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AUTHOR Ashcraft, Mark H.; And Others
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ABSTRACT

Research on the structures and processes involved in simple mental arithmetic is reviewed. Results suggest that mental arithmetic is a memory retrieval phenomenon, which develops with the experiences of elementary school and can be understood in general as retrieval from organized network representations. Simple mental addition with basic facts was examined in children from grades 1 and 5 and in college students. The data indicate that two models are necessary to describe the development of addition performance. These models are a simple counting framework for children in the early stages of mastery, followed by the network retrieval model from about the fourth or fifth grade on. The most basic result in studies of mental arithmetic is the problem size effect--a regular increase in reaction time (RT) as the numerical size of the problem increases. The RT performance varies with task (verbal production vs. verification), problem size, and age. The report recommends use of the true/false verification task in future studies of the psychology of arithmetic performance, as data generated from such tasks is more amenable to information processing models. (MP)

The Development of Network Retrieval in Addition

Mark H. Ashcraft, Bennett A. Fierman, and Mary S. Hamann

Cleveland State University

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Abstract

Simple mental addition was examined in grade school children (grades 1 and 5) and college students. Reaction time performance varied with task (verbal production vs. verification), problem size, and age. Priming manipulations also permitted a developmental evaluation of the Ashcraft network-retrieval model of mental arithmetic.

Note. Experiment 1 formed the basis of an M.A. thesis by Bennett A. Fierman, Department of Psychology, Cleveland State University, August, 1980. We wish to thank the principal and teachers of Boulevard School, Cleveland Heights, Ohio, for their cooperation in this project.

For the past several years, we have been conducting research on the structures and processes involved in simple mental arithmetic. Our paper at last year's Psychonomics convention, entitled "Network Representations of Mental Arithmetic", summarized much of this research, and as the title implies, suggested that mental arithmetic can be fruitfully investigated as an instance of retrieval from organized, long-term memory. To quote from our conclusion, "Taken as a whole, these results suggest that mental arithmetic is a memory retrieval phenomenon, one which takes time, one which develops with the experiences of elementary school, and one which can be understood in general as retrieval from organized network representations." (Ashcraft, Stazyk, & Fierman, Note 1).

Today's presentation focuses on the development of this memory retrieval phenomenon in children, and does so in the context of ordinary simple addition. We are interested in addition here for a variety of reasons: It is the first formal mathematics topic encountered in most school curricula; it is the most heavily investigated operation in psychological studies, and it has given rise to the major models of arithmetic performance. In general, the research we are presenting today indicates that two models are necessary to describe the development of addition performance, a simple counting model for children in the early stages of mastery, then the network retrieval model from about the fourth or fifth grade on. We also will present evidence which indicates that a standard true/false verification task yields relatively more analytic results than the seemingly more natural task of verbal production.

The most basic result in studies of mental arithmetic is called the problem size effect--there is a regular increase in reaction time (RT) as the numerical size of the problem increases. In fact, we know of no study which fails to demonstrate this effect in one fashion or another. The form of this increase in RT, and the associated variables and slopes which predict it, lead of course to inferences about the underlying mental processes.

Groen and Parkman, in 1972, found a linear problem size effect for a sample of first grade children. These children were shown simple addition problems like 4 + 3, and were required to press one of ten buttons to indicate the correct sum. In multiple regression analysis, their RTs were best predicted by the smaller or minimum addend, the 3 in the problem 4 + 3. This pattern of results suggested a counting model, termed the min model due to the importance of minimum addend. For the problem 4 + 3, the mental processes correspond essentially to the following sequence: 4.., 5,6,7. In other words, a counter is set to 4, then incremented by ones as a function of the min. Incrementing time in this study was estimated at 400 msec per increment.

This "counting-on" model works very poorly for adults, however. Our results in a verification task (Ashcraft & Battaglia, 1978; Ashcraft & Stazyk, in press), as well as those of Groen and Parkman (1972), show much more rapid processing for adults, with unreasonably fast time estimates for incrementing under the min model assumptions. Furthermore, our research has consistently yielded exponentially increasing RT patterns, rather than linear increases, quite damaging to the counting model. Accordingly, we have proposed a memory retrieval model, in which the size of the problem is a rough index of distance traversed during search through a network structure.

Adherents to the min model have suggested that our persistent use of the verification task may be introducing an artifact into the RT patterns; that the need to compare a correct sum with a possibly wrong one in the stimulus may in some way be producing the exponential function. In fact, there is this systematic confounding in the literature--studies with adults typically use true/false verification, and those with children use production.

Ben Fierman and I decided to tackle this confounding directly, testing first graders, fifth graders, and college students, with both a production and a verification task (see Fierman, Note 2). Our intention was not simply to clear up a confusion in the literature, but more importantly to try to determine which of the two tasks yields more representative and useful information about arithmetic processing.

Consequently, this first experiment tested children and adults in a two session experiment. One of the sessions involved production of the correct answer--a stimulus was shown, and the subject had to produce the sum verbally. Notice that this production task avoided the button-response difficulties inherent in Groen and Parkman's 1972 study, and instead presented the children with a reasonably natural, classroom-like task. The other session was a true/false verification task, in which the subjects responded verbally to problems which included either a correct or incorrect sum. We expected a constant time difference between the tasks, in that verification requires a decision/comparison stage which is absent in production; but we were curious about possible interactions of task with both age and problem size.

Figure 1 presents the significant interaction of just these variables. Notice the excessively large RT range on the ordinate--this gives the impression of no problem size effects for fifth graders and college students, although in fact this effect was highly significant at all three grade levels. The obvious source of this interaction was the departure from parallel functions at the first grade level. At older ages, these data suggest that there is in fact only a constant time difference between production and verification, probably due to the decision stage. At the first grade level, however, the production task seems to yield an advantage in RT, but only for small problems.

This makes perfectly good sense; first grade teachers in this school system must teach the small problems, that is those problems with sums up to 9, and typically rely heavily on verbal drill for this purpose. Thus at the first grade level, the production task matched normal classroom procedure for small problems.

At this point, you might think that we would conclude that the production task is preferable--after all, it matches first graders' familiar experiences, and yields comparable and slightly faster results at older ages. In fact, we conclude just the opposite, based on the more detailed analyses provided by multiple regression. For each grade and task combination, RTs and a set of structural variables were analyzed with forward, step-wise multiple regression, in order to determine specific variables and slopes which account for the problem size effects we found.

When RTs for the true problems under verification were analyzed, we found strong evidence for counting processes by first graders, and retrieval processes for older subjects. That is, the minimum addend provides the best RT prediction at the first grade level, accounting for 62% of the variance, with a slope of 516 msec. For fifth graders and adults, the exponential factor we have found before, correct sum squared, was the best predictor, beating out minimum addend by a significant amount. This pattern is critically important, since the significant exponential factor is both a disconfirmation of the major prediction from the counting model as well as an important prediction of the network retrieval model. Putting it bluntly, fifth graders seem to resemble adults, and both groups appear to be retrieving addition facts from a network representation (see also Ashcraft & Fierman, Note 3).

Turning to the data from the production task, we find that the best predictor variable at all three age levels was minimum addend. By itself, such an outcome would suggest that some min-like counting model accounts for the entire developmental range quite nicely. When considered in the light of the verification condition here and other studies from our lab, a different implication emerges. This different implication is that the production task, rather than verification, may be suspect. In other words, the production task here seemed to generate data which were consistent with a counting model at all ages, in direct contradiction to other research findings. The verification task, on the other hand, yielded data which are entirely consistent with other research--counting processes early in the school years, and memory retrieval later on.

Leaving aside the issue of possible artifacts, there is another dimension on which to base a preference for verification. Our feeling is that the verification task in many ways provides more useful and analytic data with which to understand arithmetic processes. Its advantage is due to the simple fact that an answer is presented in the stimulus; we can manipulate the onset of the answer, and in false problems its closeness to the correct answer.

We have predicted elsewhere (Ashcraft & Stazyk, in press) that a network representation of arithmetic, and a fact retrieval approach to processing, imply a set of results analogous to those found in studies of semantic memory. To take a particularly clear example, we have found very compelling evidence of network relatedness/similarity effects (Stazyk, Note 4). For today's presentation, we were concerned with another kind of semantic memory variable, priming.

Mary Sue Hamann and I decided to use a priming manipulation which involves providing advance information to the subject, in order to determine how much of the typical RT can be attributed to search or retrieval. We presented the addends of a problem either 500 or 1000 msec before the sum, in order that an approximation to search time could be made. Of course, this sort of manipulation is only possible in a verification task. We further manipulated the split or incorrectness in the false problems--the wrong answer could be off by a small, medium, or large amount (± 1,2 vs. 6,7 vs. 12,13). This manipulation is also only possible in a verification task.

The left panel of Figure 2 reveals the RT patterns to simple addition problems when adults were tested with no advance information. Reaction times for true problems range from 800 to about 1100 msec. These times are elevated when an incorrect problem has a small split--that is when its answer is wrong by only 1 or 2; when the problem has a large split, we typically find performance which is as fast, if not faster, than that to true problems. What we find fascinating in these results are first the patterns obtained with advance information in the other two panels, and second the implications of those patterns.

As expected, a full second of advance information prior to the onset of the sum yields hardly any variation in RT. These trials seem to reveal complete preparation for the upcoming sum, complete enough that the distractor problems, even with small splits, are in fact not particularly distracting. Focusing on the center panel, however, we are confronted with much more complex performance. The function for true problems here seems to show reasonably complete preparation--there is hardly any problem size effect, and one might be tempted to claim that preparation was in fact complete within the half second interval. The dotted function labeled SS (for small split) suggests that such a conclusion would be in error. In fact, preparation was not so complete that the subjects avoided the confusion of a slightly incorrect answer. Imagine how different our thinking about addition would be if there were no split conditions here--that is, if there were only the data from a production task. While the two tasks do yield comparable data on true problems, it is clear from these verification data that preparation is not entirely completed within 500 msec--the answer may have been retrieved, but there is still an element of uncertainty about it within that short an interval.

We draw two basic conclusions from these studies, one methodological and one developmental. First, in psychological studies of arithmetic performance, we recommend using the true/false verification task. It permits useful manipulations to be made, and yields data which are more amenable to information processing models. While verification does add a decision stage to the processes found in a production task, this decision stage is quite analytic for other purposes. Second, simple mental addition is initially a counting process in young children, thus capitalizing on the informal numerical knowledge brought into first grade (Ginsburg, 1977). Further development of addition, however, is not a speeding up of the counting procedure. Instead, by at least the fifth grade, if not earlier, children's performance seems best characterized as a process of memory retrieval from organized, network structures.

Reference Notes

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4. Stazyk, E.H. A Network Approach to Mental Multiplication. Unpublished M.A. Thesis, Department of Psychology, Cleveland State University, June, 1980.

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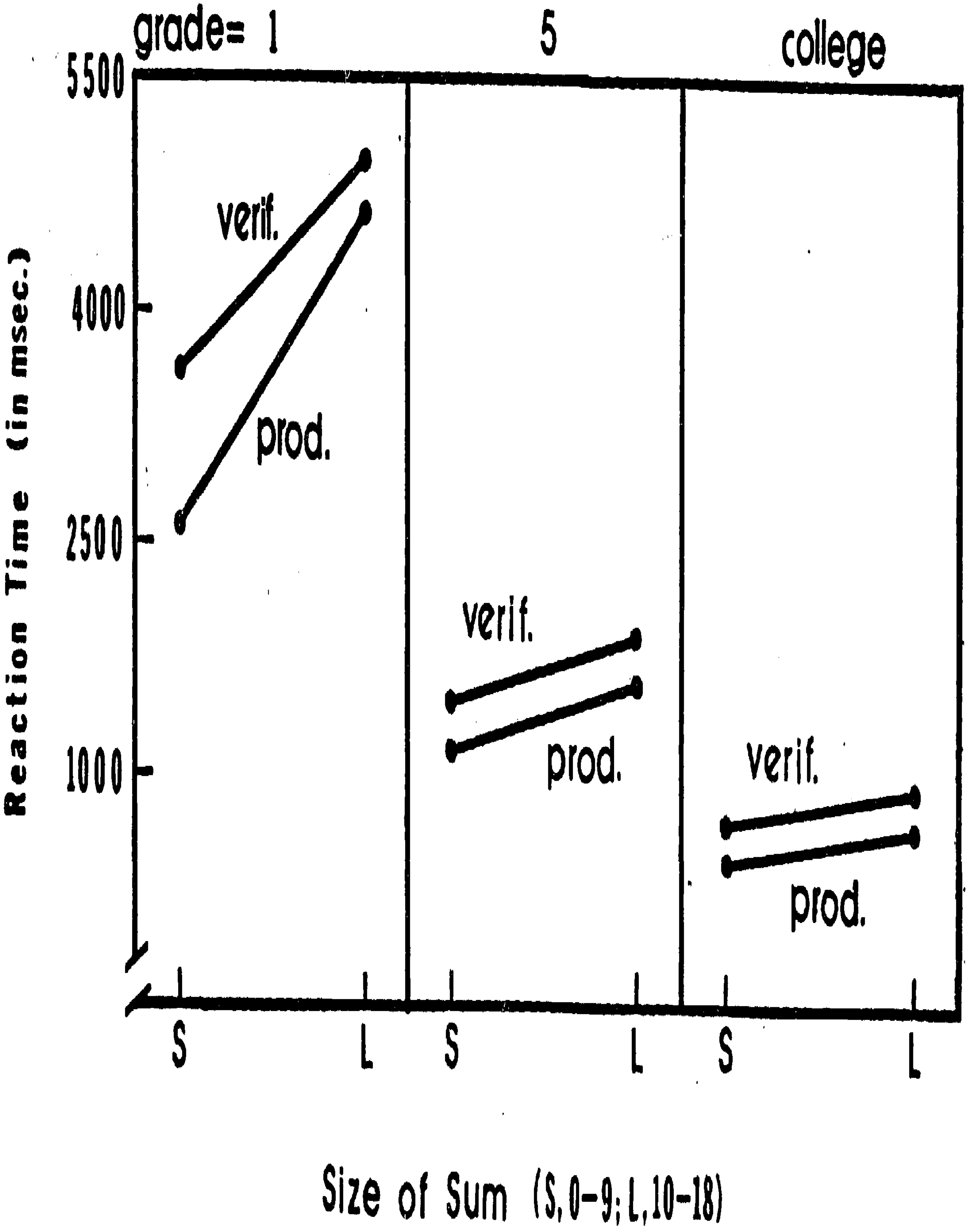
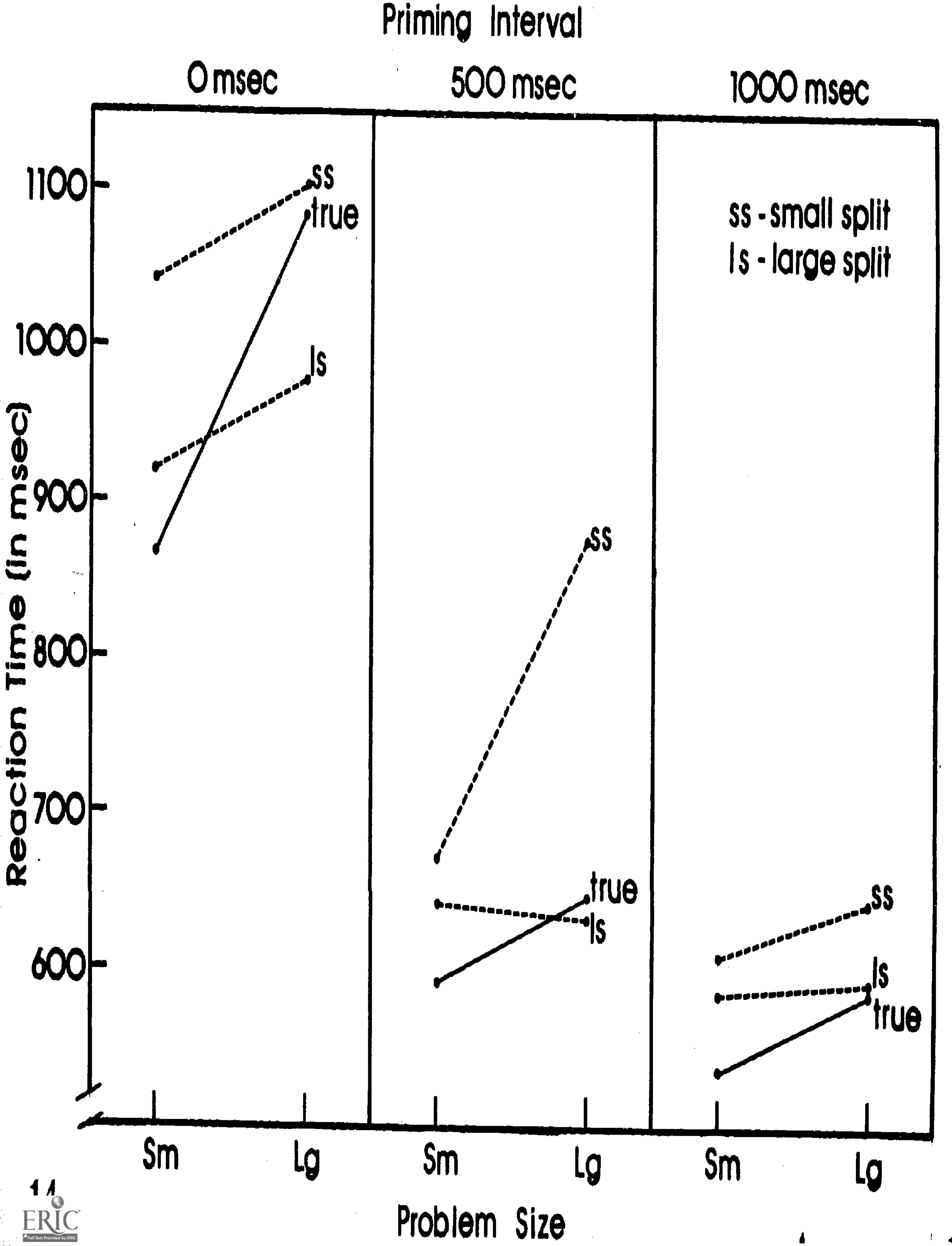
Figure Captions

1. Mean RT to simple addition problems presented either for verbal production of the sum or verbal verification of correct/incorrect.
2. Mean RT to simple addition problems presented for verification by adults; sums were presented simultaneous with the addends (0 msec), or 500/1000 msec after the addends. The medium split condition functions fell between the large and small split functions, and were omitted for clarity of comparisons.


grade= 1
5
college
5500
4000
2500
1000
Reaction Time (in msec.)
verif.
prod.
verif.
prod.
verif.
prod.
S
L
S
L
S
L
Size of Sum (S, 0–9; L, 10–18)



Priming Interval
0 msec
500 msec
1000 msec
Reaction Time (in msec)
1100
1000
900
800
700
600
ss
true
ls
ss - small split
ls - large split
Sm
Lg
Problem Size