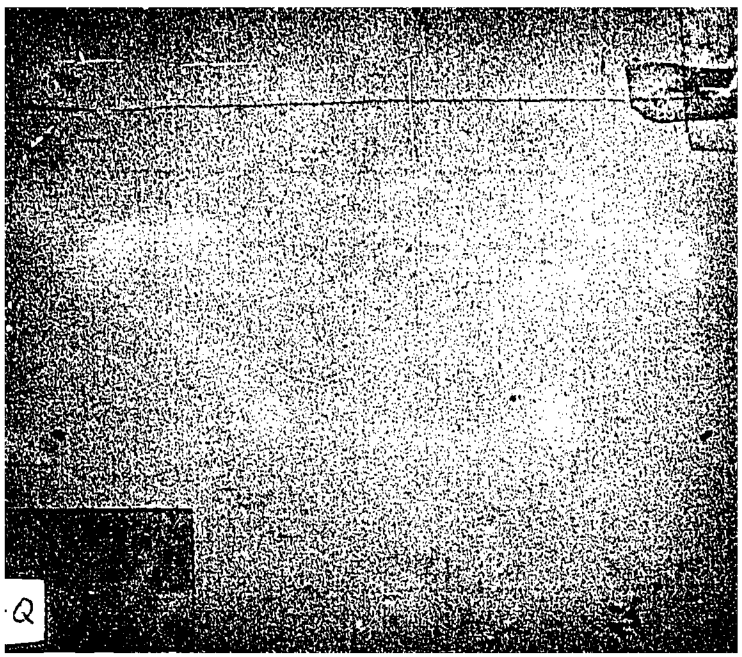

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ABSTRACT

These materials were designed to be used by life science students for instruction in the application of physical theory to ecosystem operation. Most modules contain computer programs which are built around a particular application of a physical process. Specifically, this module develops a method for calculating the exchange of heat between an animal and its physical environment. The approach used is to develop a mathematical model that represents the energy balance of an animal, to discuss the components of the model, and to apply these theories by working out the details of the energy budget of a familiar animal. Examples are used to illustrate that if the sheep's environment is given in terms of temperature, wind speed, and incoming radiation, it is possible to determine the rates of body metabolism and evaporative losses that will allow the animal to maintain thermal equilibrium. The model which is developed can be applied to other animals and environments and can be used to evaluate household fuel bills. A knowledge of differential equations is a prerequisite to the course. (Author/CS)

HEAT BALANCE OF A SHEEP IN THE SUN

by

W.H. Hatheway

This instructional module is part of a series on Physical Processes in Terrestrial and Aquatic Ecosystems supported by National Science Foundation Training Grant No. GZ-2980.

August 1979

PREFACE

Using the principle of conservation of energy and Fourier's Law, Laplace's equation is derived and solved to show the distribution of temperature in a two-layer model of a sheep. The basic heat transfer processes are reviewed so that the model can be applied to other animals and environments. The problem set forms an important extension, allowing the student to gain insight into the importance of thermal balance to normal animal physiology. This module assumes a mathematical background obtained in a differential equations course.

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Introduction

In the present module, we will calculate the exchange of heat between an animal and its physical environment. This is a problem of considerable biological importance. An organism's energy balance is crucial to its survival. If it loses heat faster than it takes it or other forms of energy in, then eventually its temperature will drop to that of its surroundings. On the other hand, if the organism takes in heat faster than it gets rid of it, eventually it will become overheated. In both cases, the temperature extremes to which the organism is subjected may be outside the range in which it is able to function effectively or even to survive.

Moreover, it is clear that there is a good deal of variation in temperature regimes to which animals are adapted. Few tropical mammals and birds can survive prolonged periods of freezing weather. On the other hand, animals like sheep and polar bears may not be able to withstand high ambient temperatures, especially in combination with exposure to sunlight. Every animal has a preferred range of body temperatures. How it manages to maintain its body temperature within this preferred range is largely determined by its ability to control the factors which affect its heat balance.

For this reason, it is exceedingly important to identify the factors which determine an animal's temperature and to understand the relative magnitude of their contributions. How important is sweating as a cooling process? Is a layer of fat as effective as one of feathers or fur in preventing bodily heat loss? Is there a relationship between body size and heat loss, and if so, how large must a warm-blooded aquatic animal be if it is to survive, say, in the waters surrounding Antarctica? It is only by carefully calculating the contributions of sweating, insulation, body size, ambient temperature, radiation, wind, humidity, and other factors to an animal's heat balance that we can make useful answers to these questions.

Accordingly, it is the objective of this module to provide the reader with some of the intellectual equipment he will need to calculate the heat balance of an organism. Our approach will be to develop a mathematical model that represents the energy balance of an animal, to discuss briefly the theory of some of the components of the model (e.g., heat conduction, convection, and absorption and emission of radiation), and to apply these theories by working out the details of the energy budget of a familiar animal. Since the principles are perfectly general, the techniques developed here may be used for any organism, plant or animal, or the reader may even wish to apply them to such practical problems as determining the effect of additional insulation on his yearly household fuel bill or the practicality of attempting to capture solar energy in Seattle, Chicago, or New York!

An animal can exchange heat with its environment by absorption or emission of radiation, by evaporation of water from its surface, and by conductive or convective exchanges of heat with its surroundings. Moreover, heat moves from the animal's body through its layers of insulation (fat and fleece) by the process of conduction. We will confine our attention to the equilibrium or "steady state" case, in which rates of heat loss and gain are equal. A solution to the more complicated situation, in which the rate at which the organism gains or loses heat in response to changing environmental conditions, is available in the computer model of Vera et al. (1975).

If we assume that the animal is in thermal equilibrium with its environment, our analysis boils down to an identification of all sources of energy loss and gain. Our assumption of thermal equilibrium, which greatly simplifies the analysis, is not entirely unrealistic. Thermal equilibrium implies that the total energy gained by the animal equals total energy lost. If an animal is to maintain its body temperature within a relatively narrow range, it cannot afford to gain or lose substantial amounts of heat. If it finds itself out of thermal equilibrium, it must make suitable adjustments. These may include moving into the shade or increasing its rate of energy loss by panting or sweating if heat gains are excessive or increasing rate of metabolism in case heat losses are very great. Having made these adjustments successfully, the animal returns to its "average" state of equilibrium. Any prolonged, substantial departure from this state constitutes a crisis which, of course, may terminate in the animal's death.

To fix our ideas, we choose an unshorn domestic sheep as our study organism. The sheep is convenient because it is a familiar animal which has been thoroughly studied by a number of veterinary scientists (e.g., Blaxter, 1962; Brockway, McDonald and Pullar, 1965; Joyce, Blaxter and Park, 1966). In addition, the energy balance of the sheep has been studied by Priestley (1957), Monteith (1973), Porter and Gates (1969), and Vera et al. (1975). Experimental work by Walsberg et al. (1978) and the theoretical studies of Kowalski (1978) have shown that under certain environmental conditions it becomes important to take into consideration the penetration of radiation and wind into the wool. These complications are not included in the present model, which must therefore be considered only a first approximation to a more correct approach. In the present module, we adopt the general method of analysis put forward by Porter and Gates, but in may details we follow Priestley and, in some cases, Monteith.

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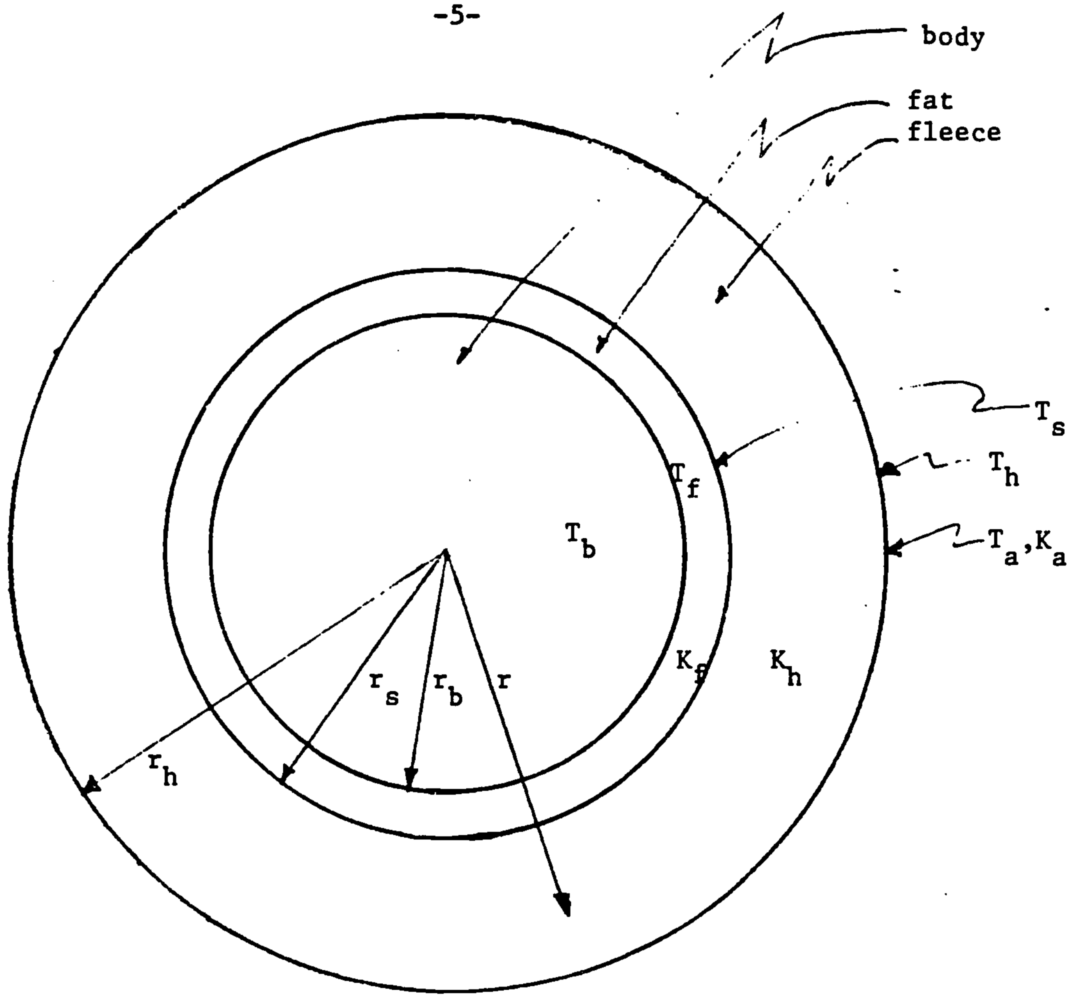

Figure 1. Sheep idealized as a cylinder. In the diagram, T_a, T_b, T_s, T_h and T_f are the temperatures of the air, body, skin, hair tips and fat; r_b, r_s and r_h the corresponding radii of the cylinder; and K_a, K_h and K_f the thermal conductivities of air, fleece and fat.

present, we will assume that $T_b > T_h$, so that metabolic heat is lost to the environment, and will defer a discussion of the factors which combine to determine the surface temperature, T_h, which is of paramount importance in our model.

The means by which metabolic energy is transported in the form of heat from the body to the fleece tips is conduction. It was observed by Fourier in 1822 that the rate of movement of heat Q (W) perpendicular to surface of area A through a material such as body fat or wool is proportional to the gradient of temperature T (K) in the material:

$$Q = -kA\ dT/dr \quad . \tag{4}$$

Here k ($W\ m^{-1}\ K^{-1}$) is a thermal conductivity coefficient, a property of the conducting material, and r (m) is the distance from the center of the cylinder measured along a radius. The steady-state temperature in a cylinder, such as our idealized sheep, is given by Laplace's equation,

$$d^2T/dr^2 + \frac{1}{r}\ dT/dr = 0 \quad , \tag{5}$$

which we derive in the following way. Consider two concentric cylinders, such as those in Fig. 1. Heat moves outward across the inner cylinder at a rate of $q_b = \frac{Q_b}{A_b} = -k\ \frac{dT}{dr}$ ($W\ m^{-2}$). Total heat loss from this cylinder is $Q_b = q_b(2\pi r_b L)$ (W). In a steady-state situation, total heat loss across a larger concentric cylinder is the same:

$$q_s\ A_s = q_s(2\pi r_s L) = q_h(2\pi r_h L)$$

so that

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$$Z_f = \frac{1}{2\pi k_f L} \ln\left(\frac{r_s}{r_b}\right) \quad . \tag{15}$$

Eliminating T_s from (12) and (14), we obtain

$$Z_h\, Q_h + Z_f\, Q_f = T_b - T_h \quad . \tag{16}$$

At this point, it is important to point out that in general Q_h will not be equal to Q_f. In the fat, rate of heat flow is exactly equal to that leaving the body,

$$Q_f = M - E_r \quad .$$

At the surface of the skin, however, heat is lost by sweating, so that

$$Q_h = Q_f - E_s = M - E_r - E_s \quad .$$

Substituting these results in (16), we obtain

$$Z_h(M-E_r-E_s) + Z_f(M-E_r) = T_h - T_b \quad ,$$

or

$$M = E_r - \frac{E_s}{1 + \dfrac{Z_f}{Z_h}} \quad \frac{T_b - T_h}{Z_h + Z_f} \quad . \tag{17}$$

This important equation governs the rate of heat movement away from the animal's body. Body heat produced by metabolism M is removed by evaporative losses due to respiration E_r and sweating $E_s/(1 + Z_f/Z_h)$ and by conduction of sensible heat to the fleece tips $(T_b - T_h)/(Z_h + Z_f)$. Total evaporation is then

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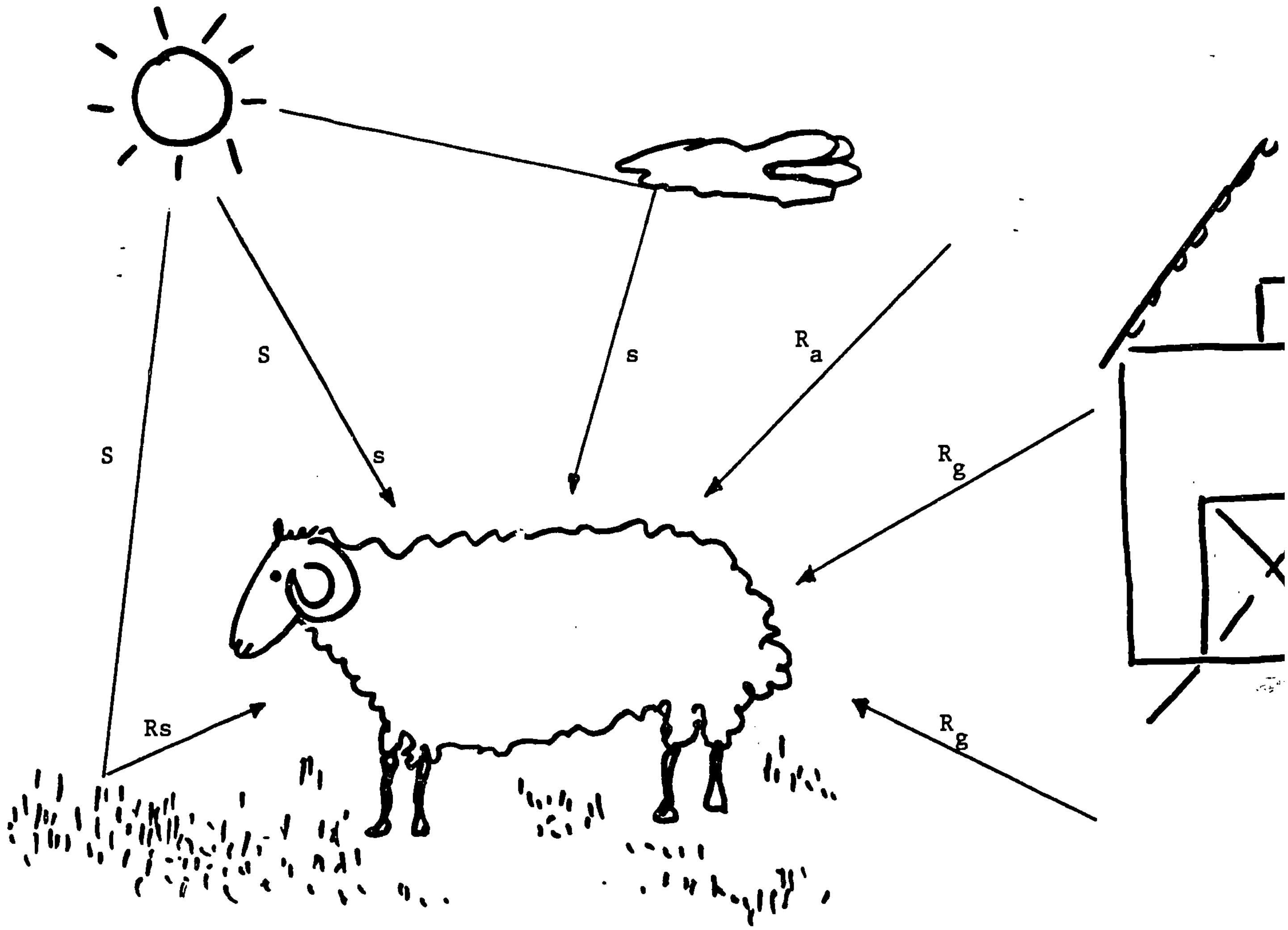

Figure 2. Energy gained by sheep from envrionment. Symbols are as follows: S = direct solar radiation, s = sky radiation, rS = reflected solar radiation; R_g, R_a = long-wave (thermal) radiation from ground and air, respectively.

past the solid surface, and it is to this effect that we now direct our attention.

It is customary to rewrite Eqn. 18 in the form

$$H = (\frac{d}{\delta})\ k_a\ A_h\ (T_h - T_a)/d \tag{19}$$

in which A_h is the surface area of the animal at the fleece tips and d is a characteristic dimension of the body past which fluid is moving, in our case the diameter of the cylinder that represents the sheep's body and fleece. The ratio d/δ is called the Nusselt number, Nu. Since Nu is a function of boundary layer thickness, it clearly depends on the Reynolds number, which, it will be recalled, may be written

$$Re = \frac{vd}{\nu} \quad .$$

In this expression v ($m\ s^{-1}$) is the velocity of the fluid relative to the body and ν is the kinematic viscosity of the fluid, about $1.5 \times 10^{-5}\ m^2\ s^{-1}$ for air at 20°C. Tables for Nu are provided in various texts on heat transfer. For gases, the empirical formula $Nu = a\ Re^n$ may be used (Goldstein, 1938). The coefficient a and the exponent n vary considerably with the Reynolds number. Monteith (1973) provides the following table for cylinders whose axes are at right angles to the wind:

Re	Nu
1 - 4	$0.89\ Re^{0.33}$
4 - 40	$0.82\ Re^{0.39}$
$40 - 4 \times 10^3$	$0.63\ Re^{0.47}$
$4\times10^3 - 4\times10^4$	$0.17\ Re^{0.62}$
$4\times10^4 - 4\times10^5$	$0.024\ Re^{0.81}$

To obtain some feeling for the magnitude of the convective term H, let us assume that our wind speed is $v = 2.0\ m\ s^{-1} = 4.5$ miles hr^{-1}. Then $Re = (2.0)(0.5)/(1.5\times10^{-5}) = 6.67\times10^{4}$, and we calculate $Nu = (0.024)(6.67\times10^{4})^{0.81} = 194$. The thermal conductivity k_a of air at 20°C is approximately 2.57×10^{-2} W m^{-1} °C^{-1}, and since the surface area A_h of the sheep is $5\pi \times 10^{-1}\ m^2$,

$$H = (194)(2.57\times10^{-2})(5\pi\times10^{-1})(T_h-T_a)/.50 = 15.7(T_h-T_a)\ W$$

$$= 2.35\ W \quad , \quad \text{if } T_h = 25°C \text{ and } T_a = 10°C \quad .$$

It is also possible to calculate the heat lost by "free convection," which is important when wind speeds are less than 1 m s^{-1}. In this case, the buoyancy of heated air is a dominant factor in the transfer of heat away from the animal. Calculation of H in this "unstable" case involves determination of the "Grashof number," which must be used in place of the Nusselt number. A problem involving free convection is offered as one of the exercises for this module.

Heat Loss by Radiation

All warm bodies lose heat energy by emitting long-wave thermal radiation according to the Stefan-Boltzmann law:

$$R_- = \varepsilon A_h \sigma T_h^{4} \quad . \tag{20}$$

Here T_h is the absolute (Kelvin) temperature of the surface of the body--in our case, the fleece tips of the sheep, σ is the Stefan-Boltzmann constant, and ε is the emissivity of the body. Total heat energy lost this way by the sheep is then $R_- = (\pi dL)(\varepsilon\sigma T_h^{4})$. To obtain a rough

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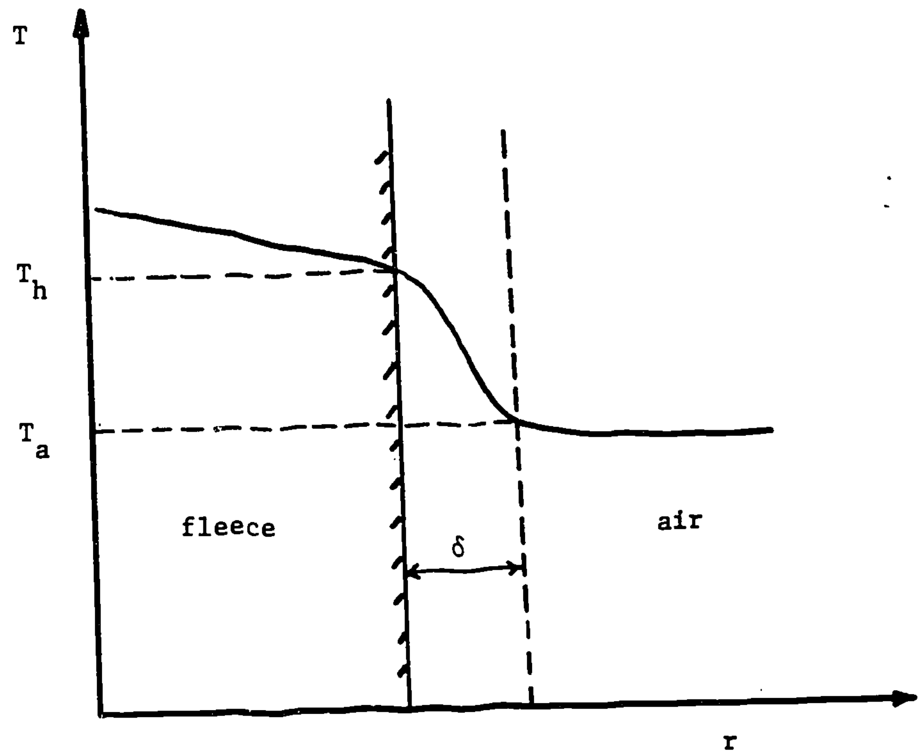

Figure 3. Temperature profile across fleece and air. δ represents boundary layer thickness, T_h = fleece-tip temperature, T_a = air temperature.

Direct Solar Radiation

We may make an approximate calculation of a_1A_1S, the direct solar radiation absorbed by the sheep's body in unit time, if we consider the amount of sunlight intercepted by a cylinder of diameter d and length L. Clearly, if the sun is directly overhead, the shadow cast by the cylinder on the ground has area dL. It follows immediately that the surface of the cylinder absorbs the same amount of radiation as would a plane of area dL. (The average amount of energy absorbed per unit area of the cylinder is then a_1 (dL) $S/(\pi dL/2) = 2a_1 S/\pi (W\ m^{-2})$, since the cylinder presents half its area to the sun.) As long as the sheep maintains its body at right angles to the sun, it intercepts the same fraction of incoming solar radiation.

We obtain an estimate of the total direct solar radiation absorbed by the animal by substituting the following values for our parameters: $a_1 = 0.74$; $A_1 = dL = 0.5 \cdot 1.0$; and $S = 1046.7\ W\ m^{-2}$, the value corresponding to a clear June 21 day at latitude 40°N at about 10 A.M. or 2 P.M. (Bartlett and Gates, 1967). Then $a_1A_1S = 387$ W.

Sky Radiation

The sheep exposes an area $A_2 = \pi dL/2\ m^2$ of its upper surface (half the area of the cylinder, neglecting the ends) to diffuse radiation coming in from the sky. Then the total sky radiation absorbed is $a_2A_2s = a_2\pi dLs/2$. Since sky radiation intensity is about 27.9 $W\ m^{-2}$ (Bartlett and Gates, 1967), we calculate this component of $A\ Q_{abs}$ as $(0.74)(\pi \cdot 0.5 \cdot 1.0)(27.9)/2 = 16$ W.

Reflected Short-Wave Radiation

The sheep's lower surface absorbs direct and scattered solar radiation that has been reflected from the grassy surface on which it stands (Fig. 3). If the grass reflectivity is 0.25 (Monteith, 1973), and if one-third of the solid angle is occupied by the sheep's shadow, the radiation received will be approximately $(0.25)(2/3)(0.74)(\pi \cdot 0.5 \cdot 1.0/2)(1047.7+27.9) =$ 104 W.

Long-Wave Radiation

The sheep is also heated by thermal radiation emitted from sky (R_a) and ground (R_g). As Priestley (1957) pointed out, development of a correct model for long-wave radiation absorbed by the sheep is a difficult problem. The major complication arises from sky radiation, which is imperfectly understood. Both Priestley (1957) and Bartlett and Gates (1967) rely on an empirical formula due to Brunt (1939) to calculate sky thermal radiation. Brunt's formula is $R_a = 1.04\ B$ where $B = (0.44 + 0.08\ \sqrt{e})\sigma\ T_a{}^4$. In this expression, e is the vapor pressure of the atmosphere (millibars), σ is Stefan's constant $(5.67\times10^{-8}\ W\ m^{-2}\ K^{-4})$ and T_a is the air temperature (K). Then the sky component of the long-wave radiation absorbed by the sheep is $a_4A_4R_a = 1.04\ BA_4$, in which $A_4 = \pi dL/2$. To calculate B, we take $e = 10$ mb, and $T_a = 10°C = 283$ K. Then $B = (.69)(5.67\times10^{-8})(283^4) = 251\ W\ m^{-2}$ and the component due to atmospheric long-wave radiation is 205 W.

Long-wave radiation from the ground absorbed by the sheep is given by $a_5A_5R_g = A_5\sigma T_g{}^4$, where T_g is the ground temperature (K). Here A_5 is again one-half the surface area of the cylinder. For our calculation, we will follow Priestley (1957) and make the convenient assumption that $T_g = T_h$, the average temperature of the tips of the sheep's hair. Since the energy loss from the upper and lower parts of the sheep's body due to emitted long-wave radiation is $(\pi dL)(\sigma T_h{}^4)$, the net energy gain due to long-wave radiation is

$$1.04BA_4 + A_5\sigma T_g{}^4 - 2A_5\sigma T_h{}^4 = (\pi dL/2)(1.04B - \sigma T_h{}^4).$$

Although we have not yet shown how T_h is to be calculated, if we assume again that $T_h = 20°C = 293$ K, then we calculate the net gain as

$$(\pi \cdot 0.5 \cdot 1.0/2)(1.04 \cdot 251 - 5.67 \times 10^{-8} \cdot 283^4) = -81\ \mathrm{W}\ .$$

Since net gain is negative, we conclude that the animal suffers a net long-wave radiative loss of 81 W at its surface.

Metabolic Heat

Porter and Gates (1969) have calculated estimates of M for several animals, including sheep, based on published data for animal oxygen consumption. Consumption of O_2 implies oxidation of protein, fat, or carbohydrate. Since the heat released in each of these reactions is known, the Porter-Gates method consists essentially of identification of the animal's diet and conversion of quantity of O_2 consumed to amount of metabolic energy produced. This method is most useful in studying animal species in which metabolic data are not readily available.

It probably comes as no surprise to the reader to learn that metabolic rates of domestic animals such as sheep have been thoroughly studied by veterinary scientists. Brockway, McDonald, and Pullar (1965) describe a fascinating series of experiments in which total, sensible, and evaporative heat losses in sheep were carefully measured by direct calorimetry. For an animal maintained at 15°C with a fleece depth of 0.1 m, total heat loss by all routes was 132 kJ kg^{-1} day^{-1}. Total evaporative heat loss was 52.3 kJ kg^{-1} day^{-1}, of which 41.9 kJ was due to respiratory evaporation. Sweating heat loss was approximately 12.6 to 20.9 kJ kg^{-1} day^{-1} regardless of temperature. The authors suggest that, since sheep have little or no physiological control over sensible heat loss, variation in respiratory evaporation heat loss is the chief means by which sheep adjust to their environment. When the weather is hot, they are obliged to pant!

An Energy Balance Calculation

In making the calculations that follow, we assume fleect thickness of 0.1 m, which is the depth of wool usually attained by Cheviot sheep (Brockway et al., 1965). Then skin diameter for our "standard sheep" is 0.3 m. The layer of subcutaneous fat is about 0.01 m thick (Porter and Gates, 1969). We take thermal conductivity of sheep's wool as 5.91×10^{-2} W m^{-2} C^{-1} (Blaxter, Graham, and Wainman, 1959) and that of fat as 20.51 10^{-2} W m^{-2} C^{-1}(Porter and Gates, 1969). Body temperature of the sheep is taken as 39°C (Priestley, 1957). We assume a body weight of 70 kg.

To consolidate our calculation, we return to our general heat balance relation (Eqn. 1):

$$M + R_{+} = R_{-} + C + H + E \quad .$$

Our objective at this point is to calculate T_h, the fleece-tip temperature. We now define net radiation received by the animal at the fleece tips as

$$\begin{aligned} R_{Nh} = R_{+} - R_{-} &= a_1A_1S + a_2A_2s + a_3A_s(S + s) \\ &\quad - a_4A_4R_a + a_5A_5R_g - \varepsilon A_h\sigma T_h^{\,4} \end{aligned} \tag{22}$$

and R_{Na} as the net radiation which would be received in the same environment if the fleece tips were at air temperature, so that

$$\begin{aligned} R_{Na} &= a_1A_1S + a_2A_2s + a_3A_3(S + s) + a_4A_4R_a \\ &\quad + a_5A_5\sigma T_a^{\,4} - \varepsilon A_h\sigma T_a^{\,4} \quad . \end{aligned}$$

Then

$$R_{Nh} - R_{Na} = a_5A_5\sigma T_h^{\,4} - \varepsilon A_h\sigma T_h^{\,4} - (a_5A_5\sigma T_a^{\,4} - \varepsilon A_h\sigma T_a^{\,4})$$

and, since $a_5 = 1$, $A_h = \pi dL$ and $A_5 = \frac{1}{2}A_h$,

$$R_{Nh} - R_{Na} = -\pi dL\sigma(T_h{}^4 - T_a{}^4)(\tfrac{1}{2} - \varepsilon) \quad . \tag{23}$$

We now write

$$R_{Nh} \doteq R_{Na} = 2\ T_a{}^3(T_h - T_a)A_h \quad . \tag{24}$$

To obtain this relationship, we have assumed $\varepsilon \doteq 1$ and have used an approximation based on the Taylor Series expansion of $T_h{}^4 = g(T_h)$, say, near the point $T_h = T_a$:

$$\begin{aligned} T_h{}^4 = g(T_h) &= g(T_a) + g'(T_a)(T_h - T_a) + \ldots . \\ &= T_a{}^4 + 4T_a{}^3(T_h - T_a) + \ldots . \end{aligned}$$

Since from Eqns. 1 and

$$M - E + R_{Nh} = H + C \quad ,$$

we obtain, using Eqn. 17a,

$$\frac{T_b - T_h}{Z_n + Z_f} + R_{Nh} = H \quad . \tag{25}$$

We have dropped the conductive term C, since heat conduction from the animal to the ground is small in comparison to other heat loss if the sheep is not lying down. We now substitute Eqn. 24 into Eqn. 25 and, remembering that H is given by Eqn. 19, we obtain an expression which we will use to.

determine T_h:

$$\frac{T_b - T_h}{Z_h + Z_f} + R_{Na} - 2T_a{}^3\sigma A_h(T_h - T_a) = \frac{Nu}{d} k_a A_h(T_h - T_a) \quad . \tag{26}$$

Rearranging, we obtain

$$T_h = \frac{\alpha T_a + \beta T_b}{\alpha + \beta} + \frac{R_{Na}}{\alpha + \beta} \quad , \tag{27}$$

where

$$\alpha = A_h(Nuk_a/d + 2\sigma T_a{}^3) \tag{28}$$

and

$$\beta = 1/(Z_h + Z_f) \quad . \tag{29}$$

Formula 27 states that the temperature at the fleece tips T_h is a weighted average of air and body temperatures T_a and T_b, increased by a contribution from all sources of radiation. The weights α and β have the units of "conductances," that is, of conductivities multiplied by the areas across which heat flows. The conductance α has components associated with convection and radiation, β with conduction in fleece and fat.

To illustrate the calculation of T_h, we take $T_a = 10°C$, $T_b = 39°C$, $A_h = 0.5\ \pi\ m^2$, Nu = 194, $k_a = 2.48\times10^{-2}$ W m^{-1} °C^{-1}, $k_h = 5.91\ 10^{-2}$ W m^{-1} °C^{-1}, $k_f = 2.51\times10^{-1}$ W m^{-1} °C^{-1}, d = 0.5 m, L = 1.0 m, $r_b = 0.145$ m, $r_s = 0.15$ m, and $r_h = 0.25$ m.

Then $\alpha = 1.571(9.638 + 2.570) = 19.19$ W °C^{-1}, $Z_h + Z_f = 1.3748 + 0.05431 = 1.428$, so that $\beta = 0.700$ W °C^{-1}. Also,

$$R_{Na} = a_1A_1S + a_2A_2s + a_3A_3r(S + s) + \tfrac{1}{2} A_h(1.04B - \sigma T_a{}^4)$$

$$= 387.3 + 16.2 + 104.1 - 80.6$$

$$= 427 \text{ W} \quad .$$

Then

$$T_h = \frac{(19.18)(283) + (0.7)(312) + 427}{19.88}$$

$$= 305.5 \text{ K} = 32.5\ ^\circ\text{C} \quad .$$

Thus, in this case, the temperature of the fleece tips is approximately 22°C higher than that of the air. We make the following observations.

1. The contribution of body temperature to fleece-tip temperature is much smaller than that of the air temperature, because the weight α is much larger than β. In turn, β is small because of the very large resistance to heat conduction offered by the wool.
2. The radiative contribution to fleece-tip temperature, which is appreciable (21°C), depends very largely on the size of the short-wave component.
3. The radiative contribution to the weight α is considerably smaller than the convective one.
4. Contribution of fat to total resistance to heat flow from the animal is very small compared to that of the wool.

Once we have determined T_h, a number of very important calculations become possible. The first of these is the total heat of metabolic origin that passes outward through the fleece tips, which is given by Eqn. 17:

$$\frac{T_b - T_h}{Z_h + Z_f} = M - E_r - E_s/(1 + \frac{Z_f}{Z_h}) \quad .$$

In the present case, the quantity on the left works out to be $(39 - 32.5)/1.428 = 4.55\ W = 393\ kJ\ day^{-1}$. Checking the other side, we have already noted that sweating heat loss is in the neighborhood of $16.8\ kJ\ kg^{-1}\ day^{-1} = 1{,}170\ kJ\ day^{-1}$ for a 70 kg animal, independent of air temperature (Brockway et al., 1965). Many experiments have shown that 70 kg sheep lose between 8,800 and 10,000 kJ day^{-1} heat. If we take $M = 8{,}800$ kJ, then $E_r = 7{,}630\ kJ\ day^{-1}$, which is about twice as great as the 3,660 kJ day reported by Brockway et al. (1965). In their experiment, however, there was no net absorption of radiation at the fleece tips, so that higher conductive heat losses were possible.

For comparative purposes, it is also useful to calculate the magnitudes of the terms in the energy balance equations 24 and 25. These are

1. metabolic sensible heat loss

$$\frac{T_b - T_h}{Z_h + Z_f} = 4.6\ W \quad ,$$

2. radiation heat gain

$$R_{Nh} = R_+ - R_- = 387.3 + 16.2 + 104.1 + 2.05 + 386.2 - 757.7$$
$$= 336\ W \quad ,$$

3. convective heat loss

$$\frac{Nu}{d}\, k_a\, A_h\, (T_h - T_a) = 341\ W \quad .$$

Thus, in this case, net incoming radiation at the fleece tips is approximately equal to convective heat loss and metabolic heat loss by conduction from the body through fat and fleece is very small. This is due in part to the high "radiant" temperature at the fleece tips and the resulting low temperature gradient between body and fleece tips.

These examples illustrate only some of the types of problems we can solve with this technique. We have shown that, if the sheep's environment is given in terms of temperature, wind speed, and incoming radiation, it is possible to determine the range of rates of body metabolism and evaporative losses due to respiration and sweating that will allow the animal to maintain thermal equilibrium. Porter and Gates (1969) have worked out the converse problem for a number of animals: given a range of metabolic rates that the animal can maintain as well as its physical echaracteristics (size, thickness of fur, etc.), the range of enviromental conditions within which it can survive can be calculated.

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Cena, K. and J.A. Clark. 1974. Heat balance and thermal resistance of sheep's fleece. Phys. Med. Biol. 19:51-65.

Cena, K. and J.L. Monteith. 1975. Transfer processes in animal coats. I. Radiative transfer. II. Conduction and convection. III. Water vapour diffusion. Proc. Roy. Soc. London Biol. 188:377-393; 395-411; 413-42[illegible].

Clark, J.A., K. Cena and J.L. Monteith. 1973. Measurements of the local heat balance of animal coats and human clothing. J. Appl. Physiol. 35:751-754.

Davis. L.B., Jr. and R.C. Birkebak. 1975. Convective energy transfer in fur. In David M. Gates and Rudolf B. Schmerl (eds.), Perspectives on Biophysical Ecology, pp. 525-548. Springer Verlag, New York.

Doney, J.M. and J. Slee. 1970. General meteorological effects on sheep. Prog. Biometeorol. 1:59-65.

Eckert, E.R.G. and R.M. Drake, Jr. 1972. Analysis of Heat and Mass Transfer. McGraw-Hill, New York. 806 pp.

Gates, D.M. 1968. Toward understanding ecosystems. Adv. Ecol. Res. 5:1-35.

Goldstein, S. (ed.). 1938. Modern Developments in Fluid Dynamics. Oxford Univ. Press. (Republished, 1965, by Dover Publ. Inc.).

Graham, N.M., F.W. Wainman, K.L. Blaxter and D.G. Armstrong. 1959. Environmental temperature, energy metabolism and heat regulation in sheep. I. Energy metabolism in closely clipped sheep. J. Agr. Sci. 52:13-24.

Hales, J.R. and G.D. Brown. 1974. Net energetic and thermoregulatory efficiency during panting in the sheep. Comp. Bioch. Physiol. A 49:413-422.

ena, K. and J.A. Clark. 1974. Heat balance and thermal resistance of sheep's fleece. Phys. Med. Biol. 19:51-65.

ena, K. and J.L. Monteith. 1975. Transfer processes in animal coats. I. Radiative transfer. II. Conduction and convection. III. Water vapour diffusion. Proc. Roy. Soc. London Biol. 188:377-393; 395-411; 413-42?.

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oldstein, S. (ed.). 1938. Modern Developments in Fluid Dynamics. Oxford Univ. Press. (Republished, 1965, by Dover Publ. Inc.).

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ena, K. and J.A. Clark. 1974. Heat balance and thermal resistance of sheep's fleece. Phys. Med. Biol. 19:51-65.

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lark, J.A., K. Cena and J.L. Monteith. 1973. Measurements of the local heat balance of animal coats and human clothing. J. Appl. Physiol. 35:751-754.

avis. L.B., Jr. and R.C. Birkebak. 1975. Convective energy transfer in fur. In David M. Gates and Rudolf B. Schmerl (eds.), Perspectives on Biophysical Ecology, pp. 525-548. Springer Verlag, New York.

oney, J.M. and J. Slee. 1970. General meteorological effects on sheep. Prog. Biometeorol. 1:59-65.

ckert, E.R.G. and R.M. Drake, Jr. 1972. Analysis of Heat and Mass Transfer. McGraw-Hill, New York. 806 pp.

ates, D.M. 1968. Toward understanding ecosystems. Adv. Ecol. Res. 5:1-35.

oldstein, S. (ed.). 1938. Modern Developments in Fluid Dynamics. Oxford Univ. Press. (Republished, 1965, by Dover Publ. Inc.).

raham, N.M., F.W. Wainman, K.L. Blaxter and D.G. Armstrong. 1959. Environmental temperature, energy metabolism and heat regulation in sheep. I. Energy metabolism in closely clipped sheep. J. Agr. Sci. 52:13-24.

ales, J.R. and G.D. Brown. 1974. Net energetic and thermoregulatory efficiency during panting in the sheep. Comp. Bioch. Physiol. A 49:413-422.

ena, K. and J.A. Clark. 1974. Heat balance and the
sheep's fleece. Phys. Med. Biol. 19:51-65.

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I. Radiative transfer. II. Conduction and
III. Water vapour diffusion. Proc. Roy. Soc
188:377-393; 395-411; 413-42?.

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heat balance of animal coats and human clothi
35:751-754.

avis. L.B., Jr. and R.C. Birkebak. 1975. Convectiv
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Prog. Biometeorol. 1:59-65.

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from clouds). Suppose your sheep is standing in a pasture in which reflectivity of the grass is 25 percent. Wind speed is 1 km hr^{-1}. Assume cutaneous evaporative heat loss equals 4 kcal kg^{-1} day^{-1}, but respiratory heat loss can vary. It is known that sheep can breathe as few as 20 times or as many as 250 times per minute, and the corresponding respiratory heat loss varies from 2.0 to 18.0 kcal kg^{-1} day^{-1}. Calculate rate of heat loss if the sheep is shorn (fleece 1 cm thick) and unshorn (fleece 10 cm thick). Assume that total metabolic heat produced is 1900 kcal day^{-1}.

5. Now assume that instead of a sheep your study animal is a blue whale, which you can idealize as a cylinder 25 m long and 5 m in diameter, with a layer of blubber 50 cm thick. Its body temperature is maintained at 33°C. If the whale swims at a rate of 15 km hr^{-1} through 5°C water, how much food (in kcal day^{-1}) must it consume to maintain thermal equilibrium? Take the Nusselt number Nu as $0.332\ (Re_x)^{1/2} \doteq 1.6 \times 10^4$ (since $V = 15$ km hr^{-1}, $x \doteq 10$ m, $\nu = 1.7 \times 10^{-5}$ m^2 sec^{-1}).

6. Calculate heat loss in the blue whale when its velocity relative to the water is zero. The thermal conductivity of water is approximately 5.7×10^{-3} J cm^{-1} s^{-1} $°C^{-1}$.

7. Calculate T_h for a rat whose length is 18 cm, total diameter is 7 cm with a fat thickness of 0.3 cm and a hair thickness of 0.5 cm. Take $\varepsilon = 1.0$ and $T_b = 39°C$. Assume the same environmental conditions as in problem 1, but make calculations for $T_a = 10$ and 30°C only. If evaporative heat loss is 40 W m^{-2}, what is the metabolic heat production? Repeat your calculations for a shrew whose body length is 5 cm, hair thickness of 0.20 cm and total body diameter of 2 cm. What percentage

is evaporative water loss of the total heat loss? Of the sheep, the rat and the shrew, who has the largest per-unit-surface-area heat production?

8. a. Derive equation 5 from equation 6.

   b. Derive equation 7 from equation 6.

   c. Derive equation 23 in the text.

9. When heat transfer depends on air movements caused by differences in fluid density (i.e., buoyancy effects), the Nusselt number can be shown to depend on another dimensionless parameter, the Grashof number,

$$Gr = \frac{\beta g d^3}{\nu^2} (T_h - T_a)$$

where β is the coefficient of thermal expansion of the fluid, g is the acceleration due to gravity, and d is a characteristic length. For air at 20°C moving at right angles to the axis of a cylinder and $10^4 < Gr < 10^9$, $Nu = 0.48\ Gr^{0.25}$ (Monteith, 1973). For the sheep, calculate rate of heat loss due to free convection, if $T_h = 25$ C and $T_a = 20$ C. (Take $\beta = 3.48 \times 10^{-3}\ deg^{-1}$ C, $g = 981$ cm sec^{-2}, $\nu = 0.145$ cm^2 sec^{-1}.)

10. Suppose you did not know T_h and wished to solve for it before proceeding further with your analysis. (Recall that we linearized terms involving $T_h{}^4$ by writing $T_h{}^4 \doteq T_a{}^4 + 4T_a{}^3\ (T_h - T_a)$, supposing that T_h is sufficiently close to T_a, which is taken as known.) How would you linearize the term containing $Nu = 0.48\ Gr^{0.25}$?

Problem Solutions

1. We need to find the total and evaporative heat losses and compare them with the results of Brockway et al. Since there are three air temperatures and three fleece thicknesses, there will be nine separate calculations. We need to calculate T_h using equation 24. To do this, we need the α's and the Nusselt numbers. Here $v = 0.2778 \text{ m s}^{-1}$.

T_a (°C)	10	20	30
ν $m^2s^{-1} \times 10^{-5}$	1.42	1.51	1.60
k_a $W\ m^{-1}\ °C^{-1} \times 10^{-2}$	2.50	2.57	2.64

Nusselt Numbers

Diameter (m)

Air Temperature (°C)	0.32	0.36	0.50
10	38.40	41.31	50.64
20	36.96	39.76	48.75
30	35.66	38.36	47.03

α and β Values (W $°C^{-1}$)

Diameter (m)

Air Temperature (°C)	0.32	0.36	0.50
10	5.644	6.200	8.079
20	5.898	6.447	8.484
30	6.178	6.803	8.927
$\beta(W°C^{-1})$	4.43	1.850	0.705

R_{Na} (W) Values
Diameter (m)

Air Temperature (°C)	0.32	0.36	0.50
10	-51.45	-57.86	-80.36
20	-59.09	-66.48	-92.33
30	-67.58	-76.03	-105.60

T_h (°C) Values
Diameter (m)

Air Temperature (°C)	0.32	0.36	0.50
10	17.65	9.48	3.18
20	22.43	16.24	11.41
30	27.39	23.14	19.69

From equation 17, the total heat loss can be estimated at M since T_b is always greater than T_h. At moderate air temperatures (15 to 30°C), metabolic rate will be high and fairly constant which from the text we will take as 8800 kJ day^{-1} = 101.8 W. At 10°C, however, it is assumed that evaporative water loss is 20.4 (W) and total heat loss is calculated.

Total Heat Loss (W)
Diameter (m)

Air Temperature (°C)	0.32	0.36	0.50
10	114.3	75.0	45.7
20	101.8	101.8	101.8
30	101.8	101.8	101.8

Evaporative Heat Loss (W)

Diameter (m)

Air Temperature (°C)	0.32	0.36	0.5
10	20.4	20.4	20.4
20	28.4	59.7	82.4
30	50.4	72.5	88.2

Brockway, McDonald and Pullar found that metabolic and evaporative heat losses were fairly insensitive to fleece thickness. Exceptions were for a sheep with (a) 1 cm fleece where the percentage of evaporative water loss was generally lower and these animals had a higher metabolic rate especially at lower air temperatures, and (b) with 10 cm fleece metabolic rate was higher at higher air temperatures. At 10°C, they measured metabolic rate to be 140 W for a sheep wi[illegible] cm fleece and 109 W for other sheep. This is higher than the model pr[illegible]ts (114.3 to 45.7 W). At 20 and 30°C, evaporative water loss was measured to be 40.7 and 91.5 W; then values should be compared with model predictions of 28.4 to 82.4 (W) and of 50.4 to 88.2 (W). The comparisons suggest that the model is excessively sensitive to changes in fleece thickness.

Evidently, heat transfer through the fleece is not accurately represented by our model. An improved model might take into account factors such as change in thermal conductivity of the fleece as a function of its water content. Such refinements are always possible. A point is always reached, however, where for all practical purposes the costs involved in making the improvement may exceed the benefits gained from it. In the present case, benefits consist chiefly in improved understanding of the process of heat transfer.

2. $k_a = 2.37 \times 10^{-2}$ W m^{-1} C^{-1}

$\nu = 1.24 \times 10^{-5}$ m^2 s^{-1}

$v = 2.778$ m s^{-1}

$E = 20.4$ W

Diameter (m)

	0.32	0.50
Re	71690	112016
Nu	205.65	295.21
α(W°C^{-1})	17.387	25.22
β(W°C^{-1})	4.43	0.705
R_{Na}(W)	-38.08	-59.51
T_h (°C)	-1.85	-10.97
$\beta(T_b-T_h)$(W)	180.96	35.23
M(W)	201.4	55.6

3. 5060 kcal day^{-1} = 245.25 W

990 kcal day^{-1} = 47.98 W

The conversion is 5.30 g of grass day^{-1} W^{-1}.

Diameter (m)

	0.32	0.50
Required M (W)	228.9	83.21
Required grass (g day^{-1})	1213	441

4. The problem assumes the same conditions as problem 1 for $T_a = 30°C$ and d = 0.32 and 0.50 m, but R_{Na} will be larger.

$$R_{Na} = \frac{\pi dL}{2}\left(\frac{2}{\pi} \cdot 0.74 \cdot 800 + 0.74 \cdot 150 + 0.25 \cdot 0.75 \cdot \frac{2}{3}(950) - 2 \cdot .1396\sigma T_a^{\,4}\right)$$

Diameter (m)

	0.32	0.50
R_{Na} (W)	236.7	369.8
T_h (°C)	56.27	69.4
$\beta(T_b-T_h)$	-76.51	-21.36
$\dfrac{E_s}{1 + (Z_f/Z_h)}$	10.4	13.1
E_r (W)	158.2	100.4

$E_s = 13.6$ W   M = 92.1 W

E_r range was 8.93 W to 60.9 W.

The animals need to lower their metabolic rate or seek shade if they are to prevent a large increase in body temperature.

5. For a whale

$$M = H$$

or

$$\frac{T_b - T_s}{Z_f} = \frac{Nu\ k_w}{d} A_s\,(T_s - T_w)$$

$$\alpha = \frac{Nu\ k_w\ A_s}{d}$$

$$T_s = \frac{T_b + Z_f \alpha T_w}{1 + Z_f \alpha}$$

$k_f = 2.052 \times 10^{-1}$ W m^{-1} C^{-1}

$r_s = 2.5$ m $r_b = 2.0$ m $A = 392.7$ m^2

$Z_f = 6.923 \times 10^{-3}$ °C W^{-1} $k_w = 0.57$ W m^{-1} °C^{-1}

$$\alpha = \frac{1.64 \cdot 10^3 \cdot 392.7 \cdot 0.57}{10} = 3.571 \times 10^4 \text{ W °C}^{-1}$$

$$T_s = \frac{33 + 254.1 \quad 5}{1 + 254.1} = 5.11 \text{ °C}$$

$$M = \frac{33-5.11}{.006923} = 4028 \text{ W}$$

$= 3.48 \times 10^5$ kJ day^{-1} with perfect efficiency

$= 2.32 \times 10^6$ kJ day^{-1} at 15% conversion efficiency

6. There are several ways one might attack this problem. One could simply assume a conduction model in which the water surrounding the animal offered no resistance to heat flow (T_s = 5.0 °C). A Nusselt number calculation could be made using the Grashof number as in problem 9. Or finally, as is done here, one could assume that still water is equivalent to some very slow velocity (in air this is about 0.10 m s^{-1}).

$Nu = 80.5$ $\alpha = 1.802 \times 10^3$

$$T_s = \frac{33 + 6.923 \times 10^{-3} \cdot 1.802 \cdot 10^3 \; 5}{1 + 6.923 \cdot 10^{-3} \cdot 1.802 \cdot 10^3}$$

$$M = \frac{33 - 7.08}{0.006923} = 3.744 \times 10^3 \text{ W}$$

= 93% of metabolic costs while swimming at over 4 m s^{-1}.

7.

		0.02	0.07
A_n	(m^2)	0.003142	.039584
Nusselt number	10°C	10.25	18.47
	30°C	9.69	17.47
Z_f			0.4544
Z_n		12.018	2.3063
β (W °C^{-1})		0.08321	0.36222
α (W °C^{-1})	10°	$\frac{.04833}{.13154}$	$\frac{0.36285}{.72507}$
	30°	$\frac{.05009}{.13330}$	$\frac{0.38567}{.74789}$
R_{Na}	10	-0.13812	-1.7404
	30	-0.2096	-2.6409
T_h (°C)	10	27.29	22.09
	30	34.05	30.82
Sensible heat loss (W)	10	0.9744	6.126
	30	0.4122	2.963
Evaporative water loss (W)		0.1257	1.583
% Evaporative water loss	10	11.4	20.5
	30	23.4	34.8
Metabolic rate per unit area (W m^{-2})	10	350.1	194.8
	30	171.2	114.9

The sheep had a per-unit-area metabolic rate of 72.8 and 64.8 W m^{-2} at 10 and 30°C, respectively. Increasing size increased absolute metabolic rates but decreased per-unit-area rates.

8. (a) $$d\left(r\,\frac{dT}{dr}\right) = \frac{d\left(\frac{\text{const}}{-k}\right)}{dr}$$

$$\frac{dT}{dr} + r\,\frac{d^2T}{dr^2} = 0$$

(b) $$r\left(-k\,\frac{dT}{dr}\right) = \text{const}$$

$$dT = c_1\,\frac{1}{r}\,dr$$

$$T(r) = c_1 \ln r + c$$

(c) Using Eqn. 22 and

$$R_{Na} = a_1A_1S + a_2A_2s + a_3A_3\,(S+s) + a_4A_4R + a_5A_5\sigma T_a^{\,4} - \varepsilon\sigma A_nT_a^{\,4}\ ,$$

we subtract the two

$$R_{Nh} - R_{Na} = a_5A_5\sigma T_h^{\,4} - \varepsilon\sigma A_hT_h^{\,4} - a_5A_5\sigma T_a^{\,4} - \varepsilon\sigma A_hT_a^{\,4}\ ,$$

making the following assumptions:

$$a_5 = 1,\ A_h = \pi dL,\ A_5 = \tfrac{1}{2}A_h$$

$$R_{Nh} - R_{Na} = \sigma\pi dL[\tfrac{1}{2}(T_n^{\,4}-T_a^{\,4}) - \varepsilon(T_n^{\,4}-T_a^{\,4})]$$

$$= \sigma\pi dL[(T_n^{\,4}-T_a^{\,4})(\tfrac{1}{2}-\varepsilon)]$$

9. $$Gr = \frac{3.48\times10^{-3}\cdot 9.81\cdot(0.5)^3}{(1.45\times10^{-5})^2}\ (5)$$

$$= 1.015\times10^{8}\ ,\quad Nu = 48.18$$

$$H = \frac{48.18\cdot 0.0257\cdot\pi\cdot 1.0\cdot 0.5}{0.5}\ (5) = 19.45\ \text{W}$$

10.

$$H = \frac{Nu\ k_a\ A_h\ (T_h - T_a)}{d}$$

$$X = \frac{k_a A_h}{d}\ 0.48\ Gr^{0.25}\ (T_h - T_a)$$

$$= \frac{k_a A_h}{d}\ 0.48\ \left(\frac{\beta g d^3}{\nu^2}\right)^{0.25}\ (T_h - T_a)^{1.25}$$

$$= k'\ (T_h - T_a)^{1.25}$$

$$= k'\ [(\overline{T}_h - T_a)^{1.25} + 1.25(\overline{T}_h - T_a)^{0.25}\ (T_h - T_a)]$$

APPENDIX I. OPERATIVE ENVIRONMENTAL TEMPERATURE

Symbols, Units and Dimensions

Symbol	Quantity	Unit	Dimension
A	Area across which heat flows	m^2	L^2
A_h	Area of cylinder representing animal's body, measured at fleece tips	m^2	L^2
A_b	Area of cylinder representing animal's body, measured inside fat	m^2	L^2
A_s	Area of cylinder representing animal's body, measured at the skin	m^2	L^2
a	Absorptivity	dimensionless	---
B	Sky thermal radiation	$W\ m^{-2}$	HL^{-2}
C	Total heat exchanged by conduction between animal and environment	W	H
d	Diameter of cylinder representing animal's body	m	L
e	Vapor pressure of atmosphere	millibar	$M\ L^{-1}T^{-2}$
E	Rate of heat exchange between animal and environment by evaporation	W	H
E_r	Rate of loss of heat by respiratory mechanisms	W	H
E_s	Rate of loss of heat by sweating	W	H
H	Rate of heat exchange by convection between animal and environment	W	H
k	Thermal conductivity coefficient	$W\ m^{-2}\ K^{-1}$	$HL^{-2}\theta^{-1}$
k_a	Thermal conductivity of air	$W\ m^{-2}\ K^{-1}$	$HL^{-2}\theta^{-1}$
k_f	Thermal conductivity of fat	$W\ m^{-2}\ K^{-1}$	$HL^{-2}\theta^{-1}$
k_h	Thermal conductivity of hair (fleece)	$W\ m^{-2}\ K^{-1}$	$HL^{-2}\theta^{-1}$
L	Length of cylinder representing the body length of the animal	m	L
M	Rate of metabolic heat production by the animal	Q	H
Nu	Nusselt number	dimensionless	---
Q	Rate of heat loss	W	H
Q_h	Rate of loss of heat across hair (fleece)	W	H

APPENDIX I (cont.)

Symbol	Quantity	Unit	Dimension
Q_f	Rate of loss of heat across fat	W	H
Q_H	Rate of heat loss from fleect tips by convection	W	H
q	Rate of heat loss per unit area	W m^{-2}	HL^{-2}
R_{+}	Total long- and short-wave radiation absorbed by the animal per unit time	W	H
R_{-}	Total long-wave radiation emitted by the animal per unit time	W	H
R_a	Thermal radiation per unit area of animal surface originating in atmosphere	W m^{-2}	HL^{-2}
R_g	Thermal radiation per unit area of animal surface originating from earth's surface (including vegetation)	W m^{-2}	HL^{-2}
Re	Reynolds number	dimensionless	---
R_{Nh}	Net radiation absorbed or emitted at hair tips	W	H
R_{Na}	Net radiation absorbed or emitted at hair tips if temperature of hair tips were the same as that of the air	W	H
r	Reflectivity	dimensionless	---
r	Distance along a radius of a cylinder measured from its center	m	L
r_b	Body radius (not including fat)	m	L
r_s	Skin radius	m	L
r_h	Hair (fleece-tip) radius	m	L
S	Direct solar radiation per unit area impinging on the animal	W m^{-2}	HL^{-2}
s	Sky (scattered and reflected solar) radiation per unit area impinging on animal	W m^{-2}	H
T	Temperature	K	θ
T_b	Body temperature of animal	K	θ
T_h	Surface, "radiant" or hair (fleece-tip) temperature of animal	K	θ

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APPENDIX I (cont.)

Symbol	Quantity	Unit	Dimension
T_s	Skin temperature of animal	K	θ
v	Wind speed	$m\ s^{-1}$	LT^{-1}
Z_f	"Impedance" to flow of heat through fat	$s\ K\ J^{-1}$	θW^{-1}
Z_h	"Impedance" to flow of heat through hair	$s\ K\ J^{-1}$	θW^{-1}
α	"Conductance" associated with radiation and convection	$W\ {}^{o}C^{-1}$	W^{-1}
β	"Conductance" associated with animal's insulation (fat and hair)	$W\ {}^{o}C^{-1}$	W^{-1}
δ	Boundary layer thickness	m	L
ε	Emissivity	dimensionless	---
ν	Kinematic viscosity	$m^2\ s^{-1}$	L^2T^{-1}
σ	Stefan-Boltzmann constant	$W\ m^{-2}\ K^{-4}$	$HL^{-2}\theta^{-4}$