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ABSTRACT

This paper argues that if conclusions about children's grasp of logical concepts are to be reached and acceptable lines of research followed, then more precise definitions of the concept "logical necessity" must be formulated. The paper defines logical necessity as "the unconditional guarantee of truth that accompanies valid deduction from axiomatic premises." It then shows that whereas hypothetical conservation and transitivity problems have solutions that are accompanied by logical necessity, physical embodiments of those problems as tasks do not. The remainder of the paper outlines problems associated with learning logical concepts from physical examples, and discusses the ambiguity of the word "must." It suggests a set of criteria for inferring a grasp of logical necessity and briefly discusses the consonance of the ideas presented with those of Jean Piaget. (FL)

On What Must Be –

More Than Just Associations

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On What Must Be - More Than Just Associations

There has been a considerable shift in views of the logical competence of small children. Much of this has accompanied a re-examination of tasks, typically ones devised by Piaget and his co-workers. From the work of inter alia, Bryant and Trabasso (1971), McGarrigle and Donaldson (1974), Rose & Blank (1974) a lot has been learnt about tasks such as transitivity and conservation, in particular about how children's performance of these tasks may be affected by factors not previously considered.

What these researchers have done is argue that the method of presentation of Piaget's problems can lead to inappropriate estimates of the child's competence. They have explicitly or implicitly regarded these tasks as requiring certain logical abilities for their solution. So once the transitivity task had been tidied up, by preventing correct responses based on parroting and ensuring that the comparisons had been remembered then it tested deductive transitive inference.

Similarly if suitable steps are taken to ensure the child's understanding of the conservation task in the way intended by the experimenter then the child's answer results from deductive reasoning and may be seen as following with logical necessity.

The concept of logical necessity has received a lot of attention from philosophers (Kneale & Kneale, 1962). The feature of it that has most troubled associationists, or empiricists, is its unconditional guarantee of truth which seems impossible to derive from experience. One of the most common examples of a conclusion accompaned by logical necessity is " 1+1 = 2". Given the definitions of the number symbols, number base and signs this statement can be deduced with logical necessity. The statement is independent of experience in that no one could reasonably persuade us to doubt it with any form of demonstration.

What makes this statement independent of experience is that it is not principally a statement about the world of objects although it can be applied to that world. Principally it is a statement in a closed system of definitions and rules which allows it to be validly deduced.

In the next section a definition of logical necessity will be given that takes account of the above properties. It will be shown that whereas hypothetical conservation and transitivity problems have solutions which are accompanied by logical necessity, physical embodiments of these problems as tasks do not. In subsequent sections problems associated with learning logical concepts from physical examples are outlined and the ambiguity of the word 'must' discussed. A set of criteria for inferring a grasp of logical necessity is suggested and the consonance of the ideas presented with Piaget's ideas briefly discussed.

<u>What is logical necessity?</u>

Logical necessity can be defined as the 'unconditional guarantee of truth that accompanies valid deduction from axiomatic premises'. The virtues of this definition are that it makes explicit the restrictions of logical necessity a) to the world of statements and b) to deductive as opposed to inductive reasoning.

Conservation and transitivity only follow with logical necessity in the abstract or the hypothetical. No physical demonstration or embodiment of the abstract forms carries with it logical necessity. Success or failure of these tasks is independent of a grasp of logical necessity. A more explicit treatment of these tasks follows.

Transitivity

In the length transitivity task a child may be shown a series of length comparisons between sticks A,B,C,D,& E. From seeing that stick B was longer than stick C and that stick C was longer than stick D, he may infer that stick B is longer than stick D. This would be a transitive inference but not necessarily a deductive inference. It would not be a deductive inference if it was based on the child's experiences of relations of length. For although, such experiences may increase confidence even to the point of subjective certainty, they do not and cannot yield logical necessity.

If the child gives a wrong judgement, it is possible that the child understands logical necessity, but is uncertain whether the physical embodiment justifies a straightforward application as the experimenter intends. Either mistrust of the validity of the comparisons or a belief that the lengths of one or more of the relevant sticks had changed would be sufficient to justify such uncertainty. Such uncertainty would be fully justified in the following example. Take three ductile rods A,B,&C, show that A is longer than B; covertly apply tension to B so that it is now considerably longer than A and slightly longer than C. Show that B is longer than C. It would then be that A was longer than B, B is now longer than C, but A is shorter than C.

Now it might be said that this is trickery because of the difference between the initial and final length of B. In an ordinary length transitivity task the materials used are not so ductile and no such underhand action is carried out. Of course experimenters may know the irrelevance of ductility in the physical embodiment of the transitivity problem they use but why should their subjects. It seems open to question whether children understand the properties of materials such as elasticity and ductility as they apply to the materials used.

The example was intended to bring out the difference between the propositional form of a transitivity problem and its translation into the world of real objects. Two important differences between propositional form and physical embodiment emerge. Unlike the propositional form the physical embodiment involves temporal reference and additional information is required. The validity of the conclusion about the lengths in the task depends on the nature of the materials in a way that is quite different from the validity of the conclusion in the propositional form. The truth of the task judgement is contingent in contrast to the logical necessity of the propositional conclusion ($B > C$, $C > D$, $\therefore B > D$).

Conservation

The propositional form of conservation problems is the same whether the domain is number or continuous quantity. Elkind's (1967) symbolism will serve: let S stand for the standard stimulus, V for the variable stimulus and V' for V that has been transformed in some perceptually salient but quantitatively irrelevant way. If $S = V$, and $V = V'$, then $S = V'$. This conclusion based on the transitivity of equality follows with logical necessity in this hypothetical form. It is not subject to revision in the light of new evidence nor could one accept that it was merely a matter of opinion over which people may argue while sharing the same definitions of the terms.

In continuous quantity conservation tasks however there is considerable scope for argument because of

1. extra conditions that have to be made explicit.

2. the fundamental imprecision of measurement of continuous quantity.

3. the necessity for temporal reference.

Take the traditional conservation of liquid quantity task. Two similar sized beakers A and B are filled to similar levels with some liquid. The contents of the two beakers are judged to be equal. Then the contents of one beaker (B) are poured into another beaker of a different shape, say a wider one (C). Not only small children and unschooled adults from other cultures but also physical scientists might judge that the quantities are no longer the same.

To get the recalcitrant physical scientist to agree that the quantities were the same several steps may need to be taken. Firstly one might need to agree to define the 'same amount' in a practical way. Because of the fundamental imprecision of measurement this definition would probable cite confidence limits - e.g. 'the same amount' meaning within 2 ml - reflecting the believed accuracy of the available method of measurement.

Next one would have to agree to consider the amount of liquid left in B as negligible. This might be unreasonable if the liquid was highly viscous, such as treacle.

Furthermore evaporation would have to be ignored. More of the liquid in B would have evaporated by the time it was in C and during the time it was in C because of increased rate of exchange due to the increased surface area brought about by pouring and by leaving in a wider container. Whether it was reasonable to ignore evaporation would depend on the definition of equality, the duration of the task, the ambient temperature, and any local differences in temperature, and the boiling point of the liquid.

In each of these steps definitions have had to be agreed or deviations from the real world allowed in order to preserve the psychologist's view of what the correct answer is to the conservation question - 'What is the relation between the quantities in A and C?' The physical scientist might justifiably contend that it is the psychologist who should change his view of the physical world. It is the psychologist's ignorance of the physical world that lets him regard the static temporally independent picture implicit in his logical analysis as a realistic and general description of the world of liquids. If the psychologist was a physics student the physical scientist might regard him as being in need of remedial teaching.

Should the psychologist dare to say that the quantities in A and C must be the same because of the reversibility of pouring, the physical scientist might raise his eyebrows at this failure to comprehend the increased rate of exchange during pouring.

In practice the transformations used in continuous quantity conservations tasks do alter the quantities, albeit generally minimally. Conservation of volume will not be a reasonable expectation if the substance is compressible and/or the liquid has a high density. Conservation of weight hardly ever happens when a ball of clay is rolled out or cut into smaller pieces - the water content of clay tends to evaporate, and clay tends to stick to one's hands.

The view of conservation suggested here is one of a logical problem with distinct difficulties in its translation into the physical world. This is very different from Margaret Mead (1960) who considered the ability to conserve as basic to survival and different from Greenfield (1966) who regarded conservation as a basic law of the physical world. Cole and Scribner's (1974) assertion that 'in desert communities where water is a treasured commodity everyone can be expected to conform to certain laws of conservation'. (p.152) can certainly be challenged. Pouring water from a small bucket into a large one, and hence increasing the surface area, can reasonably be expected to result in a decrease of the amount of water with a temperature as high as that in the Aborigines' world. It is perhaps not so surprising then that unschooled Aborigines with little contact with Western thought judge the quantities to have changed in a conservation task.

If the analysis of conservation tasks and problems outlined here suggests a reappraisal of the findings of psychologists using such tasks in the study of people from different cultures, it demands a revaluation of the studies following the resistance to extinction study by Smedslund (1961) and reviewed by Hall and Kaye (1978) and Shultz, Dover and Amsel (1979). What has typically happened in these studies is that children and/or college students have been faced with nonconservation. Their comments and explanations have been taken to indicate whether or not they view conservation as a matter of empirical belief or a matter of logical necessity. From the position taken here this is a false dichotomy. Conservation problems in their propositional form allow conclusions accompanied by logical necessity. Conservation tasks are principally matters requiring empirical belief. Whereas the conclusion of the propositional form of the problem is not subject to revision in the light of new evidence the conservation judgement in the task is, or should be. What disconfirming evidence should do is make the subject re-examine his assumptions about the integrity of the experimenter, the validity of the measuring devices, or the goodness of fit of his model of the physical world, in particular his representation of the task. This is very different to denying logical necessity which Hall & Kaye (1978) advocate as the strategy of a maximally adaptive organism.

Reluctance to c. ment or accuse the experimenter of cheating is, as Smedslund (1969) pointed out, not easy to interpret. However such reluctance has been used as the criterion for extinction of conservation in the majority of studies (see Hall & Kaye, 1978). It is notable, and slightly puzzling, that in none of these studies were nonconserving results of number conservation tasks given. There are some grounds for viewing number conservation tasks as more direct translations of conservation problems than conservation tasks involving continuous quantities. Firstly, the subject can be certain of the initial equality, if the items used are presented in such a way that they can be counted, and the subject is a competent counter. So, unlike continuous quantity conservation tasks, number conservation taaks do not suffer from the fundamental imprecision of measurement.

Secondly the transformations used are not ones which future research is liable to suggest do alter number. However there are extra conditions that have to be made explicit, concerning the physical properties of the items. If the items were rain drops or drops of mercury and one row was bunched together it is not clear that the number of drops in the two rows would remain the same. If the items were male and female rabbits originally presented in individual cages and the transformation consisted of putting one group in communal cages after some time the number of rabbits in the communal cage might very well be different.

Even in number conservation tasks conservation only occurs, given the traditional transformations, with specific classes of objects.

In this section much has been made of the distinction between the propositional form and physical embodiment of a problem. It has been claimed that logical necessity, as defined here, only accompanies conclusions in the propositional form of the problem. Furthermore the model of the physical world implicitly assumed in a straightforward application of the propositional form to the physical world is inadequate. A revaluation of cross-cultural studies of cognition where transitivity or conservation tasks have been used has been suggested. Resistance to extinction studies were claimed not to assess a grasp of logical necessity.

In spite of the examples where conservation does not really occur and in spite of the distinction between propositional form and physical embodiment, there may still be reluctance on the part of the reader to accept that the objections here are real. It may be felt that children and adults in conservation tasks are more likely to be treating the problem as a logical hypothetical one than as a particular problem concerning the items used. In other words children and adults know what experimenters intend to convey by their tasks. The question to be asked here is 'How do experimenters know that their subjects are responding to the hypothetical problem rather than the physical task?' It seems more reasonable to accept this as a sensible question, if somewhat difficult to answer, than simply to assume the question does not arise.

In the following section this issue will be placed in another context, the use of physical examples in teaching mathematical or logical concepts.

You can see what I mean

Teachers and psychologists, may select with care or 'common sense' the items they use in providing physical demonstrations or illustrations of hypothetical problems and mathematical concepts.

Unless children have some notions of why the items used are good examples there are likely to be several possible problems. They may not generalize at all or they may generalize to contexts that are inappropriate. If they notice the discrepancy between what they think ought to happen and what does, they may become confused and in the absence of a teacher resolve this confusion adversely. For example a child may attribute the discrepancy to personal failure 'I can't do this', 'I'm no good at numbers'. Alternatively the child may come to view mathematical generalizations which do follow with logical necessity, as no more immune from revision than those that do not. A child may come to interpret the word 'must' by which the teacher intended to convey logical necessity as instead simply conveying the moral or social obligation sense of 'must' as in cases where he is told he 'must' go to bed, have a bath, or stop hitting his sister. The use of 'must' in these latter contexts does not, as most if not all children are aware, preclude argument or infringement. Indeed it is this use of 'must' that would seem from casual observation to be the prevalent, if not only, sense in the young child's world.

The case of games appears counter to this. Games such as chess, draughts, and noughts and crosses do have systems of rules and definitions that allow deductive reasoning to take place and hence conclusions with logical necessity can be derived. The difficulty is to ensure the child distinguishes between the logical necessity of conclusions based on the rules and the social obligation of the players to abide by the rules. Just as one cannot assume that children understand 'must' as implying logical necessity when it is said by adults, so one cannot assume that children mean to imply logical necessity when they use the word 'must' even though the context is appropriate. Nor need it imply that the statement including the word 'must' is a product of deductive reasoning. For these reasons Donaldson's (1978) examples of children's comments made while listening to stories are not convincing evidence of their deductive powers.

Just as teachers are in danger if they rely on words such as 'must' to convey logical necessity so are psychologists if they rely on the child's use of 'must' to infer logical necessity. A solution to the teacher's problem may lie in the use of examples and non-examples from a wide range of contexts to encourage the abstraction of the concept. In the next section an attempt is made to solve the psychologist's problem of a criterion or a set of criteria for the grasp of logical necessity.

A set of criteria for the understanding of logical necessity

Logical necessity as defined in this paper is a complex concept. It is restricted to the world of hypothetical propositions and deductive reasoning. Such 'disembedded' thinking is, as Donaldson (1978) claims, not likely to be expected of, or experienced by, a child before going to school. Conversations with pre-schoolers at home seem invariably to refer to numbers of items when numbers are mentioned (Corran, Note 1).

A grasp of logical necessity is more than an ability to reason.

It is metacognitive in that it involves a recognition of different types of reasoning, i.e. deductive and inductive and an appreciation of the differences between the closed, humanly constructed, conceptual systems of pure mathematics and formal logic and the open systems of applied science, natural and social, and everyday life. Furthermore it requires an ability to distinguish valid from invalid reasoning. It makes no sense to talk of an unconscious grasp of logical necessity. It does however, as the writer is painfully aware, make sense to say that someone has a grasp of logical necessity but has difficulty in expressing what it is. For this reason devising a series of examples and nonexamples of reasoning with conclusions accompanied by logical necessity and asking people to classify them may be a fair test.

It is quite probable that not many people will pass such a test. They may fail to recognize the difference between theoretical scientific reasoning and its application. This is possibly an indictment of the way science is taught in schools. Confusions between logical necessity and psychological certainty are also to be expected as are confusions between the validity of reasoning and the truth of the conclusion (Henle, 1962).

So what?

On the one hand logical necessity is a complex concept, on the other we expect children to grasp it at a fairly early point in their learning of mathematics. If they do not grasp it they would be justifiably puzzled at what counts for proof in geometry or the impressive regularity of arithmetic. Indeed they think in a hypothetical way when they do arithmetic without having objects to count and with an understanding of what they are doing.

Perhaps the distaste for mathematics that many adults still express stems from a lack of explicit treatment of the self-contained closed systems aspects of pure mathematics and logic in their education. The well-meaning attempts by teachers to give mathematical concepts a physical form are if anything likely to impede this appreciation.

Finally how does the position adopted here relate to Piaget? It may be thought that there is considerable disagreement, certainly by those who claim that both children and Piaget regard conservation as logically necessary even in its embodiment in a task (e.g. Hall & Kaye 1978).

This however seems to be open to doubt and indeed so many of the ideas in this paper are consonant with ideas expressed by Piaget that it is not clear that this is a critique.

Piaget (1952) makes the distinction between logical and physical necessity in introducing the intermediary reactions of children to the task of conservation of continuous quantity:

> Between the children who fail to grasp the notion of conservation of quantity and those who assume it as a physical and logical necessity.... (Piaget, 1952, p.13)

Just because concrete operational children may not distinguish between the logical necessity of the conservation conclusion in the propositional form and the validity of such a conclusion in the task does not mean that Piaget has ever been unaware of the distinction. Piaget (1971) discussed the gradual development of the feeling of the logical necessity of transitivity but to claim that he regards this as pertaining to the task as opposed to the propositional form is unjustifiable.

Piaget (in Gruber & Vonèche, 1977) explicitly deals with logical necessity and physical laws:

> For a general law is not, as such, necessary. The child may very well discover the absolute regularity of a given physical law (such as that light bodies float and that heavy ones sink, etc) but there is no physical necessity that can account in his eyes for this regularity...... What makes a law necessary in our eyes is its deducibility: a law is necessary if it can be deduced with a sufficient degree of logical necessity from another law, or from sufficient geometrical reasons.
>
> (Gruber & Vonèche, 1977, p149)

In 'The Growth of Logical Thinking' Inhelder & Piaget (1958) distinguish between the child and the adolescent in terms of the former's lack of powers of reflection. For this reason alone children would not be expected to appreciate logical necessity.

Finally Piaget & Inhelder (1969) comment on training studies as follows:

> ....these short-term learning processes are insufficient to give rise to the operating structures or to achieve those closed systems which make possible a method that may properly be called deductive.
>
> (Piaget & Inhelder, 1969, p,100)

One can imagine that this selective quotation will elicit controversy as to what Piaget over the years has or has not meant to convey. Such controversy deserves the pejorative connotations of 'academic' as the word is used by people. The fundamental task for developmental psychologists is not to understand Piaget but to understand children.

Reference Notes

1. Corran, G. Personal communication. August 1980

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Footnotes

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