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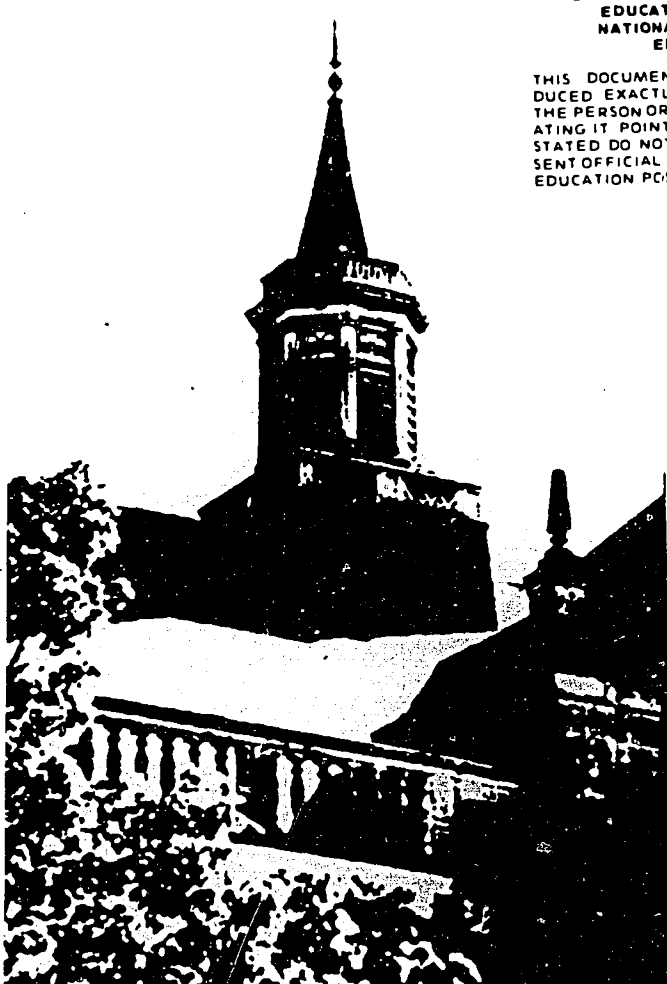
ABSTRACT

An attempt is made to define the meaning of invention and innovation in mathematics education in order to help develop guidelines for policies of research and development in this area. Spectrums of invention in the realms of geometry and group theory are proposed as models of ordering understanding of creative activity within mathematics. This document outlines areas for future research and study, and promotes the concept of international cooperation as critical in the development of criteria or standards promotion and administration of creative activity. (MP)



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Creative Activity in Mathematics Education: A First Attempt at Definition and Problem-Identification

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CREATIVE ACTIVITY IN MATHEMATICS EDUCATION: A FIRST ATTEMPT AT DEFINITION AND PROBLEM-IDENTIFICATION*

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I. Innovation and Invention in Mathematics Education

. . .elementary algebra (EMA) can be considered as reconstruction of modern algebra on an elementary level. This reconstruction is not a trivial by-product of the development of mathematics in general, but needs particular effort, the organization and performance of which is the task of didactics of mathematics.

H. G. Steiner (1974)

To begin writing [Elements de Mathematique], Bourbaki had to devise mathematical goals which his readers were expected to reach, and then he had to devise appropriate mathematical paths to these goals. From existing mathematical treatments he had to devise suitable language, especially by selecting suitable definitions, so that he could then deduce all of the mathematics he wanted, exposing it for his readers in a logical and beautiful way. To do all this involved immense toil and presumably some false starts. . .

H. B. Griffiths and A. G. Howson (1974)

Each of the above statements describes activity displaying what can be termed creative, mathematically-derived innovation and discovery within mathematics education. They refer to exploration of principles, structures and conclusions within mathematics which, when rendered in a form appropriate to the realities of the class-

*This paper draws on suggestions made at a working group on invention in mathematics education consisting of P. Damerow, H. G. Steiner, H. J. Vollrath, A. G. Howson, and I. Westbury. F. Schrag of the University of Wisconsin at Madison offered helpful comments on an earlier version of this paper.

room and school, have educational or pedagogical utility. Of course, different conceptions of education and different traditions of curriculum development have their different accompanying forms of creative activity, but in any case the task that the creative worker undertakes might be regarded as analogous to the processes of innovation and invention within technology. Such an analogy suggests an analytical separation of 'invention' and 'innovation' from research and applied research, with the implication that it is the aim of invention and innovation to effect durable, workable changes in ongoing states of affairs. By exploiting the 'inventive' connotations of the term 'invention' such an analogy focusses on the difference between routine and creative development. As Usher (1955) suggests

> the distinction between acts of skill and inventions is suggestively drawn by Gestalt psychology. Novelty is to be found in the more complex acts of skill, but it is of a lower order than at the level of invention. As long as action remains within the limits of an act of skill, the insight required is within the capacity of a trained individual and can be performed at will at any time. At the level of invention, however, the act of insight can be achieved only by superior persons under special constellations of circumstances. Such acts of insight frequently emerge in the course of performing acts of skill, though characteristically the act of insight is induced by the conscious perception of an unsatisfactory gap in knowledge or mode of action.

Assuming that these terms describe something, the case for considering 'invention' and 'innovation' as important concerns for R & D policy within education is clear: Are there policies that would encourage and support such activities within mathematics education? Are there rates of invention discernible within mathematical education that are associated with organizational factors within the social systems that surround the field? Is there a set of common processes, or a spectrum of operations ('discovery, through processing, to application'), which can order our understanding of such inventive activity? And do findings about such matters have implications for how research and development in mathematics education is ordered and administered?

While these might be good questions, their answers depend on a clear definition of what we mean by 'invention' and 'innovation'. Reaching such agreement upon a definition has been a problem whenever attempts have been made to ask questions of these kinds. While it is seemingly easy to articulate distinctions that have reasonable intuitive power, it is hard to pin these intuitions down:

> . . .it is transparently clear that our present knowledge of the historiography and social studies of technology are inadequate to satisfy the needs of policy-making theories. Current ideas are often naive and contradictory even in such basic theoretical infrastructures as 'technology and technique', 'invention and innovation', and above all in the still mystical relationship between science and technology. What we need is knowledge . . . to provide a means of relating to the social environment all that we presently understand about the substantive developmental changes in technical practice (Layton, 1977)

How can we begin this task of breaking through these ambiguities within mathematics education? Clearly the first task is the development of a firmer understanding of what it is that we might be exploring.

II. Invention with mathematics education

Before undertaking any attempt at a definition of 'invention' within mathematics education, let us consider two particular fields of mathematics, geometry and group theory and see how one might develop a spectrum of 'invention'.

Hilbert's Grundlagen der Geometrie (1909) was surely an 'invention' within mathematics.* It sets out to answer questions raised within mathematics and it offers solutions within mathematics. It contains, however, no concessions to pedagogy: there are, for example, no exercises for the reader intended to increase and test his understanding.

*Platonists might prefer 'discovery' here; yet even if one takes a Platonic view, the mathematician still has to 'invent' the language needed to communicate the concept.

Halsted's *Rational Geometry* (1907) is an attempt to recast school geometry from a Hilbertian rather than an Euclidean point of view. It is therefore a pedagogical and curricular response to Hilbert, and so would appear to lie within the domain of mathematics education. But is it an 'invention'? 'Yes' is the obvious answer, but we are forced to examine our terminology. Is 'invention' with its technological connotations exactly the right word? Would 'creative activity' serve our purposes better? What we have in Halsted's *Rational Geometry* is a 'translation', a 'reinterpretation', and a 'filling-out'. Are these the activities which are at the core of 'creative activity' in mathematics education? Let us explore that possibility.

In 1932 Birkhoff published his paper "A set of postulates for plane geometry" in *Annals of Mathematics*. His choice of a medium of communication places this 'invention' firmly at the mathematical end of a spectrum of creative activity in mathematics education. The message is addressed to fellow mathematicians and is couched in their language. Nevertheless, there is an essential difference between Hilbert's and Birkhoff's axioms: Those of Birkhoff appear to have been chosen with pedagogical considerations in mind. Not surprisingly, therefore, these axioms have lent themselves more readily than Hilbert's to *translation* into classroom terms, first by Birkhoff and Beatley (1941), then by SMSG.

We can, therefore, construct the following rough spectrum of this 'creative activity' within geometry:

Halsted	Birkhoff	Hilbert
Birkhoff and Beatley		
SMSG		

If we wished, we might attempt to find phrases to describe positions on this spectrum: on the right we have 'mathematical ideas' in the center 'the drawing together and elaboration of mathematical ideas' and on the left 'the adaptation/translation of mathematical ideas'. However, our choice of examples--an axiomatic approach to geometry--has led to a filtering at the bottom end of our spectrum! What happens if we consider transformation geometry?

Here, perhaps, we can take Klein's Erlangen program (1872) as a right-hand marker. Somewhere to his left comes G. Thomsen and his 'The treatment of elementary geometry by a group-calculus' (1933) -- a paper which expresses mathematical facts in a form still remote from the classroom, but in language which should reach a wider audience than 'the professional mathematician'.* Thus, the paper appears not in the Annals of Mathematics but in the Mathematical Gazette. But the dangers inherent in any attempt to linearize invention (or most other things), become apparent when one attempts to find a place for Bachmann's Aufbau der Geometrie aus dem Spiegelungsbegriff (1959). For the moment let us not worry over much about how this book stands in relation to Thomsen: it can be agreed that Bachmann's work fits well to the right of our spectrum, to the right of, say Jeger's Konstruktive Abbildungsgeometrie which is, in its turn, to the right of, for example, Maxwell's Geometry through Transformations (1975) on the grounds that pedagogical considerations are given more emphasis in the latter.

SMP Books 1-5 represent a different kind of creative activity. These books reflect a concern for the provision of practical, real-life embodiments for mathematical ideas. Similar work can be readily found in books by other authors. Another type of invention resulted from the manner in which early attempts to teach transformation geometry were hindered by the difficulty children found in drawing and observing 'reflections'. This problem was eased by the 'invention' (in the conventional sense) of Mira-math

*See also Thomsen (1932).

-- a simple piece of classroom apparatus intended to facilitate practical work. But what kind of 'invention' is this, or Marion Walter's 'mirror cards'? In both cases pedagogical considerations are foremost; they must then take up positions on the far left of the spectrum. (How, also, are we to classify Reflection, a game produced by Otto Maier Verlag, Ravensburg? This makes no claim to be connected with mathematics education, but probably teaches certain aspects of geometry far more effectively than most text-books.)

A transformation geometry spectrum could, then, be:

Finding new embodiments	Adaptation of ideas	Drawing together of ideas	Mathematical ideas

Walters Mira-math	SMP 1-5	Maxwell	Jeger	Bachmann	Klein (1872)

That the kind of classification suggested above the line on our 'transformation geometry' spectrum might be a useful way of ordering creative and inventive activity in mathematics and mathematics education is seen when we try to construct a similar spectrum for group theory. Here we have certain mathematicians, Cayley, Lie, Schur, Schreier, etc. who supply mathematical ideas. Other mathematicians, Jordan, Burnside, Kurosh, etc. draw together these ideas and present them in a form which their mathematical peers find useful or in a form which helps train a new generation of specialists. Other authors, van der Waerden, Birkhoff and MacLane, etc. incorporate such work in books intended to reach and inform wider audiences. (We recall that the first edition of Birkhoff and MacLane was not reviewed in the American Mathematical Monthly on the grounds that such advanced mathematics was not the concern of its readership.) In recent years any number of 'new math' texts have attempted to translate group theory into schools (This reminds us that all such creative activity is not necessarily 'good' and that there is a need to consider more closely what is meant by 'translation' and, to borrow from the parlance of group theory, what constitutes a 'faithful translation'.) Finally, we have had several attempts to provide embodiments of group theory, ranging from the simple observation that when turning a mattress we are exemplifying Klein's four group, to embodiments and games -- occasionally extremely contrived --to be found in, for example, the work of Dienes or in 'permutation' cards.

Our considerations so far would seem to suggest that within mathematical education curricular 'invention' takes two major forms:

a. translation - the process of 'making accessible' (See Kirsch, 1977)

b. the provision of embodiments

These, in their turn, can have varying relationships to different parts of mathematics and can exploit the potentialities of different media and modalities (e.g. enactive, iconic and symbolic). And, further, all such attempts at translation and/or embodying must be considered in terms of their classroom practicality. Thus, authentic curricular 'invention' must confront pedagogical considerations and must face the problem of working out its propositions in classroom detail, e.g., the question of what kind of exercises we expect students to be able to attempt.

In other words, it is clear perhaps that there is a distinctive kind of creative activity associated with work in mathematical education but it is also evident that much more must be done before we can say that we have even sketched all of its forms and located all of its dimensions.

III. Questions About Invention in Mathematics Education.

Any consideration of inventive or creative activity within mathematics education rests on a series of assumptions. We assume, for example, that it is desirable that developments in mathematics in schools resonate with developments in mathematical understanding and that such resonance authentically matches the needs of schools. The analysis outlined above suggests further that the development which leads to such advances in school mathematics is not a trivial task; the 'facile translation' of advanced mathematics will not serve the schools and 'inventive' or creative work is required to bring advances in mathematics within the orbit of the schools. And, as we seek to bring out understanding of these issues into a structure of discussion, we need to invoke the further assumption that it is possible and necessary to talk about the 'productivity' of creative activities by mathematics educators.

If we can assume that such assumptions as these have merit, and if the notion of productivity in particular can be explored in terms of the "inventive productivity" of communities or institutions of mathematicians and mathematics educators, a set of considerations bearing on policies for the organization of research and development in mathematics might be seen to emerge. Two sets of such considerations would seem to be important. First, what is the relationship between the state and health of research in mathematics in a social system or institutional structure and the state and health of mathematics education in that same system. Second, what is the relatonship between mathematics and mathematics education, and, in particular, is creative activity in mathematics educational a quite different thing -- with all of the implications that might follow for training programs and careers -- than mathematics research?

Given the ways in which the spectrum of creative activity outlined above has been developed it would seem clear that mathematics itself is an indispensable resource for approaching problems in mathematics curricula: It is less clear perhaps that there is a necessary relationship between active inquiry in mathematics in a particular intellectual community and developments in mathematics education. This kind of issue would seem to be worth investigating. One might, for instance, speculate that creative activity centering on the curriculum is restricted de facto to countries and/or institutions that are significant contributors to research in mathematics and puzzle over the implication that such a relationship is the result of causal factors.

However, it would seem likely that, while such relationships might be causal, something more than creative mathematical activity is required to stimulate systematic productivity in mathematics education. It is this consideration which brings us to the issue of the terms which one might use to describe creative curriculum development. To this point we have oscillated between the terms 'creative activity' and 'invention'. The appropriateness of one such term over another as a summary descriptor of the activity that interests us is not easily resolvable in principle but in practice an emphasis on the utility of 'invention' permits the recognition that to be successful creative curricular activity involves linkage activity analogous to that between different subsystems in technology: In the case of technology these can be thought of in terms of science, technology (or ways and means) and the economy (Aitken, 1976). In the case of curriculum in general these subsystems would be the research disciplines, the ways and means and constraints belonging to the school as a sociotechnical system, and the contextual milieu and the consequent demands it makes on a subject like mathematics. In either case an act of invention involves a transfer of information, and an individual or a group of individuals making a transfer, between these subsystems towards some practical and practicable end. (Smolimowski, 1974).

But these are speculations that call for investigation by way of

1. historical case studies of particular inventions and episodes of invention, and
2. studies thrusting at the characterizaton of conditions which affect processes of invention with their implications for institutional and system development.*

One immediate outcome of (2) could, for example, be a investigation in the light of a criterion of inventive activity of a nation's (or an international) R and D system within mathematics education:

International studies would seem, prima facie, to be critical to such endeavors. There would appear to be significant differences between countries in patterns of academic and 'mathematics education-system' organization which would affect propensities to inventive activity. Such patterns should yield understanding of the effectiveness of the different organizations and research centers of different kinds) and so permit an examination of a large variety of different organizational forms for their utility for different tasks. Likewise such comparative studies would facilitate the pursuit of cultural factors which have different effects on the developmental and inventive function. Figure 1, for example, outlines a set of institutional inputs to science, not all of which might be found in any country or at any moment of time but which might be salient as one considers the overall picture. Needless to say, such an array of possibilities bears on policy issues -- given a determination by individuals, institutions, and/or governments that invention and inventive activity is an important task of the social systems which surround mathematics education.

*For models of such investigations, see Battelle (1974) and Lemaine et al. (1976).

These are, of course, issues for the future. We have, little systematic understanding within education of the nature of inventive activity and inevitably an even more limited understanding of how planning and organization might facilitate improvements in rates of invention or subsequent differentiation and implementation. The hope is that these questions might be opened up along the lines suggested by a working group on 'science indicator studies' (Elkana et al., 1978) as they planned a conference on science indicators:

> we were far from expert either in social indicators or in the quantative appraisal of current science. . . Our reluctance was mitigated by the thought that this would be the first venture into the applied historical sociology of scientific knowledge. Our invitation to the conference stated: We should like to pose the question, "What must one look at in order to estimate the condition of science as an intellectual activity or as a social institution?" We think of this question within a broad historical and sociological frame rather than from a delimited point of view dealing with the present inputs and outputs of science measured in terms of men, money and materials. We think that our discussions of Science Indicators should be problem-oriented. At best, we will be starting an on-going activity, designed to enlarge the scope and conceptual framework of thinking about science. (p. 14)

Our goals must, of course, be even more tentative than these; education does not have a clear analogue to the somewhat halting field of science policy. We could not, for example, write a sentence such as Ziman's in Towards a Metric of Science Indicators (1978):

> No significant systematic effort has been made so far, to determine whether the studies of, say, Joseph Ben-David on relations between socio-organizational characteristics of academic systems and the growth of science, or of Robert Merton on relations between the institutionalization of science and prevailing social values, or of Warren Hagstrom on social mechanisms regulating scientific activity can help identify indicators for the state of science and the ways in which their state could be affected by alternative public policies.

The understanding of the understanding Ziman is reflecting is the task facing us and perhaps this goal is best put in terms of developing a brief for later work that is more considered than this one. The details of that brief are likewise probably best expressed by a paraphrase of the terms used by the Palo Alto conference: the goal of the consequent deliberation should be enlargment of the scope and conceptual framework of thinking about mathematics education as a creative activity affecting teaching and curricula in the schools.

References

Aitken, H. G. J — Science, Technology and Economics: The Invention of Radio as a Case Study. In: W. Krohn, E. T. Layton and O. Weingart (eds.) The Dynamics of Science and Technology. Sociology of the Sciences: A Yearbook, Vol. 2 Dodrecht 1976.

Bachmann, F. — Aufbau der Geometrie aus dem Spiegelungsbegriff. Berlin 1959.

Battelle — Interactions of Sciences and Technology in the Innovative Process: Some Case Studies. Final Report prepared for the U.S. National Science Foundation. Columbus, Ohio, 1973.

Birkhoff, G. D. — A Set of Postulates for Plane Geometry. Annals of Mathematics 33(1932), 329-45.

Birkhoff, G. D. and Beatley, R. — Basic Geometry. Chicago 1941.

Elkana, Y. et al. — Towards a Metric of Science: The Advent of Scientific Indicators. New York 1978.

Griffiths, H.B. and Howson, A. G. — Mathematics: Society and Curricula. Cambridge 1974.

Halsted, G. B. — Rational Geometry: A Text Book. New York 1907.

Hilbert, D. — Grundlagen der Geometrie. Leipzig 1909

Kirsch, A. — Aspects of Simplification in Mathematics Teaching. Proceedings of the Third International Congress on Mathematical Education, ICME, Karlsruhe, 1976.

Kochen, M. — Models of Scientific Output. In: Y. Elkana et al. (Eds.) Towards a Metric of Science: The Advent of Scientific Indicators. New York 1978.

Layton, E. Conditions of Technological Development. In: Ina Spiegel-Ross and Derek de Solla Price (Eds.) Science, Technology and Society: A Cross-Disciplinary Perspective. London 1977, 197-222.

Lemaine, G. et al Perspectives on the Emergence of Scientific Disciplines. Maison des Sciences de l'Homme, 4. The Hague, 1976.

Maxwell, E.A. Geometry through Transformations. Cambridge, 1975.

Smolinowski, H. The Structure of Thinking in Technology. In: F. Rapp (Ed) Contributions to a Philosophy of Technology. Dodrecht, 1974.

Steiner, H. G. Elementary Modern Algebra as a Significant Compontent in Secondary School Mathematics Curricula. Proceedings of ICME-JSME Regional Conference on Curriculum and Teacher Training for Mathematics Education. Tokyo 1974.

Thomsen, G. Ubereinen neuen Zweig geometrischer Axiomatik und eine neue Art von analytischer Geometrie. Mat. Zeitschift 34(1932), 667-720.

Thomsen, G. The Treatment of Elementary Geometry by a Group-Calculus. Mathematical Gazette 17 (1933), 230-42.

Usher, A. Technical Chage and Capital Formation. In National Bureau of Economic Research. Capital Formation and Economic Growth. New York, 1955.

Ziman, J. From Parameters to Portents -- And Back. In: Y. Elkana, et al (Eds) Towards a Metric of Science: The Advent of Science Indicators. New York 1978.