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THE DEVELOPMENT OF MENTAL ADDITION

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Students in grades 1, 3, 4, 5, 6, and college were timed as they produced the answers to simple addition problems or verified a given problem as true or false. First graders clearly relied on a counting process for their performance, as advanced by the Groen and Parkman "min" (for minimum addend) model. Third grade appears to be a transitional age with respect to addition processing; from fourth grade on, however, performance in both tasks yields evidence of a memory retrieval process as the major vehicle of mental addition. The results are discussed in terms of the Ashcraft and Battaglia (1978) "network-retrieval" model, a model which asserts that simple arithmetic facts are stored in a network representation, and are accessed in a spreading activation-like fashion.

THE DEVELOPMENT OF MENTAL ADDITION

The topic of our presentation today is "mental addition", quite simply how numbers are added mentally, without the aid of pencil and paper. This overwhelmingly common ability is of course crucially important to the mathematics curriculum in school, serving as a foundation for all of higher mathematics. From our perspective, mental addition is equally important to an understanding of human cognition, and more specifically to the issues of mental structures and processes and their development.

Until recently, the most widely accepted view of children's mental addition processes was a model advanced in 1972 by Groen and Parkman. Their model asserted that mental addition is a "reconstructive" memory process, that is that the sum of two numbers was literally computed or reconstructed whenever it was needed. The source of this conclusion was an experiment with first graders, in which reaction time differences to simple addition problems were best predicted by the smaller of the to-be-added numbers. In the jargon of this area, the smaller number of a "basic fact" problem, the 3 in 5+3 for example, is called the minimum addend, or simply the "min". The Groen and Parkman model says that some internal counter is first set to the larger value, and then is incremented a number of times equal to the minimum addend. Since the time to set the counter was assumed to be constant, RT in the model should be a linear function of the min -- the RT should vary directly with the number of increments added in the computation. Groen and Parkman found this min factor to account for nearly 80% of the variance in RT in their sample of first graders.

There is clearly some merit to the reconstructive counting approach that Groen and Parkman advocated. Their model fit their first graders' data very well, and a great deal of other evidence shows how heavily such small children rely on counting as they add simple numbers (Ginsburg, 1977; Groen & Resnick, 1977). The min model begins to falter, however, when older children are tested; the data we are presenting today bear directly on this matter. Before turning to the data, however, I'd like to sketch briefly the alternative model that we are proposing, and mention in passing some of the evidence we feel supports it.

Our approach to the topic of mental addition has been in the opposite direction to that taken by Groen and Parkman. They began by describing the initial mechanisms found in young children, then projecting those through development. We have instead tested adults quite extensively, to see where addition "ends up", and now are searching for the beginnings of these final mechanisms. Our reaction time results with adults are very supportive of the following conclusion: Simple mental addition in adults is a memory retrieval phenomenon, not a reconstructive counting process. Adults seem to have the basic facts of addition (and multiplication, for that matter) stored in an organized network or associative structure. Retrieval from this structure is believed to be an intersection search process, as found in semantic memory models, and the time for such memory retrieval is a positively accelerated function of the size of the problem (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, in press).

This last statement bears rephrasing here, since it is quite important to our later results. Groen and Parkman's models clearly predict that RT will be a linear function of the minimum addend. In five separate studies from our lab, however, RT has been an exponential function of the problem's sum. The square of the sum provides a reasonable fit to these RT patterns, so the emergence of 'sum squared' is viewed as evidence for memory retrieval from a network representation. Other evidence for the network structure we are proposing includes associative confusion or interference effects, as found by Stazyk, Ashcraft, and Hamann (Note 1) and Winkelman and Schmidt (1974), priming effects (Hamann & Ashcraft, Note 2), and the general implausibility of counting approaches to other arithmetic operations like multiplication.

At this point, it would seem that the two extremes of the developmental continuum are tied down. At the earliest stages, children count when they do addition in their heads, and seem to do so by "adding on" the smaller addend or min to the larger number. They require nearly 3 seconds, on the average, for even the simple facts up through 4+5=9. Adults, on the other hand, retrieve these overlearned basic facts from an organized LTM structure; their memory retrieval for the same small problems averages 950 msec. The question of current interest then becomes the following: When do children begin to abandon the less efficient and less accurate counting processes? Putting it differently, when do children begin to formulate and use an adult-like memory structure for addition fact retrieval?

Figure 1 shows the average RT results in a study with 3rd, 4th, and 6th graders. In this study, half of the stimulus problems were presented with the correct answer, the solid lines marked true, and half were presented with an incorrect sum, the dashed false functions. The curves are plotted simply across small vs. large problem size, sums 0 - 9 vs. 10 - 18. The slopes of these functions for true problems, 850, 600, and 400 msec across grades 3, 4, and 6, begin to shed some light on the underlying processes. Clearly, large problems require more time than small problems, the standard "problem size effect"; this effect is a benchmark result in this area of research, and all the models predict it. Of equally great importance, the graph shows how the problem size effect diminishes across grade levels.

What is more interesting in these data are the patterns of significance when the RTs are analyzed with multiple regression. When such analyses are performed, letting the best predictor variables enter the equation in free order, the problem size effect at the 4th and 6th grade levels is best predicted by sum squared, the exponential function we have found repeatedly with adults. This variable accounts for 68% of the variance at 4th grade, and 78% at 6th grade, with standard errors of 246 and 216 msec respectively. The single best predictor at the 3rd grade level was the small/large factor on the graph, accounting for 56% of the variance with a standard error of 450 msec. These results, plus a higher order interaction not shown on the graph, suggsted two things -- first that 4th and 6th graders do addition in a fashion very similar to that of adults, and second that marked individual differences in processing may characterize the children in 3rd grade. Individual subject analyses tended to confirm both of these conclusions. Specifically, 40% of the 3rd graders yielded RT patterns which were best fit by counting variables, and 40% showed some evidence of memory retrieval patterns. At 4th grade, 20% seemed to be counting, and 70% using retrieval; at 6th grade, only 10% of the patterns were best fit by counting variables, but 90% of the records were best fit by the sum squared retrieval factor.

In a second experiment, we decided to focus on three critical points in the developmental continuum under consideration. Since about two thirds of the fourth graders in study 1 seemed to be using a retrieval process, we reasoned that children in the fifth grade would not only be using retrieval the majority of the time, but also would be doing so more efficiently than our 4th graders had. College students were tested, to allow for specific comparisons to full-fledged retrieval, and 1st graders were tested, to examine their performance under identical procedures. It had been suggested to us that using the true/false verification task might be introducing some sort of artifact into our results, possibly accounting for the specific patterns of predictor variables we had observed. Accordingly, we tested our 1st grade, 5th grade, and college students under two task conditions, verbal verification and verbal production. In the former, subjects indicated their decisions by saying "true" or "false" out loud; in the latter, they stated the sum of the problem out loud.

Figure 2 shows the averaged RT results across grades for small and large problems. The excessively large range of RT on the ordinate gives the misleading impression that no problem size effect was found beyond the 1st grade; in fact, all three age levels yielded very significant effects of problem size. With the exception of 1st grade, production and verification revealed only a constant time difference, undoubtedly due to the decision stage in verification. The relative facilitation of RT for 1st graders' production performance is probably due to the heavy emphasis on verbal drill of the basic facts in the first grade classroom.

As before, we turn to multiple regression results to examine the details of processing in these subjects. In the analyses on verification performance, first graders' RTs were best predicted by minimum addend, in agreement with Groen and Parkman's data. Fifth graders' and college students' RTs were best predicted by sum squared.

Turning to the data from the production task, we find that the best predictor variable at all three age levels was minimum addend. By itself, such an outcome should suggest that some min-like counting model accounts for the entire developmental range quite nicely. When considered in the light of the verification condition here, which tested the same subjects, and of other studies from our lab, a different implication emerges. This different implication is that the production task, rather than verification, may be suspect. In other words, the production task here seemed to generate data which were consistent with a counting model at all ages, in direct contradiction to other research findings. The verification task, on the other hand, yielded data which are entirely consistent with other research -- counting processes early in the school years, and memory retrieval later on.

Let me set the stage for a summary of the research I've presented today by showing you the following graph. In this figure, adults' RTs to the simple addition problems are plotted against the sum of the problem, to illustrate the exponential function we have found. The important curve for today's presentation is the one marked true -- adults' RTs increase exponentially with the sum of the problem, from about 900 msec to about 1250 or 1300 msec. The slope of this function is about 1.2. The final figure shows the adult curve plotted on the same scale as the curves obtained from the developmental studies just presented. The bottom four curves, for 4th, 5th, and 6th grades and college, all show the significant exponential function of sum squared. The two linear functions are the curves for 1st and 3rd graders' RTs plotted across minimum addend. There is also an exponential curve plotted for 3rd graders since this factor provided a best fit for half of the 3rd grade sample. This family of curves suggests strongly that 3rd grade is a transitional stage in addition processing -- absolute RTs are much faster than 1st graders' times, but the retrieval function is not yet reflecting consistent memory retrieval. One final and amazing aspect of these data should be mentioned here. The 5th grade curve fits beautifully between the 4th and 6th grades, despite the fact that it came from a different experiment, with different apparatus, and with a vocal response instead of a button press.

We have probably overstated the importance of 'sum squared' as an indicator of memory retrieval. It must be admitted that in the collection of 100 basic addition facts, structural variables like the min, sum, and square of the sum are all highly intercorrelated. We have claimed that sum squared is important for two basic reasons, first because it disconfirms the only hard-and-fast prediction of the min model, a linear increase in RT, and second because the exponential function is difficult if not impossible to reconcile with any counting or incrementing-based model. There are powerful pre-theoretical reasons, on the other hand, for choosing a network approach to memory retrieval. Wickelgren (1981) has recently argued very forcefully that long-term memory is now known to be associative in structure, associations being the building blocks of any network model. The interference and priming results mentioned previously are clearly indicative of a network structure. Finally, a network approach is flexible enough to be compatible with other arithmetic information as well as other long-term memory information. As Resnick (1981) has pointed out, what remains to be achieved is a detailed theoretical acount of network structures in arithmetic. We view the research presented today as an early step in working towards that goal.

Reference Notes

1. Stazyk, E.H., Ashcraft, M.H., & Hamann, M.S. A network approach to mental multiplication. Manuscript submitted for publication, 1981.

2. Hamann, M.S. & Ashcraft, M.H. Priming in Mental Addition: Decoupling Retrieval and Decision Processes. Unpublished manuscript, 1981.

References

Ashcraft, M.H. & Battaglia, J. Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. Journal of Experimental Psychology, 1978, 4, 527-538.

Ashcraft, M.H. & Stazyk, E.H. Mental addition: A test of three verification models. Memory & Cognition, in press.

Ginsburg, H. Children's Arithmetic: The Learning Process. New York: Van Nostrand, 1977.

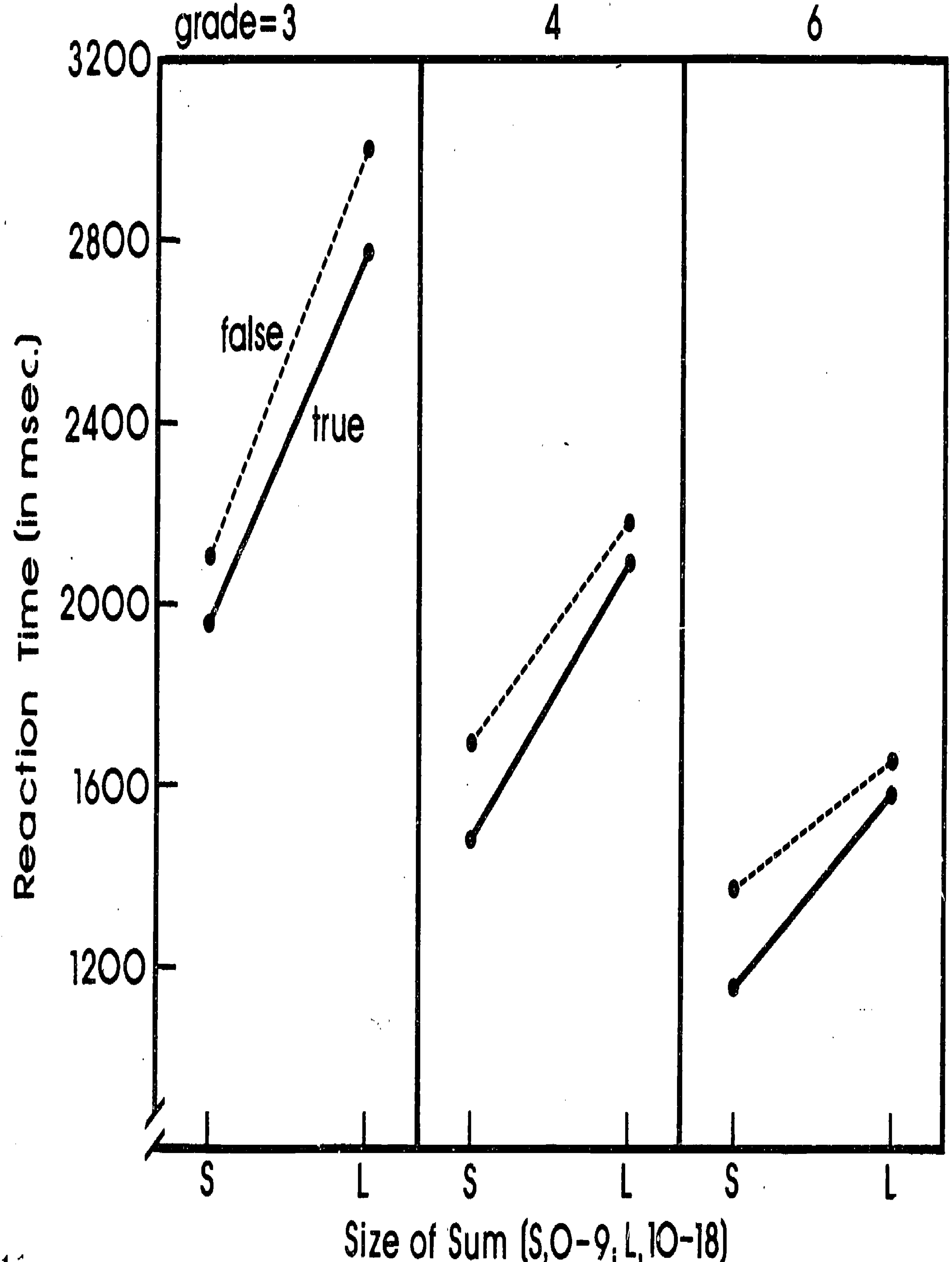
Groen, G.J. & Parkman, J.M. A chronometric analysis of simple addition. Psychological Review, 1972, 79, 329-343.

Groen, G.J. & Resnick, L.B. Can preschool children invent addition algorithms? Journal of Educational Psychology, 1977, 69, 292-302.

Resnick, L.B. Instructional psychology. Annual Reviews of Psychology, 1981, 32, 659-704.

Wickelgren, W.A. Human learning and memory. Annual Reviews of Psychology, 1981, 32, 21-52.

Winkelman, J.H. & Schmidt, J. Associative confusions in mental arithmetic. Journal of Experimental Psychology, 1974, 102, 734-736.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S
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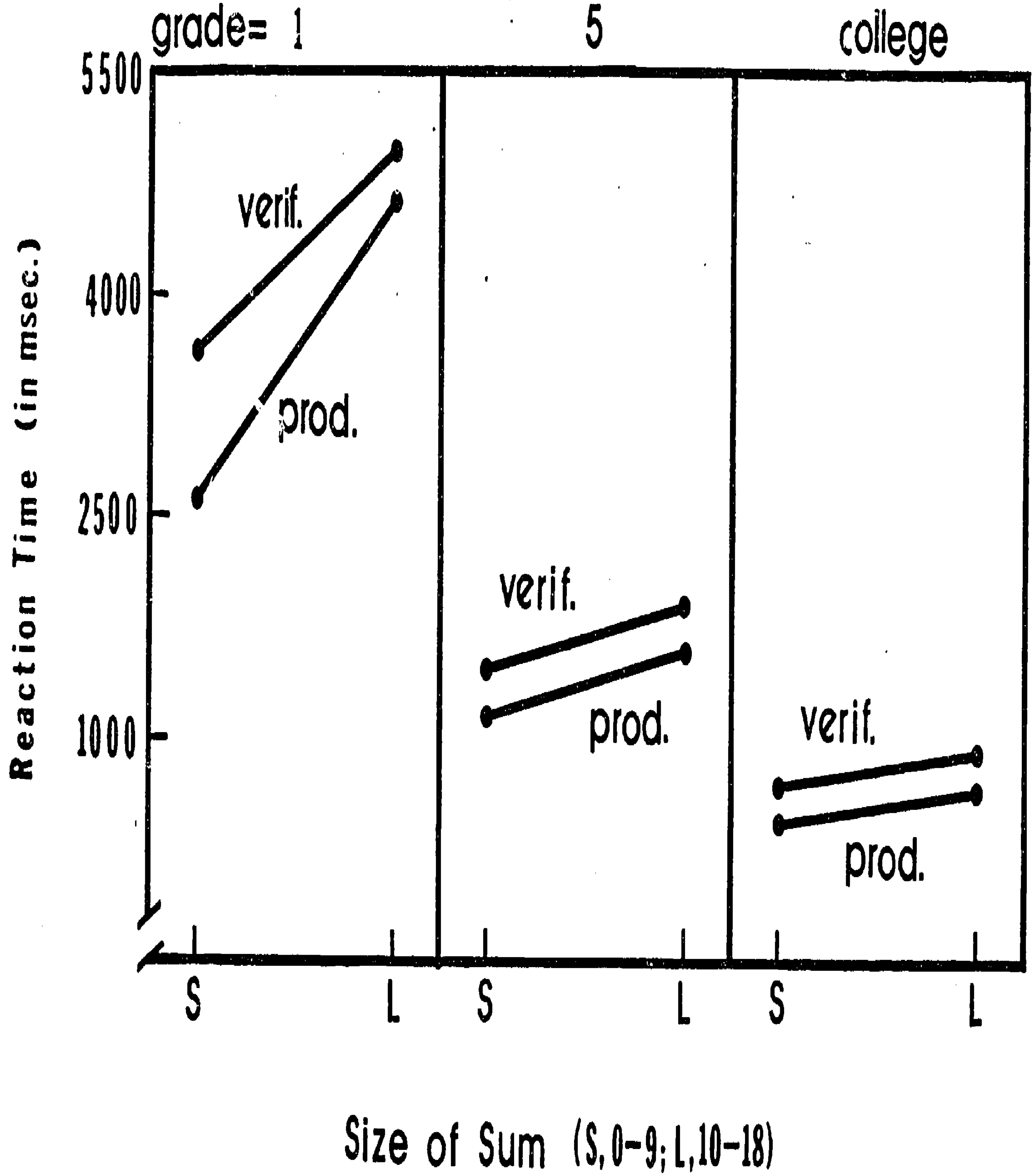
Size of Sum (S,0-9; L,10-18)

11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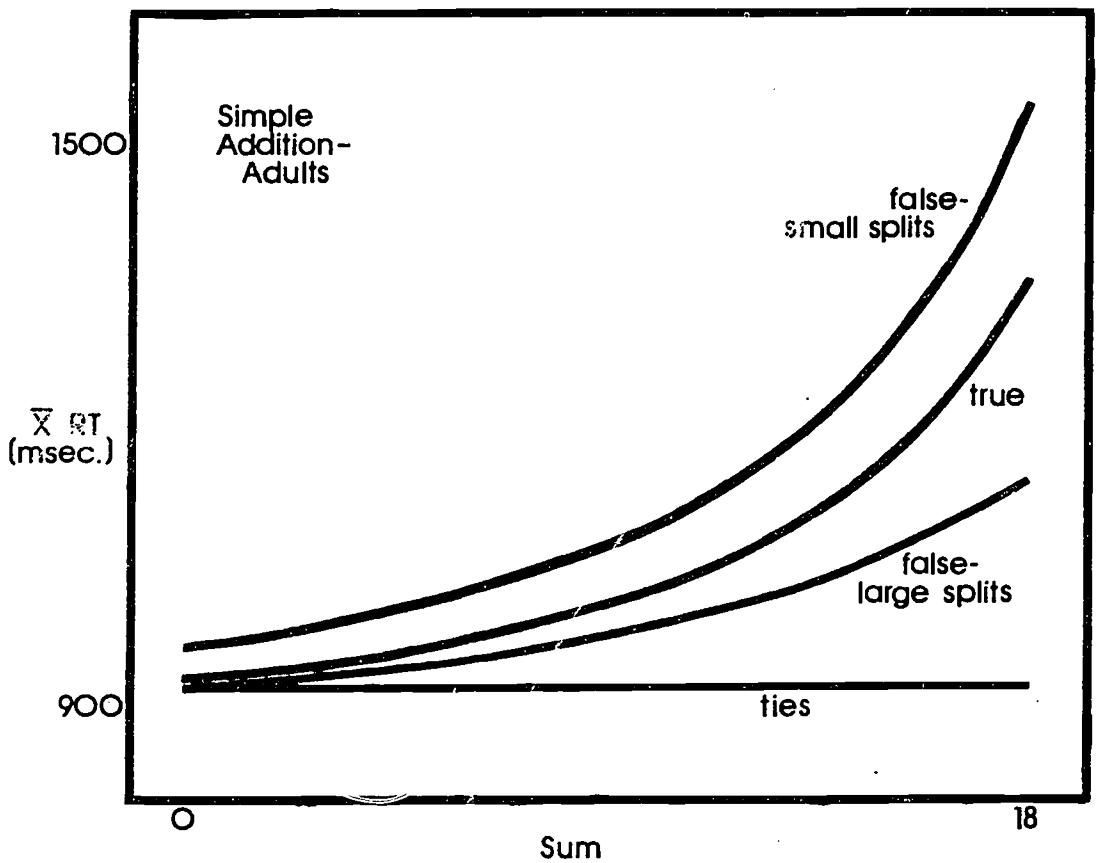

Simple
Addition-
Adults
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900
X̄ RT
(msec.)
false-
small splits
true
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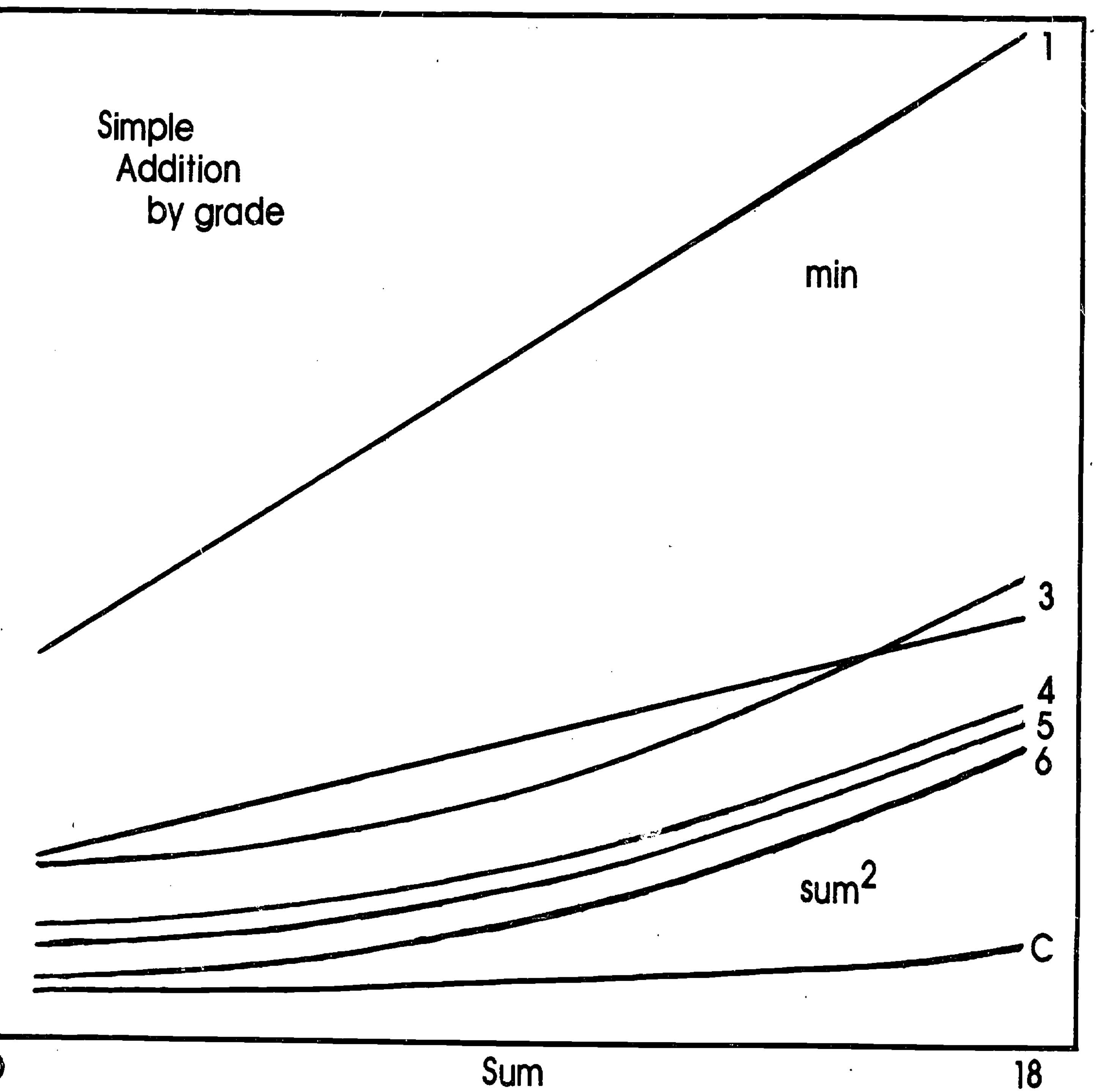
large splits
ties
0
18
Sum


Simple
Addition
by grade
min
1
3
4
5
6
sum2
C
Sum
18