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ABSTRACT

Thirteen research reports related to mathematics education are abstracted and critiqued in this publication. The topics of the research include counting, addition, subtraction, ratio, proportion, geometry, problem solving, and teaching strategies. Also included is an editorial comment by T. Kieren on mathematics education research. Research related to mathematics education which was reported in CIJE and RIE between April and June 1980 is listed. (MS)

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An editorial comment . . .

Mathematics Education Research - A Triangle

Tom Kieren
University of Alberta

It is the central purpose of Investigations in Mathematics Education to provide useful critiques of reported research in mathematics education. It is always a question what should be considered for criticism here. Partly this is a question of editorial choice. What should be the balance between critiques of easily available and more-difficult-to-find work? Should IME be identifying work which would otherwise go unnoticed by the general mathematics education community? Should IME provide critiques mainly of those reports which have not undergone a refereeing process? As suggested, such questions are always before the Editor and panel.

There is a much larger question of what constitutes mathematics education research. Of course, such a question could be considered vacuous, or answerable by "whatever mathematics education researchers do." However, as one looks at the research critiqued in IME, one is prompted to ask about the dimensions which characterize this research, the possibilities for such research, and the sources of theories in mathematics education. Answers to such questions might open up new research avenues and sharpen our criticism, in the positive sense, of current research results and practices.

Some have identified mathematics education research as lying at the confluence of, or at least defined in terms of, a relationship between the fields of psychology and mathematics. The existence of an active international study group in this area attests to such a definition. Yet even members of this group would not put a narrow construction on the dimensions of mathematics education research.

Bauersfeld, at the 1976 ICME meeting in Karlsruhe, described three concerns of mathematics education - matter meant, matter taught, and matter learnt. Using these three ideas we can use the analogy of a triangle to look at mathematics education research.

What is the "geometry" of this traingle? The three concerns defined above could be elaborated to include related actors (not necessarily distinct) - curriculum makers, teachers, and learners. Research which focused on one of the three matter-actor combinations could be thought of as a vertex of the mathematics education research triangle. Thus persons investigating mathematics teaching would be identified with the matter taught-teacher vertex, with the other vertices representing learning and curriculum research.

Of course, many disciplines and persons are interested in the research activities defined above and researchers, say in psychology, often use mathematics subject matter. What differentiates mathematics education research from this other research can be thought of as the "interest center" of our triangle - the actual mathematics learning experiences of children and adults. All mathematics education research "faces" toward these experiences and researchers would hope their work would directly or indirectly enhance such experiences.

It is rare that any one of the three matters of mathematics education concern is considered individually. Analogous to the side of a triangle are research efforts inter-relating the curriculum and the learner, or learner and teacher, for example. Such efforts are by no means new, but learner-teacher studies have received a new emphasis with recent sociologically inspired work. As well as representing an area of research, the relationships characterized by the sides of our triangle also suggest ways in which implications of research efforts at the vertices might be made. Thus, although ones' major focus might be on how a person learns mathematics, such research is informed by the curriculum "vertex" (what mathematics, what interpretation, or what experience might be used) and by the instructional "vertex" (what is the nature of the teaching or experimenter-subjects interaction). The results of learning research also informs curriculum development as well as instructional strategies and basic instructor acts.

From a topological point of view, a triangle has an interior and exterior. Even if our research triangle is not a closed figure, it is of interest to explore the analogs of interior and exterior. Because of its closeness to daily learning experiences, one might characterize the informal research done by teachers of mathematics as they prepare for, teach, and analyze their classroom activities as being one kind of "interior" research. (Perhaps its central position also indicates its relative importance). Two other kinds of current mathematics education research might be thought of in terms of the interior analog because they focus directly on learning experiences (sometimes naturally occuring and sometimes carefully defined) and because they (should) take into account all three of the vertex interests. These are the teaching experiment and the phenomenologically oriented participant observer work.

One might think of the exterior of the mathematics research triangle in terms of work in a variety of fields which have mathematics education relationships, but probably do not share its interest centre. The area of psychology has been mentioned above, and the ideas drawn from the Piagetian or the behaviorist traditions in mathematics education research are evident in almost any issue of IME. This psychological work has been most closely related to learning and instruction, but curriculum research (e.g., uses of behavioral objectives or enactive representations) has also felt an impact.

The relationship between mathematics itself and mathematics curriculum research is nominally obvious and observable in the curriculum development work of mathematicians. However, the contribution of mathematical analysis to mathematics education research can go beyond this because of the very fact that mathematical ideas are human creations. While various interpretations have varying consequences for a learner's personal knowledge, they may also imply or demand alternative teaching approaches.

If mathematics and psychology represent traditional exterior fields which influence mathematics education, they are not the only ones. The very statement above on mathematics as a human creation is a philosophical proposition. Different philosophies provide different frameworks for doing research. A recent example of this is the impact of phenomenology on the analysis of the meaning of teacher-learner interaction in a mathematics classroom.

Two other related areas are symbolic communication and linguistics. Because of its nature, mathematics learning eventually entails learning various levels of symbolic representation. How a person learns and uses these symbols can be studied with the help of appropriate symbolic and linguistic analysis which is also useful in both development and research in mathematics teaching and curriculum.

One further external field which represents the impact of society on mathematics instruction is the whole field of technology, probably best represented today by various micro-processing devices. Here the researcher can look for tools in the study of various aspects of mathematics education, but technological knowledge might require that instruction, curriculum, and learning be thought of in different ways. It is also of research interest that technological applications help elaborate mathematics education rather than limit it.

The discussion above has presented dimensions of mathematics education research using the analogy of a triangle. This triangle has several implications. One is that a researcher should take into account the relationships described in developing a research project. Second, the "exterior" fields can be used as a source of research ideas, theoretical propositions, or research methodologies. However, the mathematics education researcher must make sure that these exterior ideas are transformed into mathematics education ideas, facing the learning experience. Finally, the interest center of the triangle suggests that if research interests are always related to this center, then the likelihood of acceptability by the broader mathematics education community will be higher.

Fall 1980

Arehart, John E. STUDENT OPPORTUNITIES TO LEARN RELATED TO STUDENT ACHIEVEMENT OF OBJECTIVES IN A PROBABILITY UNIT. Journal of Educational Research 72: 253-259; May/June 1979.

Abstract and comments prepared for I.M.E. by SAMUAL P. BUCHANAN, The University of Central Arkansas and CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

"The purpose of this investigation was to explore the relationship between pupil achievement of specified objectives and the opportunity to learn them." (p. 253)

2. Rationale

There has been increased interest in teaching strategies research in the recent past. A number of process variables which tentatively relate to student achievement have been identified. However, these variables are still not precisely defined and their relationship to student achievement is somewhat ambigious. The present study was conducted in order to further validate such variables.

One such variable, opportunity to learn, was the focus of this particular study. Low-inference variables were used as measures of opportunity to learn, to test the hypothesis that the amount of opportunity to learn specific objectives contributes to the achievement of those objectives.

3. Research Design and Procedures

A total of twenty-six classes of eighth, ninth, tenth, and eleventh grades taught by twenty-three teachers was used as subjects in this experiment. A pre-test was administered to the students prior to a unit in probability being taught. The results of this test were compared to a post-test which was administered following the unit on probability to determine the number of objectives successfully mastered as a result of the teaching. Each of the teachers received from the investigator a packet which contained a list of eighteen objectives to be taught in the unit on probability, a set of homework problems keyed to the set of objectives, audio tape reels for recording the instructional sessions, and a set of explanations for the teacher.

The teachers then taught the unit on probability using the list of objectives as a guide. During the instructional portion of each class, the teacher recorded on an audio tape the instructional sequence of each of the three class periods used for this unit.

The audio tapes, pre- and post-tests, and the amount of probability homework/classwork assigned and completed by each student were returned to the investigator. OScAR-5V (Observational Scale and Report-5 Verbal) was used to code the audio tapes. Each utterance was determined to be either teacher or student originated and to be either substantive or non-substantive in nature. The investigator defines a substantive utterance as one which deals specifically with the content of objectives of a lesson.

The variables identified by the investigator as being related to student achievement were: achievement gains; emphasis; number of problems attempted by students; substantive events; percent total substantive events; informing statements; percent informing statements; teacher-initiated questions; percent teacher-initiated questions; pupil-initiated interchanges; and percent of pupil-initiated interchanges.

The data were analyzed using a two-way nested analysis of covariance. The two main bases of classification were the twenty-six classes and the sixteen objectives (two were deleted). The dependent variable was achievement gain, while each of the other variables was used as a covariate. This design controlled variance due to the following components: differences in judged levels of difficulty of the objective, differences in actual difficulties of objectives within each judged difficulty level, differences in overall achievement of different classes, and interaction between levels of difficulty and overall teacher effectiveness.

4. Findings

The four control variables accounted for 74 percent of the total variance. Difference in levels of difficulty and differences in objectives within levels accounted for 32 percent of the variance. Differences in classes accounted for 32 percent of the variance and interaction between levels of difficulty and classes accounted for the last 10 percent.

The two non-OScAR variables, Emphasis and Problems Attempted, showed a significant relation to achievement. The variable showing the strongest relationship to student achievement was Teacher-informing Statements. Pupil-initiated Interchanges showed the weakest relationship to student achievement.

5. Interpretations

The results are supportive of the hypothesis that opportunity to learn specific content as measured by emphasis and exposure is positively related to achievement.

In addition, evidence was found that (a) teachers' perceptions of what has been emphasized and what students have learned are rather accurate; (b) the amount of student work and achievement are related; (c) the amount of teacher information and questions about objectives of a lesson related positively to their achievement; (d) teacher information-giving was as effective as teacher questioning behavior, and both showed a significant relation to achievement; and (e) the number of substantive questions pupils asked was not related to how much they learned. (p. 258)

Abstractor's Comments

1. The study investigates a topic of importance to teacher education in general, and mathematics education in particular.
2. The report discusses the statistical procedures under the section labeled "Results". This information should probably come in the "Procedures" section.
3. OScAR-5V is a classroom observation instrument. Audio tapes may have hindered the collection of data. However, reliability for interpreting the data is reported.
4. The author suggests that teachers should be precise and specific concerning their teaching of specified objectives. More discussion of this topic might have been helpful.
5. It might have been interesting to use an attitude scale as another measure for the study.
6. Interested readers might find it useful to read other studies related to teaching process variables; for example, see: Evertson, C. M., Emmer, E. T., and Brophy, J. R. Predictors of effective teaching in junior high mathematics classrooms. Journal for Research in Mathematics Education, 1980, 11, 167-178.

Ayers, Jerry B.; Cannella, Gail S.; and Search, Janie M. GEOMETRIC EMBEDDED FIGURE IDENTIFICATION AND CONSTRUCTION BY LOWER GRADE CHILDREN. School Science and Mathematics 79: 677-689; December 1979.

Abstract and comments prepared for I.M.E. by WILLIAM E. GEESLIN, University of New Hampshire.

1. Purpose

The primary aim of this study was to investigate possible sex and grade differences among early elementary school children's abilities to identify and construct simple figures composed of (a) embedded squares, (b) embedded rectangles, and (c) overlapping circles. In addition, the effect of a brief instructional sequence on identification and construction of embedded figures was examined.

2. Rationale

Previous research indicated that a child's visual perception developed rapidly between the ages of four and seven. Early development of visual skills, in particular identification of embedded figures, appeared to be related directly to school performance. Therefore, investigation of children's ability to identify and construct figures involving overlapping squares, rectangles, and circles was pursued.

3. Research Design and Procedures

Subjects (N = 55) were obtained from a university-operated summer school designed for enrichment and remedial work for local school children. Table 1 gives the distribution of subjects by sex and grade level the child was entering the following fall.

Table 1

Distribution of Subjects

Grade	Boys	Girls
K	6	6
1	14	10
2	5	14

Each child was interviewed individually following a set protocol. Briefly stated: children were shown each of the three shapes (square, rectangle, and circle); shown three figures containing two connected but non-overlapping shapes; and shown a series of figures containing overlapping shapes. Each figure contained only one type of shape. The child was asked to determine the number of shapes in each drawing. Next the investigator pointed out the component shapes in three drawings consisting of overlapping figures. The child was asked again to determine the number of shapes in a series of overlapping drawings. Finally, the child was given nine incomplete drawings (one at a time) and asked to complete them with straight (or curved lines) so that a given number of shapes comprised the final figure.

4. Findings

The chi-square statistic was used to test for sex and for grade level differences on each drawing. No statistically significant sex differences were found. Grade level differences with respect to squares and rectangles were statistically significant prior to instruction but not significant following instruction. With one exception there were no differences in performance on circle drawings.

5. Interpretations

The fact that instruction eradicated grade-level differences was contrary to a previous study concerning triangles. Increases in the percentage of correct responses between first- and second-grade subjects supports the notion that a child's perceptual ability develops in a quasi-Piagetian manner. The difficulty of identification tasks appears to be related to the type of geometric figure. The emphasis in art programs and lack of perceptual-motor coordination may explain the fact that tasks involving circles were so much easier than tasks requiring use of straight lines. In general the tasks were not difficult for the children. The poorer performance of kindergarten children may result solely from the fact that these children were just beginning their formal instruction experiences. Consideration should be given to a coordinated geometry curriculum beginning at an early grade level with the study of triangles following the study of circles, shapes, and squares.

Abstractor's Comments

At first glance the article would seem to be yet another example of an isolated, irrelevant study pursued to satisfy someone's need to publish. However, this investigation exemplifies one reasonable manner in which to proceed at this stage of our knowledge concerning children's spatial development. The authors provide further information indicating that (1) children are more able than many developmental psychologists (and, consequently, many elementary teachers/mathematics educators) imply; (2) Piaget's theories concerning the conception of space need revision and/or explication; (3) students learn what they are taught; (4) a need for synthesis of research, theory, and models of spatial development still exists; (4) this area of research may eventually have a large impact on curricula and teaching; and (5) one or two studies are unlikely to answer many questions about spatial development and if viewed in isolation, may be quite misleading.

As noted by the authors, the sample was small and many have been biased in an unknown manner since it was taken from a summer program. Of greater concern is the terseness with which the study was reported. The theoretical framework, if any, was unclear. In particular, the authors did not distinquish between perception and cognition. Given certain developmental perspectives, the identification tasks and the construction tasks would be viewed as involving qualitatively different mental processes. No discussion leading toward an overall hierarchy of spatial tasks was given. Does the fact that children performed better with curved lines than straight lines lend credence to the work of Piaget? The authors claimed art programs and motor coordination were plausible reasons for this performance difference. Yet children who have yet to enter kindergarten surely are not influenced by school programs. Do art programs really emphasize curvature?

Apparently the "instruction" in this study was extremely brief. Administering a new set of figures after instruction to a subject group would have been useful. Does instruction or repeated testing influence performance most? In either case elementary teachers hardly need worry about delaying introduction of these materials. More information on the key aspect(s) of the instructional sequence is needed. Some conclusions concerning elementary curricula are inferences that go well beyond the data.

In summary, another reminder that children's abilities on spatial tasks may depend more on the task than some broadly defined mental stage is useful. Much remains to be learned about the development of children's conception of space. It is unfortunate that the authors did not include a paragraph connecting their study to a broader psychological theory or a paragraph speculating on what research should be done next. A proposed hierarchy of spatial tasks is a possible next step. Likewise, a good synthesis of <u>all</u> the research related to spatial development would be welcome.

Fischbein E.; Tirosh, D.; and Hess, P. THE INTUITION OF INFINITY. Educational Studies in Mathematics 10: 3-40; February 1979.

Abstract prepared for I.M.E. by ALAN OSBORNE,
The Ohio State University
Comments prepared for I.M.E. by H. LAVERNE THOMAS,
State University College, New York

1. Purpose

The concept of infinity can be considered as a purely logical construct, an ideal of mathematical reality. It may also be considered in terms of the psychological contexts in the processes of thinking, uses, and interpretations that constitute the intuitions about the concept. The purpose of this study was to investigate the contradictory nature of infinity in the dynamics of thinking, with particular attention given to variables such as the problem setting, age, mathematical achievement, general school achievement, and sex.

2. Rationale

The concept of infinity is clearly a formal-level concept in the Piagetian sense. How learners deal with the contradictory perceptual and conceptual intuitions had received almost no attention by researchers in psychology or mathematics education. Preliminary evidence collected by Fischbein in the mid-1960s indicated a stability of intuitive conceptions of infinity transcending age and teaching. Interesting and useful in terms of its potential for revealing factors that affect the learning process, the conflicts and ambiguities provide a setting for the productive examination of the role of intuition.

3. Research Design and Procedures

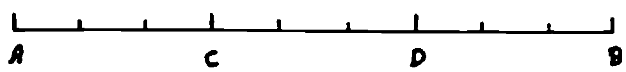
A ten-item questionnaire was administered to classrooms of students from fifth through ninth grades. At each grade level, students were selected to represent high and low SES levels. For each grade level the number of students tested was:

fifth	sixth	seventh	eighth	ninth
46	58	152	104	110

In the midst of the project, it was discovered that a portion of both of the sixth-grade classes were in a supplementary program for talented children in mathematics. The general achievement data were available only for the children in the upper three grades in this Israeli school system. One class period was required for the administration of the questionnaire.

The ten items referred to the divisibility of segments, transfinite cardinal numbers, and limits. Nine of the items had a geometrical basis. Four concerned the divisibility of line segments, one concerned cardinal number arithmetic, and three involved area concepts. Three items are displayed below in order to communicate the character of the questionnaire:

(2) Consider again segment AB of question 1. This time, instead of dividing the segment into two equal parts we will divide it into three equal parts (Figure 2). Will we arrive at a situation such that the segments will be so small that we will be unable to continue dividing? Explain your answer.



(8a) Construct a semicircle with segment AB as a diameter. Divide AB into two equal parts, AC and CB. and construct two semicircles on AC and CB as in Figure 7. Continue dividing and constructing semicircles (see Figure 7). Question: What will happen to the length of the wavy line as we shorten the length of each sub-segment? Explain your answer.

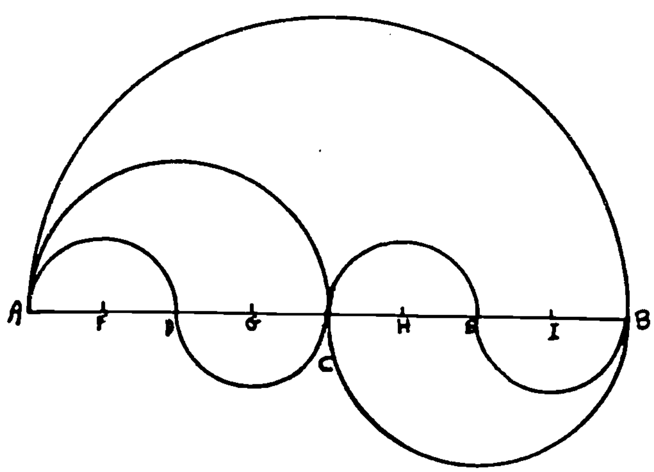


Fig. 7.

(8b) What will happen to the *sum of the areas* determined by the semicircles as we shorten the length of each sub-segment? Explain your answer.

(9) Let us consider the rectangle ABCD (Figure 8). Construct new rectangles by increasing its length and decreasing its width in such a way that the

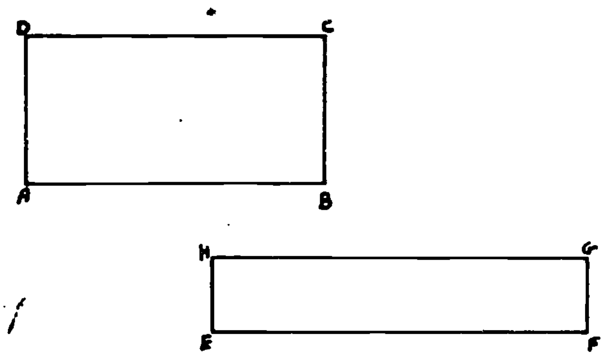


Fig. 8.

perimeter remains constant. What happens to the areas of the rectangles as the process continues?

Responses to each item were categorized to fit the content of the item. For example, for problem 8b given above, the categories of response noted were: 1) The sum of the areas is constant, 2) The sum of the areas decreases, 3) The sum of the areas increases, and 4) no answer. In particular, it was noted whether children were "finitist" or not in their responses. For many items, an infinite process response was classified in terms of whether the child additionally noted that, practically, the process must come to an end. Chi-square statistical techniques were used in examining the data per item across grade levels. Each item was cross-tabulated with five independent variables.

4. <u>Findings</u>

1. None of the chi-square tests of the answers was significant by sex.
2. For items involving infinite divisibility, "finitist" answers accounted for approximately 60 percent of the responses.
3. Seventy-one percent claimed that the set of natural numbers was larger than the set of even numbers with the primary reason given being that the even numbers were contained in the set of natural numbers.
4. The portion of "infinitist" answers varied greatly from question to question.
5. The percentage of the main categories of answers, especially "finitists" and "infinitists", was generally stable across age levels, with a slight dominance of the "infinitist" developing by the upper grades.

5. Interpretations

The authors concluded that subjects possessed an intuitive idea of the concept of infinity and that their responses were not random. This, coupled with the response patterns differing from problem to problem and with the progression of increasingly correct responses for higher levels in the curriculum, yielded the interpretation that understanding of infinity was bound to mathematical knowledge. This argument was supported by the performance data of the sixth-grade children who were participating in the special program for gifted children. However, the data suggest that mathematical training "has, then, two divergent and apparently contradictory effects" (p. 37). Experience is highly positive in some types of problems (for example, the division of line segments), but is negative or absent on other types of problems such as those involving limit processes. "Finitist" thinking tends to prevail with age and teaching effects tending to explain only a portion of children's performance. "Regular mathematical training affects only the formal, superficial understanding of the concept of infinity" (p. 38). Some incorrect interpretations tend to be maintained in well-trained pupils.

Abstractor's Comments (1)

This is a fascinating topic for research and inquiry into children's processes of thinking. The data and interpretations yield convincing support for the value of watching children deal with ambiquity and contradiction as a mechanism to observe the thinking processes. The value of the research derives primarily from examination of the response patterns for indivdual items, detail of too great an extent to be appropriate for presentation herein. The authors provided thoughtful and provocative interpretations of the response patterns.

The following points need to be considered in examining the results of the testing of children concerning their conceptions of infinity:

1. The questionnaire format responded to in a group setting may have hidden some interesting response patterns. In the majority of school settings with which I have familiarity, many children at these age levels would not have the patience to respond fully to the items or to provide the detail in written responses necessary for drawing sound conclusions.

I would have preferred some analysis of the validity attendant to this form of testing in comparison to an interview format.

2. Given the variation in response patterns from one problem setting to another, I would have preferred more extensive information concerning the curricular history of the children. Although some information of a sketchy nature was presented, it was insufficient to allow some of the interpretations the authors wanted to make. For example, three items depended on the child's understanding of area as well as infinity. I would have preferred information concerning when area was dealt with in the curriculum. Better, of course, would have been performance data concerning some of the important prerequisite concepts. A faulty concept of area might have interferred with children's correctly responding to the items concerning infinity that involved area.
3. All but one of the problems depended on children's having an appropriate store of geometrical knowledge. I would have preferred to have seen more items of a symbolic nature. Given that infinity is a formal-level concept, I think it is important to examine children's coping with the concept on a symbolic as well as a figural footing. However, trying to create some problem settings that are geometry-free in the last few days has created a healthy sympathy for the authors of the questionnaire.

Generally, the study is stimulating and well-done. It suggests interesting avenues for future research in a challenging arena of thinking. The study raises more questions then it answers concerning the nature of intuition.

Alan Osborne

Abstractor's Comments (2)

The investigators in this study have tackled a very difficult and knotty concept. Indeed, the very lateness historically of a reasonably satisfactory treatment of infinity, as is remarked in the report, attests to the difficuties in dealing with "infinity." Further, to this day there are sharp differences in points of view among persons interested in the foundations of mathematics.

Nevertheless, on a practical plane in the teaching of mathematics we do deal with that complex of concepts constituting and involving infinite sets and infinite processes. For this reason, and because there are so few studies bearing on these concepts, the research reported is of importance to mathematics educators. It is also of importance in gaining knowledge of the ways in which developmental processes interact with outside influences, specifically, systematic instruction.

Having said this, it is necessary to point out some of the difficulties that such a very anecdotal and observational form of research engender. These observations are not intended to denigrate such research. On the contrary, this writer supports research based upon careful observation of subjects performance of tasks relevant to important mathematical concepts and processes.

The first difficulty is that the study deals with global concepts and notions. The fact that these are difficult to put in operational terms does not obviate the need to make clear, in reasonably operational terms, what aspects of these concepts or ideas are being investigated. To this writer, it was never really clear what aspects of the concept of infinity were being investigated other than as this was defined by the tasks, the response categories, and even the discussion of the results. As for intuition, as distinct from other aspects of mental functioning, the descriptions given were not very satisfying nor operational.

Indeed, the entire argument separating "intuition" from "conceptualization" is not clear. In any event, is such a distinction helpful? In terms of explanatory power, would not a concept formation model serve equally well and be more economical? Clearly, the investigators do not believe so but it would seem that most of the apparent contradictions as well as other results could be so explained.

The second difficulty related to the whole process of task selection, administration, scoring, and analysis. Since no clear delineation of the particular aspects of infinity being studies is given, the reader is left to infer the purpose that particular tasks are to serve. As the report is studied, it becomes clear that tasks and subject responses are used in multiple ways to reach conclusions.

In studies of this nature, categories of task solution as well as categories of subject explanations are often utilized. That these categories are formed, and to some extent must be formed, after the fact is also typical. But it is problematic for several reasons - one being that creation of response categories can interact with hypotheses to be tested. The investigators endeavored to report categories of task solution completely. Reporting explanations is more difficult, if not impossible in a publication of this nature. Many were reported but not all. Hence, to a certain extent, the investigators are privy to important information, not accessible to the reader, that may influence conclusions drawn.

Thus, in studies of this nature, extreme care must be taken in drawing conclusions and even then appropriate reservations must be made as to their strength. It was disconcerting to find conclusions being drawn with considerable certainty.

Some specific questions about the tasks, solutions, and explanations are:

1. How were the solution categories for item 3a obtained? They don't seem to fit the question.
2. Would responses have been similar if positive even and positive odd integers had been compared in item 4?
3. How, directly, is item 5 related to transfinite cardinals? Is not, indeed, the notion of limit point, more germane?
4. In item 8b, why were a wrong and a right response combined to compare with a wrong response in the analysis?
5. Why is a "more than" comparison asked for in item 4 while an "equivalence" comparison is asked for in items 6 and 7?
6. Is the categorization of subject explanations as "concrete" and "abstract" justified and meaningful? Similarly, the categorization of subjects as "finitist" and "infinitist"?
7. Tasks deal with both denumerable and uncountable sets, but do they ask subjects in any way to differentiate them?

The tasks, as a whole, do deal with the global notion of infinity. However, would it not be useful, and inform the analysis, to consider at least some differentiation of subconcepts within the global concept? For example, one differentiation that could be made would be between infinite, or endless, processes and the property of a set of objects as having transfinite cardinality. Indeed, most of the tasks deal with infinite processes. Perhaps some differential response patterns are present in the data that reflect these two aspects of infinity.

One of the basic conclusions of the study appears to be that the "intuition" of infinity is quite resistant to schooling and maturation over the time span studied. But study of the data for grades 5 and 7 shows consistent increase in correct responses from grades 5 to 7, some showing significant values of X^2. This is noted, the influence of formal operational thought is recognized, and yet it is somewhat discounted by the investigators. Some relevant questions are:

1. Was the study conducted at the end of the school year?
2. What had present seventh graders studied in grade six?
3. Are there any curricular elements that might account for some declines from grade seven to grade nine?

Certainly this result seems worthy of further exploration in terms of the relationship of concepts of infinity and formal operational thought. In this connection the reader might be interested in exploring a closely related study, on the child's concept of limit (Taback, 1975).

Finally, in carefully studying this investigation on infinity, the reviewer was alternately perplexed and enlightened, frustrated and inspired. The reading is not easy and much of the argumentation is involved, based on multiple aspects of the data, and, to a considerable extent, inductive in nature. The study is a rich and complex investigation of a rich and complex construct. In perusing it, objections, alternative explanations, and research ideas were continually distrupting any orderly train of thought. Those interested in tackling big, complex mathematical concepts such as infinity should read this study and see what objections, alternative explanations, and research ideas are suggested.

Reference

Taback, Stanley F. The Concept of Limit. In Children's Mathematical Concepts Six Piagetian Studies in Mathematics Education (Myron F. Rosskopf, Editor). New York: Teachers College Press, 1975.

Hamrick, Kathy B. ORAL LANGUAGE AND READINESS FOR THE WRITTEN SYMBOLIZATION OF ADDITION AND SUBTRACTION. Journal for Research in Mathematics Education 10: 188-194; May 1979.

Abstract and comments prepared for I.M.E. by J. LARRY MARTIN, Missouri Southern State College.

1. Purpose

To determine the effect of delay versus immediate introduction of written symbolization of addition and subtraction symbols on students classified as ready or not ready by the investigator.

2. Rationale

Reading and language educators emphasize the importance of verbal facility as a readiness factor for reading. The spoken word must precede the written word so that meaning can be attached to the written symbol. Similarly, verbal facility with the spoken phrases of arithmetic may be an important readiness factor for the meaningful learning of the symbolization of those phrases.

3. Research Design and Procedures

The sample consisted of 38 subjects selected from 66 first graders who had been given an investigator-constructed readiness test. Students classified as ready were paired using the Key Math test, giving 8 pairs of ready students. Students classified as not ready were paired using the Key Math test and their readiness test scores, giving 11 pairs of not-ready students. The remainder of the 66 original students could not be matched and thus were not included in the sample of 38. Readiness was determined using an individually administered test designed by the investigator based on a list of the school's first grade objectives for addition and subtraction. Objectives related to reading, writing, or speed of response were either discarded or modified to require verbal responses. "Essentially, a student was considered to be ready for the introduction of the written symbolization of addition, for example, when he or she could add verbally" (p. 188).

One student from each pair was assigned to an immediate symbolization group and one to a delayed symbolization group. This gave rise to four instruction groups: ready-immediate symbolization (RI), ready-delayed symbolization (RD), not ready-immediate symbolization (NI), and not ready-delayed symbolization (ND). These four treatment groups were further divided into two halves for instructional purposes. The four treatments were conducted simultaneously over a period of 12 weeks.

The treatments consisted of a series of 63 activities on addition and subtraction based on the school's first-grade objectives mentioned above. Each activity contained three sections: concrete-pictorial, verbal, and symbolic. Students in the RI and NI groups covered the sections in order, with the written symbolization introduced simultaneously with each topic. Students in the RD group delayed written symbolization for 5 weeks. That is, they initially covered only concrete-pictorial and verbal sections of each activity. For the ND group, written symbolization was also delayed, but delayed until the student demonstrated readiness for it.

The treatments were concluded with a posttest administered to all students. Each student received 12 scores measuring his or her ability to: interpret addition, subtraction, and addition and subtraction number sentences (3 scores); produce addition, subtraction, and addition and subtraction number sentences (3 scores); state answers to addition, subtraction, and addition and subtraction number sentences (3 scores); interpret, produce, and state answers to addition, subtraction, and addition and subtraction number sentences (3 scores).

Twelve null hypotheses were tested for the ready students and 12 for the not ready students. They were that the time of introduction of written symbolization would have no affect on students' ability to perform the 12 tasks as measured by the posttest. The Wilcoxon Signed Rank Test was used to test the hypotheses.

4. Findings

The results showed no differences in abilities between ready students who had immediate symbolization and those who had delayed symbolization. Significant ($\alpha = .05$) differences were found between NI and ND students in favor of the delayed symbolization group in their ability to interpret addition, subtraction, and addition and subtraction number sentences.

5. Interpretations

Based on review of audiotapes and videotapes, discussions with participating instructors and the test data analysis, the investigator offered five conclusions:

> (1) The time of introduction of written symbolization does not affect ready students' meaningful learning of the symbolization of addition and subtraction ... (2) A delay of symbolization may cause symptoms of boredom and frustration among ready students ... (3) The time of introduction of symbolization does affect not ready students' meaningful learning of the symbolization of addition and subtraction ... (4) If a student is not ready for the introduction of written symbolization of addition and subtraction, the student's learning of the symbolization will be more meaningful if the symbolization is delayed until the student is ready ... (5) Students' readiness ... influences the students' meaningful learning of the written symbolization of addition and subtraction. (pp. 193-94)

Abstractor's Comments

More about the criteria for determining readiness might have been helpful to the reader. As is often the case in reported research, the instrument is described but not exemplified. Examples can sometimes paint a clearer picture. Another question related to readiness comes to mind. For those students classified as ready, how does one know how long they have been in that state? If the effect of a delay in symbolization is under investigation, doesn't this become critical?

The research questions raised in this study are legitimate ones. The nature of the student is a major factor in curriculum development. However, attention to readiness is most often focused on mathematical prerequisites. Verbal facility with the words and mathematical phrases can be overlooked. If mathematics is a language, then we should expect that speaking the language would facilitate reading and writing the language. Both teachers and teacher trainers would do well to keep this in mind.

Hart, Kathleen. THE UNDERSTANDING OF RATIO IN THE SECONDARY SCHOOL. Mathematics in School 7: 4-6; January 1978.

Abstract and comments prepared for I.M.E. by JOHN R. KOLB, North Carolina State University.

1. Purpose

The stated purpose of this study was to seek answers to the following questions:

a. "How well do secondary school students understand the topic of ratio?"

b. "When they don't understand ratio, what thinking strategies or answer-getting methods do they employ when faced with ratio problems?"

These questions were investigated by administering a test to a nationwide sample of two thousand English children.

2. Rationale

British Secondary School Mathematics textbooks were surveyed to produce a list of specific aspects of the topic of ratio. From this list, questions were constructed and used in interviewing thirty-five secondary children on a 1-1 basis. After the interviews, a written test was constructed and administered to one hundred children. Finally, a revised version of the test containing 27 items was given to the final sample of 2000.

3. Research Design and Procedures

The students are described as being 2nd, 3rd, and 4th year students in secondary school. Thus, they should be comparable to students in grades 7, 8, and 9, respectively, in the United States. The report indicates the sample consisted of 690 2nd year students in 8 schools, 767 3rd year students in 7 schools and 800 4th year students in 7 schools. The report does not indicate exactly how many different schools were involved nor does it indicate whether the students were in the secondary grammar, secondary modern, or secondary technical streams. The sample was described as being representative of the distribution of I.Q. in each year group.

4. Findings

Results are reported in terms of the percent of youngsters in each year-group who successfully responded to items on the test. In general, 4th year students performed better than 3rd year students who in turn had a higher success rate than 2nd year students. The means for each year-group and percentages obtaining 23 or more out of a possible 27 items are:

Mean	Percentage Obtaining 23/27 or more
2nd year 12.13	3.2
3rd year 12.88	8.0
4th year 14.65	13.9

5. Interpretations

Although 4th year students obtained the highest mean on the test, it was concluded that few students in each level really understood or could cope with ratio. The easiest problems were those that involved doubling or halving, but this ability is no indicator of ability to tackle other ratio questions. Children used a variety of methods to tackle the ratio problems, particularly those involving more than merely doubling or halving. One common method was an addition strategy in which the child concentrates on the difference between the numbers rather than the ratio of the numbers. Students performed operations on fractions and percentages without recognizing their inappropriateness. They would seek some method - any method - and attempt to consistently apply it without regard to its reasonableness in the problem.

Abstractor's Comments

This study can best be described as an assessment or testing program on a specific topic rather than an investigation of a research hypothesis. From the standpoint of research results, this study adds little to our understanding of how children deal with ratio. However, the items used to elicit student responses are quite clever. They penetrate to student understanding and comprehension and not just technique and manipulation. These items could be very helpful to any researcher who is interested in studying and understanding student's thinking with respect to ratio.

Kulm, Gerald and Days, Harold. INFORMATION TRANSFER IN SOLVING PROBLEMS. Journal for Research in Mathematics Education 10: 94-101; March 1979.

Abstract and comments prepared for I.M.E. by AARON BUCHANAN,
SWRL Educational Research and Development, Los Alamitos, California.

1. Purpose

The main purpose was to measure transfer between related types of verbal problems using a special technique of information analysis adapted from research on information processing. The study is exploratory and no hypotheses are stated explicitly. However, there is an underlying hypothesis that the amount of information which becomes functional in solving a specific problem is affected in different ways by different types of related problems that have been solved previously.

2. Rationale

There are two models that provide primary support for the rationale.

i) Transfer between related types of verbal problems

It is possible to identify different types of related problems for a specific target problem that is supposed to be solved. This study involved five types of related problems which, it was thought, would differentially affect the amount of information that was perceived and used in the target problem. The five types of related problems for the target problem were (a) equivalent problem, (b) a special case, (c) a similar problem, (d) a generalization problem, and (e) an unrelated problem. The idea was to explore how information transfers from each of the five types to the target problem. A "guessing" technique was developed to measure the amount of information in the target problem for different groups of subjects who had just previously worked through one of the related problems.

ii) Information processing theory

The study involves a "guessing" technique normally used in information analysis where subjects try to guess succeeding signs which have been deleted from a message. The adaptation of the technique for this study required subjects to guess succeeding information units (words, symbols, etc.) or steps deleted from a verbal problem. The guessing technique produces a measure of the subjective information (1) obtained from the problem statement, a set-up of the solution, and the step-by-step solution, itself. This measure is given by the formula:

$$1 = \sum_{i=1}^{k} \log_2 P_i$$

where P_i is the proportion of subjects who guess correctly the i^{th} missing step or unit of information.

3. <u>Research Design and Procedures</u>

The basic procedure called for a group of subjects to solve one of the five <u>related</u> problems and then "guess" missing steps and/or units of information in the target problem. Overall, the design involved six groups of prospective elementary teachers (N = 5 in each group), one group for each of the related problems and an additional group that actually solved the target problem before they guessed it. (Presumably, the latter group provided a kind of baseline data where guessing involved direct recall more than transfer.) The only dependent variable was (1), the value of subjective information obtained by each of the six groups on the target problem based on results of the guessing technique.

The design actually included two replications of the experiment, since each group repeated the related-problem/target-problem process twice, once for an algebraic target problem and once for a puzzle* target problem.

4. <u>Findings</u>

The major results involve subjective information values obtained on the target problem by each of the different groups that had previously solved one of the five types of related problems. The logic is tricky, since a low information value occurs when missing units of information in the target problem could be guessed with a relatively high probability.

*Missionaries and Cannibals problem.

Thus, when there is a low information value on the target problem for a particular group, it is taken to mean that there is little new actual information implicit in the problem itself. High information values, on the other hand, mean there must be a great deal of information in the problem, since it is hard for subjects to guess parts that are missing.

Algebraic Problem

The information in the target problem was analyzed in four parts: The problem statement, the solution set-up, the problem solution, and the total problem. In general, the main results showed that groups who had previously solved the *generalization* problem or the *similar* problem obtained lower information values on the target problem (i.e., guessed with a relatively high probability), the solution set-up, and the solution itself, than groups who had previously solved the *special case* or the *equivalent* problems. However, these differences were only significant in one case where the *generalization* problem and *similar* problem groups had significantly lower information values on the target problem than the *equivalent* problem group. The *generalization* and *similar* problem groups did obtain significantly lower information values in most cases than the group which had previously solved an *unrelated* problem.

Puzzle Problem

On the total solution (which was divided into 11 steps) groups which had previously solved the *generalization* problem or the *equivalent* problem obtained lower information values on the target problem than groups who had previously solved the *special case* problem or the *similar* problem. The *equivalent* problem group had significantly lower values on the target problem than the *similar* problem group, and both the *equivalent* and the *generalization* problem groups had significantly lower values on the target problem than the *unrelated* problem group.

5. Interpretations

The authors interpreted a low information value as an indicator that information had been transferred from the related problem to the target problem. Otherwise, subjects in this group would have had more difficulty replacing missing information in the target problem during the "guessing" procedure. Therefore, on the algebraic problem, they interpreted the results to mean that there was more transfer to the target problem from a *generalization* or a *similar* problem than from an *equivalent* problem or a *special case*. On the puzzle problem, the authors concluded that more information was transferred to the target problem from the *generalization* or the *equivalent* problem than from a *special case*. In general, they interpreted the data from this study to support the conjecture that a general-specific sequence may be a better one for learning how to solve problems than a more traditional sequence where instruction tends to flow from specifics to a general case.

Abstractor's Comments

The most significant contribution of this study is likely to be its use of techniques for information analysis to produce an alternative form of dependent measure in problem-solving research. The objective of this kind of analysis is extremely appealing, especially in studying the use of information embedded in the verbal problem and methods for its solution. It seems to have potential as a more sensitive tool for measuring transfer than measures used in conventional methods, which are too often limited to the frequency with which the target problem is actually solved. Intuitively, it seems that this technique, or one like it, allows the researcher to, in a sense, measure partial effects of one kind of problem on another, even in cases where subjects may not actually be able to bring about a complete solution. On the other hand, the "guessing" technique appears to have its own peculiar constraints which create a certain uneasiness over what characteristics of the problem-solving process are actually being measured.

Since the study is exploratory, it isn't proper to hold it rigorously accountable for the standards of more formal research. The study is basically well-designed and, as nearly as one can tell from the report, procedures are appropriately adequate. It does seem particularly important, even in this exploratory mode, to have some confidence that the different groups involved in problem-solving protocols were reasonably equivalent to start with; otherwise, results would be mostly meaningless.

Although there is no specific statement to the effect that group assignments were random, the procedures lead one to believe that this was probably the case.

The report, itself, is <u>much</u> too short, especially where the logic is tricky and interpretations seem counter-intuitive to the reader who may not be pretty well rehearsed on techniques of research on information processing. Too many important details are either missing or too inadvertently laid between the lines. For example, the authors used the Wilcoxin Matched Pairs Sign Ranks Test to test the significance of subjective information values, but it's hard to tell positively just what kind of values were actually matched.

Lawson, Anton E. and Wollman, Warren T. DEVELOPMENTAL LEVEL AND LEARNING TO SOLVE PROBLEMS OF PROPORTIONALITY IN THE CLASSROOM. School Science and Mathematics 80: 69-75; January 1980.

Abstract and comments prepared for I.M.E. by THEODORE EISENBERG and KEVIN GALLAGHER, Northern Michigan University.

1. Purpose

To test the Piagetian hypothesis that if "the development of formal operations must precede the learning of certain mathematical concepts is correct, then instruction in such concepts may be beneficial only for some students" (p. 69).

2. Rationale

Piaget claims that the sequence through which a person develops intellectually is invariant. However, the rate through which this development occurs is often influenced by intelligence, the environment, and other factors. High-IQ children are often at different stages in the intellectual development sequence than their low-IQ classmates. Thus, the mathematics teacher is often faced with the task of presenting certain concepts to students for which some may not be intellectually ready.

3. Research Design and Procedures

Twenty-eight seventh-grade students (parenthetically described as "12 males and 18 females" [p.70]) were categorized into one of three groups based upon their responses to a set of three Piagetian tasks: early concrete, late concrete, or early formal operational levels of intellectual development. The groups had 4, 8, and 16 students in them, respectively. They were collectively instructed over 10 periods by their classroom teacher on how to solve problems of proportionality. The instruction was based upon the text, Modern School Mathematics 7: Structure and Method by Dolciani et al., and on the teacher's own written materials.

Success with proportionality problems, according to Piagetain theory, implies that one is operating at the formal operational level. After instruction, the students were presented with a variety of problems dealing with proportionality; some were Piagetian in nature (e.g., balance-beam problems), others were straightforward paper-and-pencil tests. Based upon the researchers' discretion, partial credit was given for incomplete solutions. Evidence in support of the hypothesis that the early formal operational group benefited from the instructional unit more than the other two groups would be obtained if the three groups of students performed in the obvious expected manner for all tasks administered.

4. Findings

Using the Kruskal-Wallis ANOVA rank test on the mean scores for the post-instructional tasks, the students performed as expected on all but one of the tasks (see Table 1 reproduced below).

TABLE 1
COMPARISON OF PRETEST LEVEL OF INTELLECTUAL DEVELOPMENT WITH POSTTEST TASK PERFORMANCE

Posttest Task	Level of Intellectual Development[a]						H
	Early Concrete		Late Concrete		Early Formal		
	Mean (n = 4)	SD	Mean (n = 8)	SD	Mean (n = 16)	SD	
Balance Beam (4)[b]	.50	1.00	1.75	.71	2.50	.73	11.22**
Disc (4)	.75	.95	2.13	1.13	2.44	.89	6.89*
Math Quiz (2)	1.25	.96	.75	.46	1.81	.40	13.70**
Mr. Tall-Mr. Short (4)	1.00	2.00	1.75	1.73	2.00	1.55	2.36
The Machine (8)	3.50	3.51	5.63	3.50	7.00	1.50	6.39*

* $p < .05$
** $p < .01$
a As determined by conservation task performance.
b Numbers in parentheses represent the maximum number of points possible on each task. (p.73)

Students categorized as early concrete generally did less well than late concrete students, who in turn did less well than the early formal students.

5. Interpretations

"These results lend support to the Piagetain hypothesis that the development of formal operations is a prerequisite for the effective learning of the proportions concept.

"[Students who have trouble with lessons should receive] special assistance [which] must be much more than simply a repetition of the lesson, a rephrasing of directions, or the assignment of special homework problems. What may be called for is reorganization of the subject matter and a reemphasis on first-hand manipulative experiences so that the subject matter is taught not only from a logical point of view, but from a psychological one as well" (p. 75).

Abstractor's Comments

A fundamental hypothesis of this study is that students must be at the formal operations stage in order to successfully understand proportionality, as usually encountered in the school setting. But this was not upheld on the math quiz, which was the most conventional test given after the instructional unit. Here the early concrete students did quite well when compared to the early formal students and quite a bit better than the late concrete students. In other words, the students did not perform as expected. (The hypothesis that such categorizations of Piagetian tasks might help classroom teachers in predicting which students might have trouble with specific lessons was not upheld--nor was it discussed.) The authors simply comment that: "The Math Quiz problems gave a curious non-monotonic result. This is perhaps due to the relatively small number of subjects in the three levels" (p. 73).

Oddly, the authors do not indicate concern about the small cell sizes when the students performed in the expected direction.

The Kruskal-Wallis test was performed five times on five different sets of data. This has a severe impact upon the overall α-level of the study. Since the authors chose an α-level of .05 for each of the five tests, the overall α-level of the study is

$$1 - (.95)^5 \simeq .226$$

Even if the Kruskal-Wallis test were appropriate here (which it is not), an overall α-level in excess of .10 is generally unacceptable.

> [The basic situation for which the Kruskal-Wallis test is appropriate is one] in which the data consist of k (≥ 2) random samples, one sample from each of k treatment populations. The basic hypothesis is that of no treatment differences; that is, the k samples can be thought of as a single (combined) sample from one population. The alternatives of major interest relate to differences in location. (Hollander and Wolfe, 1973, p. 114)

For this study, the three samples were not chosen randomly. Sample differences were known <u>prior</u> to data collection and, in fact, it was these differences that formed the basis upon which the samples were constituted. Thus the authors introduced a confounding factor that was not controlled in the subsequent analysis. The three samples did not experience different treatments. Hence, the data analysis offered by the authors is inappropriate.

One experimental design appropriate to the stated purpose of the study is a randomized block design:

	Treatments	
	Instruction	Control
Early Concrete		
Late Concrete		
Early Formal		

We need to know the characteristics of the population and that some semblance of random sampling from the population took place. To aid reliability of test results, the control group should be given a placebo instruction unit unrelated to proportions.

The conclusions of this study are not warranted by the data, nor does the data analysis seem to be related to the hypothesis in question. The authors have shown (although they seem unaware of it) a correlation between the Piagetian tasks used for the initial categorization and those used after the instruction was completed. An initial categorization based upon other criteria (e.g., IQ or previous grades) probably would have yielded similar results on the post-instructional tasks. Overall, it appears a hypothesis was stated, data were collected, a statistical analysis was performed, and conclusions were made. Connections between these events are tenuous at best.

Reference

Hollander, M. and Wolfe, D. A. Nonparametric Statistical Methods. New York: John Wiley and Sons, 1973.

Lepore, Angela V. A COMPARISON OF COMPUTATIONAL ERRORS BETWEEN EDUCABLE MENTALLY HANDICAPPED AND LEARNING DISABILITY CHILDREN. <u>Focus on Learning Problems in Mathematics</u> 1: 12-33; January 1979.

Abstract and comments prepared for I.M.E. by A. EDWARD UPRICHARD, University of South Florida.

1. Purpose

This study was designed to analyze differences and similarities in whole number computation errors of four different groups of slow learning children: young mentally handicapped (YMH), older mentally handicapped (OMH), young learning disability (YLD), and older learning disability (OLD). It was hypothesized that errors made by groups would be generic rather than group-specific.

2. Rationale

The inability of children to do arithmetic may be related to environmental factors, cognitive factors, and/or affective factors. Because of the complexities involved, the grading of a child's arithmetic paper is not sufficient to identify the inefficient strategies and/or misconceptions underlying errors. To analyze errors properly requires direct interaction with the child while he or she is performing the arithmetic procedure. That is, an interview technique needs to be used to ensure appropriate diagnosis. The focus of this study was on the diagnosis of whole number computation errors. Whole number computation was selected for study since it is a prerequisite for problem solving (Pace, 1959).

3. Research Design and Procedures

Four groups of students comprised the sample for this study: 28 YMH, 30 OMH, 8 YLD, and 12 OLD. A test-retest procedure was employed. An arithmetic test measuring whole number computation (addition, subtraction, multiplication, and division) was developed based on an outline of basic skills presented in a teachers handbook on mathematics (Wilson, 1939). The test had four sections, one per operation, and each section was scored separately.

The total test was <u>group</u>-administered to all children; any child who scored between 20 and 65 percent correct on a section(s) (operation) was selected for individual retesting on that section(s) only. The retesting used different problems measuring the same skill. After each problem attempted on the retest, the examiner asked the child to explain how he or she arrived at the answer. Detailed notes of the child's explanation were taken and errors analyzed using a diagnostic checklist of errors prepared by Wilson (1939). Errors not on the checklist were also recorded and a master list of errors organized by operation was developed. There were four examiners.

4. Findings

Most of the retesting focused on subtraction and multiplication; addition was too easy, division too difficult. The MH groups were retested on subtraction more than any other operation; the LD groups were retested most on multiplication. In general MH groups made more types of errors than LD groups. More specific findings by operation are as follows:

Addition - errors were classified as Procedure, Facts, Renaming, Reading, Gaps, Process Reversal, Decimal, and Wrong Process. The number of addition errors by group were YMH-30, OMH-37, YLD-7, and OLD-2. The YMH group made the most errors in Procedure (13%), Facts (23%), Renaming (20%), Reading (17%), and Process Reversal (17%). The basic error made by OMH and YLD children was renaming (59% and 71%).

Subtraction - errors were classified as Process, Renaming, Zero, Gaps, No Subtraction, Reversals, Facts, and Lefts. The number of subtraction errors by group were YMH-237, OMH-151, YLD-24, and OLD-36. Renaming was clearly the most difficult task for children in all groups (YMH-54%, OMR-46%, YLD-58%, OLD-67%). The Zero error also caused considerable difficulty for three groups - YMH (17%), OMH (23%), and YLD (25%). Facts caused some problems for the OMH (18%) and OLD (19%) groups.

Multiplication - errors were classified as Renaming, Multiplies by 1 Digit Only, Multiples by Wrong Number, Add Errors, Zeros, Facts, Partial Sums, Reversals, Process. The number of multiplication errors by group were YMH-94, OMH-114, YLD-49, and OLD-43. Multiplies by Wrong Number was the most common error across groups (YMH-56%, OMH-28%, YLD-45%, OLD-30%). Renaming and Facts also caused some difficulty across groups.

Division - errors were classified as Process, Quotients, Remainder, Zero, Subtraction/Multiplication, Partial Dividend, Incomplete, and Facts. The number of division errors by group were YMH-30, OMH-45, YLD-0, and OLD-10. The most frequent errors made by the YMH group were Quotients (13%), Partial Dividend (40%), Incomplete (13%), and Facts (23%). Errors of OMH children were Process (22%), Zero (18%), Partial Dividend (18%), and Facts (20%). Quotients (40%), and Zero (30%) caused most difficulty for OLD children. The number of children retested in division was small; thus interpretations are difficult to make.

5. Interpretations

The types of errors found across groups vary. This is counter to the original hypothesis. However, when examining types of errors by mental age (MA) of subjects rather than groups, there seems to be a direct relationship between error type and MA. Computation errors made by MH and LD children are similar if the MA of subjects is examined. Computation errors are not unique to the type of disability.

Abstractor's Comment

There are a number of comments I would like to make on Lepore's research report. Some are simply editorial in nature, while others more substantive.

The editoral comments relate to group labels, pagination, and spelling. These comments might be considered "picky", but they jump out at the reader.

1. The labels used in the text of the report to represent young mentally handicapped and older mentally handicapped are YMH and OMH. In the tables used to summarize data YMH is MYR and OMH is OMR. Thus, there is a lack of consistency between tables and text which is confusing for the reader.
2. In the section of the report describing the multiplication results, pages 28 and 29 are out of order. The text on page 29 should precede that on page 28.
3. In Table 10 - Percentage of Division Errors, the word "Remainder" is spelled "Reminader".

The editors of Focus on Learning Problems in Mathematics must be more careful if they intend to publish a scholarly journal.

The research reported by Lepore was clinical and/or exploratory in nature. The purpose of this type of research is to generate hypotheses rather than to to confirm or deny hypotheses. The hypotheses generated in clinical research are based for the most part on the subjective judgment(s) of the researcher(s) after careful analysis of the data.

In the present study I would question the author's interpretation of the data. She claims that the errors made by the four different slow-learning groups varied, and thus her original assumption that errors made by the groups would be similar was not valid. This conclusion is an over-generalization! The data presented in Tables 7-10 clearly indicate similarities in errors in some or all four groups with respect to a specific operation. For example, in subtraction, Renaming was the most difficult error across all four groups; forty-six percent or more the errors within any group occurred in this category. Two categories, Renaming and Zero, accounted for at least seventy-one percent of the errors within any of the four groups. Other examples using data from addition and multiplication can be given to challenge Lepore's interpretation of the data.

Lindsall, C. Mauritz and Ibarra, Cheryl Gibbons. INCORRECT PROCEDURES USED BY PRIMARY GRADE PUPILS IN SOLVING OPEN ADDITION AND SUBTRACTION SENTENCES. Journal for Research in Mathematics Education 11: 50-62; January 1980.

Abstract and comments prepared for I.M.E. by WERNER LIEDTKE,
University of Victoria, Canada.

1. Purpose

As part of the exploratory investigation an attempt was made to:

a) identify, tabulate relative frequency of occurrence, and offer explanations for incorrect solution procedures used by first and second graders solving a variety of forms of open addition and subtraction sentences.

b) investigate prerequisites to the correct solution of open sentences and to their application in story problems.

2. Rationale

The investigation was based on the following assumptions:

a) Open sentences are an important topic and they serve a useful purpose, even at the first grade level, since they are essential for the understanding of addition, subtraction, equals signs, and equations. Open sentences are also useful for solving story problems.

b) Open sentences are difficult to learn. Research is cited to support this statement.

c) Knowledge of incorrect solution procedures is of value as far as diagnostic purposes and planning for instruction are concerned.

3. Research Design and Procedures

Three first-grade and two second-grade classes, each consisting of about 20 pupils, were selected from five communities which differed as far as type and size were concerned. The first graders were tested in spring, the second graders in October. Seven types of open sentences ($a + \square = c$; $\square + b = c$; $a - \square = c$; $\square - b = c$; $c = a + \square$; $c = \square + b$; $c = a - \square$) were presented in four different test-settings to determine types of incorrect procedures used by the subjects. The test-settings and response requirements included:

Written Test (W-T) - filling in missing numbers.
Oral Test (O-T) - reading sentences aloud.
Story Problem Test (S-T) - supplying answers to orally presented problems.
Demonstration Test (D-T) - illustrating sentences with blocks.

Two frequency tables, one for sentences showing the operation on the left, the other for sentences showing the operation on the right, and four contingency tables for the sentence type $\square - b = c$ were constructed to report the results.

4. Findings

a) For sentences showing the operation on the left:
For the 101 subjects, the number of correct responses ranged from 83 to 96 for the W-T, O-T, and D-T settings. For the S-T setting, 67 and 69 correct responses were obtained. The most common incorrect procedure used by the subjects was the addition of the numbers given (givens) in the open sentence. The number of correct responses for the subtraction sentence with the missing term in the second position was high for all four test settings (77 to 96), since performing the operation indicated in the sentence on the given numbers (givens) does yield the correct answer.

The number of correct responses for subtraction sentences with the missing term in the first position were 80, 57, 36, and 26 for the O-T, S-T, D-T, and W-T setting respectively. The most common errors included the subtraction of givens for the W-T setting; the subtraction of givens and solving backwards (i.e., 4 minus 1 equals 3 for $\square - 4 = 3$) for the O-T and D-T settings; and reporting one of the numbers from the story as the answer for the S-T setting.

b) For sentences showing the operation on the right:
Three types of sentences were used: $c = a + \square$; $c = \square + b$; $c = a - \square$. Since the story problems would be similar to those for sentences showing the operation on the left, only the W-T, O-T, and D-T settings were used.

For the addition sentences, the number of correct responses ranged from 44 to 80. The subjects found the O-T to be the most difficult setting. The most common errors included the addition of givens for the W-T and D-T settings. Reading the sentence from left to right was the most common procedure for the O-T setting.

For the subtraction sentence, the number of correct responses were 46 for the W-T setting, and 56 for both the O-T and D-T settings. For all three settings the most common error was the reading of the sentence from right to left (i.e., c = a - $\boxed{}$, 'something minus a equals c').

c) Factors associated with ability to solve open sentences: Since the majority of incorrect responses were interpreted to be a result of incorrect reading, it was hypothesized that the ability to solve a number sentence will depend on the ability to read and on the ability to explain (demonstrate) what the sentence means. The data for the sentence type $\boxed{}$ - b = c were judged to be most suitable to test these relationships. The results were: "The data seem to suggest that reading the sentence is probably an essential prerequisite to being able to solve it"; "being able to verbalize the sentence, although a necessary prerequisite, is not a sufficient prerequisite"; and being able to demonstrate (explain) the sentence was not a prerequisite for being able to solve it.

5. Interpretations

Since reading a sentence was found to be an important determiner of the ability to solve it, it is suggested by the authors that the role of careful and correct reading could be given more emphasis during the teaching process. The fact that the subjects were found to change from a correct reading to an incorrect one to illustrate their own incorrect solution procedures was used in support of this statement. A similar outcome for the demonstrating of an open sentence contributed to the conclusion by the authors that it is important to teach students "to comprehend number sentences in terms of all the meanings that are essential to their solution and to their application to real problems."

The importance of reading is reiterated under the discussion on implications for instructional practice. One instructional strategy that is suggested is that students should be exposed to reading and study activities that involve correct as well as incorrect solutions to open sentences. The authors make the comment that not all of the sentences considered in this study should be taught at the early grade levels which were included as part of this investigation.

Suggestions for further research include suggestions for identifying: measures of understanding open sentences; the extent to which the ability to identify correct and incorrrect solutions of number sentences is prerequisite to being able to solve them; and the extent to which pupils are able to comprehend stories that could be described by various types of open sentences.

Abstractor's Comments

The published report on the investigation would have been more complete had information on the following points been included:

a) Was the written test part of the inverview setting or was it a group test?

b) What kind of notes or records were kept during the interviews?

c) Why were second graders part of the sample? What was the reason for the particular first- and second-grade split?

d) What was the specific make-up of the open sentences? Were doubles or near doubles included for the addition sentences? Was one or zero included in any of the sentences? For the addition sentences, was the given addend greater or less than the missing addend? For the sentence $\square - b = c$, was $b > c$ or was $b < c$? For any of the sentences was $c < 10$, $c = 10$ or $c > 10$?

e) What kind of story problems were presented? What terms were used in these problems?

f) Which of the open sentences had been taught before the testing? If any of the sentences had been taught, how were they presented? What mathematics program or text was used by the subjects?

The high number of correct responses to the tasks involving the open sentences is an amazing result, especially since first graders and beginning second graders made up the sample. Many teachers, some from as high as the third grade, will classify missing addends or open sentences as one of the more difficult ideas to teach. (Some delay the topic until the arrival of the student teachers!) Why were the tasks so easy for these subjects? The answers to some of the questions raised in d above could provide some sort of clue. Perhaps the subjects had been taught the type of responses expected in these test settings? Could it be that the tasks of the test-settings were so easy that the subject could answer them correctly without having acquired an understanding of the actions associated with these open sentences?

Specific information about the make-up of the items that were used is needed before this last question can be answered for the W-T setting. Since the word 'box' was accepted as a correct response, this answers the question for the O-T setting in the affirmative and accounts for the fact that almost all subjects responded correctly. As far as the D-T setting is concerned, other difficulties exist. It is almost impossible to imagine how a young child can demonstrate with blocks the action associated with an open sentence of the type $a + \square = b$. (This is a difficult task for an adult!) The only thing a child can demonstrate is something that is representative of $a + b = c$. If the demonstration is done incorrectly, it may not be because the open sentence is not understood, but because a basic fact may have been forgotten. On the other hand, a correct demonstration may have nothing to do with understanding what the open sentence is all about. The expectations of the observer and the evaluation procedures that were used for the D-T setting are difficult to imagine.

Since it seems possible for a subject to respond correctly to the tasks of the test settings without actually being able to visualize or understand the actions associated with open sentences, the interpretations based on the results of the study come into question. The report begins with the statement that 'most elementary school arithmetic curricula introduce children to the language of number sentences and to the solution of open sentences early in their study of mathematics.' A quick on-the-spot survey of five available first-grade books showed that four of them did not include open sentences at all, and the authors of one restricted themselves to one type of sentence ($a + \square = c$).

The authors of the report state the assumption that open sentences are needed for the understanding of addition and subtraction. From the study and the results it is difficult to see why this should be the case. Wouldn't it be more appropriate to present these open sentences in the later grades when the students are able to distinguish between using a number sentence to summarize ar action for a given description or representation and using a number sentence to calculate the answer to a question? Even at this later stage, open sentences would enhance the understanding rather than be necessary for the understanding of addition and subtraction.

Throughout the discussion a heavy emphasis is placed on the ability to read open sentences. In a hierarchy of tasks that are required for the understanding of a given idea (skill or concept), shouldn't the importance of reading be de-emphasized, especially at the early stages of learning about the idea? Isn't there a danger that if reading is emphasized the result may be that learning takes place at nothing other than the verbal level? Over one hundred young children were interviewed as part of the investigation. Any task that involves young children is a rewarding experience since something unpredictable or unexpected always occurs. It was disappointing that in the whole report not even one small paragraph was included to report on something that is representative or typical of the ages of the subjects used in the study.

The report of the study was rather long and a bit too speculative. The most valuable outcome of the investigation are the suggestions for further research.

Robinson, M. A. and Straley, H. W. THE EFFECT OF TEACHER-AIDE EXPERIENCE ON MATHEMATICS APTITUDE. School Science and Mathematics 80: 245-250; March 1980.

Abstract and comments prepared for I.M.E. by TERRY A. GOODMAN, Central Missouri State University.

1. Purpose

The purpose of this study was to determine the effect of supervised teaching experience on secondary students' mathematics aptitude. In this study, mathematics aptitude was measured by student scores on the Scholastic Aptitude Test-Mathematics.

2. Rationale

There have been many efforts to improve students' mathematics aptitude and reverse the decline in SAT-M scores. A teacher aide program was designed to provide a student-teaching experience for senior high school mathematics students. It was proposed that this program would help improve participants' mathematics aptitude.

The program involved juniors and seniors in a college preparatory school. These students acted as aides, assisting classroom teachers with basic mathematics sections. Each aide conducted weekly extra-help sessions, helped make and grade tests, checked homework, worked with individual students, and taught short lessons to entire classes. Aides received independent study credit (1 Carnegie unit) for participation in the program.

3. Research Design and Procedures

An experimental group was composed of eighteen students who participated in the Teacher Aide Program during the period 1971--197[illegible]. These subjects satisfied two additional requirements:

1. Initial participation in the program was during the fall of their senior year.
2. They took the SAT-M in both junior and senior years.

A list was formed of students from the same period who had not participated in the aide program. Each experimental subject was matched with a student from this list so that:

1. the members of each matched pair graduated within one year of each other;
2. the members of each matched pair took the same senior mathematics course; and
3. the junior SAT-M score of the control subject was as close as possible to that of the corresponding experimental student.

A gain score was calculated for each subject. This gain score was the difference between the junior and senior SAT-M scores. The mean gain scores for the experimental and control groups were compared using a matched t-test.

4. Findings

The results of the study showed that the mean gain of the experimental group was significantly larger (by 37 points) than the mean gain of the control group ($p < .01$).

To further support the initial equivalence of the control and experimental groups, the groups were compared with respect to two other measures: junior SAT-Verbal scores and senior grade point average. The former was to be a measure of intellectual ability, while the latter was to represent academic ability and motivation. T-tests indicated no significant differences between the groups on either of these measures.

5. Interpretations

Based on the results of this study it was concluded that participants in the student-teaching program significantly improved the mathematics aptitude of the experimental group.

Several specific aspects of the Teacher Aide Program were suggested as possible reasons for the significant gain in SAT-M scores. First, the teacher aide was forced to understand concepts more completely in order to teach them to other students. Perhaps the students were more motivated than they would be in a student-learning setting.

Second, the aides were required to explain concepts in several ways in order to work with different students. Making up problems for tests also encouraged the aides to view concepts from more than one perspective.

Overall, then, the student-teaching experience involved relearning of basic mathematics concepts and skills. The major advantage of the program was that the student was more motivated to study, review, and explain basic mathematics.

Several recommendations were also suggested by the researchers. It was proposed that teacher aide programs involving students with lower mathematics aptitude be initiated. It was also proposed that further studies be conducted with particular emphasis on the following questions:

1. What specific factors in a teacher aide experience have effect on aptitude?
2. What kind of school environment is necessary for the success of such a program?
3. What effect do these programs have on teachers, students, and aides?

Abstractor's Comments

This study has some appeal since it was conducted in a rather normal school setting. It would be feasible for many schools to initiate programs similar to the one described in the study. There is a need for studies conducted in realistic school settings and the experimenters are to be commended for their effort. There are, however, some related issues that need to be considered.

While the experimental design for this study was appropriate, there are several questions about the analysis. Since the number of subjects was rather small, an analysis of covariance might have been more appropriate than the matched t-test. The former would provide a more precise analysis than simple gain-score comparisons. Also, the researchers mentioned that a Chi-square test was used, but did not report how or what the results were.

The researchers conclude that "participation in the Teacher Aide Program improves a student's mathematics aptitude as measured by the SAT-M." Comparison of mean gain scores is used to support this conclusion. Consideration of individual pairs of subjects raises some interesting points. In five of eighteen cases, the control subject gained as much or more in SAT-M scores as the experimental subject. Also, there was one rather extreme case in which the experimental subject had a gain score 180 points higher than the corresponding control subject. The next largest difference was 90 points. With only eighteen cases, this extreme case may have had a strong effect on the overall results. Thus, the experimenters' conclusion may be somewhat overstated. From these results, it appears that for <u>some</u> students participation in this program improves mathematics aptitude. It could be useful to consider for which students the program is most beneficial.

The experimenters claim that there was no significant difference between the groups with respect to motivation. Why, then, did some students choose to participate in the teacher aide program? The program required more work, more time, and more preparation. Perhaps the experimental students (at least in their senior year) were more motivated to review, relearn, and pursue mathematics in more depth and this higher level of motivation was more crucial than the nature of the program.

Finally, even though the program appears to have been beneficial for some students, it is not clear whether this was due to the specific nature of the program. Could students' aptitude be increased through other means, such as extra courses, independent study, or special topics? This is not to argue against the program described in the study, but some questions need to be considered. Could more students be involved through other approaches (relatively few students participated in this program over a six-year period)? What experiences should be provided for less able and/or less motivated students? Are there other approaches that could be more efficient in terms of time and money? Could the program be structured more toward individual student needs and interests?

This program shows some promise. Careful investigation of the questions raised here and those of the experimenters is needed to describe better the benefits and limitations of such programs.

Saxe, Geoffrey B. COUNTING AND NUMBER CONSERVATION. Child Development 50: 180-187; March 1979.

Abstract and comments prepared for I.M.E. by EDWARD C. RATHMELL, University of Northern Iowa.

1. Purpose

The major purpose of the studies reported was to determine the relationship between the development of (1) children's logical reasoning to use counting to compare and reproduce arrays and (2) their understanding of number conservation. Accuracy in counting was also examined as it related to the development of these counting strategies and number conservation.

2. Rationale

Views concerning the relationship between counting and the conservation of number differ widely. Both counting-based models and non-counting-based models have been proposed for the development of number conservation concepts. Advocates of the counting-based models suggest that counting experiences help children learn that spatial transformations do not alter the number of a set. Others have formulated non-counting-based models. Piaget has indicated that "...counting can never cause the development of logico-physical concepts such as number conservation." (p. 180) In fact, he goes on to suggest that "...it is only once the child reaches an understanding of number conservation that counting can acquire meaning as a 'symbolic tool' to represent numerical relationships" (pp. 180-181).

These studies were designed to determine if children develop the use of counting as a means of comparing and reproducing sets before, at the same time as, or after the development of number conservation. Predictions made from a counting-based model would include (1) an invariant sequence with children learning to use counting as a means to compare and reproduce sets prior to understanding number conservation, and (2) children who fail to develop these uses of counting will also lack number conservation. On the other hand, predictions made from a non-counting-based model would include (1) varying sequences of development between counting and number conservation and (2) children who fail to develop counting skills will still develop number conservation.

3. Research Design and Procedures

The first study was conducted with 13 four-year-olds, 13 five-year-olds, and 40 six-year-olds from middle-class nursery and elementary schools. Age groups were selected to represent children in the beginning and transitional stages of development to mature counting strategies and number conservation. The subjects of the second study included 20 seven-year-olds, 13 eight-year-olds, and 11 nine-year-olds. These children had been categorized as learning-disabled students whom teachers judged to have atypical counting skills.

In the first study, the interviewer presented two sets of tasks to individual children. The order of presentation was balanced within each age group. The accuracy of all counting was recorded for each of the tasks.

The notational counting tasks included (1) comparing a long row of 9 dots to a parallel but shorter row of 11 dots, and (2) placing a card with 9 dots on the floor beneath a table and asking the children to place the same number of beads on top of the table without moving the card. In both cases, children were encouraged to count if they did not count spontaneously.

The number conservation tasks involved comparisons of two arrays. The interviewer presented an array of 9 beads and had the children make another row of beads in one-to-one correspondence. After first asking if both sets had the same number, the children were than asked to compare the sets when one row had been spread and again after it had been compressed.

The subjects in the second study were asked to perform counting and number conservation tasks similar to those in the first study. In order to learn more about their counting skills, they were also asked (1) to count four sets of 9 or 11 dots, (2) to draw "the same number" of circles as a linear array of 10 circles, and (3) to place "the same number" of beads on a table as a linear array of 9 beads that was shown. The Piagetian tasks of class-inclusion and seriation were also presented. The reason for including these Piagetian tasks was to determine if there was a general delay of cognitive development in these students or if the lack of counting skills might have delayed only the development of number conservation.

4. Findings

Of the 21 children who conserved number in the first study, all but one had already developed quantitative counting strategies. The one exception was a subject who was at the transitional stage of developing quantitative counting. On the other hand, only 20 of the 46 who had developed quantitative counting were number conservers.

Only 10 of the 44 children interviewed for the second study were found to have atypical counting skills. The two who used prequantitative counting did not conserve number. The other eight all counted at less than 60 percent accuracy, but were quantitative counters and conservers.

There was a significant difference ($p < 0.01$) in counting accuracy favoring children who used quantitative counting as opposed to those who used prequantitative counting. However, not all quantitative counters were accurate. Six of the 45 quantitative counters in the first study were accurate less than two-thirds of the time. There was not a significant difference ($p < 0.80$) in counting accuracy between the conservers and the nonconservers.

5. Interpretations

"The present findings indicate that counting undergoes a significant intellectual construction prior to the development of number conservation" (p. 186). Since the use of quantitative counting strategies developed before number conservation for both normal and atypical subjects, "...it is likely that this constitutes an invariant developmental sequence" (p. 186).

The findings are consistent with the predictions of the counting-based models. They suggest that children use counting to discover number conservation. However, there is an inherent difference in the tasks that were used in these studies. The counting tasks require an understanding of correspondence with a static set while the conservation tasks require an understanding of a dynamic correspondence between two different arrangements of a set. "This difference suggests that the invariant order between the acquisition of counting and conservation is determined by progress in the child's understanding of one-to-one correspondence relations from static to dynamic forms. Moreover, this formulation suggest that number conservation and notational counting are not directly linked to one another..." (p. 187). The latter hypothesis would help explain the fact that counting accuracy is apparently not crucial in the development of number conservation. "Whether the child uses notational counting as a means of gathering information to help understand dynamic correspondences is simply not known..." (p. 187).

Abstractor's Comments

The studies reported here provide evidence that children develop strategies to use counting prior to the development of number conservation. Counting is used in their system of logic to compare and reproduce sets before they conserve number. The studies did not provide evidence that can be used to determine if counting-based models or non-counting-based models are appropriate for the development of number conservation.

There is much to be learned before the relationship between counting and number conservation can be established. These studies, together with an earlier study by Saxe (Child Development, 1977, 48, 1512-1520) and the work of Gelman and Gallistel (The Child's Understanding of Number. Cambridge: Harvard University Press, 1978), provide a beginning for our understanding of the development of counting and the uses of counting.

It is the belief of the abstractor that continued investigation of the development of one-to-one correspondence and counting will lead to improved learning experiences for young children. In the past, investigations emphasizing number conservation have been attractive to researchers, but the impact on the curriculum has been minimal at best. Questions such as the following are at least as important and show more promise for being relevant to the immediate needs of teachers.

1. How can we help children improve their accuracy in counting? In particular, how can we help children prevent the partitioning and coordination errors described in Gelman and Gallistel? Would a highly structured approach to matching the number names to the objects being counted help? Is the movement of objects one by one necessary for some students?
2. How can we help children learn to use the results of counting in their system of logic? There are many tasks in the primary grades which would be much easier for children if they would make use of the knowledge they have obtained through counting. The comparison and reproduction tasks that Saxe investigated are examples of where this logic is useful. Another more common problem for teachers involves the counting that children use for basic addition and subtraction facts. Most children make three separate counts when they answer these basic fact questions. It has been found that

hiding one of the sets being joined in an addition problem is an effective way for children to learn to answer the question more efficiently by counting on. Is the hidden set approach an effective way to help children learn to use the results of counting in other situations? What other approaches will help?

Smith, Lyle R. TASK-ORIENTED LESSONS AND STUDENT ACHIEVEMENT. Journal of Educational Research 73: 16-19; September/October 1979.

Abstract and comments prepared for I.M.E. by DOUGLAS A. GROUWS and JOHN J. ENGELHARDT, University of Missouri-Columbia.

1. Purpose

The primary purpose was to investigate the effect of two teaching behaviors--Content-Relevant Questions and Content-Relevant Activities--on mathematics learning. The secondary purpose was to determine whether these two teaching variables could be quantified reliably by analysis of audio tape-recordings of the lessons.

2. Rationale

Since student achievement has been shown to be related to time on task, the investigation attempted to explore to what degree two specific teacher behaviors could influence time on task and thus student achievement.

3. Research Design and Procedures

Twenty experienced high school algebra teachers were randomly selected. Each teacher taught one 20- to 30-minute lesson on direct variation in one class. A common textbook was used across schools and teachers were given lesson objectives prior to instruction. Teachers were, however, free to present the lesson in their own way.

Each lesson was audiotape-recorded, and three coders were trained to quantify the two teacher variables by listening to the recordings. After 90 minutes of training, coder reliability ranged from .81 to .92 for Content-Relevant questions and from .89 to .97 for Content-Relevant Activities.

Immediately after each lesson, a 14-item posttest (reliability of .83) covering direct variation was administered. California Achievement Test (CAT) scores were used to adjust for initial differences among students. These adjusted posttest scores were then used as the criterion of teacher effectiveness.

Content-Relevant Questions were defined as those directly related to the lesson objectives given each teacher. Each teacher received a percentage score for this variable. Three levels were defined (post hoc): HQ required that

100% of teacher questions be content-relevant, MQ required between 67% and 100% be content-relevant, and LQ required that 0% to 67% be content-relevant.

Content-Relevant Activity was defined to be the percentage of lesson time allowed for students to work individually (seatwork) or in small groups on problems involving direct variation. The degree of actual student involvement in the work was not considered. Again three levels were defined (post hoc): HA required 20% of lesson time be used for content-relevant activities, MA required 10% to 20%, and LA less than 10%.

These two teacher variables were defined to be "mutually exclusive" because questions asked during the "seatwork" time were not counted in the Content-Relevant Question scoring.

Levels were defined post hoc in order to reflect the variability of the teachers participating in the study. No specific rationale was given for the specific cut-off values for the levels. Cell sizes were unequal.

A 3 x 3 least squares analysis of variance was performed on the "covariance-adjusted" achievement scores using the student as the unit of analysis.

4. Findings

"Covariance-adjusted" means were reported for each of the nine cells in design. ANOVA showed the two main effects (Relevant Questions and Relevant Activity) and their interaction (Q x A) to be significant at the .01 level. Post hoc analysis using the Newman-Keuls method revealed the following significant differences ($\alpha < .01$): HQ>LQ, MQ>LQ, HA>MA. At the .05 level, HQ>MQ and HA>LA.

A multiple regression analysis using the adjusted posttest scores as the dependent measure was performed with class (teacher) means on the two variables entered.

5. Interpretations

The author acknowledged several limitations: The lesson taught by each teacher was on the average only 23 minutes, the posttest measured only short-term retention, causal inferences are difficult to draw as no actual manipulation was present, and coder reliability was perhaps higher than might be expected due to the background of the coders.

The author concluded that a high degree of relevant classroom activity positively influenced achievement, while extraneous teacher questions impeded short-term retention. "Since the MA and LA conditions were not significantly different in terms of influencing achievement, it may be that the percentage of lesson time the teacher designates for content relevant activities needs to be at a relatively high level before achievement is influenced positively" (p. 19).

The author also concluded that coding recordings of lessons directly (as opposed to transcripts of recordings) may lead to greater use of low-inference measures in investigating teacher effectiveness by those responsible for teacher evaluations.

Abstractor's Comments

Several points need to be made in considering this study--some dealing with statistics, some with conceptualization, and some with interpretation.

With regard to statistics, the author's primary analysis used the student as the unit of analysis. In a classroom setting where students are not randomly assigned and teacher behavior associated with the class as a whole is the major concern, the appropriate independent data points for analysis are class means. Secondly, the use of analysis of covariance followed by analysis of variance dces not make sense. A covariance model removes the effects of a covariate while testing the main and interaction effects. The author should have presented a table with the covariance analysis and a table showing raw and adjusted means.

Given that the interaction between the two teacher variables was significant, the main effects should have been looked at more carefully. The author ignores all discussion of this interaction and does not consider its influence in his conclusions.

There is a mistake in the article in Figure 1 which, given that the means are reported correctly, can be corrected by interchanging the labels LA and MA on the graph.

It is worth noting that the Low-Activity group was higher in achievement than the Middle-Activity group under both the HQ and LQ conditions. This raises a question about the generalization that a high degree of relevant classroom activity positively influences achievement. It seems that relevant classroom activity (as defined in this study) is moderated by the amount of relevant classroom questions and probably by many other instructional conditions that have not been considered nor controlled.

The variable of Content-Relevant Activity was a measure of the amount of time set aside for seatwork on the subject of direct variation. There is no indication of the degree to which students were on task. Furthermore, only 6 minutes separates the activity categories: HA roughly 6 minutes or more; MA, 3 to 6 minutes; LA, 0 to 3 minutes. This is hardly enough variation to warrant a conclusion that relevant classroom activity positively influenced achievement.

The length of time the study encompassed (23-minute for the average lesson) and the lack of consideration and control of instructional methods and classroom organization make any type of conclusion from the study suspect and preclude making any meaningful generalizations.

Several other areas of concern are: First, are the labels for the teacher behaviors descriptive? For example, is Content-Relevant Activity too broad a label to use for seatwork? Second, why wasn't the research related to seatwork and classroom questions reviewed? Third, why wasn't attention given to the validity of the codings of the teacher behaviors? Finally, could the instructional practices of one teacher have overly influenced the results of the study since in one case only one teacher represented a cell in the design?

Smith, Lyle R. and Land, Michael L. STUDENT PERCEPTION OF TEACHER CLARITY IN MATHEMATICS. Journal for Research in Mathematics Education 11: 137-146; March 1980.

Abstract and comments prepared for I.M.E. by DOUGLAS T. OWENS, University of British Columbia.

1. Purpose

"The present study addresses the following questions: (a) What are the combined effects of teacher vagueness terms, mazes, and additional unexplained content on student evaluations of instruction and on student achievement? (b) What is the relation between student perception of teacher clarity and student achievement?" (p. 138).

2. Rationale

"A number of low inference indicators of teacher clarity have been identified" (p. 137). This study goes beyond previous research on evaluating teacher clarity in that particular indicators are operationally defined and controlled. Specifically vagueness terms are defined to be "... words or phrases indicating approximation, unclarity, a lack of assurance" (p. 137). Frequency of teacher vagueness terms has been shown to correlate negatively with student achievement in social studies (Hiller, Fisher, and Kaess, 1969) and mathematics (Smith, 1977).

Mazes are identified "... as false starts or halts in speech, redundantly spoken words, and combinations of words that do not make semantic sense" (Smith, 1977, p. 137). On a previous study, the authors (Land and Smith, 1979) "... reported that student achievement in mathematics is negatively affected by a high frequency of teacher mazes" (p. 137). Additional unexplained content is composed of terms that are not directly related to the material in the lesson and which are left undefined.

3. Research Design and Procedures

The experimental design of the study was a 2 (teacher vagueness terms vs. no teacher vagueness terms) X 2 (teacher mazes vs. no teacher mazes) X 2 (additional unexplained content vs. no additional unexplained content). The following quotation containing [vagueness terms], [mazes], and additional content is illustrative.

> We, [of course], say 5 is not a (divi, uh,) division of 14, since we (we, uh, we) get a remainder of 4, [you know], when we divide 5 into 14. <u>This ties in with modular arithmetic, since 14 is congruent to 4 mod 5</u>. How many divisions does 15 have? Write them down. (Pause for 8 seconds.) It [seems] that 15 has 4 divisions: 1, 3, 5 and 15.

The eight 19-to-21 minute lessons on sums of consecutive positive integers were videotaped by one teacher. The teacher read from a script while the camera focused on relevant material from overhead projection. Time in the lessons varied due to the presence or absence of vagueness terms (7.5 per minute on average), mazes (average of 5.1 per minute), and additional unexplained terms (0.75 per minute on average). Otherwise the lessons were substantively equivalent.

The subjects, 160 college students who had no prior knowledge of the content, were randomly assigned to the eight experimental conditions ($n = 20$ per cell). After viewing the lesson and taking notes, the students took a 17-item test on the content (K-R 20 = 0.97). Following the test, each student completed a 13-item evaluation of the lesson by responding to "Definite no," "No," "Yes" and "Definite yes" on each item. The responses were coded 1-4, with 4 being the presence of a desirable quality (e.g., "1. The teacher was confident") or the absence of an undesirable one (e.g., "4. The lesson frustrated me").

"A 2 x 2 x 2 analysis of variance was performed on each of 15 dependent variables: the scores for each of the 13 response items on the Lesson Evaluation Form, the scores for the combined totals on the Lesson Evaluation Form and the post test student achievement scores" (p. 139).

4. <u>Findings</u>

"The no-vagueness condition produced higher achievement scores than the vagueness condition, although significance did not quite reach the 0.05 level. The no-mazes condition produced significantly higher achievement ($p < .02$) than the mazes condition" (p. 139). Additional content produced no main effect nor were interactions evident.

The no-mazes condition produced significantly better student response scores ($p < .001$) than the mazes condition on all 13 response items on the total. The no-vagueness condition produced significantly better student response scores (beyond the 0.05 level) than the vagueness condition for 7 of the 13 items and for the total. On 4 items, the no-vagueness condition produced (non-significantly) better response scores, and virtually the same response was produced on 2 items by the vagueness and no-vagueness conditions. The main effect due to additional content was not significant for any response items or for total, although the responses for 3 items showed slight trends in favor of no-extra-content and 2 items showed small trends in favor of the extra-content condition.

The vagueness variable interacted significantly with the mazes variable on 9 response items. A figure showing the nature of the interaction for one item is given in the report to serve as an illustration of the type of all such interactions.

The vagueness variable interacted significantly with the additional-content variable on one item. Mazes interacted significantly with additional content on 2 items. Interactions among all three variables were found for 3 items.

"To determine whether the student evaluations of instruction were related to student achievement, the mean scores for the combined totals on the Lesson Evaluation Form were ranked from 1 to 8 and were compared with the rankings of the mean achievement scores for each of the eight experimental conditions. ... The Spearman rank-order correlation was 0.786, ($p < .05$)" (p. 144).

5. Interpretations

"The results of this study indicate a cause-effect relationship between teacher mazes and student achievement. To a less significant degree, there is an indication of a cause-effect relationship between teacher vagueness terms and student achievement" (p. 144). Additional unexplained content had no significant effect and perhaps it partially counterbalanced the use of vagueness terms and mazes by inducing students to perceive the teacher as competent. This interpretation is consistent with previous research and, moreover, Item 7 ("The teacher really knew what he was talking about") was the only item which approached 0.05 significance in favor of additional content.

The results indicate that student evaluations can be useful indicators of instructional effectiveness. "Students generally were able to discriminate between lessons that contained vagueness terms and/or mazes and lessons that did not contain such phrases. Further, student evaluations were reasonably accurate predictors of student achievement as evidenced ..." (p. 145) by the rank-order correlation and the rankings.

Abstractor's Comments

This was a very interesting and relevant piece of research. Indeed, the investigators cite research (Smith, 1977) which indicates that some mathematics teachers frequently use vagueness terms and mazes.

The investigators have taken considerable care in the design and control of the experiment. For example, while a video-taped lesson may not be identical to a live lesson, it would appear to be the best simulation which would control for any extraneous factors. However, their interpretation of the statistical results is open to question. For example, was there a cause-effect relationship, even a "less significant" one, between teacher vagueness terms and student achievement when the F-ratio did not quite reach the 0.05 level of significance?

Perhaps more should have been done to display and explore the nature of the interactions. Should the main effects of Vagueness and Mazes on the 13 response items have been interpreted in the face of 9 significant interactions? Was it appropriate to have the amount of notice given to near significant differences, slight trends, and non-significant differences?

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