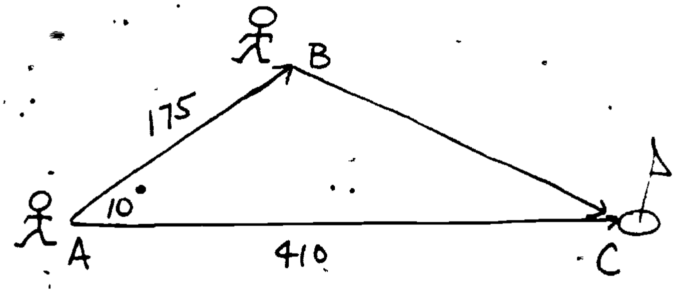
$.43 = AC$

In $\triangle BCD$, $\tan 3.6 = \frac{CD}{4}$

$.25 = CD$

$AD = AC + CD = .69$

The painting is .7 meters wide.

4) 

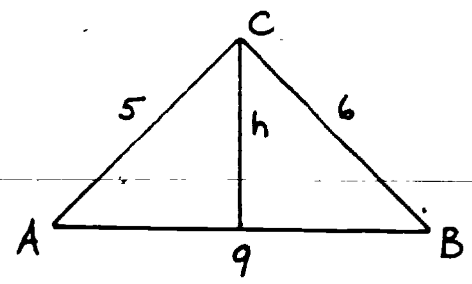
Using the law of cosines

$a^2 = b^2 + c^2 - 2 \cdot bc \cdot \cos A$

$BC = \sqrt{410^2 + 175^2 - 2(410)(175) \cos 10^\circ}$

$BC = 239.59$ yards

Wally is approximately 240 yards from the hole.

5) 

Using the law of cosines

$a^2 = b^2 + c^2 - 2\ bc \cos A$

$6^2 = 5^2 + 9^2 - 2(5\ (9) \cos A$

$$\frac{-6^2 + 5^2 + 9^2}{2(5)(9)} = \cos A$$

$$\cos^{-1}\left(\frac{-a^2 + b^2 + c^2}{2\ bc}\right) = A$$

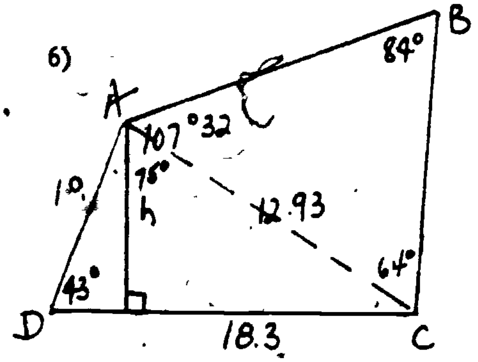
h = 5 sin A

h = b sin A

$$\text{Area } \triangle ABC = \tfrac{1}{2}\ (c)(b)\ \sin\left[\cos^{-1}\left(\frac{-a^2 + b^2 + c^2}{2\ bc}\right)\right]$$

Area of a triangle having sides 5,6 and 9 is 14.14.

6)



In $\triangle$ ADC

$$\sin 43° = \frac{h}{10}$$

$$10 \sin 43° = h$$

$$\text{Area } \triangle ADC = \tfrac{1}{2}(10)(18.3) \sin 43°$$

$$= 62$$

Using the law of Cosines

$$AC = \sqrt{10^2 + (18.3)^2 - 2(10)(18.3)\cos 43°} = 12.93$$

Using the law of Sines

$$m\angle DAC = \sin^{-1}\left(\frac{18.3 \sin 43}{AC}\right)$$

$$= 75°$$

$$m\angle BAC = 32°$$

$$m\angle BCA = 64°$$

$$AB = \frac{AC \sin\angle ACB}{\sin 84°}$$

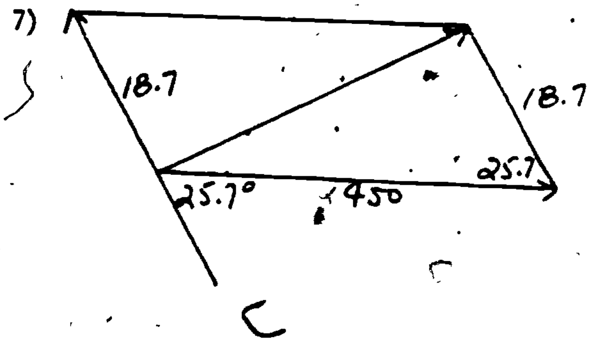
$$= 11.67$$

$$\text{Area } \triangle ABC = \tfrac{1}{2}\ AC(AB)\ \sin\angle BAC$$

$$= 40$$

Area Quad ABCD = 102

7)



Using the law of Cosines

$C^2 = 18.7^2 + 450^2 - 2(18.7)(450) \cos 25.7$

C = 433.23 kph

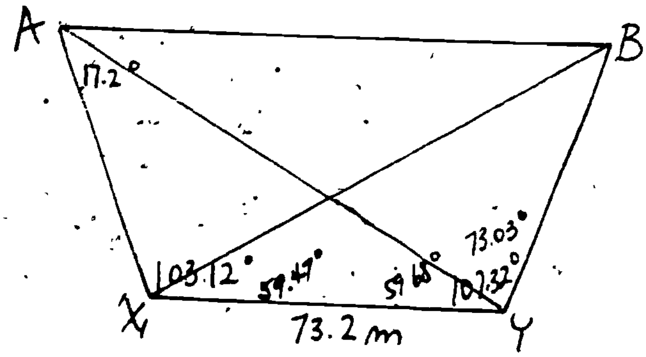
719: x = 433.23: 1

1.66 hrs. = x

1 hr 39 min 35 sec = x

It will take the plane approximately 1 hr and 40 minutes to travel 719 kilometers against the wind.

8)



In $\triangle$ AXY,

m $\measuredangle$ XAY = 17.2°

Using the law of Sines

$$\frac{73.2}{\sin 17.2} = \frac{AY}{\sin 103.12}$$

241.08 = AY

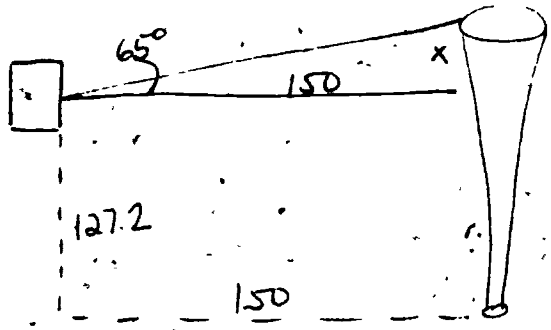
In $\triangle$ XBY.

$$\frac{BY}{\sin 59.47} = \frac{73.2}{\sin 13.21}$$ (using m $\measuredangle$ XYB = 107.32°)

BY = 275.91

In $\triangle$ ABY $AB = \sqrt{AY^2 + BY^2 - 2(AY)(BY) \cos 73.03^\circ}$

AB = 308.9 meters

9) 

$x = 150 \tan 65$

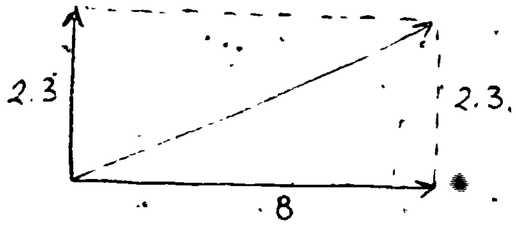
ceiling $= 127.2 + 150 \tan 65$

$= 448.88$ meters

10) A program to establish the following chart is

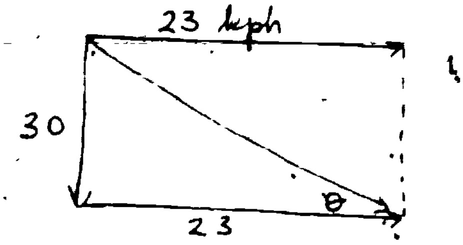
01	STO 1	10	.	
02	f TAN	11	2	
03	1	12	+	
04	5	13	R/S	← ceiling
05	0	14	RCL 1	
06	x	15	5	
07	1	16	+	
08	2	17	R/S	← displays new angle
09	7	18	GTO 01	

angle	ceiling	angle	ceiling
10°	153.65	50°	305.96
15°	167.39	55°	341.42
20°	181.80	60°	387.01
25°	197.15	65°	448.88
30°	213.80	70°	539.32
35°	232.23	75°	687.01
40°	253.06	80°	977.89
45°	277.20	85°	1841.71

11) 

Your rate is 2.4 kph.

You can use the Pythagorean Theorem here!

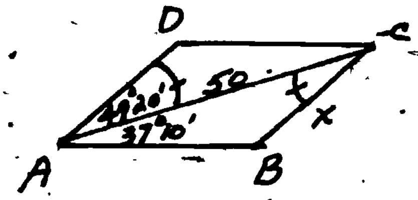
12) 

$$\theta = \tan^{-1} \left(\frac{30}{23}\right)$$

$$= 52.52^{\circ}$$

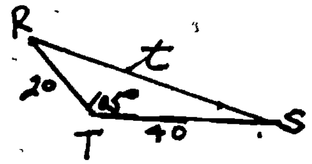
The raindrops are hitting the ground at an angle of 52.5°.

Solutions CHAPTER 6 Test

1) 

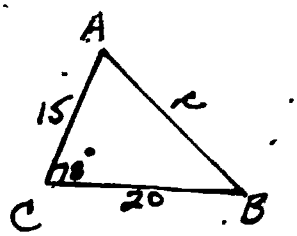
$m\angle B = 93°30'$

$$\frac{x}{\sin 37°10'} = \frac{50}{\sin 93°30'} \qquad x = 30$$

2) 

$t^2 = 20^2 + 40^2 - 2(20)(40)\cos 105°$

$t = 49$

3) 

$$\frac{20-15}{20+15} = \frac{\tan 1/2(A-B)}{\tan 1/2\,(102)°}$$

$\tan 1/2(A-B) = 5 \cdot \tan 51° - 35$

$1/2(A-B) = 10°$

$m\angle A - m\angle B = 20°$ & $m\angle A + m\angle B = 102°$ $\therefore$ $m\angle A = 61°$

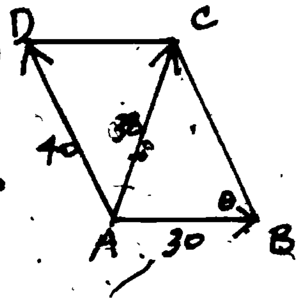
Alternate solution:

$c^2 = 15^2 + 20^2 - 2(15)(20)\cos 78°$

$c \doteq 22.4$

$$\frac{\sin 78°}{22.4} = \frac{\sin A}{20}$$

$m\angle A = 61°$

4) 

$$38^2 = 30^2 + 40^2 - 2(30)(40)\cos\theta$$

$$\cos\theta = .4400$$

$$\theta = 64^\circ$$

$$m\angle DAB = 116^\circ$$

5 a) .75

b) .7220

c) .4

d) .15

e) .(2)

Exercise Set 7.1

1) degree 3, coefficients 2, 0, -6, 4

2) degree 2, coefficients -2, 3, 0

3) degree 0, coefficients 9

4) degree 4, coefficients 1, 0, -5, -4, +10

5) degree 3, coefficients 1, 0, -1, -1

6) (a-1) $f(0) = 4$ (b-1) $f(1) = 0$ (c-1) $f(-1) = 8$

(a-2) $f(0) = 0$ (b-2) $f(1) = 1$ (c-2) $f(-1) = -5$

(a-3) $f(0) = 9$ (b-3) $f(1) = 9$ (c-3) $f(-1) = 9$

(a-4) $f(0) = 10$ (b-4) $f(1) = +2$ (c-4) $f(-1) = 10$

(a-5) $f(0) = -1$ (b-5) $f(1) = -1$ (c-5) $f(-1) = -1$

(d-1) $f(1/2) = 1.25$ (e-1) $f(a) = 2a^3 - 6a + 4$

(d-2) $f(1/2) = 1$ (e-2) $f(a) = 3a - 2a^2$

(d-3) $f(1/2) = 9$ (e-3) $f(a) = 9$

(d-4) $f(1/2) = 6\frac{13}{16} = 6.8125$ (e-4) $f(a) = a^4 - 5a^2 - 4a + 10$

(d-5) $f(1/2) = -1.375$ (e-5) $f(a) = a^3 - a - 1$

(f-1) $f(x + h) = 2x^3 + 6x^2h + 6xh^2 + 2h^3 - 6x - 6h + 4$

(f-2) $f(x + h) = -2x^2 - 4xh - 2h^2 + 3x + 3h$

(f-3) $f(x + h) = 9$

(f-4) $f(x + h) = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 5x^2$
$- 10xh - 5h^2 - 4x - 4h + 10$

(f-5) $f(x+h) = x^3 + 3x^2h + 3xh^2 + h^3 - x - h - 1$

(g-1) $f(-x) = -2x^3 + 6x + 4$

(g-2) $f(-x) = -3x - 2x^2$

(g-3) $f(-x) = 9$

(g-4) $f(-x) = x^4 - 5x^2 + 4x + 10$

(g-5) $f(-x) = -x^3 + x - 1$

Exercise Set 7.2

1) $y = x^2 - 3x - 10$: $[-3, 6]$; $d = .5$

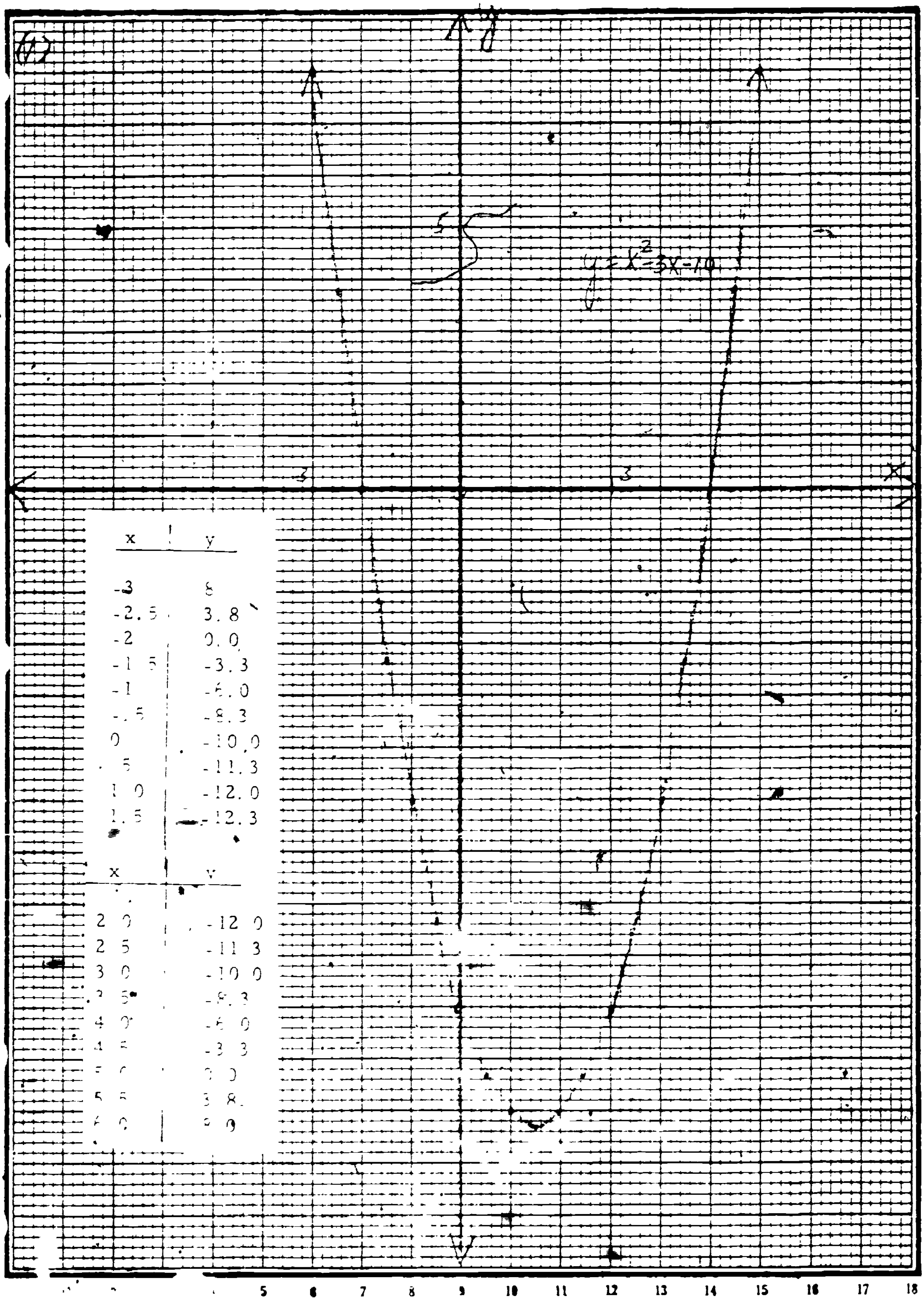
$y = x(x-3) - 10$

HP Program

01	STO 2 (d)
02	R/S
03	STO 1 (x)
04	R/S
05	RCL 1
06	3
07	-
08	RCL 1
09	x
10	1
11	0
12	-
13	f fix 1
14	R/S
15	RCL 1
16	RCL 2
17	+
18	GTO 03

TI Program

00	STO 2 (d)
01	R/S
02	STO 1 (x)
03	2nd Lbl 1
04	R/S
05	(
06	RCL 1
07	-
08	3
09	)
10	X
11	RCL 1
12	-
13	1
14	0
15	=
16	2nd fix 1
17	R/S
18	RCL 1
19	+
20	RCL 2
21	=
22	STO 1
23	GTO 1



x	y
-3	8
-2.5	3.8
-2	0.0
-1.5	-3.3
-1	-6.0
-.5	-8.3
0	-10.0
.5	-11.3
1.0	-12.0
1.5	-12.3

x	y
2.0	-12.0
2.5	-11.3
3.0	-10.0
3.5	-8.3
4.0	-6.0
4.5	-3.3
5.0	0.0
5.5	3.8
6.0	8.0

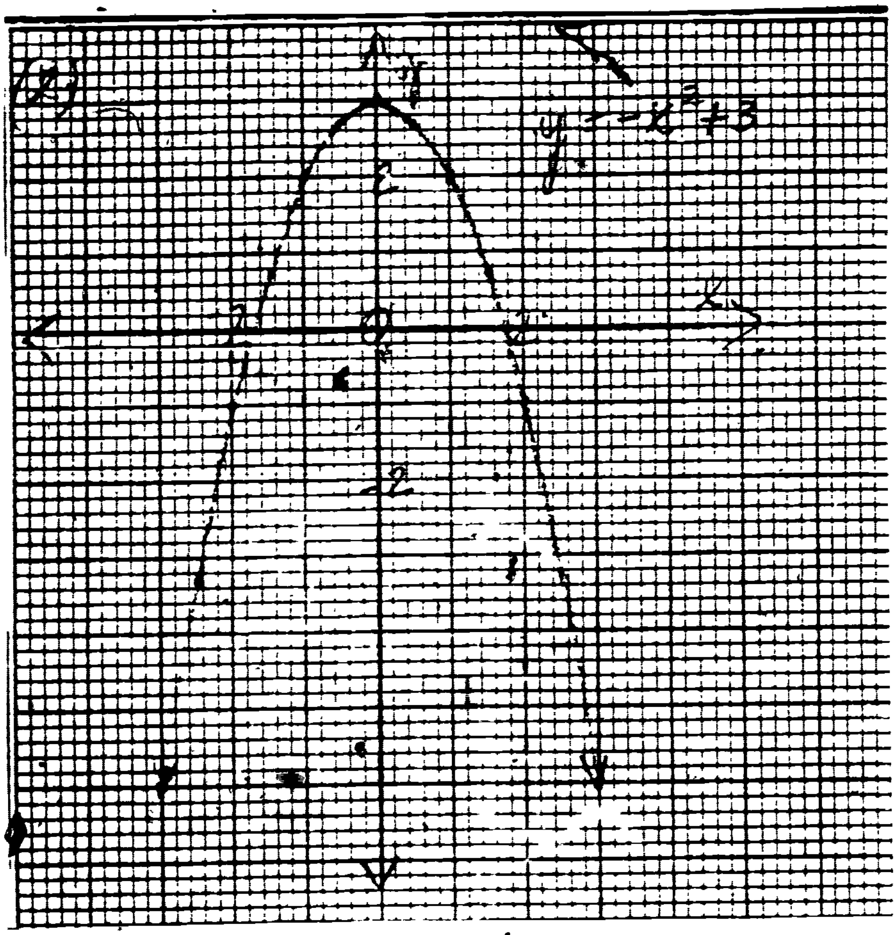
2) $y = -x^2 + 3 : [-3, 3]; d = .25$

HP Program		TI Program	
01	STO 2 (d)	00	STO 2 (d)
02	R/S	01	R/S
03	STO 1 (x)	02	STO 1 (x)
04	R/S	03	2nd Lbl 1
05	RCL 1	04	R/S
06	gx^2	05	RCL 1
07	CHS	06	x^2
08	3	07	+/-
09	+	08	+
10	f fix 2	09	3
11	R/S	10	=
12	RCL 1	11	2nd fix 2
13	RCL 2	12	R/S
14	+	13	RCL 1
15	GTO 03	14	+
		15	RCL 2
		16	=
		17	STO 1
		18	GTO 1

2) continued

x	y
-3.0	-6.0
-2.75	-4.56
-2.5	-3.25
-2.25	-2.06
-2.0	-1.0
-1.75	-0.06
-1.50	0.75
-1.25	1.44
-1.00	2
-0.75	2.44
-0.50	2.75
-0.25	2.94
0.0	3

x	y
0.25	2.94
0.50	2.75
0.75	2.44
1.00	2.00
1.25	1.44
1.50	0.75
1.75	-0.06
2.00	-1.00
2.25	-2.06
2 50	-3.25
2.75	-4.56
3.00	-6.00



3) $y = -x^2 + 6x - 8 : [0, 6]; d = .25$

$y = -x(x-6) - 8$

HP Program

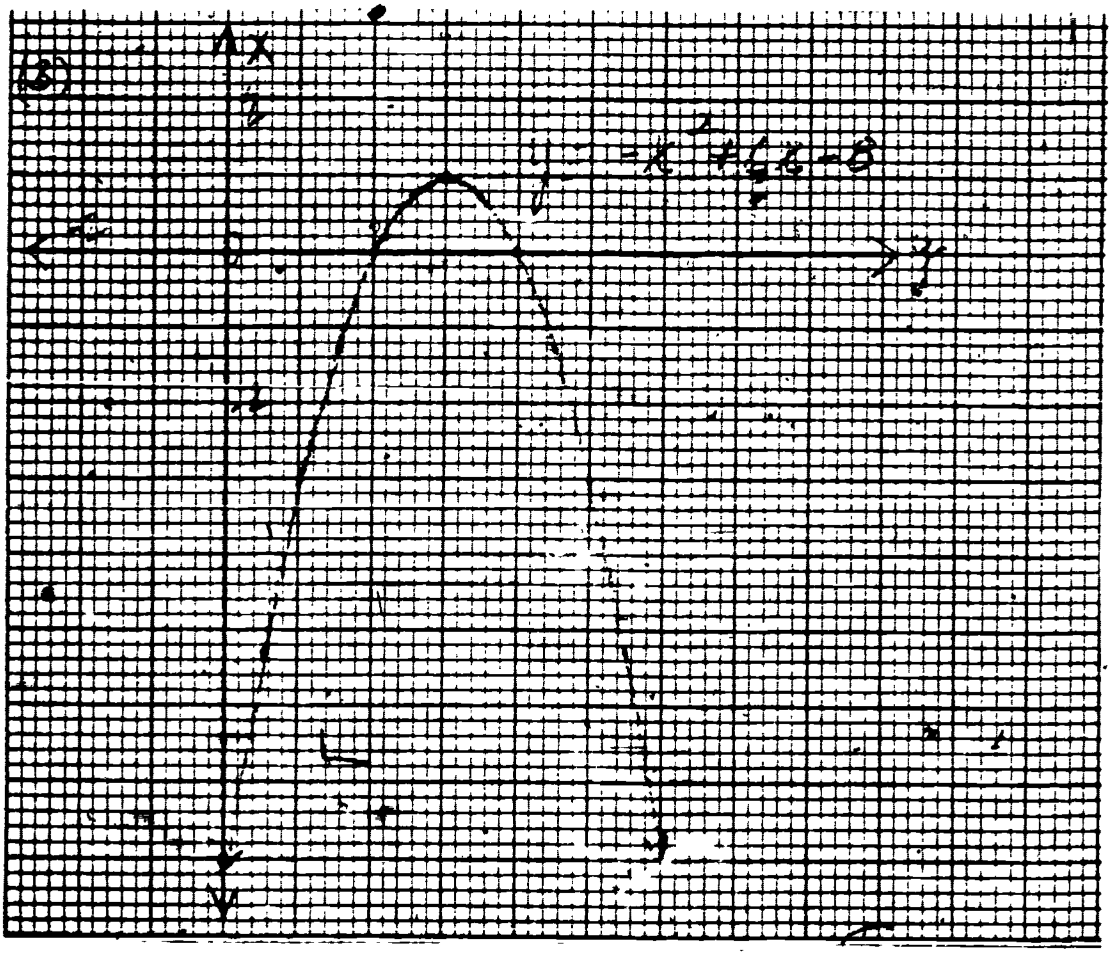
01	STO 2 (d)
02	R/S
03	STO 1 (x)
04	R/S
05	RCL 1
06	6
07	-
08	RCL 1
09	X
10	CHS
11	8
12	-
13	f fix 2
14	R/S
15	RCL 1
16	RCL 2
17	+
18	GTO 03

TI Program

00	STO 2 (d)
01	R/S
02	STO 1 (x)
03	2nd Lbl 1
04	R/S
05	(
06	RCL 1
07	-
08	6
09	)
10	X
11	RCL 1
12	+/-
13	-
14	8
15	=
16	2nd fix 2
17	R/S
18	RCL 1
19	+
20	RCL 2
21	=
22	STO 1
23	GTO 1

3) continued

x	y	x	y
0	-8	3.25	0.94
0.25	-6.56	3.50	0.75
0.50	-5.25	3.75	0.44
0.75	-4.06	4.00	0.00
1.00	-3.00	4.25	-0.56
1.25	-2.06	4.50	-1.25
1.50	-1.25	4.75	-2.06
1.75	-0.56	5.00	-3.00
2.00	0.00	5.25	-4.06
2.25	0.44	5.50	-5.25
2.50	0.75	5.75	-6.56
2.75	0.94	6.00	-8.00
3.00	1.00		



4) $y = 2x^2 - 5x - 12$: [-2.5, 5]; d = .5

$y = x(2x - 5) - 12$

HP Program

01	STO 2 (d)
02	R/S
03	STO 1 (x)
04	R/S
05	RCL 1
06	2
07	X
08	5
09	-
10	RCL 1
11	X
12	1
13	2
14	-
15	f fix 1
16	R/S
17	RCL 1
18	RCL 2
19	+
20	GTO 03

TI Program

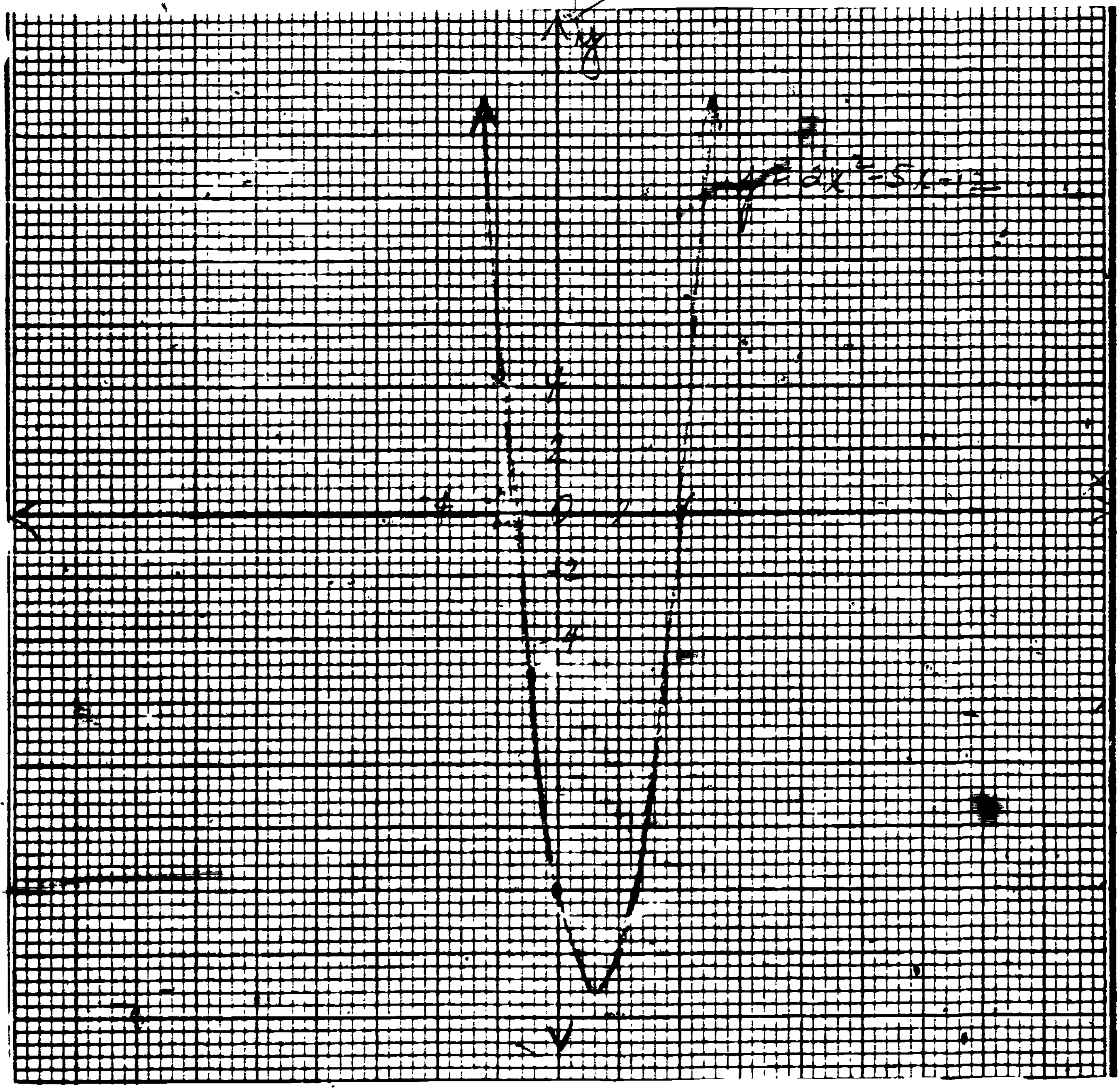
00	STO 2 (d)
01	R/S
02	STO 1 (x)
03	2nd Lbl 1
04	R/S
05	(
06	RCL 1
07	X
08	2
09	-
10	5
11	)
12	X
13	RCL 1
14	-
15	1
16	2
17	=
18	2nd fix 1
19	R/S
20	RCL 1
21	+
22	RGL 2
23	=
24	STO 1
25	GTO 1

4) continued

x	y
-2.5	13.0
-2.0	6.0
-1.5	0
-1.0	-5.0
-0.5	-9.0
0	-12.0

x	y
0.5	-14.0
1	-15.0
1.5	-15.0
2.0	-14.0
2.5	-12.0
3	-9.0

x	y
3.5	-5.0
4.0	0.0
4.5	6.0
5.0	13.0



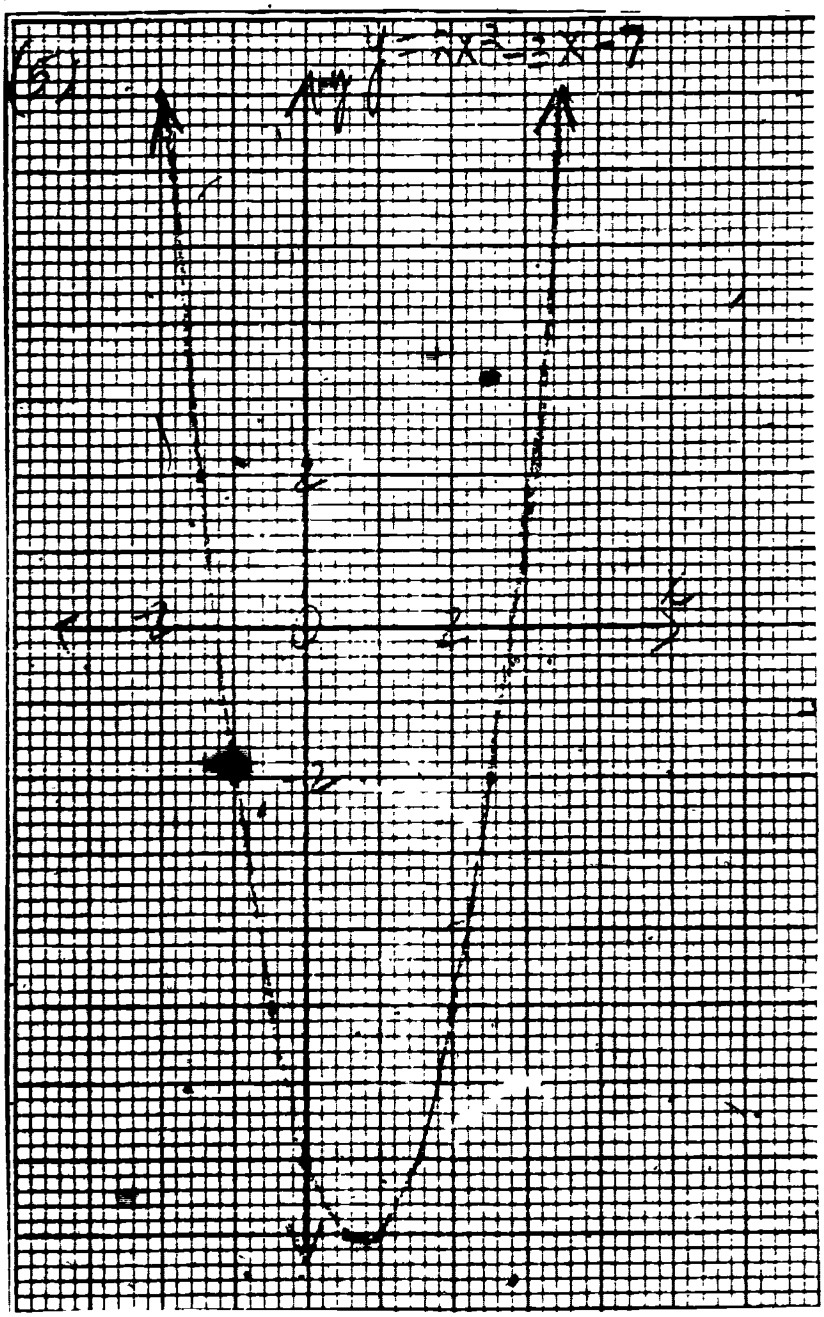
5) $y = 2x^2 - 3x - 7$: [-2, 3]; d = .25

$y = x(2x - 3) - 7$

HP Program		TI Program	
01	STO 2 (d)	00	STO 2 (d)
02	R/S	01	R/S
03	STO 1 (x)	02	STO 1 (x)
04	R/S	03	2nd Lbl 1
05	RCL 1	04	R/S
06	2	05	(
07	X	06	RCL 1
08	3	07	X
09	-	08	2
10	RCL 1	09	-
11	X	10	3
12	7	11	)
13	-	12	X
14	f fix 2	13	RCL 1
15	R/S	14	-
16	RCL 1	15	7
17	RCL 2	16	=
18	+	17	2nd fix 2
19	GTO 03	18	R/S
		19	RCL 1
		20	+
		21	RCL 2
		22	=
		23	STO 1
		24	GTO 1

5) continued

x	y	x	y	x	y
-2.00	7.00	-0.25	-6.13	1.50	-7.00
-1.75	4.38	0.00	-7.00	1.75	-6.13
-1.50	2.00	0.25	-7.63	2.00	-5.00
-1.25	-0.13	0.50	-8.00	2.25	-3.63
-1.00	-2.00	0.75	-8.13	2.50	-2.00
-0.75	-3.63	1.00	-8.00	2.75	-0.13
-0.50	-5.00	1.25	-7.63	3.00	2.00



6) $y = -x^3$: [-2, 2]; d = .25

HP Program *		TI Program	
01	STO 2 (d)	00	STO 2 (d)
02	R/S	01	R/S
03	STO 1 (X)	02	STO 1 (x)
04	R/S	03	2nd Lbl 1
05	RCL 1	04	R/S
06	ENTER	05	RCL 1
07	ENTER	06	x^2
08	X	07	X
09	X	08	RCL 1
10	CHS	09	=
11	f fix 2	10	+/-
12	R/S	11	2nd fix 2
13	RCL 1	12	R/S
14	RCL 2	13	RCL 1
15	+	14	+
16	GTO 03	15	RCL 2
		16	=
		17	STO 1
		18	GTO 1

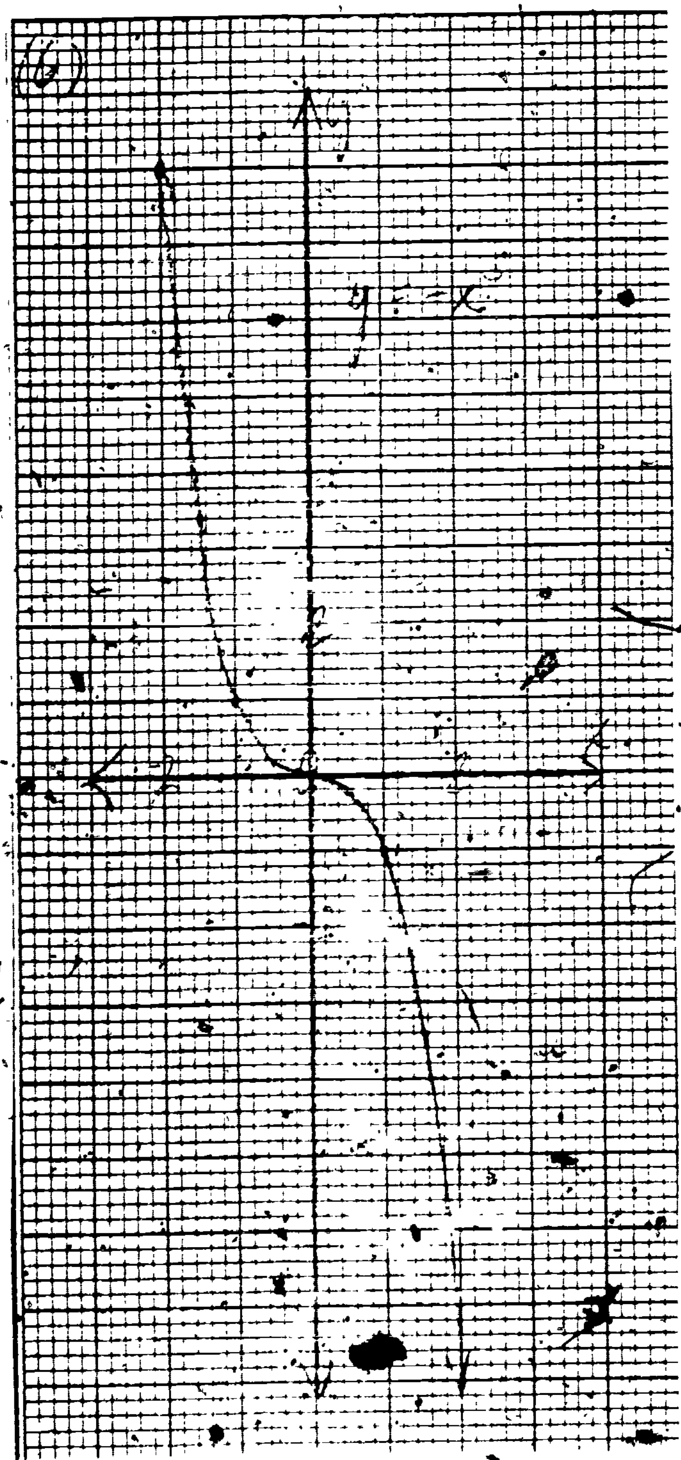
* The f $\boxed{y^x}$ key gives error messages for $y < 0$ on some HP models.

6) continued

x	y
-2.00	8
-1.75	5.36
-1.50	3.38
-1.25	1.95
-1.00	1.00
-0.75	0.42
-0.50	0.13

x	y
-0.25	0.02
0.00	0.00
0.25	-0.02
0.50	-0.13
0.75	-0.42
1.00	-1.00
1.25	-1.95

x	y
1.50	-3.38
1.75	-5.36
2.00	-8.00



7) $y = x^3 - 3x + 3 : [-3, 3];\ d = .5$

$y = x(x^2 - 3) + 3$

HP - Program

01	STO 2 (d)
02	R/S
03	STO 1 (x)
04	R/S
05	RCL 1
06	g x^2
07	3
08	-
09	RCL 1
10	X
11	3
12	+
13.	f fix 1
14	R/S
15	RCL 1
16	RCL 2
17	+
18	GTO 03

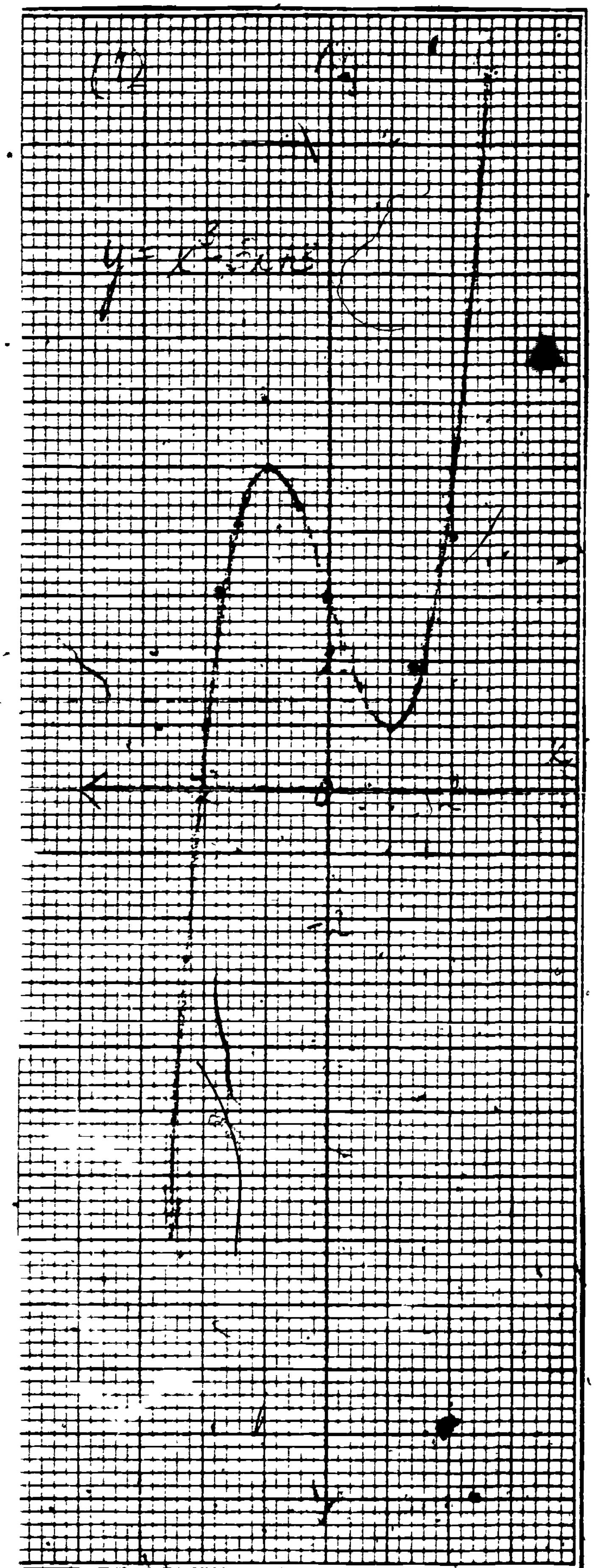
TI - Program

00	STO 2 (d)
01	R/S
02	STO 1 (x)
03	2nd Lbl 1
04	R/S
05	(
06	RCL 1
07	x^2
08	-
09	3
10	)
11	X
12	RCL 1
13	+
14	3
15	=
16	2nd fix 1
17	R/S
18	RCL 1
19	+
20	RCL 2
21	÷
22	STO 1
23	GTO 1

7) continued

x	y
-3.0	-15.0
-2.5	-5.1
-2.0	1.0
-1.5	4.1
-1.0	5.0
-0.5	4.4
0.0	3.0

x	y
0.5	1.6
1.0	1.0
1.5	1.9
2.0	5.0
2.5	11.1
3.0	21.0



8) $y = x^3 + 2x^2 - 5x - 6$; [-4, 3]; d = .5

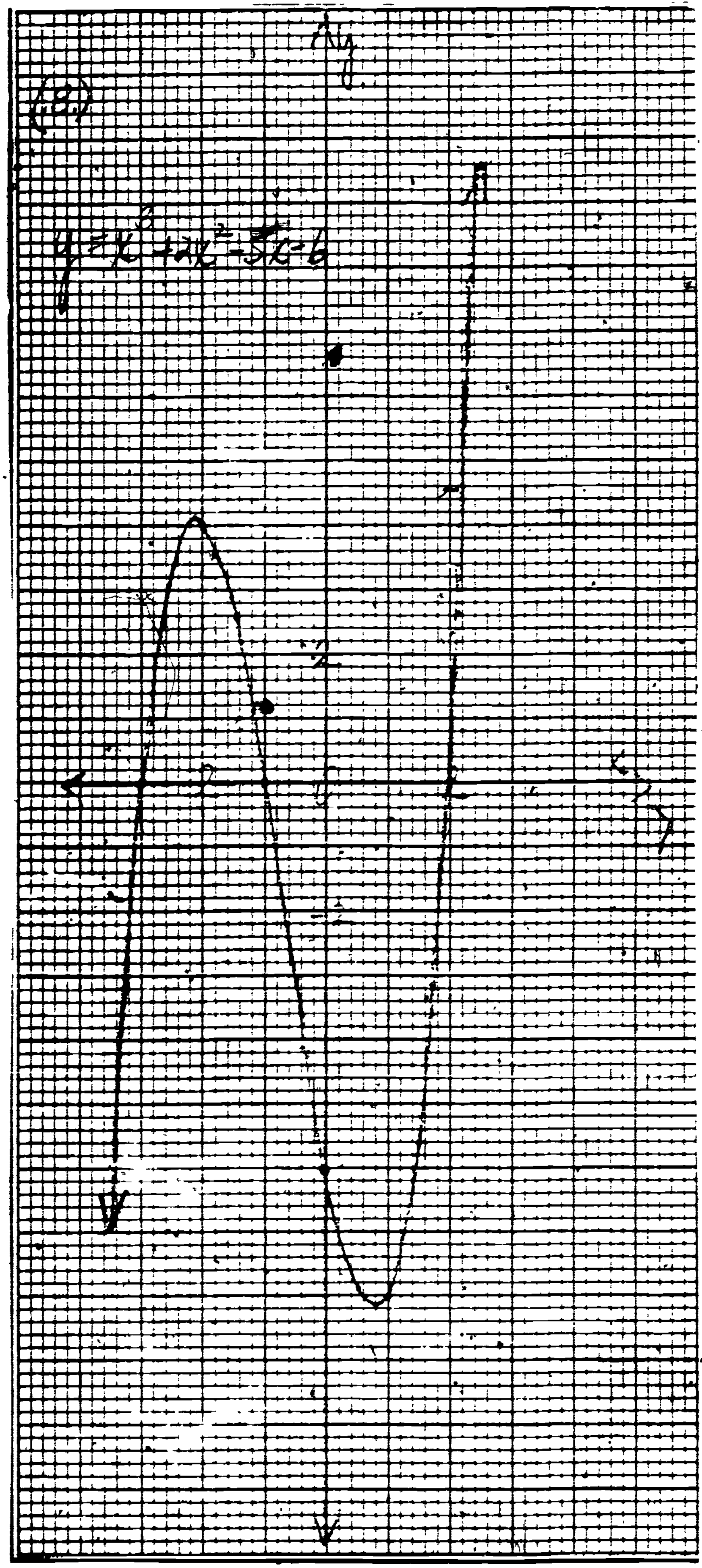
$y = x(x^2 + 2x - 5) - 6$

$y = x(x(x+2) - 5) - 6$

HP - Program		TI - Program	
01	STO 2 (d)	00	STO 2 (d)
02	R/S	01	R/S
03	STO 1 (x)	02	STO 1 (x)
04	R/S	03	2nd Lbl 1
05	RCL 1	04	R/S
06	2	05	RCL 1
07	+	06	X
08	RCL 1	07	(
09	X	08	RCL 1
10	5	09	X
11	-	10	(
12	RCL 1	11	RCL 1
13	X	12	+
14	6	13	2
15	-	14	)
16	f fix 1	15	-
17	R/S	16	5
18	RCL 1	17	)
19	RCL 2	18	-
20	+	19	6
21	GTO 03	20	=
		21	2nd fix 1
		22	R/S
		23	RCL 1
		24	+
		25	RCL 2
		26	=
		27	STO 1
		28	GTO 1

8) continued

x	y
-4.0	-18.0
-3.5	-6.9
-3.0	0.0
-2.5	3.4
-2.0	4.0
-1.5	3.6
-1.0	0.0
-.5	-3.1
0.0	-6.0
0.5	-7.9
1.0	-8.0
1.5	-5.6
2.0	0.0
2.5	9.6
3.0	24.0
3.5	43.9
4.0	70.0

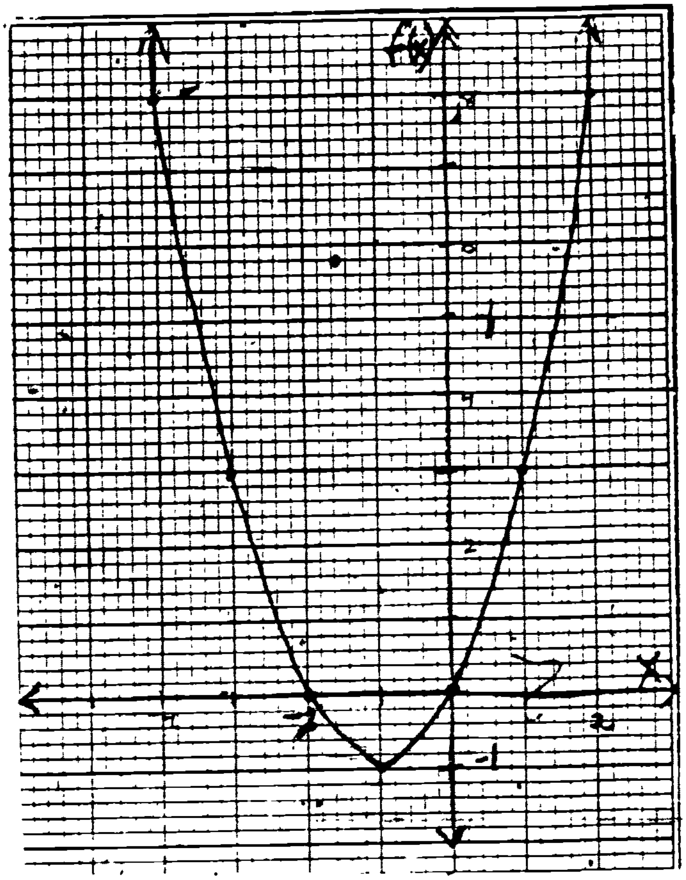


Exercise Set 7.3

1) $f(x) = x^2 + 2x$

a) (0, 0), (-2, 0)

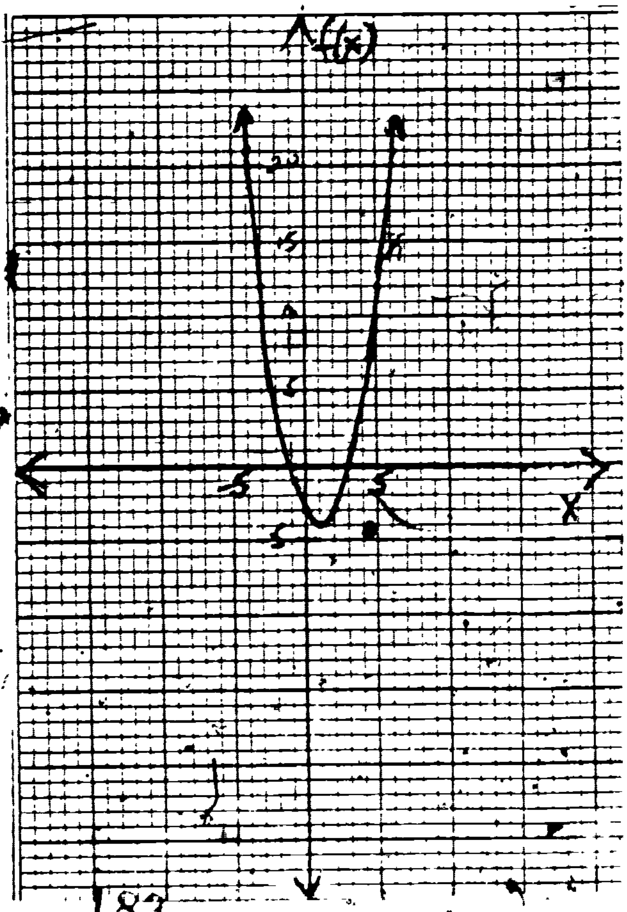
b) -1



2) $f(x) = x^2 - 2x - 3$

a) (-1, 0), (3, 0)

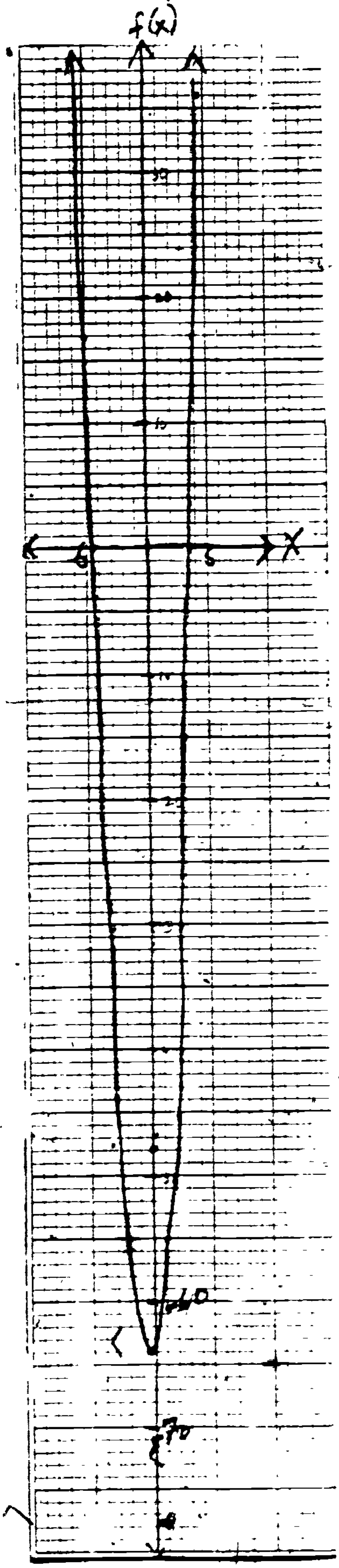
b) -4



3) $f(x) = 4x^2 + 4x - 63$ →

a) (3.5, 0), (-4.5, 0)

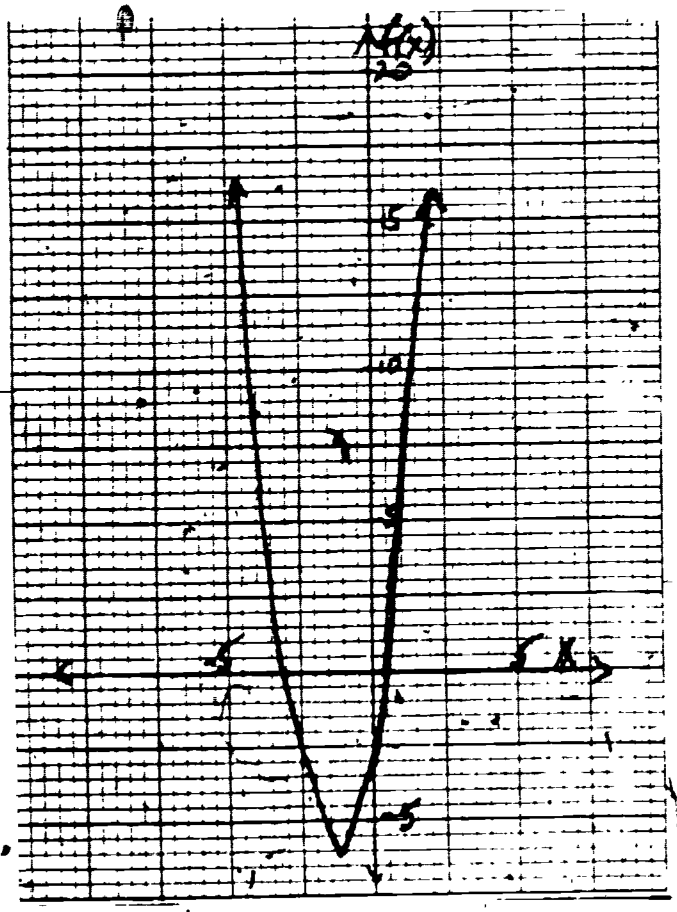
b) -64



4) $f(x) = 2x^2 + 5x - 3$

a) (-3, 0), (.5, 0)

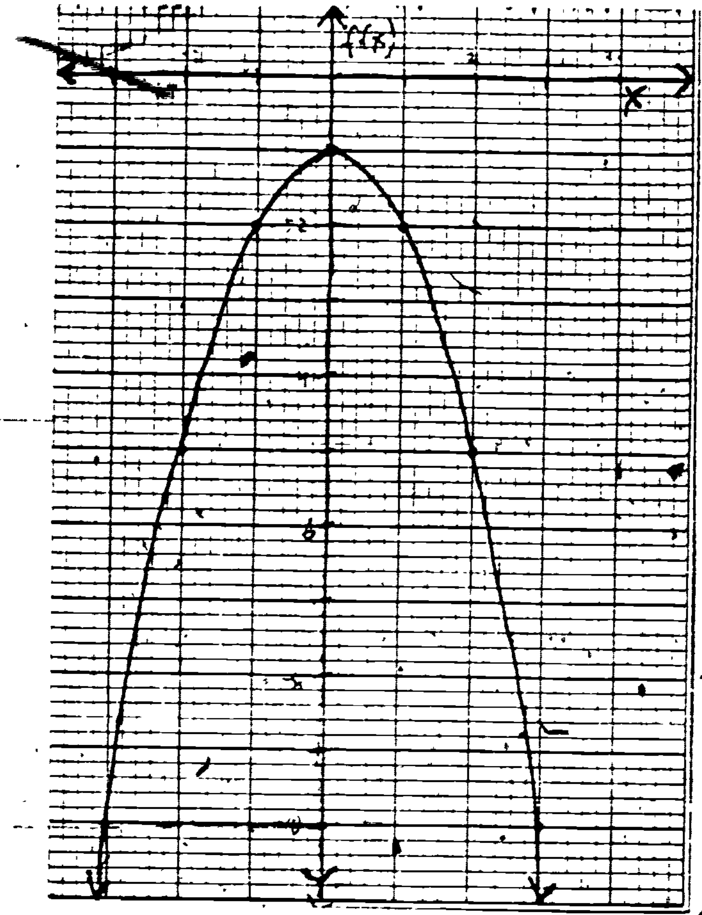
b) -6.125



5) $f(x) = -x^2 - 1$

a) none

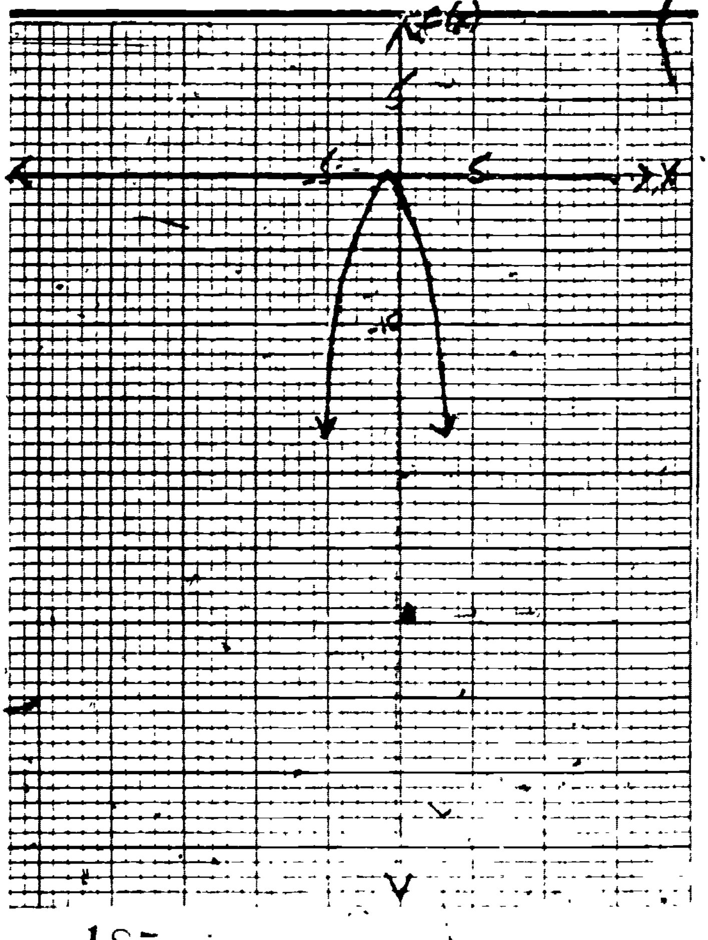
b) -1



6) $f(x) = -x^2 - 2x - 1$

a) (-1, 0)

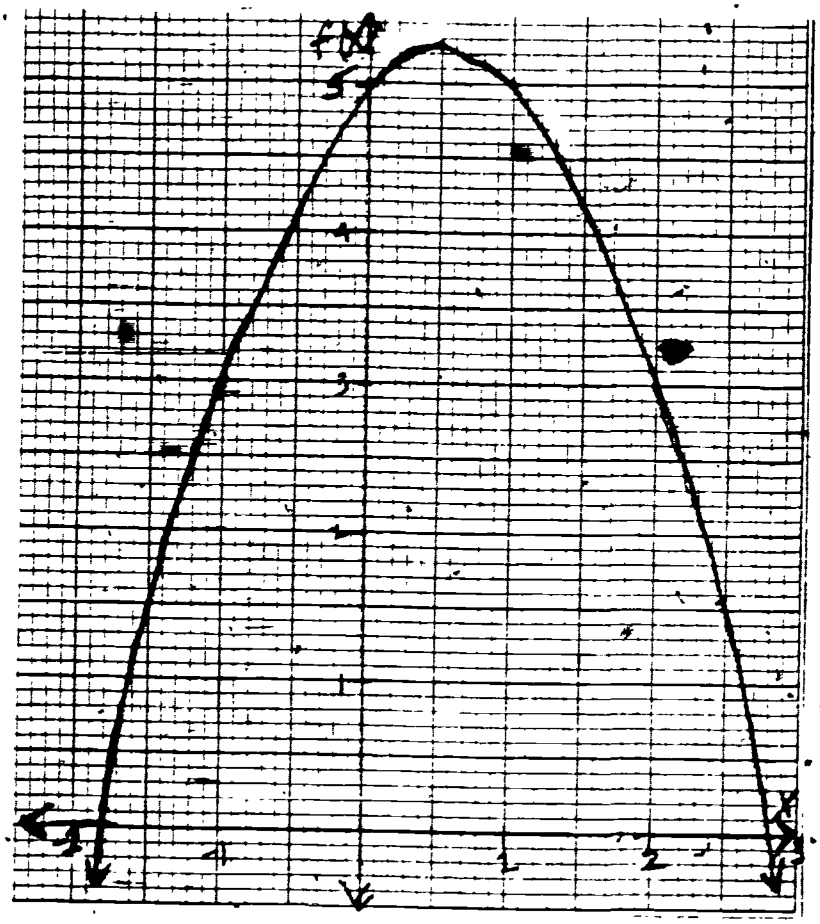
b) 0



7) $f(x) = -x^2 + x + 5$

a) (2.8, 0), (-1.8, 0)

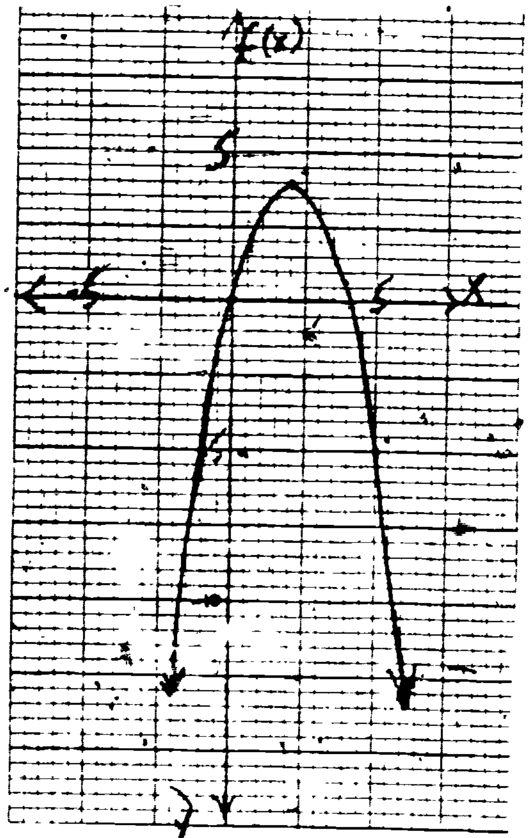
b) 5.25



8) $f(x) = -x^2 + 4x$

a) (0, 0), (4, 0)

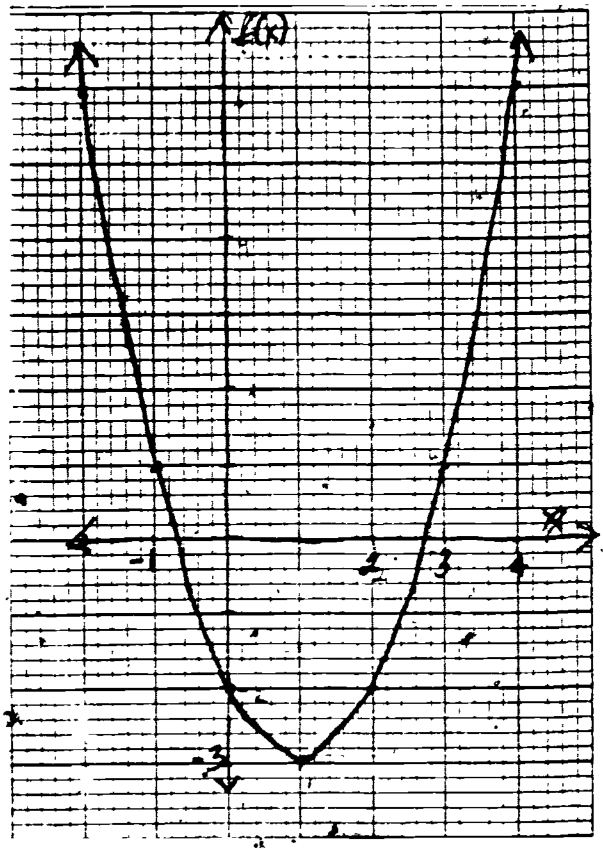
b) 4



9) $f(x) = x^2 - 2x - 2$

a) (2.7, 0) (-.7, 0)

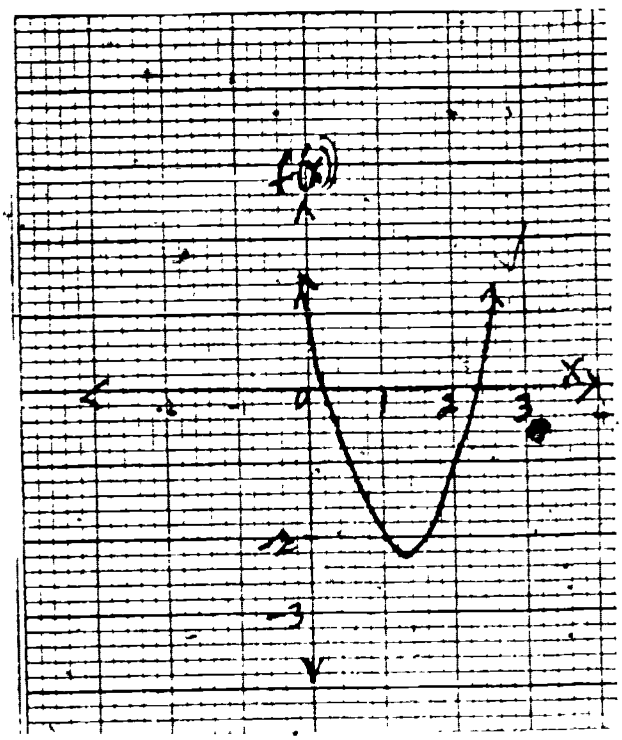
b) -3



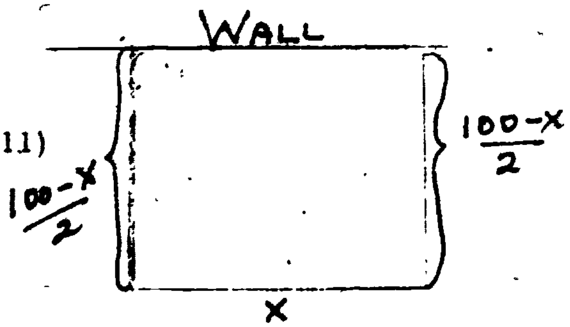
10) $f(x) = 2x^2 - 5x + 1$

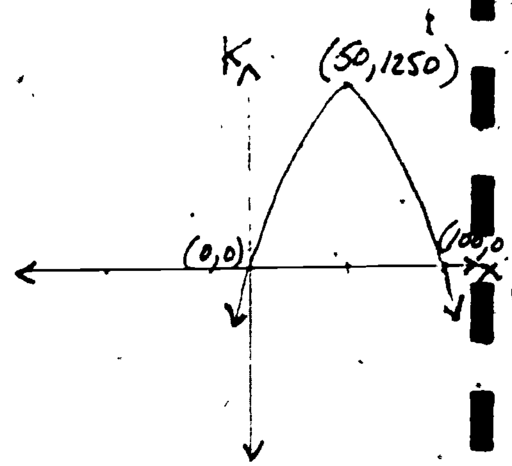
a) (.2, 0), (2.3, 0)

b) -2.125



11)





$K = x\left(\frac{100-x}{2}\right)$

$K = -\frac{1}{2}x^2 + 50x$

max. 1250

12) $h = 100 + 128t - 16t^2$

max $h = 356$ when $t = 4$

13) $a + b = 18 \rightarrow b = 18 - a$

max $= ab$

$m = a(18 - a)$

$m = -a^2 + 18a$

when $a = 9$ $m = 81$ maximum

$\{9, 9\}$

14) $a + b = 20 \rightarrow b = 20 - a$

min $= a^2 + b^2$

$m = a^2 + (20 - a)^2$

$m = 2a^2 - 40a + 400$

when $a = 10$ $m = 200$

$\{200\}$

Exercise Set 7.4

1 a) $x = 6$

b) $(6, 5)$

2 a) $x = 3$

b) $(3, -4)$

3 a) $x = -5$

b) $(-5, 0)$

4 a) $x = -2$

b) $(-2, -1)$

5 a) $x = -3/2$

b) $(-1.5, 6.75)$

6 a) $x = 7$

b) $(7, 0)$

7 a) $x = 1.5$

b) $(1.5, 0)$

8 a) $x = -2.5$

b) $(-2.5, 0)$

9 a) $x = -1.5$

b) $(-1.5, 0)$

10 a) $x = -5/3$

b) $(-5/3, 0)$

11 a) $x = 3$

b) $(3, -17)$

12 a) $x = 1$

b) $(1, 2)$

13 a) $x = .5$

b) $(.5, -3.25)$

14 a) $x = -1.5$

b) $(-1.5, -11.25)$

15 a) $x = 1.5$

b) $(1.5, -2.5)$

16 a) $x = 1$

b) $(1, 9)$

17 a) $x = -1.75$

b) $(-1.75, -9.125)$

18 a) $x = 5/6$

b) $(5/6, -49/12)$

19 a) $x = 2$

b) $(2, -23)$

20 a) $x = 1$

b) $(1, 5)$

21 a) $x = .75$

b) $(.75, 2.125)$

22 a) $x = -1.25$

b) $(-1.25, 5.125)$

23) $y = ax^2 + bx + \frac{b^2}{4a}$

$y = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2})$

$y = a(x + \frac{b}{2a})^2$

$x = \frac{-b}{2a}$ axis of symmetry

24) $y = ax^2 + bx + c$

$y = ax^2 + bx + (\frac{b^2}{4a}) + c - (\frac{b^2}{4a})$

$y = a(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}) + (\frac{4ac}{4a} - \frac{b^2}{4a})$

$y = a(x + \frac{b}{2a})^2 + (\frac{4ac - b^2}{4a})$

Turning point $(\frac{-b}{2a}, \frac{4ac-b^2}{4a})$

Exercise Set 7.5

1) $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ is the combined form of

$x = \dfrac{-b + \sqrt{b^2-4ac}}{2a}$ or $x = \dfrac{-b - \sqrt{b^2-4ac}}{2a}$

2) When the discriminant equals zero the roots are equal.
When the discriminant is greater than zero the roots are unequal.

3) If the discriminant is negative the function does not intersect the x-axis.

4) a) Roots are rational
b) Roots are irrational

5)

Discriminant	Roots	Graph in relation to x-axis
If $b^2 - 4ac = 0$	real and equal	tangent
If $b^2 - 4ac < 0$	not real (complex conjugates)	doesn't intersect x-axis
If $b^2 - 4ac > 0$ and a perfect square	two rational roots real and unequal	intersects the x-axis in two distinct points
If $b^2 - 4ac > 0$ not a perfect square	two irrational roots real and unequal	

6) $p^2 - 4nq$ 7) $k = 9$

8) $(-\infty, -6) \cup (6, \infty)$ 9) $(9/4, \infty)$

10) $\{0.5, .25\}$ 11) $\{3, -0.5\}$

12) $\{-0.5, 1.3\}$ 13) $\{0.4, -1\}$

14) $\{-5/3\}$ 15) $\{-.25\}$

16) $\{-1.7, 0.4\}$ 17) $\{3.9, 0.1\}$

18) $\{8.2, 1.8\}$ 19) $\{1.5, -0.5\}$

20) $\{6.2, 1.8\}$ 21) $\{2.3, -0.6\}$

22) $\{0^\circ, 60^\circ, 120^\circ, 180^\circ, 360^\circ\}$ 23) $\{120^\circ, 240^\circ\}$

24) $\{45^\circ, 225^\circ, 126^\circ\ 50', 306^\circ 50'\}$

25) $\{210^\circ, 330^\circ\}$

26) $\{51^\circ 10', 107^\circ 10', 231^\circ 10, 287^\circ 10'\}$

27) $\{32^\circ 30', 126^\circ 20', 233^\circ 40', 327^\circ 30'\}$

28) $\{70^\circ 40', 169^\circ 10', 290^\circ 40, 349^\circ 10'\}$

29)

Trace	Display
after Step 7	$\frac{-b}{2a}$
after Step 9	$\frac{b^2}{4a^2}$
after Step 12	$\frac{c}{a}$
after Step 14	$\frac{\sqrt{b^2-4ac}}{2a}$
after Step 18	1st root
after Step 21	2nd root

30)

Step # →		01	02	03	04	05		06
T								
Z								
Y			$-b$	$-b$			$-\frac{b}{2}$	
X	(b)	$-b$	$-b$	2	$\frac{-b}{2}$ →		(a)	$\frac{-b}{2a}$
		CHS	ENT	2	÷	R/S		÷

	07	08	09		10	11	
T					$\frac{-b}{2a}$ →		$\frac{-b}{2a}$
Z				$-\frac{b}{2a}$	$\frac{b^2}{4a^2}$ →		$\frac{b^2}{4a^2}$
Y	$-\frac{b}{2a}$	$-\frac{b}{2a}$ →		$\frac{b^2}{4a^2}$	c →		c
X	$-\frac{b}{2a}$	$\frac{b^2}{4a^2}$ →		(c)	c →		(a)
	ENT	x^2	R/S		ENT	R/S	[illegible]

	12	13	14	15	16	17	18
T	$\frac{-b}{2a}$	$\frac{-b}{2a}$	$\frac{-b}{2a}$				
Z	$\frac{-b}{2a}$	$\frac{-b}{2a}$	$\frac{-b}{2a}$	$\frac{-b}{2a}$			
Y	$b^2/4a^2$	$\frac{-b}{2a}$	$\frac{-b}{2a}$	$\frac{-b}{2a}$		$\frac{-b}{2a}$	
X	$\frac{c}{a}$	$\frac{b^2-4ac}{4a^2}$	$\frac{\sqrt{b^2-4ac}}{2a}$	$\frac{-b+\sqrt{b^2-4ac}}{2a}$		$\frac{\sqrt{b^2-4ac}}{2a}$	$\frac{-b-\sqrt{b^2-4ac}}{2a}$
	$\div$	$-$	$\sqrt{x}$	$+$	R/S	$-$	$+$

31) If an error signal appeared, the discriminant is negative and the roots are complex.

32) $\{11\}$

33) $\{3, 6\}$

34) $\{3\}$

35) $\{6\}$

36) $\{4\}$

37) $\{4\}$

Exercise Set 7.6

1 a) sum: 8
product: 12

b) sum: -6
product: -4

c) sum: 6
product: -16

d) sum: 1
product: -1/3

2) -6

3) $-3x^2 + x + 4 = 0$

4) $x^2 - 4x - 5 = 0$

5) $p = 6$

6) $c = 12$ $\{3\}$

7) $b = 0$

8) $a = -1/2$

9) $c = 36$

10) $b = 3$

11) sum: p
product: p
They are equal.

12) $y = -5$

13) $b = -8$ or $b = 13$

14 a) $x^2 + \frac{7}{3}x - 2 = 0 \rightarrow 3x^2 + 7x - 6 = 0$

b) $x^2 - 12 = 0$

c) $x^2 + 5x + 3.25 = 0 \rightarrow 4x^2 + 20x + 13 = 0$

d) $x^2 + \frac{1}{7}x - \frac{12}{49} = 0 \rightarrow 49x^2 + 7x - 12 = 0$

15) sum: -4

$\frac{-b}{a} = \frac{4}{-1} = -4$

product; $(-2 + \sqrt{5})(-2 - \sqrt{5}) = -1$

$\frac{c}{a} = -1$

16)

	axis of symmetry	vertex
a)	$x = 1$	$(1, -4)$
b)	$x = 3$	$(3, 2)$
c)	$x = 4$	$(4, 0)$
d)	$y = 0$	$(-16, 0)$
e)	$y = -5/4$	$(-1/8, -5/4)$
f)	$y = 5/2$	$(-9/4, 5/2)$
g)	$x = \frac{-b}{2a}$	$\left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$
h)	$y = \frac{-b}{2a}$	$\left(\frac{4ac-b^2}{4a}, \frac{-b}{2a}\right)$

Exercise Set 7.7

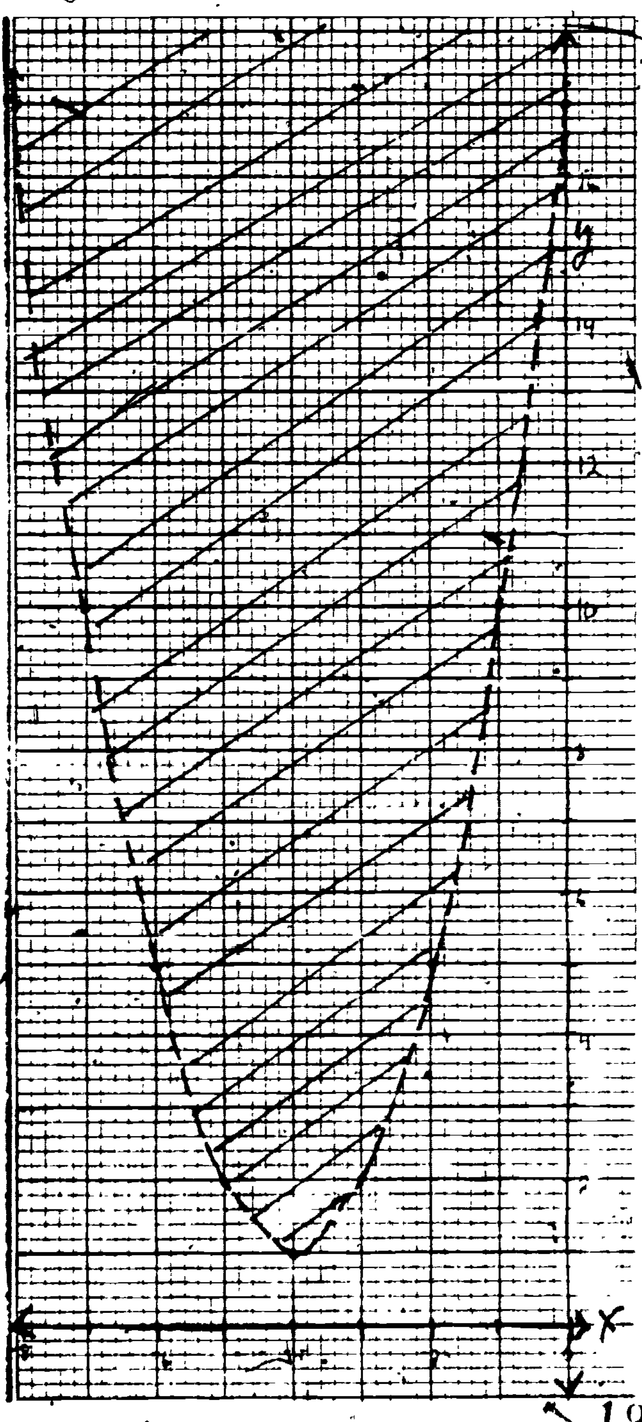
1) $0 \le x \le 2$

2) all x

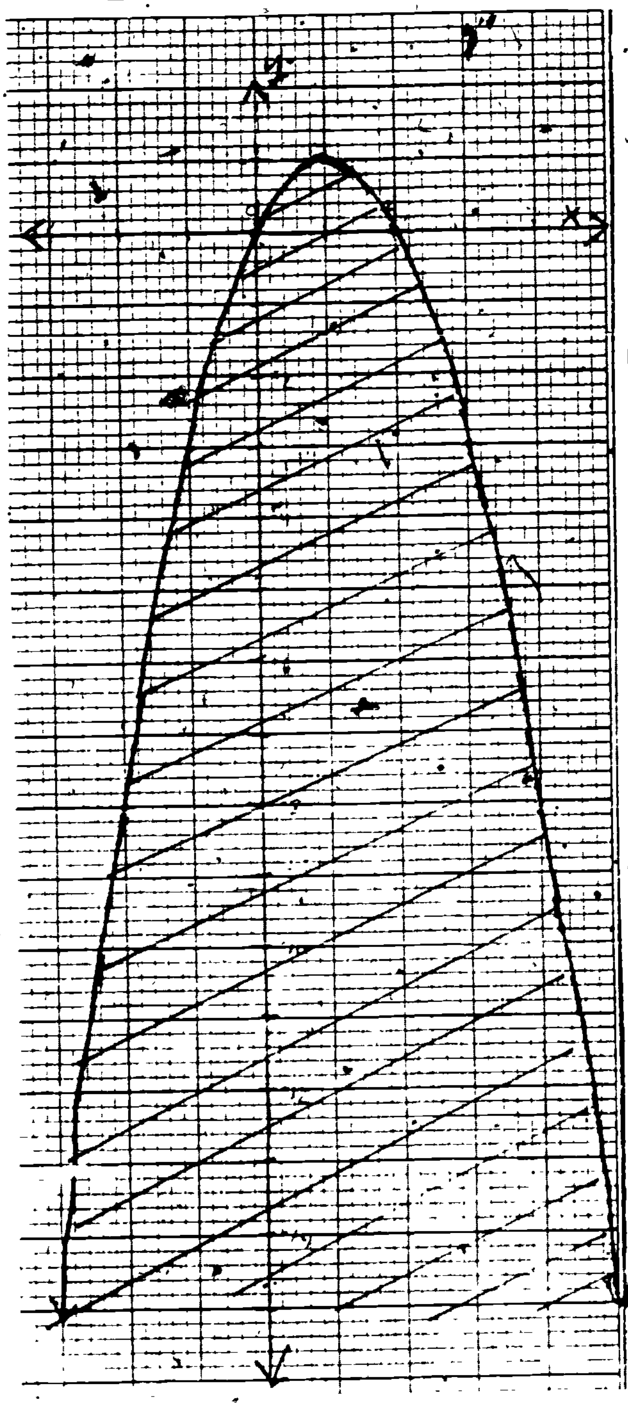
3) -8

4) $x < 0$ or $x > 2$

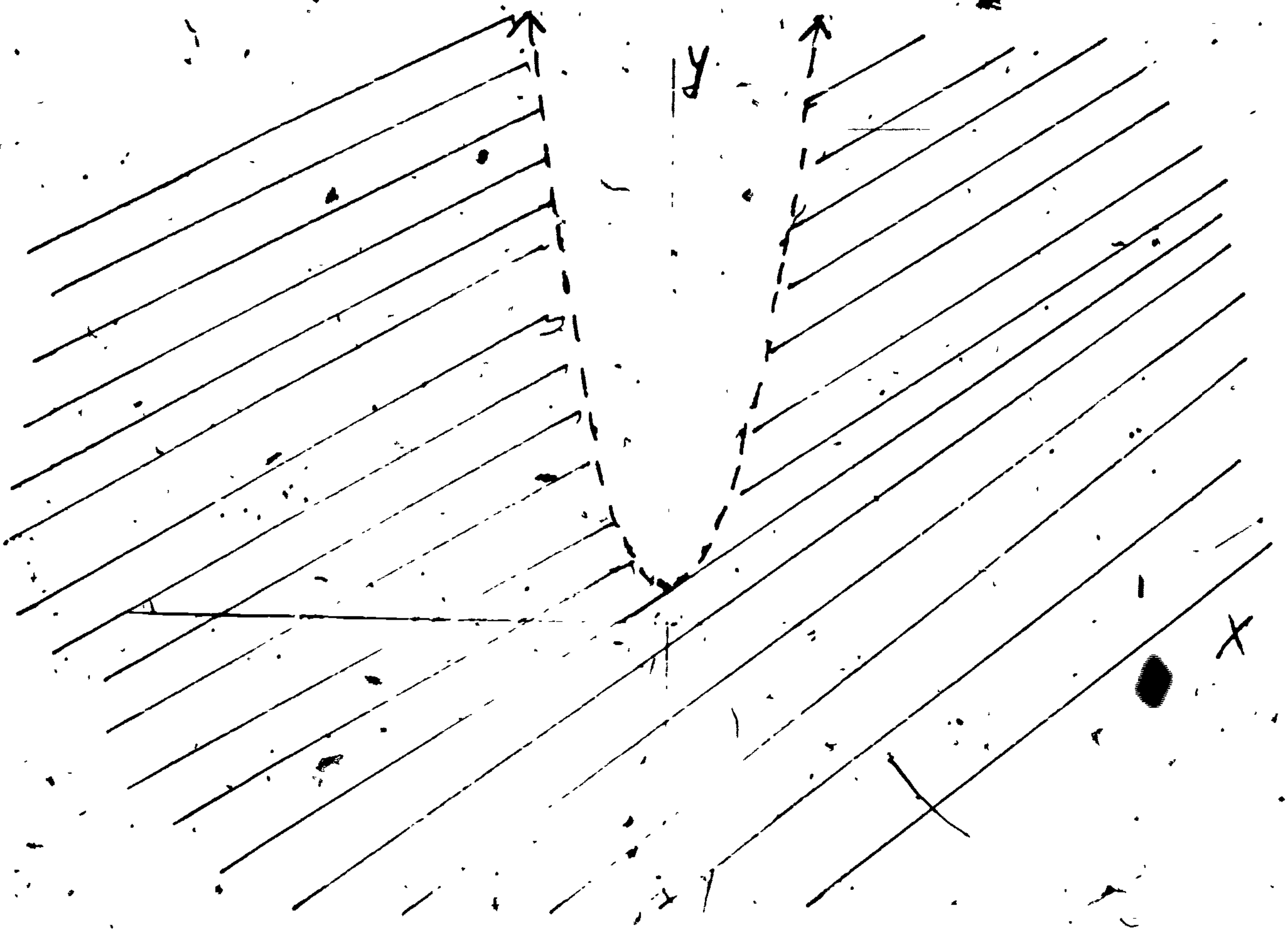
5) $y > x^2 + 8x + 17$



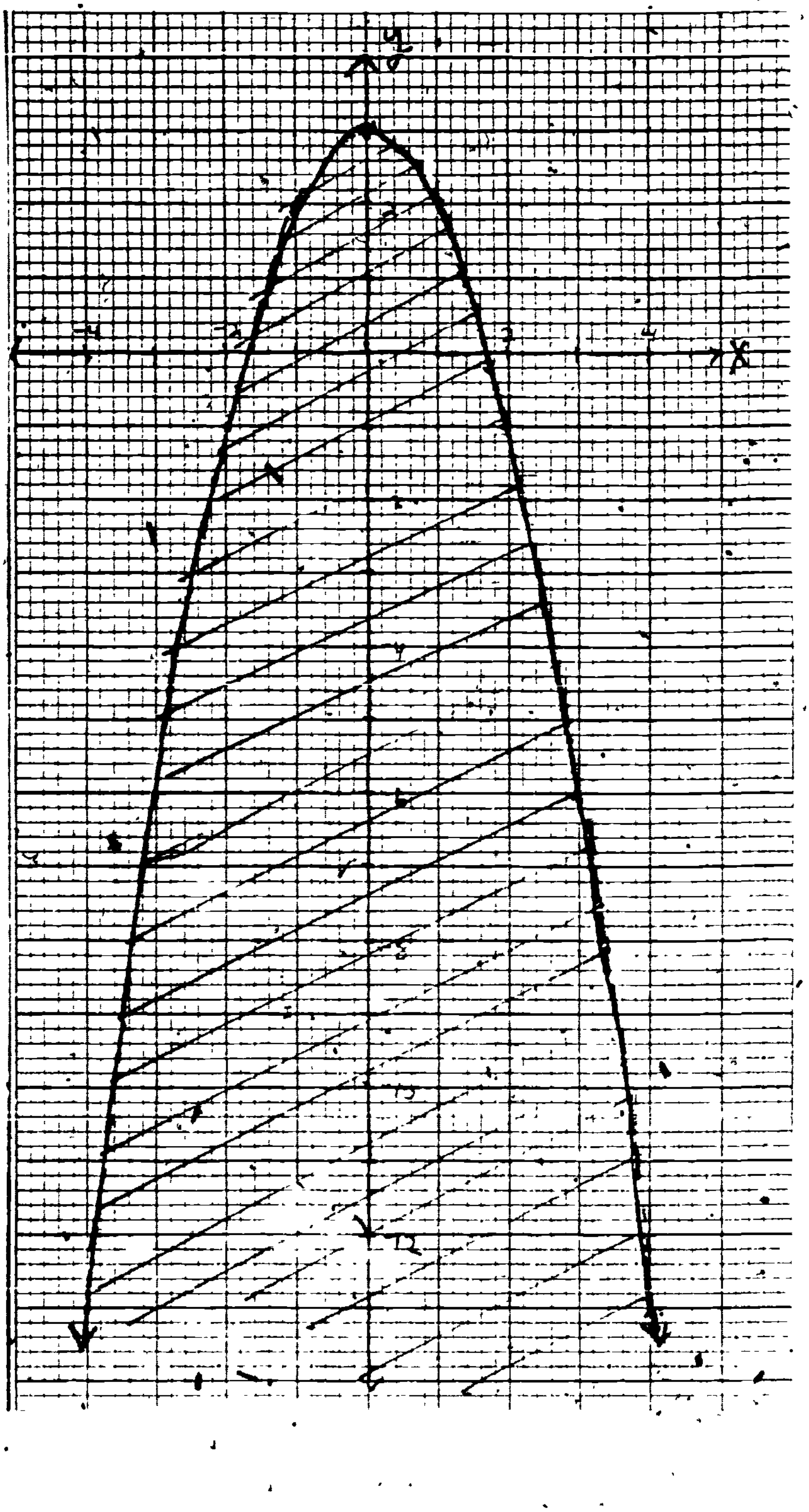
6) $y \le -x^2 + 2x$



7) $y < x^2 + 1$

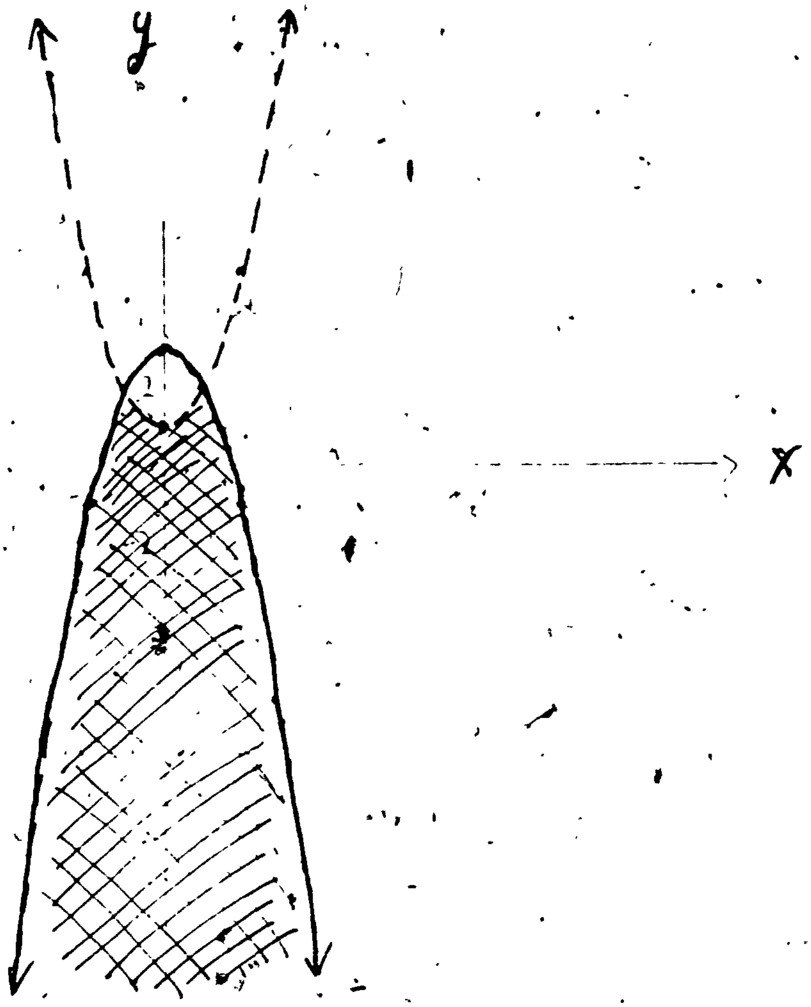


8) $y \leq -x^2 + 3$



9) $y < x^2 + 1$ and

$y \leq -x^2 + 3$



Solutions to Chapter 7 TEST

1) 3

2) (2)

3) (2)

4) (2)

5) (1)

6) (4)

7) 2

8) 120°

9) (3)

10) [illegible]

11 a) $\{1.7, -.4\}$

b) II, III

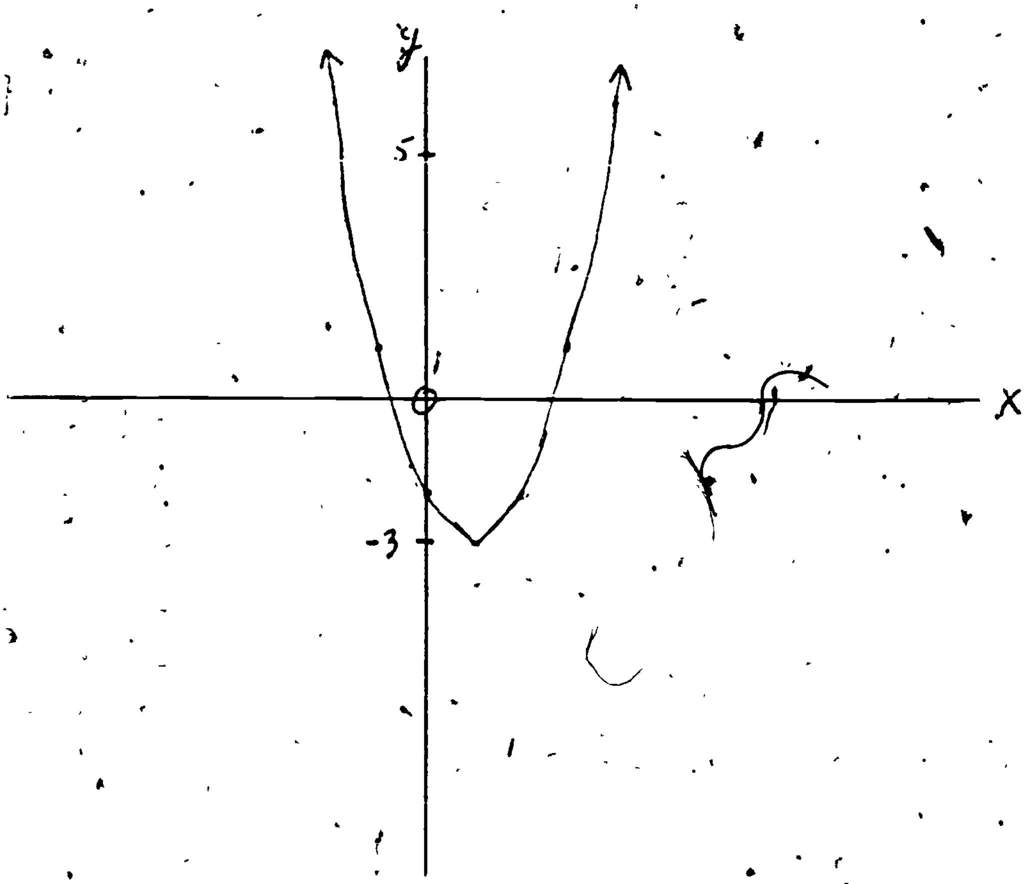
12) a) $\{1.2, -3.2\}$

b) 107°

13) (see next page)

13) b) $\{2; 7, -7\}$

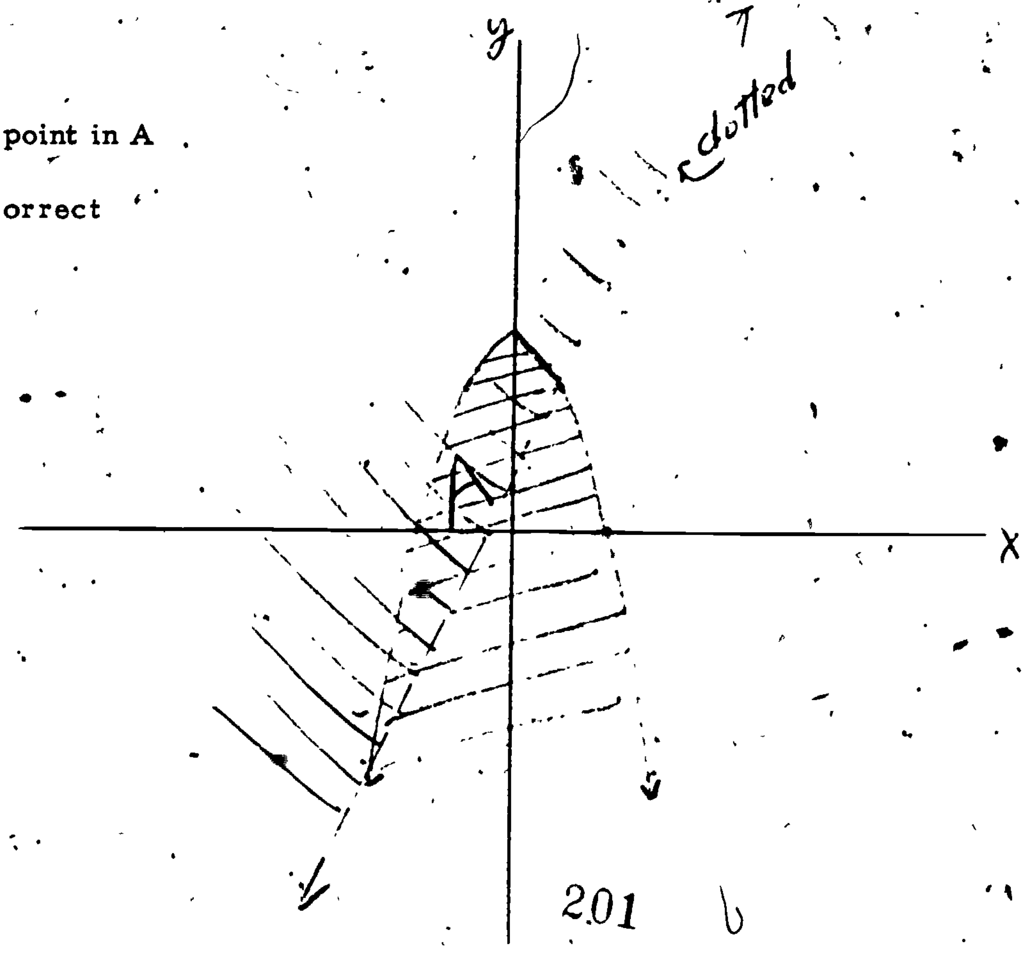
c) -3



14)

b) any point in A

is correct



Exercise Set 8.1

1) 104, 107, 110, 113, 116

2) Common difference of 3 between terms

3) 3, 6, 12, 24, 48

4) r = 2

5) 384, 768, 1536

6) common ratio of 2 between terms

7) 3

8) 48

9) NO

10) NO

11) 4, 5, 7, 11, 19

12) Neither

13) a) 6, 9, 15, 27, 51
    b) 2, 1, -1, -5, -13
    c) 3, 3, 3, 3, 3

14) Consecutive terms remain constant (no change)

15) a) neither
    b) neither
    c) arithmetic d = 0
       geometric r = 1

16) Various answers
    a, 3a+5, 3(3a+5) +5, . . .

    $(1)^3, (2)^3, (3)^3 \ldots$

    $(1 + \frac{1}{1}), (2 + \frac{1}{2}), (3 + \frac{1}{3}), \ldots$

Exercise Set 8.2

1) 9

2) 1

3) 20

4) 1.25

5) 9

6) 1

7) 20

8) 1.25

9)

HP-33E	TI-57
STO 1	STO 1
STO 2	STO 2
PAUSE	Lbl 1
PAUSE	PAUSE
RCL 1	PAUSE
2	RCL 1
X	X
STO 1	2
STO + 2	=
RCL 2	STO 1
GTO 03	SUM 2
	RCL 2
	GTO 1

S(G) = {~~3, 9,~~ 21, 45, 93, 189 ...}

10) S(B) = {4, 9, 16, 27, 46, 81 ...}

11) S(c) = {3, 6, 9, 12, 15, 18, ...}

12) Arithmetic; common difference of 3

13) 24

14) 21

15) 16

16) 9

Exercise Set 8.3

1) P 37

Q 8

R -6

S 6.75

T 10

2) $S_{10}(P) = 5(1+37) = 190$

$S_{10}(Q) = 5(-19+8) = -55$

$S_{10}(R) = 5(12+(-6)) = 30$

$S_{10}(S) = 5(0+6.75) = 33.75$

$S_{10}(T) = 5(1+10) = 55$

3) $S_{15}(S) = \frac{15}{2}[0 + 14(.75)] = 78.75$

4) $S_{20}(Q) = \frac{20}{2}[-38 + 19(3)] = 190$

5) $S_{30}(P) = \frac{30}{2}[2 + 29(4)] \doteq 1770$

6) $S_{30}(R) = \frac{30}{2}[24 + 29(-2)] = -510$

7) n

8) ℓ is negative in R, therefore after a sufficient number of terms the sum will be negative. (14 terms)

d is positive in Q, likewise after a sufficient number of terms the series and sum will be positive (14 terms again)

9) $S_n(T) = \frac{n}{2}(1 + n)$

10) 30th term is 92
approx. 75 - 80 sec. with HP33.

11) Time should be less than in #10. The calculator takes longer in #10 because it must generate each term of the series.

12) Calculator: $S_{30}(A) = 1455$ 85-90 sec. by HP33

Formula: $S_{30}(A) = \frac{30}{2}(5 + 92) = 1455$ approx 20 sec

13)
$$\begin{aligned} S &= 1 + 2 + 3 + \dots + 98 + 99 + 100 \\ S &= 100 + 99 + 98 + \dots + 3 + 2 + 1 \\ \hline 2S &= \underbrace{101 + 101 + 101 + \dots + 101 + 101 + 101}_{100 \quad 101\text{'s}} \end{aligned}$$

$\therefore 2S = 100 \cdot 101$. $\therefore S = 50 \cdot 101 = 5050$

14) Using the formula (*)

$S_1 = a$

$S_2 = 2a + d$

$S_3 = 3a + 3d$

etc.

Exercise Set 8.4

1) $r = .2,\ a_{10} = 50\cdot(.2)^9 = .0000256$

2) $d = -.12,\ a_{41} = 1.2 + 40(-.12)$

$a_{41} = -3.6$

3) $r = .4,\ a_7 = 1\cdot(.4)^6 = .004096$

4) $r = -\frac{1}{3},\ a_{12} = 18\cdot(-\frac{1}{3})^{11} = -.0001016$ or $\frac{2}{3^9}$

5) $r = \sin\theta,\ a_8 = \sin^7\theta$

6) $r = \frac{1}{\tan\theta},\ a_{30} = \tan\theta\,(\frac{1}{\tan\theta})^{29} = \frac{1}{\tan^{28}\theta}$ or $\mathrm{Ctn}^{28}\theta$

7) $a_8 = 29$

8) $r = \frac{1}{4},\ a_{12} = \frac{3}{16}\cdot(\frac{1}{4})^{11} = \frac{3}{4^{13}}$ or .000000045

9) $r = \frac{1}{2},\ S_{15} = \dfrac{40 - 40\cdot\left(\frac{1}{2}\right)^{15}}{1 - \frac{1}{2}} \doteq 79.9976$

10) $r = \frac{1}{2},\ S_{20} = \dfrac{1 - 1\cdot\frac{1}{2}^{20}}{1 - \frac{1}{2}} \doteq (\frac{1}{2})^{19} \doteq .000001907$

11) $r = -\frac{1}{5},\ S_8 = \dfrac{1 - (-\frac{1}{5})^8}{1 + \frac{1}{5}} \doteq .8333$

12) $d = -1.5,\ S_{12} = \frac{12}{2}[8 + 11(-1.5)] = -51$

13) $r = \frac{1}{3},\ S_{13} = \dfrac{-18 - (-18)(\frac{1}{3})^{13}}{1 - \frac{1}{3}} = -27$

14) $d = .25,\ S_{16} = \frac{16}{2}[6 + 15(.25)] = 78$

15) $r = 2,\ S_n = \dfrac{2\cdot 16384 - 1}{2 - 1} = 32767$

16) $r = \frac{1}{2}$, $S_n = \dfrac{500 - \frac{1}{2}(3.90625)}{1 - \frac{1}{2}} = 996.09375$

17) $a_{20} = 1,572,864$ approx. 5 sec. by HP-33 program

18) $a_{20} = 3 \cdot 2^{19} = 1,572,864$ - less than 20 sec. by formula on HP-33

19) program: $2.621440 \cdot 10^{-13}$, over 1 min. by HP 33

formula: $\ell \doteq 5 \cdot (\frac{1}{5})^{19} \doteq 5^{-18} \doteq 2.6214 \cdot 10^{-13}$. less than 1 min. by HP 33

20) $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

$S_n = a(1 + r + r^2 + \ldots + r^{n-1})$

$S_n = a\left(\dfrac{1 - r^n}{1 - r}\right)$

$S_n = \dfrac{a - ar^n}{1 - r}$

$$
\begin{array}{r}
1 + r + r^2 + r^3 \\
1 - r \overline{)\,1 + 0r + 0r^2 + 0r^3 - r^4} \\
\underline{1 - r} \qquad\qquad\qquad \\
r + 0r^2 \qquad\quad \\
\underline{r - r^2} \qquad\quad \\
r^2 + 0r^3 \quad \\
\underline{r^2 - r^3} \quad \\
r^3 - r^4 \\
\underline{r^3 - r^4}
\end{array}
$$

21) $\{0, 1, 2, 3, \ldots\}$
arithmetic

22) $\{1, 0, -1, -2, \ldots\}$
arithmetic

23) $\boxed{\log_b \ell} = \log_b a + (n-1)\log_b r$

$\boxed{a_n} \quad\; = \quad a_1 + (n-1)\,d$

In a geometric sequence, the logarithms of each term will form an arithmetic sequence with the common difference being equal to the logarithm of the geometric sequence's common ratio.

24) $\{4^1, 4^3, 4^5, 4^7, \ldots\}$ Geometric $r = 4^2$

25) $\{x^a, x^{a+d}, x^{a+2d}, \ldots\}$ $\dfrac{x^{a+d}}{x^a} = \dfrac{x^{a+2d}}{x^{a+d}} = x^d$

common ratio is x^d

26) $r = x^d$

Exercise Set 8.5

1) $a_{10} = 29$ 2) $a_{12} = 35$

3) $a_{30} = 89$ 4) $a_{40} = 119$ approx 2 min, 4 sec. on HP-33E

5) approx 3.1 sec. on HP-33E

6) $a_{10} = 1536$ 7) $a_{15} = 49,152$

8) $a_{30} = 1610612736$ 9) $a_{40} \doteq 1.649267 \times 10^{12}$ approx. 2+ min. on HP-33E

10) 17^{th} 11) 26

12)

counter

	HP-33E		TI-57
01	STO 0	00	STO 0
02	STO 1	01	STO 1
03	0	02	0
04	STO 2	03	STO 2
05	1	04	Lbl 1
06	STO + 2	05	1
07	RCL 2	06	SUM 2
08	PAUSE	07	RCL 2
09	RCL 0	08	PAUSE
10	PAUSE	09	RCL 0
11	RCL 1	10	PAUSE
12	3	11	RCL 1
13	+	12	+
14	STO 1	13	3
15	STO + 0	14	=
16	GTO 05	15	STO 1
	RTN	16	SUM 0
	5	17	GTO 1
	R/S		LRN, RST
			5, R/S

$\boxed{S_{10}(X) = 185}$

13) In both programs replace steps 12 and 13 with 2, X. In RUN position 3, R/S $S_{10}(Y) = 3069$

Exercise Set 8.6

1) $x \leq y$?   2) $x < y$?   3) $x \geq 0$?

4) HP 33 $x > y$: 21 is displayed before stopping
   TI 57 $x \geq t$: No change

5)

HP 33E

PRGM		RUN
01	1	RTN
		(a)
02	-	ENT
03	y^x	(r)
		ENT
04	X	(n)
		R/S

The eight term is
.0234375

TI-57

LRN		
00	-	LRN
01	1	RST
02	=	(a)
03	STO 3	STO 1
04	RCL 2	(r)
05	y^x	STO 2
06	RCL 3	(n)
07	=	R/S
08	x	
09	RCL 1	
10	=	
11	R/S	
12	RST	

6)

PRGM		RUN
01	+	RTN
		50 (n)
02	2	ENT
03	÷	201 (a)
		ENT
04	X	299 (ℓ)
		R/S

LRN		
00	+	LRN, RST
01	RCL 2	50 (n)
02	=	STO 1
03	X	201 (a)
04	RCL 1	STO 2
05	=	299 (ℓ)
06	÷	
07	2	
08	=	
09	R/S	
10	RST	

There are 50 add numbers between 200 and 300.

The sum of the odd numbers between 200 and 300 is 12,500.

7)

HP 33

PRGM		RUN
01	STO 0	RTN
02	STO 3	10
03	0	STO 1
04	STO 2	3
05	1	R/S
06	STO + 2	
07	RCL 1	
08	RCL 2	
09	x = y	
10	GTO 20	
11	PAUSE	
12	RCL 3	
13	PAUSE	
14	RCL 0	
15	2	
16	÷	
17	STO 0	
18	STO + 3	
19	GTO 05	
20	PAUSE	
21	RCL 3	
22	R/S	

TI 57

LRN		
00	STO 0	LRN, RST
01	STO 3	10
02	0	x ⇌ t
03	STO 2	3
04	Lbl 1	R/S
05	1	
06	SUM 2	
07	RCL 2	
08	x = t	
09	GTO 2	
10	PAUSE	
11	RCL 3	
12	PAUSE	
13	RCL 0	
14	÷	
15	2	
16	=	
17	STO 0	
18	SUM 3	
19	GTO 1	
20	Lbl 2	
21	PAUSE	
22	RCL 3	
23	R/S	
24	RST	

8) Store r in R_4

Replace steps 15 and 16 in HP-33 with RCL 4, X. in RUN position Store r in R_4

in TI-57 replace steps 13 and 14 with RCL 4, X.

9) Experience hunger, pain, elation, etc.

1) -32

2) 70

3) -5

4) .98304

5) 6300

6) 15.984375

7) -530

8) $152.\overline{518}$

9) \$9,737,418.23 more by 1¢, 2¢, etc.

10) 20,100

11) 1,073,741.824" or approx. 16.9 miles

12) 9.6