DOCUMENT RESUME

ED 206 068 EA 013 805

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TITLE The Explanatory Power of Two Idealized Models of Educational and Social Attainment.
PUB DATE Apr 81
NOTE 47p.: Paper presented at the Annual Meeting of the American Educational Research Association (Los Angeles, CA, April 13-17, 1981).

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Educational Attainment; *Educational Benefits; Educational Demand; *Mathematical Models; Socioeconomic Status; Statistical Analysis; Statistical Distributions
IDENTIFIERS *Aggregate Model; *Probabilistic Utility Model

ABSTRACT

Increasing the level of educational attainment of the population may not always increase the social benefits for those who receive the education. In fact, when a high percentage of the population attains a certain level of education, that attainment ceases to provide socioeconomic benefits; instead, those few who have not reached that educational level find themselves suffering from socioeconomic liabilities. Some of the relationships between educational attainment and socioeconomic benefits are explored in the two mathematical models discussed in this paper. The Aggregate Model rests upon idealized normal distributions of benefits and educational attainment within a meritocratic society. The Probabilistic Utility Model assumes more flexible distributions. The models indicate that, at least in theory, educational policies that seem to promote equity may in fact cause socioeconomic hardships and disequilibrium. (Author/PGD)

THE EXPLANATORY POWER OF TWO IDEALIZED MODELS OF EDUCATIONAL AND SOCIAL ATTAINMENT

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American Educational Research Association
Annual Meeting
Los Angeles, California
April 16, 1981

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I

THE SHIFTING SOCIAL BENEFITS AND LIABILITIES OF EDUCATIONAL ATTAINMENT[1]

A. Introduction

Conventional wisdom, along with aspects of economic theory, has it that as the level of educational attainment in society increases so does the aggregate and individual social benefits for those attaining the more advanced educational levels. This reflects, in great measure, the cherished American belief in the efficacy of educational attainment: educational attainment pays. Recently, this proposition has been thrown into serious doubt.

Raymond Boudon's models of inequality of educational and social opportunities suggest that educational growth has the effect of increasing economic inequality, assuming that income is dependent upon educational attainment. Boudon warns that we should not expect the development of the educational system to have positive effects upon economic equality. The effect is more likely to be a negative one and the growth of the educational system may be to some extent responsible for the "persistence of economic inequality" in certain Western industrial societies.[2]

Boudon's models also suggest that as inequality of educational opportunity decreases, the educational system expands and that this expansion leads to increased inequality of social opportunity. While the average level of educational attainment in the population increases, the educational levels that are associated with particular status expectations are "simultaneously moving upward." Thus, as individuals demand more and more education over time, the individual return tends to be nil while the aggregate return on this demand is high. The lower socioeconomic classes are compelled to demand more education, for not to do so condemns them to constantly falling social status expectations. However, more educational demand only retards this diminution in status and does not increase the lower classes' chances of achieving increased social status. Educational attainment becomes socially compulsory.

Lester C. Thurow reaches some of the same conclusions from a different perspective. He uses a "wage competition" model (in contrast to the standard "job competition" model) to explain why the distribution of education in the American population has moved in the direction of greater equality since 1945 while the distribution of income has not followed suit. His empirical data and job competition model illustrate the way in which educational attainment becomes a "defensive necessity." According to Thurow's analysis, the more rapidly the class of educated labor grows, the more such "defensive expenditures" on educational attainment becomes imperative. Rather than depend upon educational programs to stimulate more social equity, Thurow would initiate a "frontal attack on wage differentials."[3]

These two analyses suggest that there might be something inherent within the structure and logic of the educational system itself which mitigates against a concomitant increase in economic/social equity with increasing educational attainment and which makes educational attainment socially compulsory. It is this "inherent something" that I explore in this paper.

What Boudon and Thurow lack, and which is now available, is a comprehensive account of the logic and behavior of national educational systems.[4] This paper explores just how the growth of the educational system, in and of itself, affects the relationship between educational and social attainment. I develop two logico-mathematical models which illustrate the interaction between two systemic laws of behavior and a normative principle connecting the social and educational systems.[5]

The remainder of this Part is devoted to an explication of the systemic laws and normative principle. Part II presents the Aggregate Model and Part III develops the Probabilistic Utility Model. My aim is to render in a quantitative manner aspects of a conceptual descriptive/explanatory theory of the educational system. I want to examine the power that systemic growth has over the relationship between educational and social attainment.

B. A Distributive Perspective

A student who leaves school in the middle of the school year in one part of the country and who enters the same grade in a distant part of the country can generally find nearly identical curricula, procedures and facilities. Clearly, some kind of system exists. It is useful to distinguish between a system of education, which comprises all of the many ways a society educates and socializes its citizens, and an educational system, which satisfies the primary and secondary properties described below.

The educational system's primary features are threefold. First, the system is composed of schools and colleges, but not all schools and colleges. Second, these schools and colleges are related by a "medium of exchange" which includes those certificates, degrees, diplomas, letters of recommendation and the like, which allow persons to leave any level of the system in one locality and enter the same level in another. They are all instruments by which activities carried out in one place can be recognized and "exchanged" for similar activities of a school or college in some other place.

Third, by "educational system," I mean those schools and colleges that are connected by a medium of exchange and that are arranged by the Principle of Sequence. The principle states that these schools and colleges are organized into levels so that if a person has attained (i.e., completes) level N, then he or she has attained level N-1, but not necessarily level N+1. This principle allows us to speak of persons progressing through the system and appears to be a necessary property of any educational system due, in part, to differing levels of skill accomplishment, knowledge acquisition, and cognitive development of individuals. Completing a level of the educational system is what I mean by educational attainment at that level.

In addition to these primary elements, the educational system has three secondary or derivative properties. The system will have a definite size, a system of control, and will create a distribution of educational goods and second-order educational goods. The perspective for this analysis is a distributive one that includes the notion of systemic size. I will not consider the system of control at this formal level of analysis.

Every society makes some arrangement for the distribution of its goods (i.e., benefits). The educational system distributes educational goods such as knowledge, skills, and certain kinds of taste. In addition to these goods, the educational system distributes their surrogates, called second-order educational goods, such as grades, diplomas and certificates. Some persons, because of their greater ability (however it is defined within the system), tenacity, and acuity of choice, will come to possess a larger share of educational goods than other persons. If it is the case that non-educational social goods such as income, earning opportunities, status and the like are distributed by the socioeconomic system on the basis of the distribution of educational goods (through the instrumentality of second-order educational goods), then there exists a normative principle that links the educational and socioeconomic systems.

This normative principle can be rendered as: "those having a greater share of educational goods merit or deserve a greater share of non-educational social goods." The power or strength of this normative principle can be viewed as a function of the size of the educational system. For the purposes of this analysis, size is taken to be the attainment ratio at the twelfth level of the system: the percentage of 17- or 18-year-olds attaining the high school diploma.[6]

When the system is small (say, 10% attainment ratio), the socioeconomic rewards of high school attainment are likely to be quite negligible. For example, the high school diploma is not likely to be used as a screening prerequisite for job entry. In the aggregate, high school attainers do not monopolize economic opportunities simply because of attainment. Thus, the strength of the normative principle is low. To be a high school dropout when 90% of your age-cohort drops out presents no serious personal or social problem.

As the size of the educational system increases, the strength of the normative principle also increases, in part, because employers begin to utilize high school attainment as a selection criterion. There are, however, logical constraints on the strength of this principle. When all 17-year-olds attain the high school diploma (100% attainment ratio), its mere possession cannot guarantee socioeconomic advantage for anyone in this group. This is due to the tautological Law of Zero Correlation, which is necessarily true at any level of the educational system where the attainment ratio is 100% (or 0%). The Law states that: "There is a point in the growth of the system at which there is no longer any correlation between educational attainment and either the distribution of educationally relevant attributes in the population or the distribution of non-educational social goods ordinarily associated with educational attainment."

This law makes a logical claim and not an empirical one. In order for there to be a correlation between any two variables, both must be distributed in the population under consideration. If one of these variables is uniformly distributed, no correlation can occur. For instance, a society could not distribute any of its goods based upon eye color if everyone had green eyes. Thus, an empirical claim is not only unnecessary, it is inappropriate. It is equally important to note that the Law of Zero Correlation makes a claim about educational attainment and not about educational achievement, which is another matter altogether.

One corollary of the Law of Zero Correlation, the Law of Shifting Benefits and Liabilities, assures that high school attainment will have a declining social value and that, concomitantly, a failure to attain the high school diploma will have an increasing social liability, as the high school attainment ratio moves toward the zero correlation point. Thus, as zero correlation is approached (at 100% attainment), the aggregate social benefits associated with high school attainment decline for members of the attainment group and the aggregate social liabilities of non-attainment increase. These notions are illustrated in Figure 1.

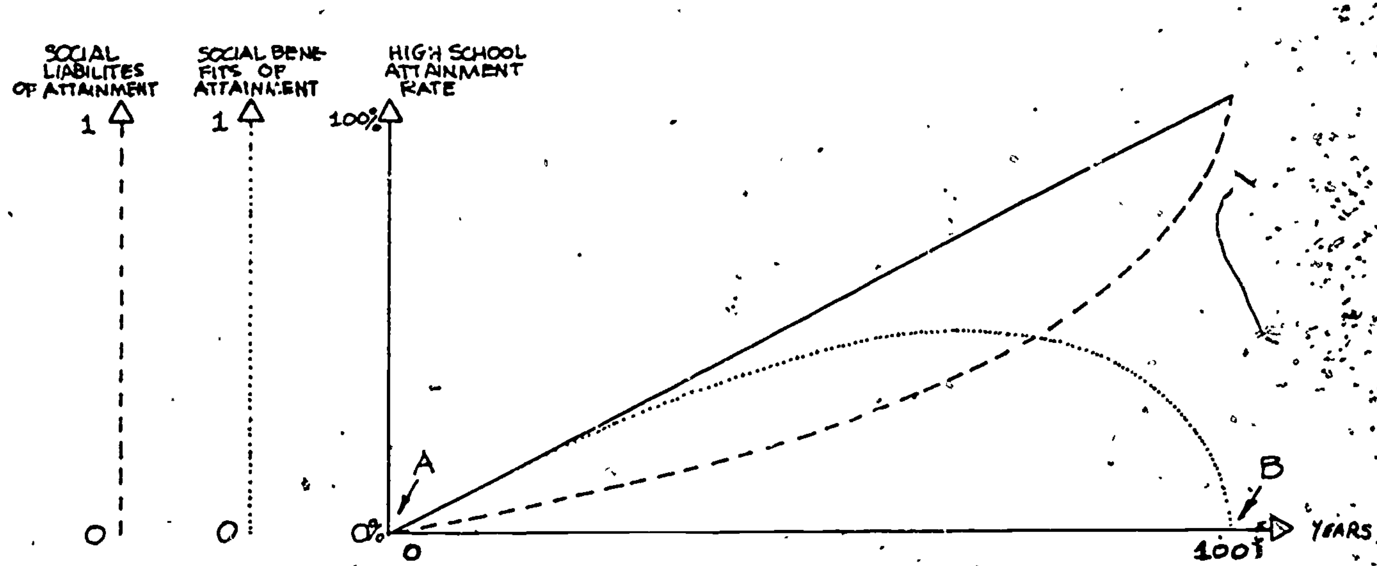


Figure 1. Social Benefit and Liability Curves and a Hypothetical Uniform Growth Line of the High School Attainment Ratio (adapted from Green, op. cit., Figures 6.1, 6.2, 6.3)

In Figure 1, the horizontal axis represents 100 years of time. The solid line, "High School Attainment Ratio," represents the assumption that the educational system grows at a uniform rate of 10% per decade. The dotted curve, "Social Benefits of Attainment," represents the strength of second-order educational goods in securing non-educational social goods. At points A and B, this strength must be zero, due to the Law of Zero Correlation. These two points are conceptually derived, while the actual shape of the benefit curve is a contingent matter. As the attainment ratio increases from 0% to 100%, the benefit curve rises until it peaks and then declines to zero. This illustrates how the power of the normative principle increases and then decreases, although the personal and social belief in it (i.e., the efficacy of educational attainment) may remain steady.

The other side of the benefit-liability coin is the social liability associated with systemic growth. The dashed curve in Figure 1, "Social Liabilities of Attainment," represents the conjecture that at the lower attainment ratios, not having a diploma is not a serious problem. However, as more and more of the age-cohort attains the twelfth level of the system, non-attainment becomes an increasing liability. Even though the benefits once associated with the high school diploma begin to decline, the liabilities of not having it increase. The precise shape of this liability curve is a contingent matter; that it rises is the point I wish to make.[7]

Figure 1 exposes a peculiar paradox. As zero correlation is approached, the aggregate social benefits once associated with high school attainment decline and the aggregate social liabilities of non-attainment increase. Where high school attainment was once a highly sought after good, it now becomes a necessity to be endured. Where school leaving was once a possible and viable consideration, it now becomes an evil to be avoided at all costs. These shifting benefits and liabilities make high school attendance and attainment increasingly compulsory in ways that were surely never meant to be. The personal and social consequences of such a situation can be devastating.

The central point here is that schooling is compulsory because it is (nearly) universal and not universal because it is compulsory. Historical data support this claim.[8] The American system appears to have reached the point of declining benefits, and I suspect that if all compulsory attendence statutes were repealed today, schooling would be just as compulsory tomorrow. Schooling has become defensive.[9]

This analysis can help to shed light on two interesting phenomenon: 1) the growing disparity between high school attainer and non-attainer job entry-level income over time, despite the increase in the attainment ratio and 2) a constant U.S. high school attainment ratio of about 75% since 1965. To help illustrate these phenomena, I utilize a logico-mathamatical model that generates data derived deductively from the properties of a mathematical distribution. I call this the Aggregate Model since the focus is on the aggregate social benefits and liabilities of educational attainment.

II

THE AGGREGATE MODEL AND APPLICATIONS

A. The Model

The following Aggregate Model rests upon three idealized assumptions:

1. non-educational social benefits are always normally distributed in the population under consideration and remain so over time - a change in the high school attainment ratio does not affect the overall normal shape of this distribution;

2. this distribution encompases those who have attained the high school diploma, but who have not gone on in formal schooling (attainers), and those who have not attained the high school diploma (non-attainers);

3. society allocates its social benefits in such a way that the attainers monopolize the upper end of the normal distribution.

The first assumption fixes the overall shape of the distribution and offers a particular view of distributed justice. Perhaps this distribution reflects some overall normally distributed attribute or attributes in the total population under consideration. Assumptions 2 and 3 tell us that the high school attainers can be found, as a group, lumped at the upper end of the distribution. The third assumption, which will be altered in Part III, represents an overly rigid meritocratic society.

These three assumptions are realized in Figure 2, which is a normal distribution in standardized normal form having a grand mean (μ_σ) of zero and a standard deviation (σ) of one. Each asymptote is truncated, for computational purposes, at 3.9 standard deviations from the mean. The high school attainment ratio (ϕ) is represented by the shaded area under the curve. This is the proportion of the total population under consideration that has attained the high school diploma. The median value of the social benefits of this group is μ_ϕ .

The unshaded portion under the curve is the proportion of the total population that has not attained the high school degree ($\bar{\phi}$) and is equal to $1-\phi$. The median value of the social benefit for this group is $\mu_{\bar{\phi}}$.

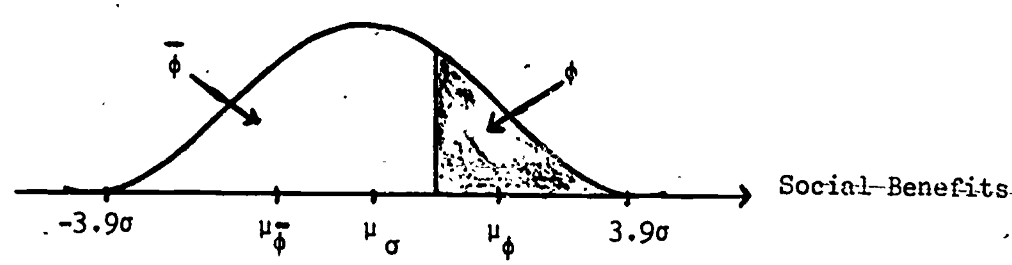


Figure 2. Standardized Normal Curve for the Distribution of Social Benefits (ϕ = high school attainment ratio; $\bar{\phi}$ = non-attainment ratio; $\mu_{\sigma} = 0$ = grand median; μ_{ϕ} = median social benefit for attainer group; $\mu_{\bar{\phi}}$ = median social benefit for non-attainer group; $\sigma = 1$ = standard deviation)

TABLE 1

Median Social Benefits, Their Differences, and Their Rates of Change For Attainer and Non-attainer Groups by High School Attainment Ratio

(1) Size of Attainment Group (ϕ)	(2) Attainer Group Median (μ_{ϕ})	(3) Non-Attainer Group Median ($\mu_{\bar{\phi}}$)	(4) $\mu_{\phi} - \mu_{\bar{\phi}}$	(5) Rate of Change of μ_{ϕ}	(6) Rate of Change of $\mu_{\bar{\phi}}$
0.01	2.575	-0.012	2.587		
0.05	1.960	-0.063	2.023	0.2388	4.2500
0.10	1.645	-0.126	1.771	0.1607	1.0000
0.15	1.440	-0.189	1.629	0.1246	0.5000
0.20	1.283	-0.253	1.536	0.1090	0.3386
0.25	1.150	-0.319	1.469	0.1037	0.2609
0.30	1.037	-0.385	1.422	0.0983	0.2069
0.35	0.935	-0.454	1.389	0.0984	0.1792
0.40	0.842	-0.524	1.366	0.0995	0.1542
0.45	0.755	-0.598	1.353	0.1033	0.1412
0.50	0.675	-0.675	1.350	0.1060	0.1288
0.55	0.598	-0.755	1.353	0.1141	0.1185
0.60	0.524	-0.842	1.366	0.1217	0.1152
0.65	0.454	-0.935	1.389	0.1336	0.1105
0.70	0.385	-1.037	1.422	0.1520	0.1091
0.75	0.319	-1.150	1.469	0.1714	0.1090
0.80	0.253	-1.283	1.536	0.2069	0.1157
0.85	0.189	-1.440	1.629	0.2530	0.1224
0.90	0.126	-1.645	1.771	0.3333	0.1424
0.95	0.063	-1.960	2.023	0.5000	0.1915
0.99	0.012	-2.575	2.587	0.8095	0.3138

Note that the attainer and non-attainer medians change as a function of the attainment ratio. When the ratio (ϕ) is zero, the non-attainer median is equal to the grand median (μ_σ). When the ratio approaches its limit of one, the attainer median approaches the grand median and the non-attainer median approaches -3.9 standard deviations from the grand median. We can easily calculate the values of the attainer and non-attainer medians for different values of the attainment ratio.[10] Table 1 shows their values, their differences and their rates of change for attainment ratios ranging from 0.01 to 0.99. Figure 3 is a plot of the attainer and non-attainer medians by the attainment ratio.

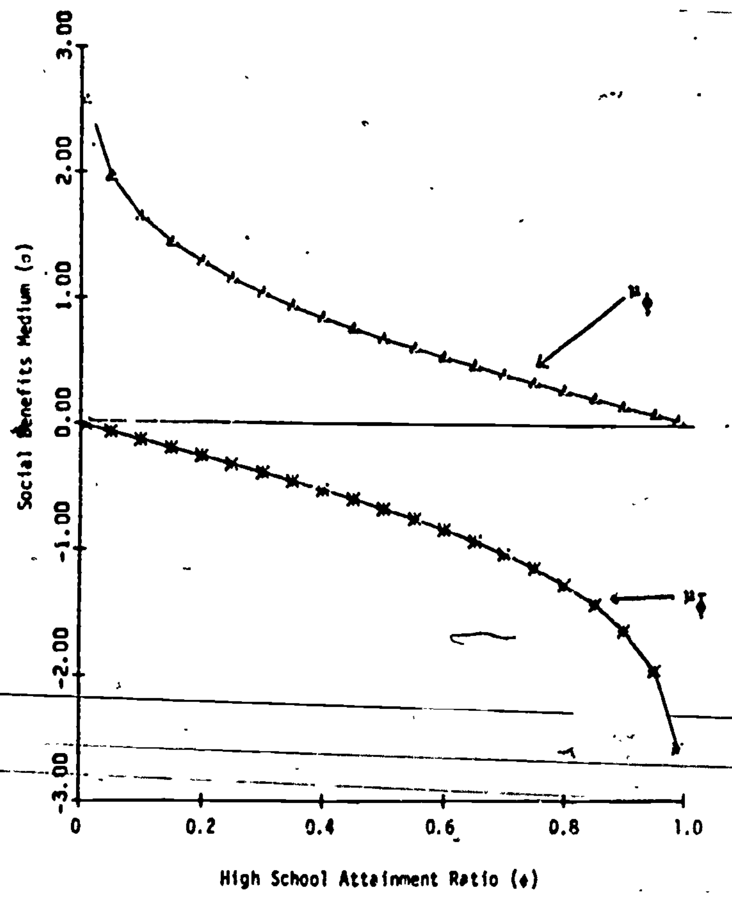


Figure 3. Median Social Benefit of Attainer Group (μ_ϕ) and Non-Attainer Group ($\mu_{\bar{\phi}}$) by High School Attainment Ratio (ϕ) (from Table 1, Columns 2 and 3)

B. An Income Disparity Analysis

A conventional analysis of high school attainer and non-attainer income disparities considers whatever is gained by the attainers to be the magnitude of the liability experienced by the non-attainers. If, for example, the median income of the attainer group is 150% of the non-attainer median income (at a particular attainment ratio), then the benefit to the former group is 50% and the liability to the latter group (in foregone income and earnings opportunities, etc.) is 50%. This approach tends to conceal the full impact of the shifting benefits and liabilities of educational attainment.

Table 1 and Figure 2 display another approach to this situation. Here we find the difference between the median benefit of the attainer group and the median benefit of the entire population under consideration (Table 1, column 2). We do the same for the non-attainer group (Table 1, column 3). The difference between these two grand-median-dispersions is a measure of the relative position of one group with respect to the other (Table 1, column 4).

If we think of such social benefits as income, salary and wages, a conventional supply and demand analysis suggests that as the supply of high school graduates increases, the relative social benefits realized by these graduates, with respect to those with no high school degree, will decline (given a constant market demand for attainers). This is just what happens in the Aggregate Model as the attainment ratio grows from 0.01 to 0.50. However, as the attainment ratio exceeds 50%, the relative advantage of the attainers over the non-attainers increases. See Figure 3.[11]

These latter results of the Model are consistent with certain empirical findings. Time-series U.S. Census data for 18- to 24-year-old males from 1939 (when the national high school attainment ratio was 50%) to 1975 display this phenomenon.[12] A U.S. Senate report which examined the incomes of 24- to 34-year-old males expressed surprise at the "paradox" of increasing relative income for high school attainers over non-attainers.[13]

The interaction between the Law of Zero Correlation and the Law of Shifting Benefits and Liabilities has certain explanatory power when the data areexamined as illustrated in the Aggregate Model. The "paradox," cited above, evaporates in light of these systemic dynamics which show the declining benefits associated with attainment and the increasing liabilities associated with non-attainment as the zero correlation point is approached.[14]

C. Stabilization of the High School Attainment Ratio

What is the meaning of the "intersection" of the benefit and liability curves in Figure 1? Although the two curves do not actually intersect (they have different vertical axes), the "intersection" shown in Figure 1 does illustrate certain interactive systemic effects. This "intersection" can be viewed as an equilibrium point in the growth of the system beyond which it no longer pays (in aggregate social benefit terms) to finish high school but is quite a serious social disaster not to do so. In a way, it is an aggregate recognition of the Law of Zero Correlation and the Law of Shifting Benefits and Liabilities. This phenomenon can be illustrated by the Aggregate Model.

Figure 4 is a plot of the rate of decline of the social benefits of attainment generated by the model. Note that after an attainment ratio of 0.20 the median value declines at a fairly constant rate until the high school attainment ratio reaches 50%. At this point in the growth of the educational system, the rate of decline increases and increases sharply at 75% attainment.

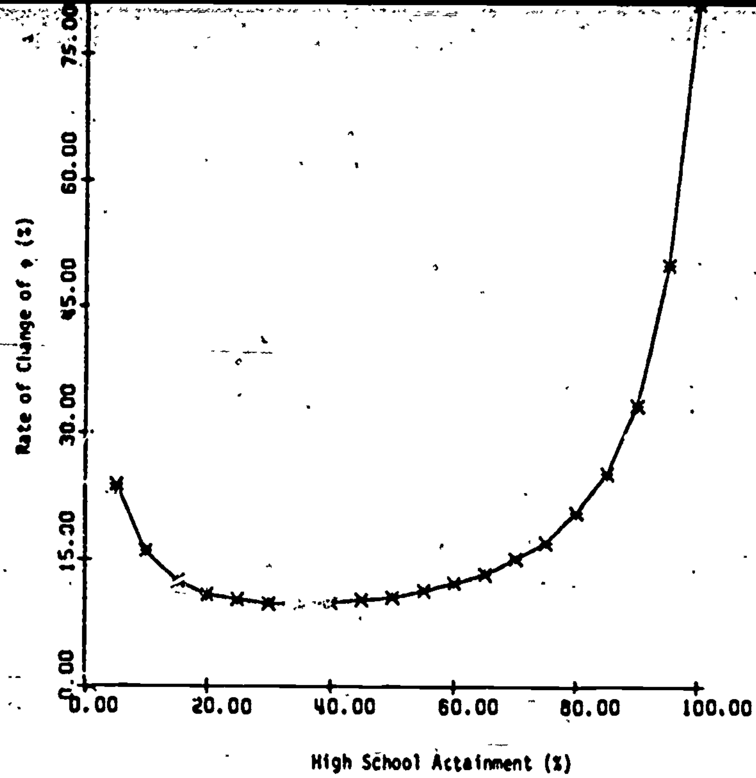


Figure 4. Rate of Change of Attainer Group Median by High School Attainment Ratio (from Table 1, Column 5)

Figure 5 is a plot of the rate of decline of the non-attainer median. Here the median declines at a decreasing rate until 75% attainment at which point the rate begins to increase and then increases sharply at 80% attainment.

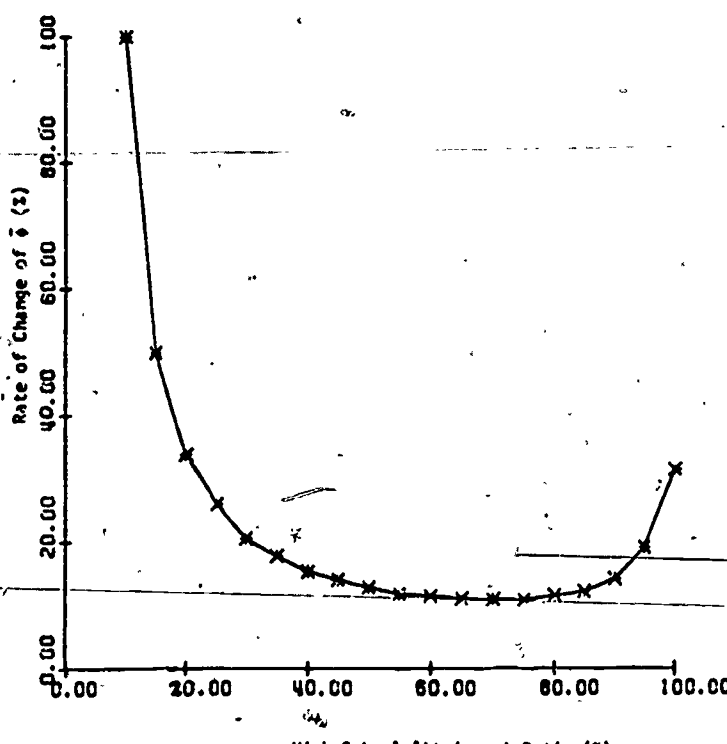


Figure 5. Rate of Change of Non-Attainer Group Median by High School Attainment Ratio (from Table 1, Column 6).

Thus, the two curves shown in Figure 3 can be said to contain inflection points which occur in the growth of the system where the high school attainment ratio is about 75%. The stabilization of the national attainment ratio at around 75% may be the social recognition of the phenomenon described by the model.[15]

It may be purely coincidental that the inflection points in the model and the national high school attainment ratio occur at about 75%. Nevertheless, the model does serve to illustrate the phenomenon of systemic "equilibrium" reflecting the interactive dynamics between certain systemic laws. The interaction between these laws offers an account of certain systemic phenomena.[16]

The behavior of the educational system described above is based upon these systemic features: the Principle of Sequence, the distribution of second-order educational goods and the size of the system as measured by the attainment ratio at the twelfth level. Systemic behavior was driven by the power of a logical tautology, its corollary and a normative principle linking the educational and social systems. It is ironic that the successful growth of the system, as measured by an increasing high school attainment ratio, appears to sow the seeds of a particular brand of failure.

III

THE PROBABILISTIC UTILITY MODEL

The idealized society reflected in the three assumptions underlying the Aggregate Model is a rigidly meritocratic one. By altering the first and third assumptions, we can build a model that reflects a society that distributes its non-educational social goods in a somewhat more flexible manner. Like the Aggregate Model, let us assume that the population under consideration is dichotomized into those who have attained the high school diploma (and nothing beyond it) and those who have not attained the degree. Furthermore, let us assume two independent normal distributions of social goods, one for the attainment group and the other for the non-attainment group. This state of affairs is illustrated in Figure 6.

Now let us assume that both of these normal distributions have identical standard deviations. Thus, we can normalize each of the distributions and leave them superimposed, one upon the other, on the social benefits axis. Note that the relative position of the two normal curve means remains unaffected by the standardization (i.e., the standardized and unstandardized means remain stationary). These standardized distributions are shown in Figure 6.

A. The Standardized Normal Distributions

Consider the two standardized normal distributions shown in Figure 6, below. Curves X_{ϕ} and $X_{\bar{\phi}}$ represent the distributions of earnings opportunities of high school attainers and non-attainers, respectively. Both curves have their asymptotes truncated, to facilitate the computations to follow, at 3.0 standard deviations above and below their respective means of zero and are

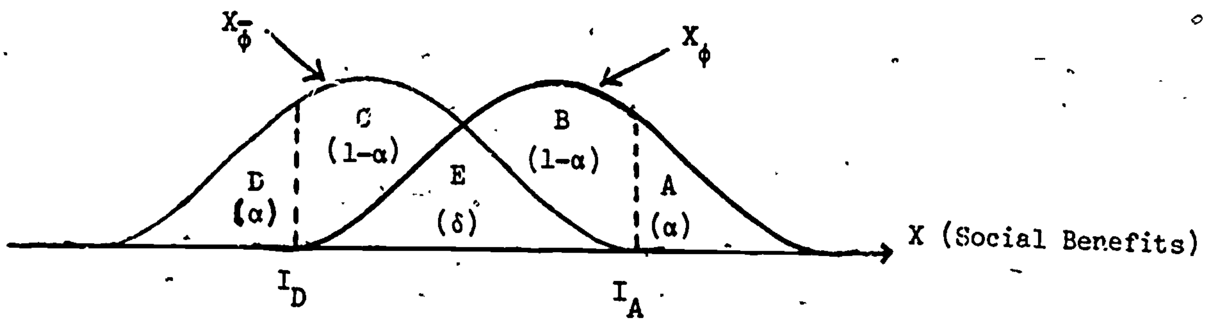


Figure 6. Two Overlapping Standardized Normal Curves

superimposed upon a common axis, X, showing an apparent overlap, area E: that area under both curves which has a common X-axis range.

We let ϕ stand for the ratio of high school attainers to the total population under consideration and let α stand for the meritocratic parameter. This parameter represents those in the total population, and in particular that proportion of distribution X_{ϕ}, which monopolizes the highest values of X. It is clear from Figure 6 that this parameter imposes an upper-bound on the range of distribuion $X_{\bar{\phi}}$ (i.e., I_A) and concomitantly places a lower-bound on the range of X_{ϕ} (i.e., I_D). Except where $\alpha=0$, the ranges of X_{ϕ} and $X_{\bar{\phi}}$ differ.

Let us assume that despite changes in the size of ϕ, the original non-standardized normal distributions retain their normal shapes and continue to have identical standard deviations and unchanged means. The X_{ϕ} mean remains forever fixed and thus for any given ϕ, only a change in α can shift the $X_{\bar{\phi}}$ curve. A mean/medium analysis of these curves is presented in Appendix B.

Unlike the Aggregate Model, individuals in X_ϕ (i.e., high school attainers) are no longer guaranteed an advantage over persons in $X_{\bar{\phi}}$ (i.e., non-attainers), with respect to some value of X (level of social benefit). The question now shifts from one of absolute advantage (as in the Aggregate Model) to one of relative advantage. We now ask, what is the probability that an individual will be advantaged with respect to X, over changes in ϕ and in α?

The symbols in Figure 6 refer to proportions and are explained in Table 2, below.

TABLE 2

PROPORTIONAL VALUES OF SECTIONS IN FIGURE 6

Section	Symbol	Meaning
A	(α)	The proportion of the population which is in X_ϕ and which monopolizes the highest X values. This is the value of the meritocratic parameter.
B	($1-\alpha$)	The proportion of the population which is in X_ϕ and which does not monopolize the highest X values.
C	($1-\alpha$)	The proportion of the population which is in $X_{\bar{\phi}}$ and which is not relegated to the lowest X values.
D	(α)	The proportion of the popultion which is in $X_{\bar{\phi}}$ and is relegated to the lowest X values.
E	(δ)	The area of intersection of Section B of X_ϕ and Section C of $X_{\bar{\phi}}$.

The above conceptualization allows us to calculate the probabilities of persons falling in any of the five sections of Figure 6 as a function of α and ϕ. These probabilities are conditional probabilities of independent events. Table 3 gives the formulae for these calculations.

TABLE 3

PROBABILITIES

Section	Probability	Meaning
A	$Pr(A\|X_\phi) = \alpha\phi$	The probability of residing in Section A is the conditional probability of residing in A (i.e., α) given that one already resides in X_ϕ (i.e., ϕ).
B	$Pr(B\|X_\phi) = (1-\alpha)\phi$	The probability of residing in Section B is the conditional probability of <u>not</u> residing in A (i.e., $1-\alpha$) given that one resides in X_ϕ (i.e., ϕ).
C	$Pr(C\|X_{\bar{\phi}}) = (1-\alpha)(1-\phi)$	The probability of residing in Section C is the conditional probability of <u>not</u> residing in D (i.e., $1-\alpha$) given that one resides in $X_{\bar{\phi}}$ (i.e., $1-\phi$).
D	$Pr(D\|X_{\bar{\phi}}) = \alpha(1-\phi)$	The probability of residing in Section D is the conditional probability of residing in D (i.e, α) given that one resides in $X_{\bar{\phi}}$ (i.e, $1-\phi$).
E_1	$Pr(E\|C\|X_{\bar{\phi}}) = (1-\alpha)(1-\phi)\delta$	The probability of residing in Section E given that one is already in $X_{\bar{\phi}}$, is the conditional probability of residing in E (i.e., δ) given that one resides in $X_{\bar{\phi}}$ (i.e., $1-\phi$) <u>and</u> resides in Section C (i.e, $1-\alpha$).
E_2	$Pr(E\|B\|X_\phi) = (1-\alpha)\phi\delta$	The probability of residing in Section E given that one is already in X_ϕ is the conditional probability of residing in E (i.e., δ) given that one resides in X_ϕ <u>and</u> resides in B (i.e., $1-\alpha$).

B. Interpretation of Area E

The move from proportions in Table 2 to probabilities in Table 3 is a crucial one. Recall that each distribution represents one part of the dichotomized total population under consideration. The overlapping area, E, is not a shared population between the two groups. It simply illustrates the common range of X shared by area B in X_{ϕ} and C in $X_{\bar{\phi}}$.

Each person in the total population under consideration has a probability of ending up in one of the two distributions. Since ϕ is the proportion of the total population that has attained the twelfth level, any individual has probability ϕ of falling under distribution X_{ϕ} (all other things being equal). Similarly, the probability of not attaining at level 12 is equal to $(1-\phi)$. Of course, $\phi+(1-\phi)$ equals 1.0, which is the total population under consideration. All of this follows from the laws of proportions.

Consider Figure 6. As Section A changes in size, $X_{\bar{\phi}}$ shifts to the left or to the right (recall that we have assumed that changes in ϕ do not affect the shape or position of the distributions). The entire area under any one of the two distributions is equal to 1.0. Thus, if α represents the value of the area of Section A, then $1-\alpha$ is the area of Section B. From this we can see that the conditional probability of an individual being an attainer and being a monopolizer of the higher values of X is $\alpha\phi$. The laws of symmetry make Section D equal to Section A. Thus, the probability of an individual being a non-attainer and being relegated to the lowest values of X is $\alpha(1-\phi)$. Similar arguments can be made for Sections B and C. The probabilistic interpretation of Section E is a more complicated matter, however.

Although Sections B and C do not actually have an area in common, they do share the common X-axis range I_D to I_A. It is useful to think of Section E as if it is the area of overlap between the two distributions. Recall that the probability of being in C is simply $(1-\phi)(1-\alpha)$. Now, the probability of being in C and at the same time being within the scope of distribution X_{ϕ} is just the probability of being in C times the area of Section E. Similarly, the probability of being in B is $(1-\alpha)\phi$. The probability of being in B and within the scope of distribution $X_{\bar{\phi}}$ is just the probability of being in B times the area of Section E.

It should now be clear that $Pr(E|C|X_{\bar{\phi}})$ is the probability of any individual non-attainer falling in the same range with and being under the same scope as an attainer. Likewise, $Pr(E|B|X_{\phi})$ is the probability of any individual attainer falling in the same range with and being under the same scope as a non-attainer. These two probabilities need not always be equal. In fact, they are equal only when ϕ=0.50.

What remains is to calculate the area of Section E, i.e, δ. This is done in Appendix A.

C. Results of the Analysis

Tables 4 and 5 give the probabilities of falling in Section E given attainment and of falling in Section E given non-attainment, respectively. These Tables are derived from the probability formulae in Table 3. To obtain the probabilistic marginal utilities of attainment, we simply perform a matrix subtraction, Table 5 minus Table 4. The results of this subtraction are shown in Table 6.

Note that the marginal utilities decrease for constant ϕ and increasing α, and decrease for constant α and increasing ϕ. Furthermore, each column reflects about the row where ϕ=0.50 so that each column below this row is the negative converse of the column above.

TABLE 4

PROBABILITY OF FALLING IN SECTION E GIVEN ATTAINMENT OF LEVEL 12

Proportion of 12th Level Attainers (α)	Meritocratic Parameter (α)									
	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
0.01	0.0035	0.0022	0.0015	0.0010	0.0007	0.0004	0.0002	0.0001	0.0000	0.0000
0.05	0.0176	0.0112	0.0076	0.0051	0.0033	0.0021	0.0012	0.0005	0.0002	0.0001
0.10	0.0353	0.0224	0.0152	0.0101	0.0067	0.0042	0.0023	0.0011	0.0003	0.0001
0.15	0.0529	0.0336	0.0228	0.0152	0.0100	0.0062	0.0035	0.0016	0.0005	0.0002
0.20	0.0706	0.0448	0.0304	0.0203	0.0134	0.0083	0.0047	0.0022	0.0006	0.0002
0.25	0.0882	0.0560	0.0380	0.0254	0.0167	0.0104	0.0058	0.0027	0.0008	0.0003
0.30	0.1058	0.0672	0.0456	0.0304	0.0200	0.0125	0.0070	0.0033	0.0010	0.0003
0.35	0.1235	0.0785	0.0532	0.0355	0.0234	0.0146	0.0082	0.0038	0.0011	0.0004
0.40	0.1411	0.0897	0.0608	0.0406	0.0267	0.0166	0.0094	0.0044	0.0013	0.0004
0.45	0.1588	0.1009	0.0684	0.0456	0.0301	0.0187	0.0105	0.0049	0.0014	0.0005
0.50	0.1764	0.1121	0.0759	0.0507	0.0334	0.0208	0.0117	0.0055	0.0016	0.0005
0.55	0.1940	0.1233	0.0835	0.0558	0.0367	0.0229	0.0129	0.0060	0.0018	0.0006
0.60	0.2117	0.1345	0.0911	0.0608	0.0401	0.0250	0.0140	0.0066	0.0019	0.0006
0.65	0.2293	0.1457	0.0987	0.0659	0.0434	0.0270	0.0152	0.0071	0.0021	0.0007
0.70	0.2470	0.1569	0.1063	0.0710	0.0468	0.0291	0.0164	0.0077	0.0023	0.0007
0.75	0.2646	0.1681	0.1139	0.0760	0.0501	0.0312	0.0176	0.0082	0.0024	0.0008
0.80	0.2822	0.1793	0.1215	0.0811	0.0534	0.0333	0.0187	0.0088	0.0026	0.0008
0.85	0.2999	0.1905	0.1291	0.0862	0.0568	0.0354	0.0199	0.0093	0.0027	0.0009
0.90	0.3175	0.2017	0.1367	0.0913	0.0601	0.0374	0.0211	0.0099	0.0029	0.0009
0.95	0.3352	0.2130	0.1443	0.0963	0.0635	0.0395	0.0222	0.0104	0.0031	0.0010

TABLE 5

PROBABILITY OF FALLING IN SECTION E GIVEN ATTAINMENT BELOW LEVEL 12

Proportion of 12th Level Attainers (α)	Meritocratic Parameter (α)									
	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.9500
0.01	0.3493	0.2219	0.1504	0.1004	0.0661	0.0412	0.0232	0.0109	0.0032	0.0010
0.05	0.3352	0.2130	0.1443	0.0963	0.0635	0.0395	0.0222	0.0104	0.0031	0.0010
0.10	0.3175	0.2017	0.1367	0.0913	0.0601	0.0374	0.0211	0.0099	0.0029	0.0009
0.15	0.2999	0.1905	0.1291	0.0862	0.0568	0.0354	0.0199	0.0093	0.0027	0.0009
0.20	0.2822	0.1793	0.1215	0.0811	0.0534	0.0333	0.0187	0.0088	0.0026	0.0008
0.25	0.2646	0.1681	0.1139	0.0760	0.0501	0.0312	0.0176	0.0082	0.0024	0.0008
0.30	0.2470	0.1569	0.1063	0.0710	0.0468	0.0291	0.0164	0.0077	0.0023	0.0007
0.35	0.2293	0.1457	0.0987	0.0659	0.0434	0.0270	0.0152	0.0071	0.0021	0.0007
0.40	0.2117	0.1345	0.0911	0.0608	0.0401	0.0250	0.0140	0.0066	0.0019	0.0006
0.45	0.1940	0.1233	0.0835	0.0558	0.0367	0.0229	0.0129	0.0060	0.0018	0.0006
0.50	0.1764	0.1121	0.0759	0.0507	0.0334	0.0208	0.0117	0.0055	0.0016	0.0005
0.55	0.1588	0.1009	0.0684	0.0456	0.0301	0.0187	0.0105	0.0049	0.0014	0.0005
0.60	0.1411	0.0897	0.0608	0.0406	0.0267	0.0166	0.0094	0.0044	0.0013	0.0004
0.65	0.1235	0.0785	0.0532	0.0355	0.0234	0.0146	0.0082	0.0038	0.0011	0.0004
0.70	0.1058	0.0672	0.0456	0.0304	0.0200	0.0125	0.0070	0.0033	0.0010	0.0003
0.75	0.0882	0.0560	0.0380	0.0254	0.0167	0.0104	0.0058	0.0027	0.0008	0.0003
0.80	0.0706	0.0448	0.0304	0.0203	0.0134	0.0083	0.0047	0.0022	0.0006	0.0002
0.85	0.0529	0.0336	0.0228	0.0152	0.0100	0.0062	0.0035	0.0016	0.0005	0.0002
0.90	0.0353	0.0224	0.0152	0.0101	0.0067	0.0042	0.0023	0.0011	0.0003	0.0001
0.95	0.0176	0.0112	0.0076	0.0051	0.0033	0.0021	0.0012	0.0005	0.0002	0.0001

TABLE 6

PROBABILISTIC MARGINAL UTILITIES OF ATTAINMENT OF LEVEL 12

Meritocratic Parameter (α)

Proportion of 12th Level Attainers (φ)	0.1000	0.2000	0.3000	0.4000	0.5000	0.6000	0.7000	0.8000	0.9000	0.950
0.01	0.346	0.220	0.149	0.099	0.065	0.041	0.023	0.011	0.003	0.001
0.05	0.318	0.202	0.137	0.091	0.060	0.037	0.021	0.010	0.003	0.001
0.10	0.282	0.179	0.122	0.081	0.053	0.033	0.019	0.009	0.003	0.001
0.15	0.247	0.157	0.106	0.071	0.047	0.029	0.016	0.008	0.002	0.001
0.20	0.212	0.134	0.091	0.061	0.040	0.025	0.014	0.007	0.002	0.001
0.25	0.176	0.112	0.076	0.051	0.033	0.021	0.012	0.005	0.002	0.001
0.30	0.141	0.090	0.061	0.041	0.027	0.017	0.009	0.004	0.001	0.000
0.35	0.106	0.067	0.046	0.030	0.020	0.012	0.007	0.003	0.001	0.000
0.40	0.071	0.045	0.030	0.020	0.013	0.008	0.005	0.002	0.001	0.000
0.45	0.035	0.022	0.015	0.010	0.007	0.004	0.002	0.001	0.000	0.000
0.50	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.55	-0.035	-0.022	-0.015	-0.010	-0.007	-0.004	-0.002	-0.001	0.000	0.000
0.60	-0.071	-0.045	-0.030	-0.020	-0.013	-0.008	-0.005	-0.002	-0.001	0.000
0.65	-0.106	-0.067	-0.046	-0.030	-0.020	-0.012	-0.007	-0.003	-0.001	0.000
0.70	-0.141	-0.090	-0.061	-0.041	-0.027	-0.017	-0.009	-0.004	-0.001	0.000
0.75	-0.176	-0.112	-0.076	-0.051	-0.033	-0.021	-0.012	-0.005	-0.002	-0.001
0.80	-0.212	-0.134	-0.091	-0.061	-0.040	-0.025	-0.014	-0.007	-0.002	-0.001
0.85	-0.247	-0.157	-0.106	-0.071	-0.047	-0.029	-0.016	-0.008	-0.002	-0.001
0.90	-0.282	-0.179	-0.122	-0.081	-0.053	-0.033	-0.019	-0.009	-0.003	-0.001
0.95	-0.318	-0.202	-0.137	-0.091	-0.060	-0.037	-0.021	-0.010	-0.003	-0.001

An inspection of Table 6 shows that it is not individually advantageous to obtain the high school diploma until 55% of the population under consideration (17-year old age cohort) does so. The row where ϕ=0.50 can be considered to be the indifference level. However, a mean/median analysis shows that, in the aggregate, it is always advantageous to be an attainer rather than a non-attainer. This is so because for all values of α, μ_{ϕ} is greater than $\mu_{\bar{\phi}}$ (except when they are equal, when α=0). A complete mean/median analysis is given in Appendix B. See columns 4 and 6 in Table B-1.

This analysis of the Probabilistic Utility Model exposes an interesting paradox: in the aggregate it is more advantageous to be an attainer no matter what ϕ and α are; individually this is not always the case. Furthermore, Table 6 indicates that the marginal disutility of not attaining the high school degree increases as attainment increases and also increases as the meritocratic parameter decreases! This phenomenon can be vividly seen in the lower left-hand quadrant of Table 6. As we move from the upper right-hand to the lower left-hand corner on the diagonal, disutilities can be seen to double, triple and even quadruple at various steps.

IV

CONCLUSIONS

The idealized models developed here have their limitations. The Aggregate Model seems, on the face of it, too meritocratic for our present society. The distribution of social benefits may not, in reality, be normal, and their means (as shown in the Utility Model) may not remain constant with systemic growth (which is clearly not the case in the Aggregate Model). Nevertheless, these models can serve as "benchmarks" against which to measure other logico-mathematical models containing different assumptions, and empirically derived models.

The high school graduation ratio is but one indicator of systemic growth and maturity.[16] It is the last systemic level that is non-selective. One not only chooses to go on to college, but in most cases one is also chosen. Along with the normative principle and with level 12 approaching zero correlation, the post-secondary system ordinarily would respond by expanding. However, it must balance this tendency against its selectivity principle. This conflict might account, in part, for the rapid rise of non-selective junior colleges over the last twenty years.

Both models are generalizable over systemic levels and make most sense with regard to initial social benefit distributions such as job-entry level income. Very simply, it is likely that the role that educational attainment plays in getting a job is quite different from the role that it plays in keeping or advancing in one.[17]

The dynamic account of the educational system that is the basis for the models presented here is a formal one, and this is reflected in the models' non-empirical grounding. Like Boudon's efforts, the models avoid the cross-sectional and variable confounding effects of survey data. This is not to deny the role and influence of human actors and social and political movements. The models merely illustrate, in a formal way, the power of a logical tautology—in conjunction with a normative principle.[18]

These models help illustrate the limitations of educational policy. Pushing the attainent level to 100% to cancel out the effects of the Law of Shifting Benefits and Liabilities, merely shifts the problem onto another level of the system: post-secondary. Merely trying to raise the high school attainment ratio might cause unnecessary hardship to those who choose not to complete the level and at the same time, diminish the benefits for attainers. We have seen how the same policy can have differential consequences at different stages of systemic growth.

Perhaps lowering the ratio 50-60% might have some effect upon the power of the normative principle to disproportionately reward formal schooling. This might be a level below the "equilibrium point" - the point at which the effects of the decline in the social benefits of attainment and the precipitous rise in the social liabilities of non-attainment seem to be maximally felt. Careful consideration should be given for providing ample opportunities for all to continue their education (i.e., to pursue learning). Such a policy must avoid an inequitable distribution of non-attainers on the basis of class, race, sex and ethnic background (i.e, educationally irrelevent attributes).

It is unlikely that such an overtly articulated policy would gain much political support. In any case, it would surely prove to be very difficult to implement. Nevertheless, such a policy is already being carried out to a certain extent by the movement towards competency-based curricula. If competency achievement (not attainment) levels are enforced, a sizable proportion of those who now normally complete high school may never do so. And a large percentage of this group will likely be composed of persons from lower socioeconomic groups. This state of affairs is hardly an adequate solution to the problem of inequality of socioeconomic opportunities.

The abandonment of the normative principle might be the most efficacious, but politically and socially the most difficult, way to reduce educational and social inequality (no guarantee, of course). If educational attainment is no longer used as an instrument for the distribution of non-educational social goods, then perhaps education could once again be pursued for the benefits that are intrinsic in the educational goods themselves and not for the socioeconomic advantages that disappear with ever increasing rates and levels of attainment.

Such a move would mean the abandonment of the illusion that the educational system is a solution for almost every societal ill. It is not clear just what new instruments for the distribution of social benefits might arise. However, if the system's logic is intractable (how does one repeal the Law of Zero Correlation?), then perhaps a reconsideration of a socioeconomic principle that disproportionately rewards formal educational attainment is in order.

APPENDICES.

APPENDIX A

CALCULATION OF SECTION E AREA

To calculate δ, we begin by truncating the asymptotes of the two standardized normal curves (Figure 6) at 3.0 standard deviations above and below their respective means. As a result, we lose 0.26% of the population of any one curve.

Since the two curves are identical (i.e., both are standardized normal curves), the point on the X-axis (μ_I directly below the point of intersection (I) lies midway between the X_ϕ and $X_{\bar{\phi}}$ distribution means, μ_ϕ and $\mu_{\bar{\phi}}$, respectively. This follows from the laws of symmetry, since Section D is always equal to Section A in area. Figure A-1 emphasizes the area of intersection in Figure 6.

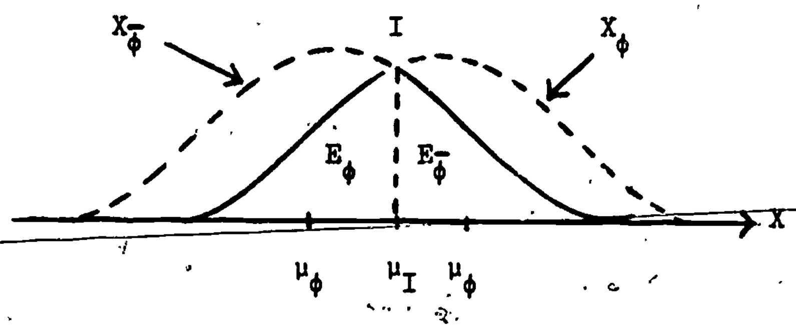


Figure A-1. Section E Area Emphasized
($E_{\bar{\phi}}$ and E_ϕ correspond to E_1 and E_2, respectively, in Table 3)

We know by symmetry, that the area to the right of the vertical line $I\mu_I$ on curve $X_{\bar{\phi}}$ (i.e., area $E_{\bar{\phi}}$) is equal to the area to the left of $I\mu_I$ on curve X_{ϕ} (i.e., area E_{ϕ}). Thus, twice $E_{\bar{\phi}}$ or twice E_{ϕ} gives us δ, the area of Section E.

Now we can proceed to develop a pair of algorithms that enable us to calculate area $E_{\bar{\phi}}$.

The area δ, equals 1.0 when α equals zero. In this situation, $X_{\bar{\phi}}$ and X_{ϕ} are superimposed one upon the other. Since $\mu_{\bar{\phi}} = \mu_{\phi}$, their relative difference, ψ, is equal to $|\mu_{\bar{\phi}} - \mu_{\phi}|$ which is equal to zero. When $\alpha = 1.0$, area δ equals zero. In this case, $X_{\bar{\phi}}$ and X_{ϕ} are mutually exclusive and ψ equals 6.0. Between these two extremes, α ranges from zero to 1.0.

We shall first examine the case where α ranges from zero to 0.5 and then the case where α ranges from 0.5 to 1.0. (Note that 0.5 is used throughout the text as an approximation to 0.4987, which is used in the calculations due to truncation.)

<u>CASE 1</u>: $(0 \leqslant \alpha \geqslant 0.5)$

Consider Figure A-2. The relative distance, ψ, between the two means, $\mu_{\bar{\phi}}$ and μ_{ϕ}, is equal to the distance on the X-axis under area A (i.e., the area corresponing to the value of α).

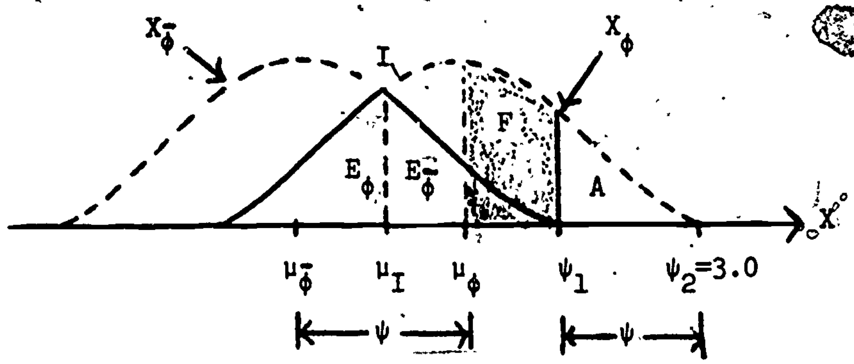


Figure A-2. Case 1 Where α Ranges from 0 to 0.5

Note that when $\alpha=0$, the two means, $\mu_{\bar{\phi}}$ and μ_{ϕ}, coincide simply because the two curves, $X_{\bar{\phi}}$ and X_{ϕ}, are superimposed one upon the other. As the value of α increases, the $X_{\bar{\phi}}$ curve is shifted to the left, a distance equal to the distance on the X-axis under Section A. Call this distance ψ, which is the value of the $X_{\bar{\phi}}$ curve translation.

Since $\psi_2=3.0$, we need only find ψ_1 in order to find ψ (i.e., $\psi = \psi_2 - \psi_1$. Area F is equal to $0.4987-\alpha$ and ψ_1 is found from a standardized normal curve table. Once we have computed ψ, we can locate μ_I with respect to $\mu_{\bar{\phi}}$. See Figure A-3, below.

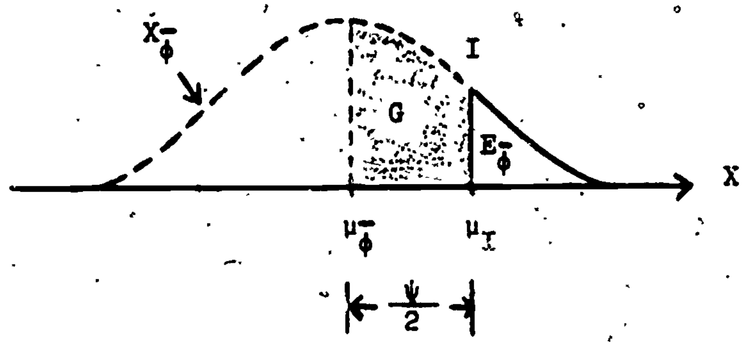


Figure A-3. The Parameters for Finding δ

Note that μ_I lies $\psi/2$ above $\mu_{\bar{\phi}}$. Area G is found from a standardized normal curve table. Area $E_{\bar{\phi}}$ is equal to 0.4987-G. The area δ, is simply twice area $E_{\bar{\phi}}$. The algorithm for this computation appears below.

ALGORITHM A-1

CASE 1 WHERE α RANGES FROM 0 to 0.5
(Refer to Figures A-2 and A-3)

Step	
1	$F = 0.4987 - \alpha$
2	ψ_1 from standardized normal curve table
3	$\psi = \psi_2 - \psi_1$
4	$\mu_I = \psi/2$ with respect to $\mu_{\bar{\phi}}$
5	G from standardized normal curve table
6	$E_{\bar{\phi}} = 0.4987 - G$
7	$\delta = 2(E_{\bar{\phi}})$

CASE 2: ($0.5 \leqslant \alpha > 1.0$)

Figure A-4 depicts the situation for this case, and the algorithm for the computation of δ follows it.

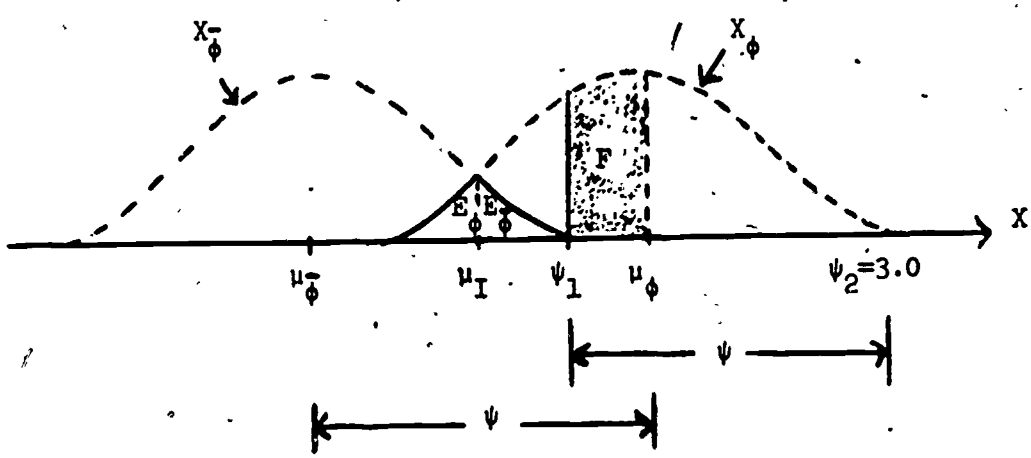


Figure A-4. Case 2 Where α Ranges from 0.5 to 1.0

ALGORITHM A-2

CASE 2 WHERE α RANGES FROM 0.5 TO 1.0
(Refer to Figures A-3 & A-4)

Step	
1	$F = \alpha - 0.4987$
2	ψ_1 from standardized normal curve table
3	$\psi = \psi_2 + \psi_1$
4	$\mu_I = \psi/2$ with respect to $\mu_{\bar{\phi}}$
5	G from standardized ormal curve table
6	$E_{\bar{\phi}} = 0.4987 - G$
7	$\delta = 2(E_{\bar{\phi}})$

Table A-1, below, gives the values of δ for α values in steps of 0.1. Table A-2 gives the intermediate values of F, ψ_1, ψ, μ_I, G, $\mu_{\bar{\phi}}$ for α values in steps of 0.1.

TABLE A-1

VALUES OF δ AS A FUNCTION OF α

α	δ
0	1.0000
0.10	0.3872
0.20	0.2776
0.30	0.2124
0.40	0.1666
0.50	0.1310
0.60	0.1006
0.70	0.0750
0.80	0.0516
0.90	0.0294
0.95	0.0172
1.00	0

TABLE A-2

INTERMEDIATE VALUES CALCULATED BY ALGORITHMS A-1 and A-2

α	F	ψ_1	ψ	μ_I	G	$E_{\bar{\phi}}$
0			0			
0.10	0.3987	1.275	1.725	0.8625	0.3051	0.1936
0.20	0.2987	0.835	2.165	1.0825	0.3599	0.1388
0.30	0.1987	0.520	2.480	1.2400	0.3925	0.1062
0.40	0.0987	0.250	2.750	1.3750	0.4154	0.0833
0.50	0	0	3.000	1.5000	0.4332	0.0655
0.60	0.1013	0.255	3.255	1.6275	0.4484	0.0503
0.70	0.2013	0.530	3.530	1.7650	0.4612	0.0375
0.80	0.3013	0.850	3.850	1.9250	0.4729	0.0258
0.90	0.4013	1.290	4.290	2.1450	0.4840	0.0147
0.95	0.4513	1.660	4.660	2.3300	0.4901	0.0086
1			6.000			

APPENDIX B

MEAN/MEDIAN ANALYSIS OF PROBABILISTIC UTILITY MODEL

We can set the Model in motion. See Figure B-1. Note that when $\alpha = 0$, the following equalities hold:

$$\mu_B = \mu_C = \mu_I = \mu_{\tilde{\phi}} = \mu_\phi \qquad (1)$$

$$|\mu_A - \mu_I| = |\mu_D - \mu_I| \qquad (2)$$

When $\alpha = 1$, another set of equalities hold:

$$\mu_C = \mu_B = \mu_I \qquad (3)$$

$$\mu_A = \mu_\phi \qquad (4)$$

$$\mu_D = \mu_{\tilde{\phi}} \qquad (5)$$

$$|\mu_A - \mu_I| = |\mu_D - \mu_I| \qquad (6)$$

Between these two extremes, it is possible to calculate the relative differences between medians (μ_ϕ and $\mu_{\tilde{\phi}}$ are the grand means and grand medians of their respective distributions) of the various sections of the two curves shown in Figure B-1.

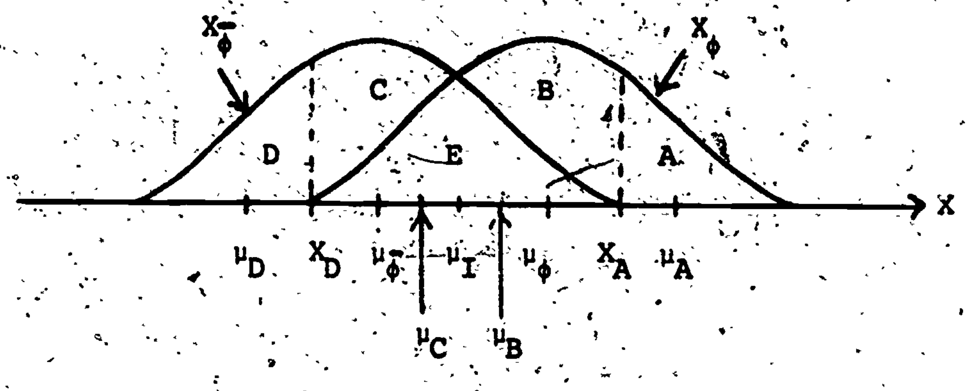


Figure B-1. Medians/Means for Sections of Curves

Assume that μ_ϕ remains constant and that both curves retain their normal shapes as the size of ϕ (and concomitantly, $\bar{\phi}$) and α change. We take μ_ϕ as our point of reference, since it remains constant, and calculate the other medians with respect to it.

1. Schemas for Median Calculations for Changing Values of α

We begin, as we did in Appendix A, by truncating the asymptotes of the two standardized normal curves at 3.0 standard deviations above and below their respective means. Medians μ_A and μ_B have already been calculated in the Aggregate Model and can be found in columns 2 and 3 of Table 1.

$\mu_{\bar{\phi}}$ is the distance on the X-axis under Section A. This distance is the ψ value computed as an intermediate step by Algorithms 1 and 2. See Table A-2. μ_I is simply one half $\mu_{\bar{\phi}}$ and is also computed as an intermediate step by Algorithms 1 and 2. See Table A-2.

We now develop schemas that compute the values of μ_C and μ_D, for changing values of α.

Due to the symmetry of the two curves and the equality of Sections A and D, median μ_C will always be as much to the right of $\mu_{\bar{\phi}}$ as μ_B is to the left of μ_{ϕ}. Thus,

$$\mu_C = \mu_{\bar{\phi}} - \mu_B \tag{7}$$

In a similar fashion, μ_D will always be as much to the left of $\mu_{\bar{\phi}}$ as μ_A is to right of μ_{ϕ}. Thus,

$$\mu_D = \mu_{\bar{\phi}} - \mu_A \tag{8}$$

Table B-1 displays the results of these computations.

2. <u>Changing Means (μ_{ϕ} and $\mu_{\bar{\phi}}$) With Changing ϕ and Constant α</u>

We have assumed throughout that the size of ϕ has no effect upon the means of the dichotomized populations. Furthermore, for computational purposes, we have assumed that only $\mu_{\bar{\phi}}$ was affected by changing α and that μ_{ϕ} remains permanently anchored.

It is not unreasonable to assume that both means change with changing ϕ and that both means change with changing α. However, both of these cases reduce to the analysis that has already been performed for the probability distributions generated by the formulae in Table 3 (constant μ_{ϕ} for changing ϕ and changing α).

TABLE B-1

INTERMEDIATE VALUES FROM ALGORITHMS 1 AND 2

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	μ_ϕ	μ_A	μ_B	μ_I	μ_C	$\mu_{\bar{\phi}}$	μ_D
0	0	3.0	0	0	0	0	-3.0
0.10	0	1.645	-0.126	-0.8625	-1.5990	-1.725	-3.370
0.20	0	1.283	-0.253	-1.0825	-1.9120	-2.165	-3.448
0.30	0	1.037	-0.385	-1.2400	-2.0950	-2.480	-3.517
0.40	0	0.842	-0.524	-1.3750	-2.2260	-2.750	-3.592
0.50	0	0.675	-0.675	-1.5000	-2.3250	-3.000	-3.675
0.60	0	0.524	-0.842	-1.6275	-2.4130	-3.255	-3.779
0.70	0	0.385	-1.037	-1.7650	-2.4930	-3.530	-3.915
0.80	0	0.253	-1.283	-1.9250	-2.5670	-3.850	-4.103
0.90	0	0.126	-1.645	-2.1450	-2.6450	-4.290	-4.416
0.95	0	0.063	-1.960	-2.3300	-2.7000	-4.660	-4.723
1.0	0	0	-3.0	-3.0	-3.0	-6.0	-6.0

To construct the probability tables for changing means, we can use the probability distributions generated by the formulae in Table 3. We need only know the sizes of ϕ and α, and the relative difference between the two dichotomized population means (see Appendix A). This relative difference, $|\mu_{\phi} - \mu_{\bar{\phi}}|$, is a function only of the size of α. Thus, if both means change with changing ϕ and with changing α, and if we know the relative difference between the means, we can calculate the new α. We can then consult the existing probability tables produced by the formulae in Table 3.

3. Non-normal Distributions with Equal and Unequal Ranges

The same sort of mean/median and probability analyses that have been performed for normal distributions can be performed for non-normal distributions. One must, however, first derive the formulae for the various curves and utilize the calculus to obtain the areas in questions and their shifting means and medians. The mathematics involved in this kind of analysis is somewhat complex.

NOTES

1. Some of the ideas presented in Part I are given a much more detailed and complete description in Thomas F. Green, with assistance of David P. Ericson and Robert H. Seidman, Predicting the Behavior of the Educational System. Syracuse: Syracuse University Press, 1980. I have borrowed from this work and make no reference to specific parts in the text of this paper. I am grateful to have had the opportunity to collaborate with Green and Ericson. Naturally, I am fully responsible for any shortcomings of this paper.

Portions of Parts I and II appear within a different context in: Robert H. Seidman, "The Logic and Behavioral Principles of Educational Systems: Social Independence or Dependence?", Keynote Paper, International Sociological Association - Sociology of Education Conference on the Origins and Operations of Educational Systems, Paris, 1980. Published as a chapter in: The Sociology of Educational Expansion, Margaret S. Archer (ed.), London and Beverley Hills: Sage, Studies in International Sociology Series, 1982.

2. See Raymond Boudon, Education, Opportunity, and Social Inequality. New York: John Wiley and Sons, 1974. Also, "Educational Growth and Economic Equality." Quality and Quantity, 8, 1974, pp. 1-10.

3. See Lester C. Thurow, "Measuring the Economic Benefits of Education," in Margaret S. Gordon, ed., Higher Education and the Labor Market, New York: McGraw-Hill Book Co., 1974; "Education and Economic Equality," The Public Interest, no. 43, Spring 1976, pp. 66-81; Generating Inequality: Mechanisms of Distribution in the U.S. Economy, New York: Basic Books, 1975.

4. This account is contained in Green, op. cit.

5. For a formal and general mathematical explication of these two models see: Robert H. Seidman, "The General Educational System: Some Aspects of Benefit and Liability Curves," Unified Theory of Educational Systems and Public Policy Project, Report V-1.04, Syracuse University, Syracuse, N.Y., 1976.

6. Growth in attainment is only one of the eight modes of systemic growth presented in Green, op. cit.

7. But why should the benefit curve decline at the upper attainment values? After all, avoiding a disaster is in a way, a benefit. Thus, there should be no decline in the demand to obtain the high school diploma - here supply creates demand! However, the empirical evidence shows that this is not the case. Indeed, it is argued in Chapter 6 of Green, op. cit., that the marginal growth of attainment will be slowed by 1) the transformation of attainment to achievement and 2) the rise in socioeconomic compulsion for more formal schooling.

8. Beginning in Massachusetts in 1852 and ending with Alaska in 1929, a compulsory attendance law was enacted in every state and territory. Prior to the time of statue enactment, only Louisiana and two territories had less than 60% enrolment of the school age population. See, Report of the Commissioner of Education, 1870-1916 and The Bi-Ennial Survey of Education, 1916-1918 (Alaska statistics were unavailable). The data are put into a social justice perspective in David P. Ericson with Thomas F. Green and Robert H. Seidman, "Justice and Compulsion in the Educational System" (unpublished paper).

9. The claim here is that variable attainment ratios and not compulsory attendance statutes compel school going. See David P. Ericson and Robert H. Seidman, "Compulsory Schooling Without Compulsory Attendance Laws: Reflections on the Behavior of Educational Systems," Proceedings of the Philosophy of Education Society, 1978, pp. 316-324.

This is a view that is in sharp contrast to the revisionist version (e.g., Samuel Bowles and Herbert Gintis, Schooling in Capitalist America, New York: Basic Books, 1976) and to the historical explanation offered by David Tyack "Ways of Seeing: An Essay on the History of Compulsory Schooling," Harvard Educational Review, 46, August 1976.

10. Here is a sample calculation of the median value of the social benefits for high school attainers and non-attainers.

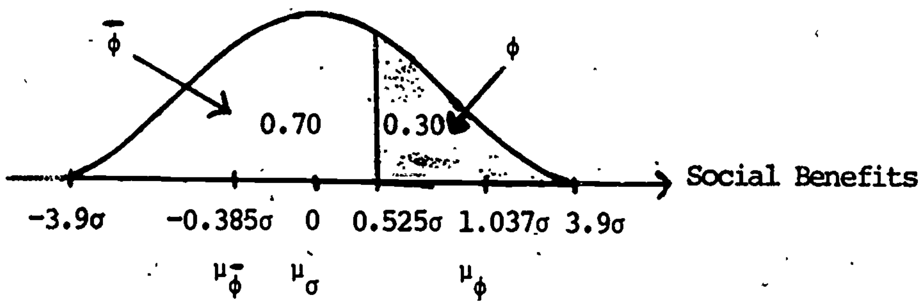
Suppose that the attainment ratio stands at 30% (see the Figure below). We know that the attainer group monopolizes the social benefits ranging in value from 0.52 to 3.9 standard deviations from the grand mean. See M. R. Speigal, "Areas Under the Standard Normal Curve from 0 to Z," Theory and Problems of Statistics, New York: McGraw-Hill Book Co., 1961, p. 343.

The median benefit for this group is thus $\mu_\phi = 1.037\sigma$. This is the point under the ϕ portion of the total distribution where half of the high school attainers (i.e., 15%) lie to the right and where the other half lie to the left.

The median social benefits for the remaining 70% of the total population (i.e., the non-attainer group) is $\mu_{\bar{\phi}} = -0.385\,\sigma$. This is the point under the $\bar{\phi}$ portion of the total distribution where one half of the high school non-attainers (i.e., 35%) lie to the right and the other half lie to the left.

These median social benefit values are derived from the standardized normal distribution, which represents a particular normal distribution of social benefits. If it turns out that, for this particular normal distribution, the median of the total distribution is \$8,000 with a standard deviation of \$2,500, we can easily calculate the medians (in dollars) of the attainer and non-attainer groups.

Attainer Group Median: \$10,593 = \$8,000 + (1.037 x \$2,500). Non-Attainer Group Median: \$7,038 = \$8,000 + (-0.385 x \$2,500).



Standardized Normal Curve for the Distribution of Social Benefits

Note: ϕ = high school attainment ratio; $\bar{\phi}$ = non-attainment ratio; $\mu_\sigma = 0$ = grand median; μ_ϕ = median social benefit for attainer group; $\mu_{\bar{\phi}}$ = median social benefit for non-attainer group; $\sigma = 1$ = standard deviation.

11. It is probably unreasonable to apply the model at the lower attainment ratios where the power of the normative principle is very low. However, the model does serve to illustrate the idea that the relative benefit disparity between the two groups first decreases and then increases. This phenomenon suggests that a particular educational policy appropriate for one stage of systemic growth might not be appropriate for another stage.

12. U.S. Bureau of the Census, Decennial Census Reports for 1940, 1950, 1960, 1970; Current Population Reports, P-60, nos. 85, 90, 92, 97, 101, Washington D.C.: U.S. Government Printing Office.

13. Henry M. Levin, et al., The Costs to the Nation of Inadequate Education. A Report Prepared for the Select Committee on Equal Educational Opportunity of the U.S. Senate, Washington, D.C.: U.S. Government Printing Office, January 1972.

14. For an extended analysis from another methodological perspective, see Appendix C in Green, op. cit.

15. See National Center for Education Statistics, Digest of Educational Statistics - 1979, Washington, D.C.: U.S. Government Printing Office, 1979, Table 60.

It is of some interest to note that U.S. Government projections of this attainment ratio to the year 1989 keep it constant at about 74%. Why? The "assumption" is that the high school attainment ratio for 18-year-olds will remain constant at the 1977-78 level through 1988! No reason is given for embracing such an assumption. See National Center for Education Statistics, Projections of Educational Statistics to 1988-89, Washington, D.C.: U.S. Government Printing Office, 1980, Tables 13 and A-2.

16. It can be argued that the ratio of attainers at the last systemic level that is non-selective is a good measure of the maturity of the educational system. It is but only one of eight modes of growth in Green, op. cit.

17. See Vincent Tinto, "Does Schooling Matter? A Retrospective Assessment," Review of Research in Education, 5, 1977, pp. 201-235.

18. For another perspective, one that focuses upon actors and events, see Margaret S. Archer, Social Origins of Educational Systems, London: Sage Publications, 1979. Chapter 1 contains an excellent exposition of "methodological individualism." Notice that the Utility Model (aggregate) attainer mean was always higher than the non-attainer mean. However, it was not always individually advantageous to attain level 12. The problems of methodological individualism require an analysis beyond the scope of this paper. Possible time lags in feeling the effects of the two models have not been considered.