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ABSTRACT

Forty-five children aged four-and-a-half to five-and-a-half years old were given number conservation tasks in three conditions: (1) a count condition in which children were helped to count each set after the transformation: (2) a match condition in which children were helped to connect by a string each animal with its peanut: and (3) the standard condition in which no help was given. On the first task, the percentage of children giving judgments of equivalence differed significantly across conditions (69%, 80%, and 14%, for the count, match, and standard conditions). Furthermore, all of the children who began with the count condition sustained conservation responses when subsequently presented the task under standard conditions, while only some of the children who began with the match condition did so. These results indicate that imposed empirical strategies different from those generally reported (visual-perceptual and Piagetian transformational strategies) will lead preoperational children to make judgments of equivalence and that, once used, the counting strategy (at least) evidently is adopted spontaneously by the children for subsequent problems of the same type. (Author)

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The relationship between counting and conservation of number has been the focus of recent theoretical (Klahr & Wallace, 1976; Gelman and Gallistel, 1978) and empirical (Russac, 1978; Saxe, 1979) work. The method used in these latter studies was to assess conservation of number and counting skill (or counting accuracy) independently and to compare the numbers of children who had adequate levels of performance on each of these. The findings were generally that many children scored well on counting but not on conservation while all those scoring well on conservation also scored well on counting. These findings imply that some levels of counting ability develop before conservation of number, but they do not directly implicate counting as involved in conservation. For example, Saxe (1979) presents a sub-skill analysis of counting and number conservation and argues that the order found between these two is due to the child's understanding one-to-one correspondence in static (counted) arrays before he understands one-to-one correspondence in dynamic arrays (as required by the standard conservation of number task). A more direct assessment of the relationship between counting and conservation of number would seem to be necessary. One such approach would be to induce counting during the conservation task and see whether it contributes to improved performance. This was done in the present study.

The Piagetian analysis of conservation of number rests heavily on one-to-one correspondence (Piaget, 1952). However, the type of one-to-one correspondence initially established in the conservation of number situation is a correspondence between the places where the objects are located, i.e., in the two rows of objects, each corresponding object is across from its mate in the same place in its row. It is this perceptual correspondence of places that is broken when one of the rows is stretched out or moved closer, i.e., the place across from an object may now no longer be filled with an object. After the transformation has broken this "nice" perceptual place correspondence, a more active means of establishing a new correspondence must be used. Imaginary lines between pairs of objects or some other way of matching, or actively pairing, objects must now be used by the child. Such active pairing is not an activity that a child will have seen very often. Thus the failure to use such a matching process in the conservation situation might only reflect a production deficiency, i.e., the child might simply fail to produce a strategy in a situation even though that strategy would in fact help him to solve the problem. This possibility was examined in the present study by helping the child to use a matching strategy. If such a matching process was new to a child, it seemed possible that by the time a child finished the matching process he might have forgotten the results of the early part of the process. Thus it seemed necessary to record the matching process as it occurred so that the child could look back over and reflect upon the whole process after he had finished it. Therefore the matching process used strings which were placed between pairs of objects.

Counting and matching can be viewed as two empirical means of establishing an equivalence relation between two sets (as required by the conservation of number situation). These two means differ in the situation in which they can be used, in their accuracy, and perhaps in their generalizability to larger and larger sets, Counting can be used in identity conservation (where one set is transformed) or in any situation in which two sets are not spatially simultaneous, while matching requires the simultaneous physical presence of two sets. No data are available on relative accuracy of counting and matching, but task manipulations seem possible to make either easier than the other. Counting would probably be more accurate in any situation in which sets to be matched were fixed and very disorganized, while matching would be more accurate (especially for very large sets) if it were accomplished by placing together movable objects two at a time.

The relative strengths of these two methods of establishing equivalence as sources of adult intuitions about relations of equivalence are not known, but the usual initial adult resistance to the definition of equivalence of infinite sets would argue that for finite sets, adult intuitions derive heavily from counting, rather than matching. For example, when asked whether there are more whole numbers or more even whole numbers, adults usually respond that there are more whole numbers![1] This reaction is based upon a counting view of equivalence and is only relinquished when adults realize that it is not possible to extend the counting process to infinite sets and that some other definition of equivalence must be used. The matching definition is then accepted (there are the same numbers of even whole numbers and whole numbers because for every whole number we can give you an even whole number -- therefore there must be the same number of them). It can then be used, though it still often co-exists with counting-based intuitions that there "really" are more whole numbers then even numbers. The question then arises as to the relative effectiveness for young children of counting and matching as methods of establishing equivalence. Russac (1978) examined both counting and matching in children, but as he pointed out, his study suffered from the methodological problem of having directed children to count in the counting task but of not similarly having directed them to match in the matching task. The present study both directed children to use counting and matching and helped them to do so correctly.

The present study was designed to examine the extent to which both counting and matching facilitated children's performance on conservation of number and to compare the relative effectiveness of these two methods.

Method

Subjects

The age range of 4½ to 5½ years was selected because children of this age are usually in the pre-conservation or transitional levels on conservation of number tasks. Forty-eight middle-class children from three pre-schools and kindergartens served as subjects. Half were between 4½ and 5 and half were between 5 and 5½; mean ages of these two groups were 4-10 and 5-4.

Tasks and Design

The conservation task was set within a context of "feeding the animals at the

1. This observation stems from the first author's experience of several years of teaching undergraduates and in-service teachers about infinite cardinalities. These observations have been checked with those of others who have taught such subject matter and have been found to be consonant with their own observations. To our knowledge, no research has been done on this question of relative strengths of adult number intuitions about equivalence.

zoo". Seven toy animals were lined up in a row facing the child, and a row of peanuts was made in front of the animals so that each animal had a peanut.[1] The child was then asked whether there were more animals than peanuts, the same number of animals and peanuts, or more peanuts than animals. After the child responded that there were the same number of animals as peanuts, the E said that the animals had to move to new places, and the E and the child together spread apart the animals to make a longer row. The child was then asked the conservation question, "Are there more animals than peanuts, the same number of animals and peanuts, or more peanuts than animals?" After the response, the child was asked "How do you know?" The above procedure was used with children in the Standard conservation condition. The same procedure was used with the Counting condition except for an extra step between the spreading out of the animals and the asking of the conservation question. After the animals were spread out, the E asked the child to count the animals and to count the peanuts. If the child made a mistake on either count, the E would tell the child: "Let's count them together" and make sure the correct answer was obtained. To ensure that the child remembered the result of both of his counts, after their completion, the E asked, "How many animals are there?" and "How many peanuts are there?" and reminded the child of the answer obtained if the child did not remember. Then the E asked the conservation question.

The Matching condition was the same as the control condition except that prior to the spreading out of the animals the E helped the child put a string connecting each animal to a peanut and ensured that the animals and the peanuts remained attached during the transformation. After the transformation the E ensured that the child attended to the strings and understood their correspondence meaning by asking the child three times which peanut went with a particular animal. If the child got this answer wrong for any of the three animals, the child was instructed to "look at the string".

A within-subjects design with complete counterbalancing of task order was used. The subjects were randomly assigned to one of the six possible orders of presentation within the half-year age cells and within each pre-school sample.

Scoring

Behaviors exhibited during the session were recorded by the E on a recording sheet. The session was tape-recorded and the tapes were transcribed. Three scoring systems were used. The first was addressed simply to performance on the conservation task. Children in each condition were scored as conservers if they responded that there were the same number of animals as peanuts and did not change their answers; otherwise they were scored as non-conservers. The second scoring system examined possible condition differences in the justifications children gave for their responses to the conservation question. Table 1 lists the classification of justifications used in this study. There were three main types: empirical justifications (counting and active pairing-matching), Piagetian operational justifications (Addition/Subtraction, Reversibility, Identity, Compensation), and visual-perceptual justifications (Length, Place Correspondence). The last two types are those usually reported in the conservation literature. Table 1 gives

1. The usual practice of having the child make the second set equal to the first was not followed because during piloting children ate some of the peanuts, distracting them from the task.

examples of each type of justification. Under the justification scoring system, children were scored as correct if they gave correct answers and used an empirical or Piagetian operational justification and incorrect if they gave incorrect answers or gave visual-perceptual justifications. The third scoring system was designed to enable age and order as well as condition effects to be examined. In this scoring system scores ranged from 0 to 2. One point was given for a correct answer and another point was given for any correct (empirical or operational) justification. Three children were dropped from the sample during the scoring process, one because the audio-tape was lost, and the other two because they failed to complete all three conditions.

Results

The Standard condition was compared successively to each of the others in the number of children who gave a correct response to the conservation question on the first task administered. The latter restriction permitted a "pure" evaluation of each treatment, in that no previous treatment had been experienced. Significantly more children gave correct answers in the Count (11 of 16) and in the Match (12 of 15) than in the Standard (2 of 14) condition, $\chi^2 = 6.94$ and 10.03 respectively, $p < .01$ for both. Thus instructing children either to count or to match was effective in achieving conservation of number.

To get a measure of the extent to which the counting and matching strategies were maintained in the subsequent control condition, the number of children giving correct responses in the Standard condition when it came first was compared to the number of children giving correct responses in the Standard condition when it followed the Count condition and when it followed the Match condition. Significantly more ($\chi^2 = 7.20$, $p < .01$) children gave correct responses in the Standard condition when the Count condition had preceded it (7 of 8) than when the Standard condition was first (2 of 14). The Standard condition resulted in 3 of 7 correct responses when it was preceded by the Match condition. That did not differ from the Standard condition when it came first (2 of 14). All of the children who answered correctly in the Count condition continued to answer correctly in the Standard condition while only about half of the children who began in the Match condition did so. Thus instructions to count not only facilitated immediate performance but the superiority was maintained in a subsequent standard task condition.

When performance was evaluated using the justification scoring (children had to give an empirical or an operational justification as well as the correct answer to be scored as correct), first trial comparisons indicated significantly superior performance for the Counting as compared to the Standard condition (10 of 16 vs. 2 of 14; $\chi^2 = 5.36$, $p < .05$), but the performance in the Matching condition was no longer superior to that in the Standard condition (6 of 15 vs. 2 of 14). Thus counting and matching did not differ in their effectiveness in producing correct answers to the conservation question (both are quite effective), but they did differ in the subsequent ability of children to describe how they knew the answer to the question. Children were able to articulate that they had counted or that counting had helped them to know that the sets were the same, but they were unable to articulate to the same degree the way in which the matching strategy helped them to know the answer. Whether that difference reflects any fundamental difference in comprehension of the respective solutions versus the greater ease with which one can be expressed is not clear.

To assess age and order as well as possible condition interactions with these variables, a 2 (Age) by 3 (Task) by 6 (Order) analysis of variance was done on the response and justification summed scores (1 point for a correct answer and 1 point for an appropriate empirical or operational justification). The BMD P2V statistical package (repeated measures, unequal cell sizes; Dixon and Brown, 1977) was used. A significant main effect of Age was found: the old four-year-olds had a mean score of 1.05 (out of 2) and the young five-year-olds had a mean score of 1.44 $\underline{F}$ (1,33) = 4.23, $\underline{p} < .05$. The main effect of task was also significant, $\underline{F}$ (2,66) = 13.82, $\underline{p} < .001$; mean scores for the Standard condition were lower (0.98) than were scores for Match (1.38) or Count (1.47). The Age by Task interaction was (see Figure 1) significant, $\underline{F}$ (2,66) = 3.06, $\underline{p} < .05$. Young five-year-olds did considerably better than the old four-year-olds in the Matching (1.67 vs. 1.00) and in the Standard (1.13 vs. 0.72) conditions but not in the Count condition (1.54 vs. 1.42). Thus children ages 4½ to 5 benefit more from counting than from matching instructions, while children ages 5 to 5½ benefit equally from counting and matching instructions. The Order by Task interaction (see Figure 2) was also significant, $\underline{F}$ (10,66) = 5.04, $\underline{p} < .001$. This reflects the fact that the large superiority in scores in the Count and Match conditions compared to the Standard condition on the first trial decreases in the second and third trials. This decrease occurs when children had had the Standard condition after a Count or a Match condition and spontaneously use a counting or a matching strategy in this Standard condition.

In order to ascertain whether the increase in Standard condition scores on the second and third trial resulted primarily from the use of counting or of matching, the justifications of the children receiving the Standard condition on the third trial (and thus after both the Count and the Match conditions) were examined. Of the 16 children in this condition, 7 gave a count or number justification, 1 gave an active pairing (matching) justification, and 2 gave an operational justification, suggesting a greater impact for the previous counting experience compared to the matching experience.

Conclusions

Instructing and helping children to use counting and matching in a number conservation task leads to a significantly greater number of correct responses than in a standard condition. Evidently both counting and matching help young children answer the conservation question accurately, but children simply do not think of using these strategies themselves. The latter conclusion is suggested by the ease with which the children use each when so instructed. Counting was as effective for 4½-5-year-olds as for 5-5½-year-olds, while matching was effective mainly for the older age group. When given the conservation task in the standard manner after a count or match condition, children continued to use counting but did not continue to use matching to as great an extent. That difference should be interpreted cautiously at this point, since it simply may reflect a greater familiarity with counting or a greater effort required by the matching with strings process rather than a more fundamental superiority of counting. In addition, the children were less likely to articulate the matching explanation—even when matching was used - than to articulate the counting explanations when counting was used. Again, this difference may be due simply to the greater demands of the matching explanation. This suggests that the child's spontaneous use of a matching strategy may be underestimated from verbal report data. Thus it also suggests that scoring systems which require a correct justification for a conservation judgement will not be as likely to show a significant effect for a matching instruction condition as for a counting condition. Furthermore, scoring systems utilizing only Piagetian operational justifications will obviously fail to show any of these effects. The results of this study indicate that counting is not necessary for correct performance on a conservation of number task (matching was also effective in producing correct performance), and neither is it sufficient (5 of the 16 children on the first trial Counting condition answered the conservation question incorrectly). However, counting apparently was sufficient for a fairly large percentage of the children, in that conservation was achieved by 55% more children in the Count than in the Standard condition. Furthermore, children continued to use counting in subsequent standard conservation situations. Thus the relationships between counting and conservation of number would seem to be complex, and a future search for a simple pre-requisite relationship is probably doomed to failure. But neither does it seem sensible to ignore in the future the considerable effectiveness of counting in the conservation situation. Counting has been dismissed in the past as soon as some demonstration of its insufficiency was made. Thus, for example, Flavell (1963) emphasized the examples reported in Greco (1962) indicating that counting was not sufficient for conservation. Flavell included the following quote:

> "the child will count two sets and state that there are "seven" here and "seven" there, whatever their spatial arrangement, but he may nonetheless argue that there are more here than there, if the arrangement is perceptually compelling in favor of inequality." (pp. 360-1)

What was neglected here were the findings reported by Greco that having the child count increased correct responses to the conservation question considerably. Correct answers for 65 children aged 4½ to 7 increased from 13 to 21 on one task, from 22 to 44 on a second, and from 20 to 32 on a third task.[1]

The present study has examined only one aspect of the conservation of number situation -- the ability to respond correctly to the equivalence question. Piaget was chiefly concerned with conservation as a test of children's conviction of the logical necessity of the maintenance of equivalence over the transformation. The use of an empirical strategy to establish the post-transformation equivalence in particular cases does not reflect such a conviction of logical necessity. Nor does the demonstration of the usefulness of these empirical strategies in establishing the equivalence in these particular cases establish that either of these strategies are spontaneously used by children to construct this conviction of logical necessity. However, the demonstrated effectiveness of counting and matching indicates that either of them might well be used to construct such a conviction. The existence of both of these effective means of establishing equivalences also indicates that children may, in fact, traverse alternative routes to this conviction of the logical necessity of the maintenance of equivalence after a spatial transformation.

The effectiveness of these two empirical strategies raises another more general possibility. The conservation of number task is the earliest conservation task to be acquired. It is also the only task involving discrete rather than continuous quantities, and thus the only task which can be answered using counting or matching strategies (except for special methods in, for example, conservation of matter and liquid in which small bits of clay or beads in glass containers are used). The

1. We are indebted to Dr. Constance Kamii for the table from the Greco work from which these numbers were calculated.

availability of these strategies may account for this earlier acquisition of conservation of number, and their use may pave the way for the acquisition of the other conservation concepts.

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Table 1

EMPIRICAL JUSTIFICATIONS

Counting: the child counts -
a notation that the child was counting on the experimenter record
the child counts out loud (on tape)
"there's seven (7)"
"I counted"

Active Pairing: the child matches individual objects in the sets when they are not in physical/perceptual correspondence -
notation on record that says "child matches" (when the animals were pulled apart;
notation that child indicated pairs (when animals were pulled apart)
"I used the strings"
"Each has got one" (and the animals are pulled apart)

PIAGETIAN JUSTIFICATIONS

Add/Sub: states that a specific action which would have changed the number of either set did not happen -
"you didn't add any"
"you didn't take away any"

Reversibility: the action which occurred can be undone, and nothing will be changed -
"you could move them back together again"

Identity: that things have remained the same
"they're still the same"
"you didn't change them"

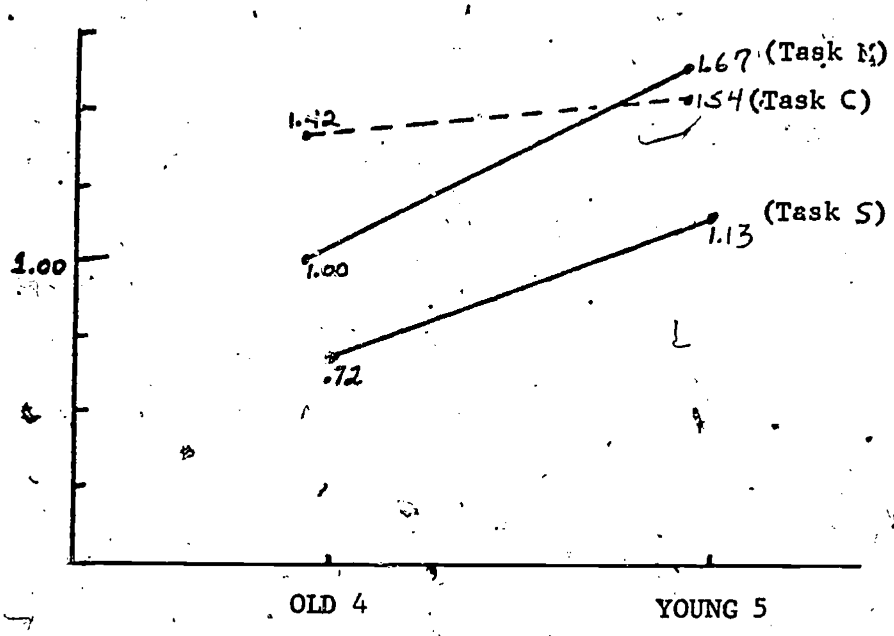
Compensation: a perceptual change is compensated by another perceptual change
"you moved them and this one doesn't have a peanut; but look at this hole, this peanut doesn't have an animal."

VISUAL JUSTIFICATIONS

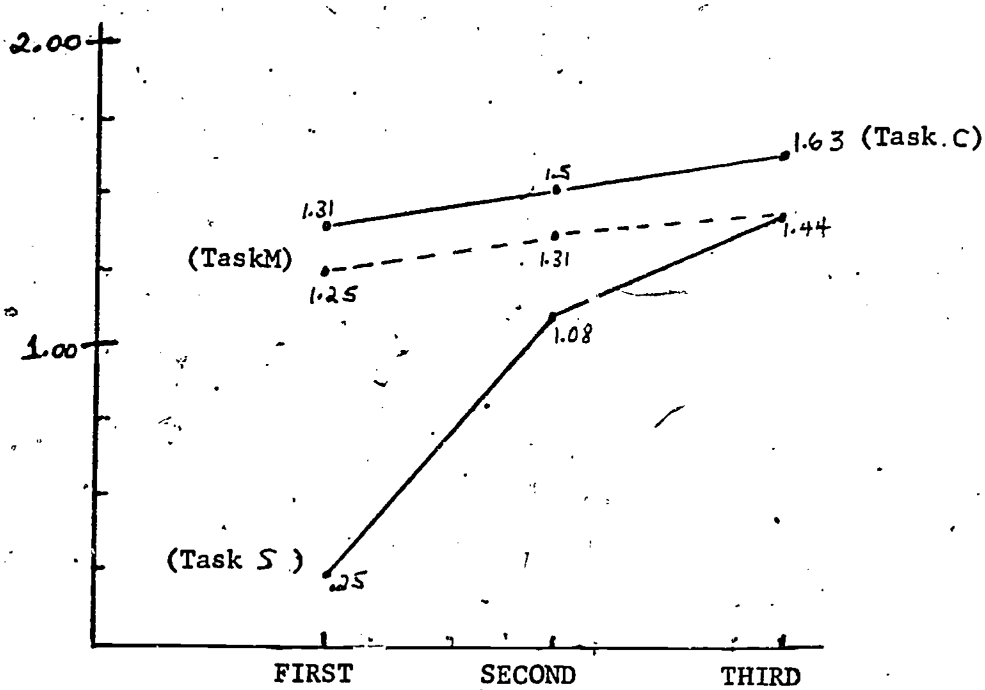
Length: the child attends to the length of the array -
"they're both just as long"
"this one's longer"
"this one's shorter"
"there's more room here" (indicating the end)
"there's less room here" (indicating the end)

Place Correspondence: the visual array is what is attended to -
"they each have one" (when the arrays are put in obvious correspondence)
"there are more/less here." (pointing to a place in the line)
"this animal (or its name) doesn't have a peanut"
"this peanut doesn't have an animal"

AMBIGUOUS/CANNOT TELL: where we really cannot tell what the child was doing -
Child answers "I don't know"
"cause I'm smart"
tape is unintelligible



<u>Figure</u> <u>1</u>: Age x Task Interaction(F=3.06; p= .054)



<u>Figure</u> <u>2</u>:Order x Task Interaction (F=5.04; p< .001)