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ABSTRACT

Results of research on the acquisition of counting skills in children is presented. The material, originally published in the Soviet Journal "Questions of Psychology," was deemed an important document that should be translated and made accessible to English readers. The question of how basic, extended processes are "abbreviated" when they become part of new, more complex ones is addressed. The subjects of this study were children between the ages of four and seven attending day care centers. All children studied were presented with a series of addition tasks. Data from the investigation provide a basis for a general outline of the way the mathematical operation of addition is internalized, and a psychological explanation is offered. More research is called for so that more detailed explanations of the origin of mental actions may be developed. (MP)

Project Paper 81-2

# THE PSYCHOLOGICAL CONDITIONS FOR THE ORIGINATION OF IDEAL ACTIONS





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## Preface

This paper contains the results of research on the acquisition of counting skills. It was originally published in the Soviet journal, Questions of Psychology, 1979, Volume 5, pages 46-54. An initial translation of this manuscript was shared with participants at a Conference on Initial Acquisition of Addition and Subtraction Skills held at the Wingspread Conference Center in Racine, Wisconsin, November 1979. Professor V. V. Davydov was one of the participants in that conference. It is an important paper on the topic of counting and should be accessible to the English readers. We had the original retranslated by James Brown. Then Mary Pulliam and I prepared this edition based on the original copy and the two translations.

Thomas A. Romberg
Editor

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## Research Objectives

Over the years research on the fundamental psychological problem of the way in which a succession of perceptual and mental processes is transformed into a single representative act has often led to the related question of how basic, extended processes are abbreviated when they become part of new, more complex ones. Among the authors who discuss this transformation are Rubinstein, 1959; Shekhter, 1967; Shevarev, 1959; Sokolov, 1968; E. Sokolov, 1960; Zaporozhets, Venger, Zinchenko, and Ruzskaya, 1967; and Zinchenko, 1958.

In studying the stage-by-stage formation of mental actions, psychologists who have given special attention to the phenomenon of abbreviation include Galperin and Kobylnitskaya, 1974; Nechaev and Podolsky, 1975; Nepomnyashchaya, 1956; Obukhova, 1972; Podolsky, 1978; and Talyzina, 1975. Galperin, characterizing the experimental research in this area and discussing the psychological nature of abbreviated actions, writes,

> Abbreviated actions are not simply eliminated but are transferred to the position of actions which are considered as if they were already done and are now "implicit." This imparts a particular aspect to the action .... Both in the perceptual and mental plane the concrete action on the object is not performed but merely "implied" outside of what is actually done, while the action really performed is generally represented ... by a movement which goes directly from the starting point to the final point without regard to the objective relations of the problem, as if to demonstrate by this its distinctness from the object-action and its disregard of the latter's objective logic and its difficulties. (Galperin, 1966, p. 253-254).

To make the following analysis of abbreviation as clear as possible, we will begin by discussing some of its basic ideas. First of all, we must differentiate the concrete action on an object and the objective logic of the problem being considered, from the abbreviated act, which is a gesture contradicting this logic and incompatible with it. Beyond these abbreviated acts lie still other acts which are merely implied, inferred to be already done. These implicit operations take the form of mental gestures or ideas, whose logic and potential again differ from those in the original concrete action. The presence of implicit operations like these is evidence of man's faculty for rationally solving problems (see for instance Galperin, p. 274).

Psychologists have now accumulated considerable data making possible a comprehensive description of the patterns of these implicit (mental) abbreviations, as distinct from their concrete prototypes. Nevertheless, we think there are many unsolved questions about the inner conditions and laws governing the origins of these abbreviations. The following questions are central:

1. By what logico-psychological method can we determine which concrete act as best correspond to the objective relationships in the problem being solved?

2. What criteria can we use to distinguish concrete actions, which are in accordance with the objective logic of a problem, from the various levels of abbreviated actions, whose logic apparently contradicts objectivity? It is extremely important to determine the essence and major characteristics of both the objective logic of concrete actions and of the other different types of "logics" which correspond to abbreviated mental acts.

3. By what psychological process are actual operations on concrete objects transformed into implied, mental operations?

4. What is the psychological and psychophysiological form of the internalized movements that represent abbreviated actions?

5. How can we study this process of internal representation and abbreviation, in the light of our answers to the previous questions?

The elaboration of all these interrelated problems will, in our view, contribute to the development of a modern theory of internalization. As Galperin noted, "This involves the immediate task of studying the laws of abbreviation as such, as well as investigating the truly enormous possibilities that are opened to thinking by abbreviated ideal actions." (p. 275).

However, developing such a theory is hindered by the serious difficulty of determining the actual content of an abbreviated or internalized action. We can describe people's actions in some detail, but their overall logic must be determined not so much by external evidence (such as observations of performance in the absence of manipulatives, or records of the subjects talking to themselves) as by the way the initial problem is solved. We must determine the structure of the solution, not simply observe a person's acts performed during the solution process.

Data on the psychology of perception and recognition shows that the abbreviation of perceptual actions is made possible by a person's ability to use new means to obtain more information about a problem. However, this data is not accompanied by an analysis of the nature of these new means, which vary according to the problem a person is attempting to solve, nor by an analysis of the general logic and physical characteristics of the problems themselves (for example, the specific geometric features of an object provided as part of a measurement problem in mathematics).

In other words, not only must we characterize the basic levels and forms of performance (the material used, talking out loud, audible muttering to oneself, internal dialogue), but also, we must show how these forms are used, and most important, how the transition from one form of performance to another takes place. What is needed, essentially, is a special method by which we can analyze the genesis and transformation of the meaningful acts people perform in solving a problem, and the way these acts change form. No such method is yet available, but without it no theory of internalization can be developed. It is noteworthy that the same line of research is pursued in Piagetian theory. However, the Piagetian approach emphasizes the importance of the logical development of internalized actions.

The general method we wish to construct demands a thorough study of the internalization of concrete actions in a particular content area (in mathematics, linguistics, history, etc.). Although some specialists believe this approach might distract psychologists from working out an overall theory of internalization, the results of our investigation of the formation of certain mathematical operations (measuring, counting, multiplication, etc; see Davydov, 1957, 1966, 1969) show that no general theory can be created without such preparatory work in a content area.

Relatively early in our research on the stage-by-stage formation of mental actions, using the example of mathematical addition, we determined that appropriate mental actions would allow a person to operate with objects irrespective of the way the original problem stated they were to be manipulated or changed (Davydov, 1959). In addition, we showed that a person's abbreviated mental action can be demonstrated by presenting that person with manipulatives to work with. Under these conditions, actions will develop quite differently from noninternalized concrete actions under the same circumstances (Davydov, 1957; Nepomnyashchaya, 1956).

In the context of these findings we decided to return, after a long interruption, to studying the internalization of addition procedures in preschool children. This involved rechecking our earlier observations, which we expanded and elaborated using videotape recordings to collect new material. The results of the earlier and later experiments are described below.*

Descriptions of our observations of the development of addition in preschool children will help demonstrate the relationship between the abbreviation of action and our inferences as to its mental form. The data will also help describe some of the psychological conditions that permit the development of abbreviated mental actions.

Experimental Results

Our subjects were children age 4-7 attending day care centers. We presented them with a series of addition tasks. All subjects were able to count forward from one and backward from any given number, and could name the preceding and succeeding numbers for a cue number. They also had to be able to identify written numerals and to understand the meaning of addition. We had them show their understanding of addition by obtaining, in one way or another, the sum of addends presented with physical counters. They were permitted to obtain the sums either by bringing the objects together and joining them, or without doing this.

* In this article we use data obtained by L.A. Khristich and V. Ja. Dobrokhotova, students working on their graduate theses at the Moscow State Extramural Pedagogical Institute.

Initially, we gave the children the first addend in each task verbally, but through a series of steps we gradually substituted objects for the verbal expression. The second addend was always expressed with objects, except in the first purely verbal task, and was not subsequently designated by a number word. All tasks except the first one required adding on by single units, as answering with a memorized formula was not possible.

The following tasks were given:

1. Purely verbal: "Add three to four. How many will this be altogether?"

2. Verbal, but the number name for the first addend is accompanied by a gesture in the direction of a definite point on the table: "Here are five (gesture). Add this many (point to objects without naming how many there are) How many are there altogether?"

3. The first addend is presented as a numeral: "Here are so many (the appropriate number is written). Add so many (present the objects). How many are there altogether?"

4. The first addend is given as a group of objects hidden in a box: "Here are six (indicate the box). Add so many (present objects). How many are there altogether?"

5. The first addend is presented as a designated set: "Here are four (present four objects). Add so many (present objects). How many are there altogether?"

For all observations, we recorded the following:

1. The child's method of performing addition (counting all, counting on);
2. The form of presentation of the addends (as described above);
3. The character of the hand and finger movements used by the child in

counting (tapping a finger on the table, feeling the objects hidden in the box, making a uniform, smooth hand movement along a line of objects, etc.);

4. Peculiarities in the way counting words are uttered (rapid and even, with hesitation and repetition, accentuated or drawn out).

Our results showed three major levels in the performance of addition, each of which shows a characteristic connection between the form in which the quantity is expressed and the child's counting performance.

At the first level, the object sets are added by counting all. The child touches each object in counting; in some children, a tendency to suit the movement of the hand to the shape of the objects (sticks, circles) was observed. This counting all procedure is imposed by the fact the quantity is expressed with objects. (From the point of view of the interconnection between the way the quantity is expressed and the child's addition performance, this level is preceded by two other stages: the acceptance by the child of the very task of addition, and the use of a counting procedure in which all the object addends are first brought together physically. This paper will not consider these two stages, however.) In our 1977-79 data, this level was recorded in 30 of 220 children.

At the second level, the task is performed without objects, but with a verbal counting all of addends one-by-one, starting with one. Forty-four of our subjects performed at the second level. These subjects performed the task "to 4 add 2," for example, by saying "One, two, three, four -- five, six. It's six." Their counting was usually combined with movements of hands, fingers, head, or body. These movements had several forms: a sweeping motion of the finger over the table with the application of pressure; rapping at one point on the table, or making a series of nods which varied from distinct to hardly discernible. In tasks 2-4 as the first addend was expressed more and more in the form of a set of objects these children's movements became more extended. When the movements were forbidden, the children's counting was disturbed.

In order to learn more about the function of these movements, we carried out a special experiment. We tried to trace the origin of these counting gestures to the point at which children pass from addition by counting present objects to addition in which the first addend is not represented by objects. Seven children who were able to add only if objects were physically present were given tasks in which the objects were hidden in a box. The experimenter began by stressing the number name of the first addend and opening the box to show the children the objects inside. The experimenter then closed the box and reformulated the task, saying, "Here in the box are five. You add two." After practicing variations of this task the children began to draw a finger over the box while uttering the successive number words. They then continued to count the second addend, which was available as a set of objects. The experimenter next gave these children tasks with an empty box, a written number, and a gesture instead of counters. All the children succeeded in learning to perform this series of tasks.

Our observations of these children's behavior led us to assume that the movement of the finger over the box served them as a means for reconstructing the initially hidden and then absent object sets, while still using them as addition counters. When they were required to operate with a quantity not physically available, the children started to use the same movements they had used with real objects, which they had touched with a finger. Now each movement (pressing, pushing, tapping) was itself a substitute or representation of the missing counter. The counter now had the role only of an object of action.

With the transition from action with actual objects to action with absent objects, the counting movements themselves changed from slow, extended pressures, shifting the position of the finger each time as if moving over a collection of items, to slight, quick tapping at one point on the table. In the transition to a purely verbal problem the movements continued to diminish. They did not disappear entirely, but were replaced by the subjects' whispering and talking to themselves, indicated by laryngeal movement. All of these movements expressed the ways in which the physical addition was originally performed, that is, by enumeration by single units either aloud, in a whisper, or to oneself.

At the third level of addition performance, the child adds by counting on the elements of the second addend to the first addend taken as a whole. The children at this level (50 subjects) carried out the addition task in the following way. They made a characteristic continuous, smooth hand movement over all the objects for the first addend while uttering the given number word in a drawn-out manner. They then added the elements of the second addend one by one ("six -- seven, eight"). This uniform, smooth movement permits the children to operate with all the elements of the addend, but without singling out each separate unit. This method of addition replaces the one-by-one method characteristic of children who still operate entirely at a material level, and is evidence that the child is capable of the actual mental action of addition.

In our investigations we discovered another group of students who also solved addition problems by counting on objects, but who did not use the continouous hand movement along the elements of the first addend. Instead, these children pointed their finger at only one element of the set, uttered the number word (without exmphasis), and then counted on the second addend ("four -- five, six"). Then the experimenter asked, "Is it really four? You have just pointed to one, and we're trying to add two to four." Some of the children immediately answered the question by a continuous hand gesture along all the objects of the first addend, pronounced the number word in a drawn-out manner, and then added the second addend. Thirty subjects responded in this way; another 40, when confronted with the same question, shifted their finger to the next object of the first addend and again named the corresponding number word. When the experimenter repeated the question, they shifted their finger to the third object of this addend and said the same word, and so on. If discouraged from using this mode of operation, the children either refused to perform the task, or began to count the elements of the addend one by one.

Outside of the situation of addition, when the experimenter asked the children to indicate four, they always related the number word to the entire group of objects. It was only when faced with an addition problem that these children would point to one element of the first addend and designate it as "four."

A kind of synthesis of counting all and counting on was observed in some children. These children would touch one object of the first set and designate it by a number word corresponding to the quantity of objects constituting the first addend. Instead of passing to the second addend, however, they would continue to enumerate the rest of the elements of the first before starting on the second. Thus for the task "to four add three" these children would respond "four -- five, six seven -- eight, nine, ten."

At first we thought the children in this group possessed counting on as a way of addition. Further investigation showed, however, that their counting on consisted only of counting on further from a given number word. This was evidenced by their pronunciation of number words in this "formal" counting-on procedure, which was characterized by the intonation and accentuation usually associated with simple counting. In actual counting on the number word for the first addend receives special emphasis. Thus, we designated this formal counting on as one variety of "imaginary" counting on.

Other types of imaginary counting on were observed as well. In an investigation of the addition of abstract sets by counting on, we discovered a group of 17 children who, when given numerals to add, performed the task by counting on, and did all the other tasks (with gestures, uncovered objects, etc.) by counting all. We gave these children a special task in which a numeral was alternated with a set of physically present objects to add. As soon as the children started to count the objects of the first addend, the experimenter wrote the appropriate numeral. The children at once rapidly and abruptly changed their method to counting on. If the numeral was taken away, they returned to counting all the objects. These subjects too were unable to correlate the number word with the total object addend. We designated this form of imaginary counting "counting onward."

In order to find out how stable these children's methods of addition were, we gave them two tasks. These tasks were presented to those who used imaginary counting on and to those who were capable of actual counting on. In one task, the first addend was given without objects and designated by the word "million." In the other task, synthetic words were used as number words: ar for one, ur for two, ir for three, and so on.

We found that none of the children capable of actual counting o changed their way of performing addition in either new task. They still shifted their hand in a continuous smooth move ent along the table (in the task with "million") or along the line of objects for the first addend (in the task with synthetic words). They continued to draw out the first number word before passing to the second addend, as well.

The children who used imaginary counting on also did not change their method in the task with the "million." They touched the table with their finger quickly as in simple counting, uttered the number word "million" and continued counting the second addend, saying "a million and one, a million and two, a million and three," etc. However, in the task with artificial words these children formed two groups. The first group (18 children) solved the task only by counting all. The second group (22 children) continued to count on, but still in the imaginary form. These children touched one of the objects of the first addend, uttered the number word (quickly and evenly as in simple counting) and continued with the second addend. If the experimenter asked, "Is this really ir?" You have just pointed to ar, and we are supposed to add so many to ir," the children indicated another object of the same addend, then a third one, or they turned to counting all.

In the next stage of our investigation, we studied the general intellectual development of children who possessed actual and imaginary counting on. We assumed that the defects of imaginary counting on as a mathematical procedure are difficult to discover by studying only the children's behavior in addition and subtraction tasks. As a matter of fact, by performing addition in the form of imaginary counting on, the child can arrive at the same result as in actual counting on, since imaginary counting on does not always distort the formal result of addition. Therefore, we decided to correlate the properties of imaginary counting on and of actual counting on with some general characteristics of the children's intellectual development. We proceeded from the assumption that children performing actual counting on must have a higher level of intellectual development.

We believe that an essential index of high intellectual development is the ability of reflection or introspection -- the special faculty of considering the basis of one's own ways of acting.* Introspection as an element of thinking is intrinsically associated with the ability to resolve contradictions. Consequently, we assumed that a child who has already found a way of reconciling a certain contradiction possesses a definite level of introspection. Without introspection a child would probably be unable to resolve any contradiction whatever.

Our children were faced with a contradiction when the experimenter asked, "Is this really four? You have just pointed to one!" The contradiction arose because the child related a certain number word both to the entire group and to one of its elements. As we described, some of these children did not need a leading question to resolve this contradiction, for they had themselves already discovered a resolution in the use of their continuous hand movement over

* A detailed psychological investigation of introspection as a basic component of thinking was carried out by Zak, 1978; Maksimov, 1979; and Nosatov, 1978.

all the elements of the first addend and their special emphasized pronunciation of the number word designating the entire set.

For some of the children who related the number word to only one element of the corresponding set and did not include the whole set by use of a gesture, the leading question helped to elicit the fact that they too actually possessed a way of reconciling the contradiction. However, those children who could not deal with the contradiction typically answered the experimenter's question by indicating once more only one element of the first addend, or refused to continue the task. Imaginary counting on was characteristic of this group of children.

To verify our assumptions we carried out a special investigation to determine the presence of introspection in the children who performed imaginary and actual counting on and those who solved the tasks with abstract sets only by counting all. We used the method developed by Zak (1978), modified with respect to the age of the subjects. The results are given in Table 1.

A high percentage of children solving addition problems by actual counting on were found to possess introspection. The percentage of children possessing introspection was very low among those who used imaginary counting on and counting all. Thus, actual counting on was found to be in some way associated with a higher level of intellectual development, indicated by the faculty of introspection.

The next step in our research was to study the conditions under which actual counting on is formed. We began by teaching such counting on to five children who were already able to perform stable verbal counting all (the second level of addition),

Table 1

Possession of Introspection at Various

Levels of Addition Ability

| | | Subjects possessing introspection | |
|---|---|---|---|
| Mode of addition | N | n | percent |
| Actual counting on | 80 | 72 | 90 |
| Imaginary counting on | 40 | 3 | 8 |
| Counting all | 44 | 6 | 14 |

These children were given a task with a gesture or written number. When they started to count the first addend, they were interrupted by a series of questions and prompts from the experimenter: "How many are here? (pointing at the numeral or at a place on the table already indicated by a gesture) Four? Then to four you have to add two." After several such prompts the children began to use counting on. They touched the numeral or the place indicated on the table with their finger, uttered the number word designating the first addend, and counted on the elements of the second addend. All of the other tasks in which the first addend was presented with objects absent were performed in the same way.

Then we gave this group of children a task with objects present, which they again carried out solely by counting all. When this method of addition was forbidden by the experimenter, some of the children turned back to counting on, relating the number word for the first addend to one element of that group. However, when posed with a contradiction of the type, "Is it really four? You have just pointed to one," they could not resolve it. The rest of the children simply refused to perform the task at all. Thus the counting on we had taught the children in this way turned out to be the imaginary type.

We successfully formed actual counting on procedures in 20 children by using objects-present tasks. These children had already mastered the addition of both present and abstract sets by counting all. We taught them counting on by the following experimental procedure.

When the child was counting the first addend, the experimenter asked "How many are there?" just as the child was about to point to the last element of the first addend, which was somewhat removed from the other objects. In this case the continuation of the movement along the line of objects coincided with the child's uttering of the number word designating the entire group of the first addend. After this the children continued counting the second addend. Following two or three such tasks, the experimenter asked "How many are there?" just as the child reached the <u>second</u> to last element, then the <u>third</u> to last, and so forth. The child's answer had to be coordinated with the continuing movement of the hand along the row of counters.

Gradually, counting on was formed in all 20 children. Characteristic of their counting on was the emphasized or drawn out pronunciation of the number word for the first addend, combined with a hand movement. The counting on ability formed in this way was easily applied to the other tasks using an empty box, a numeral, or a hand gesture. With the transition to objects-absent tasks the hand movement gradually diminished and the emphasis on the counting word became less and less marked.

Discussion

Our experimental results permit us to set forth a general outline of the way the mathematical operation of addition is internalized, and to offer a psychological explanation. The addition task is first presented to a child by an adult who sets out two object sets (addends) and asks the child to determine the number that refers to the total group (the sum). The child's own action consists of physically bringing together or joining the two separate sets. The adult indicates to the child how to do this. However, during this joining, the two sets lose their individual numerical identity and the child can obtain the result of the addition only by counting one by one all the elements of the sum. This constitutes the initial physical operation of addition. (Figure 1, a)

An important stage in internalizing the operation of addition is the child's mastery of the process without having to bring the two addend sets together. An interesting situation arises here. The child has to find the sum when the number words corresponding to the addends are already known. With this information, the child will be able to count on to the first addend the elements of the second one. (In principle the sum could even be determined with the aid of a table.) However, this does not happen. In order to obtain the sum, the child still counts one by one the elements of the first addend, which seemingly is already known. (Figure 1, b)

This is paradoxical since the child, on the one hand, has already mastered counting from any given number and, on the other hand, can correctly correlate a number word with an entire set outside the situation of addition. However, in adding, neither "mastery" or "knowledge" will assure the acceptance of a

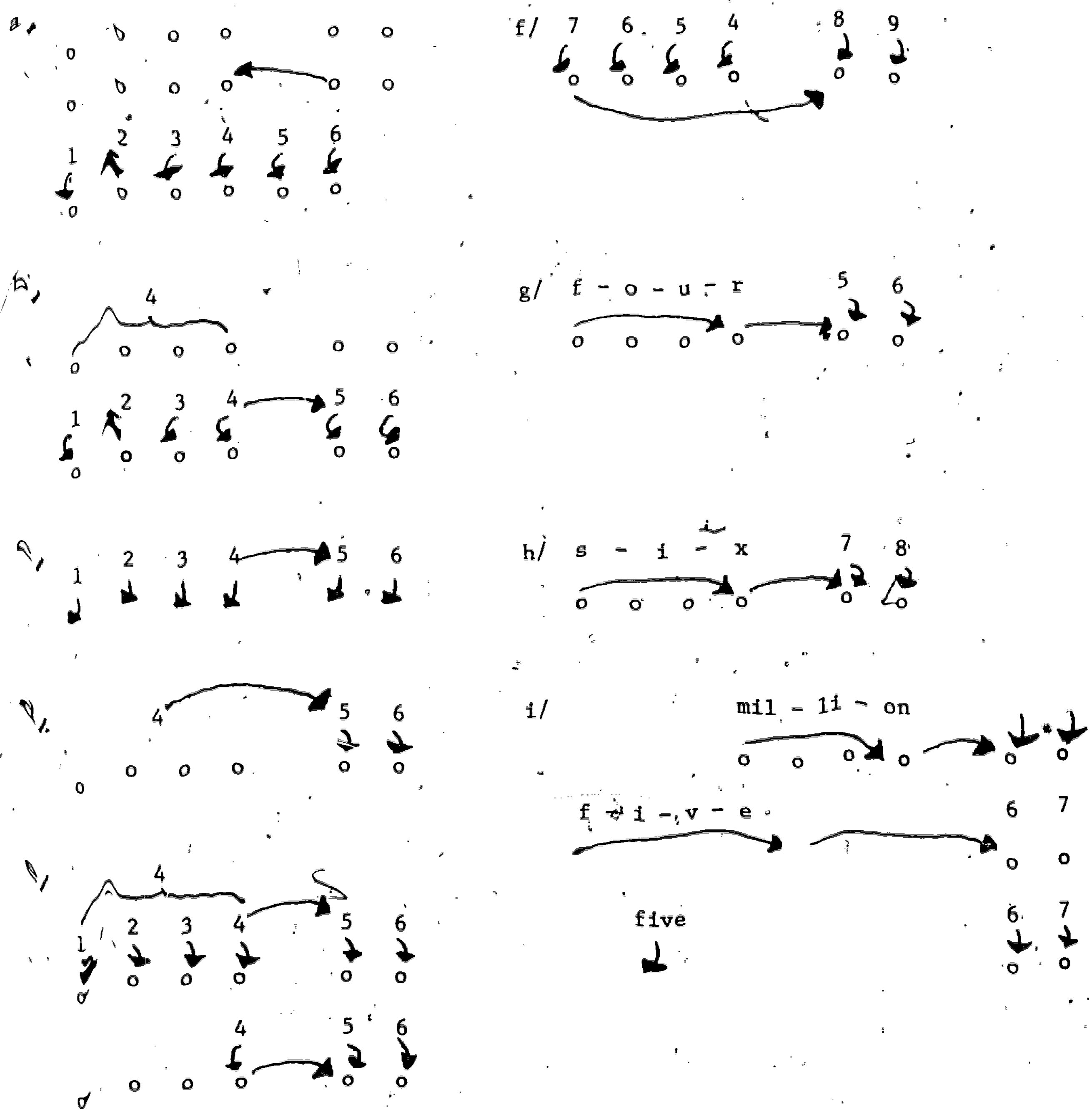


Figure 1. A general scheme showing change in the ways of adding numbers.

certain set of objects as an integral addend. Is this because the child is able to correlate number words only with object groups that are actually present? Even with objects-absent, though, children still perform addition by counting on, in this case by the use of a reduced hand movement which "restores" the objects to be counted.

Some children do consider the possibility of counting on when given only a verbal addend. These children use their mastery of counting to immediately count "onward" from a given number word. While this looks like counting on, when these children are asked to correlate the number word with the set of objects they turn back to counting all or relate the number word to only one element of the addend, not the whole group of items. (Figure 1, d)

What psychological explanation can be given for the paradoxical enumeration of an addend which is already known to a child, and for the fact of imaginary counting on? We think the explanation lies in the content upon which the child is operating. Counting on involves counting from a given number based on the ordinal characteristic of that number. In the process of addition, the immediate physical representation of the addends is useless and irrelevant to the child until the sets are brought together. Nonetheless, the child also has to take into account the number word which indicates a value irrespective of the object set. This is the cardinal aspect of the word. The number word also has an ordinal value, which is manifested in its relation to the total object set. A child is very early taught to consider this aspect, but when the number of objects in the first addend actually coincides with the number word that defines it, the child has a feeling of inconsistency, of contradiction - for there is indeed a contradiction between the cardinal and ordinal aspects of the number.*

For this reason, a child may ignore the ordinal aspect of the number word and thus be obliged to recount the already determined addend. On the other hand, the child may ignore the cardinal meaning of the number and perform the counting of the elements of the second addend, correlating the number name for the first addend to only one of its elements. (This becomes obvious when the child is questioned, "Is this really four? You have just pointed to one!") Still other children synthesize these two procedures, apparently trying to consider both the ordinal and cardinal aspects of the number. They designate one element of the addend by the given number word and then proceed to count the entire first addend beginning with this number. (For example, counting 4 plus 2 as "Four, five, six, seven; eight, nine - it's nine.") (Figure 1, f)

In our opinion, the contradiction children meet in performing their addition procedures can be resolved only when they change the way they operate with the objects-present addend, while continuing to preserve the hand gesture along the objects representing that addend (considering the cardinal aspect). In this new way of adding, the child no longer stops at each element of the first addend, but immediately names the result with a number word and passes to the second addend (implying consideration of the cardinal aspect). The child's characteristic hand gesture and emphasized pronunciation satisfy the requirement of the simultaneous consideration of both aspects of the number. The smooth gesture reveals the real unity of the aspects in the child's own activity. (Figure 1, g) The child is able to physically characterize one group of objects by <u>different</u>

* This contradiction between the cardinal and ordinal aspects of the number and the psychological difficulties it presents to the child have been described by Piaget, 1969 (pp. 386-390).

numbers, that is by <u>any</u> number, since when the number is given beforehand there is no need to ascertain all the elements of the group. Therefore when the same group of objects is designated by the experimenter by different numbers (up to a "million" - little children of course have only a vague notion of what this is), the child is able to treat that group as a representation of any possible number by means of the smooth hand gesture which designates the number given. (Figure 1, h)

For these children, it is not the number which represents the corresponding set of objects, but instead a certain set which can represent any given number. We think it is this inversion of the relationship between the object group and the number that gives us a key to the way children discover by themselves the unity and coincidence of the cardinal and ordinal aspects of the number, and the way children use this discovery in the form of a continuous hand gesture along the objects of the first addend. We can say that this very movement gives rise to the "mental plan" by which actual addition is performed, because only in this movement does the object group begin to appear to the child as a unit, an <u>addend</u>. At this point the child becomes aware of the action of addition, as distinct from the action of counting which previously sufficed to solve addition problems. The addition task may now be performed by an action that is adequate for it and this action has to express the relationship between the object group and the number. In practical addition, numbers serve as representatives of definite groups of objects, whereas in mental addition the group of objects serves to symbolize the number. The transition from counting all to counting on, based on the smooth movement of the hand along an object group, takes place when the child grasps the relationship between a group of obj..cts and a number which is characteristic of mental adding.

This explanation of the mys..ry of counting on implies that its formation must be based on physically present addends, whether children spontaneously and independently discover it for themselves or whether they must be specially taught. It would be impossible to change the functional relationship between the object set and number and obtain a transition to a new way of understanding this relationship if the object set were absent. This is why when a given number word is detached from an object group (for example, when the first addend is given as a numeral) and this new way of understanding has not yet developed, we observe the phenomenon of imaginary counting on. The action of counting is not yet transformed into the mental process of addition. When the child makes this transition, the characteristic hand movement diminishes and the value of the number for the first addend become irrelevant, because any such movement correlated with any number now can express the new relationship between the object set and the number. Since the child no longer needs object ad... 's to perform addition, the smooth movement is soon reduced and the ac... .. pronunciation of the number word symbolizes its use as an addend. Such s. bolism is an important element of internalization. (Figure 1, i)

We arrived at this explanation by observing the conditions and causes of change i.. he form and content of an action as it was internalized. How then can we d..cribe mental action proper, as distinct from a corresponding physical action? The mental action is based upon a change in the method of performance based on a new relationship between the physical and the verbal (symbolic) approaches to an object. Without a thorough understanding of this psychological reality, no adequate method can be developed for studying the processes of internalization.

Our investigation showed that children performing imaginary counting on are devoid of introspection and that introspection is present in those who can perform true counting on. This is understandable because the sense of contradiction and the search for ways to reconcile it presuppose that people have the ability to examine the bases for their own actions and to analyze the conditions for effective performance; that is, that people are endowed with introspection. Introspection thus plays an important psychological rôle as children come to accept true counting on as a way of reconciling the contradiction between the cardinal and ordinal aspects of numbers they encounter in addition.

A detailed analysis of the origin of counting on is justified if it contributes to the solution of basic psychological issues of how abbreviated mental actions develop. How does our research help elucidate these issues? Because our results were obtained from actual experimental data, they provide a useful empirical basis for discussing the theoretical questions about internalization we set forth in the first pages of this paper.

First of all it is important to discriminate between the logic of material actions and the logic of ideal actions. We have demonstrated their different psychological characteristics and possibilities using the example of a mathematical operation. The creation of an integrated theory of these logics must be grounded in the principles of dialectical logic which, according to Ilyenkov, is "not only a general scheme for subjective activity ... but also a general scheme for change in any natural and socio-historical material in which this activity is performed and whose objective requirements always bind it." (p. 5)*

In the study of the origin of internalized, mental actions the logico-psychological method can be used only structurally; we must constantly draw on data from experiments set up to investigate the actual behavior of subjects faced with various real-world tasks. Thus, without information about the way children perform addition problems, it would be impossible to really understand the way their performance changes and develops.

Our description of the formation of mental mathematical operations involves not only the origin of particular actions but also the initiation of thinking as an "ideal component of the real activity of social man." (Ilyenkov, p. 5)**

The transition of physical acts to ideas is closely connected with the use of symbols. In our example of addition this is clearly visible. Only by transforming the object set into a symbol for any number was it possible for a child to reconcile the contradiction between the cardinal and ordinal aspects of number. It was this transformation that brought about the mental action of addition. The acceptance of a symbol opens to a child the immense possibilities of using numbers in the logic of mental actions.

As brilliantly expressed by Losev: "...the essence of a symbol is never a thing's or a reality's givenness, but its assignedness; not the thing itself or reality itself as something engendered, but their engendering principle; not something's 'proposedness,' but its 'supposedness'" (1979, p. 12). And further, "As concerns the symbol of a thing, it contains in a hidden form all of the thing's possible manifestations"(p. 17).

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* A significant contribution concerning the problems of the content-genetic method in logic and psychology has been made in the investigations of Shchedrovitsky (1964).

** This idea of the initiation of thinking was thoroughly developed in psychology by Leontiev (1977), Rubinstein (1959), and others.

The logic of ideas as actions with symbols concerns the notion that an idea is "none other than the form of a thing but outside the thing" (Ilyenkov, p. 189). Viewed from this standpoint, mental actions exists only in an ever recurring cycle as "thing-deed-word-deed-thing" (p. 193).

When children begin to use a symbol in their addition of numbers, we see that these children can feel the contradiction in their own actions and find by themselves an adequate way of reconciling it. This fact is evidence of the profoundly dialectical essence of children's thinking.

In our example, mental addition based on a physical symbol is realized in the form of an actual act of motion - here, a smooth hand movement and a drawn out pronunciation of a number word. The movement, in an abbreviated, reduced form, then itself becomes the symbol of a number. The study of the beginning and transformation of this movement as a basic component of mental action may become important to psychologists in further investigations of the process of internalization. (In particular, an interesting question is the eventual fate of this reduced movement.)

The transformation of symbolic gestures is of principal importance. In our example, the hand movements together with characteristic speech formed a symbol; later only an abbreviated articulation remained to denote the number. These transformations must be kept in mind when dealing with already formed symbols and their role in mental activity. "The spontaneously ideal," writes Ilyenkov, "is realized in the symbol and through the symbol ... through the external, sensorially perceptible visible or audible body of the word"(p. 193).

This analysis of our investigation of the origin of mental actions merely hints at the way other similar logico-psychological problems may be solved.

Considerable data is needed in order to arrive at detailed explanations; it is our task to collect this material using general logico-psychological theories about the nature of ideas.

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