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AUTHOR	Connor, Jane M.; Serbin, Lisa A.	
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ABSTRACT

Several studies that were directed towards examining the influence of cognitive factors in sex-related differences in mathematics achievement are reported. Specifically, two lines of research were pursued examining: (1) the relationship between different types of visual-spatial skill and mathematics achievement, and (2) the trainability of visual-spatial skill in junior high school students. Part I of this report discusses two studies that examine the relationship between mathematics achievement (including computation, algebra, and geometry) and visual-spatial skill. The results of the two studies indicated that visual skill and spatial orientation skill are somewhat distinct and both contribute to predicting mathematics achievement. Further research examining the development and trainability of these skills is promoted. Part II focuses on visual-spatial training research that was carried on in 1977-1980. The 1979-1980 investigation is the focus, with the two earlier years of work ending with negative results. This earlier work is documented in the hope that others may profit in reading about these experiences and conclusions. (MP)

MATHEMATICS, VISUAL-SPATIAL ABILITY, AND SEX-ROLES

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Jane M. Connor
State University of New York at Binghamton

and

Lisa A. Serbin
Concordia University

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MATHEMATICS, SPATIAL ABILITY, AND SEX-ROLES

INTRODUCTION

The fact that large numbers of women avoid mathematics in the latter years of high school and on some measures don't achieve as well as males even when they continue with mathematics (Fennema & Sherman, 1977) has been of increasing concern to educators, psychologists, sociologists, and other students of human behavior. This deficit in mathematics education has serious implications for the career options available to both women who attend four-year colleges and to those who choose to enroll in more vocationally oriented programs (Sells, 1973).

A variety of potential sources for this deficit have been examined by researchers working under the recent NIE program on Women and Mathematics and by others. These include the role played by mathematics anxiety (Tobias & Donady, 1976), sexist wording of mathematics problems and text books (Carey, 1958; Graf & Ruddell, 1972), lack of parental encouragement (Fox, 1975), teacher influences (Ernest, 1976; Fennema, 1976; Pederson, Shinedling, & Johnson, 1975), and attitudes toward mathematics (Fennema & Sherman, 1977).

The present research was directed towards examining the influence of cognitive factors in sex-related differences in mathematics achievement. More specifically, two lines of research were pursued examining: (1) the relationship between different types of visual-spatial skill and mathematics achievement, and (2) the trainability of visual-spatial skill in junior high school students.

PART I

THE RELATIONSHIP BETWEEN VISUAL-SPATIAL SKILL AND MATHEMATICS ACHIEVEMENT

Several types of evidence suggest an important role of visual-spatial skill in mathematics achievement and for understanding sex-related differences in mathematics achievement. Developmentally, sex-related differences in visual-spatial test performance and mathematics achievement appear to emerge at roughly the same age, 12 - 14 years (Maccoby & Jacklin, 1974). In correlational studies visual-spatial skill is frequently found to be a significant predictor of mathematics achievement. Fennema and Sherman (1977), for example, obtained significant correlations between math achievement and a spatial-visualization measure in four secondary schools. These correlations tended to be higher than correlations between math achievement and general intelligence (verbal) measure. Further evidence is furnished by I. Macfarlane Smith in his book Spatial Ability: Its Educational and Social Significance, in which he describes a great many studies that demonstrate the importance of spatial ability for success in mathematics.

Lastly, a logical analysis of the nature of visual-spatial ability and of mathematics achievement indicates that they should bear an important relationship to one another. Visual-spatial ability is a cognitive skill involving the ability to perceive spatial relationships and to mentally manipulate visual material. According to several mathematicians, the nature of mathematical thinking is highly dependent on such a cognitive skill. Hamley, a mathematician and psychologist, states that, "Mathematical ability is probably a compound of general intelligence, visual imagery, ability to perceive number and space configurations and to retain such configurations as mental patterns" (Smith, 1964). Another mathematician, Meserve, notes the extensive use of geometrical models in all areas of mathematics and says that "...geometrical thinking must retain some link... with spatial intuition" (Fennema, 1976).

Despite the findings relating visual-spatial skill to mathematical achievement, research in this area is complicated by the fact that neither the domain of skills represented by the term "visual-spatial" nor the domain represented by the term "mathematics achievement" is uni-dimensional. Potentially certain types of visual-spatial skills may be related to certain types of mathematics problems whereas other types of visual-spatial skills may be totally unrelated to any aspect of mathematics achievement.

The two studies that follow were designed to examine the relationship between mathematics achievement (including computation, algebra, and geometry) and visual-spatial skill. The visual-spatial tests that were used included tests from the subdivision of skills referred to as "spatial orientation-visualization" and from the subdivision referred to as "closure" (Ekstrom, French, Harman & Dermen, 1976). In this way mathematics achievement could be related to specific types of visual-spatial skills.

Study 1

Subjects

Research participants were the entire seventh and tenth grades of a suburban-rural school district, almost all of whom were white. This included 134 seventh graders (71 male, 63 female) and 205 tenth graders (108 male, 97 female).

Testing materials

Three sets of measures were obtained from each grade level. These included: (a) Six measures of visual-spatial skill and a measure of verbal skill, (b) a mathematics achievement test formulated for this project, and (c) various standardized test scores and school grades obtained from school records. Each of these sets of measures is now discussed in detail.

The visual-spatial tests. Five visual-spatial tests were selected from the Educational Testing Service Kit of Factor Referenced Tests (Ekstrom, et. al., 1976). The Cube Comparisons test consists of 21 problems each one of which is a picture of two cubes with letters on the three visible faces of the cubes. The subject must decide which of the two pictures represents images of the same three-dimensional cube. This test is thought to be a measure of the skill of Spatial Orientation. The Card Rotations Test is also thought to be a measure of the same skill. This test consists of 80 problems for each of which the subject must decide if a given symbol can be rotated in a two-dimensional plane to match another symbol. For the Hidden Patterns test, a measure of flexibility of closure, the subject must decide if a shape which looks somewhat like an upside-down Y with an extra line attached is or is not embedded in each of 200 line drawings. The Gestalt Completion Test, a measure of speed of closure consists of 10 incomplete black and white drawings. The subject tries to determine what each is a picture of. The Paper Folding Test, a measure of visualization skill contains 10 problems each one depicting how a piece of paper is folded in a specific way and then a hole punched in the folded paper. The task is to decide how the holes would appear on the paper if it were unfolded. The vocabulary test that was used was also taken from the ETS kit and consisted of 18 vocabulary items. The sixth visual-spatial test that was used was an abbreviated version (only 10 items) from the Space Relations part of the Differential Aptitude Test (Bennett, Seashore, & Ivesman, 1973). Each item consists of a drawing of a two dimensional shape which if folded along indicated lines could make a three-dimensional shape. The subject must choose from four alternatives which shape this would be. The test involves both the skills of spatial orientation and visualization.

The mathematics test. This test consisted of 48 problems adapted from a number of standardized mathematics tests. The purpose of the adaptation was to construct a test which included 16 problems each from the areas of arithmetic, algebra, and geometry. The classification of problems was determined by agreement of at least four out of five mathematics majors as to the content area of each problem used. Separate tests appropriate for seventh and tenth graders were so constructed.

Standardized scores and school grades. With the use of coded subject numbers to ensure confidentiality, the following information from subjects' permanent school records was obtained:

Seventh graders

a. Sixth-grade results on the Stanford Achievement Tests--Vocabulary, Reading Comprehension, Word Studies, Math Concepts, Math Computation, Math Applications, Spelling, Language, Social Science, and Science

b. Sixth-grade I.Q., New York State Reading, and New York State Mathematics scores

c. Sixth-grade English and mathematics grades, first-quarter seventh-grade English and mathematics grades

Tenth graders

a. Sixth-grade results on the Stanford Achievement Tests--Word meaning, Paragraph meaning, Spelling, Language, Arithmetic Computation, Arithmetic Concepts, Arithmetic Applications, Social Studies, and Science

b. Sixth-grade I.Q., third grade New York State Reading and sixth-grade New York State mathematics scores

c. Ninth-grade English and mathematics grades, first-quarter tenth-grade English and mathematics grades

Results

Sex differences in Mean Performance. The means, standard deviations, sample sizes, and F-ratios for all measures for seventh and tenth graders are shown in Tables 1 and 2.

Among the seventh graders females performed significantly better than males on the SAT Word Studies section, on the SAT Language section, and in sixth and seventh grade English class ($p < .01$). They also tended ($p < .10$) to do better on the SAT Spelling section, the SAT Math Concepts section, and in sixth grade math class. Seventh grade males tended to do better than females on the DAT Space Relations Test. There were no significant differences between seventh grade males and females on any of the ETS factor-referenced cognitive tests.

Among the tenth graders males performed significantly better than females on the Geometry subscale of the math achievement test ($p < .05$) and on the following Stanford Achievement Tests: Arithmetic Applications, Social Studies,

TABLE 1

MEANS AND STANDARD DEVIATIONS FOR ALL MEASURES
SEVENTH GRADE STUDENTS

Test	Males $\bar{X}$	Males S.D.	Males N	Females $\bar{X}$	Females S.D.	Females N	F-Ratio
Mathematics Achievement Tests							
Geometry	6.89	2.47	71	6.79	2.35	63	< 1
Algebra	7.56	2.66	71	7.90	2.81	63	< 1
Arithmetic	8.77	3.11	71	9.62	2.93	63	1.73
ETS Tests							
Cube comparisons	2.98	3.97	64	2.87	4.18	55	< 1
Hidden patterns	62.28	29.37	64	62.91	28.20	55	< 1
Gestalt completion	4.78	1.88	64	5.09	1.91	55	< 1
Paper folding	3.45	2.65	64	3.46	2.57	55	< 1
Vocabulary	3.88	3.12	64	3.62	2.53	55	< 1
Card rotations	38.06	18.19	64	37.40	16.39	55	< 1
DAT Test							
Space relations	5.63	2.50	64	4.81	2.40	55	3.31*
SAT Tests							
Vocabulary	57.74	24.75	57	60.04	21.65	53	< 1
Reading comprehension	60.12	26.56	57	63.81	22.31	53	1.1
Word study	62.30	28.41	56	72.29	19.41	52	4.48**
Math concepts	66.30	23.75	56	72.62	18.63	53	2.36
Math computation	63.89	22.39	56	68.91	21.06	53	1.78
Math application	65.75	22.38	56	61.66	21.27	53	< 1
Spelling	52.46	29.65	57	59.77	23.86	53	2.43
Language	56.46	24.56	57	66.28	21.26	53	5.70**
Social Sciences	62.75	25.19	57	68.13	18.56	53	1.93
Science	67.39	24.70	57	66.96	20.46	53	< 1

* $p < .10$
** $p < .05$

(This table is continued on the next page.)

TABLE 1 (CONT.)

Test	Males			Females			
	$\bar{X}$	S.D.	N	$\bar{X}$	S.D.	N	F-Ratio
Intelligence Quotient	108.27	13.91	55	111.00	10.15	52	< 1
New York State Tests							
Reading	57.09	23.54	54	61.60	19.18	53	< 1
Mathematics	54.89	23.23	54	53.77	17.15	53	< 1
School Grades							
English - 6th grade	81.18	8.78	55	85.22	5.41	51	7.54**
Mathematics - 6th grade	78.36	10.96	55	81.37	8.88	51	2.49
English - 7th grade	74.12	11.65	60	84.43	8.93	56	25.83**
Mathematics - 7th grade	78.02	12.00	60	82.63	8.99	56	3.42*

* $p < .10$
** $p < .05$

TABLE

MEANS AND STANDARD DEVIATIONS FOR ALL MEASURES
TENTH GRADE STUDENTS

Test	Males $\bar{X}$	Males S.D.	Males N	Females $\bar{X}$	Females S.D.	Females N	F-Ratio
Mathematics Achievement Tests							
Geometry	7.15	5.74	108	5.46	2.81	97	5.52 **
Algebra	5.63	2.63	108	5.05	2.48	97	1.29
Arithmetic	8.19	3.48	108	8.09	2.70	97	< 1
ETS Tests							
Cube comparisons	6.52	5.91	102	6.48	5.77	89	< 1
Hidden patterns	70.94	37.54	102	69.04	35.71	89	< 1
Gestalt completion	7.19	1.68	102	7.18	1.74	89	< 1
Paper folding	4.15	2.58	102	4.39	2.58	89	< 1
Vocabulary	7.30	3.73	102	7.06	4.42	89	< 1
Card rotations	49.64	20.91	102	47.78	21.43	89	< 1
DAT Test							
Space relations	7.47	2.08	102	6.83	2.95	89	3.13 *
Sixth-grade Stanford Achievement							
Word meaning	50.23	26.49	91	44.68	28.83	71	1.30
Paragraph meaning	49.58	27.84	91	46.86	27.11	70	< 1
Spelling	37.96	26.33	91	46.34	25.97	70	4.06 **
Language	36.48	23.50	91	42.59	24.14	70	2.60
Arithmetic computation	23.34	19.01	91	20.72	14.31	71	< 1
Arithmetic concepts	49.89	27.02	91	42.99	21.76	70	3.05 *
Arithmetic application	48.60	28.45	91	30.14	23.43	70	19.37 **
Social studies	46.79	27.91	91	37.63	25.64	71	4.09 **
Science	49.76	29.18	91	40.01	24.15	71	4.66 **
TOTAL	57.02	13.70	91	56.23	11.40	70	< 1

* $p < .10$
** $p < .05$

(This table is continued on the next page.)

TABLE (CONT.)

Test	Males $\bar{X}$	Males S.D.	Males N	Females $\bar{X}$	Females S.D.	Females N	F-Ratio
Intelligence Quotient	109.02	14.69	93	107.97	12.27	71	< 1
New York State Tests							
Reading	51.78	22.34	81	56.97	25.08	63	1.99
Mathematics	48.10	25.67	91	39.37	22.05	71	1.89
School Grades							
English - 9th grade	77.35	10.94	98	78.98	11.42	83	1.54
Mathematics - 9th grade	78.09	11.42	96	78.46	10.55	82	< 1
English - 10th grade	77.71	14.26	97	78.53	11.96	83	1.24
Mathematics - 10th grade	81.07	13.05	69	77.92	12.79	52	1.50

and Science. Males also tended to perform better on the abridged DAT Space Relations Test, on the SAT Arithmetic Concepts section, and on the New York State Mathematics Test.

Females performed significantly better than males on the Spelling section of the SAT ($p < .05$) and tended to do better on the SAT Language section and New York State Reading Test ($p < .10$). There were no significant differences between tenth grade males and females on any of the ETS factor-referenced tests.

Factor analysis. In order to understand the relations among the large number of measures obtained in this study, separate factor analyses (principal components type) were completed on the seventh and tenth grade data.

The tenth grade data set appeared to contain five distinct factors which accounted for 66% of the variance (see Table 3). Factor 1 had high loadings on the ETS Vocabulary Test, SAT Word Meaning, SAT Paragraph Meaning, SAT Spelling, SAT Language, SAT Social Studies, SAT Science, I.Q., and New York State Reading. Factor 1 was clearly a verbal-general intelligence factor. Factor 2 had high loadings on the Geometry, Algebra, and Arithmetic subscales, SAT Arithmetic Computation, SAT Arithmetic Concepts, SAT Arithmetic Applications, and New York State Mathematics. Factor 2 was clearly a mathematical ability factor. Factor 3 had high loadings on grades obtained in English and mathematics in the ninth and tenth grades. This appeared to be a school achievement factor. The visual-spatial tests divided into two factors. (In a 3-factor solution, shown in Table 4, they were all heavily loaded on the same factor which was distinct from the verbal and mathematics factors.) The Cube Comparison Test, Paper Folding Test, Card Rotations Test, and DAT Space Relations Test had high loadings

TABLE 3

PRINCIPAL COMPONENTS ANALYSIS - FACTOR LOADINGS
TENTH GRADE DATA
5 - FACTOR SOLUTION

Test	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Mathematics Achievement Tests					
Geometry	-0.01	0.75	0.00	0.11	0.11
Algebra	0.30	0.57	0.18	0.07	0.21
Arithmetic	0.14	0.76	0.04	0.17	0.25
ETS Tests					
Cube comparisons	0.25	0.19	0.06	0.70	-0.02
Hidden patterns	0.22	0.18	0.19	0.06	0.63
Gestalt completion	0.02	0.06	-0.10	0.16	0.70
Paper folding	0.02	0.21	0.14	0.72	0.14
Vocabulary	0.67	-0.03	0.13	0.33	0.05
Card rotations	0.32	0.15	-0.05	0.50	0.47
DAT Test					
Space relations	0.08	0.04	0.21	0.68	0.16
Sixth-grade Stanford Achievement					
Word meaning	0.78	0.23	0.23	0.18	-0.04
Paragraph meaning	0.78	0.37	0.20	0.15	0.10
Spelling	0.78	0.16	0.10	-0.11	0.21
Language	0.74	0.32	0.13	0.12	0.14
Arithmetic computation	0.36	0.58	0.16	-0.04	0.11
Arithmetic concepts	0.41	0.63	0.19	0.22	-0.09
Arithmetic application	0.46	0.66	0.25	0.24	-0.17
Social studies	0.63	0.47	0.26	0.32	-0.05
Science	0.67	0.39	0.16	0.25	-0.04

(This table is continued on the next page.)

TABLE 3 (CONT.)

Test	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
Intelligence Quotient	0.61	0.38	0.25	0.29	0.04
New York State Tests					
Reading	0.86	0.09	0.12	0.05	0.13
Mathematics	0.50	0.59	0.29	0.27	-0.05
School Grades					
English - 9th grade	0.45	0.11	0.70	0.09	0.15
Mathematics - 9th grade	0.15	0.14	0.76	0.23	0.17
English - 10th grade	0.56	0.09	0.42	0.03	0.22
Mathematics - 10th grade	0.22	0.22	0.80	0.12	-0.17

TABLE 4

PRINCIPAL COMPONENTS ANALYSIS - FACTOR LOADINGS
TENTH GRADE DATA
3 - FACTOR SOLUTION

Test	Factor 1	Factor 2	Factor 3
Mathematics Achievement Tests			
Geometry	-0.06	0.68	0.17
Algebra	0.31	0.63	0.20
Arithmetic	0.10	0.69	0.31
ETS Tests			
Cube comparisons	0.19	0.35	0.50
Hidden patterns	0.31	0.08	0.46
Gestalt completion	0.02	-0.10	0.60
Paper folding	0.02	0.36	0.60
Vocabulary	0.65	0.06	-0.32
Card rotations	0.27	0.13	0.70
DAT Test			
Space relations	0.11	0.20	0.58
Sixth-grade Stanford Achievement			
Word meaning	0.77	0.41	0.12
Paragraph meaning	0.77	0.40	0.20
Spelling	0.78	0.10	0.09
Language	0.72	0.32	0.21
Arithmetic computation	0.37	0.54	0.05
Arithmetic concepts	0.39	0.69	0.10
Arithmetic application	0.45	0.76	0.06
Social studies	0.62	0.57	0.21
Science	0.64	0.47	0.18

(This table is continued on the next page.)

TABLE 4 (CONT.)

Test	Factor 1	Factor 2	Factor 3
Intelligence Quotient	0.62	0.47	0.24
New York State Tests			
Reading	0.84	0.10	0.15
Mathematics	0.51	0.68	0.16
School Grades			
English - 9th grade	0.66	0.25	0.10
Mathematics - 9th grade	0.39	0.34	0.12
English - 10th grade	0.68	0.14	0.14
Mathematics - 10th grade	0.44	0.45	-0.12

on Factor 4. This factor represents what ETS has referred to as the "spatial-visualization" subdivision of visual-spatial ability. Factor 5 had high loadings on Hidden Patterns, Gestalt Completion, and Card Rotations. This factor represents what ETS has referred to as the "closure" subdivision of visual-spatial ability.

The results of the factor analyses for the seventh graders were similar to those for the tenth graders but the factor structures were not as sharply outlined. The clearest solution was the one with 3-factors which accounted for 51% of the variance. As seen in Table 5, in this solution several mathematics measures loaded more highly than the ETS Vocabulary Test on the verbal-general intelligence factor. Similarly, several verbal measures loaded as highly as SAT Mathematics Concepts and Mathematics Applications did on the mathematics ability factor.

Correlations. In order to determine the extent to which the specific skills measured by the cognitive tests were related to mathematics achievement, univariate correlations between each of the ETS tests and each of the mathematics measures were calculated separately for boys and girls at each grade level.

a. Seventh grade boys. The ETS Card Rotations Test and DAT Space Relations Test were the overall best predictors of seventh grade boys' mathematics achievement. (See Table 6.) Card Rotations and Space Relations correlated at the .005 significance level with almost every mathematics measure and surpassed the Vocabulary Test as predictors of mathematics achievement.

The Cube Comparisons Test appeared to be a better predictor than Vocabulary on the Geometry and Arithmetic subscales, while Paper Folding was a better predictor than Vocabulary on the

TABLE 5

PRINCIPAL COMPONENTS ANALYSIS - FACTOR LOADINGS
SEVENTH GRADE DATA
3 - FACTOR SOLUTION

Test	Factor 1	Factor 2	Factor 3
Mathematics Achievement Tests			
Geometry	0.03	0.65	0.41
Algebra	0.27	0.68	0.31
Arithmetic	0.13	0.77	0.18
ETS Tests			
Cube comparisons	0.03	0.05	0.62
Hidden patterns	0.39	-0.01	0.36
Gestalt completion	0.32	-0.17	0.36
Paper folding	-0.03	0.12	0.61
Vocabulary	0.47	0.02	0.32
Card rotations	0.22	0.22	0.54
DAT Test			
Space relations	0.22	0.26	0.48
SAT Tests			
Vocabulary	0.72	0.08	0.35
Reading comprehension	0.74	0.42	0.16
Word study	0.67	0.41	0.09
Math concepts	0.57	0.55	0.24
Math computation	0.25	0.78	0.07
Math application	0.44	0.54	0.44
Spelling	0.67	0.46	-0.17
Language	0.67	0.57	-0.01
Social sciences	0.77	0.33	0.24
Science	0.73	0.40	0.23

(This table is continued on the next page.)

TABLE 5 (CONT.)

Test	Factor 1	Factor 2	Factor 3
Intelligence Quotient	0.66	0.44	0.29
New York State Tests			
Reading	0.72	0.25	0.10
Mathematics	0.26	0.71	0.21
School Grades			
English - 6th grade	0.68	0.49	-0.05
Mathematics - 6th grade	0.41	0.75	0.03
English - 7th grade	0.47	0.56	-0.28
Mathematics - 7th grade	0.27	0.76	0.01

TABLE 6

CORRELATIONS BETWEEN COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT MEASURES
SEVENTH GRADE BOYS

Test	Mathematics Achievement			SAT Tests			New York State
	Geometry	Algebra	Arithmetic	Math Concepts	Math Computation	Math Application	Mathematics
ETS Tests							
Cube comparison	0.31*	0.18	0.25*	0.25	0.87	0.30*	0.21
Hidden patterns	0.16	0.39**	0.10	0.31*	0.19	0.24	0.19
Gestalt completion	0.17	0.15	0.09	0.24	0.13	0.23	0.07
Paper folding	0.37**	0.18	-0.01	0.11	0.06	0.20	0.01
Vocabulary	0.23	0.35**	0.10	0.28*	0.21	0.35*	0.25
Card rotations	0.36**	0.46**	0.39**	0.55**	0.39**	0.62**	0.40**
DAT Test							
Paper folding	0.39**	0.42**	0.23	0.57**	0.31**	0.63**	0.55**

* $p < .05$
** $p < .01$

Geometry subscale. The Hidden Patterns Test and Gestalt Completion Test tended to be very poor visual-spatial measures for predicting mathematics achievement. Hidden Patterns had high correlations with math measures only when the Vocabulary Test did not correlate significantly with any of the mathematics measures.

b. Seventh grade girls. The only visual-spatial measure which was a generally good predictor of mathematics achievement in seventh grade girls was the Paper Folding Test. (See Table 7.) Paper Folding correlated more highly than did any of the other cognitive tests with the Geometry and Algebra subscales, SAT Mathematics Applications and the New York State Mathematics Test. None of the other visual-spatial tests were good predictors of mathematics achievement in seventh grade girls.

Among seventh grade girls, the Vocabulary Test was the best predictor of achievement on the Arithmetic subscale and on SAT Mathematics Concepts, surpassing all of the visual-spatial tests.

c. Tenth grade boys. Cube Comparisons was the overall best predictor of mathematics performance among tenth grade boys. (See Table 8.) It correlated very highly with every mathematics measure except for SAT Arithmetic Computation. The Paper Folding and Card Rotations Tests also tended to be very good predictors of mathematics achievement.

The Hidden Patterns Test, Gestalt Completion Test, and DAT Space Relations Test were relatively poor predictors of tenth grade boys' math achievement. Gestalt Completion did not correlate significantly with any of the mathematics measures.

TABLE 7

CORRELATIONS BETWEEN COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT MEASURES
SEVENTH GRADE GIRLS

Test	Mathematics Achievement			SAT Tests			New York State Mathematics
	Geometry	Algebra	Arithmetic	Math Concepts	Math Computation	Math Application	
ETS Tests							
Cube comparison	0.05	0.04	0.09	0.15	0.06	0.22	0.04
Hidden patterns	0.01	0.13	0.20	0.31*	0.17	0.21	0.19
Gestalt completion	-0.06	0.04	0.00	0.17	-0.08	0.12	-0.25
Paper folding	0.35**	0.34*	0.24	0.16	0.13	0.32*	0.29*
Vocabulary	0.16	0.24	0.43**	0.40**	0.04	0.24	0.10
Card rotations	0.09	0.22	0.21	0.20	0.11	0.11	-0.15
DAT Test							
Paper folding	0.22	0.31*	0.22	0.12	0.17	0.26	0.15

* $p < .05$
** $p < .01$

TABLE 8

CORRELATIONS BETWEEN COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT MEASURES
TENTH GRADE BOYS

Test	Mathematics Achievement			Stanford Achievement			New York State
	Geometry	Algebra	Arithmetic	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application	Mathematics
ETS Tests							
Cube comparison	0.24*	0.40**	0.31**	0.13	0.39**	0.46**	0.47**
Hidden patterns	0.19	0.38**	0.27**	0.16	0.19	0.22*	0.27*
Gestalt completion	0.07	-0.04	0.10	0.05	0.04	0.03	0.02
Paper folding	0.23*	0.35**	0.31**	0.15	0.41**	0.35**	0.41**
Vocabulary	0.14	0.29**	0.26**	0.16	0.29**	0.33**	0.37**
Card rotations	0.14	0.30**	0.36**	0.21	0.32**	0.29**	0.31**
DAT Test							
Paper folding	0.12	0.10	0.14	0.09	0.27*	0.17	0.21

* $p < .05$
** $p < .01$

d. Tenth grade girls. The Card Rotations and Cube Comparisons Tests were the best visual-spatial measures for predicting mathematics achievement in tenth grade girls. (See Table 9.) Card Rotations correlated very highly with the Algebra and Arithmetic subscales, SAT Arithmetic Computation, and the New York State Mathematics Test. Cube Comparisons correlated very highly with the Geometry and Arithmetic subscales and the New York State Mathematics Test.

The Vocabulary Test was among the best predictors of performance on SAT Arithmetic Applications, Arithmetic Concepts, Arithmetic Applications, and the New York State Mathematics Test. Hidden Patterns, Paper Folding, and DAT Space Relations were occasionally good predictors of mathematics performance, but in general only when vocabulary was a good predictor as well. The Gestalt Completion Test was the worst of the cognitive tests for predicting mathmatics achievement among tenth grade girls.

Canonical correlations. In order to determine the relationship between the set of mathematics measures as a whole and the specific skills measured by the cognitive tests (the visual-spatial and vocabulary tests), canonical correlations between these two sets of measures were calculated separately for boys and girls and for seventh and tenth graders (see Table 10).

The canonical correlation between the two sets of measures for seventh grade boys was .78, statistically significant at the .05 level. The largest canonical weights for this case were on the DAT Space Relations Test with a loading of -.71 and the SAT Mathematics Applications Test with a loading of -.79. The canonical correlation for seventh grade girls was .61, which was not statistically significant.

The canonical correlation for tenth grade boys was .73, statistically

TABLE 9

CORRELATIONS BETWEEN COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT MEASURES
TENTH GRADE GIRLS

Test	Mathematics Achievement			Stanford Achievement			New York State
	Geometry	Algebra	Arithmetic	Arithmetic Computation	Arithmetic Concepts	Arithmetic Application	Mathematics
ETS Tests							
Cube comparison	0.30**	0.23*	0.36**	0.15	0.25*	0.38**	0.40**
Hidden patterns	0.13	0.26*	0.18	0.32**	0.33**	0.11	0.36**
Gestalt completion	0.10	0.18	0.25*	0.14	0.18	0.19	0.16
Paper folding	0.25*	0.24*	0.31**	0.26*	0.14	0.22	0.33**
Vocabulary	0.24*	0.23*	0.23*	0.32**	0.33**	0.45**	0.49**
Card rotations	0.22*	0.31**	0.36**	0.39**	0.24	0.30*	0.39**
DAT Test							
Paper folding	0.14	0.21*	0.19	0.27*	0.24	0.38**	0.39**

* $p < .05$
** $p < .01$

TABLE 10

COEFFICIENTS FOR THE CANONICAL CORRELATION
BETWEEN
COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT

Group	Coefficients for the First Set		Coefficients for the Second Set	
Seventh grade boys	Mathematics Achievement Tests		ETS Tests	
	Geometry	-0.11	Cube comparisons	0.01
	Algebra	-0.31	Hidden patterns	-0.10
	Arithmetic	0.36	Gestalt completion	-0.10
	SAT Tests		Paper folding	-0.04
	Math concepts	-0.37	Vocabulary	-0.24
	Math computation	0.33	Card rotations	-0.23
	Math application	-0.79	DAT Test	
	New York State Mathematics	-0.04	Space relations	-0.71
Seventh grade girls	No significant ($p < .05$) canonical correlation was obtained.			
Tenth grade boys	Mathematics Achievement Tests		ETS Tests	
	Geometry	0.24	Cube comparisons	-0.22
	Algebra	-0.37	Hidden patterns	-0.17
	Arithmetic	-0.43	Gestalt completion	0.24
	Sixth-grade Stanford Achievement		Paper folding	-0.48
	Arithmetic computation	0.21	Vocabulary	-0.20
	Arithmetic concepts	-0.20	Card rotations	-0.47
	Arithmetic application	-0.06	DAT Test	
	New York State Mathematics	-0.49	Space relations	0.10
Tenth grade girls	Mathematics Achievement Tests		ETS Tests	
	Geometry	-0.24	Cube comparisons	0.00
	Algebra	0.15	Hidden patterns	-0.02
	Arithmetic	-0.16	Gestalt completion	0.28
	Sixth-grade Stanford Achievement		Paper folding	0.11
	Arithmetic computation	0.10	Vocabulary	0.64
	Arithmetic concepts	-0.15	Card rotations	0.23
	Arithmetic application	0.59	DAT Test	
	New York State Mathematics	0.63	Space relations	0.31

significant at the .01 level. The largest canonical weights for this case were on the Paper Folding and Card Rotations Tests (loadings of -.48 and -.46, respectively) and the New York State Mathematics Test (loading of -.49). The canonical correlation for tenth grade girls was .67, statistically significant at the .05 level. The largest canonical weights were on the ETS Vocabulary Test with a loading of .64 and the New York State Mathematics Test with a loading of .62.

Discussion

The results of the analyses of variance indicate that males tended to perform better than females on mathematics measures, while females tended to perform better on verbal measures. The male advantage in mathematics did not emerge until tenth grade. Females tended to do better on the verbal measures in both seventh and tenth grades. Although sex differences in mathematics achievement and verbal skill are not found in all studies, when they are obtained they are almost invariably in the direction observed in this study (Maccoby & Jacklin, 1974).

While males performed consistently better on the DAT Space Relations test, there were no sex differences for either grade on any of the ETS visual-spatial tests. These results indicate that the discovery of a sex difference in visual-spatial skill is highly dependent on the type of visual-spatial measure used.

The results of the factor analyses showed support for a general visual-spatial ability factor apart from verbal and mathematical ability. The results also indicated an increasing differentiation of skills with age; among the tenth graders two separate visual-spatial factors were identified whereas only one such factor was clearly evident among the seventh graders. This is consistent with what some cognitive ability researchers have labeled the Age Differentiation Hypothesis. This hypothesis postulates that "human intellective ability differentiates from one global undifferentiated ability, g, in early childhood to more general verbal and quantitative factors and ultimately to the complex pattern of abilities repeatedly found in college students" (Dye & Very, 1968).

The first of the two visual-spatial factors identified for the tenth graders had high loadings on the ETS Cube Comparisons Test, the ETS Paper Folding Test, the ETS Card Rotation Test, and the DAT Space Relations Test. The Cube Comparisons and Card Rotations Tests are considered by the ETS to be marker tests for skill in spatial orientation while Paper Folding is a marker test for visualization skill. The DAT Test is usually considered to involve skills in both spatial orientation and visualization. Thus the first visual-spatial factor can be considered a general spatial orientation-visualization factor.

The second visual-spatial ability subfactor loaded highly on the ETS Hidden Patterns Test, the ETS Gestalt Completion Test, and to a lesser extent on the ETS Card Rotations Test. The Hidden Patterns Test and Gestalt Completion Test represent what ETS has referred to as the "closure" subdivision of visual-spatial ability. Hidden Patterns is a marker test for the "flexibility of closure" factor, while Gestalt Completion is a marker test for the "speed of closure" factor. One reason that the Card Rotations Test may have loaded somewhat on the "closure" subdivision of visual-spatial ability is that it is extremely similar in "form" to the Hidden Patterns Test. The Hidden Patterns Test and Card Rotations Test both consist of a large number of items which must be completed in a short amount of time. Both tests therefore emphasize a type of clerical, high-speed perceptual skill. Thus, even if the actual cognitive ability that each test measures is quite different, a certain degree of association would still exist between them.

To summarize, the factor analyses performed yielded three main factors that distinguished between verbal, mathematical, and visual-spatial ability. Confirming evidence was supplied for the ETS-referenced subdivision of visual-spatial ability into a "spatial-visualization" factor and a "closure" factor. Finally, the DAT Space Relations Test was found to load highly on the "spatial-visualization" subdivision of visual-spatial ability.

The results of the bivariate correlations showed that for boys various types of visual-spatial skills were highly correlated with math achievement. Generally those tests with high loadings in the "spatial-visualization" factor were good predictors whereas those tests with high loadings in the "closure" were poor predictors. The Gestalt Completion test was especially poor as a predictor of mathematics achievement for boys. The conclusion that visual-spatial ability and mathematics achievement were closely related for boys was supported by the results of the canonical correlations. Strong statistically significant canonical correlations were obtained between the cognitive tests and the mathematics measures with the major weights on the DAT Space Relations Test for the seventh grade boys and the Paper Folding and Card Rotations Tests for the tenth grade boys.

In general, the relationshiop between visual-spatial skill and mathematics achievement was markedly less for girls. The bivariate correlations showed that some math measures were more highly correlated with scores on the visual-spatial tests than on the vocabulary tests whereas for other math measures the reverse was true. It should be noted that those visual-spatial tests which were good predictors of math achievement among girls were all from the "spatial-visualization" subdivision of visual-spatial ability. Just as was the case for boys, those tests from the "closure" subdivision of visula-spatial ability were poor predictors of math achievement among girls.

The canonical correlations for the girls yielded markedly different results than the parallel analysis for the boys. For the seventh grade girls, no significant correlates were found while for the tenth grade girls the only cognitive test with a substantial weight on the canonical variable was the Vocabulary Test. These results suggest that for girls verbal skills may play a more important role than visual-spatial skills in mathematics achievement. Since these results, however, have not been reported in the literature previously, and since it is not possible to make a statistical test of the difference in canonical weights, this conclusion must be ocnsidered tentative at this time, pending replication.

Study 2

Study 2 was planned as a replication of Study 1. The purpose was to evaluate the generalizability of the results obtained in the first study by collecting additional data from other schools. Since the tests measuring the visual-spatial skills of flexibility and speed of closure (the Hidden Patterns and Gestalt Completion Tests) appeared to contribute relatively little to predicting mathematics achievement, these were dropped from the test battery. Instead two tests of visualization were included (the ETS Form Board Test and the ETS Surface Development Test described below). By employing three tests of visualization, two tests of spatial orientation and one test involving both skills (the DAT Space Relations Test), we hoped to find empirical support for the distinction between these two skills and an evaluation of the relative importance of each skill for mathematics achievement.

Subjects

Subjects were 374 seventh graders (189 boys, 185 girls) enrolled in two junior high schools in a small city in Upstate New York and 560 tenth graders (277 boys, 283 girls) enrolled in various mathematics courses in neighboring high schools.

Testing materials

The mathematics achievement measures used in this study were the seventh and tenth grade math tests described in Study 1. Since the analysis of the algebra, geometry, and arithmetic scales added relatively little to the findings of Study 1, only a combined score was used.

The ETS Cube Comparisons Test, Vocabulary Test, Card Rotations Test, PaperFolding Test, and DAT Space Relations Test, described in Study 1, were included in the battery of cognitive tests given to the students. (Due to time constraints several of the tenth grade classes were unable to do the Paper Folding Test.) The Hidden Patterns Test and the Gestalt Completion Test were replaced by the ETS Form Board and Surface Development Test. The Form Board Test contains 24 items each, one of which shows a geometric shape followed by five smaller geometric shapes. The subjects' task is to indicate which ones of the smaller shapes could together be arranged to form the larger shape. The Surface Development Test is similar to the DAT Space Relations Test. "In this test, drawings are presented of solid forms that could be made with paper or sheet metal. With each drawing there is a diagram showing how a piece of paper might be cut and folded so as to make the solid form. Dotted lines show where the paper is folded. One part of the diagram is marked to correspond to a marked surface in the drawing. The subject is to indicate which lettered edges in the drawing correspond to numbered edges or dotted lines in the diagram" (Ekstrom, et al., 1976). The test consists of five items in each of six drawings.

Procedure

Testing was carried out on two consecutive days in individual math classes in all schools except one, in which the entire seventh grade student body was tested as a group. On the first day, five tests were administered in a 30-minute session, in the order listed: the Surface Development, Form Board, Cube Comparisons, Vocabulary, Card Rotations, and Spatial Relations tests. On the second day, the appropriate mathematics achievement test was administered along with the Paper Folding test. The math test took 30 minutes to administer and the Paper Folding, 8 minutes. For test administration in individual math classes ranging in size from 19 to 31 students, two experimenters were continuously present, while in the mass testing session, 10 proctors supervised adminstration of the tests to 126 students.

Results

Among the seventh graders boys obtained a higher mean score on the Form Board Test ($p < .05$); no other reliable sex differences were obtained. Among the tenth graders the males had higher mean scores than the females on three of the visual-spatial tests: the Surface Development Test, the Form Board Test, and the Paper Folding Test. There was also a trend for boys to do better on the DAT Space Relations Test. Girls tended to perform better than boys on the Vocabulary Test. (See Table 11.)

Correlations between the set of six visual-spatial measures and the vocabulary test with the mathematics achievement test are shown separately for boys and girls in Table 12. All of the correlations were positive, and with two exceptions, statistically significant. In four instances, the correlation of visual-spatial performance with mathematics achievement was significantly higher for boys than for girls. (This was seen for seventh graders on the Surface Development and Space Relations Tests and for tenth graders on the Form Board and Card Rotations Tests.) In most other instances the correlation coefficient was higher for boys but not significantly so. There was no sex difference in the strength of the association between performance on the Vocabulary Test and Mathematics Achievement.

In order to examine the predictability of performance on the Mathematics Test from the set of cognitive measures as a whole, a step-wise multiple regression procedure was used. The results of this procedure for the two grades and for boys and girls separately are shown in Table 13. Greater predictability as indicated by the magnitude of the multiple correlation coefficient (R^2) appeared to be obtained for the seventh graders than for the tenth graders. A substantial beta weight was obtained for the Vocabulary Test in all of the analyses; it was the second variable entered into the regression equation for all of the groups except the tenth grade girls, for whom it was the first variable entered.

For three of the groups the regression analyses yielded only three variables with significant beta weights. Interestingly, in each case one test from the spatial orientation domain (Form Board, Paper Folding, Space Relations, Surface Development), in addition to the Vocabulary Test, was always indicated in the final set of predictor variables.

TABLE 11

MEANS, STANDARD DEVIATIONS, SAMPLE SIZES, F-RATIOS FOR BOYS AND GIRLS, SEVENTH AND TENTH GRADES

	Boys			Girls			
	$\bar{X}$	S	n	$\bar{X}$	S	n	F-ratio
Seventh Grade							
Vocabulary	3.26	2.86	174	3.55	2.77	182	< 1
Surface Development	4.16	2.90	174	4.00	2.77	180	< 1
Form Board	2.64	2.07	173	2.19	1.90	180	4.43**
Paper Folding	2.10	2.42	179	2.33	2.25	181	< 1
Cube Comparisons	3.26	3.50	176	3.30	3.93	181	< 1
Card Rotation	34.99	21.09	177	37.56	20.0	177	1.38
DAT Space Relations	5.35	2.65	170	5.22	2.85	177	< 1
Mathematics Test	12.18	8.07	182	12.70	6.85	184	< 1
Tenth Grade							
Vocabulary	6.84	3.89	260	7.47	3.81	264	3.51*
Surface Development	9.00	5.56	259	8.09	4.84	261	3.96**
Form Board	4.20	2.63	259	3.48	2.31	261	10.99**
Paper Folding	4.77	2.63	188	4.15	2.44	218	6.00**
Cube Comparisons	6.10	4.65	260	5.76	4.37	262	< 1
Card Rotations	47.18	20.21	261	49.20	18.35	264	1.41
DAT Space Relations	6.62	2.37	252	6.25	2.43	256	3.02*
Mathematics Test	9.04	6.55	256	9.04	5.70	218	< 1

* $p < .10$

** $p < .05$

TABLE 12

CORRELATIONS BETWEEN
COGNITIVE MEASURES AND MATHEMATICS ACHIEVEMENT

	Seventh Graders		Tenth Graders	
	Boys	Girls	Boys	Girls
Vocabulary	.40	.44	.48	.40
(n)	(172)	(177)	(240)	(248)
Surface Development	.40	.12[a,b]	.30	.29
(n)	(171)	(175)	(239)	(248)
Form Board	.44	.34	.40	.25[b]
(n)	(170)	(175)	(239)	(246)
Paper Folding	.51	.48	.39	.32
(n)	(178)	(181)	(187)	(218)
Cube Compairons	.46	.33	.24	.19
(n)	(173)	(177)	(239)	(247)
Card Rotations	.38	.35	.32	.12[a,b]
(n)	(174)	(178)	(240)	(248)
DAT Space Relations	.42	.17[b]	.19	.30
(n)	(167)	(172)	(232)	(241)

[a]Correlation not significant at .05 level.

[b]Difference in correlation coefficient for boys and girls is statistically significant ($Z < 1.96$, $p < .05$.

TABLE 13

RESULTS OF STEP-WISE REGRESSION ANALYSIS

DEPENDENT VARIABLE IS PERFORMANCE ON MATHEMATICS TEST

Group	Variable	Beta weight	Standard error	F-ratio	p value
Seventh grade boys	Form board	1.90	.27	3.61	.05
	Cube comparisons	1.96	.16	3.83	.05
	Vocabulary	3.65	.19	13.31	.001
	DAT Space relations	2.06	.21	4.26	.05
R^2=.466	Paper folding	4.82	.22	23.10	.001
Seventh grade girls	Cube comparisons	1.77	.11	3.09	.08
	Vocabulary	4.24	.16	17.94	.001
	Paper folding	6.13	.19	37.51	.001
R^2=.362					
Tenth grade boys	Form board	3.46	.14	11.99	.001
	Vocabulary	2.74	.10	7.44	.01
	Card rotations	3.37	.02	11.36	.001
R^2=.262					
Tenth grade girls	Surface development	2.45	.06	5.99	.05
	Cube comparisons	2.16	.07	4.65	.05
	Vocabulary	3.42	.08	11.82	.001
R^2=.220	DAT Space relations	2.60	.12	6.68	.01

Discussion

The results of Study 1 and 2 together show some consistency in patterns, as well as certain inconsistencies. With respect to sex differences in performance on visual-spatial tests, all of the differences that were significant favored males. There were many comparisons, however, on visual-spatial test performance which were not significant, despite the relatively large sample sizes that were employed. Differences favoring males were more noticeable among tenth graders than seventh graders, but even among the tenth graders no significant male advantage was found on six of the visual-spatial measures of Study 1 and two of the visual-spatial measures of Study 2. The tests which most consistently differentiated the sexes were the DAT Space Relation Test and the Form Board Test, both measures of visualization and with a very similar content. Nevertheless even on these tests statistical significances at the .05 level were only obtained in two of the four relevant comparisons. In sum, it seems fair to conclude that junior and senior high school males will perform better than females on some visual-spatial measures, some of the time.

Sex differences in performance on the mathematics and verbal measures, when observed, were always in the direction of superior male performance on mathematics measures and superior female measures on verbal measures. As with the visual-spatial measures, however, there were many instances in which the relevant comparisons were not even close to approaching a level of statistical significance. Although the reliability of the different measures used in the two studies may be one factor in explaining these inconsistencies (the lower the reliability of a test the less sensitive it is to "true" group differences), this one factor does not appear to be sufficient to explain some of the different results that emerged from the two studies when sex differences in performance on the same test were compared. An alternative interpretation is that sex differences in visual-spatial skill, verbal skill, and quantitative skill may be very greatly influenced by the measures used, the conditions of testing, and the learning experiences that boys and girls bring with them to the testing situation. We are unable to say, at this time, exactly how all of these variables interact to result in a significant or non-significant sex difference in performance. What is clear is that it is seriously misleading to simply summarize the state of knowledge at this time by saying boys do better in mathematics and the visual-spatial domain while girls do better in the verbal area.

The analyses which examined the nature of the relationship between visual-spatial skill and mathematics achievement yielded some additional sex differences of interest. (The analyses were not exactly parallel in the two studies because of differences in the data sets available.) The results of the canonical correlations on the tenth grade data in Study 1 and the bivariate correlations in Study 2 suggested that there may be a closer association between mathematics achievement and visual-spatial skill for boys than for girls. Again, we note that sex differences in strength of association were found in only a minority of the correlations calculated, but when obtained were in the direction just stated. One possible interpretation is that girls rely more upon verbal approaches to the solution of mathematics problems. However, this interpretation is questionable given the lack of a finding of a stronger relationship between the verbal measures and mathematics achievement for girls than for boys (except in the canonical correlation for tenth graders in Study 1). At this point we can only tentatively conclude that certain visual-spatial skills do not appear to be as relevant to mathematics achievement for girls as for boys.

The results of the factor analysis and the bivariate correlations in Study 1 showed that the visual-spatial skill of closure was not as closely related to mathematics achievement as the skills of visualization and spatial orientation. The results of the two studies suggest that visualization skill and spatial orientation skill are somewhat distinct and both contribute to predicting mathematics achievement. Further research examining the development and trainability of these skills, the focus of the second part of this project, thus appears warranted.

PART II

TRAINING VISUAL-SPATIAL SKILLS

BACKGROUND

Although a variety of hormonal (Broverman, Klaiber, Kobayashi, & Vogel, 1968), genetic (Bock, 1967; O'Connor, 1943), physiological (Hyde & Rosenberg, 1976; Buffery & Gray, 1972; Bakan, 1971), and other biological theories have been proposed to explain sex-related differences in visual-spatial skill, support for these theories is weak and inconsistent (Sherman, 1976). It is also clear that the existence of a biological contribution to sex-related differences in visual-spatial skill does not preclude the possibility of strong environmental and social contributions to such differences as are observed. Theorists proposing social and cultural explanations of these differences include Berry (1971), Maccoby (1966), and Sherman (1967, 1971). Sherman's view, which at this time appears to have the best support, is that sex-related differences in visual-spatial skill are at least partially due to sex-related differences in the extent to which males and females have the opportunity to engage in activities which foster the development of visual-spatial ability. Correlational work which provides some evidence for this view is the finding that there is a relationship between the activity preferences of preschoolers, as observed over an extended period of time, and visual-spatial skill (Connor & Serbin, 1977; Serbin & Connor, 1979). In these studies, higher visual-spatial performance has been associated with a preference for "masculine" activities such as climbing, building with blocks, and playing with balls and trucks, as opposed to "feminine" activities such as cooking, doll play, and housekeeping.

An experimental approach which may be a useful way to investigate the validity of Sherman's hypothesis involves the examination of the effects of visual-spatial training on the performance of males and females. If the visual-spatial aptitudes of females are relatively undeveloped as a result of deficits in practice created by social and environmental limitations, then appropriate training or practice experiences might be expected to have a greater impact on the visual-spatial performance of females than of males. Three research studies, including two completed in our laboratory do, in fact, show this to be the case (Connor, Serbin, & Schackman, 1977; Connor, Schackman, & Serbin, 1978; Goldstein & Chance, 1965). Since these training studies have important psychological and educational implications for: (a) the teaching of mathematics, (b) understanding sex-related differences in mathematics, and (c) understanding possible causes and remedies for sex-related differences in visual-spatial ability, this research and related training studies will be reviewed in some detail.

THE TRAINABILITY OF VISUAL-SPATIAL SKILLS

One of the earliest reports of training effects on visual-spatial ability is a report of the effect of one year's experience in engineering school on performance on the College Entrance Examination Board Space Relations Test (Blade & Watson, 1955). Despite the lack of a true control group (i.e., random assignment was not used), the use of reasonable comparison groups and the existence of independent replications (see also Myers, 1953) lend support to the findings and interpretations made by the authors. Students in two engineering programs were observed to improve an average of one standard deviation in performance on the DAT Space Relations Test between the time they entered the program and the beginning of their second year. Students in two non-engineering programs showed a gain of half as much during the same period. The improvement made by the engineering students is substantial in view of their already high level of performance of the first test administration, approximately one-half a standard deviation above the norm. Sex differences in response to training were not reported in this study; there were presumably few, if any female engineering students at the school.

The effectiveness of a more systematic visual-spatial training program is described by Brinkmann (1966). Brinkmann developed a short course in elementary geometry emphasizing visual-spatial problem solving and using programmed instruction techniques. (He cites the results of three unpublished studies showing that more traditional geometry courses which emphasize formal proofs have no effect on visual-spatial performance.) Extensive discrimination training was provided as well as practice with the visual and physical manipulation of geometric shapes. Twenty-five eighth-grade students who participated in this training program once a day for three weeks showed an average increase of 18 raw score points, or approximately 1½ standard deviations, on the Space Relations part of the Differential Aptitude Test. The control group, which participated in standard math classes during the training period, showed an increase of only 3 raw score points. There was also evidence of a significant increase in knowledge of geometry as a result of the program. There was no evidence of poorer performance by females on the post-test (in fact, females averaged two points higher than males on the Spatial Relations post-test). Unfortunately, Brinkman did not report separate means for males and females on the pre-test, so that it is not clear if a sex-related difference was eradicated by the training procedure or if no sex difference was apparent in his sample prior to training. However, Brinkmann's conclusion that "girls can at least hold their own when provided with the opportunity to learn something about a particular area in which they are often assumed to possess less ability" does appear to be a reasonable interpretation of these data.

Less structured approaches to visual-spatial training were adopted by Ciganko (1973) and Rennels (1970). Ciganko gave two groups of ninth graders drawing practice over a six-day period, drawing either from directly observed stimuli or visualized stimuli. Both groups showed significant and similar increases on the Spatial Relations part of the Multiple Aptitude Test from pre-test to post-test. Rennels worked with eighth-grade disadvantaged youths and developed two five-week training programs on the topic of linear perspective in drawing. One of the training programs involved what Rennels called a "synthetic" approach to the topic while the other involved an approach he labeled "analytic". Students receiving the "analytic" training program showed a greater improvement from pre-test to post-test on standard measures of visual-spatial ability than the student receiving the "synthetic" method or than students in the control group. Unfortunately, it is unclear from the written report of this experiment exactly what specific components distinguished the two training methods, whether there were sex-related differences in response to training, or on which measures of visual-spatial ability significant increases were observed.

Studies showing no effect of a training experience have been reported by Lolla (1974) and Mendicino (1958). Lolla exposed 30 ninth grade students to a two-hour training procedure in tactual-visual perception involving wooden blocks and line drawings. No beneficial effect of the training was observed on the Space Relations part of the Differential Aptitude Test. In fact, a post hoc analysis indicated that for students initially high in visual imagery the training procedure actually interfered with subsequent performance on the Space Relations Test. Mendicino's study, which was similar to the approach taken by Blade and Watson (1955), showed that students who took a one year course in machine shop during the tenth grade performed virtually identically on the DAT Space Relations Test to a matched group of students who did not take the course.

A small number of studies have investigated sex-related differences in response to training. Goldstein and Chance (1965) gave adult males and females extensive practice on the Embedded Figures Test and additional comparable stimuli. They found that both males and females reduced their discovery time substantially over the series of trials. However, while females responded significantly more slowly than males in the first block of trials, by the last block of trials there were no significant sex-related differences. It appears that both males and females may have reached a biological limit in the speed with which they could respond.

In research by the present writers (Connor, Serbin, & Schackman, 1977; Connor, Schackman, & Serbin, 1978), sex differences in the response of children to training or practice on a visual-spatial test in children have been found. The first study involved the random assignment of first, third, and fifth graders to either one of two training conditions, which were designed to teach visual-spatial disembedding, or to a control group. One of the training procedures (the overlay condition) consisted of practice locating a diamond shape in each of five specially constructed complex pictures. After a few moments exposure to the complex figure (in which few of the children were able to locate the diamond), an overlay was removed from the picture which decreased the amount of complexity and detail in the picture. Shortly thereafter a second overlay and a third overlay were removed, at which point the diamond, with few extraneous details, was readily apparent to all children. The overlays were then repositioned so that the child could observe changes in the appearance of the diamond shape as more details were added. This procedure was followed with all five pictures. The second training procedure (the flat figures condition) consisted of practice in locating the diamond in identical pictures without the overlays.

The results of this study indicated that girls who received the overlay procedure before being tested on the Children's Embedded Figures Test scored significantly higher on the test than girls in the Flat Figures and the Control conditions. The test performance of the boys was not affected by either type of training. Comparing the performance of the two sexes, there was a tendency for boys in the control condition to receive higher scores than girls in the control condition, while among the children receiving the overlay training the direction of the means was reversed. It appeared, in other words, that the slight deficiencies girls sometimes show in visual-spatial disembedding during the elementary school years are readily changed with even a brief amount of appropriate training.

In a second study with first graders, Connor, Schackman, & Serbin (1978) explored these findings further with a pre-test post-test design and a measure of generalization to a related test. This design permitted the evaluation of both practice and training effects. On the pre-test girls tended to score slightly less than boys. This tendency was not observed on the post-test, which was given four days later, for either the control or the training group. Thus the effect of the pre-test (i.e., practice) was to eliminate the sex difference as well as to increase the scores of both boys and girls. In addition to the practice effect, an effect of training was found for both boys and girls. However, there were no significant differences between the training and control group on the generalization measure, which consisted of an adaptation of the DAT Space Relations test appropriate for use with young children (Sternglanz, 1977).

A final study in this series (Connor, Serbin, & Freeman, 1978) employed the identical procedures with a group of educable retarded children whose performance and verbal I.Q.'s on the Wechsler Intelligence Scale for Children were both below 80. On the pre-test the performance of these children was similar to that of children of comparable mental age rather than comparable chronological age. On the post-test the children in the control condition showed no significant increase in performance, while the children in the overlay training condition were performing at age-appropriate levels, an increase greater than one standard deviation. No sex differences were observed in this study for mean level of performance or response to training. The implication that visual-spatial training is particularly effective in increasing the performance of students who initially obtain relatively low scores on tests of visual-spatial ability is consistent with our other findings.

In sum, the results of these studies indicate that visual-spatial ability is trainable at a variety of age levels. Beneficial effects of training have been found in elementary school children (Connor, et al., 1977; Connor, et al., 1978), junior high school students (Brinkmann, 1966; Ciganko, 1973; Rennels, 1970), and college students (Blade & Watson, 1955; Goldstein & Chance, 1965; Myers, 1953). Such procedures have been found effective both with students relatively high in visual-spatial ability (Blade & Watson, 1955) as well as with students relatively low in visual-spatial ability (Connor, Serbin, & Freeman, 1978; Rennels, 1970). Three studies have found that training and/or practice effects are relatively stronger for females than for males (Connor, Schackman & Serbin, 1978; Goldstein & Chance, 1965).

VISUAL-SPATIAL TRAINING RESEARCH, 1977 - 1979

The research described in detail below was carried on in 1979 - 1980. The positive results obtained in this search, however, were preceded by two years of work with negative results. Although traditions in publishing are such that little encouragement is given to the documentation of negative results, we believe it is important to share with other researchers and educators some of the experiences and conclusions about this earlier work in the hope that others may also profit from them.

On the basis of Brinkmann's (1966) work, all of the materials developed and evaluated during 1977 - 1979 were designed with a programmed instruction format. The magnitude of the gains obtained in Brinkmann's study was impressive and the target population (junior high school students) was the same as for this project. It thus appeared that programmed instruction materials which would allow each child to procede at his or her own pace might be a suitable format for the materials we were to develop. Such an approach would have the further advantage of requiring less active or didactic involvement from the teacher and would thus facilitate the statistical evaluation of effectiveness by minimizing variables due to teacher variables. Our current evaluation, however, on the basis of work with over 1000 children in two geographically distant locations is that the use of the programmed instruction format was a mistake, given other constraints and features of this project. The junior high schools in the school districts that participated in this project (and presumably in many other districts as well) were typically taught in traditional lecture or lecture and discussion styles with a strong teacher presence. They were not used to working consistently on their own for whole class periods despite the opportunity for checking their answers as they worked through the materials. The dissimilarity between the content of the visual-spatial training materials and the usual content of their mathematics classes, in which these materials were given, also seemed to highlight the distinction between the present programmed learning materials and their regular work. The result was that many students did not approach the learning task with much motivation or seriousness. In sum, we believe that the programmed instruction format is one which must be introduced gradually and consistently to the students by their regular classroom teachers over a period of time if junior high school students are to accept it. For the final year of work on this project we employed small instructional groups instead.

A second problem which emerged concerned the evaluation of the training materials as they were being developed. We developed three sets of materials in conjunction with our graduate assistants and obtained preliminary data on the effectiveness, intelligibility, attractiveness, etc., of the materials with college students, who were highly accessible. We found a minimum of evidence for the effectiveness of the materials with this population. However, the college students represent a select sub-group, and it was apparent that we could draw few conclusions about th utility of the materials for a non-selected younger group on the basis of the college student data. Our next step, then, was to test the materials with a smaller group of junior high school students in a private school system as well as some paid volunteers from the public school system. Students had no difficulty with the materials and appeared to understand and learn from them. Minor modifications were made on the basis of their feedback. With a great deal of anticipation and much negotiation we proceeded to a full-scale evaluation of the materials. This evaluation was conducted with several hundred students randomly assigned to training and control groups. Between three and five classes were assigned to each set of materials. Much to our dismay the results of this evaluation were negative. In no case did the group receiving the training materials perform better on the visual-spatial tests than the control group, which had its usual classroom activities instead of the training materials. Indeed, in some instances the control group actually appeared to perform somewhat better than the training group. Despite what we thought was a reasonable amount of pilot-testing of the materials there was no objective evidence that they were effective teaching tools. In retrospect we believe we erred in relying too heavily upon the encouraging preliminary work which employed groups and settings not representative of the conditions and population which the full-scale evaluation involved. We conducted the preliminary evaluations since we considered it important to have some reason to believe our training materials were suitable and effective before requesting substantial amounts of classroom time from a school system. However, inherent situational differences in the nature of preliminary evaluations and regular classroom evaluations make it difficult to generalize from one to the other. In the final year of this project we decided we needed more input from experienced classroom teachers who would be able to evaluate the materials as a whole, taking into account their knowledge of students' attention span, motivation, and classroom factors in their recommendations about the materials. We also asked the teachers to meet as a group so that the reflections and evaluations of one teacher could be checked out with the others. Since any teaching package, including our training materials, contains a large number of components, it is typically not feasible to evaluate separately each component, and the informed judgments of our experienced teacher consultants were critical for this work. Based upon our experience of these two years, the data collected, and the input of the teachers and teacher-consultants, the training materials were completely re-done and five sets of visual-spatial training materials were evaluated in 1979 - 1980. The report of this work follows.

TRAINING STUDY, 1979 - 1980

Method

Subjects

Subjects were 231 boys and 203 girls from eighth grade mathematics classes from two suburban junior high schools in upstate New York.

Materials

Five sets of visual-spatial skills training materials were evaluated. Each set was designed to progress from simple, more concrete tasks to more complex, demanding tasks over the course of a half-hour training session. Each set emphasized training a specific type of visual-spatial skill that was measured by a particular test of visual-spatial skill. The relevant test was administered immediately following the training session.

Materials training spatial orientation and visualization as measured by the Differential Aptitude Test. This set of materials was divided into two sections. In the first section, a set of 10 three-dimensional geometrical objects and 13 two-dimensional patterns were presented to the students. The students were asked to match each three-dimensional object with the two-dimensional patterns which could be folded to make it. Although the sides of the objects were painted different colors, the students were asked to ignore the colors and make their matches on the basis of shape only. Students could then fold the two-dimensional patterns to confirm their decisions.

In the second part, a page of drawings of three-dimensional structures having features such as windows, doors, etc., and a page of drawings of two-dimensional featureless patterns which corresponded to "unfoldings" of the solid structures on the first page were presented to the students. Students were asked to match drawn structures to the drawn pattern which could be folded to construct it, and then to draw the features of the structure in the appropriate positions on the unfolded blank pattern. Cut-outs of the patterns were provided so the students could confirm their matches by actually constructing in three dimensions the structure, originally pictured in two-dimensions, from the two-dimensional pattern.

Materials training spatial orientation as measured by the Cube Comparisons Test. This set consisted of two sections. The first group of tasks involved constructing three-dimensional lattices or "trellises" using four wooden popsicle sticks or six strips of paper. Six different model lattices of increasing complexity were shown to the students, who had to construct each of the same lattices as viewed from the back. Then the students had to choose which one of eight drawings of lattices represented a picture of a model lattice seen from the back.

A series of exercises in which students were asked to visualize a single die being rotated completed the training session. These exercises progressed from simply determining where each number pattern of dots would end up on the die after a single rotation in a specific direction, to drawing on a blank picture of a die the pattern of dots which would be visible on each face following a sequence of three rotations in specified directions from a given original position.

Materials training visualization as measured by the Form Board Test. This set of materials made use of Tangrams, a set of plastic geometrical pieces including one large triangle, two medium right-angle triangles, an acute angle triangle, a medium square, and a parallelogram. Students were given these pieces and a set of 10 worksheets with drawings of complex geometrical shapes which could be made from combinations of the Tangram pieces. The task was to create the pictured shapes using the Tangram pieces combined in as many different ways as possible. As training progressed, the student was asked to simply draw lines on the figures indicating where the Tangram shapes could be placed and to use the actual pieces only to confirm their decisions. Again, they also had to come up with as many different arrangements of the different pieces as were possible to make the worksheet figure.

Materials training visualization as measured by the Paper Folding Test. One part of this set of materials consisted of paper-and-pencil exercises in which the student was asked to determine lines of symmetry in drawings of a wide variety of objects differing in visual complexity. The second task required the student to draw the mirror image of a figure about a given axis. There were 8 different figures of varying levels of complexity. The final training item in this set of materials was a square divided into 36 smaller squares, each containing a number from 1 through 36. Arrows were drawn along the vertical midline, horizontal midline, and the diagonal. Students were given problems of the form:

$$2 \;\rightarrow\; \uparrow\; \underline{\ ?\ }$$

with the arrows indicating independent folding operations performed on the large square about the line indicated by the arrow. Starting with the square number indicated (in this example, 2), the child was to mentally perform the folding operations represented by the arrows and then respond as to which square number was now covered by the original square number (in this case, 2). Problems containing up to five arrows, that is, five independent operations, were included.

Materials training spatial orientation as measured by the Card Rotations Test. The first part of this group of materials consisted of drawing tasks in which the students had to imagine rotating letters of the alphabet through different degrees of rotation and draw them in their final positions. Actual rotation of the pages themselves could be used to confirm the student's answers. The second and most difficult part of this set consisted of a series of small squares containing a dot grid-work. Some of the dots were connected together to create a line figure. The student's task was to imagine the entire square rotated 180^{o} and to draw in the physically unrotated square exactly where the line figure would end up after the rotation. Transparent overlays of the square and line figure, which could be actually rotated on top of the original, were used to confirm the student's answers.

Control group materials. The control groups worked on a set of verbal exercises while the experimental subjects were receiving visual-spatial training. These verbal exercises consisted of: (1) a reading comprehension task, in which a short piece was read and questions asked about its content and meaning; (2) a verbal analysis task; (3) a sentence generating task, for which students composed sentences of a specified length in which some words began with predetermined letters; (4) a sentence completion task, in which students supplied the second half of descriptive similes. These exercises were similar to items on the Scholastic Aptitude Tests, General Educational Development (high school equivalency) Tests, and the Making Sentences (FE-1) and Figures of Speech (FA-3) sections of the ETS Factor-Referenced Cognitive Tests.

Trainers

Ten undergraduate students at SUNY - Binghamton served as trainers. The trainers were completely familiar with all sets of training materials and practiced at administering them to each other several times prior to administering them to the experimental subjects. Each of the trainers administered each set of training materials, including the control set, at least twice.

Procedure

Training sessions were carried out in individual mathematics classes ranging in size from 18 to 27 students. Groups of two or three students worked directly with one experimenter. Half of each class was assigned to the control group; the other half of the class received training on one set of the visual-spatial training materials. After the half-hour training session, all members of the class were administered the appropriate visual-spatial skills test. A minimum of 38 students, 20 boys and 18 girls, were trained with each of the five sets of materials.

During the training sessions trainers were directed to encourage participation and to maintain the motivation and interest of the students in the training materials. They were also instructed to actively use the training materials to try to teach students to accomplish the required tasks. Students were always encouraged to solve the problems in their heads. The use of physical manipulation was restricted to confirming or correcting the students' answers, or to instructing students having difficulty with the task as to what was required. Trainers were instructed to limit the amount of physical manipulation of the materials by the student as much and as early in the training session as was feasible, depending on the ability of the individual student and the amount of coaching he or she required.

RESULTS

The score of each student on the test that he or she received was corrected for guessing by calculating

$$\text{number correct} - \left(\frac{1}{n-1} \times \text{number incorrect}\right)$$

where n was the number of alternative choices for each response. Two factor analyses of variance were then conducted for each of the five sets of training materials-test groupings with sex of subject and experimental condition (training or control) as between-subjects factors. The significant effects obtained in these analyses, as well as the means, standard deviations, and sample sizes for each sub-group are shown in Table 14.

Significant training effects were found for the materials teaching the skills of spatial-orientation and visualization as measured by the Differential Aptitude Test and the skill of spatial orientation as measured by the Card Rotations Test. There were no significant training effects for the other three sets of materials. The interaction of sex of subject and experimental condition was significant on the Card Rotations Test. The training effect in this case was due to a marked training effect for boys and none for girls. Among the children in the control group, females tended to receive higher scores on the Card Rotations Test than males ($p < .10$). The opposite was true for the children in the training group.

Table

MEANS, STANDARD DEVIATION, AND SAMPLE SIZES

FOR TRAINING AND CONTROL GROUPS

Test	Males Training	Males Control	Females Training	Females Control	Significance Effects ($p < .05$)
Differential Aptitude					
$\bar{X}$	6.67	3.74	4.73	3.02	TREATMENT, SEX
s	2.85	3.14	2.63	2.01	
n	21	33	20	15	
Cube Comparisons					
$\bar{X}$	6.52	5.93	7.28	6.00	------
s	4.19	4.26	4.65	4.28	
n	21	28	18	22	
Paper Folding					
$\bar{X}$	3.19	3.79	4.00	3.74	------
s	2.30	2.14	2.52	2.24	
n	22	25	19	17	
Card Rotations					
$\bar{X}$	54.75	34.40	44.05	44.58	TREATMENT, INTERACTION
s	14.87	20.70	15.29	17.03	
n	20	15	19	31	
Form Board					
$\bar{X}$	3.69	3.60	2.94	4.04	------
s	2.26	1.88	1.82	2.01	
n	26	20	17	25	

DISCUSSION

Thirty minutes of exposure to visual-spatial training materials resulted in significant increases in performance on a visual-spatial test for two of the five sets of materials developed. These results support the conclusion that visual-spatial skills are teachable in a classroom setting with junior high school students. The negative aspects of the results (i.e., that three of the sets of materials were not demonstrated to be effective) points to the difficulty of designing effective materials as well as the difficulty in having an impact on skills in brief training periods.

An examination of the tests on which an effect was demonstrated and the tests on which no effect was discernible suggests that visualization may be a more difficult skill to teach than spatial orientation. The two tests which have been defined as visualization tests (Form Board and Paper Folding) showed no training effect. Of the two tests defined as measures of spatial orientation (Cube Comparisons and Card Rotations) one showed a training effect and the results for the other were in the appropriate direction for each sex, though not statistically significant. Lastly, the Differential Aptitude Test, which is thought to be a measure of both spatial orientation and visualization showed a training effect. The training results with the other tests suggest the possibility that scores on this test were elevated as a result of students' improvement in spatial orientation rather than visualization skill.

There was no indication in this study (as opposed to the findings of Connor, et al., 1977, 1978) that females profited more from training than males. In fact, the only treatment by sex interaction that was obtained was in the opposite direction; i.e., on the Card Rotations Test the males profited from the training while the females did not. It is interesting to note, however, that in this case the males in the control group were performing less well than the females in the control group. What these findings suggest is that the sex which is performing less well without training is likely to benefit more from training.

The results of this training study are consistent with those of the correlational studies in the patterns of sex-related differences in overall scores. That is, the Differential Aptitude Test is a relatively consistent discriminator of male and female performance, but other measures do not yield sex-related differences consistently at all. This does not appear to be a function of the reliability of the tests, as the reliability for the ETS tests appears to be quite similar to that for the Differential Aptitude Test (Ekstrom, et al, 1976; Bennett, et al., 1973). The Space Relations section of the Differential Aptitude Test, however, is the visual-spatial test most widely used for vocational guidance to high school students and for admission selection of students to technical programs such as engineering and dentistry. This implies that the use of this visual-spatial test may be having a more negative effect on females' pursuing carrers in technical fields than would be the case if a different visual-spatial test (or several visual-spatial tests) were used more widely.

CONCLUSIONS

The results of this research lead to the following conclusions:

1) It is noted that among junior and senior high school students the appearance and magnitude of sex-related differences in visual-spatial skill are quite variable. While we have always found these differences to favor males when they appear, with many groups and on different tests they frequently did not appear at all. In the psychological and educational literature, references to sex-related differences in visual-spatial skill frequently imply that such differences are both more universal and more substantial than we find them to be. It would be appropriate for such references to be qualified by modifiers such as "On some tests..." or "As is sometimes found...".

2) The skills of "flexibility of closure" (disembedding) and "speed of closure" appear to have little relationship to mathematics achievement. The skills of "spatial orientation" and "visualization" do appear to contribute meaningfully to predicting mathematics achievement. We recommend that researchers interested in examining visual-spatial skills related to mathematics achievement or educators concerned with the development of these skills concentrate their efforts on the latter two skills rather than the former two.

3) There was some indication from the results of this project that the association between visual-spatial skills and mathematics achievement is stronger for males than for females. To the extent that mathematics problems are solvable in different ways, the implication is that females may be less likely to use a visual-spatial approach than males.

The implications of such choices are unknown, nor is it clear that a difference in approach represents a deficit on the part of either sex rather than a preference. Further research on the use of different approaches to mathematics problem-solving appears to be a fruitful area to pursue.

4) The results of the training studies showed that junior high school students can improve their visual-spatial skills with brief training sessions. However, effective materials are not easy to design and cannot be assumed to be effective on the basis of content or face validity alone. It also appears that it may be easier to teach the skill of spatial orientation than the skill of visualization.

5) There was no consistent pattern of sex-related differences in response to training. The hypothesis that students who perform relatively poorly on visual-spatial tests may improve more as a result of training than students who perform well received some support from the results on the Card Rotations Test. (In this case, however, it was the males in the control group who performed somewhat less well than the females.)

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