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ABSTRACT

The concept that it is important for students to learn how to read the language of mathematics is promoted. Most mathematics teachers neither have the knowledge nor feel the responsibility to develop reading skills in their students. The materials in this document are viewed to be of sufficient variety and potential for making important improvements in mathematics classrooms. The teaching ideas are based on the well-accepted notion that learning results from interest in a subject. Providing that interest, through readiness, is considered the key. Individual chapter titles are: (1) The Reading Phase of Mathematics; (2) The Instructional Framework; (3) The Structured Overview; (4) The Development of Mathematics Vocabulary; (5) The Dilemma of Word Problems; (6) The Readability of Mathematics Materials; and (7) Concluding Remarks. Additional Mathematics/Reading Resources - Teacher Focus; Recreational/Supplementary Resources - Student Focus; and References are found at the conclusion of the document. (MP)

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FOREWORD

Most mathematics teachers neither have the knowledge nor feel the responsibility to develop reading skills in their students. On the other hand, they understand that much of their responsibility is to serve as mediator between what students find incomprehensible in mathematics instructional materials and what they hope students will learn. The central thesis of this monograph is that it is important that students learn how to read the language of mathematics. To avoid the task either by mediating it or by choosing instructional materials that contain very little reading or by any other technique that renders the student continually dependent upon others is fundamentally wrong.

This thesis, I confess, is one that I have been preaching for many years, and the fact that the ideas here are so well stated further tends to bias me in favor of the monograph. The material presented here is of great potential value to mathematics teachers. I believe that those who study the monograph and apply its ideas will become better mathematics teachers. The ideas for structuring mathematics lessons in such a way that reading comprehension is enhanced are entirely workable. The mathematics teacher who feels completely incompetent as a reading teacher should feel quite at home in attempting to implement many of the suggestions here. I am delighted that Cyrus Smith and Henry Kepner have written this book, and I hope that many mathematics teachers will benefit by studying it.

The materials in the monograph are of sufficient variety and potential for making important improvements in mathematics classrooms. Furthermore, the references offer ample additional material. The style is clear, simple, straightforward, and comprehensible. It neither talks down to the mathematics teacher who feels insecure in teaching reading, nor does it employ jargon.

Individual teachers would do well to own copies of this monograph. It would also be a valuable addition to the libraries of teacher education institutions and to the professional collections of schools.

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PREFACE

Just as reading is more than pronouncing symbols and attaching meaning and understanding to those symbols, mathematics entails more than a mechanical or manipulative approach to numbers. Mathematical competence requires an understanding of symbols in order to master two basic processes—classification and the study of relationships. Therefore any approach to improving reading skills in mathematics must focus primarily on comprehension, on understanding abstract ideas in order to improve the study of sets and functions.

Traditionally, discussions about improving reading in a mathematics classroom have dealt almost exclusively with either word problems or specific vocabulary. Although the authors of this text, Cyrus F. Smith, Jr., and Henry S. Kepner, Jr. deal with these traditional areas, they employ a broader focus. They offer the reader a bright new perspective, combining the insight of a former high school reading teacher with that of a mathematics teacher. Thus they confront the problems identified by both language and mathematics professionals and then provide a methodology and teaching strategy to help overcome these stumbling blocks.

This book's teaching ideas are based on the well-accepted notion that learning results from interest in a subject. Providing that interest, through readiness, is the key. Such an approach involves rethinking and evaluating the curriculum. It also identifies reasons for lack of student achievement as well as appropriate remedies. It begins by not assuming what students know, but by assessing what they know. And it provides for a range of abilities.

One of the more innovative features of the text is its approach to evaluating mathematics materials. Rather than assessing readability by such factors as length of sentence, number of syllables, and frequency of words, the authors propose a cloze procedure and a textbook survey, and they provide excellent examples of these instruments. In addition, they suggest specific activities such as the structured overview as either an advanced or postgraphic organizer.

In short, this text should provide mathematics teachers with a new and enlightened orientation that will help them develop individual strategies to assist students in the reading phase of mathematics instruction.

Alfred J. Ciani
Series Editor

1. THE READING PHASE OF MATHEMATICS

There are many operational definitions of reading. Most center on the skills and abilities needed to translate and understand ordinary prose. That is, the reader applies pronunciation strategies to word symbols which guide thought and allow for the attachment of meaning. Herber (1978, p. 9) gives a definition of reading that is appropriate for reading mathematics. He states that ". . . reading is a thinking process which includes decoding symbols, interpreting the meaning of symbols, and applying the ideas derived from the symbols." While these factors are necessary conditions for reading mathematical language, they are by no means sufficient. That is, each individual learns mathematics in a variety of ways that are based upon skills and background.

Initially, many concepts are developed through observation and exploration of physical experiences. As students learn and mature, they become capable of handling mathematical ideas through the manipulation of highly abstract symbols and notation. It is these efficient, but abstract, symbols and notation that present special concern to the mathematics teacher. The need to decode words and symbols, to associate meaning with them, and to use meanings can be exemplified as each reader tries to grasp the following statement:

$$\forall \epsilon > 0, \exists \delta > 0 \ni |f(x) - f(a)| < \epsilon$$
$$\text{whenever } |x - a| < \delta$$

In ordinary language, this mathematical sentence states that a function f(x) is *continuous* at the point x = a. Not only is the translation from written symbols to spoken words difficult, it also takes considerable mathematical background to grasp the concept that is expressed.

Undoubtedly, some readers experienced considerable difficulty in the pronunciation as well as in the meaning of the statement listed above. Students at all levels of study in mathematics face similar problems when reading mathematical language. Because mathematical language is so precise and frequently contains high concept density, a few words and symbols often convey a very complex idea. In this sense, mathematical statements resemble logical expressions more so than ordinary prose. Consider the complexity of "iff" representing the exceedingly involved concept of "if and only if." Or, consider the confusion that many students have in pre-algebra and early algebra instruction with the varied interpretations of "−." Initially, "−" represents the binary operation in subtraction; *i.e.*, 5 − 3 means 5 subtract 3. With the introduction of integers, −7 represents a particular seven, namely "negative 7." In this context, the horizontal bar describes which 7 is being considered. Finally, −x refers to the "opposite of x" or the "additive inverse of x." Here −x identifies the monary operation, *i.e.*, an operation of a single number. If students are confused about the exact meaning of a word, symbol, or number, the solution to a sentence like "What is the value of −x when x = −7?" becomes an exercise in frustration.

The inability of students to read mathematical notation correctly was recently demonstrated (NAEP, *Changes*, 1979, p. 53). As part of this assessment students were given a substitution exercise in two forms. In one form, the exercise asked students to find the value of an expression like "x − 2 when x was a small whole number."

Seventy percent of 13-year olds and 90 percent of 17-year olds responded correctly. When the exercise used standard function notation ($f(n) = n - 2$), student performance for 13-year olds dropped to 28 percent correct; 17-year olds dropped to 42 percent; and, of those 17-year olds taking Algebra II, less than two-thirds responded correctly.

Besides differing in symbolic form from ordinary prose, mathematics also uses vocabulary in very special ways. This is done to accommodate the unique terms and symbols of mathematical language. For example, "integer," "perpendicular," and "sine" have specific mathematical meanings; and, $|x - a|$, $\sqrt{9}$, and y^2 represent symbols with mathematical interpretations. Words like "mean," "base," and "revolution" have meanings in common usage which are different in mathematical usage. Further, some words will have different meanings within the mathematics curriculum. The words "base" and "median" have different connotations in statistical, geometric, and arithmetic contexts. Therefore, the teacher and the student must be made aware of the differences in mathematical language and ordinary language.

In the mathematics class the teacher should insure that the components of reading are used and reinforced. In assuring proper decoding, or sounding, the teacher can preview key vocabulary before starting a new unit. Pronunciation and meaning can be improved through recognition of similar terms, prefixes, suffixes, word stems or the examination of a word or symbol in context.

The skill of interpreting the meaning of symbols is also crucial to mathematics. While teachers are often aware of the new vocabulary for a chapter, teachers should check student interpretation of the many other words and symbols incorporated in that topic. Mathematical concepts are developed in a spiral curriculum in which concepts, words, and symbols are developed and practiced, then followed by a period of disuse. When returning to previously learned words and symbols, teachers should verify that students know their meanings. For instance, after completing a junior high school unit in geometry, instruction typically returns to work on number concepts. Before initiating this instruction, the teacher might include a review of the terms "fraction" and "decimal." Some questions that the teacher should consider are:

- What is the student's—not the teacher's—definition for these words?
- Are there numbers described by both terms?
- Are there numbers described by one term and not the other?

In other words, the anticipation of situations which identify and teach the meaning of key terms and symbols is valuable to the learning that will be developed.

This monograph will identify several of the major problems students have in reading mathematics. And this monograph will provide examples of student difficulties and suggest classroom activities for developing vocabulary, structuring content, solving word problems, and improving the readability of materials. These strategies are not panaceas or formulas which guarantee student success. Rather, they should be viewed as instructional examples for mathematics teachers in grades 5 through 12. Their purpose is for more effective and efficient instruction that will lead to student independence in the reading phase of mathematics.

2. THE INSTRUCTIONAL FRAMEWORK

The instructional framework is a useful strategy for guiding students through an assigned reading in a subject matter classroom. Suggested by Herber (1970) and refined by Earle (1976) and Herber (1978), the instructional framework is dependent upon the subject matter teacher's analysis of the reading assignment in terms of both content and process. Content analysis is the selection and organization of information we wish to convey to students. Process analysis is the development of an instructional sequence and the inclusion of appropriate learning activities that provide the students access to the content. The combination of the content and process analyses forms the instructional framework. The result of this melding is a teacher-designed and -controlled mechanism that utilizes a variety of practical teaching strategies and techniques. A common offshoot of its use is more effective and efficient teaching. The purpose of this chapter is to discuss the instructional framework.

CONTENT ANALYSIS

The teacher begins content analysis (Earle, 1976) by examining the instructional materials used for student learning. The teacher should look for those ideas, concepts, or understandings suitable for development in the context of a lesson. The lesson could consist of teaching for one day, several days, or several weeks. Next, the ideas, concepts, or understandings are transformed into lesson statements. Some teachers may elect to write these lesson statements as behavioral objectives, others may feel more comfortable in writing simple declarative sentences. While the formats will vary from teacher to-teacher, the lesson statements should be written in language that is clear and precise. Listed below are four lesson statements which could be developed in the context of a one-week unit on quadrilaterals:

1. The median of a trapezoid (the segment joining the midpoints of the nonparallel sides) is parallel to the bases and its measure is one half the sum of the measures of the bases.
2. The diagonals of a parallelogram bisect each other.
3. If one angle of a parallelogram is a right angle, the parallelogram is a rectangle.
4. Either diagonal of a parallelogram separates the parallelogram into two congruent triangles.

An examination of these lesson statements shows them to be ordered randomly rather than positioned in the most logical sequence of lesson development. Further, student learning of the major understandings expressed in these statements requires the use of precise, competent language. Therefore, the teacher should rank the lesson statements in the order of their importance and list the vocabulary essential to the understanding of each. An example of these refinements follows:

1. Either diagonal of a parallelogram separates the parallelogram into two congruent triangles.

   (Vocabulary: diagonal, parallelogram, congruent)

2. The diagonals of a parallelogram bisect each other.
   (Vocabulary: diagonals, bisect)
3. If one angle of a parallelogram is a right angle, the parallelogram is a rectangle.
   (Vocabulary: right angle, rectangle)
4. The median of a trapezoid (the segment joining the midpoints of the nonparallel sides) is parallel to the bases and its measure is one half the sum of the measures of the bases.
   (Vocabulary: median, trapezoid, segment, measure, base).

The initial content analysis for the one-week unit on quadrilaterals produced many ideas, concepts, and understandings suitable for student study. However, as all experienced teachers know, time demands, variations in student abilities, curriculum expectations, and the like are such that not everything can be taught. Therefore, the lesson statements presented above are by no means comprehensive but were selected from a larger pool of relationships available for study. Further, the identification of the major understandings contained in each lesson statement was influenced by the teacher's own knowledge and by the overall goals of the curriculum. Similarly, it is important to identify the vocabulary essential for student understanding. This vocabulary is not limited to unfamiliar, difficult terminology but rather is representative of words and terms important to student learning. The vocabulary, as determined by the teacher, is essential to communicate the major understandings contained in the lesson statements. Content analysis, then, allows the teacher the necessary flexibility and efficiency to plan an instructional unit. The anticipation of teaching strategies and the selection of instructional activities can now be developed.

PROCESS ANALYSIS

Once the major understandings are identified and prioritized and the essential vocabulary listed, the teacher begins to formulate an instructional sequence for the lesson. Process analysis typically results in a three-part outline. The parts include a lesson introduction, an assimilation component, and follow-up activities. The introduction consists of those activities and strategies which prepare students for the lesson. In the assimilation component, the teacher provides the instruction to the major understandings selected in content analysis. The follow-up component anticipates activities which help students reinforce, internalize, or extend the major understandings.

Herber (1978) lists seven items in process analysis which should be considered in developing the outline. These are:

1. Selecting appropriate motivational activities
2. Reviewing pertinent background information
3. Setting purposes for the lesson
4. Giving directions for reading
5. Teaching essential vocabulary

6. Providing sufficient guidance for assimilation of learning
7. Anticipating appropriate activities which develop student independence.

Typically, items one through five are developed in the introduction, item six is developed within the assimilation component, and item seven is developed as the teacher selects follow-up activities. For some lessons, the seven components may all be used; in other lessons some may be combined or eliminated. The completed outline synthesizes the content/process analyses into an instructional framework. A comprehensive, general listing follows.

Instructional Framework

I. Introductory Activities—student motivation, preparation, and direction

A. Selecting appropriate motivational activities
1. Inclusion of teacher dominated or student dominated activities
2. Use of a media presentation

B. Reviewing pertinent background information
1. Teacher-led discussion of prior learnings
2. Student awareness of learning as related to real-life situations
3. Discussion of important concepts related to the lesson

C. Setting purposes for the lesson
1. Discuss with students the types of problems they will be solving
2. Give students a list of questions to answers as they do the lesson
3. Identify the types of follow-up activities students will perform

D. Giving directions for reading
1. Discuss strategies useful in reading word problems
2. Identify key sentences or paragraphs in the text
3. Eliminate unnecessary reading passages
4. Set reasonable time limits for reading, problem solving, and follow-up activities
5. Discuss important tables, diagrams, charts needed during the lesson

E. Teaching essential vocabulary
1. List important terminology (words and symbols) on the chalkboard
2. Select representative words and symbols which will be taught for meaning or pronunciation or both
3. Pronounce remaining words and symbols for students

II. Assimilation Activities—instruction in the major understandings

A. Lecture style discussion

B. Silent reading

C. Supplementary instruction
   1. media presentation

D. Supervised study

E. Guided discovery lesson

III. Follow-Up Activities—activities intended to extend, internalize, or reinforce student learning.

A. Small group discussions

B. Teacher-made content tests

C. Vocabulary games or puzzles

D. Mathematical project (individual or group)

E. Additional teaching in the meaning or pronunciation of words and symbols

F. Demonstrations of the mathematical principle being studied in different contexts

G. Written assignments

H. Model construction

I. Additional practice in reading word problems

J. Additional instruction in strategies useful in solving word problems

K. Discussion of related student experiences

L. Additional reading or study in supplementary textbooks

Our purpose is to present a comprehensive list of activities. Teachers should select activities for instruction based upon their perceptions of the material to be learned and the students who will learn it. Many of these activities can enhance what students learn when they are reading mathematics. Teachers who have been frustrated when students fail to read would do well to reconsider the directions they give when making an assignment. The authors of this monograph urge teachers not to say, "Go read," but to give purpose and direction to the assignment.

The chapters wh[illegible]h follow will present teaching ideas, suggestions, and strategies that fit within th[illegible]t of an instructional framework. Teachers are urged to experiment with [illegible]vities and to modify them through use and judgment. These activities alone will not remedy the problems students have in reading mathematics. Rather, they are intended to complement sound mathematics instruction.

3. THE STRUCTURED OVERVIEW

Educators and learning theorists have long known that abstract subject matter is more difficult to understand and learn than concrete subject matter. One reason for this is that abstract material is more difficult to relate or subsume within one's cognitive framework. Ausubel (1960, 1963, 1968) has postulated that three conditions are necessary if one is to learn about abstract ideas. These are: (1) abstract material must be organized in a meaningful way, (2) the learner must possess a cognitive foundation to which new learning can be attached, and (3) the learner must have a strategy and the will to learn the new ideas. Barron (1969) proposes a teaching strategy which meets Ausubel's three conditions. This strategy, the structured overview, has been shown to assist both the teacher and the student in learning abstract subject matter.

Consider in Figure 1 the structured overview for plane figures.

FIGURE 1
PLANE FIGURES

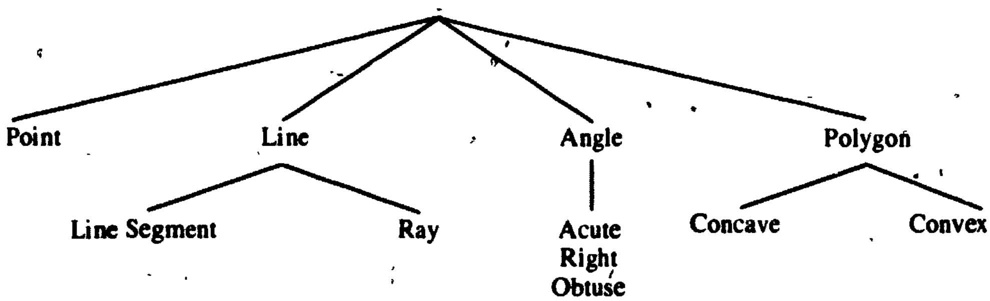

Figure 1 depicts a skeletal ordering of terms which describe the concept "plane figures." As one looks closely, it becomes clear that an organization of interrelationships exists among these terms. In one instance, a descending hierarchy is apparent. This hierarchy begins with the most general term (plane figures), moves through a level of less-general terms (e.g., point), and culminates in specific terms (e.g., line segment). An organization of interrelationships also exists as the structured overview is considered on a continuum from right to left. Specifically, the sequence of most common instruction is depicted. That is, learning of "point" and "line" are necessary before meaningful instruction about "angles" can begin. And, the teaching of "polygons" is built upon the knowledge base amassed in the prior instruction about "angles, line, and point."

The structured overview of Figure 1 has met Ausubel's first requirement for abstract learning. Specifically, this structured overview shows one logical sequence of instruction based upon the relationships of specific terms (e.g., acute angle) to the more general concept "plane figures." Through directed use of the structured overview, the learner can relate new information to what was previously learned, thereby

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fulfilling Ausubel's requirement of a cognitive foundation. Further, the structured overview becomes the strategy for both teaching and learning in that it is a planning tool for the teacher and an assimilation tool for the student. While Ausubel's demand for the willingness of the learner cannot be guaranteed, the structured overview does motivate the learner by providing a visual referent and an idea framework that is expanded in the course of the lesson. The visual nature of the structured overview can assist the teacher's plan for instruction based upon a standard textbook unit. This is accomplished as the teacher deletes unnecessary words and terms, and adds or repositions those which clarify and explain relationships. The planning is, of course, based upon the teacher's knowlege of the material and the learning characteristics of students. In effect, the structured overview allows the teacher to examine the projected scope of a unit graphically and then set lesson priorities based upon the teacher's knowledge of the material and the varying abilities of students.

The effectiveness of the structured overview has been tested in the classroom. Studies by Earle (1970), Scarnati (1973), Williams (1973), and Glynn and DiVesta (1977) concluded that the strategy enchanced student learning. In studies conducted by Hash (1974) and Bowman (1975), the use of the structured overview did not significantly influence student comprehension. From these six studies two conclusions can be drawn: (1) the structured overview assisted teachers in planning their instruction, and, (2) the use of this strategy did not inhibit student learning.

In order to construct a structured overview, Earle and Barron (1973) suggest the following six steps for teachers:

1. Analyze the vocabulary and learning task and list all words that you feel are representative of the major concepts that you want the students to understand.
2. Arrange the list of words until you have a diagram which shows the interrelationships among the concepts particular to the learning task.
3. Add to the diagram vocabulary concepts which you believe are already understood by the students in order to depict relationships between the learning task and the discipline as a whole.
4. Evaluate the overview. Have you depicted major relationships clearly? Can the overview be simplified and still effectively communicate the relationships you consider to be most important?
5. When you introduce the learning task, display the diagram to the students and explain briefly why you arranged the words as you did. Encourage the students to supply as much information as possible.
6. During the course of the learning task, relate the new information to the structured overview as it seems appropriate.

A closer inspection of these directions will reveal how the construction of a structured overview can aid the teacher to prepare a unit of instruction (steps 1–4) and to involve students actively at various stages within the lessons (steps 5–6). In a geometry classroom, for example, a teacher could plan for and involve students in a unit on convex polygons in the following way. See Figure 2.

The first step in the development of the structured overview requires the teacher to list the general vocabulary which is associated with the concept "convex polygons." This list could include such terminology as: polygon, convex, triangle, quadrilateral, pentagon, hexagon, heptagon, and octagon.

The second step asks the teacher to arrange the listed vocabulary in a diagrammatic form which shows the interrelationships among the terms.

FIGURE 2
POLYGON

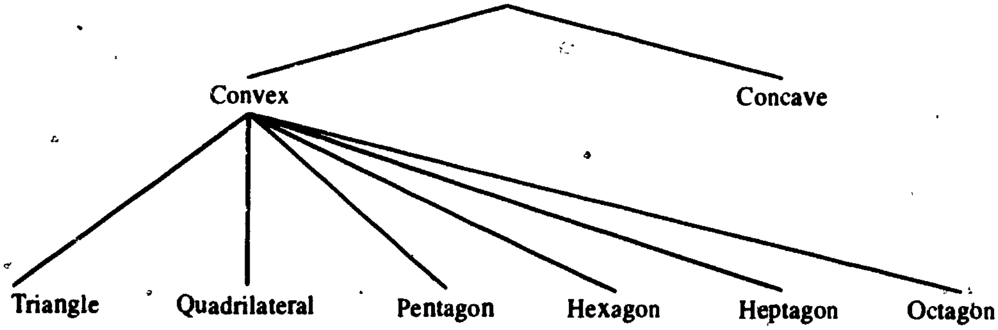
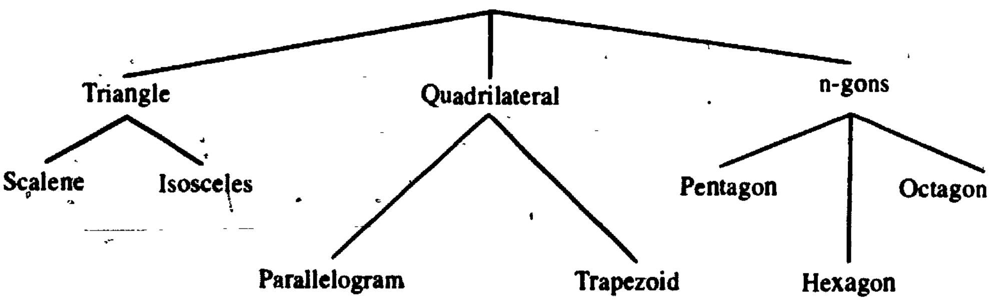


Figure 2 shows: (1) two categories of polygons—concave and convex, and (2) numerous examples of the category of convex polygons, *i.e.*, triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons to name but a few.

Steps 3 and 4 are included for pre-teaching clarification and evaluation. Step 3 provides for the addition of words and terms which will more clearly show the interrelationships that the teacher feels to be important. Step 4 calls for an evaluation of the structured overview that is intended to assist the student in discovering the interrelations. The refined structured overview might look like Figure 3.

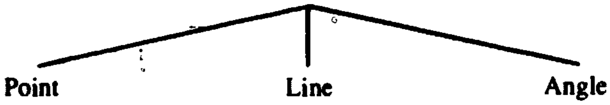
FIGURE 3
CONVEX POLYGONS



A close inspection of Figure 3 demonstrates the importance of clarification and simplification. For example, the word "concave" has been eliminated, and the most general term has been combined as "convex polygons." The more specific terms "triangle" and "quadrilateral" have been clarified with examples of each; the terms "pentagon, hexagon, and octagon" have been listed as "n-gons." Similarly, the term "heptagon" has been eliminated because of its limited application. Furthermore, the instructional sequence is identified in which "triangle" will be taught followed respectively by "quadrilateral" and "n-gons."

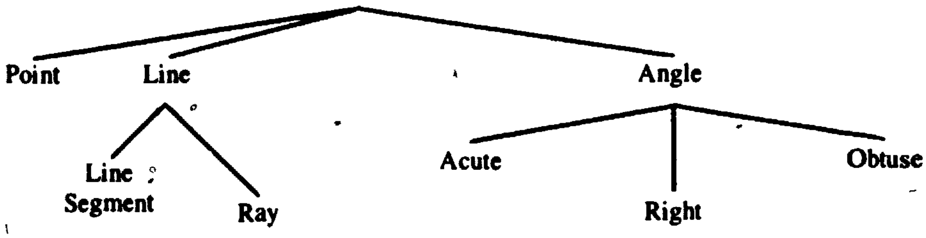
Step 5 provides the transition between teacher planning and student involvement. For most effective use, we suggest that students participate in discovering relationships rather than being told about them. The teacher should attempt to have students relate the concepts learned in previous lessons to the one at hand. Therefore, the teacher might begin the lesson writing the term "plane figures" on the chalkboard and saying, "We will begin today a unit on polygons. Previously we have studied the various components that make up these figures. Can anyone tell me the names of some of these components?" As students contribute such terms as "point, line, and angle," the teacher places them within the structured overview, as seen in Figure 4.

FIGURE 4
PLANE FIGURES



In step 6, the teacher calls for additional vocabulary which clarifies the terms "line" and "angle" and adds them to the diagram, as in Figure 5.

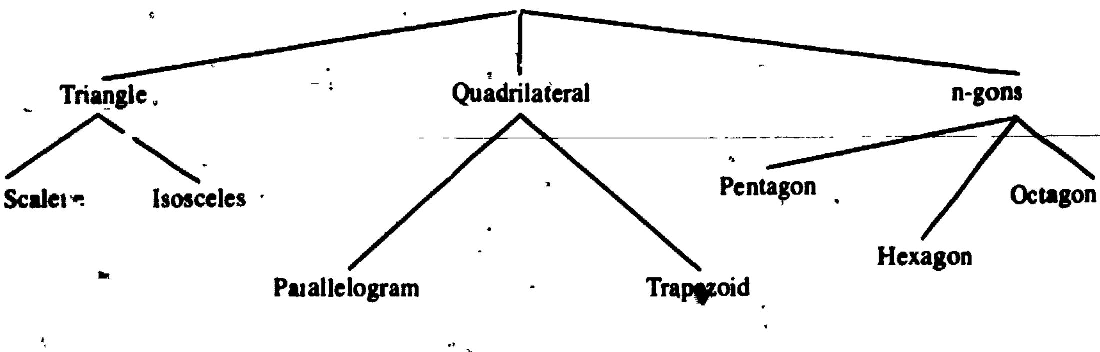
FIGURE 5
PLANE FIGURES



In effect, the teacher has given a quick review using the terms elicited from students. After an intuitive exploration of polygons in which figures are drawn, the teacher writes the term "polygon" on the chalkboard. Discussion now focuses on identifying parts of the polygon and special terminology such as "vertex," "side," and "interior angle" are listed. This activity provides a sound basis for the detailed study of polygons.

When the teacher is confident that students can identify the major parts of a polygon, the structured overview, Figure 6, is presented.

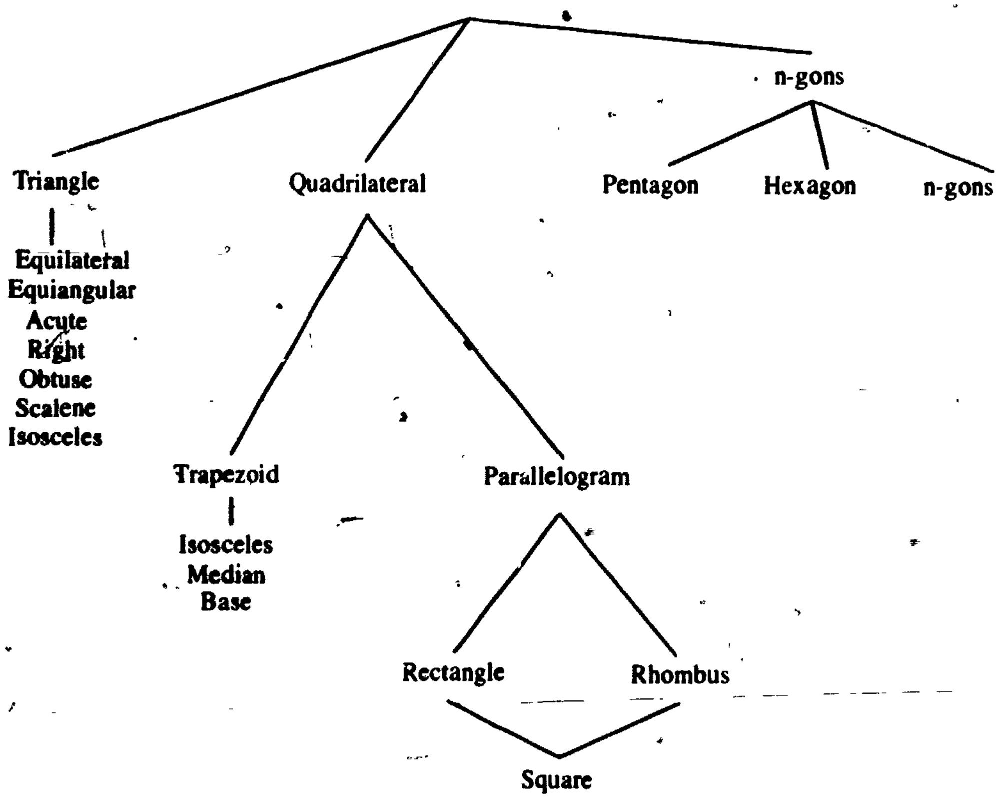
FIGURE 6
CONVEX POLYGONS



The teacher might introduce it by saying, "This unit on convex polygons contains information which is highly organized. We will use diagrams such as this to help you keep track of information and to see how ideas fit together. Don't memorize the diagrams that we develop. Rather, try to relate new information to them in much the same way as we did in reviewing the elements that make up plane figures. What can you tell me by looking at this overview?" Students should be able to tell the teacher that these are at least three major divisions of convex polygons (triangles, quadrilaterals, and n-gons) and that the overview shows two or three examples of each.

As a unit on convex polygons develops, the initial structured overview undergoes many changes as students discover new information to add to it. Figure 7 is one example of a structured overview expanded by students following lessons on triangles and quadrilaterals.

FIGURE 7
CONVEX POLYGONS



As described above, Figure 7 serves as a catalyst for selected learning and purposeful guided instruction. One recent development in the use of this strategy is the substitution of a pictorial format, such as Figures 8–10, for the tree diagrams, such as Figures 1–7. Another modification, termed a Post-Graphic Organizer, is a tool for assessing students' ability to analyze, extend, and internalize conceptual relationships. The pictorial format will be discussed immediately; the Post-Graphic Organizer will be discussed in the next chapter.

THE STRUCTURED OVERVIEW: PICTORIAL FORMAT

An interesting modification of the tree diagram type of structured overview is suggested by Snowman and Cunningham (1975), Childrey (1975), Herber (1978) and Smith (1979). Studies conducted by these individuals show that pictorial formats for the structured overview, in some instances, can enhance student learning. One advantage of the pictorial format of the structured overview is that learners can more clearly visualize the concept being taught. For example, consider Figure 8.

FIGURE 8

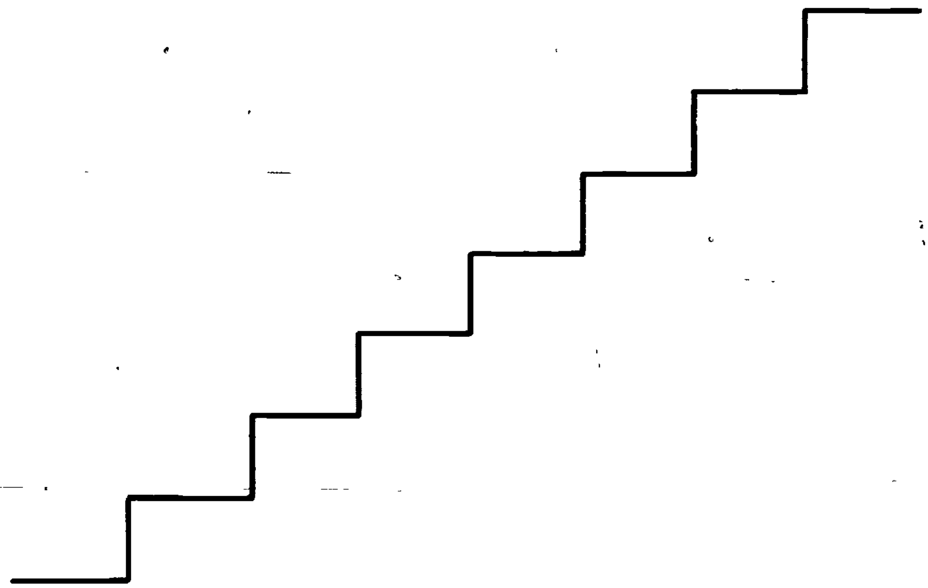


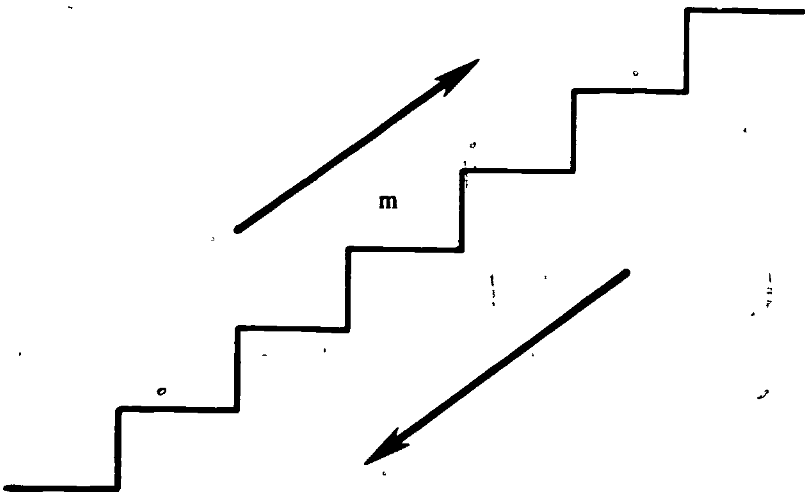
Figure 8, a staircase as viewed from the side, can be useful for those students who experience difficulty or frustration in learning the metric system. The vocabulary and symbols selected for use with this structured overview are:

meter	liter	gram	deka	×
milli	centi	deci	hecto	÷
multiplication		division	kilo	

The teacher begins the lesson by telling students that the metric system is based upon the power of 10. Next, the words for length, volume, and weight (meter, liter, gram) are introduced. Then the metric prefixes and their corresponding symbols: milli (m), centi (c), deci (d), deka (da), hecto (h), and kilo (k) are presented to the students. This discussion might be accompanied with references to common experiences or familiar words (running in a 100 meter dash; knowledge that a century is 100 years).

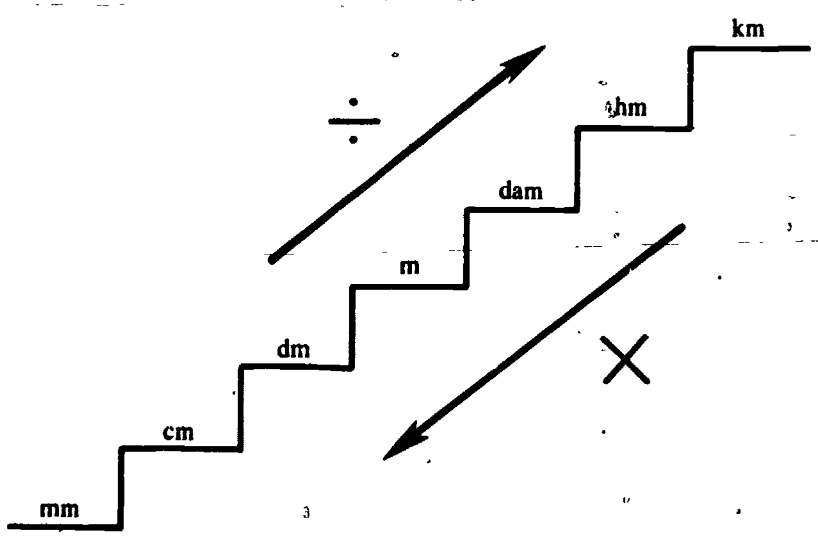
The teacher returns to the staircase diagram and adds the arrows and a standard metric unit—say, the meter.

FIGURE 9



Successive metric units are added above and below the standard unit, *i.e.*, decameter, hectometer, kilometer, and decimeter, centimeter, millimeter. In order to represent a single length in several units, the division and multiplication symbols are added which illustrate that the metric system is based on the powers of ten. For example, dividing by powers of ten provides for movement up the staircase (1000 m = 1 km) and multiplying by the powers of ten provides for downward movement (1 m = 100 cm). Student applications are numerous through the selection of different standard metric units. A completed metric staircase showing the relationships of length would look like Figure 10.

FIGURE 10



Like the format of the tree diagram, the pictorial format utilizes the organization of the material to be learned (in this case, the metric dependence on the powers of 10), and provides a cognitive foundation for the new learning (the common experience of the staircase). The combination of these factors with the teacher-led discussion becomes the strategy for sensing the relationships within the metric system.

SUMMARY

The structured overview can assist a teacher to clarify instructional objectives, sequence instruction, provide individualization, and give both purpose and direction to a lesson. These accomplishments are based upon the teacher's knowledge of the subject matter and tempered by the teacher's perceptions of the needs of the students. For example, in the second chapter, we discussed a planning strategy entitled "The Instructional Framework." That strategy provides teachers a means for combining content analysis with process analysis. The melding of content and process typically results in a three-part teaching outline which has provision for introduction, assimilation, and follow-up activities. Upon reexamination of the teaching outline presented in the second chapter, several references are made to the structured overview. This potential for multiple usage is but one suggestion of the utility inherent in this device. A second suggestion can be seen if one considers the structured overview as a practical way of applying the seven elements of process analysis and efficiently incorporating them within all of the stages of the lesson outline.

For instance, a specific example of the instructional framework strategy in the second chapter can be easily applied to a unit on quadrilaterals. Please note how the structured overview is used for maximum instructional effect. Initially, the structured overview serves to review background information by giving students a visual perspective on quadrilaterals. This is accomplished by relating the new information (quadrilaterals) to that previously studied (polygons and parallels). The structured overview is used for setting purposes by first introducing the students to those quadrilaterals the teacher selected for teaching (parallelograms and trapezoids) and then graphically depicting the relationships that exist among them. These relationships are made clearer for students in the teacher's selection of textbooks or supplementary readings, class discussions, and assignments.

The essential vocabulary can also be taught within the context of the structured overview. (This will be more fully developed in the next chapter.) For example, the words "parallelogram, quadrilateral, and rectangle" can be taught quickly for meaning and pronunciation using structural analysis (base words and affixes), and the specific meanings of the words "leg, base, and median" can be derived from drawing upon the students' knowledge of previous contextual usage from lessons on triangles and/or statistics. New words, like "rhombus" and "trapezoid," can be taught through context clues, glossary or dictionary usage, or the teacher's direct instruction in their meanings and pronunciations.

As a motivational tool, the structured overview helps the student visualize the various geometric figures and their word symbols. For this reason, the given structured overview might combine both pictorial and tree formats. Undoubtedly, this combination would provide the students with a strategy for conceptualization and word meaning associations. Further, the students' prior experiences can be incorporated within the development of the structured overview and their internalization of the relationships within its components. Hopefully, their interest and motivation in studying quadrilaterals will be enhanced by using the structured overview.

The structured overview can also be used as a follow-up activity. One such way is for the class to review key words and the relationships among quadrilaterals as a review prior to a test on the unit. Another consists of the Post-Graphic Organizer that will be discussed in the next chapter.

The reader may have noticed similarities between structured overviews, Venn diagrams, and flow charts. All of these formats have been shown to help teachers visually depict important concepts to their students. The format chosen is dictated by the content, the needs of the students, and the teacher's perceptions of both.

4. THE DEVELOPMENT OF MATHEMATICS VOCABULARY

In the previous chapter, the structured overview was discussed in some detail. This strategy manipulates vocabulary to enhance comprehension and understanding of abstract subject matter. Acquisition of vocabulary is a developmental process. Therefore, mathematics teachers have an obligation to help students acquire proficiency with words, symbols, and expressions. The purpose of this chapter is to present additional strategies that the teacher may use in developing the student's fluency in the language of mathematics. Teachers are urged to consider the following strategies for inclusion within the introductory, assimilation, and follow-up stages of the instructional framework discussed in the second chapter. Based on student needs and content requirements, some activities may seem to have limited value while others may foster productive vocabulary usage.

TEACHING VOCABULARY

There are at least six strategies that a reader uses to unlock unfamiliar words and symbols for meaning and pronunciation. These are phonetic analyses, instant recognition, syllabic analysis, structural analysis, contextual analysis, and dictionary and glossary usage (Clayton, 1968).

Phonetic Analysis. Phonetic analysis is sound-to-symbol pronunciation strategy that forms the basis of initial reading instruction (*e.g.*, the letter "a" is assigned either a long or short sound depending on its association with other letters) for translation into oral speech (*i.e.*, base, math). Typically, it is phonetic analysis that subject matter teachers think of when they consider reading instruction within their curricula. It is our opinion that basic instruction in the reading process has no place in the subject matter classroom.

Instant Recognition. Instant recognition is an automatic pronunciation or meaning response to words or symbols. These "sight" words and symbols are continually being incorporated into one's reading vocabulary. Words and symbols such as "tangent" and "sine" or "tan" and "sin" will at first be unfamiliar to the student learning trigonometry. Through regular use, these words and symbols will be recognized instantly.

Syllabic Analysis. Syllabic analysis or syllabication is a pronunciation strategy in which a speech sound is associated to a vowel(s) so that a continuous utterance results. The uttered sound may result in a complete word, *i.e.*, base, or, as recognized divisions within a word, *i.e.*, ex-po-nen-ti-a-tion.

Structural Analysis. Structural analysis utilizes word roots, prefixes, suffixes, compound words, and syllabication principles to derive both meaning and pronunciation. "Triangle," for example, is composed of a prefix (tri) and a root (angle). The combination of these elements form the conceptual representation of a three-sided polygon.

Contextual Analysis. Contextual analysis is the anticipation of the meaning of a word or symbol through its association with other words or symbols. For example, in the sentence, " 7 is a *solution* for the equation $5x - 8 = 27$ is a true statement," the meaning of the word "solution" is derived from the fact that the condition $x = 7$ makes the statement true.

Dictionary or Glossary Usage. Dictionary or glossary usage is a reference skill strategy which provides pronunciation clues, specific meaning, syntactic usage, and word origins for many mathematics words. Some of these would be: algebra, calculus congruent, ellipsoid, inverse, locus, logarithm, numerator, quotient, perimeter. Therefore, a good standard dictionary is an effective mathematics instructional tool. Because of the specialized meaning of mathematical terms, a good mathematics dictionary is desirable too.

It is important to raise a caution before proceeding further. Specifically, the mathematics teacher's primary responsibility is to teach the content of the mathematics curriculum. Instruction in vocabulary can complement any curriculum when placed within the proper perspective. It is our recommendation that teachers think of vocabulary teaching in the framework of a developmental process. That is, vocabulary teaching has far greater utility and longer lasting effects when it is done in small but regular increments. Five minutes of daily instruction is far superior to intense but infrequent use of the dictionary or excessive amounts of time spent on vocabulary recognition or memorization.

What follows is a five-minute segment of vocabulary teaching within a mathematics lesson. For example, in a unit on geometric figures, there are several opportunities for vocabulary teaching. A lesson about "triangles" can be introduced by listing the words equilateral, equiangular, acute, right, obtuse, scaline, and isosceles on the chalkboard. The word equiangular can be taught for meaning using structural analysis. The teacher first divides the word into its prefix (equi) and its stem (angular). The teacher next asks if either of these components is familiar. Since most students will have had experience with the concept of equations (the act of making a mathematical expression equal), the meaning of the prefix "equi" can be made by transposition. Similarly, the meaning of the stem word "angular" can be derived. Once the meaning of equiangular has been uncovered, it is but a short jump to solicit the meaning of equilateral. Those students familiar with a lateral pass, as in football, can supply the information that a pass of this nature is thrown to the side. Therefore, equilateral means "equal sided." A word, such as acute, can be assigned to a student volunteer to derive its meaning from context by giving an appropriate textual reference. The dictionary should be consulted if a word is to be used precisely. Acute, for example, has an invariant meaning with application to medicine, music, and mathematics. The remaining words, if not taught for meaning, are pronounced by the teacher or by the class in unison.

While the strategy for obtaining the meanings of the words described above may be obvious to the teacher, there are no assurances that every student is aware of the process. Further, the brief amount of instructional time expended here may reap some benefits when the student reads independently. Rather than glossing over unknown words, the student might apply structural clues, contextual clues, or dictionary reference to obtain the meanings of words such as equidistant (equally distant), equilibrium (equal balance), equigranular (crystals of same size), equimolar (equal concentration of moles), equanimity (evenness of mind), equatorial (about the equator), equivocate (avoid commitment).

LIST-GROUP-LABEL

Taba (1967, 1971), and Fraenkel (1973) discuss a strategy that is helpful to students in organizing concepts and analyzing relationships. This technique, entitled

List-Group-Label, differs from other strategies which explore relationships in one major respect. Specifically, in List-Group-Label, the students are responsible for contributing the vocabulary suggested by a concept rather than manipulating vocabulary provided by the teacher. The importance of this variation is significant because the vocabulary to be manipulated is generated exclusively from student experiences. This personal experiential search identifies individual cognitive foundations to which new ideas may be more readily attached. When using a structured overview, the teacher is never sure if all students understand the organizational structure of interrelated concepts. Further, when using the structured overview, the teacher dominates by controlling the experience. In contrast, List-Group-Label, dependent upon student experiences and existing cognitive foundations, allows students to dominate.

The teacher initiates List-Group-Label by writing a general word, term, or phrase on the chalkboard. Next, students are told that the general word has something to do with the next unit of study and they are asked if they know of any words or terms that are similar. As words begin to come forth, associations are frequently made which stimulate additional ones. After a sufficiently large list has been drawn up (usually 18-30 words and terms), the students are directed to cull similar words from the master list and to attach a label that will explain them.

For example, one teacher initiated a lesson solving word problems using List-Group-Label to clarify some student difficulties. The teacher explained that certain words within a problem indicated certain computational operations. The word "operation" was printed on the chalkboard and students were asked to list words or terms that indicated an operation. The students compiled the following list:

OPERATIONS

total	plus	greater than	tripled
difference	remaining	take away	quotient
product	less than	reduced by	twice
doubled	decrease	remainder	times
increase	diminished by	multiplied by	equals
less	sum	divided by	

The students grouped the words and labeled them. They were:

- total, sum, increase, greater than, plus (addition)
- decrease, less, less than, diminished by, difference, take away, reduced by, remaining (subtraction)
- times, product, doubled, tripled, twice, multiplied by (multiplication)
- divided by, quotient, remainder (division)
- difference, doubled, decrease, diminished by (words beginning with "d")
- less, plus, sum, times, equals (small words).

In the discussion that followed, the teacher pointed out that students were aware of many of the words which indicated an operation. However, many students glossed over these words when they appeared in a word problem. Therefore, in solving future word problems, they should first read the problem, underline words which indicate an operation, and, then, set up the problem for solution.

The teacher was also aware that two of the lists were indicative of syntactic structures, words beginning with "d" and small words, rather than semantic ones. The purpose of List-Group-Label is semantic association. However, in certain instances syntactic association may be a useful memory device. Teachers should be aware that List-Group-Label explores and analyzes semantic relationships, thereby reinforcing and clarifying student understanding of a concept.

RAPID RECOGNITION

If the teacher is of the opinion that students need drill in the instant recognition of certain words or symbols, the common practice of using flash cards or rapid recognition exercises may be considered. The former are easily made using 3 × 5 or 4 × 6 index cards. An example of the latter is presented as Exercise 1.

EXERCISE 1
Rapid Recognition-Perception

Directions: Underline the word or symbol in one of the four columns to the right that is the same as the key word or symbol.

	Key word or symbol				
1.	add	area	angle	add	addition
2.	>	<	+	>	÷
3.	÷	×	=	+	÷
4.	base-ten	base	add	base-ten	hasten
5.	chart	closed	chart	check	count

Because of space limitation, only five lines in Exercise 1 have been depicted. For more effective usage, it is suggested that at least twenty lines be incorporated in each exercise and that students be given steadily decreasing amounts of time to complete these exercises. Teachers are cautioned when using strategies such as this in that they quickly deteriorate into seat work with no academic purpose.

A more useful variation of the rapid recognition technique is one that stresses comprehension. Here, math symbols or words are used to reinforce mathematical equivalence. An example is Exercise 2.

EXERCISE 2
Rapid Recognition-Comprehension

Directions: Underline the words or symbols to the right that mean the same as the key word or symbol.

	Key word or symbol				
1.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{6}$	$\frac{5}{12}$	$\frac{2}{3}$
2.	>	greater than	less than	equal to	not equal to
3.	0.25	$\frac{25}{100}$	$\frac{2}{5}$	$\frac{1}{4}$	$2\frac{5}{10}$
4.	less than	−	>	<	≠
5.	$\frac{3x + 9}{3}$	$x + 9$	$3x + 9$	$x + 3$	$x + 6$
6.	∥	perpendicular	parallel	equal	similar

Much work in mathematics involves the use of equivalent expressions. Thus, rapid recognition of equivalence can be helpful in improving this capability.

WORD MAZE

Perhaps the most flagrant example of a seat work activity disguised as a vocabulary development tool is the "word search" or "word maze." Activities such as Exercise 3 are found across many subject areas. They serve to keep students occupied, but their educational impact is doubtful.

EXERCISE 3

WORD MAZE-FRACTIONS

Directions: The words in the following maze are words associated with "Fractions." You may find words horizontally, vertically, or diagonally.

Find the following words:

NUMERATOR	PROPORTION
FRACTION	DIVIDE
RATIONAL	SIMPLIFY
EQUIVALENT	INTEGER

L	L	A	S	E	[illegible]	T	I	V	I	T	C	A	K
W	R	D	G	G	B	D	H	Q	E	E	P	W	R
O	T	A	E	D	I	V	I	D	W	I	F	O	O
R	F	K	T	M	R	E	G	E	T	N	I	I	W
D	O	N	O	I	T	R	O	P	O	R	P	H	T
M	U	D	S	E	C	P	C	D	Y	L	H	V	A
A		R	A	C	T	I	O	N	I	M	Y	J	E
Z	P	G	E	L	A	N	O	I	T	A	R	Y	S
E	A	R	O	T	A	N	I	M	O	N	E	D	E
S	N	R	O	T	A	R	E	M	U	N	C	T	N
A	S	E	Q	U	I	V	A	L	E	N	T	P	A
R	A	T	S	I	M	P	L	I	F	Y	H	X	N
E	Z	V	Y	G	Z	U	V	U	D	Y	J	R	I
B	L	A	T	A	N	T	F	O	O	L	I	S	H

MODIFIED CROSSWORD PUZZLES

The crossword puzzle format provides opportunity for reinforcing the meaning of mathematics words and terms. Consider the modified crossword puzzle, "What's My Line?"—Vectors (Puzzle 1).

PUZZLE 1

WHAT'S MY LINE?—VECTORS

Directions: Look at each statement below. Consult your text or any previous assignments necessary to find a word that fits each description and has the same number of letters as the spaces indicated. Fill in those you can identify. Reference pages are given as an aid.

1. _ _ O _ _ _ _ _ _ _ _
2. _ _ _ _ _ R
3. _ _ _ _ _ _ _ _ D _ _ _ _ _ _ _
4. _ _ _ _ _ _ E _ _
5. _ R _ _ _ _ _ _ _ _ _ _
6. _ _ _ E
7. _ D _ _ _ _ _ _
8. P _ _ _ _
9. _ _ _ _ _ _ A _ _ _ _ _
10. _ _ _ _ _ I _ _ _ _ _ _ _
11. _ _ _ _ _ R _

1. X_Q and X_P are the ______________ of the points P and Q on line h. (page 466)
2. ______________ is the term usually associated with the directed distance from point P to point Q in two-dimensional space. (page 467)
3. In one-space (line) h, the __________ __________ from point P to point Q of h is $h_Q - H_P$. (page 466)
4. $\Delta X \quad X_B - X_A$ is the X– ______________ of the vector v(AB). (page 469)
5. For any 3 vectors $\overrightarrow{AB}$, $\overrightarrow{CD}$ and $\overrightarrow{EF}$, if $\overrightarrow{AB} = \overrightarrow{CD}$ and $\overrightarrow{CD} = \overrightarrow{EF}$, then $\overrightarrow{AB} = \overrightarrow{EF}$ is a statement of the ______________ Principle for vectors. (page 470)
6. One-space is represented geometrically by a ______________. (page 466)
7. For each pair of points A and B, $\overrightarrow{AB} = \overrightarrow{AB}$ is a statement of the ______________ Principle of Vectors. (page 470)
8. Two-dimensional space is represented geometrically by the XY– ______________. (page 467)
9. $\overrightarrow{AB} + \overrightarrow{CD} = \overrightarrow{CD} + \overrightarrow{AB}$ represents the ______________ Property of vectors. (page 475)
10. $(\overrightarrow{AB} + \overrightarrow{CD}) + \overrightarrow{EF} = \overrightarrow{AB} + (\overrightarrow{CD} + \overrightarrow{EF})$ represents the ______________ Property of vectors. (page 475)
11. $\overrightarrow{AB} + \overrightarrow{CD}$ is a vector, is a statement of the ______________ Property of vectors. (page 475)

Puzzle 1 is constructed for use with *Geometry Structure and Function*. 2d edition by Kenneth Henderson, McGraw-Hill, 1968.

The major difference with the crossword puzzle format is that the student need only supply missing words horizontally. It has been our experience that students are readily frustrated with the traditional crossword puzzle format. This frustration seems to result from the requirement for interrelated letter configurations (horizontal and vertical) and for the amount of time puzzles such as this take to complete. It is not uncommon to find that a majority of students complete less than half of traditional crossword puzzles. Teachers also experience frustration in constructing the traditional crossword puzzle. Again, this frustration stems from the search for vertical and horizontal words with common letters and from the time needed for construction.

"What's My Line?"—Vectors was constructed for a senior high mathematics class. The teacher used it in the assimilation and follow-up stages of an instructional framework. As an assimilation activity, students were directed to use contextual clues or the page numbers at the end of each statement. Please note that the amount of text reading was limited to 10 pages and only 11 words were needed to complete the puzzle. These factors insured reasonable time limits. When most students were finished, small group discussions were initiated and answers were compared. A whole class discussion was used to clarify responses and amplify student observations. In some instances specific references to the text were required; in others, students used past experiences. Please note that this activity was not used as a test but rather as a motivational device to teach word meanings.

A variation of this format is presented as Puzzle 2. This puzzle, entitled "What's My Line?"—Properties, requires knowledge of standard definitions and symbolic algebraic expressions. Again, please note its brevity and the inclusion of statements that range from fairly easy to fairly difficult. This provision for a range in levels of difficulty allows the teacher to challenge students at levels commensurate with their ability rather than lockstepping the instruction. Further, it is not necessary for all students to complete every item in the puzzle. Rather, it is suggested that after students have worked independently or in small groups, the teacher lead a discussion about the concept which requires students to confirm their answers. Such a discussion promotes teacher-student, student-student, and student-content interactions.

CONCENTRATION

Most teachers are familiar with a card game entitled "Concentration." A variation of this game, "Metric Concentration" (Learning Activity 1), can assist mathematics teachers teach the literal meaning of metric symbols. In "Metric Concentration," a deck of paired metric equivalents (*i.e.*, 320 m and 0.32 k[m]) and distractors (*i.e.*, 120 cm and 12 m) is given to students. After the deck has been shuffled and the cards placed face down, the first player turns over two cards. If the cards are equivalent, the student keeps them and is rewarded with an additional turn. If the cards are not equivalent, they are again placed face down and the player relinquishes the turn. The game continues until all of the cards, with the exception of the distracting pairs, are gone. The player with the most pairs is the winner.

At first glance, "Metric Concentration" appears to be little more than a memory game. The inclusion of the distracting pairs, however, prevents the student from winning without thinking about metric principles. A player cannot base a choice exclusively on the numerical digits appearing on the cards, ignore the metric symbols, and hope to win. (See Learning Activity 1.) The player who focuses only on the digits 32 and disregards the metric unit symbols mm and cm could incorrectly pair 320 m to 32 m. Knowledge of the metric unit symbols is necessary for the correct match of 32 mm to 3.2 cm. Therefore, knowledge of metric structure is inherent to winning. An expectation that players verify their matches tends to clarify and extend student learnings of the metric system.

PUZZLE 2

"WHAT'S MY LINE?"—PROPERTIES

Directions: To solve the following puzzle, look at the statements below. Think of a word which fits each description and has the same number of letters as the number of spaces provided in the corresponding line. Write the words on the line. Do this for each statement.

1. _ P _ _ _ _ _ _ _
2. _ _ _ _ R _ _ _ _ _ _ _
3. _ _ _ O _ _ _ _ _ _ _
4. _ _ P _ _ _ _ _
5. _ _ _ _ _ _ _ _ _ _ E
6. _ _ R _ _ _ _ _ _ _
7. _ _ _ _ T _ _ _
8. _ _ _ I _ _ _ _
9. _ _ _ E _ _ _ _
10. _ _ _ _ _ S _

1. Two numbers located the same distance from zero.
2. $a(b+c) = (ab) + (ac)$
3. $(a + b) + c = a + (b + c)$
4. The symbol $-a$
5. $a + b = b + a$; $ab = ba$
6. $\sqrt{7}$
7. $a \cdot 1 = a$; $a + 0 = a$
8. Number expressed in the form $\frac{a}{b}$
9. The set of all whole numbers and their opposites.
10. A number added to its opposite equals zero.

LEARNING ACTIVITY 1
SAMPLE DECK: METRIC CONCENTRATION

9.7 m	39 cm	120 dkm	0.46 dkm
320 m	8 m	3.2 cm	9700 mm
4600 mm	1.2 km	0.03 m	460 m
120 cm	4600 dm	39 m	3.9 m
8 mm	0.32 km	0.08 km	32 mm
0.039 km	97 cm	0.97 km	12 m

ANALYZING RELATIONSHIPS

So far, this chapter has presented strategies that help students recognize mathematical words and symbols or attach primary meaning to them. Students also need practice in analyzing and explaining the relationships that can exist between words or symbols.

DECIMAL DICE

A purpose of Decimal Dice is for students to order pairs of decimal values. Typically, the student must first recognize the symbols, interpret their meanings, then compare the values and be able to justify the relationship

The materials needed to play are two cubes with decimal values on each face. To start, each player rolls a die to establish the order of play. The first player then rolls the dice and must determine which die contains the larger or smaller value. Each player is given two points for a correct answer. The number of points needed to win is determined in advance.

SAMPLE FACES FOR DICE

Die A	*Die B*
0.6	0.29
0.03	0.1
0.502	0.51
0.057	0.35
0.3	0.031
0.508	0.601

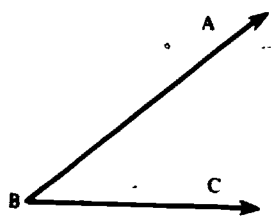
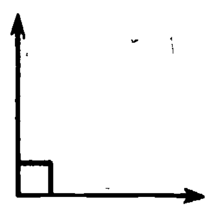
As student familiarity with decimal values becomes automatic, several variations can be incorporated, decimal values can be made more difficult, computations can be performed. Regardless of the variation, discussion of correct and incorrect answers is essential for maximum student learning.

WORD/SYMBOL RELATIONSHIPS

The purpose of Learning Activity 2 is to reinforce the meaning of mathematical words and symbols. Students are given a grid which contains either a mathematics word, term, or symbol. In the former, they are asked to draw and label the symbol suggested by the word or term. In the latter, they are asked to express in the fewest numbers of words the expression suggested by the given symbol.

LEARNING ACTIVITY 2

Directions: In each box below there is a mathematical word, term, or symbol. When a word or term is present, make a drawing and label it. When a symbol is present, write the expression or make a drawing suggested by it. The first one is done for you.

ACUTE ANGLE  <ABC	$\parallel$	 ABCD
SCALENE TRIANGLE	ALTITUDE	

IDENTIFYING AND DESCRIBING RELATIONSHIPS

The purpose of the following activities is to identify and describe relationships that exist with a group of mathematical terms or expressions. In Learning Activity 3, students identify a term or expression that is inappropriate and then write a word or phrase that correctly explains the relationship between those remaining. In the first set, the words "coefficient," "exponent," and "variable" identify components of a single term, while "polynomial" refers to a sum of terms.

LEARNING ACTIVITY 3

Directions: In each of the sets below, three of the terms are related. Circle the term that is UNRELATED. On the line at the top of the set, write the word or phrase that explains the relationship existing among those remaining.

______________	________	______________
coefficient	factor	associative property
exponent	multiple	closure property
polynomial	prime	commutative property
variable	term	distributive property

______________	________	______________
consistent system	dilation	adjacent side
dependent system	reflection	hypotenuse
inconsistent system	rotation	cosine
independent system	translation	sine

____

- integers
- irrational numbers
- rational numbers
- whole numbers

Forcing student to express themselves in writing enhances student interaction and learning. Even though the teacher might construct a list of terms with a particular relationship in mind, at times a student may observe an alternative relationship that is correct, or one that is at least worthy of class discussion.

A variation on the identification and description of relationships involves the presentation of five terms. In Learning Activity 4 the student must find two relationships. Three terms fit one relationship and the remaining two terms fit a second. In the second set the student should note that three expressions represent the number one, while the other two represent values other than one.

LEARNING ACTIVITY 4

Directions: There are five terms in each set below. You are to identify two terms which are related so that the remaining three also fit a relationship. Notice that you may select a relationship for some terms so that the remaining terms are unrelated. You must establish two relationships using all five terms.

altitude	1
angle-bisector	$0.\overline{9}$
median	$\frac{9}{9}$
opposite side	0.99999
perpendicular-bisector	$\frac{99999}{100000}$

acute	FOR I = 1 TO 20
equilateral	IF C > O THEN 80
scalene	PRINT X
segment	READ A
ray	SUM = T + SUM

33

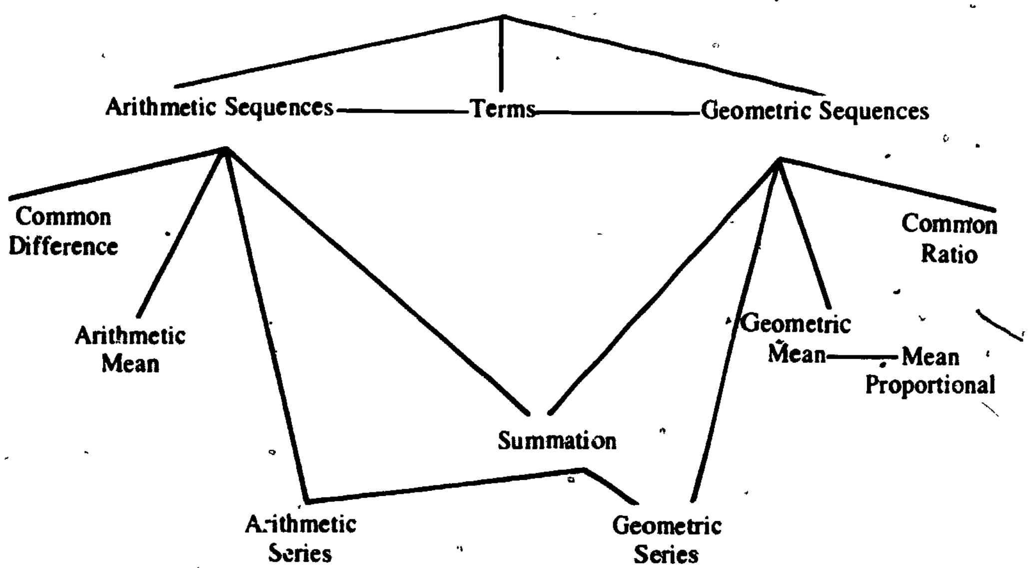
POST-GRAPHIC ORGANIZER

Barron (1978) suggests a refinement of the structured overview called the post-graphic organizer. It is typically used as a group activity in class. Its purposes are to help students explore and analyze relationships among key words and to provide teachers with immediate feedback about student understanding. An example of a post-graphic organizer is presented in Learning Activity 5. Before reading further, you are urged to complete the activity. One possible organization of terms is given as "Sequences of Real Numbers."

LEARNING ACTIVITY 5

Directions: Cut along the lines to separate the terms listed below. Choose the most general term. Arrange the remaining terms in such a way that logical relationships are depicted. Terms may be added by using the blank slips. Not all of the terms need to be used. Be prepared to explain your arrangement.

Geometric Series	Arithmetic Series
Common Difference	Arithmetic Mean
Arithmetic Sequences	Summation
Terms	Mean Proportional
Geometric Mean	Sequence of Real Numbers
Geometric Sequences	Common Ratio



When you complete Learning Activity 5, consider how to use the post-graphic organizer in the classroom. After assigning students to small groups and providing the list of terms with directions, the teacher observes group work and provides guidance as needed. If most groups are confused, then the teacher should know that some reteaching is in order before moving on to the next objective. If the confusion is not widespread, then the teacher should listen as students attempt to explain their difficulties. Based on the students' comments, the teacher suggests a reordering of a few terms.

When most students understand the key relationships, the teacher directs certain groups to put their post-graphic organizers on the chalkboard. It is suggested that students transfer the words and terms to index cards and tape them to the chalkboard. An opportunity is now present for intergroup discussion about mathematics. For instance, students can explain why terms from the master list were deleted or used more than once, or, why additional terms were added. The focus is on similarities and differences intended to solidify student learning. Since students are responsible for making inferences which result in valid conclusions, we have found this to be a productive summary activity.

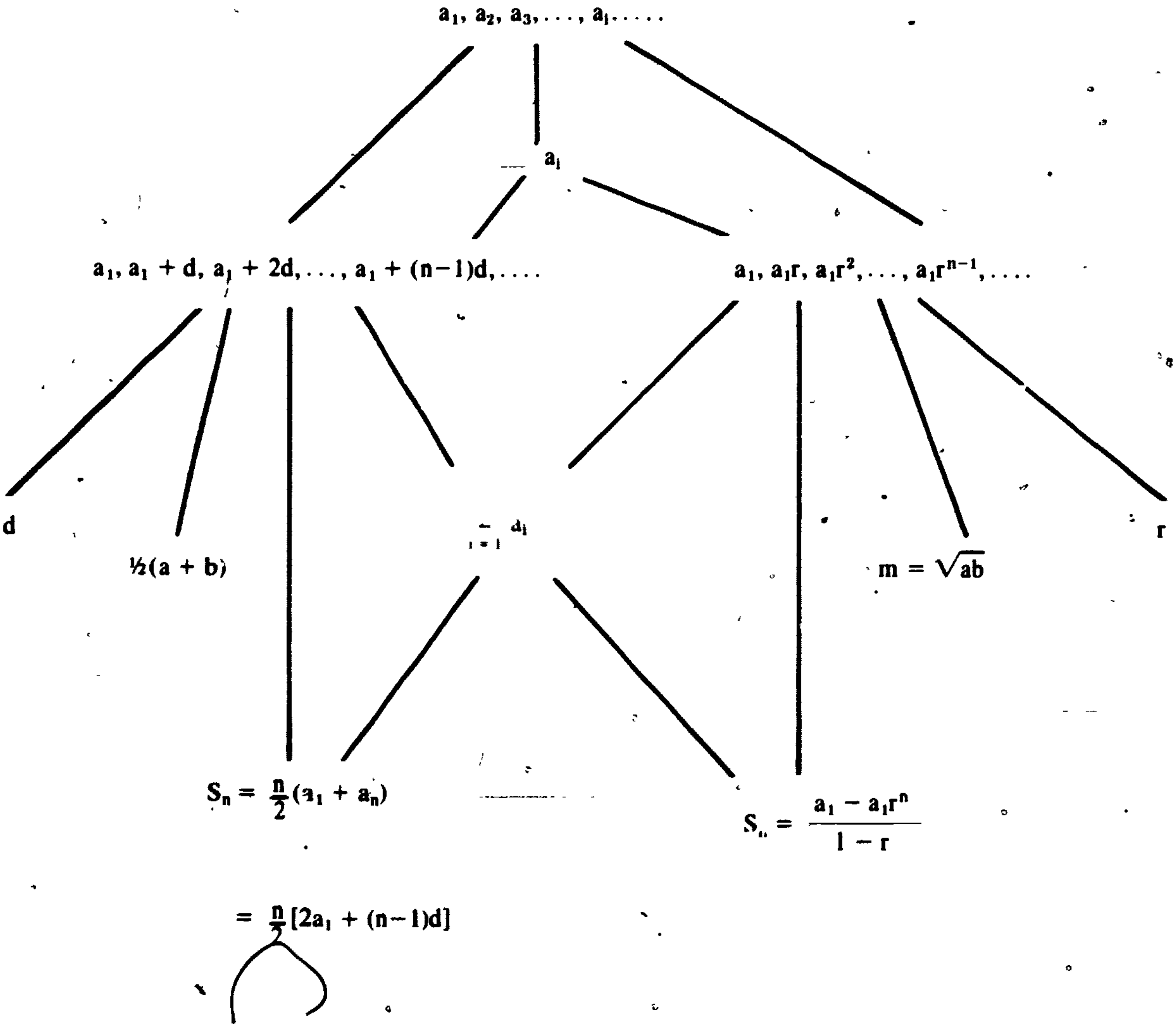
The post-graphic organizer is a meaningful activity for both students and teachers. Runyon (1978) found that students were able to analyze relationships within unit concepts and that teachers had the opportunity to view a concept from student perspectives. Runyon has shown that the post-graphic organizer activity stimulates creative thought. Further, the student discussion and organizational refinements can give the teacher new perspectives on familiar ideas.

A parallel form of the post-graphic organizer stresses relationships between key symbols and expressions. A sample is presented as Learning Activity 6, which corresponds to Learning Activity 5.

LEARNING ACTIVITY 6
POST-GRAPHIC ORGANIZER: SYMBOLIC EXPRESSION

Directions: Same as those for Learning Activity 5.

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$	$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}[2a_1 + (n-1)d]$
d	$\frac{1}{2}(a + b)$
$a_1, a_1 + d, a_1 + 2d, \ldots, a_1 + (n-1)d, \ldots$	$\sum_{i=1}^{n} a_i$
a_i	$\sqrt{ab}$
m	$a_1, a_2, a_3, \ldots, a_i, \ldots$
$a_1, a_1 r, a_1 r^2, a_1 r^3, \ldots, a_1 r^{n-1}, \ldots$	r



A careful examination of the word and symbol post-graphic organizers for sequences shows several items of importance in the reading phase of mathematics. First note that the symbol S_n represents the sum of the first n terms of an arithmetic series and also the corresponding sum for a geometric series. Each interpretation of S_n has a different meaning and a different formula. The letter m represents geometric mean, not slope. Different meanings for words or symbols within a unit or across topics must be considered by the teacher and understood by the students.

In contrast, mathematicians are not always consistent on terminology or symbols. The reader must discern the writer's meaning. How do arithmetic sequences differ from arithmetic progressions? They don't! Some authors use the term sequences; others use progressions; still others mention both terms. This is important for the teacher to note, especially in a multitext program. While developing the vocabulary and symbols for a topic, the teacher must check the student's knowledge of these words and symbols from prior content. Therefore, the post graphic organizer activity can provide clarification for both teacher and students.

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SUMMARY

The goal of vocabulary development in mathematics classrooms is precise, competent communication. In pursuit of this goal, the teacher may employ a variety of strategies which vary accordingly in their sophistication, difficulty, and intents. In some instances, the teacher should consider direct instruction. In others, manipulation activities may prove beneficial. This chapter describes representative learning strategies on this continuum that are useful when students approach fluency in the language of mathematics.

5. THE DILEMMA OF WORD PROBLEMS

The most frustrating topic in mathematics for both students and teachers is "word problems." This chapter will examine the reading of word problems and make some suggestions for their constructive utilization.

A major reason cited for the students' poor performance in solving word problems is that students cannot read them. While this purported reason is true for a few very poor readers, it is questioned by many educators as an explanation for most readers. When reading can be cited as a major factor, there is evidence that inadequate experience with specific mathematics vocabulary is a prime source of the trouble. Several suggestions for improving student use and recall of mathematics vocabulary were provided in the previous chapter. It is recommended that teachers review these strategies in order to provide appropriate vocabulary instruction when warranted.

In a detailed study of sixth graders solving word problems. Knifong and Holtran (1976) reported that 52 percent of their student failures were due to clerical or computational errors. In a follow-up study (1977), the same authors interviewed students on problems for which their errors could not be labeled as computational or clerical. In classifying student performance, Knifong and Holtran attributed only about 10 percent of the missed problems to poor reading ability. Most students who responded incorrectly could read the problem aloud, describe the situation, and identify what was to be found. However, only one-third could describe how to work the problem. "This failure cannot be claimed as evidence of reading difficulty," according to Knifong and Holtran. The difficulty is more likely centered on the students' inability to synthesize the available information. A summary of student performance is listed in Table 1.

TABLE 1

MEAN FREQUENCY OF RESPONSES TO DIRECTIVES AND QUESTIONS BY STUDENTS NOT SOLVING PROBLEMS CORRECTLY
(Knifong and Holtran, 1977)

Directive/Question	Mean Percent Responses		
	Correct	Uncertain	Incorrect
Please read the following aloud:	95%	1%	4%
What situation is the problem describing?	98	1	1
What is the problem asking you to find?	92	3	5
How would you work this problem?	36	17	47

Similarly, a review of the research literature indicates that class activity on reading instruction, other than work on mathematics vocabulary and symbols, has not resulted in improved performance in problem solving (Hollander, 1978, 63–65). Therefore, teachers should not spend the students' time on such activities.

Strategies for solving word problems. A common text strategy, often called the formal analysis method, has students respond to the following questions: (1) What is given? (2) What is to be found? (3) What is to be done? and (4) What is an estimate of the answer? This strategy has not significantly improved student performance (Hollander, 60). While responding to the first two questions, the student analyzes the problem by separating it into components. However, question three requires a synthesis which often is beyond the students' ability. Asking the student to dissect a problem may be the wrong thing to do.

The translation technique, frequently advocated in algebra, should not receive heavy emphasis (Yeshurun, 1979, pp. 6-7). The direct literal translation from English to mathematical sentences seldom provides an accurate description of the original problem. For example, contrast the sentences, "Jill's height is less than Tom's." And, "Jill has six dollars less than Tom." In the first sentence, "less than" forms the equality, $J = T - 6$. The carefully constructed textbook problem is the major exception to this shortcoming. However, the procedure often distracts the learner from the major relationships inherent to the situation.

Recent emphasis on studying a problem for its major relationships appears to be a promising practice. Instead of breaking down relations by listing components in formal analysis, Earp (1970) suggests five steps in reading word problems. They are:

1. Read first to visualize the overall situation
2. Read to get specific facts
3. Note difficult vocabulary and concepts
4. Reread to help plan the solutions
5. Reread the problem to check the procedure and solution.

Procedures such as this focus on key relationships. These procedures should be prominent in classroom work.

An initial focus on the action or process described in a word problem is crucial for the student. Consider the simple problem, "Marion has some money in his pocket. Marion finds 50 cents on the way to school. At school, Marion counts to find a total of \$1.83. How much did Marion bring from home?"

A valuable problem solving strategy is to find a major relationship. In this example, "original amount + found amount = total amount" is the key relationship. After noting this relationship, the student can return to substitute the known and unknown quantities, $x + \$0.50 = \$1.83$.

The above process reverses the initial focus of the formal analysis approach. Instead of the first step, "Let x = ..." the student seeks a relationship first. Doblaev (1969) reports that most students attempt to solve word problems by using both approaches. At this time, it is not possible to predict which approach a pupil will pick for a given problem. Therefore, teachers must help students read problems with a focus on finding the major relationship.

The most important instructional variable known to improve problem-solving ability is the number of problems attempted. For this reason students must have the continual opportunity to attempt varied word problems. In setting up word problems for students, the teacher must provide problems which allow for student success; a gradual buildup in difficulty and variety can follow. Many of the current mathematics texts are lacking in both respects. In noncontrolled situations, we can report better pupil performance and attitude toward word problems when the problems are assigned daily. Furthermore, problems of a basic level when mixed with more complex ones provide a greater degree of student success and confidence.

In attacking problems, students should experience the modeling of problem-solving behaviors. To hear the teacher think aloud while solving a problem or to work in pairs or small groups on a problem can be very instructive to the student. Too often, students view the final result as the only goal. Consequently, students center their efforts on the numbers in a problem and not on the relationships. To change this unfortunate stress, classroom activities culminating with the construction of a mathematical sentence should be used. The construction and verification of the sentence as an appropriate statement of the original problem is fundamental to mathematical problem solving.

While students' performance in computation declined slightly from the first National Assessment of Educational Progress in Mathematics in 1972–73 to the second assessment in 1977–78, students' performance in applications and problem solving showed a significant decline (NAEP, 1979). Commenting on these results and the "back-to-basics" movement, Shirley Hill, President of the National Council of Teachers of Mathematics, stated:

> Throughout the NAEP reports, there is evidence that students proceed mechanically and thoughtlessly through problems, seeking a familiar routine or a rigid rule to apply. . . . While a reliance on drill and rote memorization of rules will produce a good showing on tests of short-term retention, this reliance also creates a mind set that is antithetical to insight into the essence of a problem (Hill, 1979).

Clearly, the need to provide frequent and real problems for solving is crucial in mathematics.

In an interesting study, Zweng (1979a, 1979b) interviewed students on word problems that they could not solve. During the interview, students were given a problem they could not solve earlier. Hints were provided to help each student solve the problem. The hints were chosen to test various forms of instructional strategies. Having students focus on the action of the problem was one of the most productive strategies. On the other hand it was found that the presentation of the problem in a low verbal format helped students of very high ability but did not help the average or below average ones. The low verbal format, very popular in mathematics texts today, did not improve the solving ability of most students. Observers reported that it served as a "re-reading" for the high ability students but seldom assisted other students. Zweng concludes that the focus of students must be on the total problem and the choice of operations, not just on the answers.

Students must learn that mathematical problem solving requires concentration, effort, and original thinking. Each problem must be viewed initially as a new situation to be contemplated. Students must not confuse problem solving with the memorization process necessary for computational and algebraic algorithms. And teachers must realize that students attempt to solve problems in a variety of ways. Teachers must continually help students attack a variety of problems in different ways. Teachers should challenge students with inferential, geometric, and qualitative problems, as well as numerical ones.

Given that most current texts provide inadequate problem-solving activities, we recommend supplemental problem-solving activities continuously throughout the school year. A recent release of such materials is of importance. The Iowa Problem Solving Project (see listing in Additional Mathematics/Reading Resources: Teacher Focus) has generated eight instructional modules, each with student booklets, teacher's guides, and a 100-card problem deck. The problems are suitable for fifth through eighth grade students. The modules, with focuses such as "Problem Solving Using Resources," are valuable additions to the existing texts.

6. THE READABILITY OF MATHEMATICS MATERIALS

Recently, many individuals have become concerned about the difficulty students have in reading mathematics materials. These concerned people postulate that since text materials have been written at levels above those of most students, more readable books need be adopted. Subsequently, text adoption committees have been formed. A serious problem arises, not in the reasoning cited above, but with the method frequently used in assessing the readability of texts. This method employs the use of one or more readability formulas which provide definitive grade-level estimates. While other readability assessment tools are available, the information that they provide is often dismissed in comparison to the apparent "hard data" of readability formulas. This chapter will discuss the issue of using readability formulas with mathematics texts and describe other tools useful in determining text readability.

READABILITY FORMULAS

A major component of most readability assessment is a text analysis which utilizes readability formulas. Most formulas are regression equations based on factors judged important in reading ordinary English prose. Factors such as sentence length, syllables, word frequency, appearance of familiar words, and the like are emphasized in these formulas. Widely used formulas such as Dale-Chall, Flesch, Fog and Smog manipulate some of these factors and apply them to standard amounts of prose material.

One fallacy of using readability formulas to estimate the reading level of mathematical texts can be seen when one compares the narrative prose used in standardizing these formulas to the expository prose of mathematics. Quite simply, the former is discourse intended to represent a succession of events which includes plot, setting, theme, and so forth. The latter is discourse which expounds or explains or analyzes ideas. While it may be argued that m thematical texts contain limited amounts of narrative prose (word problems, for instance), the bulk of a mathematics text consists of numeration symbols, algebraic notation, tables, graphs, and the like.

The readability formulas cited previously do not deal correctly with mathematical vocabulary since it is not included in the common vocabulary lists (Kane, 1970). While mathematics vocabulary is an important factor for student understanding, "[it] is far outweighed by the difficulty of symbolism of mathematics" (Klum, 1973, p. 651). Unfortunately, the readability formulas commonly used ignore mathematical symbols ($\sqrt{\ }$) and sentences ($5x + 7 = 8x - 9$). Finally, as Klum (1973, p. 651) notes, "The variables that make mathematical material difficult to read are different from those affecting the reading difficulty of ordinary English." Therefore, we need to reconsider the practice of using readability formulas intended for estimating the reading levels of narrative prose in mathematical materials.

Recently, attempts have been made to develop suitable text assessment procedures for mathematical texts. One such formula was designed by Kane, Byrne, and Hater (1974, pp. 30–34) for use with textbooks in grades 6–9. In addition to counting words, this formula also counts tokens. Mathematical tokens ". . . are signs which appear in the language of mathematics which are not word tokens, punctuation, or drawings such as $\sqrt{\ }$, 2, ⊦, ², and %" (Kane, *et al.*, 1974, p. 32). The independent variables included in this formula are:

A = Number of words not on the Dale List of 3000 Words that are also not on the list of Mathematics Words Familiar to 80% of 7th-8th Grade students (Kane, Byrne, Hater, 1974, Appendix A)

B = Number of changes from a word token to a math token and vice-versa.

C = Number of different math terms not on the 80% Math List plus the number of different math symbols not on the 90% Symbols List. (Kane, *et al.*, Appendices A, G)

D = Number of questions marks

These variables are then manipulated within the following equation: Predicted Readability = $-0.15A + 0.10B - 0.42C - 0.17D + 35.52$. This result, sampled from a passage of 400 tokens (words and symbols) is designed to correlate with the number of correct responses on a 75 item Cloze procedure. (Cloze procedure will be discussed in the next section of this chapter). In general, the higher the formula score, the easier the reading level of the passage.

Klum has developed a similar readability formula specifically for algebra material (Klum, 1971). The Klum formula uses percent of math symbols, percent of reader directed sentences, percent of math vocabulary words, and the average length of sentences as independent variables.

Both formulas cited here account for mathematical symbols and mathematical expressions—the major flaw of omission in other readability formulas. While there is not sufficient data to translate predicted readability into grade equivalents, the values are seemingly appropriate for comparisons between materials.

Even though these readability formulas do not identify grade levels, their application may provide a relative comparison of reading difficulty for several texts. However, there is little data to prove or disprove this conjecture. For example, Garstka (1977) found no significant difference in student achievement in algebra content over three sets of materials constructed for consistently different levels of difficulty using the Flesch, Kane, and Klum readability formulas. Too often the results of a single formula are accepted without question when in fact different results among formulas are quite common. Instances such as this indicate the questionable validity of formula results.

While it may seem logical to apply formulas designed for mathematics reading, even these formulas provide little meaningful information useful in the selection of a textbook. Until valid formulas are developed or existing ones refined, a teacher's time will be better spent examining a text in reference to the ways students use it.

CLOZE PROCEDURE

Taylor (1953) introduced a readability technique that is remarkably different from the regression equation format of most readability formulas. Taylor called this technique the "Cloze procedure." The Cloze procedure is based upon the learning theory of Gestalt psychology. That is, a learner is able to conceptualize a complete event through the interrelationships of its component elements. In other words, the parts of an object sum to its total.

Taylor was able to apply Gestalt theory to readability. He did this by systematically deleting every n-th word from a prose passage, replacing each deletion with a blank of standard length, and then requiring the reader to supply the deleted words. The reader accomplished this task through the use of syntactic and semantic clues that remained in the passage. "Clozure" took place after the reader supplied a sufficient number of deleted words to understand the passage. Eventually, Taylor and other researchers standardized the directions for making a Cloze procedure. They are:

1. Select a sufficiently large sample (between 250 and 300 words) from a textbook.
2. Reprint the first sentence exactly as it appears in the text.
3. Randomly select one of the first 5 words in the second sentence. Delete this word and replace it with a blank space of consistent length (10 spaces). Continue to delete every fifth word so that there are 50 deletions. Complete the sentence in which the fiftieth blank occurs.
4. Reprint the next sentence exactly as it appears in the text.

The beginning of a Cloze procedure would look like this:

INTEGERS

Choose any point on a line. Label __________ __________ A and assign it

__________ number 0. Then 0 __________ called the

coordinate of __________. Choose a point to __________

right of A on __________ line. Label this point __________

and assign it the __________ 1.*

Students should be given sufficient time to complete the Cloze procedure. If students are unfamiliar with the Cloze procedure, it may be helpful to put an example on the chalkboard to show them how the use of syntax and semantics can aid them in supplying the missing words. Please note that the consistent use of 10 spaces per blank does not give students clues as to the length of the words deleted. The teachers should also tell students that each blank space represents only one word and that they should attempt to fill all of the blanks. When students finish the Cloze procedure, correct answers are totaled and multiplied by two in order to obtain a percentage score.

Research conducted by Bormuth (1967) has shown that Cloze scores are valid measures of comprehension. Bormuth then compared Cloze scores to multiple choice test scores. He concluded that (1) Cloze scores falling below 40 percent were comparable to multiple choice test scores below 75 percent, (2) Cloze scores ranging between 42 percent and 58 percent were comparable to multiple choice test scores between 76 percent and 89 percent, and, (3) Cloze scores exceeding 60 percent were comparable to multiple choice test scores above 90 percent. Bormuth inferred that (1) a Cloze score below 40 percent indicated that the reader experienced extreme difficulty in understanding the passage, (2) a Cloze score between 42 percent and 58 percent indicated that the reader understood the passage and that it was suitable for instructional purposes, and (3) a Cloze score above 60 percent indicated that the reader easily understood the passage and that it could be read independently.

*Based on K. J. Travers, L. C. Dalton, and V. F. Brunner, *Using Algebra* (River Forest, Ill.: Laidlaw Brothers, 1974), p. 8

APPLYING THE CLOZE PROCEDURE TO MATHEMATICAL TEXT

Hater (1969), Kane, Byrne and Hater (1974), and Hater and Kane (1975) suggest the use of the Cloze procedure as substitutes for tests of comprehension over mathematics material. These directions are similar to those of Taylor and other researchers that are listed in the preceding section of this chapter. These directions differ, however, in that they take into account the variations unique to mathematical writing. In constructing a mathematics Cloze test, the number of deletions is increased from 50 to either 75 or 100. This increase allows more student involvement with the passage and greater familiarity with its ideas. Math tokens as well as words are deleted. Thus, numbers and symbols—prime elements in mathematical understanding—are given proper weighting in student understanding of the passage. Deleted math tokens are replaced with shorter blanks (4 spaces rather than the standard 10 spaces which replace deleted words) to accommodate contextual demands. And, "Tokens are ordered according to the words used to read them. For example, 1/5 can be read 'one-over-five.' Therefore, these tokens are thought of as ordered 1, —, 5" (Kane, Byrne, and Hater, 1974, p. 19). A 26-item Cloze test which applies these mathematical modifications follows as Sample 1. Readers are urged to attempt it in order to acquaint themselves with the Cloze procedure. Remember that each blank represents only one word or math token.

SAMPLE 1
CLOZE PROCEDURE—EQUATIONS: $x + b = c$

We can use the addition property of equality to solve certain types of equations.

Let a, b, and ____ be any real numbers:

__________ a = b, then ____ + c = b ____ c.

We will now __________ this property to simplify ____ − 3 = 5.

____ − 3 = 5	__________
x + (− ____) = 5	Subtraction __________
[x + (____3)] + ____ = 5 + 3	__________ property of =
[____ + (−3____] + 3 = ____	Addition
x + [____ −3) + ____] = 8	Associative __________ of +
x + ____ = 8	Inverse property __________ +
x = 8	__________ property of +

So, __________ x − 3 = ____, then x = 8. __________ solution of x = ____ is obviously 8. But 8 is also the solution of *each* equation in the chain of equations above!

Cloze answers: (1) c, (2) If, (3) a, (4) +, (5) use, (6) x, (7) x, (8) given, (9) 3, (10) property, (11) —, (12) 3, (13) addition, (14) x, (15)), (16) 8, (17) (, (18) 3, (19) property, (20) 0, (21) of, (22) Identity, (23) if, (24) 5, (25) The, (26) 8.

Even though the Cloze procedure, "Equations: $x + b = c$," deviates from the standard number of items specified for mathematical texts, an examination of it will show how the reader gets information from the text to supply missing word and math tokens. Of the 26 deletions, 10 are words and 16 are math tokens. Four of the 10 words are function words [if (2), of (21), if (23), the (25)]. Function words are few in number yet high in frequency of occurrence. Therefore, the reader has an excellent chance of supplying function words when they are deleted. The remaining six are content words [use (5), given (8), property (10), addition (13), property (19), identity (22)]. "Use," deletion 5, was used previously in the first sentence. "Given" deletion 8, initiates the standard "proof" format. While it does not appear elsewhere in the Cloze test, students familiar with the proof format can supply it from prior experience. "Property," deletions 10 and 19, is repeated four times in the Cloze test. "Addition," deletion 13, is also repeated in the Cloze test. "Identity," deletion 22, can be supplied by student knowledge of the identity property of addition. This property was introduced on page 34 of the text and student progression through the textbook required numerous applications of it. Of the 16 math tokens, 7 are numbers and 9 are symbols. All of the math tokens are sufficiently repeated within the text of the Cloze procedure.

An analysis such as this points out the "close" reading that students do when they take a Cloze procedure. Student attention and concentration are at high levels, requiring careful text examination to supply missing word and math tokens. Teachers should consider using a Cloze procedure to show students that the reading demands of mathematics texts are much more rigorous than those required of ordinary English. Therefore, mathematical discourse needs to be savored rather than rushed through haphazardly.

In an earlier section of this chapter, teachers were cautioned against using readability formulas to estimate reading levels of mathematics texts. A similar caution needs to be raised regarding the use of Cloze procedure. Specifically, the Cloze procedure, when applied to ordinary English, reliably predicts student understanding of material to be read. Students are able to make clozure by applying knowledge of syntactic and semantic constraints common to ordinary English. When used with mathematical prose, the Cloze procedure may also predict student understanding. However, low Cloze scores, those which indicate frustration, may be the result of insufficient mathematical background rather than the inability to apply mathematical knowledge. For example, the Cloze procedure presented earlier was taken from Chapter 3, Section 4 of *Using Algebra*. In order for clozure, students must apply the knowledge they learned in previous chapters, through in-class work, from assignments, and so forth. A student who was not exposed to this prior learning would probably do very poorly on the Cloze selection. Therefore, if the Cloze procedure is to be used in the textbook adoption or selection process, it is important that representative Cloze passages be given to students with sufficient mathematical knowledge about the concept being addressed.

The analysis of Cloze items in this section should also serve as a caution when teachers give reading assignments. Students who are inadequately prepared for the demands of a reading task are more likely to become frustrated. Student frustration can lead to superficial attention, apathy, or outright refusal to perform reading tasks. Therefore, the length of an assignment should be reasonable and should be prefaced with a strategy for completing the reading. The second and third chapters, "The Instructional Framework" and "The Structured Overview," provide useful guidelines.

EVALUATION CRITERIA

Krause (1976) has developed a list of 18 items that teachers should consider in evaluating a textbook. These considerations are comprehensive in that some items are applicable to all reading materials while others are better suited to specific subject areas. Some of Krause's considerations, appropriate for evaluation of mathematics materials, are: (1) awareness of concept density and complexity within mathematical or prose sentences, (2) use of understandable terminology, (3) emphasis of new, specific, or difficult terminology, (4) provision of instructional strategies for teaching new, specific, or difficult words, (5) main idea clearly stated at the beginning of the chapter, and summary statements at the end, (6) inclusion of questions, problems, or activities reflecting different levels of comprehension, and, (7) text references to everyday experiences.

Krause's list also includes several items which reflect the students' knowledge of the textbook as a reference tool. Since mathematics textbooks are sources of information as well as explanations and resource tools, teachers should provide instruction in textbook usage. Student knowledge of this skill is important for their success in the mathematics curriculum. One strategy to teach textbook usage is the textbook survey.

TEXTBOOK SURVEY

The textbook survey (Shepard, 1973; Earle, 1976) is an easily constructed, informal instructional tool. Its purpose is to acquaint students with their textbooks soon after they receive them. Time spent on helping the student become familiar with a text—its content, style, and organization—is just as important as the preparatory work needed to help a student use a calculator or a protractor effectively. To use a calculator to its fullest, the student must learn the correct sequencing of keystrokes, the calculator's order of operations, and the procedures for storing and recalling data from memory.

Instruction in the use of the textbook as a learning tool and also as a reference source for previously learned material should not be overlooked. A textbook survey can provide this instruction. Items on the survey are representative of those skills needed to read a basic text. These skills (Shepard, 1973, p. 29) would incude: (1) referencing (use of the table of contents, index, glossary, appendices, etc.), (2) vocabulary knowledge, (3) symbol knowledge, (4) translation (words to symbols, symbols to words), and (5) understanding relationships in formulas and equations.

When administering the textbook survey, the teacher should explain the purpose of the survey and give students sufficient time to complete it. The teacher may wish to adminster parts of it over several days. In this way, fatigue is reduced to a minimum and attention is maximized. An example of a textbook survey for *Using Algebra* by Kenneth Travers, Leroy Dalton, and Vincent Brunner (Laidlaw Brothers, 1974) follows as Sample 2.

SAMPLE 2

PART I.

Directions: Using your textbooks, answer the following questions.

1. How many chapters are in the book? ______________________

2. Who are the authors of *Using Algebra?* ______________________

____

3. A. For an angle of 66°, what is the sine? __________ cosine? __________

   tangent? __________

   B. On what page did you find this information? ______________

4. A. What is the square root of 145? ______________________

   B. On what page did you find this information? ______________________

5. In which chapter will you learn to factor polynomials? ______________________

6. A. What is the rule for multiplying a positive and a negative number? ______________________

   B. Where did you find it in the text? ______________________

7. In which chapter will you learn about Trigonometry? ______________________

8. On what page will you look in Chapter 6 to see if you are ready to take a chapter test? ______________________

9. Although each chapter discusses a different topic, there are some things that all chapters have in common. What are they? ______________________

10. Throughout the book are puzzles, games and brainteasers. What page indicates where these "Special Topics" are found? ______________________

11. On what page will you find the following symbols introduced?

    $\cap$, $\cup$ __________ $\sqrt{\ }$ __________

    $f{:}x \rightarrow y$ __________ $0.\bar{0}$ __________

12. Are the answers to the problems in the book? __________ If so, which problems are answered and on what pages will you find them? ______________________

13. What makes the definitions in the book stand out on the page? ______________________

14. Using the Table of Contents, list five words or terms that you are not familiar with. List sections in which that phrase or word is listed.

Word	*Section*
(1) __________	__________
(2) __________	__________
(3) __________	__________
(4) __________	__________
(5) __________	__________

15. Write a definition for each of the five words or terms that you listed above.

(1) ______

(2) ______

(3) ______

(4) ______

(5) ______

16. What is the sum of $\frac{11}{24} + \frac{7}{20}$? ______

17. On what page are examples of addition of fractions found? ______

PART II.

Directions: Using the page numbers listed with each question, answer the following questions.

1. Listed alphabetically below are some operations of real numbers. Give an example of each with its Inverse. (p. 188)

A. Addition
B. Division
C. Multiplication
D. Subtraction

______

2. There are two methods to multiply polynomials. What are they? (p. 216)

(1) ______

(2) ______

3. On a coordinate plane (p. 264), which is the

x — axis? ______

y — axis? ______

4. When an equation (p. 282) is in the form $y = mx + b$,

what does m represent? ______

what does b represent? ______

5. If a and b are any real numbers, $b \neq 0$; and n is a positive integer, then what does $\left(\frac{a}{b}\right)^n$ equal? (p. 342) ______

6. There are two types of problems involving time. What are they? (p. 394)

(1) ______

(2) ______

7. Page 64 discusses the distributive property. Let a, b, and c be any real numbers, and complete the statement

$a \cdot (b + c) =$ ______________________________ (p. 64)

8. Let a, b, and c be any real numbers.
If a = b, then a + c = b + c. This is an example of what property? (p. 82) ______________________________

9. What is a prime number? (p. 155) ______________________________

10. What do the letters in FOIL stand for? (p. 219)

F — ______________ I — ______________
O — ______________ L — ______________

A similar format for teachers to consider is the chapter survey. The chapter survey, as its name implies, focuses on smaller segments of text material. It is not nearly as long, consisting of between 10 to 15 items, but serves as an effective motivational tool and study aid. The focus of the chapter survey is on such things as important vocabulary, charts, tables, formulas, references and the like. Students, in effect, preview a chapter under the teacher's direction rather than proceeding haphazardly. A teacher-led discussion, centered on the survey, thus provides an overview on the topics to come.

MATH TEXTS WITH MINIMAL READING

While the argument is appealing that reading can be omitted to help many students learn mathematics, the authors strongly condemn this reasoning as unsound and ultimately disastrous for students. Presenting mathematics to a student without the need to do some reading may help on a particular lesson, but this technique curtails the student's ability to become an independent learner of mathematics. Consequently, the student could become totally dependent on teachers or interpreters for future use of mathematics.

After the initial developmental reading instruction in the primary school, the motivation of students to read centers on the desire to learn something that can be found in print. In daily life, there are fewer tasks now than in the past that require reading. Radio, TV, and movies present much information about the world. Mathematics, because of its high density of information in symbols and notation, is not likely to be transformed from print to oral communication outside of the classroom. Thus, by avoiding the necessity to read mathematics material, teachers can limit the student's ability to learn new mathematics or relearn forgotten material outside of a school setting.

SUMMARY

The issue of the student's ability to read mathematics textbooks is one of national concern. It frequently arises when school systems go about the process of adopting or selecting a textbook. In response to this need, this chapter discusses several ways and criteria for evaluating textbooks for the purpose of determining their suitability for instruction.

The issue of readability, however, is not limited to text selection. It frequently appears as an in-service topic. Undoubtedly, some teachers reading this book have been herded into auditoriums or cafeterias, along with their colleagues in other subject areas, supposedly to analyze the reading levels of their textbooks. Unfortunately, most of these in-service programs dwell exclusively on instruction in the use of readability formulas. Such in-service instruction offers very little useful information. This chapter has attempted to point out the problem of using readability formulas to estimate reading levels of mathematical texts. These formulas at best indicate superficially the grade level and at worst provide information void in instructional application. Those planning readability in-service programs would provide teachers a greater service and more meaningful information by acquainting them with the Cloze procedure and the textbook survey.

Finally, the current trend of removing or reducing the amount of reading in mathematics texts must be seriously questioned. While texts with minimal reading appear to solve an immediate problem for the teacher, this format creates at least two critical problems for the students. First, the students are forced to rely on the teacher, or other students, for an explanation of a topic. Second, the students are not improving their chances to learn or review mathematical skills or concepts independently. Without practice in reading mathematics, the students will be less likely to review a topic or to study a new one from a written source.

7. CONCLUDING REMARKS

Earlier, reading was defined as thought guided by printed symbols. This definition is particularly valid in the reading of mathematical discourse. Reading mathematics is similar to reading ordinary English prose in that both require the knowledge of letter sound correspondence and denotative or connotative meanings. These elements are necessary conditions for understanding ordinary English prose. By themselves, however, they are not sufficient for understanding mathematical discourse.

The reading phase of mathematics utilizes a very precise notation system of words, numbers, and symbols. In most instances, understanding mathematics requires proficiency and fluency in all of them. This is particularly true as mathematics instruction develops beyond the instruction of basic arithmetic principles. Elementary instruction in the reading process is also developmental in nature. The materials in basic reading instruction reflect a predominant application of ordinary language. Unfortunately, there is a limited student transfer of reading skills from the narrative prose of basal readers to the expository prose of mathematics texts. Specific instruction for reading mathematics, when given by a reading teacher outside the mathematics classroom, typically consists of artificial workbook activities that result in a minimum of transfer to the mathematics curriculum. Therefore, it is the mathematics teacher in the mathematics classroom who should give the instruction for reading mathematics.

This monograph has identified useful teaching strategies and learning activities which enhance mathematics understanding. The strategies and activities by themselves are not a panacea. Rather, they serve as examples upon which mathematics teachers can broaden their instructional practices. This monograph also provides suggestions for instruction in the reading phase of mathematics. Hopefully, instructional emphases on reading and interpreting mathematical discourse will help students become more independent learners of mathematics.

ADDITIONAL MATHEMATICS/READING RESOURCES: TEACHER FOCUS

The listing which follows consists of journal articles, publications, skill development kits, booklists and other sources useful in helping teachers locate and select additional mathematics/reading resources. This list is by no means comprehensive. It is suggested that teachers use it as the nucleus of a resource file for mathematics/reading materials. Please note that while many of these items could be read independently by students, it is recommended that they be used in the context of an instructional setting. That is, it is the opinion of the authors that students will gain more information from them when teachers incorporate them within a mathematics lesson rather than having students read them without direction.

Aiken, Lewis R., Jr. "Mathematics as a Creative Language." *Arithmetic Teacher* 24 (March 1977):251–55.

Applications in School Mathematics. (1979 Yearbook). National Council of Teachers of Mathematics. Washington, DC: NCTM, 1979.

Audio-Reading Progress Laboratory Levels 7 and 8. Educational Progress Corporation, P.O. Box 45633, Tulsa, OK 74145.

Balow, Irving H. "Reading and Computation Ability as Determinants of Problem Solving." *Arithmetic Teacher* 11 (January 1964):18–22.

Bausch and Lomb, Inc., Rochester, NY 14602, Manufacturers of lenses, microscopes, binoculars, and other scientific equipment.

Bibliography of Recreational Mathematics, A. Useful list for building a classroom library. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Butts, Thomas. *Problem Solving in Mathematics; Elementary Number Theory and Arithmetic*. Glenview, IL: Scott, Foresman and Company, 1973.

Catalog of Free Teaching Materials. Riverside, CA: Rubidoux Printing Co., P.O. Box 1075, Ventura, CA 93001.

Consumer Buying Prospects. Commercial Credit Co., 330 St. Paul Place, Baltimore, MD (Quarterly).

Consumer Price Index, The. U.S. Department of Labor, Bureau of Labor Statistics, 441 G Street, N.W., Washington, DC 20212 (Monthly).

Creative Publications, Box 328, Palo Alto, CA 94302.

Earle, Richard A. *Teaching Reading and Mathematics.* 1976. International Reading Association, 800 Barksdale Road, Newark, DE 19711. Also available from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Earp, N. Wesley. *Reading in Mathematics.* 1970. ERIC Document Reproduction Service, Box 190, Arlington, VA 22210 (ED 036 397).

Experiences in Mathematical Ideas, Vol. I, II. National Council of Teachers of Mathematics. Reston, VA: NCTM, 1970.

Finance Facts. National Consumer Finance Association, 1000 16th St., N.W. Washington, DC 20036 (Monthly)

Free and Inexpensive Learning Materials, 15th biennial ed. Order from: George Peabody College for Teachers, Division of Surveys and Field Services, Nashville, TN.

Free and Inexpensive Pictures, Pamphlets and Packets for Air/Space Age Education, 6th ed. Order from: National Aerospace Education Council, 806 15th St., N.W. Washington, DC 20005.

Free Learning Materials for Classroom Use. An annotated list of sources with suggestions for obtaining, evaluating, classifying, and using. Order from: The Extension Service, State College of Iowa, Cedar Falls, IA.

Freeman, George F. "Reading and Mathematics." *Arithmetic Teacher* 20 (November 1973): 523–29.

Futurists, The. World Future Society, P.O. Box 19285, Twentieth St. Station, Washington, DC 20036

General Motors Corp., Public Relations Staff, Room 1-101, General Motors Building, Detroit, MI 48202.

Go Scholastic Book Services, Inc., 904 Sylvan Ave., Englewood Cliffs, NJ 07632.

Gruenberger, Fred and Jaffray, George. *Problems for Computer Solution.* New York: John Wiley and Sons, Inc., 1965.

Hater, Mary Anne, Kane, Robert B. and Byrne, Mary Ann. "Building Reading Skills in the Mathematics Class." *Arithmetic Teacher* 21 (December 1974): 662-68.

High School Mathematics Library. 800 titles, includes magazine titles and publishers directory. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Historical Topics for the Mathematics Classroom (31st Yearbook). National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091, NCTM, 1969.

Iowa Problem Solving Project, Price Laboratory School, Cedar Falls, IA 50613.

John Hancock Mutual Life Insurance Company, 200 Berkeley Street, Boston, MA 02117.

Kane, Robert B., Byrne, Mary Ann, and Hater, Mary Ann. *Helping Children Read Mathematics*. New York: American Book Company, 1974.

Kidd, Kenneth P., *et al. The Laboratory Approach to Mathematics*. Chicago: Science Research Associates, Inc., 1970.

List of Materials Available to Secondary School Instructors. Order from: B. A. Schuler, Educational Service Bureau, Dow Jones and Co., Inc., Princeton, NJ 08540.

Mapmakers. Aero Service Cap, 210 E. Courtland St., Philadelphia, PA 19120 (Quarterly).

Mosteller, Fredrick, *et al. Statistics by Example*. Reading, MA: Addison-Wesley Publishing Co., 1973.

Mathematics Library—Elementary and Junior High School. Annotated Bibliography of mathematics books for classroom library. Includes grade level K-9. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Pachtman, Andrew B., and Riley, James D. "Teaching the Vocabulary of Mathematics Through Interaction, Exposure, and Structure." *Journal of Reading* 22 (December 1978): 240–244.

Paperback Goes to School, The. Annual list of paperback titles considered useful for classroom and supplementary use by a joint committee of the National Education Association and the American Association of School Librarians. Bureau of Independent Publishers and Distributors, 122 E. 42nd St., New York, NY 10017.

Reading and Study Techniques for Academic Subjects. Baldridge Reading Instruction Materials, 14 Grigg Street, Greenwich, CN 06830.

Road Maps of Industry. National Industrial Conference Board, 845 Third Ave., New York, NY 10022 (Semi-monthly).

Science and Security. Harris, Upham & Co., 120 Broadway, New York, NY 10004 (Qu terly).

Science Booklist for Young Adults. American Association for the Advancement of Science, 1515 Massachusetts Ave., N.W., Washington, DC 20005.

Selected Free Materials for Classroom Teachers. Order from: Fearon Publishers, Inc., 2165 Park Blvd., Palo Alto, CA 94306.

Sobel, Max A. and Maletsky, Eva. M. *Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies*. Englewood Cliffs NJ: Prentice-Hall, Inc., 1975.

Sources of Free Teaching Aids. Order from: Bruce Miller Publications, Box 369, Riverside, CA 92502.

Sources of Teaching Materials. Order from: Catherine Williams, Ohio University Press, Columbus, OH 43210.

Space Mathematics: A Resource for Teachers. National Aeronautics and Space Administration (NASA). Washington, DC: Government Printing Office, 1972.

Thinking Box. Benefic Press, 10300 W. Roosevelt Road, Westchester, IL 60153.

U.S. Atomic Energy Commission, Technical Information, Oak Ridge, TN 37830.

RECREATIONAL/SUPPLEMENTARY RESOURCES: STUDENT FOCUS

The listing which follows is a small sample of mathematics sources that students have found interesting and comprehensible for a variety of specific purposes. The authors know of individual students who have learned from each source. No single source wold appeal to every student. More important, few mathematics sources are read from cover to cover. Typically, a user will seek information on a specific topic. That is, students must learn that mathematics sources are reference tools which are often read in ways very different from ordinary prose material.

Abbott, Edwin. *Flatland*. New York: Dover Publications, 1970.

Alder, Irving, ed. *Readings in Mathematics*, Books 1, 2. Lexington, Mass.: Ginn and Co., 1972.

Asimov, Isaac. *Asimov on Numbers*. New York: Pocket Books, 1977.

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Campbell, Stanley K. *Flaws and Fallacies in Statistical Thinking*. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1974.

Creative Computing. Morristown, New Jersey: Creative Computing, P.O. Box 789-M (monthly magazine).

Dwyer, Thomas A. and Critchfield, Margot. *BASIC and the Personal Computer*. Reading, Mass.: Addison-Wesley Publishing Co., 1978.

Eves, Howard W. *In Mathematical Circles—A Collection of Mathematical Stories and Anecdotes*. Boston: Prindle, Weber and Schmidt, Inc., 1969.

Fadiman, Clifton. *Mathematical Magpie*. New York: Simon and Schuster, 1962.

Gamow, George. *One, Two, Three . . . Infinity*. New York: Viking Press, 1961.

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———. *Mathematical Circus*. New York: Alfred A. Knopf, Inc. 1979.

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Henle, James M. *Numerous Numerals*. Reston, Va.: National Council of Teachers of Mathematics, 1975.

Jacobs, Harold. *Mathematics—A Human Endeavor*. San Francisco: W. H. Freeman and Co., 1970.

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Kastner, Bernice. *Applications of Secondary School Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 1978.

Kline, Morris. *Mathematics and the Physical World*. New York: Thomas Y. Crowell Co., 1959.

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Levinson, Horace. *Chance, Luck and Statistics: The Science of Cnance*. New York: Dover Publications, Inc. 1963.

Luckiesch, M. *Visual Illusions—Their Causes, Characteristics and Applications*. New York: Dover Publications, 1965.

Lyng, Merwin J. *Dancing Curves: A Demonstration of Geometric Principles*. Reston, Va: National Council of Teachers of Mathematics, 1978.

Margenau, James and Sentlowitz, Michael. *How to Study Mathematics*. Reston, Va: National Council of Teachers of Mathematics, 1977.

Mathematics in the Modern World: Readings from "Scientific American." San Francisco: W. H. Freeman & Co., 1968.

Menninger, K. W. *Mathematics in Your World*. New York: Viking Press, 1962.

National Council of Teachers of Mathematics. *The Mathematics Student*. Reston, Va: NCTM (A journal published six times per year for secondary students).

———. *Mathematical Challenges*. Reston, VA: NCTM, 1965.

———. *Mathematical Challenges II Plus Six*. Reston, VA: NCTM, 1974.

New Mathematical Library. New York: Random House, Inc., 1961–present. (A set of at least 25 individual volumes for senior high student exploration).

The Paradox Box. San Francisco: W. H. Freeman & Co., 1975. (Filmstrip-booklet series with paradoxes in logic, probability, number, geometry, statistics, and time).

Runion, E. and Lockwood, James R. *Deductive Systems: Finite and Non-Euclidean Geometries*. Reston, Va: NCTM, 1978.

Osen, Lynn M. *Women in Mathematics*. Cambridge, Mass.: MIT Press, 1974.

Polya, George. *How to Solve It*. Garden City, N.Y.: Doubleday and Co., 1957.

Sawyer, W. W. *The Search for Pattern*. Baltimore: Penguin Books, Inc., 1970.

Singh, Jagit. *Great Ideas of Modern Mathematics*. New York: Dover Publications, Inc., 1959.

Smith, David Eugene. *A Source Book in Mathematics*, Volumes 1, 2. New York: Dover, 1959.

Stevens, Peter. *Patterns in Nature*. Boston: Little, Browne Co., 1974.

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Clayton, Kathleen K. "Word Recognition Skills for the Junior High School." In *Forging Ahead in Reading*, Vol. 12, Part 1. Edited by J. Allen Figurel. Newark, Del.: International Reading Association, 1968, pp. 59–62

Doblaev, L. P. "Thought Processes Involved in Setting Up Equations." In *Soviet Studies in the Psychology of Learning and Teaching Mathematics, Volume III Problem Solving in Arithmetic and Algebra*. Edited by Kilpatrick and Wirszup. Chicago: University of Chicago, 1969. (Available through National Council of Teachers of Mathematics.)

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