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ABSTRACT

Recent research on human memory and cognition with particular emphasis on research that might have some promise for mathematics education is reviewed. The focus is on techniques for analyzing the mind in two areas: (1) the architecture of the human memory system, using the information processing model: and (2) the content and structure of knowledge that a student brings to a problem, using the comprehension, schema, process, and strategy models. The document describes each of these models in separate sections, along with some recent findings by researchers using each analysis technique. It is felt that to understand mathematics learning and problem solving, one must understand both the hardware (e.g., the information processing model) and software (e.g., comprehension, schema, process, and strategy models) that a pupil brings to a task. Promising signs are seen that there will be a continuing "fruitful interaction" between the needs of the mathematics classroom and the development of analytic theories in cognitive psychology. (MP)

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Cognitive Psychology
and Mathematical Problem Solving

Richard E. Mayer

Report No. 80-1

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It is a pleasure to address the 4th International Congress on Mathematical Education, and I appreciate Jeremy Kilpatrick's inviting me to speak in the Begle Series on Research in Mathematics Learning.

My assignment for this presentation is to review recent research on human memory and cognition with particular emphasis on research that might have some promise for education in mathematics. Trying to demonstrate the relevance of memory research for real-world problems like mathematics learning is a rather humbling experience. However, recent developments in the psychology of memory are noteworthy for two reasons: (1) psychologists have developed analytic tools for describing mental processes, structures and knowledge that may be relevant to performance in mathematics, and (2) psychologists have begun to apply these tools to real-world problems including mathematics.

Let's begin with an example of what has been called "one of those 20th century fables", an algebra story problem:

> A sleek new blue motorboat traveled downstream in 120 minutes with an 8 km/h current. The return upstream trip against the same current took 3 hours. Find the speed of the motorboat in still water.

Take a minute and try to solve this problem. As you solve it, write down each step you take, and write down each piece of your existing knowledge you needed in order to solve the problem. Certainly, this problem is not meant to represent all of the many facets of mathmatics learning, but it does serve as an example of a typical problem present in secondary school algebra textbooks. The goal of this paper is to determine whether there is any useful information from research on human memory and cognition that is relevant to understanding how a student learns and solves problems like these.

The major contribution of modern research and theory in human memory and cognition to date concerns techniques for analyzing human mental life. The most relevant analytic techniques with respect to mathematics education are:

(1) Techniques for analyzing the architecture of the human memory system, i.e., what are the characteristics of the basic memory stores and processes that are available to the student? Subsequent sections refer to these techniques as the information processing model.

(2) Techniques for analyzing acquired knowledge, i.e., what is the content and structure of knowledge that a student brings to a problem? Subsequent sections refer to comprehension models (based on linguistic and factual knowledge), schema models (based on knowledge of problem forms), process models (based on knowledge of algorithms), and strategy models (based on knowledge of heuristics).

Information Processing Model

What are the memory stores and processes that people use when solving the motorboat problem? Although theorists disagree on many details, typical information processing models analyze human memory into separate stores, such as sensory memory, short-term memory/working memory, long-term memory, and into many control processes such as attention, rehearsal, search of long-term memory, etc. (See Klatzky, 1980; Loftus & Loftus, 1976).

Analysis of individual differences in mathematics ability. Hunt, (Hunt, 1978; Hunt, Lunneborg & Lewis, 1975), has used the information processing model in order to analyze differences between students who score high vs. low on standardized tests of verbal ability. For example, high verbal students are faster than low verbal students on retrieving a letter's name from long-term memory, high verbal students can hold more letters temporarily in working memory than low verbal students, and high verbal students can make a mental decision about whether two letters match faster than low verbal students. However, low verbal students are no slower on reaction time tasks per se, and the differences that are obtained are very small (e.g., 20 to 40 msec per letter). Although, the differences in processing are small, they add up to a considerable effect when you note that they must be performed thousands of times during the course of reading. Thus, Hunt et. al. were able to characterize differences in verbal ability in terms of differences in the operating characteristics of the human informaton processing system.

Analysis of individual differences in mathematical ability. Individual differences in mathematical problem solving may be due to specific acquired knowledge (Greeno, 1980; Simon, 1980), as discussed in the subsequent four sections. In addition, there may be individual differences in the characteristics of the information processing system that are particularly important for mathematics:

(1) Holding capacity of working memory. People may differ with respect to how much information they can handle at one time. For example, the motorboat problem requires that you hold the following facts: rate of current is 8 km/h; time downstream is 120 minutes; time upstream is 3 hours; speed in still water is unknown; distance upstream equals distance downstream. Case (1978) has shown that young children may not be able to handle more than two or three items at a time, and has provided instructional procedures that do not overload working memory. In a recent experiment, we (Mayer, Larkin & Kadane, 1980) found some evidence that writing in equation form (rather than words) may help reduce load on working memory. For example, problems were presented in word form such as, "Find a number such that 11 less than three times the number is the same as if 8 more than 3 times the number was divided by 2," or problems were presented in equation form such as, $(3X + 8)/2 = 3X - 11$. Although subjects were able to solve both kinds of problems, the pattern of response times suggested that subjects given equations were able to use a planning strategy--looking a few

steps ahead--while subjects given word problems used a different strategy that required much less memory and no planning.

(2) Type of code used in working memory. Subjects may differ with respect to the mode of representation in working memory such as visual vs. verbal vs. equation representation. For example, Hayes (1978) recently interviewed subjects as they solved simple arithmetic and algebra problems. Subjects differed greatly in their use of imagery; some reported heavy reliance on imagery, including "counting points" in images for digits, while others rarely or never used imagery. In addition, some problems elicited more imagery than others; for example, few people reported imagery in solving 5 + 7 = but most subjects reported that they visually moved symbols in their working memory to solve K + 13 = 8. Individual differences in imagery are relevant to algebra story problems such as the motorboat problem, as well. In a recent study, we (Mayer & Bromage, 1980) asked subjects to either draw a picture, write an equation, or write in simple English as a translation for various story problems. Some problems were almost undrawable, such as, "Laura is 3 times as old as Maria was when Laura was as old as Maria is now. In 2 years Laura will be twice as old as Maria was 2 years ago. Find their present ages." Other problems evoked consistent pictorial representation, such as, "The area occupied by an unframed rectangular picture is 64 square inches less than the area occupied by the picture mounted in a frame 2 inches wide. What are the dimensions of the picture if it is 4 inches longer than it is wide?

(3) Speed of mental operations in working memory. People may also differ with respect to how fast they can carry out a single mental operation in working memory. For example, Groen & Parkman (1972) and Resnick (1976) have developed models of simple addition and subtraction, and subjects may differ in terms of how long it takes to perform one step. Recently, we measured response times for

simple algebraic operations such as moving a number from one side to another; this was accomplished, for example, by comparing time to solve, 3X - 8 = 22, to time to solve 3X = 30. The difference between these two problems gives an estimate of the time to move a number; there were large individual differences, and time to move was faster for equation format as compared to word format.

(4) Speed of search for target in long-term memory. Suppose you learned some formulas like: Driving time = arrival time - leaving time, Distance = speed x driving time, Distance = gas mileage x gas used, Speed = wheel size x wheel speed. Then suppose I ask, Arrival time = 6:00, Leaving time = 4:00, Average speed = 25, Find distance. You have to search your memory for the required equation, determine a value for a variable in the equation, and search for another equation, and so on. People differ with respect to how fast they can search for and find target information in long-term memory. In our research (Mayer & Greeno, 1975; Mayer, 1978) search time varies greatly from person to person; also, search time for a new equation is faster when the material is meaningful (as above) than when it is a set of corresponding nonsense equations. Thus, equation format seems to slow down the speed of search in long-term memory but is more efficient for temporarily holding information in working memory.

(5) Selective attention. When you read a problem like the riverboat problem you need to key in on crucial facts such as time to go upstream is 3 hours, but you can ignore irrelevant information such as the type or color of the boat. Recent research by Robinson & Hayes (1978) shows that subjects are quite able to distinguish between what is "important" and what is "garbage" in an algebra story problem, although there are certainly individual differences in selective attention.

(6) Pattern matching. When you read a problem like the riverboat problem, you may look at a few critical features and say, "That's a current problem."

You match some features of the problem to a set of features stored in memory. Similarly, in solving an algebra equation such as, $(8+3X)/2 = 3X - 11$, you may note that one side is divided by 2. This pattern may alert you to the necessity to multiply both sides by 2. Since pattern matching is an important component of problem solving, individual differences in the speed and efficiency of the matching process could influence performance.

Comprehension Models

According to most descriptions of mathematical problem solving, the first step is to translate the words of the problem into an internal representation such as going from the words of an algebra story problem to an equation. For example, in order to solve the motorboat problem you need at least two kinds of acquired knowledge--linguistic knowledge, such as "motorboat" is a noun, "travel" is a verb; and factual knowledge, such as "120 minutes equals 2 hours" or rivers have currents that run from upstream to downstream.

Role of linguistic and factual knowledge. Bobrow (1968) developed a computer program called STUDENT which solves simple problems such as.

> If the number of customers Tom gets is twice the square of 20 percent of the number of advertisements he runs, and the number of advertisements he runs is 45, what is the number of customers Tom gets?

The translation phase of the program involves steps such as, (1) Copy the problem word for word. (2) Substitute words like "two times" for twice. (3) Locate each word or phrase that describes a variable, such as "the number of customers Tom gets" and note if two or more phrases refer to the same variable. (4) Break the problem into simple sentences. (5) Translate each simple sentence into variables, numbers, and operators, such as, (NUMBER OF CUSTOMERS TOM GETS) = $2[.20$ (NUMBER OF ADVERTISEMENTS)$]^2$, (NUMBER OF ADVERTISEMENTS) = 45, (NUMBER OF CUSTOMERS TOM GETS) = (X).

As you can see, STUDENT performs a very literal translation of words into equations. To do even this, however, requires that STUDENT have knowledge of the English language such as the ability to distinguish between operators and variables, and some factual knowledge such as knowing that a dime equals 10 cents or there are seven days in a week. More recently, Hayes & Simon (1974) have developed a program called UNDERSTAND that translates problems into an internal representation. In a recent study, we (Johnson, Ryan, Cook & Mayer, 1980) gave students a 30 minute lesson on how to translate algebra story problems from words to equations using a modified version of Bobrow's procedure. Results indicated that instruction on translation improved the performance of low ability subjects on tests of writing equations. Further work is needed to determine to what extent deficiencies in linguistic and factual knowledge influence students' problem solving performance, and to determine means of diagnosing and remediating the lack of knowledge.

Schema Models

Let's return for a moment to the motorboat problem. Is there anything else you need to know beyond linguistic and factual knowledge? One basic idea you need to know can be expressed as, distance = rate x time. Further, the specific form of the motorboat problem is, (rate of powerboat + rate of current) x (time downstream) = (rate of powerboat - rate of current) x (time downstream). This equation represents the structure of the problem, and helps the student know what to look for; we will refer to the student's knowledge of the form of the problem as a "schema."

Understanding. You may have noticed that STUDENT does not really "understand" what it is doing, and does not care whether the variables are related to another in a logical way. Is this the way humans solve problems? Paige & Simon (1966) gave students problems such as:

> The number of quarters a man has is seven times the number of dimes he has. The value of the dimes exceeds the value of quarters by two dollars and fifty cents. How many has he of each coin?

Some subjects behaved like STUDENT by generating literal translations of the sentences into equations. Other subjects recognized that something was wrong in this problem and corrected it by assuming that the second sentence said, "The value of the quarters exceeds the value of dimes by $2.50." Finally, some students looked at the problem and said, "This is impossible." Thus, while some students may use a literal translation, there is evidence that some students try to "understand" the problem.

How can people be encouraged to successfully "understand" a story problem? Paige & Simon asked subjects to draw pictures to represent each problem. When subjects drew integrated pictures, containing all the information in one diagram, they were much more likely to arrive at the correct answer. When students produced a series of sentence by sentence translations, they were more easily led astray. Thus, in addition to linguistic and factual knowledge the student needs knowledge about how to put the variable together in a coherent way.

Role of schemas. A further breakthrough concerning how people understand story problems like the motorboat problem comes from the work of Hinsley, Hayes & Simon (1977). Subjects were given a series of algebra problems from standard textbooks and were asked to arrange them into categories. Subjects were quite able to perform this task with much agreement, yielding 18 different categories such as river current (the category for the motorboat problem), DRT, work, triangle, interest, etc. Hinsley, Hayes & Simon found that subjects were able to categorize problems almost immediately. After hearing the first few words of a problem such as, "A river steamer travels 36 miles downstream..." a subject could say, "Hey, that's a river current problem." Hayes, Waterman & Robinson

(1977) and Robinson & Hayes (1978) found that students use their schemas to make highly accurate judgments concerning what is important in a problem and what is not.

Many of the difficulties people have in solving story problems can come from using the wrong schema. For example, Hinsley, Hayes & Simon presented subjects with a problem that could be interpreted as either a triangle problem or a distance-rate-time problem. Subjects who opted for one interpretation paid attention to different information than subjects who opted for the other interpretation. Early work by Luchins (1942) demonstrated that shifting from problems that require one schema to problems that require another can cause "einstellung" (or "problem solving set"). For example, Loftus & Suppes (1972) found that a word problem was much more difficult to solve if it was a different type of problem from ones preceding it.

Greeno and his colleagues (Riley & Greeno, 1978; Heller & Greeno, 1978) have located schemas for children's word problems such as "cause/change" (Joe has 3 marbles. Tom gives him 5 more marbles. How many does Joe have now?), "combine" (Joe has 3 marbles. Tom has 5 marbles. How many do they have together?), and "compare" (Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?) Development of schemas seems to be in the order above; for example, second graders perform fine on cause/change problems but poorly on compare problems. When asked to repeat the compare problems, one-third of the children said" "Joe has 3 marbles. Tom has 5 marbles. How many marbles does Tom have?". Failure to solve word problems may, thus, be due to lack of appropriate schema rather than poor arithmetic or logical skills.

Can anything be done to make subjects more readily "understand" problems, i.e., help students find appropriate schema for problems? Our work on solving algebra equations shows that subjects are much faster at making appropriate deductions when the material is familiar (Mayer & Greeno, 1972; Mayer, 1975). Further, when instruction emphasizes familiar experiences subjects are better able to recognize unanswerable problems and engage in transfer; for example, in teaching students to solve binomial probability problems instruction could emphasize previous experience with batting averages, rainy days, and seating r people at n spaces at a dinner table (Mayer & Greeno, 1972; Mayer, 1975). Indeed, further research is needed to determine how to teach problem solvers to effectively use the schemas they have.

In order to gain a broader perspective on the nature of schemas for algebra story problems, I recently surveyed the exercise problems of major algebra textbooks used in California secondary schools (Mayer, 1980). Of the approximately 4000 problems collected, 25 general "families" of problems were located: motion, current, age, coin, work, part, dry mixture, wet mixture, percent, ratio, unit, cost, markup/discount/profit, interest, direct variation, inverse variation, digit, rectangle, circle, triangle, series, consecutive integer, physics, probability, arithmetic, and word (no story). Each type of problem has its own familiar plot line, but there is a major distinction between problems that require use of a formula (such as distance-rate-time for the motorboat problem) and problems that do not (such as the advertising problem solved by Bobrow's STUDENT). Also, for any major family, there are many distinct variants--there were 14 different basic forms located for current problems, with observed frequencies of from 3 to 14. As an example, some of the common "motion" problems were: simple distance-rate-time, vehicles approaching from opposite directions, vehicles starting from the same point and moving out in opposite directions, one vehicle overtakes another, one vehicle makes a round trip, speed changes during the trip, two vehicles take the same amount of time to travel, two vehicles cover the same distance, etc. The procedure used for de··ribing the format of any particular problem is to list the key information; there were four major types of statements: (1) assignment of a value to a variable, e.g., the time to travel downstream is 120 minutes, (2) designation of a relation between two variables, e.g., Laura's age is twice that of Anne, (3) assignment of an unknown to a variable, e.g., what is the speed of the boat in still water, (4) statement of fact, e.g., the cars took the same road. In a series of recall studies (Mayer & Bromage, 1980) it is clear that subjects tend to focus on these types of propositions when asked to remember a problem.

Process Models

Let's consider the motorboat problem again. As we have seen, in order to translate and represent the problem, the student needs appropriate linguistic, factual and schema knowledge. In order to solve the problem the student needs to know: (1) the rules of arithmetic, and (2) the rules of algebra.

Role of arithmetic algorithms. Groen & Parkman (1972) and Resnick (1976) have provided process models to represent the algorithms that children have for simple addition and subtraction. These models can be represented as flow charts, and can be fit to the reaction time data of children. One particulary interesting aspect of the process model work in arithmetic is that children tend to develop more sophisticated (e.g., larger) models as they get older. More recently, Brown & Burton (1978) have been able to model the procedural bugs students have for three digit subtraction problems. Bugs include borrowing from zero (103 - 34 = 158), subtracting the smaller from the larger number (258 - 118 = 145), and ignoring a zero (203 - 192 = 191). This work builds on previous analyses of error patterns, and allows for a precise description of the child's algorithm for subtraction.

Role of algebraic algorithms. Mayer, Larkin & Kadan (1980) have described a model for simple algebraic operations such as moving a variable from one side to the other. The process involves creating nodes, deleting nodes, and forming links among nodes. Simon (1980) has pointed out that algebra textbooks tend to emphasize algebraic algorithms (such as adding equal quantities to both sides of an equation) but fail to emphasize the conditions under which an algorithm should be applied.

Strategy Models

So far we have listed the types of knowledge needed to translate the motorboat problem (i.e., linguistic, factual, schema) and to solve the motorboat problem (i.e., algorithms). In addition, this section explores one final type of knowledge needed to control and use the knowledge at the right time--strategic knowledge. For example, Polya (1968) has emphasized general strategies such as working backwards or working forwards to solve mathematical problems, and more recently Wickelgren (1974) has offered some general problem solving strategies, some especially relevant to mathematics. Attempts to teach these strategies have met with some limited success (Lochhead & Clement, 1979).

Means-ends analysis for algebra equations. Newell & Simon (1972) have provided a technique for representing problems as a problem space. The problem space begins with a concrete description of the given (initial) state of the problem, the goal state, and all intervening states that can be generated by applying allowable operators. For example, the problem (8 + 3X) = 2 * (3X - 11) has the equation as its given state, X = some number as its goal state, and intermediate states like 8 + 3X = 6X - 22, etc. Newell & Simon offer a powerful strategy called "means-ends analysis" for guiding the problem solving process. The procedure may be represented as a production system, a list of condition action pairs. For example, typical productions could be: "If there is an X term on both sides, move the one on the right to the left side of the equation." Problem solving involves moving through the problem space by executing the relevant productions in the production system (see Mayer, Larkin & Kadone, 1980, for an example). Recent work by Bundy (1965), Matz (1979), Davis & McKnight (1979), and Carry, Lewis & Bernard (1980) is directed offering precis models of the strategic knowledge required for solving algebra equations. Further work is needed to provide models that are closer to the real-world performance of school children.

Finally, recent work by Larkin (1979) and by Simon & Simon (1978) has compared the strategic knowledge of experts and novices concerning how to solve physics problems. Experts tend to rely on better organized production systems with more actions chunked into each production. Similarly, recent work by Mayer & Bayman (1980) concerning students knowledge of how to use electronic calculators showed that experts relied on more sophisticated strategies than novices. Further work should focus on the optimistic implication that expertness involves the acquisition of great amounts of knowledge rather than special mental abilities.

Conclusion

To understand mathematics learning and problem solving, you must understand the hardward (e.g., the information processing model) and software (e.g., comprehension, schema, process, strategy models) that a student brings to a task. There are promising signs that we will continue to see what Larkin (1979) calls a "fruitful interaction" between the needs of the mathematics classroom and the development of analytic theories in cognitive psychology.

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