DOCUMENT RESUME

ED 204 151 SE 035 256

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TITLE	Problem Solving Skills of Hispanic College Students.
INSTITUTION	Massachusetts Univ., Amherst. Dept. of Physics and Astronomy.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
PUB DATE	Apr 81
GRANT	NIE-G-79-0094
NOTE	17p.: Paper presented at the Remedial and Developmental Mathematics in Colelge: Issues and Innovations, Conference (CUNY, April 9-11, 1981). Not available in hard copy due to copyright restrictions.
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Algebra: *College Mathematics: College Science: *Concept Formation: Educational Research: Engineering Education: Higher Education: *Hispanic Americans: Language Research: *Learning Problems: Mathematics Education: Performance: Problem Solving: *Verbal Ability
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

Minorities have for some time been underrepresented in the technical fields, such as engineering and computer science. This development is known to be caused by a variety of factors, but the primary purpose of this report is to help identify those factors that adversely affect the cognitive development of the technical bilingual student in terms of which are language related and which are not. The results of two projects aimed at elucidating the interconnection between linguistic and mathematical cognitive development are presented. The first is a study of translational mathematics skills among bilingual Hispanic engineering and science students. In the second study, an attempt is made to determine the dependence of algebraic performance upon verbal context. Data were obtained using both written exams and clinical interview techniques. Results indicated several mathematical misconceptions unique to Hispanic students that may originate in linguistic factors. Possible causes and remedies for these misconceptions are discussed. (MP)

Problem Solving Skills of Hispanic College Students*†

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Abstract

Problem solving strategies employed by some bilingual Hispanic technical college students in solving a variety of mathematics problems will be presented. Data is obtained using both written exams and clinical interview techniques. Results indicate several mathematical misconceptions unique to the Hispanic students that may originate in linguistic factors. Possible causes and remedies for these misconceptions will be discussed.

*Work supported by National Institute of Education Grant NIE-G-79-0094. The contents of this study do not necessarily reflect the position, policy, or endorsement of the National Institute of Education.

† Text of paper presented by the first author at the Remedial and Developmental Mathematics in College: Issues and Innovations, Conference, City University of New York, April 9-11, 1981.

Introduction

At the Bilingual Research Project at the University of Massachusetts, we have been investigating several facets of the problem of underrepresentation of bilingual Hispanics in technical fields such as engineering and computer science. Needless to say, the causes of this situation are a complex mix of cultural, economic, linguistic, environmental, and educational factors. Our efforts, of course, lie in the area of educational research, and we believe that significant progress can be achieved in this area once the specific needs of this bilingual group can be identified. We have been trying, therefore, to identify some of the factors which might adversely affect the cognitive development of the technical bilingual student, and to sort these factors into those that are language related and those that are not. One approach by which we hope to do this is through the study of the interdependence between language skills and mathematical skills. Consequently, a number of our current research projects are aimed at elucidating the interconnection between linguistic and mathematical cognitive development.

Today I will present the results of two of these projects. The first is a study of translational math skills among bilingual Hispanic engineering and science students. Translational math skills are defined for our purposes as the ability to translate verbal statements into mathematical equations, and vice-versa. In the second study we have attempted to determine the dependence of algebraic performance upon verbal context. Here we wish to investigate how the level of mathematical performance can be influenced by the degree of verbal skills necessary to process the problem. Finally, the implications of these results will be discussed, and suggestions will be made regarding the improvement of the technical education of bilingual Hispanics.

Subjects and Research Procedure

The subjects participating in the study were comprised of two groups of college technical students. The bilingual group was composed of 22 freshmen, 11 sophomores, 8 juniors, and 2 seniors, for a total of 43. Most, namely 27, were engineering majors; the remaining were science majors. All but 4 of the Hispanic group were balanced bilinguals; that is, their performance on the Spanish and English language proficiency exams were nearly equivalent.

The norm group consisted of 52 monolinguals, of which 43 were freshmen. 5 were sophomores, and 4 were juniors. Of these, 38 were engineering majors, and the rest of the students were majoring in the sciences. Both groups volunteered and were paid to participate in the study.

All of the mathematics exams used for these studies were designed locally. Specific details of the exams will be discussed later. A Spanish as well as an English version of each exam was composed not as mere translations of one another, but as distinct exams equivalent in content and difficulty. In all cases the exams were graded on the total number of problems correct.

The exams used to determine language proficiency were Test of Reading, Level 5, and Prueba de Lectura, Nivel 5, available from Guidance Testing Associates, and contain three subsections: vocabulary, speed of comprehension, and level of comprehension. The bilingual group was tested in both languages, while the monolinguals were given only the English versions.

After the results of the written exams were known, nine members of the bilingual sample were randomly selected for clinical interviews, during which students were encouraged to "think aloud" while solving selected problems from the exams. The interviews were videotaped and subsequently analyzed for common error patterns.

Results

A. Formula Translation Exams

The Formula Translation exam was designed to measure the student's ability to translate from syntactic to symbolic representations. The exam contains 14 questions, with the second half of the exam providing a redundancy check through problems that are equivalent to those in the first half in difficulty and content. The time allotted for the exam was 12 minutes.

The means and standard deviations for the Formula Translation exam and the subsections of the language proficiency exam are shown in Table 1. The reliability coefficients (Cronbach's α) for these exams are all about .9. Two noteworthy observations can be made from the results shown in Table 1. First, the monolinguals have a decided advantage over the bilingual group in language proficiency. The second observation is that the bilinguals do not perform better in their native language. They actually appear to consistently perform slightly, but not significantly, better in English. This is probably due to the fact that most of their technical training has been in English.

Table 2 contains the Pearson correlation coefficients between the results on the Formula Translation exam and a total language proficiency score obtained by summing the scores on the three subsections of the language exam. It is evident that the results on the Formula Translation exam are more strongly correlated with verbal ability for the bilinguals than for the monolinguals.

A breakdown of the Formula Translation exam results by individual questions is listed in Table 3. We will discuss some of the common error patterns later; for the moment we will consider only the number of correct and incorrect answers. The first row of Table 3 shows the number of correct responses, with the percentage of the sample that responded correctly shown in parentheses. It is clear that the monolinguals outscore the bilinguals almost two to one. A two-by-two Chi-square analysis was performed on these results problem by problem, taking into account the number right versus the number wrong for each question. These results are given in the row labelled χ^2 with significance coefficients shown in the last row. Unanswered questions were omitted from the analysis. It is interesting to note that only two of the questions do not have a significant χ^2, namely questions #10 and #11; these were concrete problems relevant to any person's daily experiences. (One dealt with a clothing store and the other with manipulating money.) A similar χ^2 analysis of the bilingual results across language did not yield a significance greater than .05 for any English-Spanish problem pair.

B. Verbal Context Exams

This pair of exams was designed to investigate how performance in algebra is influenced by the degree of verbal skills necessary to process a problem. The two exams (Terse Word Problems and Verbose Word Problems) are completely equivalent as to level of mathematics, but in one the questions are expressed tersely, while in the other the questions are embellished with irrelevant technical jargon which may prove intimidating to the student. The means and standard deviations of both exams are listed in Table 1. It is clear that increased verbiage causes poorer performance for both the monolingual and bilingual students. Since the Verbose exam requires a greater language facility, it is not surprising that there is more of a difference in the means between the Terse and Verbose versions for the bilinguals (D = 1.23) than for the monolinguals (D = .57).

A paired T-test on the difference scores yields a value of 3.80 for monolinguals ($p<.001$) and the bilingual results in English give a value of T = 5.35 ($p<.001$). These values clearly demonstrate that students tend to err more frequently as the amount of verbal material increases. A T-test on the difference mean between the two groups yields T = 2.40 ($p<.05$). We conclude, then, that although one might think that verbal skills play a secondary role in technical areas of study, language skills remain an important factor in bilingual student performance.

The Pearson correlation coefficients between the verbal score and math exam scores appearing in Table 2 support this interpretation. Although it is again true that the math scores correlate more strongly with verbal score for the bilinguals, the monolingual correlation coefficients cannot be completely trusted. For the monolinguals the reliability coefficients for the Terse and Verbose exams are extremely low, indicating that these exams were too easy for that group. The reliability coefficients for the bilingual group, however, were quite high ($\alpha \cong .75$).

C. Clinical Interviews

Since we are primarily interested in language-dependent sources of error, we wish to identify error patterns evidenced by bilinguals but not by the monolinguals. These error patterns were determined by selecting 9 members of the bilingual sample and 11 members of the monolingual sample for clinical interviews, during which students solved problems selected from the exams. Analysis of the videotapes of these interviews revealed a number of common errors; "common" being defined here to mean that at least two of the students exhibited the error.

Three of the problems we discuss here are from the Formula Translation exam, and a fourth is from a long word problem exam. These problems are listed below.

1. (FT-1)

   Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.

2. (FT-5)

   Write an equation using the variables C and P to represent the following statement: "At a certain restaurant, for every four people who ordered cheesecake, there were five who ordered pie." Let C represent the number of cheesecakes ordered and P the number of pies ordered.

3. (FT-6)

   Write an equation to represent the following statement: " A certain council has 9 more men than women on it." Use M for the number of men and W for the number of women.

4. (LWP)

   "At an engineering conference, 9 meeting rooms each had 28 participants, and there were 7 participants standing in the halls drinking coffee. How many participants were at the conference?"

Only those error patterns unique to the bilinguals will be discussed in detail. We will discount errors made by the bilinguals which are also commonly made by the monolinguals. To place the following remarks about the error patterns of bilinguals in context, we will briefly mention the errors common to both groups.

By far, the most common error made by both groups is the variable reversal error. For example, in the first problem students write 6S = P instead of the correct version, 6P = S. Recent research by Clement, Lochhead, and Monk (1981) has sought to identify the possible reasons why students do this. An important result of this study was that almost none of the mistakes were due to a misinterpretation of the problem. Indeed, our interviews with the monolingual students support this, and the only common error made by the monolinguals was of a variable reversal type.

This variable reversal error seems to derive from one or more of the following reasons:

a) Students translate incorrectly; that is, they sybolically encode "six times as many students" as 6S, without further consideration.

b) Students do not appreciate the distinction between a variable and a label; that is, they do not distinguish between an equation such as 6S = P, where the variables S and P can assume a variety of values, and an equivalence statement such as 3 ft = 1 yd.

c) Even when students do not misunderstand the points above, but just carelessly write the incorrect answer, they have such poor problem solving skills that they fail to check the answer to inserting typical values.

If we now consider Table 4, in which the number of incorrect responses are broken down further into the variable reversal error and other errors, we can see that many of the bilinguals make errors different from the variable reversal type. Although it also appears that they make the variable reversal error more frequently, this does not turn out to be statistically significant.

The interviews with the bilingual students revealed six different types of errors, which can be classified as follows:

Type 1: In response to problem 1, some students wrote 6S = 6P. Students explained this response by stating that the phrase "as many students as professors" implies an equal number of each, or S = P. The "six times" preceeding the statement was interpreted to mean that the equation should be multiplied by 6. Clearly, this is a language error.

Type 2: In response to problem 1, some students wrote 6S + P = T. The rationale of these students was that this expressed the relative numbers of students and professors in relation to the total number of people. When prompted into noting that the question requested a relationship only between S and P, the students subsequently wrote 6S = P. Apparently this error stems from not carefully determining what is being asked.

Type 3: In response to problem 2, some students wrote 4C/5P. When interrogated, students replied that this was the desired relationship between the number of pies sold and the number of cheesecakes sold. Not all students could be prompted into writing an equation. Some felt that a proportional relationship was what was requested.

Type 4: In response to problem 2, some students wrote 4C < 5P. Students justified this answer by asserting that one could never set up an equation for the two variables, explaining that if 4 cheesecakes were bought, 5 pies were; if 8 cheesecakes were bought, 10 pies were, etc. Consequently, all one could assert is that there would always be fewer cheesecakes than pies sold.

Type 5: In response to problem 3, some students wrote 9M = W. This was a frequent non-reversal type error, occurring 5 out of 9 times, and derives from interpreting "9 more men than women" as "9 times more men than women." When prompted, several of the students realized that the relationship involved addition, not multiplication, and subsequently wrote M + 9 = W, a reversal type error. Two students maintained the opinion that the relationship was multiplicative; perhaps this is an incidence of functional fixedness.

Type 6: In response to problem 4, 30% of the bilinguals wrote 245 (that is, 28 x 9 - 7), while only 8% of the monolinguals made this error. This can be explained as the compounding of two misinterpretations. If the word "participants" is interpreted as the number of people actually physically present and listening to talks, as opposed to all of those people registered for the conference, and if one assumes that the seven coffee drinkers came from the nine rooms, then 245 is the logical answer. For the bilinguals, misinterpretation such as this does not occur only in English but also in Spanish.

There are clear and unique differences in the error patterns between the bilingual and monolingual groups. Of the reasoning errors made only by the bilinguals, only two, namely types 3 and 4, seem to derive from conceptual difficulties with mathematics. The other 4 appear to be, to one degree or another, linguistic in nature.

Summary

Our findings to date can be summarized in the following points.

1) The technical skills of bilingual engineering and science majors are more strongly correlated with linguistic skills than are those for monolinguals.

2) The performance of bilinguals in technical matters at which they are competent is strongly influenced by the amount of verbal information they must process.

3) Bilingual technical students do evidence error patterns that are both similar and distinct from those of their monolingual counterparts. Several of these errors appear to be unambiguously of linguistic origin.

Discussion

There are several things I would like to discuss regarding these conclusions. The first is that our finding that language and math skills are more correlated for bilinguals does not imply, directly or indirectly, that one could necessarily improve these students' math ability by teaching them formal language structure. One might just as well conclude that one could improve their verbal skills by teaching them mathematics. Assuming, of course, that one did succeed in teaching them formal language structure, there would perhaps be an improvement in their ability to grasp certain distinctions between math concepts. Improving their potential to learn math, however, is not the same thing as improving their math ability. We think that this correlation means only that there is a strong associative, not causal, relationship between math and verbal skills - at least up to some point in the development of one's cognitive capacities. We hypothesize that, after crossing some linguistic threshold level (Mestre et al., 1981), skills can develop independently of one another. This would account for our results with the monolinguals.

The second point . would like to make is regarding our result that the bilinguals' performance is more dependent upon verbal context than the performance of the monolingual students. One might be tempted to conclude that teaching college-level math and science courses Spanish would improve the situation; however, we find no evidence that this is the case for these students. In fact, we are oppositely inclined; these students already demonstrate a slight superiority in English verbal skills. It follows, then, that at least for students who have proceeded as far in their technical training as the students in our sample, any advantage that might be gained by teaching them in Spanish, which may be a more familiar language, would be outweighed by the disadvantages of having to learn an entirely new technical vocabulary.

Finally, the last comment I wish to make concerns attempts to improve this situation through remedial instruction. Judging from the results of our Formula Translation exam, all students could benefit from being taught problem solving techniques such as defining variables, checking answers, etc. Such instruction could be particularly efficacious for the bilingual student who is generally at a lower level of language proficiency and is, therefore, more easily intimidated by an elaborate verbal context. This last point is the one which is most relevant to the topic of this conference. The fact that the bilinguals we tested were at a significantly lower level of math proficiency than their monolingual counterparts implies that many bilinguals will find themselves among the participants in remedial math courses. It is our opinion, however, that any attempt to improve their math proficiency will not be successful unless certain language factors are also considered. Particular language misconceptions, such as those revealed in this study, must be dispelled, or they will probably remain insurmountable obstacles to the development of math skills.

References

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Table 1

Test Means and Standard Deviations

	Max Score	Bilinguals (n = 43) Spanish	Bilinguals (n = 43) English	Monolinguals (n = 52)
Formula Translation	14	4.7 ± 4.4	5.1 ± 4.2	9.6 ± 4.5
Language Exams				
Vocabulary	45	28.6 ± 9.6	29.3 ± 8.2	35.9 ± 4.2
Speed of Comprehension	30	10.8 ± 4.9	12.0 ± 4.5	17.9 ± 4.4
Level of Comprehension	50	22.4 ± 8.5	25.2 ± 8.6	35.0 ± 6.4
Word Problems				
Terse	10	6.88 ± 2.13	8.14 ± 1.96	9.38 ± 0.77
Verbose	10	4.21 ± 2.44	6.91 ± 2.51	8.81 ± 1.09

Table 2

Pearson Correlation Coefficients

	Total Verbal	Fortran	Terse
Formula Translation	.37** .42** .27**		
Word Problems			
Terse	.69** .33* .23	.50*** .30* .23*	
Verbose	.65*** .53*** .48***	.53*** .47*** .31*	.76*** .81*** .32**

The three entries in each position correspond to bilinguals in Spanish, bilinguals in English, and monolinguals, respectively.

*Significance levels $p < .05$; **$p < .01$; ***$p < .001$

Table 3. Performance on Formula Translation Exams

	Problem Number						
	1	2	3	4	5	6	7
Correct	18(41.9) 14(32.6) 35(67.3)	16(37.2) 19(44.2) 39(75.0)	16(37.2) 17(39.5) 33(63.5)	24(55.8) 22(51.2) 44(84.6)	8(18.6) 8(18.6) 23(44.2)	10(23.3) 17(39.5) 45(86.5)	10(23.3) 6(14 0) 33(63.5)
Incorrect	25(58.1) 29(67.4) 17(32.7)	27(62.8) 24(55.8) 13(25.0)	27(62.8) 26(60.5) 19(36.5)	17(39.6) 19(44.2) 7(13.5)	33(76.8) 33(76.7) 29(55.8)	33(76.8) 25(58.2) 7(13.5)	32(74.4) 36(83.8) 19(36.5)
x^2	11.4	9.4	5.4	11.9	6.3	21.9	23.1
p	.001	.005	.025	.001	.025	.001	.001

	Problem Number						
	1	2	3	4	5	6	7
Correct	15(34.9) 19(44.2) 40(76.9)	14(32.6) 21(48.8) 38(73.1)	18(41.9) 19(44.2) 35(67.3)	31(72.1) 29(67.4) 44(84.6)	8(18.6) 5(11.6) 22(42.3)	10(23.3) 14(32.6) 37(71.2)	7(16.3) 8(18.6) 30(57.7)
Incorrect	27(62.8) 24(55.8) 12(23.1)	27(62.8) 21(48.8) 14(26.9)	24(55.8) 21(48.8) 17(32.7)	10(23.3) 8(18.6) 7(13.4)	31(72.1) 31(72.1) 28(53.9)	29(67.4) 20(46.5) 14(26.9)	34(79.0) 23(53.5) 21(40.4)
x^2	10.7	5.3	3.7	.9	8.8	8.4	8.5
p	.005	.025	N.S.	N.S.	.005	.005	.005

Note: The three entries correspond to bilinguals in Spanish, bilinguals in English, and monolinguals in English, respectively. The number of students is followed in parentheses by the percentage this number constitutes of the total.

Table 4. Breakdown

Problem Number

	1	2	3	4	5	6	7
Correct	18(41.9)	16(37.2)	16(37.2)	24(55.8)	8(18.6)	10(23.3)	10(23.3)
	14(32.6)	19(44.2)	17(39.5)	22(51.2)	8(18.6)	17(39.5)	6(14.0)
	35(67.3)	39(75.0)	33(63.5)	44(84.6)	23(44.2)	45(86.5)	33(63.5)
Variable Reversal Error	20(46.5)	23(53.5)	24(55.8)	11(25.6)	26(60.5)	7(16.3)	23(53.5)
	23(53.5)	20(46.5)	19(44.2)	8(18.6)	28(65.1)	6(14.0)	24(55.8)
	16(30.8)	13(25.0)	19(36.5)	4(7.7)	27(51.9)	4(7.7)	18(34.6)
Other Error	5(11.6)	4(9.3)	3(7.0)	6(14.0)	7(16.3)	26(60.5)	9(20.9)
	6(14.0)	4(9.3)	7(16.3)	11(25.6)	5(11.6)	19(44.2)	12(28.0)
	1(1.9)	0(0)	0(0)	3(5.8)	2(3.9)	3(5.8)	1(1.9)
No Answer	0(0)	0(0)	0(0)	2(4.7)	2(4.7)	0(0)	1(2.3)
	0(0)	0(0)	0(0)	2(4.7)	2(4.7)	1(2.3)	1(2.3)
	0(0)	0(0)	0(0)	1(1.9)	0(0)	0(0)	0(0)

Problem Number

	8	9	10	11	12	13	14
Correct	15(34.9)	14(32.6)	18(41.9)	31(72.1)	8(18.6)	10(23.3)	7(16.3)
	19(44.2)	21(48.8)	19(44.2)	29(67.4)	5(11.6)	14(32.6)	8(18.6)
	40(76.9)	38(73.1)	35(67.3)	44(84.6)	22(42.3)	37(71.2)	30(57.7)
Variable Reversal Error	23(53.5)	19(44.2)	20(46.5)	6(14.0)	26(60.5)	8(18.6)	25(58.1)
	19(44.2)	17(39.5)	17(39.5)	5(11.6)	28(65.1)	5(11.6)	18(41.9)
	11(21.2)	13(25.0)	17(32.7)	6(11.5)	28(53.9)	4(7.7)	19(36.5)
Other Error	4(9.3)	8(18.6)	4(9.3)	4(9.3)	5(11.6)	21(48.8)	9(20.9)
	5(11.6)	4(9.3)	4(9.3)	3(7.0)	3(7.0)	15(34.9)	5(11.6)
	1(1.9)	1(1.9)	0(0)	1(1.9)	0(0)	10(19.2)	2(3.9)
No Answer	1(2.3)	2(4.7)	1(2.3)	2(4.7)	4(9.3)	4(9.3)	2(4.7)
	0(0)	1(2.3)	3(7.0)	6(14.0)	7(16.3)	9(21.0)	12(27.9)
	0(0)	0(0)	0(0)	1(1.9)	2(3.9)	1(1.9)	1(1.9)

Note: The three entries correspond to bilinguals in Spanish, bilinguals in English, and monolinguals in English, respectively. The number of students is followed in parentheses by the percentage this number constitutes of the total.