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ABSTRACT

This study focuses on basic mathematical skills mastered by middle school students. A test designed for the investigation was administered to 400 seventh- and eighth-grade pupils in three middle schools in Houston, Texas. The test, which focuses on fractions, was administered by the regular mathematics teacher during the mathematics class period and all students were given sufficient time to finish all items on the test. Among the results, the data indicated that while the renaming of fractions to higher terms and the renaming of an improper fraction to a mixed numeral were skills mastered by many of the pupils, more students have difficulty renaming a mixed numeral to an improper fraction. (MP)

AN INVESTIGATION OF THE MASTERY OF RATIONAL NUMBER CONCEPTS AND SKILLS BY MIDDLE-SCHOOL STUDENTS

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The back-to-basics movement in mathematics education has stimulated interest in the development of criterion-referenced tests which can reliably discriminate between those students who have mastered a concept or skill from students who have not mastered it. In an earlier report (Sadowski, 1980), an extreme-types latent trait model was described and its use in determining mastery of sets of items was illustrated. Since the extreme-types model is fully described elsewhere (see Dayton and Macready, 1976; Macready and Dayton, 1977) only a brief outline of the assumption of the model and the interpretation of the parameters and test of significance yielded by the data analysiswill be described here.

The Extreme-types Model

The extreme types latent trait model is based on the assumption that, for an n-item domain, students may be placed in one of two discrete categories: 1) masters of the concept that the items in the domain are testing, or 2) non-masters of the same concept. The extreme-types model attempts to fit the domain scores from a group of Ss to this model, while estimating three parameters. The first parameter is theta, the estimated proportion of masters in the domain (i.e., those with a score of n). Two itwm parameters, alpha and beta are estimated for each item in the domain. Alpna is an estimated intrusion parameter, the conditional probability that a student who is a non-master will be able to correctly respond to the item. Beta is the estimated omission parameter, defined as the conditional probability that a student who is a master will not respond correctly to the item. A goodness-of-fit Chi-square statistic is used to determine whether the domain is homogeneous based on the internal consistency of the student responses, i.e., do the data fit the extreme-types model given the estimated item parameter values.

The fit of the model to the data is assessed by the standard Pearson Chi-square goodness-of-fit test. The "expected" or predicted frequencies for each of the score patterns of zeros and ones for the items of a domain are compared to the observed frequencies for each domain. This Pearson Chi-square statistic is evaluated as Chi-square with degrees of freedom equal to 2^k-q-1, where k = the number of items in the domain, and q is the parameters estimated (q = 9 for a four item domain). Values of p greater than .001 are evidence of fit to the model, where p is the right-tail probability of the Chi-square distribution.

The Construction of the Test Domains

The underlying assumption of the extreme-types model is that it is possible to determine homogeneous sets of test items such that students will be able to answer all of the items correctly (if the S has mastered the concept or skill) or the S will not be able to answer any correctly (if mastery has not been achieved). The suggestion has been made (Macready and Merwin, 1973) that both item content and internal consistency of student responses be considered when determining the homogenity of a domain of items, so that the all-or-none mastery assumption is more reasonable.

The number of items in a domain is determined by 1) the estimated proportion of masters in a population, 2) the reliability of the mastery decision that is desired, i.e., how many students are misclassified by the cutoff score, and 3) the "guessing rate" of the items, e.g., multiple-choice items can be correctly answered by non-masters more readily than supply items. In the article by Macready and Dayton, tables are given which show that a mastery score of 3 in a 4-item domain will misclassify about 5% of the masters for test items involving little or no guessing (such as computation problems), when the popusation is equally divided between masters and non-masters.

If four items are to be constructed for each skill or concept, the next task is to determine what item content will produce a homogeneous domain when the internal consistency of student responses is considered. For example, a concept of a fraction as a part of a region should be generalized across circles, rectangles, triangles, etc., but research has shown that students recognize parts of circular regions as models for fractions more frequently than other region models. Likewise, research has shown that addition of fractions with denominators that are composite numbers (e.g., 4,6,8,10) are easier than those addition examples with unlike denominators that are prime numbers. Thus, although the test items in a domain might be selected a priori on a logical basis assuming all the items are mathematically homogeneous, the internal consistency of student responses to the items might not support the homogenity of the domain. When this occurs, the test constructor must make a subjective judgment about the desireability of restructuring the domain, taking into consideration the interpretation of the mastery/non-mastery decisions based on an analysis of items in the domain.

The test of rational number concepts and skills used in this study consisted of 21 domains, each having four items. The placement of items into domains was based on 1) an analysis of the item content as determined by the Wilson Content Taxonomy (1976) ; 2) the skill levels of middle school students and 3) methods used in teaching non-negative rational number concepts and skills.

The test consisted of 6 domains on fraction computation, (2 on addition, 1 on subtraction, 1 on multiplication and 2 on division.). Models for fractions included 1 domain each on the region model, the set model, the number line model and the division interpretation of a fraction. Equivalent fraction domains included a region model, a set model and two domain on renaming to higher and lower terms at the symbolic level only. Comparison of fraction and understanding of terms (improper, proper and mixed numeral) were each covered by one domain.

<u>Method</u>

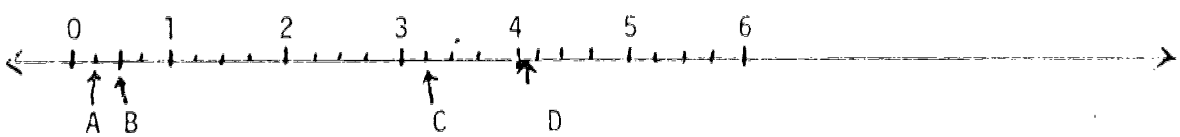
After the test was constructed, it was administered 400 Ss enrolled in middle schools in Houston, Texas. All students in the seventh and eighth grades in three schools were tested. The test was administered by the regular mathematics teacher during the mathematics class period and all students were given sufficient time to finish all items on the test.

Results

The two domain on the region and set mode's for proper fractions were homogeneous with almost all students demonstrating mastery of the concepts. The proportion of master was greater than 90% for each of these domains. In contrast, the number line model for a fraction showed almost equal numbers of masters and non-masters. The item missed by most students was item D as shown below, while item B was correctly answered by many non-masters. The reson for this is obvious.

Number Line Model Domain Items

Write a fraction for each letter:



The domain of items on the division interpretation of a fraction was not homogeneous, indicating that many students have partial knowledge of the concept. From an inspection of the items below, it could be argued that item 4 does not test the division interpretation of a fraction and support for this is found in the 41 students who were only able to answer item 4 correctly.

Division Interpretation of a Fraction Domain Items

Draw a ring around another way to write each of the following:

1) 5 ÷ 4 A) 4/5 B) 4 x 5 C) 5/4 D) 5)‾4

2) 3 is divided by 7 A) $3\overline{)7}$ B) 3/7 C) 7/3 D) 7 ÷ 3

3) the fraction 5/9 A) 9÷5 B) [illegible]/5 C) 5 x 9 D) [illegible]

4) $5\overline{)6}$ A) 6÷5 B) 1 1/6 C) 5/6 D) 5÷6

The values in Table 1 below show that the omission parameter (beta) for Item 4 in the number line model is much larger than for the other three items, while the intrusion parameter (alpha) for item 2 in the same domain indicates that many non-masters are able to do this problem correctly.

Table 1

	Number Line Model		Division Interpretation	
	Theta = .51		Theta = .48	
ITEM	ALPHA	BETA	ALPHA	BETA
1	.04	.01	.10	.15
2	.26	.02	.14	.20
3	.03	.13	.19	.25
4	.03	.39	.35	.31
	Chi-square = 46.4, *df* = 6		Chi-square = 17.2, *df* = 6	

In the division interpretation domain, Item 4 contributes to the lack of homogenity for the domain since it has both a larger omission and intrusion parameter value. Both domains are not homogeneous, although the division domain is very nearly so.

For the domains on equivalent fractions the values in Table 2 show that the item involving ½ is the easiest of the four items (alpha = .53) while the low beta values reflect the fact that masters are not likely to miss these items. This also holds true for the items in the domain for changing improper fractions for mized numerals and vice-versa. Note that the first two items are much easier than the last two items in this domain.

The equivalent fractions domain is homogeneous, the improper to mixed numeral domain is not.

Equivalent Fraction Domain Items

Write the missing numerators or denominators.

A) ½ = /4   B) [illegible]   C) [illegible]   D) 5/6 = 20/

Impro[illegible]-Mixed Numeral Domain Items

A) 11/2 = ____   B) 9/4 = ____   C) 4 1/3 = ____   D) 9 ½ = ____

Table 2

	Equivalent Fractions		Improper Fraction -Mixed Numeral	
	Theta = .81		Theta = .85	
ITEM	ALPHA	BETA	ALPHA	BETA
1	.53	.01	.33	.02
2	.25	.09	.38	.04
3	.21	.01	.13	.02
4	.17	.05	.16	.01
	Chi-square = 11.0, *df* = 6		Chi-square = 90.9, *df* = 6	

The domains on computation with fraction showed some fairly predictable patterns. Two domains on addition and subtraction are shown below with the estimated parameter values in Table 3.

Addition Domain Items   Compute. Put answers in lowest terms.

A) 4/5 + 2/3 =   B) ½ + 3/7 =   C) 7/8 + 5/6 =   D) ½ + 2/3 + 7/8 =

Subtraction Domain Items

A). 5 3/4 - 6/7 =   B) 2 1/5 - 1 3/5 =   C) 3 - ¼ =   D) 4 - 2 3/8=

Table 3

	Addition			Subtraction	
	Theta = .63			Theta = .64	
ITEM	ALPHA	BETA		ALPHA	BETA
1	.00	.16		.05	.26
2	.05	.10		.02	.21
3	.00	.26		.05	.15
4	.00	.31		.08	.18

Chi-square = 12.2, df = 6   Chi-square = 110.2, df = 6

The small alpha values are consistent with item format while the larger beta values for addition items 3 and 4 attest to the difficulty of adding fractions with unlike denominators that are not relatively prime, and the effect of 3 addends is also apparent. The fact that the correct answer requires the student to write the answer in lowest terms also effects the difficulty of the problem. This domain is homogeneous, meaning that the masters can be separated from non-masters with some degree of reliability.

The subtraction domain is not homogeneous, however and for this set of items the difficulty of items 1 and 2 is reflected in the larger beta values. An inspection of the items reveals that these items require different skills than items 3 and 4. In Table 4 below it can be seen that there were 25 Ss who had only items 3 and 4 correct, and 21 with items 2,3, and 4 correct, while 17 were able to do only items 1 and 2 correctly. Thus, although 101 Ss had zero scores and 108 had scores of 4, the domain is not homogeneous.

Table 4

PATTERN	OBS N	PRED N	CHI-SQUARE
0000	101	100.0145	0.0097
1000	6	6.4077	0.0259
0100	3	3.4961	0.0704
1100	17	3.6301	49.2429
0010	6	6.7081	0.0747
1010	1	5.3660	3.5524
0110	5	6.9873	0.5652
1110	13	19.3902	2.1060
0001	9	9.9443	0.0897
1001	6	4.6650	0.3820
0101	3	5.8578	1.3942
1101	5	15.9844	7.5484
0011	25	8.6526	30.8852
1011	11	23.2418	6.4479
0111	21	31.3393	3.4111
1111	108	88.3147	4.3878

The two domains on multiplication and division are shown below. The estimated parameter values are shown in Table 5.

Multiplication Domain Items   Compute. Put answers in lowest terms.

A) 1/8 x ½ =   B) 2/3 x 5/6 =   C) 3/8 x 4/5 =   D) 3/5 x 4/7 =

Division Domain Items

A) 6/7 ÷ 5 =   B) 3/4 ÷ 4 =   C) 2 ÷ 3/5   D) 8 ÷ 2/3 =

Table 5

	Multiplication		Division	
	Theta = .68		Theta = .57	
ITEM	ALPHA	BETA	ALPHA	BETA
1	.03	.06	.03	.05
2	.02	.12	.00	.14
3	.02	.14	.01	.26
4	.05	.07	.01	.22
	Chi-square = 28.7, *df* = 6		Chi-square = 88, *df* = 6	

The lack of homogenity for the multiplication domain was due mainly to a group of 13 Ss who had items 1 and 4 correct only. The extimated parameters could not account for this data, i.e., the predicted N was 3, while the observed N was 13. The division domain is interesting, since the last two items appear to be far more difficult than the first two. The model does not fit the data even though 144 were at zero and 107 were at 4. Only item 3 was missed by 21 Ss while 24 Ss missed items 3 and 4.. The results were similar to the subtraction domain above.

Discussion

The lack of domain homogenity for fraction domains are predictable and consistent with earlier research. The easier fractions such as ½ and ¼ are answered correctly by Ss when the same problem with other fractions is missed. Computation items reflect unlike and like denominator differences, and also denominators that are relatively prime and those that are not. The "reducing" of fractions causes greater difficulty and the need to rename from a whole number is the subtraction of fractions is more difficult when the renamed unit must be added to the fractional part. The region and set models for fractions has been mastered by most of the middle school students while the number line model is only mastered by about half of the students in middle school. The division interpretation of a fraction is one that appears to be mastered by many of the Ss used in this study. The renaming of fractions to higher terms has been mastered as has the renaming of an improper fraction to a mixed numeral. Hoewever, the renaming of a mixed numeral to an improper fraction is not mastered by as many of the middle school students tested.

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