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ABSTRACT

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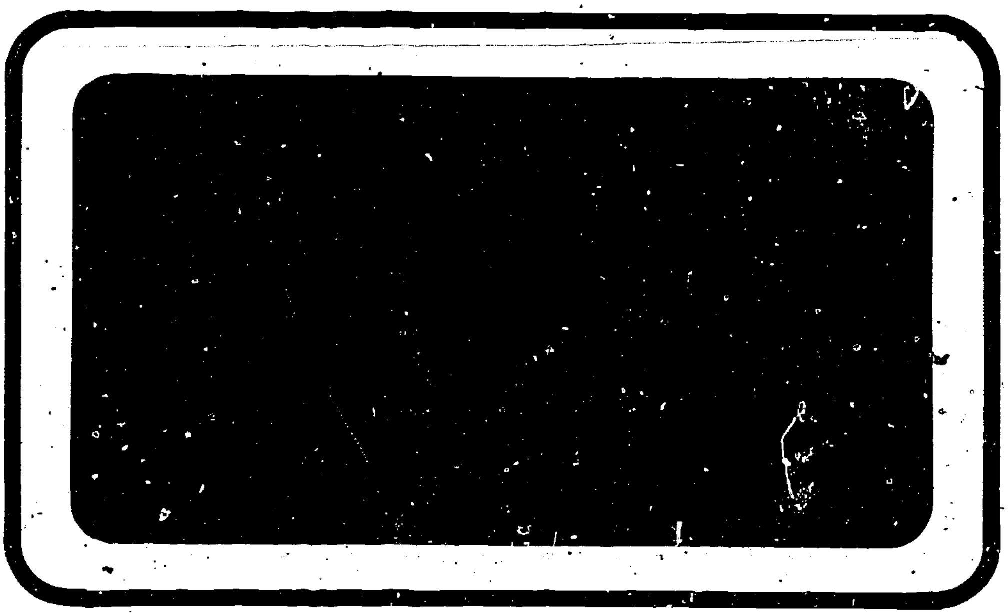

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SERIES IN LEARNING AND COGNITION

Recall of Algebra Story Problems

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Report No. 80-5

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Abstract

In Experiments 1 and 2 subjects read a series of standard algebra story problems, and were asked to recall each problem. In Experiment 3, subjects were asked to construct problems based on certain situations (such as "train leaving stations"). Results indicated that "relational propositions" (such as "the rate in still water is 12 mph more than the rate of the current") were more difficult to remember than "assignment propositions" (such as "the cost of candy is $1.70 per pound), problems with relational propositions were much harder to reproduce in coherent form than problems with assignment propositions, subjects were far more likely to convert a relation into an assignment than vice versa, and in making up problems subjects tended to use assignment propositions more than relational propositions at a ratio of 25 to 1. In addition, subjects showed a knowledge of problem schemas by recalling relevant information much better than irrelevant details, recalling high frequency problem forms better than low frequency forms, converting problems from low to high frequency forms, and by constructing problems that matched standard textbook forms.

Rationale

Algebra story problems have earned a well deserved reputation as a threat to students' achievement in secondary school mathematics courses. For example, a recent test of all 12th graders in California public schools revealed that more than half were unable to correctly solve simple story problems such as the following (California Assessment Program, 1979):

> An astronaut requires 2.2 pounds of oxygen per day while in space. How many pounds of oxygen are needed for a team of 3 astronauts for 5 days in space?

Equally troubling results have been reported in national surveys of mathematical problem solving in the U.S., such as the Assessment of Educational Progress (see Carpenter, Corbitt, Kepner, Lindquist & Reyes, 1980). For example, only 29% of a large national sample of 17 year olds were able to solve the following problem:

> Lemonade costs 95¢ for one 56 ounce bottle.
> At the school fair, Bob sold cups holding 8 ounces for 20¢ each.
> How much money did the school make on each bottle?

Why are algebra story problems so hard to solve? Why is it difficult to teach children how to solve such problems? In spite of years of training and practice in solving story problems, why are simple story problems greeted with moans, fearful faces, and incorrect answers? The answer to these questions would provide the basis for a psychological theory of human problem solving as well as a pedagogy of mathematical learning. The present paper addresses one aspect of these questions, namely, which aspects of a problem are hard to remember?

Translation vs. Solution

A recent review (Mayer, in press,a) of cognitive science research in algebra problem solving (e.g. Bobrow, 1968; Clement, Lochhead & Soloway, Note 1, Note 2; Hayes & Simon, 1974; Hayes, Waterman & Robinson, 1977; Heller & Greeno, Note 3; Hinsley, Hayes & Simon, 1977; Paige & Simon, 1966; Riley & Greeno, Note 4; Robinson & Hayes, 1978) suggests that two processes are involved in solving story problems:

translation: understanding the problem, as manifested in translating the words of the problem into an internal representation in memory,

solution: applying the legal rules of algebra and arithmetic to this internal representation, in order to deduce the answer.

Current work suggests that the major difficulty lies in the translation phase, although most instruction focuses on the solution phase (Simon, 1980). The remainder of this introduction summarizes evidence that translation is influenced by the structural properties of propositions in the problem, and that translation is influenced by the learner's schema for the problem. Then, a series of studies are presented which assesses the difficulty of representing various kinds of algebraic information in memory.

Translation influenced by propositional structure. In an early study, Loftus & Suppes (1972) located "structural variables" that affect difficulty of story problems for sixth graders. For example, difficulty of a problem was increased if it was a different type from the previous one, if it required many arithmetic operations, and if the syntactic structure of the sentences was complex. It seems likely that translation is related to the specific structural properties of the relevant sentences, although this idea was not directly tested.

More recently, Greeno and his colleagues (Heller & Greeno, Note 1; Riley & Greeno, Note 2) have explicitly tested whether certain kinds of propositions are more difficult to translate than others. For example, primary school children were quite proficient at repeating problems in which each sentence deals with one variable, such as a "cause/change" story: "Joe has 3 marbles. Then Tom gave him 5 more marbles. How many does Joe have now?" However, younger children made many errors when a sentence involved a relation between two variables such as a "compare" story: "Joe has 3 marbles. Tom has 5 more marbles than Joe. How many marbles does Tom have?" Typically, students would repeat this problem as: "Joe has 3 marbles. Tom has 5 marbles. How many marbles does Tom have?" Apparently, children had more difficulty in translating sentences that involve relational information. This finding is consistent with Loftus & Suppes' (1972) finding that the hardest problem in their set was one that contained a relational proposition: "Mary is twice as old as Betty was 2 years ago. Mary is 40 years old. How old is Betty?"

Similarly, Clement, Lochhead & Soloway (Note 1, Note 2) have shown that difficulties in translating relation propositions are not limited to primary school children. College students were asked to write equations to represent propositions such as: "There are 6 times as many students as professors at this university." One-third of the students produced the wrong equation, with the most typical error being, 6S = P. However, when students were asked to translate relational statements like this one into a computer program, the error rate fell dramatically. Such results suggest that people have difficulty in interpreting what a relational proposition means when they must use a static format such as equations or simple sentences.

<u>Translation influenced by schemas</u>. Paige & Simon (1966) presented "impossible" problems such as the following:

> The number of quarters a man has is seven times the number of dimes he has. The value of the dimes exceeds the value of the quarters by two dollars and fifty cents. How many has he of each coin?

Some subjects translated the problem into equations; some recognized the inconsistency; and some changed the problem to say that the value of the quarters exceeds the value of the dimes by $2.50, yielding the equation: $10x + 250 = 7(25x)$. Apparently, these latter approaches suggest that some subjects tried to the fit the given problem with their past knowledge about similar problems.

More recently, Hinsley, Hayes & Simon (1977) have found that subjects were able to sort story problems into consistent categories such as "work", "motion", "distance-rate-time", "triangle", "current", etc. Based on this research, Hinsley et. al. detected 18 basic categories for story problems, and suggested that people have "schemas" for each--i.e., knowledge of the structure of each type of problem. When an ambiguous problem was presented to subjects, half interpreted it as a "triangle problem" and half as a "distance-rate-time" problem. The two groups focused on entirely different information in the problem, and even misread facts in a way consistent with their categorization. For example, a "triangle" subject misread "four minutes" as "four miles", assumed this was a leg of the triangle, and applied the Pythagorean theorem. In other studies (Hayes, Waterman & Robinson, 1977; Robinson & Hayes, 1978) subjects were asked to judge which parts of a problem were relevant; subjects tended to decide what category the problem was in and then to make accurate judgments about which facts were relevant. Apparently, what is remembered from a story problem is influenced by the subject's schema for the problem.

Structural Analysis of Story Problems

A companion paper (Mayer, in press,b) summarizes a recent survey of exercise problems in major algebra textbooks used in California public schools. Of the approximately 1200 problems collected, 25 general "families" of problems were located: motion, current, age, coin, work, part, dry mixture, wet mixture, percent, ratio, unit cost, markup/discount/profit, interest, direct variation, inverse variation, digit, rectangle, circle, triangle, series, consecutive integer, physics, probability, arithmetic, and word. Each type of problem has its own familiar plot line, but there was a major distinction between problems that required use of a formula (such as "distance = rate x time" in motion problems) and problems that did not (such as "arithmetic" or "part" problems). Also, for any major family of problems, there were many distinct formats (or "templates"). For example, there were 13 different templates for motion problems such as one vehicle overtaking another ("overtake"), two vehicles converging on the same point ("closure"), speed change during a trip ("speed change"), one vehicle making a round trip ("round trip"), etc.

The relevant information for any given story problem could be described as a list of propositions, with each "template" having a unique list of propositions. One interesting outcome of this analysis was that the relevant information in nearly all of the problems in algebra textbooks could be describing using four basic types of propositions:

(1) Assignment proposition. This involves giving a single numerical value for some variable. Examples include, "the cost of the candy is $1.70 per pound", "the time to fill one pipe is 6 hours", or "total amount invested was $4000".

(2) Relation proposition. This involves giving a single numerical relationship between two variables. Examples include, "the length is 2 1/2 times

the width", "the area of one rectangle is 64 square inches less than the area of a second rectangle", or "the rate in still water is 12 mph more than the rate in the current".

(3) Question proposition. This involves the question asked in the problem, in which the goal is to find a single numerical value corresponding to a given variable. Examples include, "how much time will it take to empty the tank?" or "how many miles will the first car have gone before it is passed?".

(4) Relevant fact. This involves a fact which is necessary for the integrity of the problem. Examples include the fact that "the same route was used" in a camp trip problem and or that the "tank is full" in a pipes problem (see Table 1).

In addition, each problem contained information that was not relevant to solving the problem. For example, in the fence problem, "Mr. Zecha", "chain fencing" and "lot" are not directly relevant; in the store problem, "candy" and "gift box" are not relevant (see Table 1).

Any problem can be described as a template consisting of a list of propositions, with each proposition giving the class of variable and any numerical value. For example, the template for "river problem" in Table 1 may be represented as:

distance downstream = NUMBER
distance upstream = NUMBER
time downstream = RELATION time upstream
rate in still water = RELATION rate of current
rate of current = FIND

The first two propositions are "assignments", the next two are "relations" and the last one is a "question".

The present series of studies investigates how structural properties of problems influence student's ability to remember story problems. If problems involving relational information are more difficult to translate (and hence more difficult to answer correctly), then subjects should have more difficulty remembering relational propositions. In addition, the present series of studies investigates how students' knowledge of problem types (i.e. "schemas") influences their ability to remember problems. If schemas are used to translate problems, then subjects should recall information that is relevant to schemas more easily than irrelevant information, and should tend to produce coherent, solvable problems.

EXPERIMENTS 1 AND 2

The goal of Experiments 1 and 2 is to determine which types of information subjects remember from standard algebra story problems. First, previous research suggests that structural properties of the propositions may influence translation into an internal representation. If "relational" propositions hinder translation, one can predict that relation propositions will be more difficult to remember than assignment propositions. Second, previous research suggests that subjects use schemas for translating problems into internal representations. If subjects use schemas, one can predict that information relevant to the problem will be recalled better than information that is not relevant to the problem.

Method

Subjects and Design

Experiments 1 and 2 used identical designs. In each study there were 24 undergraduates who were recruited from the Psychology Subject Pool at the University of California, Santa Barbara. Each subject served in one of three treatment groups based on activity during problem presentation (sentence, equation, or picture). Subjects in Experiment 1 and 2 received different sets of problems (set 1 or set 2).

Materials

For each experiment, materials consisted of a subject questionnaire, three sets of instructions, a set of problem sheets, and a set of cued recall sheets.

The subject questionnaire was a one sheet typed set of questions concerning the subject's age, sex, mathematics experience, SAT scores, and related matters.

The three sets of instructions each consisted of a two page discription of the task, including an example. The sets asked subjects to rewrite each problem as a set of sentences (sentence instructions), as a set of equations (equation instructions), or to draw a labeled picture or diagram (picture instruction).

The set of problem sheets consisted of 8 half sheets of paper, with an algebra story problem and a title typed onto each. Problems were selected to be representative of the types of problems found in algebra textbooks. The problems used in Experiment 1 and the problems used in Experiment 2 are listed in Table 1.

The cued recall test consisted of 8 half sheets of paper with a title of a problem typed onto each. The list of title cues for each problem in Experiment 1 and Experiment 2 is given in Table 1.

Table 1 About Here

Procedure

Subjects were randomly assigned to treatment and were run in groups of one to three people per session. First, each subject filled out the subject questionnaire. Then, instructions for the experiment were administered, and the problem cards were given. Subjects were told that they would have two minutes to study the algebra story problem. Their job was to rewrite the problem either into basic sentences, equations or a picture (depending on the subject's treatment group). After two minutes, subjects were instructed to go on to the next problem, and so on for each of the eight problems. Then instructions were read for the test, and the eight cued recall sheets were given. Subjects were told that they should try to write down the problem exactly as it was presented for each of the cued recall sheets. Subjects responded to each recall sheet for two minutes, and were not allowed to go ahead or go back to previous sheets. After all eight sheets had been attempted, the subjects were debriefed, thanked and excused.

Results

Scoring

For purposes of scoring the recall protocols, each problem was broken down into units. As described in the introduction, there were four types of information units that were used to define each problem (assignment, relation, question, fact) as well as non-relevant information units.

Each problem was then listed as a set of essential units and non-essential units. For example, for the airways problem the essential units were: number of hours at rate 1 = 2, increase in speed for rate 2 in mph = 30, length of total trip in miles = 570, number of hours for total trip = 3 1/2 hours, length of first 2 hours of trip in miles = FIND. For the frame problem, the essential units were: area of rectangle = 64 square inches less than area of rectangle 2, frame width = 2 inches, length of rectangle 1 = 4 inches more than width of rectangle 1, length of rectangle 1 = FIND, width or rectangle 2 = FIND.

The performance of each subject on each problem was scored by recording which of the essential and non-essential units were present in the recall protocols. If the unit was correctly recalled (i.e. the variable was stated and the values were correct) the subject received full credit for that unit; if the unit was structurally correct but the specific values were wrong (e.g. "frame width = 4 inches" instead of "frame width - 2 inches") the subject was given credit for having recalled the problem. Thus, the data for each subject consisted of numbers of assignment, relation, question, fact, and non-essential information units that were recalled, and the number of correctly recalled problems.[1]

The propositional Structure Hypothesis

This section explores whether the structure of propositions in a problem influences the subjects' ability to recall the proposition. In particular, this section explores what could be called the "propositional structure hypothesis"--the idea that assignment propositions are psychologically more basic than relational propositions. For example, students may expect story problems to take the form of a list of assignments of values to variables. Thus, at encoding or comprehension, a relational proposition will be more difficult to represent; similarly, at retrieval, if there are gaps in memory a student may be more likely to reconstruct the information as an assignment than as a relation. Indeed, a frequency analysis of assignment and relation propositions in standard textbook problems shows that assignments outnumber relations by as much as 12 to 1 (Mayer, in press, b). Similarly, the previously cited research of Clement, Lochhead & Soloway (Note 1, Note 2) demonstrates that students have an unusually difficult time in translating relational propositions into equations; Greeno and his colleagues (Heller & Greeno, Note 3; Riley & Greeno, Note 4) also report that children often change relation propositions to assignment propositions when they are asked to repeat a story problem.

Levels effect analysis. The propositional structure hypothesis predicts that there should be different retention rates for assignment and relation propositions. In order to test this idea, the retention rates for assignment and relation propositions as well as question propositions, facts, details and numbers was obtained for each subject for all problems.

Table 2 shows the proportion of recall failures by type of information for all of the problems used in Experiment 1, and for all of the problems used in Experiment 2. Two analyses of variance were conducted on the error rate data for Experiment 1 and 2 with type of information as a factor; these analyses yielded a significant difference among error rates for Experiment 1, $F(4,92) = 11.37$, $p < .001$, and for Experiments 2, $F(4,92) = 30.93$, $p < .001$. Table 3 shows the proportion of recall failures by type of information for each of the seven problems that contained both assignment and relation propositions. As can be seen, there is a consistent pattern in which relation propositions are remembered less well than assignment information. T-tests were conducted comparing overall error rates on assignments and relations in Experiment 1 and in Experiment 2, yielding significant values, respectively, of $t(23) = 7.53$, $p < .001$ and $t(23) = 2.35$, $p < .05$. In addition, t-tests were conducted to compare the error rates for relation and assignment propositions for each of the seven problems shown in Table 3. The values were: for river, t(23) = 1.91; for work, t(23) = 1.58; for frame, t(23) = 2.17; for freeway, t(23) = 3.88; for TV, t(23) = 1.24; for race, t(23) = 1.17; for fence, t(23) = 1.45; for total of all seven problems, t(23) = 3.91 (with t-values above 2.069 significant at .05).

Tables 2 and 3 About Here

The foregoing analysis provides evidence that relation propositions are more difficult to remember than assignment propositions. In order to provide further information on the structural features of story problems that are related to recall difficulty, a multiple regression analysis was performed. For each problem, the following information collected: number of assignment propositions, number of relation propositions, number of question propositions, number of relevant facts, number of variables, average probability that the problem would be recalled in correct form. Correct form means that each proposition is recalled although the specific numbers need not be correct; and that the problem is coherent in the sense that all essential information is presented.

In a preliminary multiple regression, probability of coherent recall of the problem was the dependent variable while the independent variables were number of assignments, relations, unknowns, facts, and variables. Only two variables produced significant improvement in the regression function--number of assignment propositions and number of relation propositions. Therefore, a second multiple regression was conducted, using probability of coherent recall of the problem as the dependent variable, with number of assignments and relations as the independent variables. The resulting function was: (probability of correct recall of a problem) = 1.17 - .14 (number of assignment propositions) - .30 (number of relation propositions). Figure 1 gives the predicted and obtained performance on remembering each of the 16 problems in Experiments 1 and 2. As can be seen, the correlation between predicted and obtained recall probability was .942; thus, number of assignment propositions and number of relation propositions account for approximately 89% of the variance. These analyses indicate that relation propositions are weighted about twice as strongly as assignment propositions, suggesting that problems with relation propositions are as hard to remember as problems with twice as many assignment propositions.

Figure 1 About Here

Error Analysis. The previous section has provided evidence for the prediction that relation propositions should generate quantitatively more recall errors than assignment errors. An additional prediction of the propositional structure hypothesis is that relation propositions should generate qualitatively different kinds of errors than assigment propositions. In particular, if assignments are more psychologically basic than relations, one would predict that subjects might convert relations into assignments in their recall protocols but that assignments would not be converted into relations.

This section provides an analysis of errors in order to determine whether different types of errors were committed for each type of proposition. For each subject, each error in recall was classified as one of the following:

Omission Error: The proposition was not reproduced in the recall protocol for the problem.

Specification Error: The variable in the proposition was changed to a different variable in recall. For example, the proposition, "A river steamer travels 36 miles downstream," may be recalled as, "A boat travels 36 mph downstream."

Conversion Error: The form of the proposition was changed from a relation to an assignment or from an assignment (or question) to a relation. For example, the proposition, "The steamer's engines drive in still water at 12 mph more than the rate of the current," may be recalled as, "The speed of the boat in still water is 12 mph."

The major issue addressed in this section concerns whether subjects make qualitatively different errors in recall of assignment vs. relation vs. question propositions. Figure 2 gives the number of subjects (out of a total of 48 in Experiments 1 and 2) who committed at least one omission, specification, and conversion error for each of three types of propositions. Tests for differences, based on the z-distribution, and .05 significance level, were conducted among the proportion of subjects committing each type of error for each type of proposition. For omission errors, there were no significant differences among the proportion of subjects committing errors on assignment, relation or question propositions. For specification errors, subjects were significantly ($p < .05$) more likely to commit a specification error for a question proposition than for a relation proposition or for an assignment proposition. For conversion errors, subjects were significantly ($p < .05$) more likely to change a relation proposition to an assignment ($n=18$) than turn an assignment into a relation ($n=1$) or turn a question into a relation ($n=0$).

Figure 2 About Here

Figure 3 provides another way to examine the issue of qualitatively different kinds of errors in recall of different kinds of propositions. Figure 3 summarizes the average number of each kind of error to the total number of errors for each of three kinds of propositions. One-way ANOVAs were conducted for each type of error to compare the weighted number of errors of that type on assignment, relation and question propositions. As can be seen, omission errors represent a large but varying proportion of errors for each type of proposition, $F(2,46) = 3.81$, $p < .05$. For specification errors, there was a trend in which question and assignment propositions tended to generate a higher proportion of specification errors than relation propositions, $F(2,46) = 3.29$, $p < .05$. For conversion errors, there was a striking pattern in which a substantial proportion of errors were conversions for relation propositions but not for assignment or question propositions, $F(2,46) = 4.96$, $p < .025$.

Figure 3 About Here

The most striking outcome of this analysis of errors is that relation propositions tend to be converted into assignment propositions in a substantial number of cases. In all, there were 20 clear cut cases involving five different relation propositions in which subjects converted a relation proposition into an assignment, and only one case in which a subject converted an assignment into a relation. Table 4 provides examples of the typical conversions that occurred in Experiments 1 and 2. These conversions are consistent with the idea that assignment propositions often represent a more comfortable way of storing information than relations; for example, the assignment may be psychologically more basic than the relation. Hence either at time of encoding, during storage, or at time of retrieval, a relation tends to become distorted into an assignment.

Table 4 About Here

Analysis of River Problem. The river problem is interesting because it contains both assignment and relation propositions. The river problem consists of the following relevant propositions:

distance upstream = 36 miles

distance downstream = 24 miles

time upstream = SAME time downstream

rate in still water = 12 + rate of current

rate of current = FIND

Tables 5 and 6 summarize the recall performance for these five propositions.

Tables 5 and 6 About Here

The first two propositions involve assignment of a value to the distance variables. The error rate for these propositions is 50%. The most typical error, accounting for 92% of the errors, is to substitute a different kind of assignment proposition, such as: "The boat travelled at 36mph upstream and 24 mph downstream." Omission of an assignment proposition accounted for only 8% of the errors. The next two propositions express quantitative relations between two variables. The error rate for these propositions is 65%. The major type of error for the third proposition is to omit it from the problem. The major type of error for the fourth proposition is to convert it to an assignment proposition such as, "The rate in still water is 12 mph". For the relation propositions, omissions accounted for 62% of the errors, and changes to an assignment accounted for 38% of the errors. The error rate for the question proposition (the fifth proposition above) is 21%, with the major type of error being omission. Apparently, subjects are able to remember that two assignment statements are part of the problem, however a substantial portion of subjects forget which variable is being assigned. The third proposition, a relation, seems to be frequently omitted, perhaps because no obvious numerical value is involved. The fourth proposition involves an obvious numerical value, but is often remembered as an assignment. Apparently subjects "expect" numbers to be assigned to certain variables, such as m.p.h., and have difficulty when other kinds of information are presented. It is as if they have no "slot" to put relational information so they must either ignore it or convert it to assignment.

The Schema Hypothesis

The foregoing sections provided evidence that the propositional structure of statements in a problem influence its recallability. This section investigates another factor that may influence recallability, namely, how closely a problem matches prototypical story problems. Previously cited research by Hinsley, Hayes & Simon (1977) and Greeno & Riley (Note 3) and Greeno & Heller (Note 4) suggest that students are able to recognize different problem types and that some types are much more difficult to remember than others. In order to derive a more extensive listing of typical problems, Mayer (in press,b) tallied the frequency of over 100 types of problems found in standard algebra textbooks.

This section explores whether students' knowledge of typical problem forms is related to recall performance. In particular, this section investigates what could be called the "schema hypothesis"--the idea that students possess certain schematic representations for typical problem forms, and that these schematic representations influence comprehension and retrieval. In comprehension and learning, a schema for a problem can be used to determine which information is relevant and to build the propositional form of each piece of information. In retrieval, a schema for a problem can again determine the expected relevant information--perhaps filling in gaps that cannot be remembered --as well as suggesting the propositional structure of remembered information. For the current discussion, a schema for a story problem can be represented as the list of propositions (assignments, relations, questions) consisting of a slot for the type of variable (such as rate or distance or number of units), a slot for specific numbers, and slots for specific relations. Thus, the propositional analysis performed in previous sections provide a representation framework for describing each type of story problem. A type consists of all problems that possess the same form of propositions and same general story line.

Levels effect analysis. A straightforward way to test the schema hypothesis is to consider the retention rates for information that is relevant and irrelevant to the problem. If students were not aware of problem forms (i.e. schemata) they would treat relevant and irrelevant information in equivalent ways. However, if students are aware of problem forms during encoding and retrieval, they should focus preferentially on information that is relevant to the problem schema.

In order to test this idea, average error rates for recalling relevant information (i.e. assignment, relation, question propositions) was compared to recall of irrelevant information (details). Recall of a proposition requires recall of variable (such as "miles per hour", or "number of minutes", or "cost per pound" etc.) as well as the correct relations, while recall of a detail requires only recall of a specific variable that is irrelevant (such as the name of a character, the type of vehicle, etc.). As can be seen in Table 2, there is a clear pattern in which error rates are much higher for irrelevant than relevant information. A t-test comparing overall error rates for the two kinds of information revealed a strong significant in Experiment 1, $t(23) = 11.09$, $p < .001$, a nonsignificant effec in Experiment 2, $t(23) = 1.23$, n.s. and a significant effect for Experiment 1 and 2 combined, $t(47) = 5.87$, $p < .001$. Thus, as predicted, subjects tend to recall relevant information better than irrelevant information.

<u>Frequency analysis</u>. One way to test the schema hypothesis is to examine the relationship between probability that a problem is correctly recalled and the frequency with which that problem appears in standard algebra textbooks. According to the schema hypothesis, recall should be better for problems that are more typical. In order to test this idea, frequency values were obtained for each of the 16 problems in Experiments 1 and 2, using frequency data collected by Mayer (in press,b). The 16 problems were rank ordered based on their frequency of occurrence; Table 1 gives the rank orders. In addition, for each problem, the proportion of subjects (out of 24) who correctly remembered the problem was taken from the previous analyses, and these were rank ordered.

Figure 4a shows the relationship between frequency rank and recall rank for the 16 problems, with each dot in the figure corresponding to one problem. As can be seen, there is a trend in which recall performance increases as the typicality of the problem increases. A correlation yielded a value of r = .66, suggesting a moderately strong relation in which frequency rank was able to account for approximately 44& of the variance in recall among problems.

Figures 4a and 4b About Here

The strength of the relationship shown in Figure 4a may be hindered somewhat by the fact that the 16 problems used in Experiments 1 and 2 involved many different "families" of problems. Mayer (in press,b) defined a family as problems that share a certain basic underlying formula. For example, seven of the problems used in the experiments involve the formula, total = amount x rate. These include motion problems such as freeway, camp trip, race, and airways; current problems such as river; and work problems such as pipes and work. In order to investigate the relation between frequency and recall within a single "family" of problems, an additional analysis was performed. Based on observed frequencies of occurrency in algebra textbooks (Mayer, in press,b) the seven problems were rank ordered from 1 to 7. Similarly, the problems were rank ordered based on their recall probabilities.

Figure 4b shows the recall rank as a function of frequency rank with each dot corresponding to one problem. As can be seen there is a trend in which the more frequent a problem is, the easier it is to recall. For example, the pipe problem and race problem, with respective frequencies of 49 and 23, are the most frequent and also the most easily remembered; similarly, the river, freeway, and work problems are the least frequent with respective frequencies of 0, 4 and 0, and are also the most difficult to recall. A correlation between frequency rank and recall correctness rank yielded a value of r = .85, suggesting that within a family frequency can account for 72% of the variance among problems.

Error analysis. Another way to test the schema hypothesis is to investigate the types of errors that were committed. According to the schema hypothesis, conversions of propositions should be more likely to change a low frequency problem into a higher frequency problem than vice versa. In order to test this idea, norms were used that provided a listing of over 100 problem forms and their observed frequency out of approximately 1200 problems in algebra textbooks (Mayer, in press,b). These norms were used to classify each of the 21 cases of conversion observed in Experiments 1 and 2.

Using the independently established frequency norms, it was found that for the 10 cases of conversion in the river problem, the given problem was an "Equal Time 2" problem with an observed frequency of 0, while the convert version of the problem was an "Equal Time 1" problem with an observed frequency of 9. Similarly, the two cases of conversion in the freeway problem changed the given problem from a "Closure 2" problem (observed frequency = 4) to a "Closure 1R problem (observed frequency = 12). The 5 cases of conversion in the frame problem changed it from a "Frame Relative 3" (observed frequency = 0) to either a "Frame Relative 1" (observed frequency = 5) or to a "Frame 1" (observed frequency = 11). The 3 cases of conversion involving the race problem could not be analysed because the norms included both the given and the converted versions of the problem under the "overtake" category; similarly, the one case of conversion of an assignment into a relation in the freeway problem could not be analysed because the norms included both the given and the converted version under the "Closure 2" heading. Thus, of the 21 cases of conversion, 17 changed the problem from a low frequency version into a higher frequency version, and 4 conversions could not be analyzed using the existing norms. A binomial test revealed that this pattern was significant beyond $p < .05$. Thus, there is consistent support for the idea that when subjects convert problems, they tend to distort the problem towards more typical versions.

EXPERIMENT 3

The goal of Experiment 3 was to determine which types of information subjects use in constructing standard algebra story problems. one can predict that the problems will be solvable, coherent and match basic types of problems found in textbooks. If subjects have trouble working with relational propositions, one can predict that constructed problems will contain very few relation propositions.

Method

Subjects and Design

The subjects were 36 subjects recruited from the same population as in Experiments 1 and 2. All subjects received the same treatment.

Materials

The materials included the subject questionnaire from Experiments 1 and 2, 16 cued problem sheets based on the 16 problems used in Experiments 1 and 2, and an instruction sheet. The 16 cued problem sheets were each 8½ x 5½ in., and contained two or three basic keywords from each of the 16 problems used in Experiments 1 and 2. Table 7 lists the keywords given for each cued problem sheet. The instruction sheet consisted of a paragraph that asked subjects to make up typical algebra story problems using the words on each of the 16 problem sheets.

Table 7 About Here

Procedure

As in Experiments 1 and 2, subjects were run in groups of one to three people per session. First, subjects filled out the questionnaire and then instructions for the experiment were given. Subjects were told they would be given a sheet with some words from an algebra story problem printed on it and their job was to make up a typical story problem using those words. Subjects were told to try to make common problems that they would expect to see in a typical algebra textbook. After two minutes on the first sheet, subjects were instructed to go on to the next sheet and so on. The order of the sheets was randomized for each subject. After spending 2 minutes on each of the 16 sheets, subjects were thanked and excused.

Results

Scoring

The answer to each problem sheet for each subject was scored as in Experiments 1 and 2.

For each subject on each problem, a list of propositions was generated, including assignment, relation, and question. Also, for each subject on each problem, the problem was classified by type according to independently established norms (Mayer, in press,b).

The Propositional Structure Hypothesis

The propositional structure hypothesis states that assignments are psychologically more basic than relations. In order to test this idea, the number of assignment, relation, and question propositions was tallied for all 16 protocols for each subject. Figure 5 summarizes the results. ~~As can~~ be seen, the average "made-up" story problem consisted of approximately 4 propositions, containing about 3 assignments and one question. Relation propositions were very rare, accounting for less than 3% of the propositions produced.

The problems used in Experiments 1 and 2 contained approximately 51% assignments, 19% relations, and 30% questions; a sample of problems from standard algebra textbooks (Mayer, in press) contained 61% assignments, 11% relations, and 28% questions. T-tests revealed that the proportion of relative propositions produced in Experiment 3 was significantly less than the 19% rate for Experiments 1 and 2 or the 11% rate for standard textbook problems, $t(26) = 13.60$, $p < .001$, and $t(26) = 27.19$, $p < ..001$, respectively. Thus, there is evidence that subjects favor story problems that do not contain relation propositions.

Figure 5 About Here

The Schema Hypothesis

The schema hypothesis states that subjects have a knowledge of typical problem forms, and they use these in comprehension and recall of story problems. If subjects possess schemas for story problems, they should be able to generate coherent problems in Experiment 3--problems that correspond to the typical problem types found in algebra textbooks.

In order to test this idea, each problem produced by each subject was labeled as either invalid or as one of the basic forms found in an independent analysis of textbooks (Mayer, in press,b). Then, the protocols were grouped into one of the following categories:

simple story-- a story problem from the target family, that involves giving values for two variables and asking for a third unknown variable. For example, for any of the motion problems or current problems, the family formula is, distance = rate x time, and a simple story might be: "One boat travels 25 miles in 5 hours. What is its speed?"

complex story--a story problem from the target family that involves a more complex situation than plugging into a three variable formula. For example, for any of the problems that contained cues for a motion problem, the generated problem might be an overtake problem or a round trip problem, etc.

other story-- a story problem from a different family than that cued. For example, if the cues called for a motion problem, the problem generated might be a simple rectangle problem.

arithmetic-- a problem that does not require any underlying formula, but rather involves simple addition or subtraction. For example, cues for a motion problem might generate: "One car traveled 50 miles in the morning and 150 miles in the afternoon. How far did it go altogether?"

part-- a problem that does not require any underlying formula, but rather involves a situation in which some amount is broken into two parts. For example, "The total trip took 3 hours. The first part took twice as long as the second part. How long was the first part?"

invalid-- a protocol that contained an incoherent problem. The most typical invalid problem lacked a question.

Figure 6 shows the proportion of constructed problems falling into each category. As can be seen, over 75% of the constructed problems were coherent problems that matched forms typically found in algebra textbooks. A third of the constructed problems were so simple that they did not require an underlying formula--i.e., the arithmetic and part problems. These are very common in primary school mathematics and probably represent a student's first exposure to "algebra problems". When problems involving a formula were generated they often involved only the most simple form--simple story--or a simple form of another kind of problem--other story. Only 20% of the constructed problems were complex story pro...ems; of these, almost all were instances of the most frequently observed problems in algebra textbooks (Mayer, in press). Thus, Figure 6 suggests that subjects possess some very basic schemas for story problems--mainly arithmetic and part problems, and simple formula problems. Evidence of knowledge of more complex (and less frequent) schemas was less evident.

Figure 6 About Here

GENERAL DISCUSSION

The present studies provide additional information concerning students' memory processing of algebra story problems. In particular, these studies provided additional evidence concerning the "propositional structure hypothesis" and the "schema hypothesis".

Proposition Structure Hypothesis

These studies tend to confirm the results of previous investigations with children (Heller & Greeno, Note 3; Riley & Greeno, Note 4$ and college students (Clemdnt, Lochhead & Soloway, Note 1, Note 2) that relational information is difficult to remember and translate. The present experiments provided several tests of the idea that assignment information is more psychologically basic than relational information, i.e., that people are more prepared to deal with information stated as an assignment than as a relation: (1) In Experiments 1 and 2, recall of relational propositions was substially lower than recall of assignment propositions, with the error rate relation being about twice the error rate for assignments. (2) In Experiments 1 and 2, relation propositions contributed to difficulty of recall at about double the weight of assignment propositions. (3) In Experiments 1 and 2 subjects were far more likely to convert a relation proposition into an assignment than an assignment into a relation. (4) In Experiment 3, when asked to make-up problems, subjects include almost <u>no</u> relation propositions, with assignments outnumbering relations by a ratio of approximately 25 to 1.

Apparently, relational information is more difficult to mentally represent than other kinds of relevant information in a story. In a sense, relational propositions are the "weak link" in the subject's attempt to move from a story to an internal representation. An implication of this finding is that special attention should be paid to teaching children how to translate among relational propositions (in English), relational equations, and concrete manipulatives or pictures. Clement et. al. (Note 1, Note 2) have suggested that experience with computer language might be useful in teaching these translation techniques.

<u>Schema Hypothesis</u>

These studies also support the idea suggested by Hinsley, Hayes & Simon (1977) that subjects recognize and use problem categories in processing story problems. In addition, these studies tend to extend previous results (Heller & Greeno, Note 3; Riley & Greeno, Note 4) that some schemas are more basic than others. The present experiments allowed for several major tests of the idea that subjects possess knowledge of problem schemas: (1) In Experiments 1 and 2, recall of relevant information was much better than recall of information not relevant to story line. (2) In Experiments 1 and 2, problems that were in a form that is commonly observed in algebra textbooks were easier to correctly recall than problems that consisted of infrequent forms. (3) In Experiments 1 and 2, subjects rarely changed the category format of the problem; when problems were converted, they tended to be changed from a low frequency form to a higher frequency form. (4) When subjects were asked to make-up problems, more than 75% of the constructed problems were coherent problems, and most were equivalent to very simple, high frequency forms.

Apparently, students possess some very basic schemas for story problems. The results suggest that subjects are adept at learning the basic problem "categories" in algebra story problems. However, a difficulty may arise when students are given problems for which they have no schema. An implication of these findings is that explicit training in the more complex problem types, including naming of each major type, may enhance problem solving performance. In particular, students may need practice in determining which problems are of the same form and which are not. This training seems most important for complex story problems--problems that go beyond simple use of a three term equation or simple arithmetic and part problems. Further research is warrented to determine whether such training affects problem solving performance.

In addition, these studies suggest that certain problem forms are more salient than others. (1) In Experiments 1 and 2, short problems with no relational propositions were the easiest to reproduce. For example, the pipes, race, and will problems were easiest to remember in correct form. Much more difficulty was encountered in remembering longer problems with relational propositions such as the age and river problems. (2) Similarly, in Experiment 3, subjects tended to construct problems that did not require a formula (such as arithmetic or part problems) or problems that used a formula in a very simple way. For example, a simple formula story problem involves a three variable equation (such as distance = rate x time), and presents assignments for two variables, and a question for the third. Apparently, the more complex versions of each problem category are less readily accessable.

There appears to be a hierarchy of development of problem schemas in which non-formula and simple-formula schemas are learned before more complex story formats. Further research is warranted to determine the specific ordering of difficulties of standard problem types. One implication of such work concerns the sequencing of problems in the course of instruction.

Footnote

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[1]Since there were no effects or interactions due to the between subjects manipulation, this factor was not included in any subsequent analyses.

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Figure Captions

Figure 1. Predicted and Obtained Probability of Problems Being Recalled in Correct Form--Experiments 1 and 2

Note. - Each dot corresponds to one problem. Standard error is .08. Percentage of explained variance is 88.7%. "A" refers to the number of assignment propositions in the problem; "R" refers to the number of relation propositions in the problem.

Figure 2. Number of Subjects Who Committed Omission, Specification, and Conversion Errors For Each Type of Proposition--Experiments 1 and 2

Note. - Numbers indicate how many subjects out of a total of 48 committed at least one error. "A" refers to assignment proposition, "R" refers to relation proposition, "Q" refers to question proposition. Asterisk (*) after 24 indicates significantly more subjects made specification errors for questions than for other kinds of propositions; asterisk (*) after 18 indicates significantly more subjects committed conversion errors for relation propositions than for other kinds of propositions.

Figure 3. Proportion of Omission, Specification, and Conversion Errors to Total Error for Recall of Each Type of Proposition--Experiments 1 and 2

Note. - Numbers in parentheses indicate total number of errors observed for each type of proposition. The total number of to-be-recalled propositions was 984 for assignments, 360 for relations, and 552 for questions.

Figure 4. Relation Between Frequency Rank and Recall Rank for all 16 Problems and for 7 Selected Problems--Experiments 1 and 2

Note. - Each dot corresponds to one problem.

Figure 5. Average Number of Propositions Constructed per Problem by Type of Proposition--Experiment 3

Note. - 95% confidence interval is indicated around ± symbol.

Figure 6. Proportion of Constructed Problems by Format of Problem--Experiment 3

Note. - 95% confidence interval is indicated around ± symbol.

Table 1

Algebra Story Problems Used in Experiments 1 and 2

Experiment 1

Title (Recall Rank)	Problem
Age (1)	Laura is 3 times as old as Maria was when Laura was as old as Maria is now. In 2 years Laura will be twice as old as Maria was 2 years ago. Find their present ages.
River (2)	A river steamer travels 36 miles downstream in the same time that it travels 24 miles upstream. The steamer's engine drives in still water at a rate of 12 miles per hour more than the rate of the current. Find the rate of the current.
Freeway (3)	A truck leaves Los Angeles en route to San Francisco at 1 p.m. A second truck leaves San Francisco at 2 p.m. en route to Los Angeles going along the same route. Assume the two cities are 465 miles apart and that the trucks meet at 6 p.m. If the second truck travels at 15mph faster than the first truck, how fast does each truck go?
Frame (5)	The area occupied by an unframed rectangular picture is 64 square inches less than the area occupied by the picture mounted in a frame 2 inches wide. What are the dimensions of the picture if it is 4 inches longer than it is wide?
Camp Trip (8.5)	Some members of the Rocky Mountain Outing Club hiked to an overnight campsite at the rate of 3 miles per hour. The following morning they returned on horseback over the same route at 10 miles per hour. The total time spent in going and returning was 6 1/2 hours. Find the distance to the campsite.

Table 1 (continued)

Experiment 1

Title (Recall Rank)	Problem
Mixture (11.5)	One vegetable oil containes 6% saturated fats and a second contains 26% saturated fats. In making a salad dressing how many ounces of the second may be added to 10 ounces of the first in order to make 16% saturated fats?
Will (14.5)	In his will a man left his wife \$20,000 and his son \$12,000. Upon his death, his estate amounted to only \$16,400. If the court divides the estate in the ratio of the bequests in the will, what should each receive?

Experiment 2

Title (Recall Rank)	Problem
TV (4)	The entertainment portion of a 30-minute TV program lasted 4 minutes longer than 4 times the portion devoted to advertising. How many minutes were devoted to advertising?
Store (6)	One brand of candy costs \$1.70 per pound. Another costs \$1.50 per pound. The two brands are to be mixed to form one 5-pound gift box that will cost \$8.10. How many pounds of each should be included?

Table 1 (continued)

Experiment 2

Title (Recall Rank)	Problem
Coins (8.5)	On a ferry trip, the fare for each adult was 50¢ and for each child was 25¢. The number of passengers was 30 and the total paid was $12.25. How many adults and children were there?
Investment (8.5)	Mr. Brown invested a total of $4000. On part of this he earned 4%. On the remainder he lost 3%. Combining his earnings and losses, he found his annual income to be $55. How much did he have invested at each rate?
Airways (11.5)	After an airplane had been flying for 2 hours, a change in wind increased the plane's ground speed by 30 miles per hour. If the entire trip of 570 miles took 3 1/2 hours, how far did the plane go the first two hours?
Race (13)	In a sports car race, a Panther starts the course at 9:00 a.m. and averages 75 miles per hour. A Mallotti starts 4 minutes later and averages 85 miles per hour. How many miles will the first car have driven when it is passed?
Pipes (14.5)	One pipe can fill a tank in 6 hours while another can empty it in two hours. How long will it take to empty the full tank if both pipes are open at once?

Table 1 (continued)

Experiment 2

Title (Recall Rank)	Problem
Fence (16)	Mr. Zecha has just fenced his rectangular lot using 350 feet of chain fencing. If the length is 2 1/2 times the width, find the area of the lot.

Note - Recall rankings are based on a scale of 1 to 16 with 1 indicating that the fewest number of subjects were able to correctly recall the problem and 16 indicating that the most subjects were able to correctly recall the problem.

Table 2

Proportion Incorrect Recall by Type of Information

	Assignment Proposition	Relation Proposition	Question Proposition	Relevant Fact	Details
Exp. 1	.14	.35	.40	.40	.36
Exp. 2	.04	.14	.20	.43	.50
	.09	.29	.30	.41	.42

Table 3

Error Rates for Seven Key Problems by Type of Proposition

	Type of Proposition		
Problem	Assignment	Relation	Question
River (Exp. 1)	.48	.65	.62
Work (Exp. 1.)	.25	.38	.46
Frame (Exp. 1)	.13	.24	.73
Freeway (Exp. 1)	.12	.30	.52
TV (Exp. 2)	.08	.17	.08
Race (Exp. 2)	.04	.13	.33
Fence (Exp. 2)	.00	.08	.25
Overall Error Rates	.16	.28	.43

Note. - The seven problems listed above each consist of relation, assignment, and question propositions. The other nine problems contain no relation propositions.

Table 4

Examples of Converting a Relational Proposition to an Assignment Proposition

Problem	Number of subects who changed	Relational Proposition from Problem	Assignment Proposition from Recall Protocol
River	10 cases	"The steamer's engine drives in still water at a rate of 12 miles per hour more than the rate of the current."	"Assume it goes 12mph by motor alone (with no current.)" "In smooth water the en3ine causes it to move 12mph." "Its engines push the boat 12mph in still water."
Frame	4 cases	"The area occupied by an unframed rectangular picture is 64 square inches less than the area occupied by the picture mounted in a frame..."	"A picture has an area of 64 inches." "If an unframed rectangular picture is 64 sq. in..." "The area of an unframed picture is 64 inches."
Race	3 cases	"A Mallotti starts 4 minutes later..."	"M left at 9:04..." "Morittile...starts at 9:04." "A M______ leaves at 9:04."
Freeway	2 cases	"If the second truck travels 15 mph faster than the first truck..."	"and the other is going 15 mph..." "another leaves...going 15 mph..."
Frame	1 case	"...if it is 4 inches longer than it is wide?"	"...if it is 2 inches wide and 4 inches long."

50

51

Note. - There were 20 cases in which subjects converted a relational proposition into an assignment proposition and 1 case in which a subject converted an assignment into a relational proposition.

Table 5

Proportion of Repsponse by Type of Error for Five Propositions in River Problem

	Prop. 1	Prop. 2	Prop. 3	Prop. 4	Prop. 5
Correct Assignment - Same Variable (1. distance upstream = 36 miles)	.50*	.50*	-	.41	-
Modified Assignment - Different Variable (1. rate upstream = 36 mph)	.46	.46	-	-	-
Correct Relation - Same Variables (4. rate in still water = 12 mph+ rate of current)	-	-	.38*	.33*	-
Correct Question - Same Variable (5. rate of current = FIND)	-	-	-		.67*
Modified Question - Different Variable (5. rate of boat = FIND)	-	-	-		.21
Omit	.04	.04	.62	.25	.12

Note. - Asterisk (*) indicates correct answer.

Table 6

Proportion of Response by Type of Error for Three Numbers in River Problem

	36	24	12
Correct	.79*	.75*	.79*
Different Value	.17	.12	.04
Omit	.04	.12	.17

Note. – Asterisk (*) indicates correct answer.

Table 7

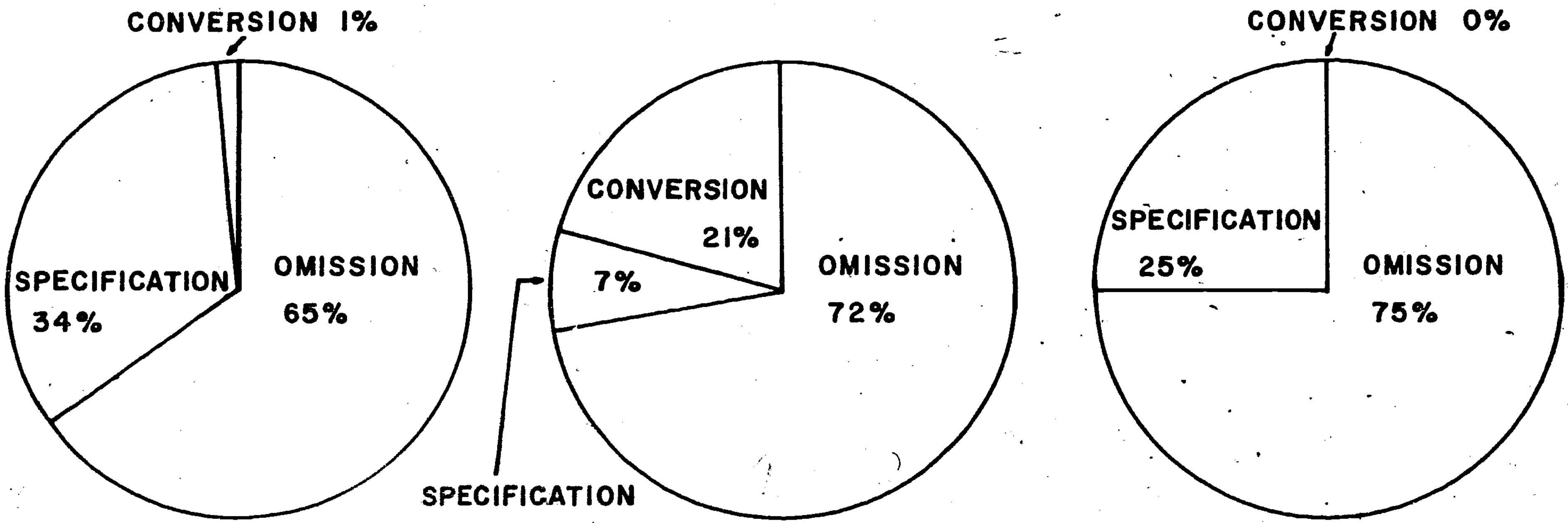
Keywords Used for 16 Problems - Experiment 3

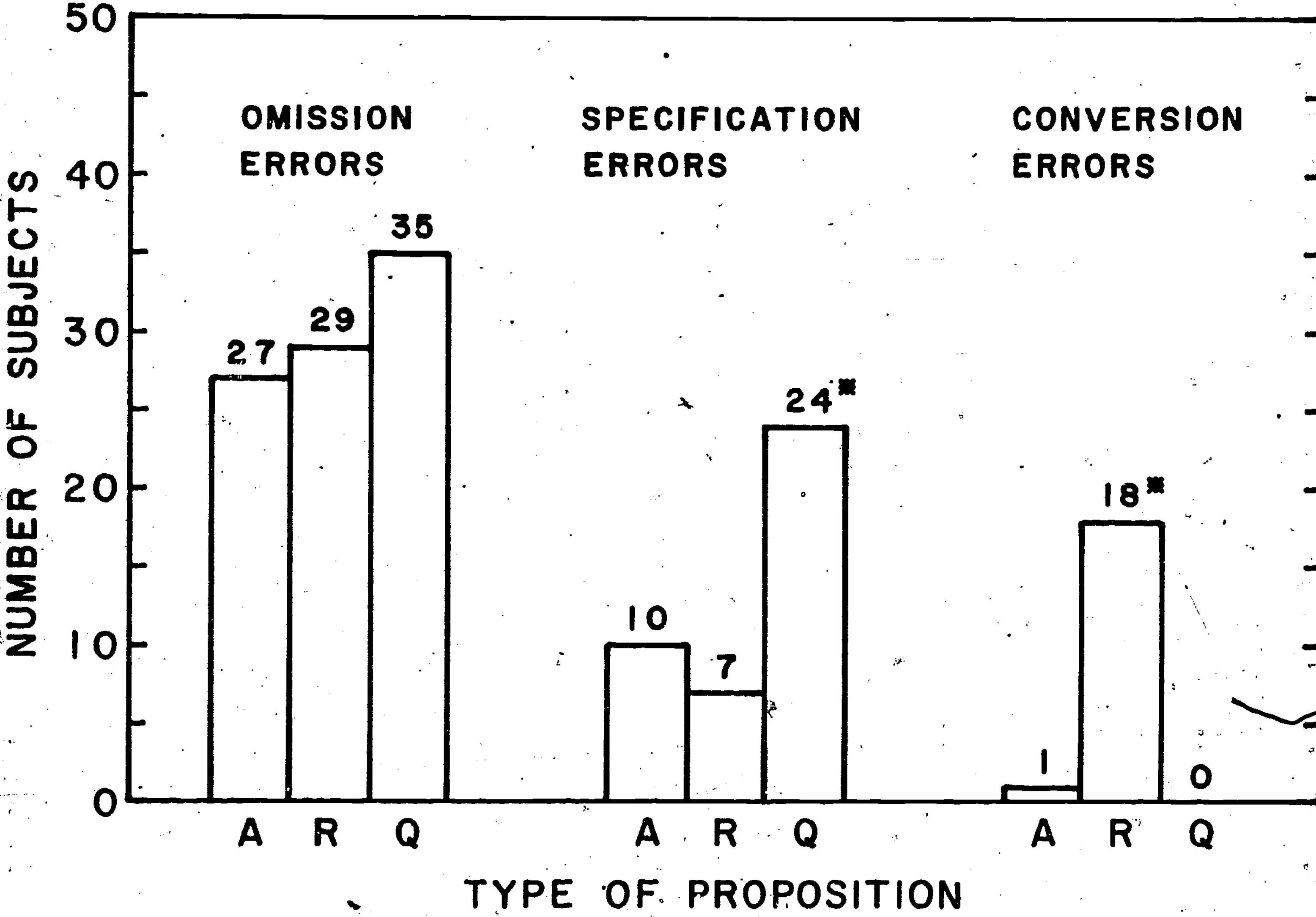
Title	Keywords
Mixture	an oil and water solution, another oil and water solution, a mixture of the two solutions
Work	one person working on first part of task, another person working on remainder of task
Will	a person's will, bequest left to one relative, bequest left to another relative, a person's estate at time of death
Age	ages, one person, another person
Frame	rectangular picture, rectangular frame and picture together, frame around picture
Freeway	first truck leaving one city, second truck leaving another city
Camp Trip	camping trip, hiking to campsite, riding back from campsite
River	boat trip upstream, boat trip downstream, river current
Store	one brand of candy, another brand of candy, gift box mix of the two candies
Pipes	one pipe, another pipe, a tank
TV	cord divided into two pieces, first part of cord, second part of cord
Investment	investment, part of the money, remainder of the money, annual income
Fence	rectangular lot, chain fencing
Airways	airplane trip, first part of trip, remainder of trip
Race	car race, first car, second car
Coins	ferry passengers, adult tickets, child tickets

ASSIGNMENT PROPOSITIONS (A) (n=68)

RELATION PROPOSITIONS (R) (n=102)

QUESTION PROPOSITIONS (Q) (n=157)




NUMBER OF SUBJECTS
50
40
30
20
10
0
OMISSION ERRORS
SPECIFICATION ERRORS
CONVERSION ERRORS
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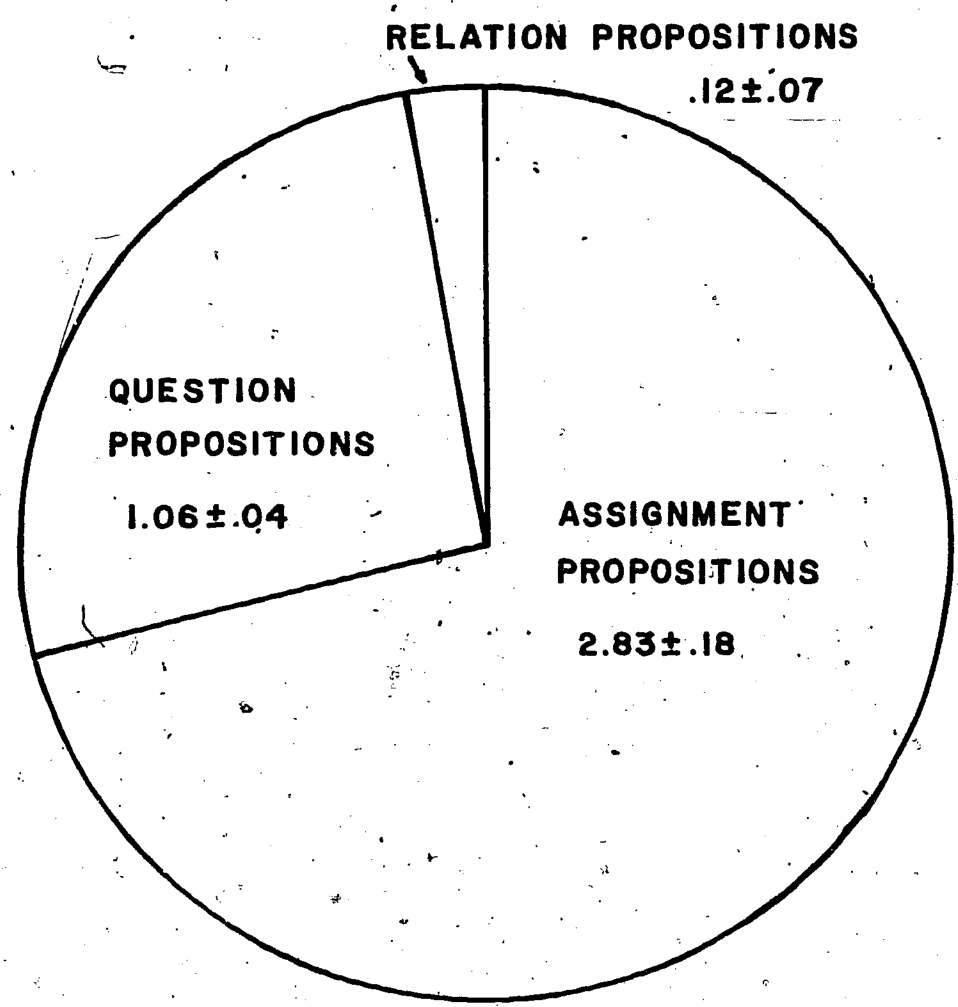
29
35
10
7
24*
1
18*
0
A
R
Q
A
R
Q
A
R
Q
TYPE OF PROPOSITION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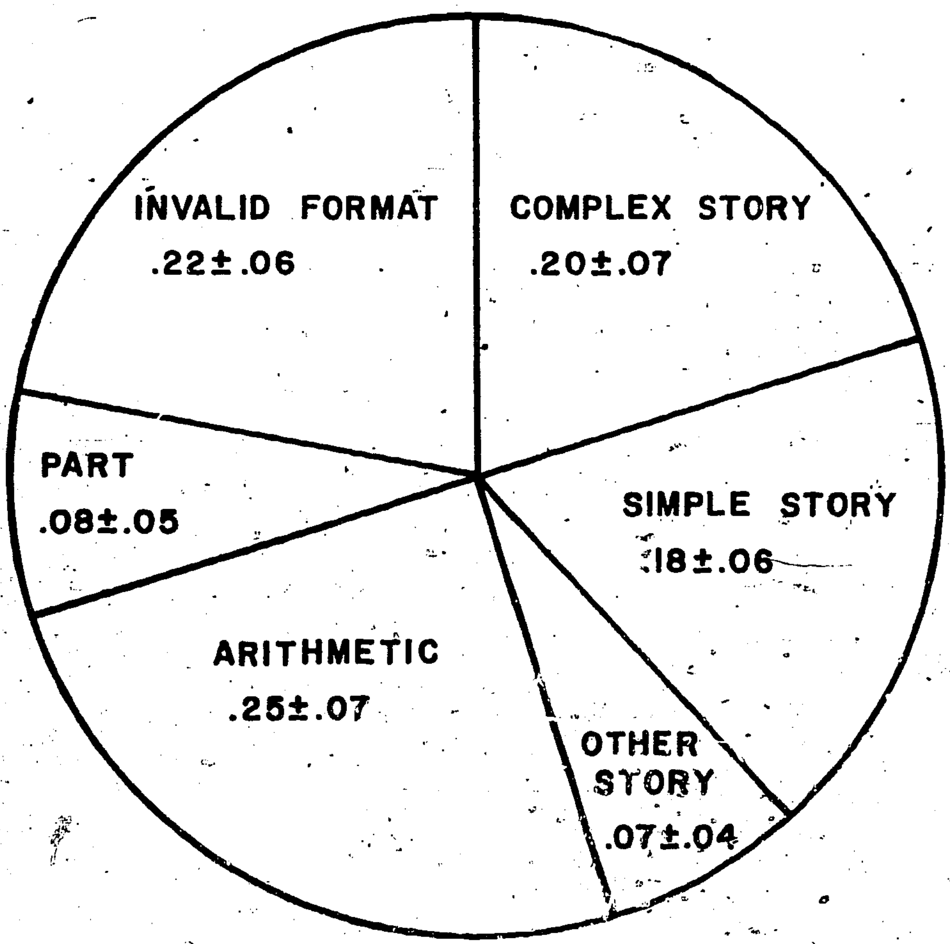
QUESTION
PROPOSITIONS
1.06±.04
ASSIGNMENT
PROPOSITIONS
2.83±.18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COMPLEX STORY
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PART
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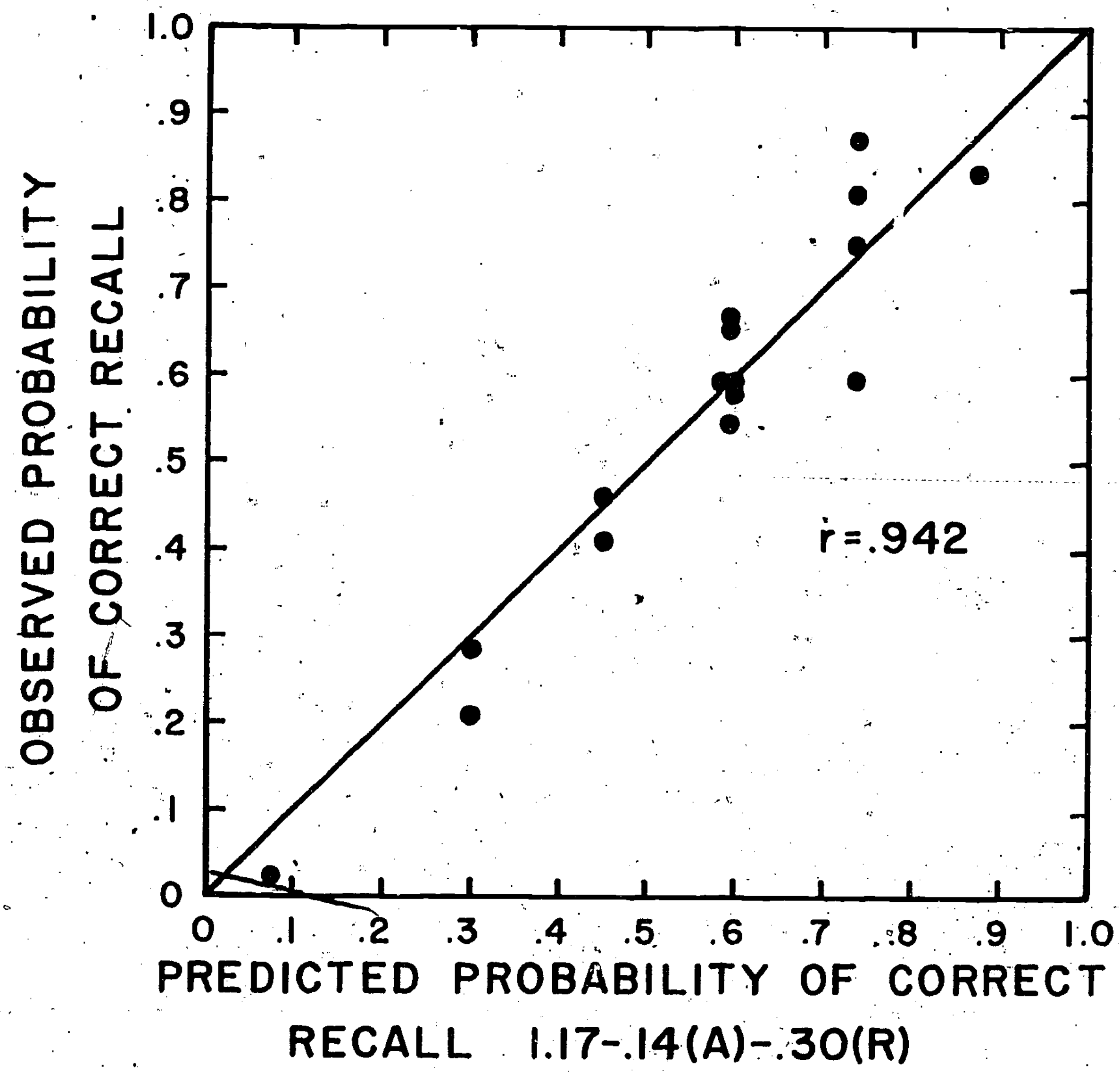
OTHER
STORY
.07±.04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OF CORRECT RECALL
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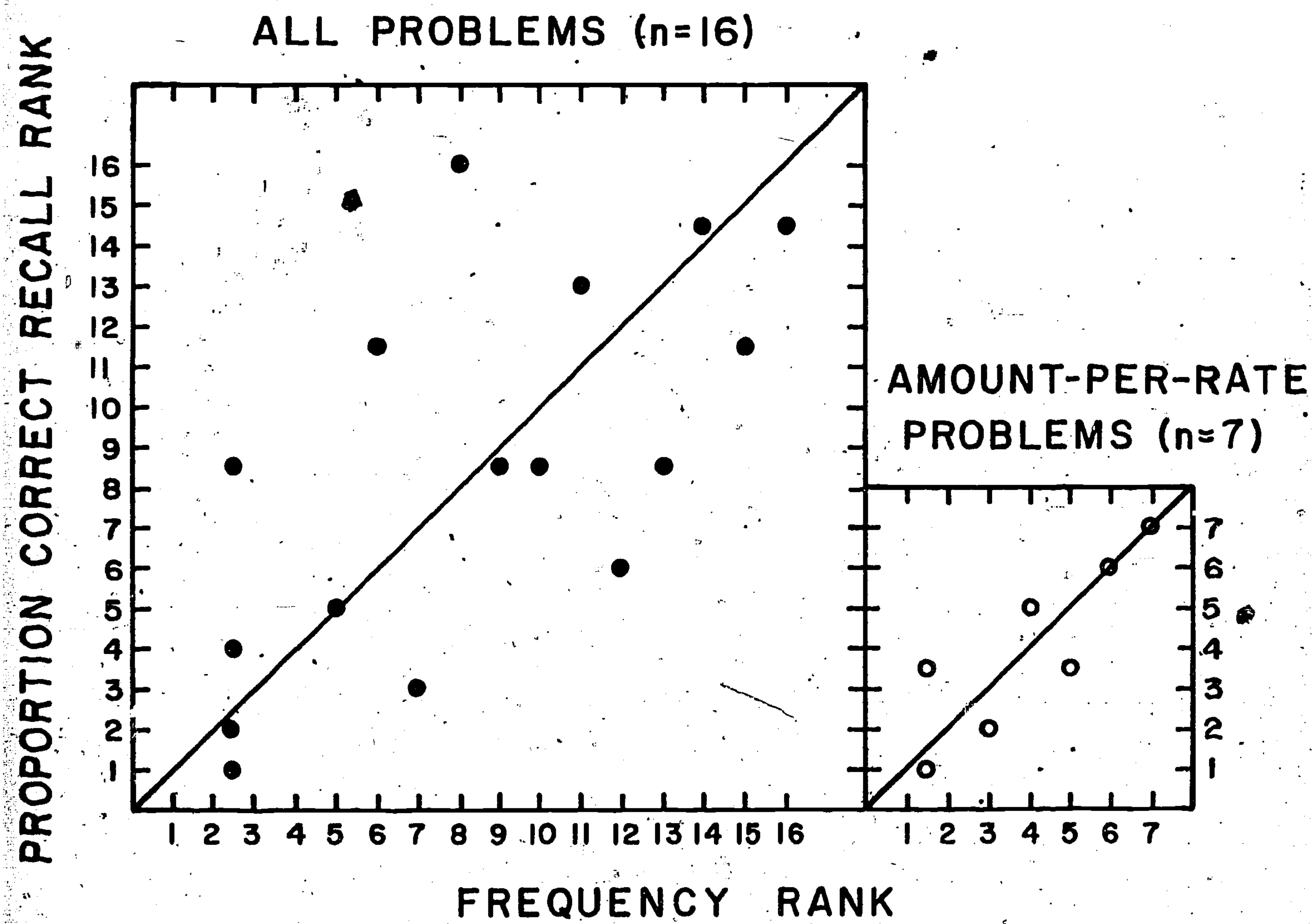
.8
.7
.6
.5
.4
.3
.2
.1
0
r=.942
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
PREDICTED PROBABILITY OF CORRECT
RECALL 1.17-.14(A)-.30(R)


ALL PROBLEMS (n=16)
AMOUNT-PER-RATE PROBLEMS (n=7)
PROPORTION CORRECT RECALL RANK
FREQUENCY RANK
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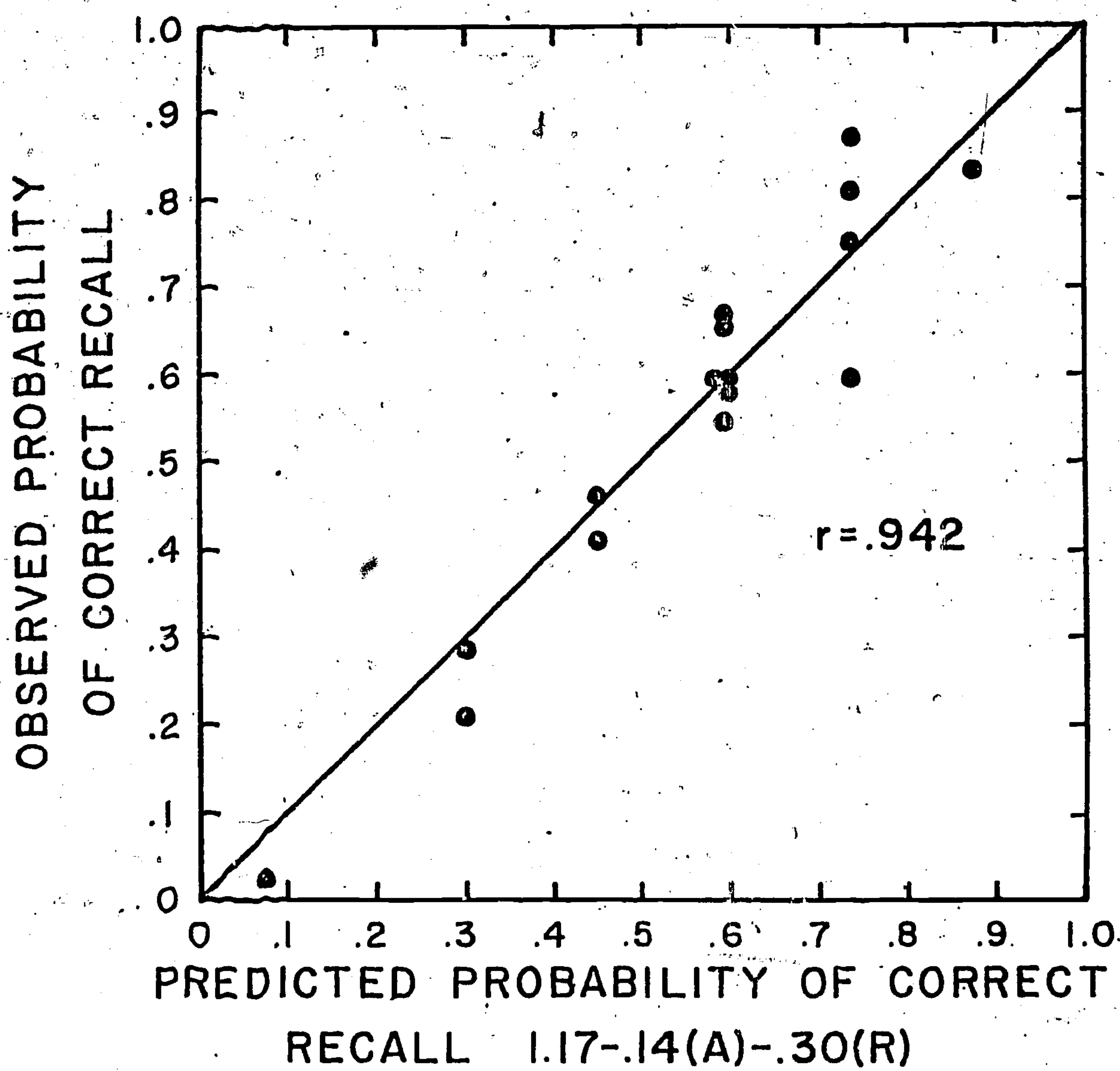
1 2 3 4 5 6 7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0
.1
.2
.3
.4
.5
.6
.7
.8
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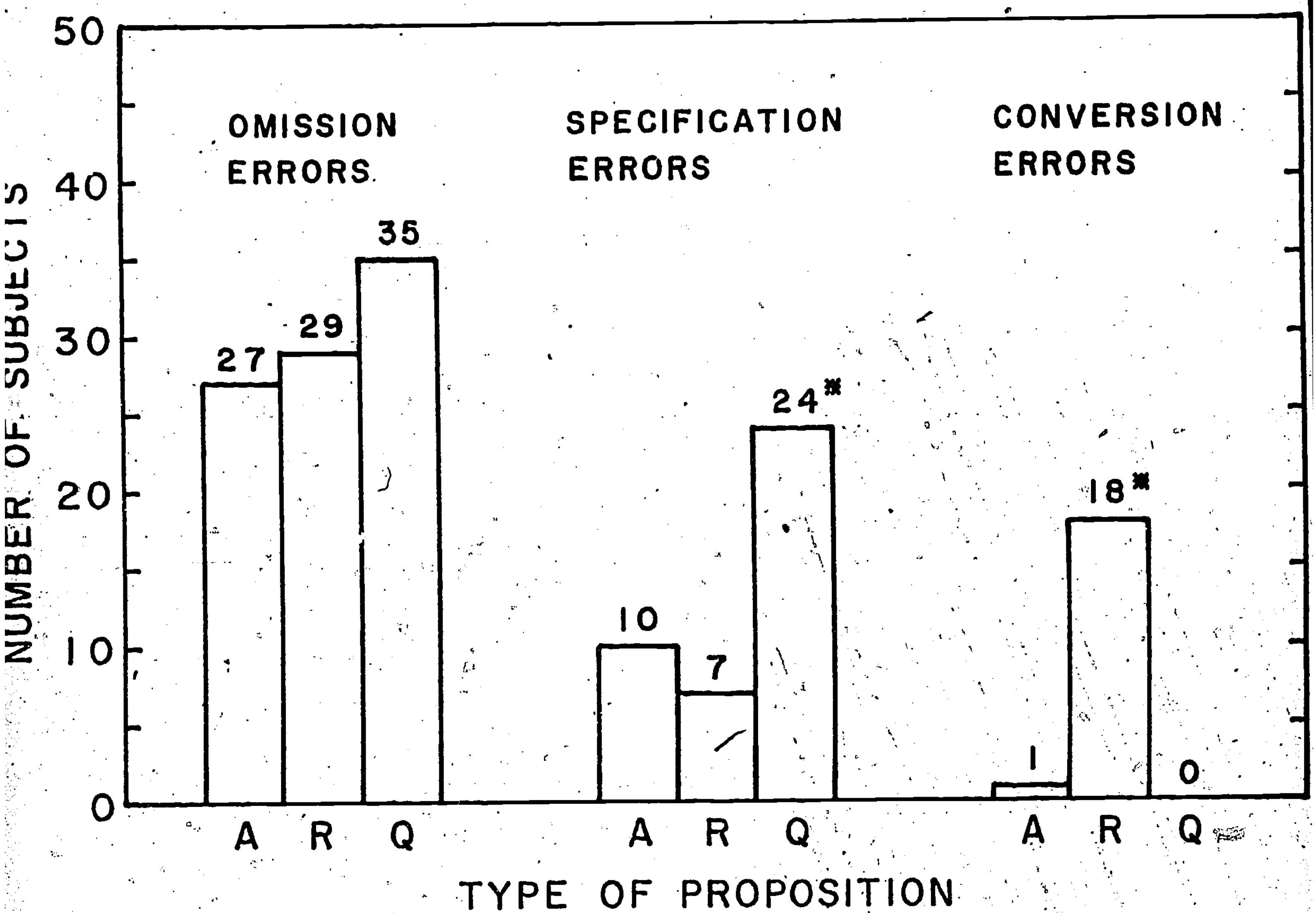
r = .942
PREDICTED PROBABILITY OF CORRECT RECALL 1.17-.14(A)-.30(R)

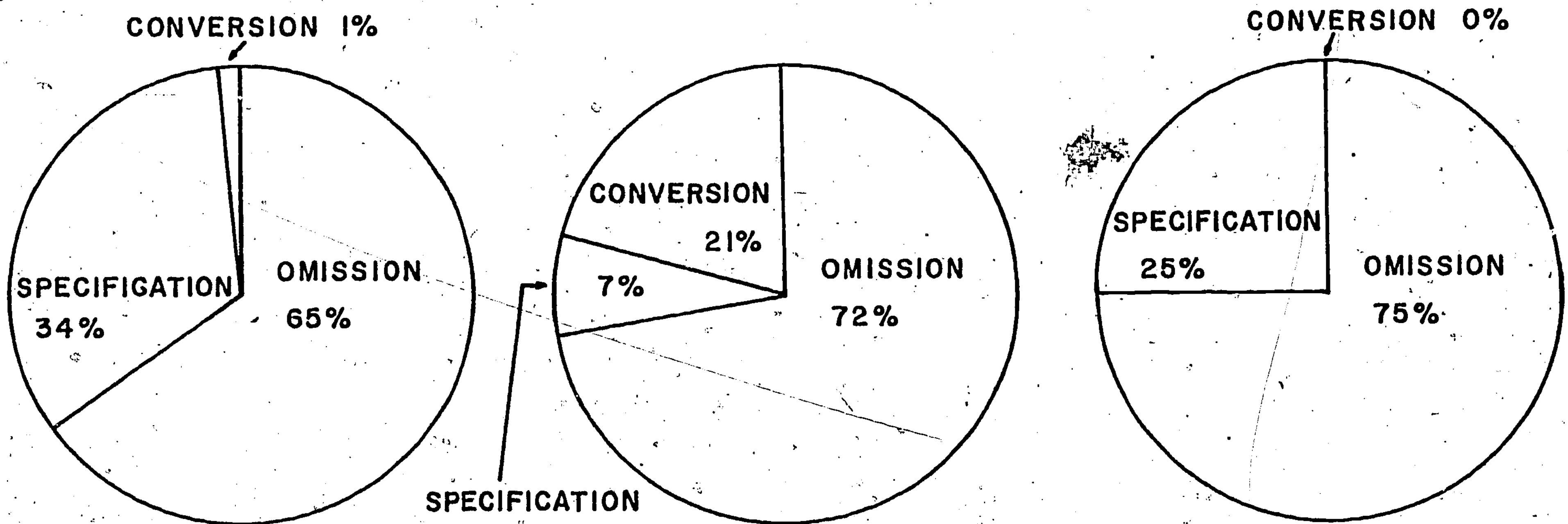


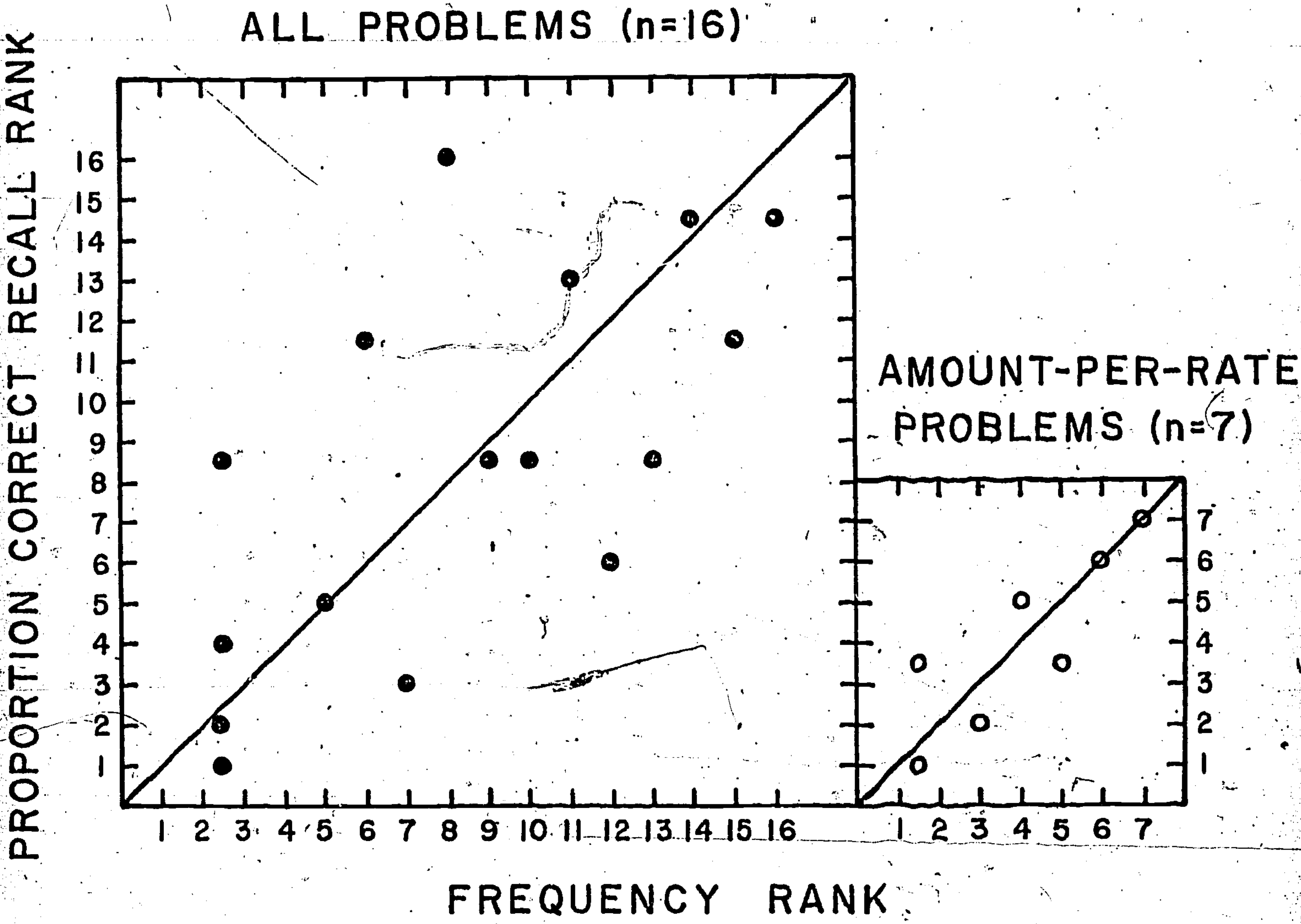
NUMBER OF SUBJECTS
50
40
30
20
10
0
OMISSION ERRORS
SPECIFICATION ERRORS
CONVERSION ERRORS
27
29
35
10
7
24*
1
18*
0
A
R
Q
A
R
Q
A
R
Q
TYPE OF PROPOSITION

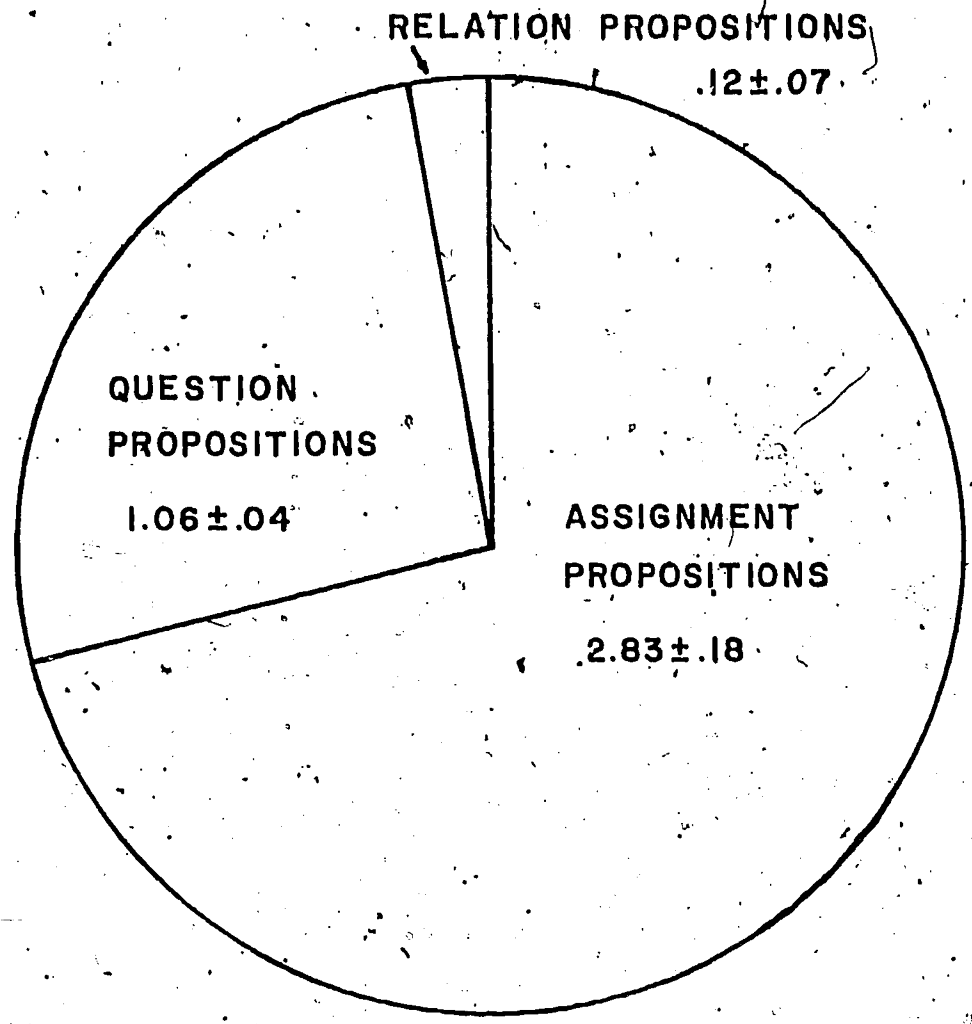
ASSIGNMENT PROPOSITIONS (A) (n=68)

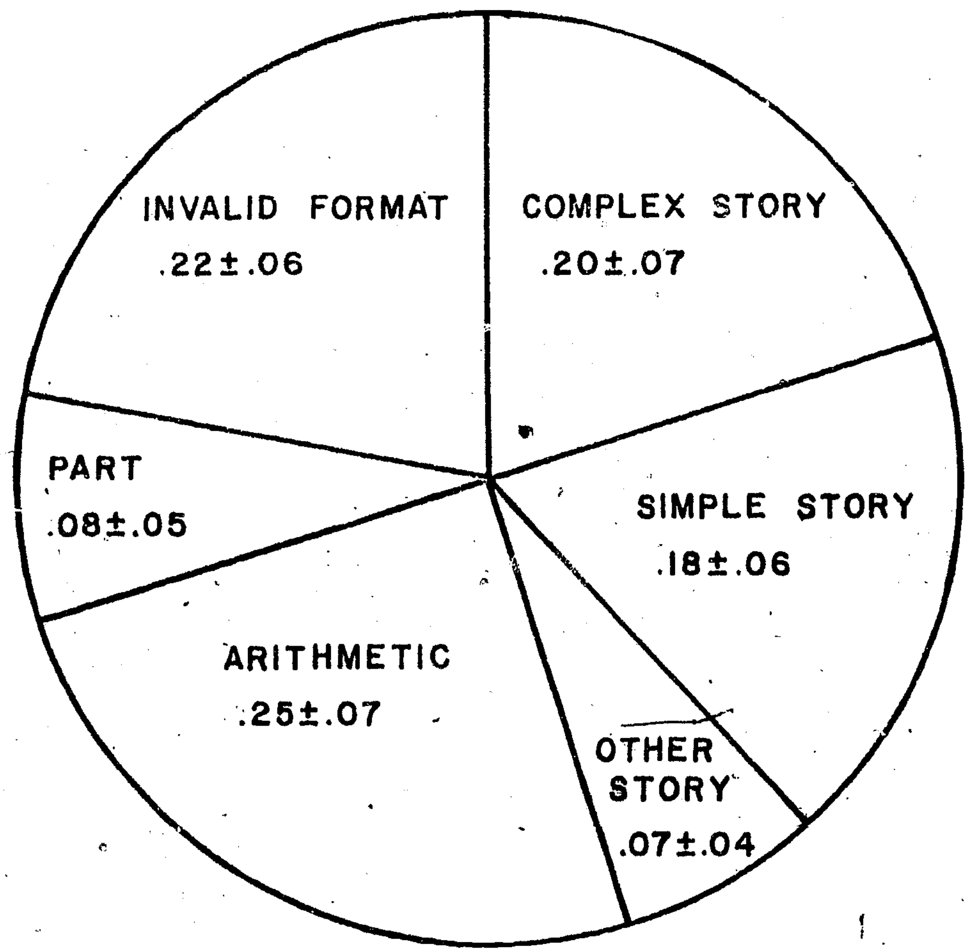
RELATION PROPOSITIONS (R) (n=102)

QUESTION PROPOSITIONS (Q) (n=157)




ALL PROBLEMS (n=16)
AMOUNT-PER-RATE PROBLEMS (n=7)
PROPORTION CORRECT RECALL RANK
FREQUENCY RANK
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16
1 2 3 4 5 6 7


RELATION PROPOSITIONS
.12±.07
QUESTION PROPOSITIONS
1.06±.04
ASSIGNMENT PROPOSITIONS
2.83±.18


INVALID FORMAT
.22±.06
COMPLEX STORY
.20±.07
PART
.08±.05
SIMPLE STORY
.18±.06
ARITHMETIC
.25±.07
OTHER
STORY
.07±.04

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