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ABSTRACT

This study was designed to investigate the relationship between the level of conservation of displaced volume and the degree to which sixth-grade children learn the volume algorithm of a cuboid, namely, volume equals weight times length times height. The problem chosen is based on an apparent discrepancy between the present school programs and Piaget's theory concerning the grade level at which the volume algorithm should be introduced. Subjects were chosen from three suburban schools in British Columbia and classified as conservers, partial conservers, and non-conservers, using a judgment-based test of volume conservation. The students were divided into three groups, two designated as experimental and one as a control group. One experimental group was taught the algorithm with the standard approach used in North America, with the second experimental group taught with a method emphasizing multiplication skills. Data showed that sixth graders could apply the volume algorithm to computation and comprehension questions regardless of their volume conservation level. There was also noted improvement of students' conservation levels regardless of their volume achievement scores or their treatments. (MP)

THE RELATIONSHIP BETWEEN VOLUME CONSERVATION AND A VOLUME ALGORITHM FOR A RECTANGULAR PARALLELEPIPED

by

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We accept this thesis as conforming
to the required standard

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Ted Hobbs ....

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ABSTRACT

Chairman: Dr. Douglas Owens

This study was designed to investigate the relationship between the level of conservation of displaced volume and the degree to which sixth grade children learn the volume algorithm of a cuboid, "Volume = Length X Width X Height (V = L x W x H)," at the knowledge and comprehension levels. The problem is a consequence of an apparent discrepancy between present school programs and Piaget's theory concerning the grade level at which this algorithm is introduced. While some school programs introduce the algorithm as early as grade 4, Piaget (1960) claims that it is not until the formal operational stage that children understand how they can find volume by multiplying the boundary measures. Very few children in grade 4 are expected to exhibit formal operations. In such a predicament there seems to be a need for research in order to justify our present school curriculum or to suggest modifications.

Subjects of three suburban schools in British Columbia were classified as nonconservers (N = 57), partial conservers (N = 16) and conservers (N = 32) using a judgement-based test of volume conservation. The subjects were then divided into two experimental groups and one control group by randomizing each

conservation group across the three treatments. One of the experimental groups (N = 36) was taught the volume algorithm using an approach (Volume Treatment) which resembles that of school programs used in North America. Activities of this treatment included comparison, ordering, and finding the volume of cuboids by counting cubes and later by using the algorithm "V = L x W x H." The other experimental group (N = 39) was taught the algorithm using a method that emphasized multiplication skills (Multiplication Treatment). This treatment included training on compensating factors with respect to variations in other factors and was supplemented by a brief discussion of the volume algorithm. The control group (N = 30) was taught a unit on numeration systems.

Four different tests were used: Volume Conservation (11 items), Volume Achievement (27 items), Multiplication Achievement (20 items) and the computation section (45 items) of the Stanford Achievement Test. The pretests were: Volume Conservation, Volume Achievement, and Computation. The posttests and retention tests were: Volume Conservation, Volume Achievement, and Multiplication Achievement. Data from the posttests and retention tests were analyzed separately using a 3 X 3 fully crossed two-way analysis of covariance.

Subjects in the volume treatment showed they were able to apply the volume algorithm to computation and comprehension questions regardless of their conservation level. On the posttest and retention test, subjects of this group showed a 65 per cent performance level. For the grade 6 students in the study, conservation level was not a significant factor in learning the volume algorithm at the computation and comprehension levels.

On the posttest, subjects of the multiplication treatment performed significantly ($F = 10.33$, $p < 0.01$) better than those in the other groups on the Multiplication Achievement Test. Subjects of the volume treatment did significantly ($F = 12.24$, $p < 0.01$) better than those in the other groups on the Volume Achievement Posttest. It seems appropriate, therefore, to teach the volume algorithm of a cuboid using a method that includes students' active involvement in manipulating physical objects.

There was, generally, an improvement of the students' conservation levels regardless of their volume achievement scores or treatments. The transition from a lower to a higher level of conservation was found a) independent of treatments between the pretest and each of the posttest ($X^2 = 0.93$, $df = 2$) and retention test ($X^2 = 0.97$, $df = 2$) and b) independent of volume achievement scores between the pretest and each of the posttest (biserial $r = 0.13$) and retention test (biserial $r = 0.09$).

In an addendum to the Conservation Test students were asked to write reasons for their judgements in items involving equal and unequal volumes. Those written reasons were more explicit on the items of unequal volumes than of equal volumes.

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CHAPTER I

THE PROBLEM

The process of selecting, ordering and timing topics in the mathematics curriculum has caught the interest of many mathematics educators. There has been, for example, major concern among curriculum analysts about the necessary cognitive abilities and appropriate age level for presenting volume concepts to elementary school students. While the majority of textbooks contain volume activities as early as grade three, many educators hold that most children do not conserve volume until about age 12 (Uzgiris, 1964; Carpenter, 1975-b; Elkind, 1961-a). There has been a need, therefore, to theoretically and experimentally examine the positions of these educators in order to justify the present curriculum or suggest its modification. Such an examination can focus on many aspects of volume presentation. The present study, however, deals particularly with the introduction of the volume algorithm for a rectangular parallelepiped i.e., "Volume = Length x Width x Height (V = L x W x H)".

Two widely used textbook series in British Columbia introduce the algorithm "V = L x W x H" in grade 5 (age 10) (Dilley et al., 1974 and Eicholz et al., 1974). Another series introduces the algorithm formally in grade 4 (age 9) (Elliot et al., 1974) and uses it informally in grade 3 (age 8) (Elliot et al., 1975). This last series, for example, involves third graders in situations in which they are to compute the volume of a rectangular parallelepiped given its length, width and height.

Most proponents of Piaget's theory would disagree with such early introduction of the algorithm and claim that most children do not develop the necessary cognitive abilities for learning it before grade 6 (age 11). Piaget (Piaget et al., 1960) himself, for example, holds that "it is not until stage IV [formal operational] that children understand how they can arrive at an area or volume simply by multiplying boundary edges" (p. 408). Piaget (1960) adds in his discussion of volume calculation that "knowledge learned in school increasingly interferes with the spontaneous development of geometrical notions as children grow older" (p. 381). Osborn (1976, pp. 27-28), likewise, warned that a premature stress of volume algorithms creates a serious learning problem.

There seems to be some discrepancy between Piaget's position and most school programs regarding the level at which the algorithm "V = L x W x H" should be introduced. This discrepancy raises the issue of whether or not present school curricula are justified in presenting the algorithm at the grade 5 level. Fabricant (1975) suggested that "teaching of geometric formulas at the elementary school level has to be seriously studied to see where such formulas would be most profitably placed in the curriculum" (pp. 6-7). It was as a result of the concerns mentioned above that the present study originated.

Definition of Terms

It is necessary to clarify the usage of certain terms which will occur throughout the study.

The term volume refers to the measure of the space displaced by a three dimensional object. The object may be any substance: solid, liquid or gas. Volume is not to be confused with capacity which refers to the measure of the space enclosed by a three dimensional object. Even though "internal volume of a hollow container ... is synonymous to capacity" (Kerslake, 1976, p. 14), the term volume usually refers to non-hollow objects.

The term conservation refers to the concept that a certain attribute of an object (or objects) remains invariant under changes of other irrelevant attributes (Wohlwill and Lowe, 1962, p. 153). For example, the volume of a substance remains invariant regardless of its shape and position as long as nothing is added or taken away. Likewise, the numerousness of a set remains unchanged during changes in the spacial arrangement of the set as long as nothing is added or taken away.

The term rectangular parallelepiped which is synonymous with cuboid (Webster's dictionary, edited by Gove, 1971, p. 550) is illustrated by the shape of filled boxes. These terms, however, have been confused with rectangular prism and a clarification is needed. A rectangular parallelepiped is a right rectangular prism rather than just a rectangular prism (James and James, 1976). Throughout this paper cuboid will be used interchangeably with rectangular parallelepiped.

The term algorithm refers to a procedure of ordered steps that guarantees a correct result if the steps are performed correctly and in the proper order (Lewis and Papadimitriou, 1978). Algorithms vary in their level of difficulty. The four basic operations, addition, subtraction, multiplication and division have rather simple algorithms, while solving systems of equations by the use of inverse matrices is a more complex one. The volume of a cuboid may be obtained by applying the algorithm of finding the measures of the three dimensions - length, width, and height - of the cuboid and computing the product of these three measures (V = L x W x H).

The term learning the volume algorithm refers to the mastery of the algorithm "V = L x W x H" at the levels of computation and comprehension. The level of computation includes, for example, situations where students are asked to state the volume given diagrams of partitioned cuboids, non-partitioned cuboids with known dimensions, or a word description of the dimensions of cuboids. The level of comprehension includes, for example, situations where students are asked to state the total volume given a diagram of attachments of cuboids with known dimensions or to state the volume of the cuboid resulting from proposed dimensional transformations on a given cuboid.

Statement of the Problem

The purpose of this study was to determine the relationship between the level of conservation of volume and the degree to which sixth grade students learn the volume algorithm for a cuboid "V = L x W x H". The level of volume conservation was determined using variations of tests employed by Piaget (Piaget, Inhelder and Szeminska, 1960). The achievement on the usage of the algorithm was based on objectives and activities found in widely used elementary textbook series. The study also provided informative data with regard to the volume conservation level of sixth grade children. Furthermore, the study showed the relationships among mathematics achievement, levels of conservation, and learning of the algorithm "V = L x W x H" for the volume of a cuboid.

In order to gain information about learning the algorithm for the volume of a cuboid, three treatments were implemented. The first treatment consisted of teaching the volume algorithm of a cuboid "V = L x W x H" using a guided discovery method based on approaches of present school programs. The second treatment consisted mainly of learning the task of varying factors when the product is constant. For example, given that 36 = 2 X 3 X 6, the student would be able to complete statements such as 36 = 4 X [ ] X [ ]. This task was supplemented by a brief discussion of the volume algoritnm of a cuboid "V = L x W x H." The third treatment served as control treatment and consisted of teaching various numeration systems.

The general aims of this study may be listed as follows:

1. To determine the various degrees to which conservers, partial conservers, and non-conservers of volume learn the volume algorithm of a cuboid "V = L x W x H."

2. To determine the degree of effectiveness for each of the two teaching methods on learning the volume algorithm for a cuboid.

3. To determine the effect of learning the volume algorithm of a cuboid on the transition from one volume conservation level to another.

4. To determine the relationship between sex and the levels of conservation of volume.

5. To determine the relationship between sex and the degree of learning the volume algorithm for a cuboid.

6. To determine the relationship between mathematics achievement and the levels of conservation of volume.

7. To determine the relationship between mathematics achievement and the degree of learning the volume algorithm for a cuboid.

Finally the results of this study will be useful in verifying aspects of the developmental theory of Piaget and according to Siegler and Atlas (1976, p. 360) training studies (not unlike this one) have become a standard method for investigating cognitive development.

Justification of the Problem

The present study is a consequence of a concern about a discrepancy between the present school programs and the cognitive theory of Piaget. Specifically, some school textbooks introduce the volume algorithm of a cuboid as early as grade 3 (age 8) while Piaget and his followers claim that most children do not develop the conservation of volume before age 11 (grade 6). In such a predicament there seems to be a need for research in order to justify our present school curriculum or suggest its modification. DeVault expressed the need for such research by noting the following:

> Needed now is the research that will make the link in the continuum between the research of the behavioral scientists and the work of the mathematicians who have designed new programs for schools ... The studies most likely to produce useful results for curriculum work would be experimental studies ... (DeVault, 1966, pp. 637-639)

Likewise, Steffe and Hirstein (1976) discussed children's thinking in measurement situations and recommended the following:

> In planning the mathematics experiences ... the teacher should consider the stages of cognitive development. The proposed content and the methods of presenting that content should also be considered. (Steffe and Hirstein, 1976, p. 35)

Implications of Piaget's Cognitive Theory

Piaget has for several decades tested, interviewed and observed children. His theory has become increasingly more influential in curriculum planning because "everybody in education realizes that Piaget is saying something that is relevant to the teaching of children" (Duckworth, 1964-b, p. 496).

A central theme in the cognitive theory of Piaget is the attainment of certain conservation tasks that are considered requirements for understanding of mathematical concepts (Piaget, 1941, p. 4). For example "3 + 5" and "8" are two names for the same number. This conservation of number is necessary for comprehension, generalization and retention of addition basic facts. It has even been reported that conservation of number is a better predictor of success in addition and subtraction problem solving than is Intelligence Quotient (I.Q.) (Van Engen, 1971). Van Engen, further, recommended that "it would seem that for these children [nonconservers of number], the school should center their attention on activities that might enhance conservation rather than on our traditional arithmetic curriculum" (p. 48).

The effect of number conservation in addition and subtraction problem solving is particularly relevant to this study. Even though care should be taken in generalizing to volume concepts, the importance of number conservation seems to suggest a possible analogy. This study was designed to find the relationship between conservation of volume and a volume algorithm. Volume conservation seems to be an apparent necessity for volume measurement. To measure volume is to compare a chosen unit of volume with the volume of an object. It is evident that volume conservation of the unit and of the object to be measured is a requirement before measurement can be meaningful.

Piaget (Piaget et al., 1960) seems to hold the same position regarding calculations in measurement including that of volume. He considers the concept of conservation to be necessary for any meaningful computation in both area and volume:

> ... Children attain a certain kind of conservation of area (and volume), based on the primitive conception of area (and volume) as that which is bounded by lines (or faces). That understanding comes long before the ability to calculate areas and volumes by mathematical multiplication, involving relations between units of different powers ... (p. 355)

Piaget's volume experiments have also revealed that most children do not conserve volume before the formal operational level and thus do not understand how they can arrive at a volume of a cuboid by simply multiplying its dimensions. Piaget argues that :

> The decomposition and redecomposition of a continuum are operations which belong to the level of formal operations. This explains why it is not until stage IV that children understand how they can arrive at an area or a volume simply by multiplying boundary edges. (Piaget et al., 1960, p. 408)

On the other hand, Piaget is not saying that the intellectual development proceeds on its own regardless of the stimuli of the surroundings. In fact, and contrary to what has been attributed to him, Piaget considers education to be a tool for cognitive development; he only questions the extent to which it is beneficial (Modgill, 1974, pp. 126-127). In other words, Piaget favors education that leads the child to discovery and rejects rote learning that forces information on the student who is not ready for it:

> This is a big danger of school - false accommodation which satisfies a child because it agrees with a verbal formula he has been given. This is a false equilibration which satisfies a child by accommodating to words, to authority and not to objects as they present themselves to him ... (Piaget, 1964, p. 4)

Validation of some of Piaget's Findings

Piaget's findings including those of volume conservation and computation have been examined by researchers throughout the world. Elkind (1961-a), Carpenter (1975-b) and Uzgiris (1964), for example, found that at least 75% of American students do not develop conservation of volume before age 11 (grade 6). Likewise, Lovell and Ogilvie (1961) found that it was not until age 14 that 50% of British students developed volume conservation. More recently, Arlin (1977) reported that only about 30% of grade five students in British Columbia conserved volume.

On the other hand, many educators believe that "acquisition of formal scientific reasoning may be far more dependent on specific instructional experiences and far less dependent on general maturation than hypothesized by Inhelder and Piaget (1960)" (Siegler and Atlas, 1976, p. 368). Graves (1972, p. 223), for example, considered education and experience to be necessary for volume conservation. Lovell (1971, p. 179) went further to suggest that even seven- and eight-year olds (grade 2 and 3) can learn how to use the algorithm "V = L x W x H" in order to calculate the volume.

Need for Research

The task of applying psychological theories to school curriculum depends, unfortunately, on policy makers rather than on educational researchers. DeVault found that often school policies "are based on theory that is never tested in instructional contexts" (DeVault, 1966, p. 636). One reason for this is that our awareness of the psychology of learning is very limited (Young, 1967, p. 40). It has been reported that the majority of students do not conserve volume before 13 or 14 years of age (Lovell and Ogilvie, 1961). Meanwhile, it has also been noted that the majority of adults do not conserve volume (Elkind, 1962; Towler and Wheatly, 1971; Graves, 1972). The averages stated above have varied considerably with cultures and communities and may be misleading if used without verification in curriculum planning (Fogelman, 1970). In any case, one should not necessarily delay the introduction of the volume algorithm "V = L x W x H" until all students conserve volume. Studies, such as the ones mentioned above, indicate that one can not expect all students to conserve volume. There is also the danger of introducing the algorithm too early and harming the process of learning. It seems, therefore, that the matter of placing activities, which depend on volume conservation, in the proper grade is presently a matter of preference rather than exactness.

It was an intention of the investigator to provide necessary data of the relationship between volume conservation and a volume algorithm. Such data was used for making recommendations related to the justification for teaching the volume algorithm prior to grade 6. DeVault advocates that "it seems reasonable ... to assert that the studies most likely to produce useful results for curriculum work would be experimental studies" (DeVault, 1966, p. 639).

CHAPTER II

REVIEW OF RELATED LITERATURE

Even though Piaget has for several decades tested, interviewed and observed children, his influence in North American educational psychology and education in general was, until the 1950's, limited to a few individuals. In the last two decades, however, it has become increasingly clear that he is "the foremost contributor to the field of intellectual development" (Ginsburg and Opper, 1969, p. ix). Only aspects of Piaget's theory which are relevant to this study are reviewed here. Interested readers may find more comprehensive summaries of Piaget's views in Flavell (1963), Maier (1965), Ginsburg and Opper (1969) and Berlyne (1957).

Summary of Piaget's Theory

Piaget's goals of education seem to be consistent with his theory in general and his philosophy of learning in particular. His educational goals consist of creating individuals who are active, critical, creative, inventive and discoverers (Piaget, 1964, p. 5). Piaget distinguishes between development of knowledge and learning. While development is a spontaneous and genuine process concerning the totality of knowledge, learning is a process limited to only a particular problem and caused by a teacher or an external stimulus (Piaget, 1964, p. 8).

Piaget explains that the essence of the development of knowledge starts first, even in young children, by forming "schemes" which are organized patterns of behavior and are based on the child's "actions on" and "experience in" his surroundings (Ginsburg and Opper, 1969, pp. 21-22). These schemes are nourished and manifested by "operations" which consist of interiorized actions that act, mentally or physically, on events or objects by modifying them, interpreting them and understanding the way they are constructed (Piaget, 1964, p. 8).

Piaget, further, perceives that a given operation does not exist independently from other operations. Rather, "it is linked to other operations and as a result it is always a part of a total structure" (Piaget, 1964, p. 9). A structure is, thus, an independent system of operations which is governed and closed, (in the mathematical sense), under certain laws of transformation. Mathematically, the interdependence of operations can be illustrated in many ways. For instance, some structures are substructures of larger structures as in the case of the natural number system being a substructure of the rational, real or complex structure (Piaget, 1970, p. 23).

Piaget did not attempt to describe the most complicated and general structures but he tried to discover the simplest ones that illustrate the acquisition of knowledge. For Piaget,

"... the central problem of development is to understand the (processes of) formation, elaboration, organization and functioning of these structures" (Piaget, 1964, p. 9).

Piaget observed that the processes of these structures fall into four main stages and substages which characterize mental growth. The first stage, the sensorimotor, starts from birth and continues until about two years of age. In this stage the child grows to distinguish between himself and his surroundings; he also develops the permanence of objects even when they can no longer be seen. The second stage, the preoperational, lasts until about the age of seven years. It is characterized by the child's use of symbols, by language development, and by growth in intuitive reasoning. In the third stage, the concrete operational, which lasts until about age 11 or 12, the child learns to group using classification as well as seriation. The child is capable also of classifying and seriating simultaneously. However, he is still limited to thinking only about objects that exist and actions that are possible. The last stage, the formal operational, is demonstrated by the child's capability to define concepts and to reason logically, systematically, and symbolically. The child can also perform operations on verbally stated propositions rather than only directly on reality (Ginsburg and Opper, 1969).

The ages mentioned in the above paragraph are not in any way fixed or universal; they are the approximate average ages noted in Piaget's studies. What is fixed, however, is the order of succession of the stages; a normal child develops through every one of the stages in the order they are mentioned. The age at which children reach a certain stage may vary, amongst children, from a few months to a few years and the period one remains in that stage depends on his degree of intelligence and his social milieu (Piaget and Inhelder, 1969, pp. 152-153).

The transition from one stage to the next is not interchangeable but integrative. "Each (stage) results from the preceding one, integrating it as a subordinate structure, and prepares for a subsequent one, into which it is sooner or later integrated" (Piaget and Inhelder, 1969, p. 153). The sensorimotor perception, for example, does not cease in the following stages but it continues to function in developed thought and it becomes integrated in its structures (Piaget, 1973, p. 122).

Piaget (Piaget and Inhelder, 1969, pp. 154-155) recognizes four factors that are responsible for the development from one stage to the next: maturation, experience, social transmission and most importantly, "equilibration". Maturation consists of organic growth, especially of the nervous system which provides possibilities for development if minimal experience is available. The second factor, experience, can be physical or logical-mathematical. The physical experience consists of knowledge acquired by abstracting the physical properties of objects while the logical-mathematical results by abstracting actions performed on the physical objects rather than the physical objects themselves. An example of a logical-mathematical experience, often given by Piaget, concerns a child who was playing with pebbles. He put his pebbles in a row and counted ten, then he counted in the other direction and also got ten. Next, he put the pebbles in a circle and still counted ten. The child, thus, discovered that his pebbles add up to ten regardless of their configuration (Piaget, 1964, p. 12, Piaget, 1970, pp. 16-17). Piaget labels this experience as logical-mathematical, which is obviously independent of and superior to the physical experience of the pebbles themselves. The third factor, social transmission and interaction, is illustrated by the verbal instructions acquired by the child in the process of development and formal education (Pulaski, 1971, pp. 9-11).

The three factors described above are considered by Piaget to be necessary but not sufficient for mental development. The most important factor for development is "equilibration" which consists of "a series of active compensations on the part of the subject in response to external disturbances and adjustment (Piaget and Inhelder, 1969, p. 157)." The term "equilibrium", in intellectual connotation, refers to an active state of an open system of structures which interacts with the environment and modifies itself accordingly (Ginsburg and Opper, 1969, p. 172). Piaget (1975, p. 170) and similarly Festinger (1957), explain that the human being strives toward equilibrium and harmony amongst his knowledge, opinions, attitudes and behaviour. Piaget claims, however, that this equilibrium is never attained but is continuously improved (1975, p. 23) through the curiosity of exploring and the construction of new information (1975, p. 170). Thus, "a cognitive system which has attained a high degree of equilibrium is not at rest. It interacts with the environment in terms of its structures [assimilation] and it can modify itself in line with environmental demands [accommodation]" (Ginsburg and Opper, 1969, p. 172). Assimilation and accommodation are two inseparable aspects of every act; on one hand the learner filters the input of the environment into his own structures and on the other hand he modifies his structures in order to fit the pressure of reality (Piaget and Inhelder, 1969, pp. 4-6).

The above description of the stages and factors influencing development seem to warn educators that not only does the child "think less efficiently than the adult, but he thinks differently" (Ginsburg and Opper, 1969, p. 8) depending upon his mental structures and capabilities. In fact, Piaget postulated that the capabilities of understanding mathematical concepts depends upon the attainment of specific "conservation" attributes (Piaget, 1941, p. 4). Conservation of an attribute refers to the realization that this attribute remains invariant under changes of other irrelevant attributes. Piaget has made a considerable effort to determine the approximate ages at which various conservation tasks are achieved and the means by which the mind constructs the notions of these tasks. He has not been nearly as successful in the latter as in the former. His findings have revealed that children in the preoperational stage rely heavily on their immediate perception in acquiring knowledge about their surroundings. He further reported that, on the average, his subjects conserved number at about age six, length at seven, substance at eight, weight at nine and volume at eleven (Piaget and Inhelder, 1969, p. 99).

Piaget is not saying, however, that the intellectual capabilities of the child develop on their own regardless of the stimuli of the surroundings. He is only saying that one can not increase the understanding of the child by just telling him; understanding necessitates conditions in which the child experiments, manipulates, and interprets (Duckworth, 1964-a, p. 2). Piaget holds, on the other hand, that the child's interpretation and response depend on his readiness level and his mental structures. Piaget, consequently, does not encourage acceleration of learning and he charges Americans of unprofitably doing so:

> Tell an American that a child develops a certain way of thinking at seven, and he immediately sets about to try to develop those same ways of thinking at six or even five years of age ... . Most of the research ... hasn't worked because experimenters have not paid attention to the equilibrium theory ... . Learning a fact by reinforcement does not in and of itself result in mental adaptation. (Piaget, 1967, p. 343)

Piaget's Studies of Volume

Piaget, Inhelder, and Szeminska (1960) studied rather thoroughly the subject of conservation and measurement of volume. They showed children a solid nonpartitioned cuboid of base 3 cm x 3 cm and height 4 cm and they told them that the block was a condemned house built on an island. They further asked the children to use centimetre cubes in order to build possible houses that have the same space; the new houses were to be built on islands of 2 cm x 2 cm, 2 cm x 3 cm, 1 cm x 1 cm, or 3 cm x 4 cm (pp. 355-357).

Piaget et al. (1960) also used an auxiliary method. They showed children several rectangular parallelepipeds and asked them to compare pairs of them: "are these two as big as one another? Is there as much wood in each of them?" (p. 357). An alternative for this method was to give children a certain cuboid and ask them to find, out of a dozen cuboids, another that had the same size or room (p. 357).

The preliminary two methods described above were followed by further questioning. The experimenters used the cubes of a partitioned rectangular parallelepiped to build another of different base while the subject watched. Then the subject was asked: (a) whether the new and old houses had the same room or which had more, (b) whether one can use the same cubes in the present house in order to build a new one that had as much space as the old one and looked exactly like it. The experimenters tried continuously to discover if the child relied totally on the conservation of the number of cubes or if he considered the total volume and conserved it (pp. 357-358).

The experimenters always checked their results by using a water displacement technique. They built a metal cuboid of 3 cm x 3 cm x 4 cm in the bottom of a container while the child looked and observed the rise in the water level. They then asked the child if he thought the level of the water would change if the arrangement of the cubes was modified to 2 cm x 1 cm x 18 cm or to 2 cm x 2 cm x 9 cm.

Piaget et al. noted in their experiments three levels of understanding the volume concept (1960, pp. 358-385). These levels are briefly described below:

Level 1 (age 8 or 9): Children conserved interior volume i.e., the quantity of matter contained inside the boundary. They also showed understanding of the logical (not mathematical) relationships between dimensions, that is, when asked to reconstruct a house on a smaller base they constructed it taller than the original, though not tall enough. The children did not show conservation of volume in the sense of space occupied or water displaced.

Level 2 (age 9 or 10): Children of this level showed progress over those of the previous one. They started to measure correctly by using the unit-cubes and expressing measures in metrical relationships such as "twice as much" and "nearly three times as high." They could also copy volume correctly but they could not meaningfully calculate it by multiplying the length, width and height. In fact, when asked about the volume or the space occupied by an object they equated the volume with the number of cubes necessary to surround the object. They still did not conserve volume in terms of space occupied or water displaced even though they recognized that the rearrangement of the unit-cubes did not alter the interior volume.

Level 3 (age 12 and above): Children of this level established a relationship between the area of the base and the volume. They discovered volume in terms of the product of length, width, and height; and that two volumes were equal if the product of their respective linear dimensions were equal. Children of this stage also conserved volume with respect to the surrounding space as in the water displacement.

The experiments and interpretations of Piaget et al. (1960) seem to say that the conservation of volume is well developed when the child acquires its three meanings i.e., (1) conservation of interior volume, (2) conservation of occupied space and (3) conservation of complementary volume or water displaced. Piaget et al. (1960) concluded that the complete notion of volume conservation and measurement involves multiplication of three lengths and necessitates the formal operational grasp of the continuity of space. "The decomposition and redecomposition of a continuum are operations which belong to the level of formal operations. This explains why it is not until stage IV that children understand how they can arrive at an area or a volume simply by multiplying boundary edges" (p. 408).

Reactions to Piaget's Theory and Experiments

The reactions of psychologists and educators, regarding Piaget's methods of testing and his conclusions, have varied from criticism to praise. Likewise the validation experiments have shown inconsistent results.

Pinard and Laurendeau (1964) looked for the existence of Piaget's general stages of mental development in a population of French-Canadians. They generally confirmed the existence of Piaget's stages of mental development but they eliminated some substages and added others. The data they collected from 700 children in Quebec revealed that French-Canadian students attained the Piagetian stages a year or two later than students used by Piaget in Geneva.

Carpenter (1975-a) reported that his own study revealed that in the case of first and second graders "measurement concepts begin to appear in young children earlier than Piaget et al. (1960) concluded" (pp. 3-13). In the same year however, he (Carpenter, 1975-b) published contradictory results. He declared that the results of the National Assessment and Michigan State Assessment showed that "on the whole measurement concepts develop somewhat later in average American students than indicated by the measurement studies of Piaget, Inhelder and Szeminska" (pp. 501-507).

Flavell (1963) reviewed Piaget's writing and questioned his work. He criticised Piaget mainly on (1) the gap between the facts described in the experiments and the theory he concluded and (2) the use of the qualitative method and the role of the language in the interpretation of data.

Piaget responded to Flavell's criticisms. He explained that his method of study is epistemological rather than psychological and he admitted that his research is far from complete. He even encouraged psychologists and educators to carry on further statistically sound studies under controlled conditions (Piaget, 1963, p. ix).

Finally, most educators seem to approve the Piagetian order of the stages of development, but they appear to be divided regarding the degree of accelerating the development and particularly the conservation tasks. Many side with Piaget in respecting the level guidelines and consider learning experiences to be necessary but not sufficient for conservation tasks. Other educators challenge Piaget's level guidelines and consider the conservation tasks to be abilities which are acquired through a process of cumulative learning. Shulman (1971) claimed that those who side with Piaget seem to outnumber those who do not. However, the question of accelerating the acquisition of conservation tasks and thus the suitable ages for presenting various activities is by no means resolved.

Training Experiments

Educators have been challenged, for decades, by the developmental theory of Piaget to take a stand regarding issues related to education; for example, the role of training in conservation tasks. A number of researchers, consequently, have seriously studied this subject and have drawn their own conclusions. Many have claimed success in their conservation training; others have reported failure. The following two sections of this paper will summarize some of these training studies in conservation tasks other than volume, and in volume.

Discussion of Training Experiments other than Volume

Piaget (1970, p. 13) set out "to explain how the transition is made from a lower level of knowledge to a level that is judged to be higher" using his equilibration theory. Unfortunately, there has been little evidence validating this theory. He himself admitted (seminar, the Catholic University of America) that the equilibration theory does not sufficiently explain the transition between cognitive development stages (Beilin, 1971, p. 84). Specifically, little seems to be known about the laws of transition from non-conservation to conservation (Wohlwill and Lowe, 1962, p. 153). Training experiments have been able to suggest and test various ways thought to aid the cognitive development from non-conservation to conservation.

Even though the ultimate purpose of this study is to investigate the learning of a volume algorithm it seems necessary to review training experiments in areas other than volume whenever they are relevant. The studies reviewed, therefore, include training for conservation of substance, weight, volume, and number. It is in number conservation training that most researchers have occupied themselves and many have claimed success.

Unfortunately, research studies have not been consistent with regard to the effectiveness of conservation training. In fact, Rothenberg and Orost (1969) reported that "for every successful study there are others testing the same types of training which report no significant transfer-to-training" (p. 70). This is not unusual in educational research because of differences in procedures and test conditions and because of varied emphasis on subject responses. Mermelstein et al. (1967), for example, employed what appeared to be successful procedures of substance conservation in four major studies (Smedslund, 1961; Bruner, 1964, Beilin, 1965; Sigel, 1966). Their results showed that none of the training procedures could induce substance conservation. Nevertheless, studies have raised important aspects of the process of acquisition of conservation. The most important of these aspects are adding-subtracting, language and verbal explanation, screening, extinction and reversibility. A summary of what research has shown regarding each of these aspects will be discussed hereafter.

Wohlwill (1959) was the first to study the effect of "addition-subtraction" treatment on the acquisition of number conservation. The treatment consisted of playing a game of matching a given set of objects with one of three pictures representing six, seven or eight elements. The experimenter changed the configuration of objects while adding or taking away one element. Wohlwill reported that this treatment was successful in inducing number conservation but he asserted that the subjects who benefited the most were those who were "on the doorstep" of conservation. Later, Wohlwill and Lowe (1962) and Rothenberg and Orost (1969) used similar "addition-subtraction" training to that of Wohlwill and also reported success in inducing number conservation.

As far as the effect of language is concerned, it does not seem to be clear how this factor contributes to intellectual development. Language has been used mainly in verbally instructing the subjects, their verbal description of the changes in an attribute, or in demanding verbal explanations for their responses. Sonstroem (1966) advocated that "it is when the child is both saying and doing that he learns not to believe fully what he is seeing" (p. 224). Beilin (1965) also reported success in conservation training when he verbally explained to the children the law of conservation after each unsuccessful response. Mermelstein et al. (1967), however, replicated the above experiments and found contradictory results. They observed that language training interfered with rather than facilitated the substance conservation processes. Moreover, the effect of most training procedures became insignificant when verbal explanation was demanded from the subjects who newly acquired conservation (Sonstroem, 1966; Wohlwill and Lowe, 1962; Roll, 1970).

The effect of screening misleading cues is closely related to the effect of language. Subjects who are led by misleading aspects are put in a situation where they respond verbally to a non-perceived action. Bruner (1966, pp. 183-207) used the screening technique in asking four- to seven-year-olds to predict the level of the water to be poured from a container to another and then watch the attained level. He claimed (p. 235) that it is when both enactive and symbolic representations agree that the ikonic yields and conservation is achieved. He further claimed success in inducing substance conservation with his five- to seven-year-olds but not with the four-year-olds. Bruner's approach has been under attack by other researchers including Piaget himself. Piaget (1968) accused Bruner of inducing a pseudoconservation and forcing the verbalization of identity. He argued that conservation shows understanding of identity and not vice versa. Moreover, Piaget advocated that the mental structure precedes language development rather than follows it. Experimentally, Strauss and Langer (1970) reported rejection of Bruner's hypothesis that screening misleading cues induces substance conservation.

The effect of extinction has also been a source of conflict in the research literature. Extinction is measured by the degree to which conservers resist deliberate confusion by the researcher. Smedslund (1961) reported that trained conservers did not resist extinction as did natural conservers.

Hall and Kingsley (1968) rejected Smedslund's claim that natural conservers resist extinction more than trained ones do. Hall and Kingsley replicated Smedslund's experiment and reported that none of their 17 natural conservers resisted extinction.

There seems to be a promising trend in inducing conservation by reversibility training. Piaget (1968) explains that reversibility has two forms, reversibility by inversion-negation and reversibility by reciprocity. Inversion-negation includes the mental operation of returning to the original state while reciprocity consists of compensating variations in related attributes. Wallach and Sprott (1964) employed what seemed to be successful application of reversibility, via inversion-negation, for number conservation. They used the technique of fitting dolls back into their beds after they have been clustered or scattered. The experimenters did not provide training for reversibility but gave those subjects who already knew it in real life a chance to apply it in the number conservation process. In a later study, Wallach, Wall and Anderson (1967) checked Wallach and Sprott's (1964) results and applied them to conservation of continuous liquid quantity. They concluded that "reversibility as well as not using misleading perceptual cues would seem to be necessary for conservation" (p. 441). Roll's (1970) findings, however, confirmed those of Wallach and Sprott (1964) and Wallach et al. (1967). Roll (1970) asserted that inversion-negation reversibility results in an increase of number conservation if no verbal explanation is required.

Discussion of Experiments Involving Volume Conservation Tasks

Elkind (1961-a) replicated some of Piaget's conservation experiments. His results agreed with Piaget's for mass and weight but not for volume. The data he collected from a sample of 175 elementary students in Newton, Massachusetts, revealed that "the conservation of volume did not in most cases (75%) appear before the age of 11" (p. 225). Carpenter (1975-b) reported similar results from the data collected in the National Assessment of Educational Progress (NAEP). Only 6% of the 9-year-olds, 21% of the 13-year olds and 43% of the 17-year-olds gave correct answers to the volume of a pictured solid which is partitioned into unit cubes.

Elkind (1961-b) extended his replication to junior and senior high school students. The data of 469 subjects from Newton, Massachusetts, showed that 95% to 100% of high school graduates attain conservation of mass and weight but only 68% of them reach the conservation of volume. On the average, however, only 47% of the secondary students conserve volume. Moreover, Elkind found that, in all secondary grades, I.Q. scores and age were positively correlated with volume conservation and a significantly higher number of boys than girls conserved volume. Nadel and Schoeppe (1973) replicated Elkind's (1961-b) study on 28 eighth-grade females in Levittown, New York. Their "results are strikingly parallel to those obtained by Elkind for the same mean age group" (p. 309).

In volume, particularly, Nadel and Shoeppe found that only 29% of their subjects, whose mean age was 13 years 5 months, had reached the conception of volume conservation (p. 309).

Elkind (1962), finally, extended his testing to young adults. He chose a sample of 240 college students from Massachusetts whose ages varied between 17 and 37 years. His results were consistent with his previous findings in that 92% of those college students conserved mass and weight but only 58% conserved volume. Age as well as I.Q. score were positively correlated to volume conservation and significantly more boys conserved volume than did girls in all age groups. Towler and Wheatley (1971) replicated Elkind's (1962) last study on 71 college students at Purdue University and they confirmed Elkind's results.

Lovell and Ogilvie (1961) used Piagetian methods to question 191 British students in grades 6 to 9 about various volume concepts. They found that Piaget (Piaget et al., 1960) was certainly correct when he asserted that a well developed concept of volume can not be attained, on the average, before age 11 or 12. In fact, "it appears that not until the fourth year (age 14) of the junior school do 50 per cent of pupils realize that the amount of water displaced by a single cube is independent of the size of the full container" (p. 124). On the other hand, the researchers believed that proper school training can speed up the acquisition of the volume concepts. They even assumed that it is possible to "learn" how to calculate interior volume as well as occupied volume, before the concept of volume (in terms of water displacement) is developed; and trusted that such activities may focus attention on conservation of both interior and occupied volume. Lovell (1971) suggested that even some seven and eight year olds can learn how to use the algorithm "V = L x W x H" in order to calculate the interior or occupied volume of a cuboid (p. 179).

Uzgiris (1964) tested Piaget's findings of the ordered sequence of conservation of substance, weight, and volume with respect to the variation of materials used in the experiments. The data collected from 120 elementary students in Illinois supported Piaget's sequence and the average ages he found in conservation of substance and weight but not of volume. Only 20% of Uzgiris' sixth graders, who had a mean age of 12 years 2 months, conserved volume. Bat-Haee (1971) also reported support of the above sequence of Piagetian conservation tasks in the data he collected from 181 pupils in Missouri. Bat-haee, further stated that conservation performance was positively related to I.Q. score and age; it was independent of sex.

Graves (1972) investigated the effect of race and sex on the degree of conservation of mass, weight, and volume. She tested 120 adults; 30 of whom were white males, 30 white females, 30 black males, and 30 black females. The subjects were all enrolled in adult basic education classes and their grade levels varied from one to eight. Graves found that in volume conservation tasks, whites scored significantly higher than blacks and males scored significantly higher than females. Moreover, 78% of all subject adults conserved mass, 67% weight but only 24% volume. Graves concluded that in the case of volume, maturation alone can not explain development. "It may be that educational or practical experiences in the sciences and mathematics are necessary before an individual can attain conservation ... of volume" (Graves, 1972, p. 223).

Price-Williams, Gordon and Raminez (1969) studied the role of experience and particularly manipulation in various conservation tasks. They administered Spanish versions of Piagetian conservation experiments in number, liquid, substance, weight, and volume. Their sample consisted of a total of 56 Mexican students whose ages varied between 6 and 9 years; half of the students lived in pottery-making families. Those children who had experience in pottery-making scored significantly higher in substance conservation tasks than the other subjects. The same children scored higher but not significantly higher than the rest of the subjects in all four remaining conservation tasks. Price-Williams et al. concluded that their study suggested "that the role of skills in cognitive growth may be a very important factor" (p. 769).

Summary and Implications of Literature Reviewed

Age of Subjects

Within certain limits, age does not seem to affect the process of conservation training. In number conservation, for example, Rotnenberg and Orost (1969) succeeded with younger children while Wohlwill and Low (1962) failed with older ones. What appears to be important is the cognitive level of the subject involved. Inhelder (cited in Modgil, 1974, pp. 125-126) reported that in one of her experiments 12.5% of the preoperational subjects progressed to an intermediate level while 75% of the intermediate level subjects advanced to the operational level. This particular finding can be seen across many studies; Wohlwill (1959), Beilin (1965), Strauss and Langer (1970) and others have noted that training is most successful with children who are "in possession of the proper cognitive structure but have not yet reached equilibration" (Kingsley and Hall, 1967). These subjects are said to be at a transitional stage and standing "on the doorstep" of conservation.

For volume, however, the reviewed studies indicate that the expected percentage of conservers among 11 and 12 year olds (grade 6 and 7) varies from 20% to 25% (Uzgiris, 1964; Elkind, 1961-a; Carpenter, 1975-b). This expected percentage of volume conservers is particularly important to this study. The nature of this study necessitates the inclusion of volume conservers in the sample. Grade 6 seems to be a reasonable choice for this study because (1) it is just one grade higher than grade 5 wherein most textbook series introduce the volume algorithm for a cuboid and (2) one can expect to find a sufficient number of conservers for the study.

Nature of Volume Conservation Tests

The value given to the subjects' justification for their responses while inferring their developmental stages (i.e., conservation levels) seems to have caused a considerable discussion in the literature. Some of Piaget's main collaborators, for example, have explained that in Piagetian experiments "special attention should be paid to the child's justification of his answers (Inhelder and Sinclair, 1969, p.5)." Others have criticized the eminance of language in Piagetian type experiments and, specifically have disagreed with Piaget's emphasis on the child's verbal explanation of his actions and decisions (Flavell, 1963). In order to eliminate this procedural problem, some researchers have conducted research in which they did not require justification for students' responses, and in fact have attempted to avoid verbal instructions through pretraining procedures (Braine, 1959, Bever, Mehler and Epstein, 1968, for example). Calhoun (1971) reported difficulty in assessing children's number conservation using their verbal responses and recommended the use of totally nonverbal procedures for such assessment.

Researchers who favour justification seem to believe that without such justification type I error may be caused by mistakenly inferring a higher developmental stage to subjects. Subjects may respond correctly by concentrating on irrelevant attributes (Smedslund, 1963 and 1969). For example, a child might respond correctly by simply chosing the first (or last) alternative or, in case of volume conservation testing, by focusing on the substance or weight attribute. Research is reported which has shown that more subjects are classified as conservers when the justification criterion was not used than when it was (Brainerd, 1973, p. 174). For example, Roll (1970) reports that few of his trained number conservers showed verbal awareness of conservation. Wohlwill and Lowe (1962) indicate that using their nonverbal training procedures for number conservation the increase in conservation responses became negligible when verbal justifications were demanded. Thus the argument is that justifications of students' responses may further reveal their developmental level and reduce type I error.

Researchers who oppose the requirement of a verbal explanation for inferring the subject's developmental level, argue that type II errors may be made by using criteria which are too stringent. The argument of those researchers seems to be based on the theory of Piaget himself. Flavell (1963), for example, observed that in the theory of Piaget "language behavior is here treated as a dependent variable with cognition as the independent variable (p. 271)." Brainerd (1973) also explains that "Piaget has long maintained that ... cognitive structures originate in logic rather than language" and further, considers adequate explanations sufficient but not necessary conditions for inference of cognitive structures (p. 177). Brainerd holds that if one chooses to employ the explanation criterion, then one unduly restricts the behavioral domain to which the theoretical construct (structure) applies (p. 177). Brainerd concludes that "from the standpoint of Piaget's theory the judgement criterion risks only the usual "extraneous" type I and type II errors but does not risk any built in source of error as does the explanation criterion (p. 178).

Hobbs (1975) sided with Brainerd while discussing the necessary and sufficient subject's behaviour for determining developmental level and applied his conclusions to volume conservation testing. He explained that even though in Piagetian type testing a mistake is not taken at face value and subjects are given repeated chances to give the correct answers, the persistent subjectivity of the experimenter is liable to cause incorrect inference of conservation levels (Hobbs, 1975, p. 272).

Piaget's position, regarding the verbal justification of the subject's action, does not seem to oppose Brainerd's (1973) or Hobbs' (1975) position. Piaget (1963) explained that the nature of his epistemological studies necessitates interaction with the subjects in order to "... unearth what is original and easily overlooked ... and to [use] methods, including verbal ones, which are as free and flexible as possible." Furthermore, he encouraged educators to conduct studies under controlled conditions (Piaget, 1963, p. ix). More recently, Piaget (1973) explained that

> In fact it is a very general psychological law that the child can do something in action long before he really becomes 'aware' of what is involved- 'awareness' occurs long after the action. In other words, the subject possesses far greater intellectual powers than he actually consciously uses. (Piaget, 1973, p. 86)

Hobbs (1975) developed and used a displaced volume conservation test based on judgement alone. In the first part of the test subjects were shown experimentally that objects occupy space in water and cause the water level to rise and that the space occupied varies directly with the size of objects. The second part of the test was designed to detect those who persistently judged that weight, rather than size, is directly related to the space occupied. The last part of the test consisted of presenting balls of the same size and glasses of water to the same level, immersing one of the balls in water, transforming each of the other balls in turn and questioning the student about the anticipated water level in the other glass if the transformed ball was immersed in it. The procedures used by Hobbs (1975) were particularly useful in the development of the Volume Conservation Test used in this study.

The position of Piaget (1973) himself and the interpretations of Flavell (1963) and Brainerd (1973) to Piaget's theory are the basis for the justification of the judgement-based Volume Conservation Test used in this study. In the development of the Volume Conservation Test, further care was taken to reduce type I error caused by students' concentration on irrelevant attributes. For example, no questions that allowed subjects' guessing were asked. In fact the response format consisted of darkening one of five broken lines to whichever the student thought the water rose. Further, an effort was made to detect the students who concentrated on weight rather than volume and classify those students as nonconservers of volume. The criteria used in developing the items of the Volume Conservation Test used in this study were based on Piaget's testing procedures of volume conservation (Piaget et al., 1960, Piaget and Inhelder, 1968). The variety of transformations to plasticine balls, the preparation of subjects for the test and detecting the conservers of weight but not volume were adaptations of Hobbs' (1975) procedures in his volume conservation testing. The protocol of the Volume Conservation Test used in this study is presented in detail in Chapter III.

Choice of Treatments

The concept underlying treatment procedures is a very important issue because this study is designed to show the relationship between training on the usage of the volume algorithm "V = L x W x H" and volume conservation. A concern has been expressed about the necessary multiplication abilities involved in the calculation of the volume of a cuboid (Spitler, 1977). It is conjectured that when students are proficient in varying factors of a fixed product they can rapidly predict, determine and compare volumes or dimensions of cuboids. For example, it is probable that students with such proficiency would successfully solve Piaget's island problem in which subjects are to predict the height of a replacement for a condemned building to be built on a base different from the original. In other words in this treatment factor manipulation is considered to be the key for volume calculation and conservation. This conjecture provides the basis for the multiplication treatment of the study. An outline is given in Chapter III and details are provided in Appendix A.

The conjecture mentioned above may be justified within the context of the cognitive theory of Piaget. Inhelder and Piaget (1958) asserted that conservation presumes reversibility by inversion-negation or by reciprocity. Reciprocity "... is analogous to compensating changes in one affirmation by equal and opposite changes in a related affirmation" (Brainerd, 1970, p. 227). The procedure proposed in the previous paragraph is intended to train students in compensating one or more factors in multiplication with respect to variations in other factors.

In fact the effectiveness of reversibility via inversion-negation seems the most promising of training procedures. Piaget cautioned, however, against inappropriate generalization across various conservations. He explained that the first-order conservation (number, length, substance, weight, and area) are an index of concrete operational level and that second-order conservations (volume, density, momentum, and rectilinear motion) are an index of formal operational level. He added that the first-order conservations require only successive application of the two aspects of reversibility (inversion-negation and compensation), while the second-order ones necessitate simultaneous application of both aspects (Inhelder and Piaget, 1958, p. 320). Inhelder (cited in Green, Ford and Flamer, 1971) also objected to the separation of various aspects of reversibility. She held that emphasizing one aspect of reversibility, at the expense of the other, could harm the subjects' learning.

Piaget further maintains that proficiency in number manipulation does not lead to understanding of volume if conservation is not achieved. He holds that "it is one thing to multiply two numbers together and quite another to multiply two lengths or three lengths and understand that their product is an area or a volume. The latter involves the continuity of space ..." (Piaget, et al., 1960, p. 408). There seems to be a possibility that the multiplication treatment would lead to a limited and temporary learning of the volume algorithm and of conservation. Piaget would examine the effectiveness of such learning with respect to three criteria: retention, generalization, and cognitive level of subjects before such training (Piaget, 1964, pp. 17-18). The results of this study are expected to reveal the effect of multiplication skills in the learning of the volume algorithm.

The other experimental treatment, labeled volume treatment, was designed to teach the volume algorithm for a cuboid "V = L x W x H" using an approach that resembles those of school programs used in North America and particularly in

British Columbia. Such resemblance was necessary in order to apply the results of the study to school programs. On the other hand, this treatment was considered an improvement over school approaches because it was more comprehensive and required more students' active involvement, for example in building with cubes, than is normal in the elementary school classroom. Furthermore, the sequential progress of activities used in this treatment, comparison, ordering, counting of cubes, algorithm, was consistent with other models for the teaching of volume in particular (Elliot et al., Teacher's guidebook, 1974, v. 4, p. 4) and of measurement in general (Thyer and Maggs, 1971).

Preliminary activities of this treatment involved macroscopical direct comparison and direct ordering of closed boxes (cuboids) with respect to their volume as well as building with unit cubes models of polyhedrals some of whose units may not be visible, counting the number of cubes and stating the volume(s) (Eicholz et al., 1974, v. 5, p. 276; Dilley et al., 1974, v. 6, p. 110; Elliot et al., 1974, v. 4, p. 145). Later, students used nonstandard then standard unit blocks to build similar cuboids to the ones given, counted the number of blocks and indirectly compared then ordered those cuboids.

The volume algorithm "V = L x W x H" was introduced as a simplification (Eicholz et al., 1974, v. 5, p. 276) of counting cubes i.e., length X width yielded the number of cubes in one layer and length X width X height gave the total number of cubes (Dilley et al., 1974, v. 5, p. 134; Elliot et al., 1974,

v. 4, p. 143). This treatment ended by applying the volume algorithm to various cases. For example, the algorithm was applied to attachments of cuboids (Dilley et al., 1974, v. 6, p. 111), to partially covered cuboids (Eicholz et al., 1974, v. 6, p. 272) and to proposed dimensional transformations (Eicholz et al., 1974, v. 6, p. 273). An outline of the main activities of this treatment is given in Chapter III and detailed lesson plans are provided in Appendix A.

Conclusion

There seems to be a growing belief among educators that intellectual development, at least for transitional subjects, can be accelerated with proper training. Flavell and Hill (1969) summarized:

> The early Piagetian training studies had negative outcomes, but the picture is now changing. If our reading of recent trends is correct, few on either side of the Atlantic would now maintain that one cannot by any pedagogic means measurably spur, solidify, or otherwise further the child's concrete-operational progress. (p. 19)

Even those who are in accord with the Piagetian theory hold that learning can accelerate development but they maintain that it does not initiate it (Smedslund, 1961; Halford and Fullerston, 1970). Piaget himself considers education to be a tool for stage acceleration:

> But it remains to be decided to what extent it [education] is beneficial... Consequently, it is highly probable that there is an optimum rate of development, to exceed or fall

behind which would be equally harmful. But we do not know its laws, and on this point as well it will be up to future research to enlighten us. (Piaget, 1972, quoted by Modgil, 1974, pp. 126-127)

CHAPTER III

PROCEDURES

This chapter includes discussion and description of four major considerations: the choice of subjects, description of the treatments, the preparation of tests and the way the tests and treatments were conducted.

Subjects

The study was conducted on sixth grade students in a suburban school district of the lower mainland of British Columbia. These subjects followed a mathematics program typical of those used in North America. In 1971, the annual average family income in that district was $11 033 while the average family income in the Vancouver metropolitan area was $10 664 (Statistics Canada, 1974). These subjects were of ages and socioeconomic status similar to those of most suburban grade 6 students in North America. The sample chosen, therefore, appeared to be representative of the population of suburban grade 6 students in North America.

The subjects consisted of 171 students of seven grade 6 classes in three schools. Each of two schools had two full classes of grade 6, while the third school had one full class of grade 6, one class of grade 5 and grade 6 combined, and one class of grade 7 and grade 6 combined. Subjects who missed any test or treatment day were eliminated from the study. The final sample was 105 students.

Description of the Treatments

The study included two experimental groups and one control group. The two experimental groups underwent two different treatments which were both aimed at arriving at the volume algorithm of a cuboid i.e., V = L x W x H. The treatment of the control group consisted of learning various numeration systems. The three treatments were believed to be of about the same level of difficulty and required about the same amount of time. Each of the three treatments is described in detail below.

Volume Treatment

The aim of this treatment was to teach the volume algorithm for a cuboid "V = L x W x H" using a guided discovery method based on approaches of present school programs. In such programs volume lessons include activities for finding or computing the volume of cuboids by counting cubes or by using the algorithm, "V = L x W x H."

The main concepts and activities of this treatment are outlined below. Complete and detailed lesson plans are provided in Appendix A.

1. Direct comparison of objects.

2. Direct ordering of objects.

3. Indirect comparison of closed boxes.

4. Standard units: m^3, dm^3 and cm^3.

5. Indirect ordering of closed boxes.

6. Volume of polyhedral models built from unit cubes.

7. Volume of partitioned and non-partitioned cuboids.

8. Algorithm for the volume of a cuboid "V = L X W X H."

9. Application of the volume algorithm to cuboids and diagrams of cuboids.

10. Application of the volume algorithm to the following cases:
    a. Word description of cuboids.
    b. Cuboids touching side by side.
    c. Diagrams of cuboids with some unit cubes attached or removed.
    d. Attachments of half cubes to cuboids.
    e. Diagrams of partially covered cuboids.

11. Application of the volume algorithm to cuboids with proposed dimensional transformations.

Multiplication Treatment

This treatment, like the volume treatment, was aimed at teaching the volume algorithm of a cuboid "V = L x W x H". The emphasis here was on developing the skills of varying two and three factors provided that their product remained fixed. For example, given that 24 = 2 X 3 X 4, the students were trained in completing statements such as 24 = 6 X [ ] X [ ]. This task was followed by a brief discussion of the volume of a cuboid "V = L x W x H". The details of this treatment are provided in Appendix A; a brief outline is given below.

1. Review of the commutative and associative principles.

2. Prescribing the range for the missing factor in an inequality involving two factors at each of its sides.

3. Prescribing the range for the missing factor in an inequality involving three factors at each of its sides.

4. Effect on the product of two factors when these factors are changed additively or multiplicatively. (Note: "additively" and "multiplicatively" will subsume decrease as well as increase.)

5. Effect on the product of three factors when these factors are changed additively or multiplicatively. (Note: "additively" and "multiplicatively" will subsume decrease as well as increase.)

6. Effect on one of two factors when the other factor is changed and the product is fixed

7. Effect on two (or one) of the three factors when one (or two) of these factors is (are) changed and the product is fixed.

8. Clarification of the concept of volume.

9. Algorithm for the volume of a cuboid "V = L X W X H."

10. Application of the volume algorithm to partitioned, partially partitioned and partially covered cuboids.

11. Application of the volume algorithm to cuboids with proposed dimensional transformations.

Control Treatment

This treatment was given to the control group for the purpose of controlling for any "Hawthorne", maturation, history and sensitization effects. The treatment consisted of an instructional unit on numeration systems and was believed to be of about the same level of difficulty as the treatment offered to the two experimental groups. The following is a general outline for this treatment (detailed lesson plans may be found in Appendix A).

1. Review of Base 10 place value concepts
2. Bundling in fives and expressing numbers in Base 5
3. Counting in Base 5
4. Converting numerals from Base 10 to Base 5
5. Bundling in sixes and expressing numbers in Base 6
6. Counting in Base 6
7. Converting numerals from Base 10 to Base 6
8. Converting numerals from Base 5 to Base 10
9. Addition in Base 5 with and without renaming
10. Subtraction in Base 5 with and without renaming —

Description of Tests

Three different tests were administered as pretests, posttests and retention tests. The pretests consisted of the Volume Achievement Test, the Volume Conservation Test and a portion of the Stanford Achievement Test (SAT). The posttests and the retention tests consisted of the Volume Achievement Test, the Volume Conservation Test and the Multiplication Achievement Test. The Volume Conservation Test, the Volume Achievement Test and the Multiplication Achievement Test were piloted using a fifth grade class, a sixth grade class and a seventh grade class. The pilot results were used to revise the classification scheme of conservation levels, to confirm the suitability of the grade level (sixth) chosen for the major study and to improve the testing instruments. Each of the revised tests will be described in turn and copies of the tests except the Volume Conservation Test are included in Appendix E. The Volume Conservation Test is completely described below and the answer sheets are given in Appendix E.

The Volume Conservation Test was based on procedures used by Piaget (1960), Piaget and Inhelder (1963) and Hobbs (1975) in their detection tests for volume conservation levels. In this test the experimenter explained the procedures and demonstrated the tasks to the class as a group. During this test an effort was made to keep minimal the interaction among students, and between students and the experimenter. Each student responded on a separate answer sheet by darkening a line to show the judgement. Terms such as "more" or "less" were avoided as much as possible. The first part (pages 2 and 3, see Appendix E) of the test was intended to give the students familiarity with the test procedures. The second part (pages 4-6) was used to identify those subjects that associate volume with weight. The third part (pages 7-11) was designed to classify the subjects in one of the three categories, nonconservation, partial conservation and conservation. In the last part (pages 11 and 12) the students were asked to give reasons for their judgement; this provided validity information for the classification in the third part of the test. The following is a description of the conservation test.

1. The experimenter displayed, side by side, three identical test tubes partially filled to the same level with coloured water. The levels were marked around the tubes. The experimenter told the group that the levels were the same. The experimenter then displayed two identical balls and a larger ball of plasticine close to

the tubes. He told the group that two of the balls were the same and the third was larger. He asked a student to come forward and confirm that the water levels in the tubes were the same, that two of the balls were the same and that the third was larger. If the student disagreed, the experimenter asked him to adjust the amount of water or plasticine by adding or deleting. Each student was given a pencil and an answer booklet which consisted of 12 different coloured answer sheets.

2. The experimenter asked, "What will happen if I put this ball (right) into this tube (right)? Where will the water level be?" The experimenter put the ball in one of the tubes, the water rose and students were instructed to turn to page 2 and observe the drawn result. This question was suggestive by nature and was intended to familiarize students with the questioning and answering process.

3. The experimenter asked, "What will happen if I put this other ball (middle) into this tube (middle)? Where will the water level be after I put the ball in? Darken the line nearest to where the water level will be after I put the ball in." The subjects darkened a line showing their judgement and turned to page 3. Then the experimenter put the ball in the tube and pointed out the level to the students.

4. Step 3 was repeated using the third tube and the larger plasticine ball than the ones in the first two tubes; students used page 3 for their responses.

5. All tubes and balls were removed. The experimenter displayed two new tubes partially filled with the same amount of water, a plasticine ball, and a steel ball of the same size as the plasticine ball. The experimenter pointed out that the balls were of the same size. A different student was called on for verification. The student was also asked to compare the weight of the steel ball and the plasticine ball using a double-pan balance scale. The experimenter put the plasticine ball in one of the tubes, the water rose. Similar questions to the ones in section 3 above were asked using the steel ball. Students responded on page 4, then the experimenter put the steel ball in the tube.

6. Step 5 was repeated using a steel ball which was smaller but heavier than a ball of plasticine. Students responded on page 5, turned to page 6 and the experimenter put the steel ball in the tube.

7. All tubes and balls were removed. Two new test tubes partially filled with water were displayed. The experimenter presented two cubes; one made of glass, the other of aluminum. Both cubes had the same size but one

> was heavier than the other. A student came forward and confirmed these facts. The experimenter put the glass cube in one of the tubes and the water rose to a certain level. The experimenter said, "From now on I will not show you the answers. If I put the aluminum cube into this other tube, where will the water level be? Darken the line nearest to where the water level will be." Students were instructed to respond and then turn to page 7. The experimenter did not put the aluminum cube in the tube.

Subjects who failed two of the questions on pages 4, 5 and 6 demonstrated evidence of associating volume with weight. These subjects were classified as nonconservers of volume. The second part of the test immediately followed.

> 8. The experimenter presented two test tubes, and five plasticine balls of the same size. He put one of the balls in one of the tubes and the water rose. He then rolled one of the balls into a sausage shape in front of the group. The experimenter then said, "If I put the sausage into this other tube, where will the water level be? Darken the line nearest to where the water level will be." The children responded on page 7, turned to page 8 but the experimenter did not put the sausage in the tube.
>
> 9. Step 8 was repeated by transforming one of the balls into nine or ten small pieces, one of the balls into a small piece and a large piece, and finally the last ball into three similar but flattened pieces. In each of the above mentioned transformations the students used a separate answer sheet (pages 8,9,10) to darken a line indicating their judgement.
>
> 10. All tubes and balls were removed. Two beakers with the same amount of water and two plasticine balls of the same size were presented. A student came forward and confirmed these facts. The experimenter put one of the balls into one of the beakers. He transformed the other ball into a "pancake" shape in front of the group. He then said, "If I put the 'pancake' into this other beaker, where will the water level be? Darken the line nearest to where the water level will be." The children responded (page 11). The experimenter did not put the 'pancake' in the beaker but he said, "If you indicated that the level of the water will be higher than the level in the other beaker, explain why it will be higher. If you indicated that the level will be lower, explain why it will be so. And if you indicated that the water level will be the same as the other beaker, explain why it will be the same." The students responded and turned to page 12.

11. The beaker containing the ball was replaced by an identical beaker with water at the same level. Step 10 was repeated using two identical boxes made of marble. The experimenter put the first box and its detached top simultaneously into one of the beakers. They sank. He then placed the top of the other box on the box and said, "If I seal the top to the box, put the box in the other beaker and it sinks, where will the water level be? Darken the line nearest to where the water level will be." The experimenter asked for reasons as in step 10.

Success in the above test was measured by scoring the five responses on sheets 7 through 11. Each item was considered correct if the correct line was darkened. Students who succeeded in all 5 responses were classified as conservers. Those who succeeded in one or none were classified as nonconservers. The rest of the students were classified as partial conservers.

The comments written by the students on pages 11 and 12 were only used to reveal the degree of consistency between the students' judgement and reasoning. However, due to difficulties in interpreting verbal communications these comments were not considered in the conservation classification. Furthermore, the question on page 12 concerning the two marble boxes was not a part of the conservation classification scheme. This question involved two unequal quantities and was not consistent with the Piagetian questioning protocol. It was included because it was believed that students are able more easily to give reasons for inequality than for equality.

The purpose of the Mathematics Achievement Test was to reveal any possible correlation between volume achievement scores and general mathematics achievement. The arithmetic computation section of the Stanford Achievement Test was used

(Madden et al., 1973). Two reliability coefficients, the split half estimate and the coefficient based on Kuder-Richardson formula, are stated in the manual to be 0.90 with a standard error of 2.9. This Mathematics Achievement Test contains 45 multiple choice items and was administered, as recommended, in a maximum of 35 minutes. The score of each subject on this test was determined by the number of correct responses.

The items of the Volume Achievement Test are variations of questions found in the three textbook series, Heath Elementary Mathematics, Investigating School Mathematics, and Project Mathematics. The test was composed of 27 questions for each of which the student was given a drawing of a cuboid, or an attachment of cuboids, and asked to find the volume. Measurements of dimensions were given in the form of number without units. Scores on this test were also determined by the number of correct responses.

The Multiplication Achievement Test consisted of 11 multiple choice items and 9 short answer items. In some of the items students were given assertions of the form a X b = c X d, a X b > c X d, a X b X c = d X e X f, a X b X c > d X e X f, where one of the factors was not given and were asked to predict this missing factor. In other items students were asked to predict changes, such as increase, decrease or doubling, in products when factors changed additively or multiplicatively. Students' scores were determined by the number of their correct responses.

Instruction and Testing Procedures

Instructors

Three male instructors who were all certified teachers were used to carry out the three treatments. The choice of the same sex instructors was made in order to exclude the teacher sex variable. The instructors were randomized in such a way that each of them taught all treatments. The investigator did not teach any of the treatments. He trained the instructors before the treatments and provided printed guides and materials for daily instruction. He also met with the instructors as a group every morning and afternoon during the treatment period in order to review lessons, handle problems and insure uniformity of instruction. The instructors only carried out the 4-day treatments; they did not administer any of the tests.

The investigator administered the pretest, posttest and retention test of volume conservation. Two female teachers assisted in administering the Mathematics Achievement Pretest and the Volume Achievement Pretest, posttest and retention test.

Schedule of Instruction and Testing

In the beginning of the experiment the students were told that the reason for including them in the study was to learn more about the way grade 6 students learn mathematics. They were also informed that the outcome of the study would not affect their grades at school nor would it serve for individual diagnosis or evaluation. The classroom teachers were requested not to teach any mathematics during the period of treatment nor to discuss the treatment topics with their students.

The experiment began by giving the pretests: mathematics achievement, volume achievement and volume conservation. The Volume Conservation Test was used to classify students as conservers, partial conservers and nonconservers. The names of the subjects of all classes at each school were listed initially according to conservation level. Then the names of pupils within each conservation group were randomized across the three treatments. Boys and girls were randomized separately in order to balance for sex. This procedure determined the subjects of each treatment at each of the schools. Three school days at the beginning of the experiment were reserved for the pretests and randomization.

The treatments began after the three pretesting days and lasted for four consecutive school days. At each school the students of both classes were taught in the three predetermined groups for the same class period. On each of the four days instructors moved to all three schools in such a way that they gave at each school one class period of instruction before recess, one after recess and one in the afternoon.

Two days after the treatments the Volume Conservation Test, the Volume Achievement Test and the Multiplication Achievement Test were administered. Three consecutive school days were reserved for these posttests. The retention tests of volume conservation, volume achievement and multiplication achievement were given seven weeks after the posttests. During these seven weeks the classroom teachers resumed regular classroom instruction in mathematics.

Design of the Study

The basic concern of this study necessitated considering two main factors. One factor was made up of the three levels of volume conservation, while the other was composed of the three treatments. There were also four dependent variables, the Volume Achievement Posttest, the Volume Achievement Retention Test, the Multiplication Achievement Posttest and the Multiplication Achievement Retention Test. Scores on the Volume Achievement Pretest and Mathematics Achievement Pretest (SAT) were used as covariates. A schematic representation of the design is given in Table 3.1.

Table 3.1
Experimental Design

Conservation levels and treatments	Multiplication	Volume	Control
Nonconservers			
Partial conservers			
Conservers			

A Pretest-Posttest Control Group design which appears in Campbell and Stanley (1963, p. 13) was used with blocking on the conservation factor. Schematically the design is as follows:

Randomized	G1	Pretest	Treatment 1	Posttest	Retention Test
assignment	G2	Pretest	Treatment 2	Posttest	Retention Test
to groups	G3	Pretest	Treatment 3	Posttest	Retention Test

Hypotheses

While searching for answers to the aims of the study certain statistical hypotheses in null form were tested. Some of the hypotheses listed below were deduced from the design described above while the others were based on the aims of the study.

H 1. There are no significant differences in volume achievement scores, on the posttest and retention test, among conservation groups.

H 2. There are no significant differences in volume achievement scores, on the posttest and retention test, among treatment groups.

H 3. There are no significant interactions in volume achievement scores, on the posttest and retention test, between conservation and treatment.

H 4. There are no significant differences in multiplication achievement scores, on the posttest and retention test, among treatment groups.

H 5. The transition from a lower level of conservation on the pretest to a higher level of conservation, on the posttest and retention test, is independent of volume achievement scores on the posttest and retention test respectively.

H 6. The transition from a lower level of conservation on the pretest to a higher level of conservation, on the posttest and retention test, is independent of treatments.

H 7. The transition from a higher level of conservation on the pretest to a lower level of conservation, on the posttest and retention test, is independent of volume achievement scores on the posttest and retention test respectively.

H 8. The transition from a higher level of conservation on the pretest to a lower level of conservation, on the posttest and retention test, is independent of treatments.

H 9. The volume achievement scores, on the posttest and retention test, are not related to mathematics achievement scores.

H10. The initial level of conservation is independent of mathematics achievement scores.

H11. The initial level of conservation is not related to sex.

H12. The volume achievement scores on the posttest are not related to sex.

Statistical Analyses

In order to test hypotheses 1-4, each dependent variable posttest score and retention test score, was analyzed separately by using a 3 X 3 fully crossed two-way analysis of covariance. The analysis was carried out using the computer program BMDP2V. In the cases where significant differences were found across treatments and conservation levels, Scheffe post hoc comparisons were made to determine which groups differed significantly.

Hypotheses 5 and 7 dealt with two variables, one of which involved a forced dichotomy while the other was measured on an interval scale. A biserial correlation coefficient as recommended by Glass and Stanley (1970, p. 168) was used for testing these hypotheses. Hypotheses 6 and 8 were tested using frequency tables and the Chi Square statistic. However, since the number of transitional cases among the three conservation levels was expected to be small it was not appropriate to use the number of transitional cases between each pair of levels. Instead transitions were classified as "change" vs "no change" for each treatment in order to allow for Yates' correction for continuity to be applied (Glass and Stanley, 1970, p. 332). Hypothesis 11 dealt with a nominal variable and an ordinal variable. The Wilcoxon two-sample test using tied scores as recommended by Marascuilo and McSweeny was used for testing of this hypothesis (1977, p. 267). Hypothesis 9 dealt with two variables on interval scales and the recommended analysis is the usage of Pearson's product-moment correlation coefficient (Glass and Stanley, 1970, pp. 109-113). Hypothesis 10 included variables on ordinal and interval scales. This hypothesis was tested using Kendall's Tau correlation coefficient (Glass and Stanley, 1970, pp. 176-178). Finally, hypothesis 12 involved a truly dichotomous variable and a variable on an interval scale. It was tested using a point-biserial correlation coefficient (Glass and Stanley, 1970, pp. 163-164).

CHAPTER IV

RESULTS

This chapter contains the results of the study. Included are sections on test analyses, preliminary study, analysis of covariance, correlation study, tests for independence and post hoc qualitative analyses. The section of preliminary study contains two parts, the variable covariates and the instructor effect.

Tests Reliabilities and Item Analysis

Three different tests were administered as pretests, posttests and retention tests. The pretests consisted of the Volume Achievement Test, the Volume Conservation Test and the arithmetic computation section of the Stanford Achievement Test (SAT). The posttests and the retention tests consisted of the Volume Achievement Test, the Volume Conservation Test and the Multiplication Achievement Test. The test reliabilities and a summary of item analysis will be reported for the Volume Achievement Test, the Volume Conservation Test and the multiplication achievement test. A complete list of item statistics can be found in Appendix B.

Volume Achievement Test

The Hoyt estimates of reliabilities (internal consistency) in the pretest, posttest and retention test were 0.94, 0.95 and 0.94 respectively. The difficulty level, as defined by the percent of subjects responding correctly, ranged from 8.2 to 80.7 in the pretest, from 23.4 to 78.4 in the posttest and from 2[illegible].6 to 87.1 in the retention test. The item point-biserial correlation coefficients ranged from 0.35 to 0.74 in the pretests, from 0.35 to 0.79 in the posttest and from 0.39 to 0.78 in the retention test.

Volume Conservation Test

The Hoyt estimate of reliabilities in the pretest, posttest and retention test were 0.78, 0.85 and 0.82 respectively. The difficulty level ranged from 35.7% to 76.6% in the pretest, from 59.6% to 86.0% in the posttest and from 68.4% to 91.2% in the retention test. The item point-biserial correlation coefficients ranged from 0.46 to 0.78 in the pretest, from 0.46 to 0.75 in the posttest and from 0.33 to 0.77 in the retention test.

Multiplication Achievement Test

The Hoyt estimate of reliabilities in the posttest and retention test were 0.86 and 0.79 respectively. The difficulty level ranged from 8.2% to 85.4% in the posttest and from 7.6% to 93.0% in the retention test. The item point-biserial correlation coefficients ranged from 0.34 to 0.67 in the posttest and from 0.34 to 0.55 in the retention test.

Preliminary Analysis

Covariates

The Stanford Achievement Test (SAT) and the Volume Achievement Pretest were considered as possible covariates for the four dependent variables, Volume Achievement Posttest, Volume Achievement Retention Test, Multiplication Achievement Posttest and Multiplication Achievement Retention Test. To determine the covariates a sequence of multiple step-wise regression analysis was conducted using a 5% inclusion and a 5% deletion levels. The analysis was carried out using the computer program BMD02R which automatically removes any variable when its significance level becomes too low.

The results of the analysis revealed that only the Volume Achievement Pretest entered as a covariate for both the Volume Achievement Posttest (F=78.63, df=1,103) and the Volume Achievement Retention Test (F=58.39, df=1,103). The Multiplication Posttest, however, had two covariates, the SAT (F=35.31, df=1,103) and the Volume Achievement Pretest (F=11.33, df=2,102). Similarly, the Multiplication Retention Test had two covariates the SAT (F=43.35, df=1,103) and the Volume Achievement Pretest (F=21.81, df=2,102). Table 4.1 summarizes these results.

Table 4.1
F Values for Entering of Covariates

Dependent Variable	Covariate	df	F Value
Vol Ach Post	Vol Ach Pre	1,103	78.63
Mul Ach Post	SAT	1,103	35.31
	Vol Ach Pre	2,102	11.33
Vol Ach Ret	Vol Ach Pre	1,103	85.39
Mul Ach Ret	SAT	1,103	43.35
	Vol Ach Pre	2,102	21.81

Vol Ach Pre: Volume Achievement Pretest
Vol Ach Post: Volume Achievement Posttest
Vol Ach Ret: Volume Achievement Retention test
Mul Ach Post: Multiplication Achievement Posttest
Mul Ach Ret: Multiplication Achievement Retention test
SAT: Stanford Achievement Test

Assumptions of Analysis of Covariance

Analysis of covariance as recommended by Winer (1971, pp. 752-753) was preferred to analysis of variance since blocking was feasible on conservation level only while past achievement in computation and volume calculation were believed to be related to future volume achievement. In the analysis of covariance adjustment of dependent variables (i.e., volume achievement and multiplication achievement) for the regression on covariates (i.e., SAT and pretest volume achievement) was intended to reduce bias and increase the accuracy of the treatment effect (Cochran, 1957, p. 262).

Further justifications for using analysis of covariance are related to the fulfillment of certain assumptions (Elashoff, 1969, p. 385): 1. Randomization was fulfilled since individuals were assigned randomly to groups and groups were assigned randomly to treatments. 2. Covariates, having been measured at the beginning of the experiment, were independent of treatments. 3. Covariates are measured accurately using the standardized SAT (Kuder-Richardson reliability coefficient = 0.90) and the Volume Achievement Test (Hoyt estimate of reliability = 0.94). 4. The regression of the dependent variables on the covariates was believed to be linear. In fact in the preliminary analysis, a linear regression model was used to determine the relationship between dependent variables and covariates. The significant relationship found confirmed the assumption of linearity. 5. Fulfillment of the assumption of homogeneity of covariance and no treatment-slope interaction was done through comparing scatter plots of the dependent variables versus covariates for each treatment group (Elashoff, 1969, p. 392). Winer (1971) further claims that "there evidence to indicate that the analysis of covariance is robust with respect to homogeneity assumptions on ... regression coefficients (p.772)." 6. The assumption of normal distribution of dependent variables within each treatment group at each conservation level was not tested because of the small number of subjects in some cells. Elashoff (1969) explains that the fulfillment of this assumption i.e., normality is required for "statistical convenience" only (p. 386).

Instructor Effect

The proposed experimental design in chapter 3 was 3X3 (Treatments X Conservation Levels). However, a behaviour problem in a treatment group in one of the schools made necessary the inclusion of instructors as a factor. The experimental design became 3X3X3 (Treatments X Conservation Levels X Instructors). Analysis of covariance were conducted to determine the effect of instructors in both the Volume Achievement Posttest and the Multiplication Achievement Posttest. Since the intent of the 3-way analysis was to insure that any instructor effect will be identified, the significance level for the rejection of the null hypothesis was set at 0.10. The main effect of instructors and the 3-way interactions were found to be nonsignificant ($p \leq 0.10$). The only 2-way interaction, found to be significant ($p \leq 0.10$) was Instructors X Treatments on the Multiplication Achievement Posttest. A summary of the analysis of covariance for the instructors effect is presented in Table 4.2.

Since the main effect of instructors was not significant and just a single 2-way interaction involving instructors was significant, the effect of instructors was not thought to be strong enough to necessitate restructuring the original 2-way design of the study. It was therefore possible to pool across instructors and reduce the design to 3X3 (Treatments X Conservation Levels) as was suggested in chapter 3.

Table 4.2
Analysis of Covariance - Instructors Effect

Source of Variation	df	MS	F	significance
Vol Ach Post				
Main Effect				
Instr	2	39.54	1.54	0.22
Tr	2	517.02	20.15	0.00
Cons	2	93.24	3.63	0.03
2-way Interactions				
Instr X Tr	4	12.77	0.50	0.74
Instr X Cons	4	52.10	2.03	0.10
Cons X Tr	4	25.58	1.00	0.41
3-way Interactions	8	21.02	0.82	0.59
Error	76	25.65		
Mul Ach Post				
Main Effect				
Instr	2	11.40	1.29	0.28
Tr	2	71.80	8.10	0.01
Cons	2	25.10	2.83	0.07
2-way Interactions				
Instr X Tr	4	20.55	2.32	0.07*
Instr X Cons	4	4.58	0.52	0.72
Cons X Tr	4	16.09	1.82	0.14
3-way Interactions	8	13.00	1.47	0.18
Error	76	8.86		

* $p < 0.10$ (Instructor effect only)
Instr: Instructor; Tr: Treatment; Cons: Conservation Level

Analysis of Covariance

Hypotheses 1-3

The test of hypothesis 1 (p. 58) revealed that a significant ($p \le 0.05$) difference was found in Volume Achievement Posttest scores among conservation groups. The ANCOVA for the Volume Achievement Posttest scores can be found in Table 4.3. Post hoc analysis using Sheffe's method of multiple comparisons showed a significant ($p \leq 0.05$) superiority of the conservers group over the partial-conservers group. No significant difference was found among any other conservation groups at the 0.05 level. There was no significant difference found in Volume Achievement Retention Test scores among conservation groups at the 0.05 level. The ANCOVA for the Volume Achievement Retention Test scores can be found in Table 4.4.

The means, standard deviations and group sizes of treatments by conservation levels for the Volume Achievement Pretest can be found in Table 4.5. The unadjusted means, standard deviations and group sizes of treatments by conservation levels for the Volume Achievement Posttest and Volume Achievement Retention Test can be found in Appendix D. The adjusted means, standard deviations and group sizes of treatments by conservation levels for the Volume Achievement Posttest and Volume Achievement Retention Test can be found in Tables 4.6 and 4.7 respectively. In all of these tables the marginal values for the means and standard deviations are determined by calculating the weighted average of the cell values.

The test of hypothesis 2 (p. 58) revealed that a significant ($p \leq 0.001$) difference was found in Volume Achievement Posttest scores among treatment groups (see Table 4.3). Post hoc analysis using Sheffe's method of multiple comparisons showed a significant superiority of volume treatment over multiplication treatment ($p \leq 0.01$) and over control treatment ($p \leq 0.01$). No significant difference was found between multiplication treatment and control treatment at the 0.05 level. Similarly, a significant ($p \leq 0.001$) difference was found in Volume Achievement Retention Test scores among treatment groups (see Table 4.4). Post hoc analysis using Sheffe's method of multiple comparisons showed a significant superiority of volume treatment over multiplication treatment ($p \leq 0.01$) and over control treatment ($p \leq 0.01$). No significant difference was found between multiplication treatment and control treatment at the 0.05 level.

The test of hypothesis 3 (p. 58) revealed that no significant interaction was found in Volume Achievement Posttest scores between conservation levels and treatments at the 0.05 level. Similarly, no significant interaction was found in Volume Achievement Retention Test scores between conservation levels and treatments at the 0.05 level.

The adjusted means on the Volume Achievement Posttest were 13.06 (48%) for the nonconservers, 10.82 (40%) for the partial conservers, 15.18 (56%) for the conservers, 17.42 (65%) for the volume treatment group, 12.30 (46%) for the multiplication treatment group and 9.87 (37%) for the control group. Likewise the adjusted means on the Volume Achievement Retention Test were 14.52 (54%) for the nonconservers, 11.23 (42%) for the partial conservers, 14.87 (55%) for the conservers, 17.49 (65%) for the volume treatment group, 13.51 (50%) for the multiplication treatment group and 10.89 (40%) for the control group.

Table 4.3
Analysis of Covariance of Volume Achievement Posttest Scores

Source	df	MS	F
Conservation Level	2	88.88	3.20*
Treatment	2	339.56	12.24**
Conservation Level X Treatment	4	31.50	1.14
Error	95	27.74	

* $p < 0.05$ ** $p < 0.001$

Table 4.4
Analysis of Covariance of Volume Achievement Retention Scores

Source	df	MS	F
Conservation Level	2	74.16	2.67
Treatment	2	217.01	7.81*
Conservation Level X Treatment	4	23.35	0.84
Error	95	27.79	

* $p < 0.001$

Table 4.5
Means, Standard Deviations, and Group Sizes of Volume Achievement Pretest Scores for Treatments by Conservation Levels (Maximum Score = 27)

Conservation	Treatments			
Level	Volume	Multiplication	Control	Total
Non-conservers	7.21	7.61	4.45	6.37
	(5.23)	(4.88)	(3.98)	(4.68)
	19	18	20	57
Partial-conservers	10.17	8.67	5.75	8.50
	(8.23)	(7.71)	(4.50)	(7.10)
	6	6	4	16
Conservers	9.18	10.60	14.00	10.75
	(6.85)	(6.17)	(8.85)	(6.91)
	11	15	6	32
Total	8.31	8.92	6.53	8.03
	(6.23)	(5.81)	(5.02)	(5.73)
	36	39	30	105

Table 4.6
Adjusted Means, Standard Deviations, and Group Sizes of Volume Achievement Posttest Scores for Treatments by Conservation Levels (Maximum Score = 27)

Conservation Level	Treatments			
	Volume	Multiplication	Control	Total
Non-conservers	18.33	11.60	9.35	13.06
	(5.63)	(7.27)	(6.87)	(6.58)
	19	18	20	57
Partial-conservers	15.49	7.83	8.29	10.82
	(7.41)	(10.17)	(7.05)	(8.36)
	6	6	4	16
Conservers	16.91	14.92	12.65	15.18
	(4.29)	(7.54)	(8.59)	(6.62)
	11	15	6	32
Total	17.42	12.30	9.87	13.36
	(5.52)	(7.82)	(7.24)	(6.86)
	36	39	30	105

Table 4.7
Adjusted Means, Standard Deviations, and Group Sizes of Volume Achievement Retention Test Scores for Treatments by Conservation Levels (Maximum Score = 27)

Conservation Level	Treatments			
	Volume	Multiplication	Control	Total
Non-conservers	19.00	13.91	10.81	14.52
	(6.51)	(6.34)	(6.90)	(6.59)
	19	18	20	57
Partial-conservers	14.04	8.96	10.41	11.23
	(8.13)	(10.00)	(8.66)	(8.96)
	6	6	4	16
Conservers	16.76	14.84	11.48	14.87
	(5.08)	(7.96)	(9.05)	(7.17)
	11	15	6	32
Total	17.49	13.51	10.89	14.12
	(6.34)	(7.53)	(7.56)	(7.13)
	36	39	30	105

Table 4.9
Analysis of Covariance of Multiplication Retention Scores

Source	df	MS	F
Conservation Level	2	4.62	0.68
Treatment	2	7.81	1.15
Conservation Level X Treatment	4	3.42	0.51
Error	94	6.76	

Table 4.10
Adjusted Means, Standard Deviations, and Group Sizes of Multiplication Achievement Posttest and Retention Test Scores for Treatments (Maximum Score = 20)

	Treatments			
Test	Volume	Multiplication	Control	Total
Posttest	9.67	12.59	10.29	10.93
	(3.69)	(3.01)	(4.08)	(3.55)
	36	39	30	105
Retention Test	10.69	11.34	10.76	10.95
	(2.99)	(3.53)	(3.1[illegible])	(3.23)
	36	39	30	105

Correlation Study

Hypotheses 5 and 7

Biserial correlation coefficients were calculated for testing of hypotheses 5 and 7 (p. 59). The test of hypothesis 5 revealed that the transition from a lower to a higher level of conservation between the pretest and the posttest was found independent of volume achievement scores at the 0.05 level. Similarly, the transition from a lower to a higher level of conservation between the pretest and the retention test was found independent of volume achievement scores at the 0.05 level. Table 4.11 shows the biserial correlation coefficients between volume achievement scores and transition to a higher or lower level of conservation at the posttest and the retention test levels. Table 4.12 shows the cell sizes of conservation levels X treatments on the pretest, posttest and retention test. Table 4.13 shows the number of students whose conservation levels went up, down or remained the same between the pretest and each of the posttest and the retention test.

Likewise, the test of hypothesis 7 revealed that the transition from a higher to a lower level of conservation is found independent of volume achievement scores between the pretest and the posttest and between the pretest and the retention test at the 0.05 level (see Table 4.11).

Even though the transition from one conservation level to another was independent of volume achievement scores there was a general improvement in the subjects' conservation levels. In the pretest there were 57 nonconservers, 16 partial conservers and 32 conservers while in the retention test there were 36 nonconservers, 13 partial conservers and 56 conservers (see Table 4.12).

Table 4.11
Biserial Correlation Coefficients Between Volume Achievement Scores on the Posttest, Retention Test and Transition to a Higher or Lower Level of Conservation

	Transition Higher	Transition Lower
Posttest-pretest	0.13 (0.18)[1]	0.03 (0.79)
Retention Test-Pretest	0.09 (0.34)	0.03 (0.77)

[1] Number in ( ) indicates the significance level

Table 4.12
Cell Sizes of Conservation Levels X Treatments on the Pretest, Posttest and Retention Test

Conservation	Treatments			
Level	Volume	Multiplication	Control	Total
Non-conservers	19[1]	18	20	57
	19	10	17	46
	13	11	12	36
Partial-conservers	6	6	4	16
	3	8	4	15
	6	5	2	13
Conservers	11	15	6	32
	14	21	9	44
	17	23	16	56
Total	36	39	30	105
	36	39	30	105
	36	39	30	105

[1] The first number refers to the pretest
The second number refers to the posttest
The third number refers to the retention test

Table 4.13
Number of Subjects Whose Conservation Levels Changed/ Did Not Change Between the Pretest and each of the Post and Retention Test

	Level Up	Level Down	Same Level	Total
Posttest	23	11	71	105
Retention Test	32	8	65	105

Hypotheses 9 and 12

Pearson's product-moment correlation coefficient was calculated for testing of hypothesis 9 (p. 59). The test revealed that the Volume Achievement Posttest scores were found to be significantly correlated ($r = 0.35$, $p \leq 0.001$) to the pretest mathematics achievement scores measured by SAT. Similarly, Volume Achievement Retention Test scores were found to be significantly correlated ($r = 0.37$, $p \leq 0.001$) to the pretest SAT scores. Table 4.14 summarizes the correlation coefficients and the significant levels between the volume achievement scores and the SAT scores.

Point biserial correlation coefficients were calculated for testing of hypothesis 12 (p. 60). The test revealed that the volume achievement scores on the posttest and the retention test were not found to be correlated to sex. Table 4.13 summarizes the point-biserial correlation coefficients and the significant levels between the volume achievement scores and sex.

Table 4.14
Pearson's Product Moment Correlation Coefficients and Significance Levels Between Volume Achievement Scores and SAT Scores on the Posttest and the Retention Test

	SAT	Significance
Volume Achievement Posttest	0.35	0.00026
Volume Achievement Retention Test	0.37	0.00011

Table 4.15
Point-Biserial Correlation Coefficients and Significance Levels Between Volume Achievement Scores and Sex on the Posttest and the Retention Test

	Sex	Significance
Volume Achievement Posttest	0.14	0.13
Volume Achievement Retention Test	0.12	0.23

Hypothesis 10

Kendall's Tau correlation coefficient was calculated for testing of hypothesis 10 (p. 60). The test showed that the initial level of conservation was found to be independent (Kendall's Tau = 0.09, p = 0.08) of mathematics achievement scores measured by the computation section of SAT. The sample used for testing of this hypothesis consisted of all 146 subjects who took the Volume Conservation Pretest regardless of their participation in other parts of the experiment. All other hypotheses were tested using the data of the 105 subjects who did not miss any test or treatment day.

Tests of Independence

Hypotheses 6 and 8

Frequency tables were made and Chi Square statistics were calculated for testing of hypotheses 6 and 8 (p. 59). The test of hypothesis 6 revealed that the transition from a lower to a higher level of conservation between the pretest and the posttest was found independent of treatments at the 0.05 level; the Chi Square for transition up or no transition up versus treatments was 0.93 with df = 2. Similarly, the transition from a lower to a higher level of conservation between the pretest and the retention test was found independent of treatments at the 0.05 level with Chi square of 0.97 and df = 2. The frequencies of transition up versus treatments between the pretest and each of the posttest and the retention test are represented in Table 4.16. The frequencies of transition up or no transition up versus treatments between the pretest and each of the posttest and the retention test are represented in Table 4.17.

Likewise, the test of hypothesis 8 revealed that the transition from a higher to a lower level of conservation between the pretest and posttest was found independent of treatments at the 0.05 level; the Chi square for transition down or no transition down versus treatments was 0.91 with df = 2. Likewise, the transition from a higher to a lower level of conservation between the pretest and the retention test was found independent of treatments at the 0.05 level with Chi square of 3.48 and df = 2. The frequencies of transition down or no transition down versus treatments between the pretest and each of the posttest and the retention test are represented in Table 4.18.

Table 4.16
Contingency Table: Transition Up, Down, or Staying the Same Versus Treatments Between Pretest-Posttest and Pretest-Retention Test

Treatment		Transition Up	Transition Down	No Transition
Volume	Post	6	5	25
	Retention	11	4	21
Multiplication	Post	10	4	25
	Retention	10	4	25
Control	Post	7	2	21
	Retention	11	0	19

Table 4.17
Contingency Table: Transition Up Versus Treatments on the Posttest and the Retention Test

Treatment		Transition Up	No Transition Up
Volume	Post	6	30
	Retention	11	25
Multiplication	Post	10	29
	Retention	10	29
Control	Post	7	23
	Retention	11	19

Posttest: Chi square = 0.93, df = 2, p = 0.63
Retention test: Chi square = 0.97, df = 2, p = 0.61

Table 4.18
Contingency Table: Transition Down Versus Treatments on the Posttest and the Retention Test

Treatment		Transition Down	No Transition
Volume	Post	5	31
	Retention	4	32
Multiplication	Post	4	35
	Retention	4	35
Control	Post	2	28
	Retention	0	30

Posttest: Chi square = 0.91, df = 2, p = 0.63
Retention test: Chi square = 3.48, df = 2, p = 0.18

The results reported in this section are related directly to the hypotheses of the study. However, additional findings about the transition between conservation levels are included in the section of Post Hoc Qualitative Analyses.

Hypothesis 11

The Wilcoxon two-sample test using tied scores was applied for testing of hypothesis 11 (p. 60). The test revealed that the initial level of conservation of the males was found significantly ($p \leq 0.05$) better than that of the females. The frequencies of the pretest conservation levels versus sex is represented in Table 4.19. The sample used for testing of this hypothesis consisted of the 150 subjects to whom the pretest

Table 4.19
Contingency Table: Pretest Conservation Level Versus Sex

Conservation Level	Males	Females	Total
Nonconservers	33	45	78
Partial Conservers	13	9	22
Conservers	30	20	50
Total	76	74	150

was administered. There were 76 males and 74 females. The nonconservers were 33 males and 45 females, the partial conservers were 13 males and 9 females and the conservers were 30 males and 20 females.

Post Hoc Qualitative Analyses

The results reported in the previous sections of this chapter relate directly to the hypotheses of this study. However, some additional findings which seem to be of significance are reported in this section.

Transition between Conservation Levels

The tests of hypotheses 6 and 8 revealed that the transition between conservation levels from pretest to posttest, pretest to retention test and posttest to retention test was independent of treatments at the 0.05 level. The following, however, are observations based on the detailed contingency tables of transition among conservation levels. These tables are included as Tables 4.20, 4.21 and 4.22.

1. The regression of conservers to nonconservers occurred very rarely. Only two of 32 (6.25%) conservers regressed to nonconservers between the pretest and the posttest. None of the 32 conservers regressed to nonconservers between the pretest and retention test. Similarly, none of the 44 conservers regressed to nonconservers between the posttest and the retention test.

2. There does not seem to be any observable difference in the progress of nonconservers and partial conservers to higher levels of conservation. For example between the pretest and the posttest 19 of 57 (33%) nonconservers and 4 of 14 (29%) partial conservers progressed to a higher level of conservation.

3. Even though there was a general improvement of conservation levels among all treatment groups, the control group seemed to have undergone a steady progress with respect to conservation levels. That is, subjects in all groups progressed and regressed but those in the control group did not seem to regress as much as the subjects in the other two treatment groups. The two of 30 subjects (6.66%) in the control group who regressed between the pretest and posttest, progressed back to their original level in the retention test. No other subjects in the control group regressed between the posttest and the retention test. In the volume and multiplication treatments there were four in each who regressed from pretest to retention test.

The high stability of the conservation level of conservers throughout the experiment is not surprising. Some research reported in chapter 2 indicated that natural conservers showed stability of their conservation level and even resisted misleading cues. However, it was curious to note that the conservation level of subjects in the control group did not seem to regress as much as the level of subjects in the other treatment groups. The instability of the other two groups could be explained by the influence of experience in volume activities on the partial conservers. Tables 4.20, 4.21, and 4.22 show that most of those who regressed were partial conservers in the volume and multiplication groups. The experience in volume activities could have disturbed the partial conservers so that they incorrectly applied knowledge acquired in the treatments to volume conservation tasks. Those who were in the control group could have used their intuitive understanding of volume conservation.

Table 4.20
Contingency Table: Pretest-Posttest Transition
Among Conservation Levels by Treatments

Pretest	Post Test Nonconservers	Partial Conservers	Conservers
Volume Treatment			
Nonconservers	15	1	3
Partial conservers	3	1	2
Conservers	1	1	9
Multiplication Treatment			
Nonconservers	8	2	8
Partial conservers	2	4	0
Conservers	0	2	13
Control Treatment			
Nonconservers	15	3	2
Partial conservers	1	1	2
Conservers	1	0	5

Table 4.21
Contingency Table: Pretest-Retention Test Transition
Among Conservation Levels by Treatments

Pretest	Retention Test Nonconservers	Partial Conservers	Conservers
Volume Treatment			
Nonconservers	11	3	5
Partial conservers	2	1	3
Conservers	0	2	9
Multiplication Treatment			
Nonconservers	8	1	9
Partial conservers	3	3	0
Conservers	0	1	14
Control Treatment			
Nonconservers	12	1	7
Partial conservers	0	1	3
Conservers	0	0	6

Table 4.22
Contingency Table: Posttest-Retention Test Transition Among Conservation Levels by Treatments

Posttest	Retention Test: Nonconservers	Partial Conservers	Conservers
Volume Treatment			
Non-conservers	12	4	3
Partial-conservers	0	2	1
Conservers	0	1	13
Multiplication Treatment			
Non-conservers	8	1	1
Partial-conservers	2	4	2
Conservers	0	1	20
Control Treatment			
Non-conservers	10	2	5
Partial-conservers	2	0	2
Conservers	0	0	9

Students' Reasons for their Responses on Question 11

Question 11 of the Volume Conservation Test was a part of the conservation classification scheme. In this question the students were asked to give reasons for their responses. The reasons were intended to provide validity information for the classification scheme. The reason given by each student for the response on question 11 was first classified as consistent, inconsistent, or unclassifiable. A reason was classified as consistent if it did not contradict the response. A reason was coded as unclassifiable if it was not possible to understand the reason given by the student.

The consistent and inconsistent responses were further classified according to the following nine attributes:

1. Same: general use of the term "same" with reference to the

two balls without specifying the attribute. Example: "The two balls are still the same."

2. Size: general use of the term "size".
3. Volume: specific use of the term "volume".
4. Amount: specific use of the term "amount".
5. Room: specific use of the term "room".
6. Mass: specific use of the term "mass".
7. Weight: specific use of the term "weight". Example: "It would be the same because it's got the same weight."
8. Shape: specific use of the term "shape".
9. Other: reference to a reason other than the above.
   Examples: "Because they look the same length". "Because the ball was heated."

There was only one unclassifiable response in the pretest, none in the posttest and none in the retention test. This unclassifiable response was eliminated from the data of pretest. Observations based on Tables 4.23, 4.24 and 4.25 can be made about 104 classifiable responses in the pretest, 105 in the posttest and 105 in the retention test. There were ten inconsistent responses in the pretest, ten in the posttest and two in the retention test. All of these responses except two consisted of an incorrect response and a reason which might support the correct response of "equivalence" or "same water level."

1. In the pretest, posttest and retention test the most frequent reason given for a correct response was related to size. The second and third most frequent reasons given for a

correct response were weight and amount respectively. The reasons for a correct response did not seem to be affected by the conservation level of students.

2. The reasons given by conservers for their correct responses seem to be more evenly distributed among the various attributes than the reasons given by nonconservers and partial conservers.

3. The reason given most frequently for incorrect responses in the pretest, posttest and retention test related to shape, room and weight.

Table 4.23
Number of Students with Respect to their Reason for their Response on Question 11 of the Volume Conservation Pretest

	Reasons								
Response	1	2	3	4	5	6	7	8	9
Question 11 Correct									
Nonconservers	2	10	0	7	0	0	7	1	0
Partial conservers	1	2	0	1	0	0	2	0	0
Conservers	0	12	4	5	0	1	9	0	1
Question 11 Incorrect									
Nonconservers	0	4[2]	0	1[1]	4	0	5[1]	8[1]	7[2]
Partial conservers	2[2]	1[1]	2	0	4	0	0	1	0
Conservers	0	0	0	0	0	0	0	0	0

Code for tables 4.21, 4.22, 4.23: 1. Same 2. Size 3. Volume 4. Amount 5. Room 6. Mass 7. Weight 8. Shape 9. Other
[1] means 1 inconsistent response included.
[2] means 2 inconsistent responses included.

Table 4.24
Number of Students with Respect to their Reason for their Response on Question 11 of the Volume Conservation Posttest

Response	Reasons 1	2	3	4	5	6	7	8	9
Question 11 Correct									
Nonconservers	2	15	1	1	0	0	5	1	2[1]
Partial conservers	1	2	0	0	0	0	3	2[1]	0
Conservers	4	17	4	10	2	0	5	0	2
Question 11 Incorrect									
Nonconservers	0	1[1]	1[1]	0	1	0	7[4]	3	0
Partial conservers	1[1]	1[1]	0	0	1	0	1	1	2
Conservers	0	0	0	0	0	0	0	0	0

[1] means 1 inconsistent response included.
[4] means 4 inconsistent responses included.

Table 4.25
Number of Students with Respect to their Reason for their Response on Question 11 of the Volume Conservation Retention Test

Response	Reasons 1	2	3	4	5	6	7	8	9
Question 11 Correct									
Nonconservers	1	8	0	1	0	0	6	0	1
Partial conservers	0	1	0	0	0	0	3	0	1
Conservers	5	24	5	8	1	2	9	0	2
Question 11 Incorrect									
Nonconservers	0	0	0	0	4	0	5[1]	6	4[1]
Partial conservers	0	1	0	0	0	0	0	3	4
Conservers	0	0	0	0	0	0	0	0	0

[1] means 1 inconsistent response included.

Most of those who answered question 11 correctly gave a reason related to size. However, a considerable number of subjects, including nine conservers in each of the pretest and retention test, who answered question 11 correctly gave a reason related to weight. If those nine classified as conservers are only weight conservers and not volume conservers there is doubt about the validity of the conservation test used in this study. However, an earlier part of the conservation test (items 4, 5 and 6) was designed to detect those who were only weight conservers. These weight conservers were classified as nonconservers of volume. Language factors could have prevented some of those subjects from expressing their reason more appropriately. The reasons given by students for their responses to question 11 do not seem to provide sufficient information for conclusive evidence about the validity of the classification scheme used in the Volume Conservation Test.

Consistency between Levels of Conservation and Responses to Question 12 of the Volume Conservation Test

Question 12 of the Volume Conservation Test concerning the two marble boxes was not a part of the conservation classification scheme. This question involved two unequal quantities and was not, therefore, typical of the usual Piagetian questioning protocol. It was included because it was believed that students might be able to give reasons for inequality more easily than for equality. The following paragraph reports observations about the consistency found between responses to question 12 and the levels of conservation. The reasons given by the students for their responses are reported in the following section of this chapter.

The numbers of students who answered question 12 correctly or incorrectly are represented in Table 4.26 for all students who took that particular test. On the pretest and posttest there was a positive relationship between the level of conservation of subjects and their responses on question 12. For the pretest Chi Square was 7.82 ($p \leq 0.05$) and for the posttest Chi Square was 15.59 ($p \leq 0.001$). It was surprising that there was not a significantly positive relationship between the level of conservation of subjects and their responses on question 12 in the retention test at the 0.05 level.

There was a general improvement of correct answers on question 12 among all conservation groups between the pretest and the posttest. In the pretest 29 of 78 (37%) nonconservers, 14 of 22 (64%) partial conservers and 29 of 50 (58%) conservers answered question 12 correctly. In the posttest 32 of 77 (42%) nonconservers, 16 of 21 (76%) partial conservers and 35 of 48 (73%) conservers answered it correctly. In the retention test, 27 of 53 (51%) nonconservers, 14 of 24 (58%) partial conservers and 60 of 93 (65%) conservers answered it correctly.

It appears that between the pretest and posttest there was a general improvement among all conservation groups in answering question 12. The conservers seem to have improved the most (15% improvement) followed by partial conservers (12%) followed by the nonconservers (5%). On the other hand the number of nonconservers who answered question 12 correctly seems to have increased steadily in the pretest, posttest and retention test. The percents of conservers and partial conservers who answered it correctly seem to have increased in the posttest and then decreased in the retention test. The increased percents of correct responses of these two groups followed by decreased percents could possibly be attributed to the tendency of regression toward the mean in successive observations.

Table 4.26
Contingency Table: Correct or Incorrect Response on Item 12 of the Volume Conservation Pretest, Posttest and Retention Test Versus Conservation Levels

Test	Correct	Incorrect
Pretest		
Nonconservers	29	49
Partial conservers	14	8
Conservers	29	21
Posttest		
Nonconservers	32	45
Partial conservers	16	5
Conservers	35	13
Retention Test		
Nonconservers	27	26
Partial conservers	14	10
Conservers	60	33

Pretest: N = 150, Chi square = 7.82, df = 2, p = 0.0201
Posttest: N = 146, Chi square = 15.59, df = 2, p = 0.0004
Retention test: N = 170, Chi square = 2.59, df = 2, p = 0.2735

Students' Reasons for their Responses on Question 12

All the reasons given by the students for their judgement on question 12 were first classified as consistent, inconsistent, unclassifiable, or no response. There were very few cases of no response. There were none in the pretest, one in the posttest and one in the retention test. There were only two unclassifiable responses in the pretest, none in the posttest and three in the retention test. The data of Tables 4.27, 4.28 and 4.29 summarize all reasons given by students excluding the no response and unclassifiable cases. Tables 4.27, 4.28 and 4.29 contain 103 classifiable responses of the pretest, 104 responses of the posttest and 101 responses of the retention test.

The consistent and inconsistent reasons were classified according to the following ten attributes:

1. Weight: specific use of the term "weight". Example: "Because it is the same weight."
2. Amount: specific use of the term "amount".
3. Size: specific use of the term "size".
4. Room: specific use of the term "room".
5. Shape: specific use of the term "shape".
6. Space: specific use of the term "space".
7. Same: general reference to the term "same" without specifying any other attribute.
8. Closed container: reference to the fact that when the box is closed water will not go in it. Example: "It will be lower because it has a lid and water won't go in."
9. Open container: reference to the fact that the water went in one of the boxes because it was open.
10. Air inside: reference to the fact that the closed box keeps the air inside it.

0. Other: reference to a reason other than the above. Examples: "Because it is bigger". "Because more pressure will be on the water than the open box."

There were four inconsistent responses in the pretest, five in the posttest and six in the retention test. All of these responses except two consisted of an incorrect response and a reason which might support the correct response of inequality or different water levels.

1. In the pretest, posttest and retention test most of those who answered correctly gave a reason related to the facts that water can not go into the closed box, that the water went in the open box, or that there is air inside the closed box. The most frequently given reason was that water can not go into the closed box.

2. In the pretest, posttest and retention test most of those who gave an incorrect response gave a reason related to weight, amount or size. The reason related to weight was the most frequently given.

3. There does not seem to be any observable difference in the frequency of reasons given by the three conservation groups for their correct or incorrect responses.

Table 4.27
Number of Students with Respect to their Reason for their Response on Question 12 of the Volume Conservation Pretest

Response	Reasons 1	2	3	4	5	6	7	8	9	10	0
Question 12 Correct											
Nonconservers	5	0	0	0	0	1	0	11	1	3	1
Partial conservers	1	0	0	0	0	0	0	6	3	1	0
Conservers	1	0	0	1	0	1	0	9	3	3	1
Question 12 Incorrect											
Nonconservers	14[2]	0	2	1	0	0	4	9[1]	1	0	2
Partial conservers	1	0	1	0	0	0	1	2	1	0	0
Conservers	7	0	2	0	0	0	0	2[1]	0	1	0

Code for tables 4.24, 4.25, 4.26: 1. Weight 2. Amount 3. Size 4. Room 5. Shape 6. Space 7. Same 8. Water does not go in, box closed, etc. 9. Water went in, box open, etc. 10. Air inside 0. Other

[1] means 1 inconsistent response included.
[2] means 2 inconsistent responses included.

Table 4.28
Number of Students with Respect to their Reason for their Response on Question 12 of the Volume Conservation Posttest

Response	Reasons 1	2	3	4	5	6	7	8	9	10	0
Question 12 Correct											
Nonconservers	0	0	0	0	0	0	0	17	5	2	2
Partial conservers	1	0	0	0	0	1	0	5	1	1	0
Conservers	1	0	0	5	0	3	0	12	4	3	0
Question 12 Incorrect											
Nonconservers	6[2]	1	2	0	0	0	2	1	2[1]	0	5[1]
Partial conservers	2	0	2	0	0	0	1	1[1]	0	0	0
Conservers	1	2	6	0	1	0	2	2	0	0	2

[1] means 1 inconsistent response included.
[2] means 2 inconsistent responses included.

Table 4.29
Number of Students with Respect to their Reason for their Response on Question 12 of the Volume Conservation Retention Test

Response	Reasons 1	2	3	4	5	6	7	8	9	10	0
Question 12 Correct											
Nonconservers	2	0	0	0	0	1	0	7	6	2	0
Partial conservers	0	0	0	0	0	1	0	5	0	2	1
Conservers	0	0	0	1	0	2	0	10	10[1]	4	6[2]
Question 12 Incorrect											
Nonconservers	7	0	1	0	0	0	1[1]	5[2]	1	0	2
Partial conservers	0	0	0	0	1	0	1	0	0	0	2
Conservers	3	3	5	0	1	1	5[1]	1	0	1	0

[1] means 1 inconsistent response included.
[2] means 2 inconsistent responses included.

Most students who responded correctly to question 12 could give an explicit reason which was related to the fact that one of the containers was open (closed) and water would (would not) go inside it. In the pretest 40 of 52 (77%) subjects who answered correctly gave reason 8, 9 or 10; in the posttest 50 of 63 (79%) subjects who answered correctly gave reason 8, 9 or 10; in the retention test 46 of 66 (77%) subjects who answered correctly gave reason 8, 9 or 10. On the other hand, it was previously noted that most of those who responded correctly to question 11 gave an imprecise reason that was related to size. It appears that it was easier for students who responded correctly to give more explicit reasons about inequality of volumes in question 12 than about equality in question 11.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This chapter contains a brief review of the problem, the findings and the conclusions, limitations of the study, implications for educational practice, and recommendations for future research.

Review of the Problem

The purpose of this study is to determine the relationship between the level of conservation of volume and the degree to which sixth grade students learn the volume algorithm of a cuboid "V = L x W x H". The problem is a consequence of an apparent discrepancy between the present school programs and the cognitive theory of Piaget concerning the time to introduce the volume algorithm of a cuboid.

Two widely used textbook series in British Columbia introduce the algorithm "V = L x W x H" in grade 5 (age 10) (Dilley et al., 1974 and Eicholz et al., 1974). Another series introduces the algorithm formally in grade 4 (age 9) (Elliot et al., 1974) and uses it informally in grade 3 (age 8).

Most proponents of Piaget's theory would disagree with such early introduction of the algorithm and claim that most children do not develop the necessary cognitive abilities for learning it before grade 6 (age 11). Piaget (1960) himself, for example, holds that "it is not until stage IV (formal operational) that children understand how they can arrive at an area or volume simply by multiplying boundary edges" (p. 408).

Piaget (1960) considers the concept of conservation to be necessary for any meaningful computation in both area and volume:

> ... Children attain a certain kind of conservation of area [and volume], based on the primitive conception of area (and volume) as that which is bounded by lines (or faces). That understanding comes long before the ability to calculate areas and volumes by mathematical multiplication, involving relations between units of different powers ... (Piaget, 1960, p. 355)

On the other hand, many educators believe that "acquisition of formal scientific reasoning may be far more dependent on specific instructional experiences and far less dependent on general maturation than hypothesized by Inhelder and Piaget (1960)" (Siegler and Atlas, 1976, p. 368). Graves (1972, p. 223), for example, considered education and experience to be necessary for volume conservation. Lovell (1971, p. 179) went further to suggest that even seven and eight year olds (grade 2 and 3) can learn how to use the algorithm "V = L x W x H" in order to calculate the volume.

There seems to be a discrepancy between the present school programs and the theory of Piaget. Some of the present programs introduce the volume algorithm of a cuboid as early as grade 3. This position seems to be backed by some educators who claim that scientific reasoning is more dependent on training and instruction than maturation. Piaget and his proponents seem to argue that conservation of volume is a prerequisite for any meaningful calculation of volume. However, one ought not necessarily to delay the introduction of the volume algorithm "V = L x W x H" until all students conserve volume. Studies have indicated that the majority of adults do not conserve volume (Elkind, 1962; Towler and Wheatley, 1971; Graves, 1972). In such a predicament there seems to be a need for research in order to justify our present school curriculum or suggest its modification. This need has been acknowledged by such educators as DeVault who advocates that "it seems reasonable ... to assert that the studies most likely to produce useful results for curriculum work would be experimental studies (DeVault, 1966, p. 639)."

Findings and Conclusions

The aims of this study which were stated in Chapter 1 will be restated in this section. Also a summary of the findings related to each of the aims will be reported.

Aim 1. To determine the various degrees to which conservers, partial conservers, and nonconservers of volume learn the volume algorithm of a cuboid "V = L x W x H".

Findings. The results of the posttest showed a significant ($p \leq 0.05$) superiority of the conservers group over the partial conservers group in the Volume Achievement Posttest. No significant difference was found between any other conservation groups at the 0.05 level. There was no significant difference found in Volume Achievement Retention Test scores between conservation groups at the 0.05 level.

Aim 2. To determine the degree of effectiveness for each of the two teaching methods on learning the volume algorithm for a cuboid.

Findings. The results showed a significant superiority of volume treatment group over multiplication treatment group ($p \le 0.01$) and over control treatment group ($p \le 0.01$) on the Volume Achievement Posttest. No significant difference was found between multiplication treatment group and control treatment group at the 0.05 level. Similarly, the results of the retention test showed a significant superiority of volume treatment group over multiplication treatment group ($p \le 0.01$) and over control treatment group ($p \le 0.01$): no significant difference was found between multiplication treatment group and control treatment group at the 0.05 level.

Aim 3. To determine the effect of learning the volume algorithm of a cuboid on the transition from one volume conservation level to another.

Findings. The transition from a lower to a higher level of conservation between the pretest and the postest was found to be independent of volume achievement scores at the 0.05 level. Similarly, the transition from a lower to a higher level of conservation between the pretest and the retention test was found to be independent of volume achievement scores at the 0.05 level. Likewise, the transition from a higher to a lower level of conservation was found to be independent of volume achievement scores between the pretest and the posttest and between the pretest and the retention test at the 0.05 level.

Even though the transition from one conservation level to another was found to be independent of volume achievement scores there was a general improvement in the subjects' conservation levels.

Aim 4. To determine the relationship between sex and the levels of conservation of volume.

Findings. The Volume Conservation Pretest revealed that the initial level of conservation of males was found to be significantly ($p \leq 0.01$) better than that of the females. Out of a total 150 students, 76 were males and 74 were females. The nonconservers were 33 males and 45 females, the partial conservers were 13 males and 9 females and the conservers were 30 males and 20 females.

Aim 5. To determine the relationship between sex and the degree of learning the volume algorithm for a cuboid.

Findings. The Volume Achievement Posttest scores were not found to be significantly ($p \leq 0.05$) correlated to sex. Similarly, the Volume Achievement Retention Test scores were not found significantly correlated with sex at the 0.05 level.

Aim 6. To determine the relationship between mathematics achievement and the levels of conservation of volume.

Findings. The initial level of conservation was found to be independent of the mathematics achievement scores of the pretest as measured by the computation section of SAT.

Aim 7. To determine the relationship between mathematics achievement and the degree of learning the volume algorithm for a cuboid.

Findings. The Volume Achievement Posttest scores were found to be significantly ($r = 0.35$, $p \leq 0.001$) correlated to the mathematics achievement pretest scores measured by SAT. Similarly, Volume Achievement Retention Test scores were found to be significantly ($r = 0.37$, $p \leq 0.001$) correlated to the pretest of SAT scores.

Summary of Conclusions and Discussion

Importance of Conservation Levels. This study was an attempt to determine the relationship between the level of conservation of volume and the degree to which sixth grade students learn the volume algorithm. The only significant result in this connection was that the conservers scored higher than the partial conservers on the Volume Achievement Posttest. The conservers did not score significantly higher than the nonconservers on the posttest; partial conservers scored lower, though not significantly lower, than the nonconservers on the posttest. On the retention test, the scores of the conservation groups on the Volume Achievement Test did not differ significantly. Furthermore, students of the volume treatment group scored 65% on each of the Volume Achievement Posttest and Volume Achievement Retention Test. Children in grade 6 seem to be able to apply the volume algorithm for a cuboid, at the computation and comprehension levels, regardless of their conservation level.

So far as students who have reached the 6th grade level are concerned, it appears, that conservation level is not an important factor in learning the volume algorithm as defined in this study. It is possible, although this study has no data to support it, that conservation level might likewise be relatively unimportant as a factor that influences successful learning of the volume algorithm by students in, say, grades 4 and 5. If this be so, then the present school programs which do present the volume algorithm in those grades may not be unreasonable. So long as the criterion for reasonability is learnability, the present study does not support the idea that the introduction of the volume algorithm should not take place before the learners have become conservers of volume. However, there may be factors other than volume conservation which would also influence the learnability of the volume algorithm. The need for further research, regarding the time to introduce the algorithm, will be discussed later in this chapter.

Effect of Treatments. Subjects who were in the volume treatment did significantly better, on the Volume Achievement Posttest and Volume Achievement Retention Test, than those subjects who were in each of the other treatments; the subjects in the other two treatments did not differ significantly in volume performance. The subjects who were in the multiplication treatment did significantly better, on the Multiplication Achievement Posttest, than those who were in the volume treatment and the control treatment; those who were in the volume treatment and those who were in the control treatment did not differ significantly. This indicated that at the posttest level the multiplication treatment was successful. That is, the students had learned the multiplication material which was taught.

In short, it may be concluded that, at the posttest level, the subjects who were in the multiplication treatment learned the multiplication material but the subjects who were in the volume treatment did significantly better than those who were in the multiplication treatment, on the Volume Achievement Posttest.

The conclusions mentioned above seem to suggest that the volume treatment is better than the multiplication treatment in teaching sixth graders the volume algorithm of a cuboid. The volume treatment included activities for determining the volume of cuboids by building them with cubes and counting the number of cubes: this method later used the algorithm "V = L x W x H" for computing the volume of a cuboid. The multiplication treatment consisted mainly of studying the effect of varying factors on their product and varying factors when their product is constant; this task was supplemented by a brief application to the volume algorithm "V = L x W x H."

The results of this study do not, therefore, support the conjecture made in Chapter 2 that students who are proficient in varying factors of a fixed product can rapidly predict and determine the volumes or dimensions of cuboids. On the contrary, the results seem to support Piaget's claim that "it is one thing to multiply two numbers together and quite another to multiply two lengths or three lengths and understand that their product is an area or a volume ... (Piaget et al., 1960, p. 408)."

Transition between Conservation Levels. Results of the study revealed that there was, generally, an improvement of the students' conservation levels regardless of their volume achievement scores or their treatments. The transition from a lower to a higher level of conservation between the pretest and each of the posttest and retention test was found independent of volume achievement scores and of treatments.

It appears that the improvement of the subjects' conservation level was influenced by some factor(s) other than treatments and volume achievement scores. Possible factors could have been growth, peer influence, test influence (sensitization) and 'Hawthorne effect'. Growth is suspected to have been a factor because the experiment lasted about two months during which subjects in grade 6, especially those who were "on the doorstep" of conservation, could have developed from one stage of cognitive development to the next. Uncontrollable students' discussions (peer influence) of the Volume Conservation Test outside the classroom could have influenced the results of the posttest and the retention test since these tests were identical to the pretest. The test itself could have influenced some subjects to think seriously about conservation tasks and to correct their own errors in later tests. Finally, the development of the students from one conservation level to the next could have been partially attributed to the fact that they were chosen for the experiment (Hawthorne effect) and consequently to the influence of feeling special and worthy.

Effect of Mathematics Achievement. Results of the study revealed that volume achievement scores on the posttest and the retention test were correlated to mathematics achievement scores measured by the computation section of SAT. The initial level of conservation of volume was found to be independent of the mathematics achievement scores measured by the computation section of SAT.

The above-mentioned results seem to suggest that a competency in mathematics computation may indicate a competency in volume achievement or vice versa. Furthermore, the mathematics achievement score and volume conservation level seem to be independent.

Effect of Sex. The effect of sex on the degree of learning the volume algorithm of a cuboid and on the initial level of conservation of volume was examined. The degree of learning the volume algorithm of a cuboid, at the posttest and retention test levels was not found to be related to sex. On the other hand, the males were found to have a significantly ($p \leq 0.01$)

higher initial level of conservation than the females.

The above-mentioned superiority of males over females in volume conservation has been also reported by other researchers such as Graves (1972), Elkind (1961-b and 1962) and Wheatley (1971). The superiority of the males to the females in the initial level of conservation could be attributed to the more active participation of males in practical experiences involving manipulative skills. (Price-Williams et al., 1969 and Grave, 1972).

Limitations of the Study

There are several limitations of the study. Some of these limitations are related to the type of subjects chosen for the experiment while other limitations are consequences of the procedures of the study.

The subjects of this study were sixth grade students of a suburban area. Their average family income was slightly higher than the average income of the greater metropolitan area. The subjects consisted originally of 171 students but when those who missed any test or treatment day were eliminated, the final sample was reduced to 105 students. There were 57 nonconservers, 32 conservers and only 16 partial conservers. Any generalization based upon the results and conclusions of this study is limited to this or to a similar population of students. On the other hand, the subjects used in this study were grade 6 students who followed a mathematics program typical of those used in North America. They were of ages and socioeconomic status similar to those of suburban grade 6 students in North America. These subjects seem, therefore, to be representative of the population of suburban North American grade 6 students and findings could be generalized to this larger population.

The limitations relating to the procedures involve constraints due to the treatments and constraints relating to the tests. One of the experimental treatments, the volume treatment, was more comprehensive and required more students' active involvement than is normal in elementary classrooms. The other experimental treatment, the multiplication treatment, was different from usual school approaches in its emphasis on multiplication skills involved in the learning of the algorithm "V = L x W x H." Furthermore, only one version of each of the Volume Achievement Test, Volume Conservation Test and Multiplication Achievement Test was used at different stages of the experiment. The classification of subjects into volume conservation levels was achieved using a judgement-based volume conservation test. The generalizations of this study are limited by the treatments and test instruments used.

Implications for Educational Practice

Implication 1. The students of the volume treatment had an adjusted mean score of 65% on both the Volume Achievement Posttest and Volume Achievement Retention Test. This seems to indicate that students in grade 6 are capable of learning the volume algorithm for a cuboid "V = L x W x H." The volume achievement scores of such students do not seem to be affected by their levels of conservation of volume.

Since the conservation level did not seem to be an important factor in learning the volume algorithm, using conservation as a criterion, the present school programs that introduce the algorithm prior to grade 6 are not proven unreasonable. This study does not, therefore, suggest the delay of introducing the volume algorithm for a cuboid "V = L x W x H." This is not to say that the prevalent school practices are justified with respect to the theory of Piaget. The section on future research outlines possible ways for pursuing the matter of justification of the school programs.

Implication 2. The results and conclusions of this study indicate that the activity-oriented volume treatment was successful. This treatment was based on determining the volume of cuboids by building them with cubes and counting the number of cubes: later the algorithm "V = L x W x H" was used for computing the volume of a cuboid. It seems, therefore, appropriate to teach the volume algorithm of a cuboid using an activity oriented method.

Implication 3. The mathematics computation scores of SAT were found to be positively correlated with volume achievement scores. This seems to suggest that a competency in mathematics computation may indicate a competency in volume achievement or vice versa.

Implication 4. Females seem to be as capable as males in learning the volume algorithm for a cuboid "V = L x W x H." However, the superiority of the males to the females in the initial level of conservation could be attributed to the more active participation of males in practical experiences involving manipulative skills. Activity-oriented programs in the teaching of volume concepts may be beneficial to the acquisition of volume conservation by females.

Recommendations for Future Research

The purpose of this study was to determine the relationship between the level of conservation of volume and the degree to which students learn the volume algorithm for a cuboid "V = L x W x H." On the basis of the findings, conclusions and implications of the study, further research is needed on this topic.

It is recommended that the experiment be replicated on a larger sample. The sample of this study consisted of 105 students only 16 of which were partial conservers. A larger sample might influence the results found in this study.

It is also recommended that the experiment be replicated on subjects in a grade lower than six. The independence of conservation level and volume achievement could have been influenced by the high grade level chosen. Students in grade 6 could have developed learning habits that were very effective, or those students could have been "on the doorstep" of volume conservation and became conservers early in the experiment. The choice of a lower grade and consequently a lower level of development might reveal the importance of volume conservation level more clearly.

A further recommendation is to conduct a study, in which volume learning at higher cognitive levels than computation and comprehension is investigated with respect to a correlation with volume conservation.

It is also recommended that a volume conservation posttest and a volume conservation retention test be developed which are parallel forms of, but not identical to, the volume conservation pretest. The identical conservation tests given in the pretest, posttest and retention test could have allowed a greater peer influence or sensitization effect than if they were not identical.

It is also recommended that the following observations which were made in the Post Hoc Qualitative Analyses be investigated further.

1. It was noted that there was a general progress in the conservation levels of subjects in all groups. However, in cases of regression, the partial conservers who were given

experiences in volume activities seem to have regressed the most. Further investigation, which includes interviewing of subjects, is needed for cases where regression occurs. Such investigation may reveal the effect of volume activities on the stability of the conservation level of subjects, particularly the partial conservers.

2. Students were asked, in question 11 of the Volume Conservation Test, to write reasons for their responses. Those responses did not seem to provide sufficient validity information for the final judgement about the test used. A considerable number of students gave an incorrect response and a reason which might support the correct response while other students gave a correct response and a reason related to weight. Further investigation is needed to validate the assessing of volume conservation. It is recommended that methodological studies be undertaken to determine the relationship between nonverbal and interrogation methods of assessing conservation of volume.

3. Question 12 of the Volume Conservation Test concerned two unequal volumes where students were asked to write reasons for their responses. It was noted that the number of partial conservers and conservers, who answered question 12 correctly, increased in the posttest and decreased in the retention test. The number of nonconservers who answered question 12 correctly seemed to have improved steadily in the pretest, posttest and retention test. On the other hand, most correct responses to question 12 were supported by explicit reasons. Further

research is needed for question 12 in particular and cases of conservation of inequality of volume in general.

Finally, in this study the relationship between volume conservation and the degree of learning a volume algorithm involving multiplication skills was investigated. It is recommended that similar research be conducted to determine the relationship between various conservation tasks and the degree of learning other algorithms involving multiplication skills in the elementary school. For example, research may be designed to investigate the relationship between area conservation and learning the algorithm for the area of a rectangular region, "A = L x W", that is, area equals length times width.

In summary, further research is needed before the prevalent school practice of introducing the volume algorithm for a cuboid (V = L x W x H) in grades earlier than grade 6 can be justified on the basis of the cognitive theory of Piaget.

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Appendix A

DESCRIPTION OF THE INSTRUCTIONAL UNITS

Treatment A

Lesson 1

Behavioral Objectives:

1. Given two models of cuboids (closed boxes or solid blocks) the volume of which differ macroscopically, the students will be able to state which one has the greater volume.

2. Given five models of cuboids (closed boxes or solid blocks) the volume of any two of which differ macroscopically, the students will be able to order the five models by volume.

3. Given two closed boxes, the volumes of which do not necessarily differ macroscopically, and given a set of decimetre cubes to be used as units, the students will be able to build models congruent to the closed boxes, and thereby to state the volume of each box.

4. Given five closed boxes, the volume of any two of which do not necessarily differ macroscopically, and given a set of decimetre cubes to be used as units, the students will be able to build models congruent to the closed boxes, and thereby to order the five boxes by volume.

5. Given a picture of polyhedral model built from unit cubes, some of which may not be visible, and given a set of unit cubes, the students will be able to build the pictured model and state its volume.

Outline:

1. Direct comparison of objects.

2. Direct ordering of objects.

3. Indirect comparison of closed boxes.

   3.1. Need for units; volume is the number of unit cubes.
   3.2. Non-standard units: Discussion.

4. Standard units: m^3, dm^3 and cm^3.

5. Indirect ordering of closed boxes.

6. Volume of polyhedral models built from unit cubes.

7. Worksheet.

Materials:

1. Cardboard boxes.
2. 1 inch-cubes.
3. Some decimetre cubes and centimetre cubes.
4. A poster of polyhedral models built from unit cubes.
5. A cm ruler.

Activities:

Give each student 25 inch-cubes (refer to these cubes as simply "cubes" and not as "inch-cubes"). Ask the students to lay the blocks aside because they will be used later in the period.

1. (2 min) Direct comparison of objects:

Display the two closed cardboard boxes A and B of sizes 6 cm X 4 cm X 2 cm and 40 cm X 20 cm X 10 cm respectively. Ask the students to guess which is bigger, which occupies more space and which has the greater volume. Conclude that box B is bigger than box A and that any one of the following sentences describes this fact.

a. Box B is bigger than box A
b. Box B takes up more room than box A
c. Box B occupies more space than box A
d. Box B has a larger volume than box A.

2. (3 min) Direct ordering of objects:

Display, in this order, the five closed boxes F, G, H, I, and J of sizes 2 dm^3, 4 dm^3, 16 dm^3, 1 dm^3 and 10 dm^3 respectively. Ask the students to help to order the boxes from largest to smallest. Allow time for responses then order the boxes by comparing any two boxes and then putting a third in its proper position between the first two and so on.

3. (5 min) Indirect comparison of closed boxes:

3.1. Need for units:

Display a closed cardboard box in each of two distant locations of the classroom (use box P and box Q) and ask the students to compare the volumes of the boxes without moving them. Lead the discussion in order to conclude that perception may be deceiving; determine and compare the volumes using the following activ`ties:

a. Use units (smaller boxes provided) to build next to each box a cuboid with the same shape and volume as that of the box.
b. Count the number of units and write the volumes of the boxes on the board. (Volume of P = 7 units, Volume of Q = 8 units).
c. Compare the volumes of the boxes using the numoers of units found in b.

Stress that in this process, <u>any</u> units of the same size <u>could</u> be used, but the same units must be used throughout.

3.2. Non-standard units: Discussion:

Encourage individual students to suggest items which could be used as units.

Discuss with the students the feasibility and convenience of scme of the suggested units.

4. (5 min) Standard units: dm^3 and cm^3:

Ask the students about the common units for measuring length (desired answer: m, dm, cm, ...). If necessary use a cm ruler to measure length. Lead the discussion in order to conclude that m^3, dm^3, cm^3 are consistent with the units we usually use to measure length. Show cubes of volume 1 dm^3 and 1 cm^3. Emphasize that these volumes are the usual metric units used in industry and commerce. Use decimetre cubes to build a congruent shape and to measure the volume of box B (4 dm X 2 dm X 1 dm) and use centimetre cubes to build a congruent shape and to measure the volume of box A (6 cm X 4 cm X 2 cm). Write on the board statements such as "Volume of box B = 8 dm^3" and "Volume of box A = 48 cm^3."

5. (5 min) Indirect ordering of closed boxes:

Display three closed boxes (I, F, K) and ask the students to suggest how to order the boxes from largest to smallest using the 1 dm^3 blocks and the method described in section 3.1 above. Determine the volume of each box and order the boxes accordingly.

6. (5 min) Volume of polyhedral models built from unit cubes:

Display a poster (#1.1) of two arrangements of cubes. The first of the arrangements includes one block "absent" although it may appear to be "present." The second of the arrangements includes one block "present" although it may appear to be "absent." Ask the students to use the cubes (inch-cubes) in order to build cuboids exactly like the ones pictured in the poster and to state the volumes in terms of the cubes.

7. (10 min) Worksheet:

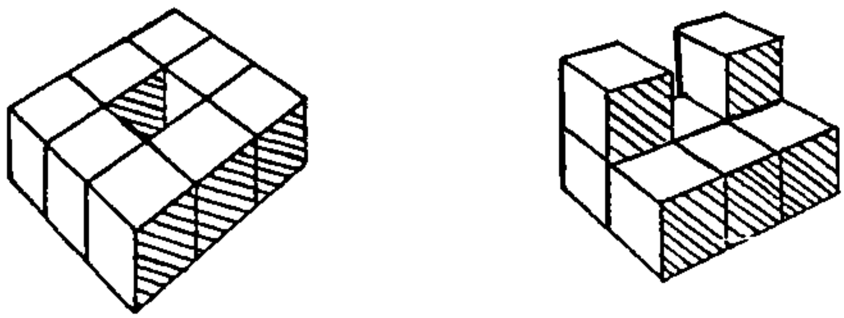
Display boxes X, Y, W and Z in one area of the classroom (station #1). Display also boxes #1 and #2 as well as 15 decimetre cubes in another area (station #2). Give each of the students a copy of the attached worksheet and explain it to them.

Divide the pupils into 3 groups. Let two of the groups each be of about 1/4 the class and the third of about 1/2 of the class. Ask each of the smaller groups to start on station #1 or #2 and the larger group on the seat work.

Instruct the students in the smaller groups to finish the work at their own station, then, with your approval, to move to the other station and finish the work there, and then to return to their seats for the seat work. Similarly instruct the students in the larger group to finish their seat work first then wait for your approval in moving to the stations. Send students from this larger group to the stations as they finish their seat work and as space and order at the stations allow. At the end of the period collect the worksheets.

Poster of Lesson A1

No. 1.1



Name: Last:_______________, First:___________

Lesson A1 - Worksheet

(Station #1)
1. Examine the two boxes lettered X and Y.

Which occupies less space, Y or X? _____

which has the smaller volume Y or X? _____

2. Examine the four boxes lettered W, X, Y, and Z; then order them from largest to smallest and record your answers below.

(largest) ______, ______, ______, ______ (smallest)

(Station #2)
3. Build stacks of cubes similar to these boxes and answer the following:

The volume of box #1 is: _______ cubes.

The volume of box #2 is: _______ cubes.

(Seat work)
4. For each of the figures below use the cubes to build a model. Count the number of cubes and record the volume.

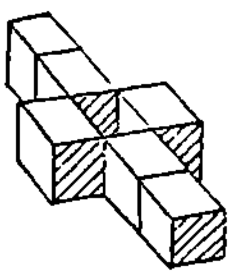


Figure A

Volume =____ cubes

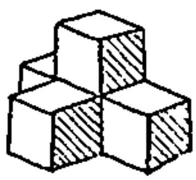


Figure B

Volume =____ cubes

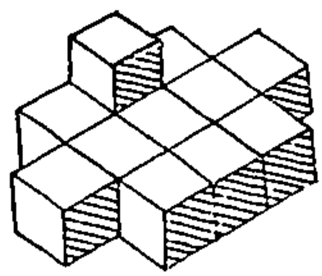


Figure C

Volume =____ cubes

5. list A, B, and C in order

(Largest) ________ ________ ________ (Smallest)

Treatment A

Lesson 2

Behavioral Objectives:

1. Given a cuboid or a diagram of a cuboid, which is completely partitioned or partially partitioned into unit cubes, the students will be able to build a layer and determine the volume of the layer, the number of layers and the total volume.

2. Given a diagram of a non-partitioned cuboid, the dimensions shown either by numerals or by the fact of its edges being marked in unit segments, the students will be able to determine the volume of a layer, the number of layers and the total volume of the cuboid.

Outline:

1. Follow up of the worksheet from the previous lesson.

2. Volume of partitioned and non-partitioned cuboids.

3. Algorithm for the volume of a cuboid "V = L X W X H."

4. Application of the volume algorithm to cuboids and diagrams of cuboids.

5. Worksheet.

Materials:

1. Three cardboard boxes.
2. Some decimetre cubes.
3. A poster of: a partitioned cuboid, a partially partitioned cuboid and a non-partitioned cuboid.

Activities:

1. (3 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson. Explain that in order to compare volumes one can count the cubes and compare the numbers obtained.

For illustration count the cubes of the shapes in #4 as each student follows on his own worksheet, report the volumes of the shapes and state their order.

2. (7 min) Volume of partitioned and non-partitioned cuboids:

Give each student 25 inch-cubes. Display a poster (#2.1) of a partitioned cuboid of dimensions 2, 2 and 5. Ask the students to build with the cubes a cuboid exactly like the one pictured in the poster, to count the number of blocks and to state the volume of the cuboid.

Draw on the board a diagram of a non-partitioned cuboid of dimensions L = 4, W = 3 and H = 2. Write these dimensions along the edges. Ask the students to guess the number of cubes necessary to build the cuboid. Partition the top layer then the rest of the cuboid. Ask the students to determine the volume by using the blocks to build a model then count the number of blocks used.

3. (12 min) Algorithm for the volume of a cuboid "V = L X W X H":

Display a closed cardboard box (H) whose dimensions are L = 4 dm, W = 2 dm and H = 2 dm. Display also about 20 decimetre cubes. Discuss with the students how one can determine the volume of the box using the available cubes. Lead the activities using the cubes in order to determine:

a. The number of decimetre cubes along the length of the bottom layer. Write on the board "L(length) = 4 dm" i.e., 4 cubes fit along the length of the bottom layer.

b. The number of decimetre cubes along the width of the bottom layer. Write on the board "W(width) = 2 dm"

c. Build a layer and conclude that the number of blocks that can fit in the bottom layer is given by L X W.

d. The number of decimetre cubes along the height of the box. Write on the board "H(height) = 2 dm"

e. Build a shape congruent to the box and conclude that the total volume is the volume of one layer (L X W) multiplied by the number of layers (H) i.e., V = L X W X H.

Display another cardboard box (R) whose dimensions are 3 dm, 3 dm and 2 dm. Display also about 10 decimetre cubes. Discuss with the students how one can determine the volume of the box using the available cubes, knowing that there are not enough cubes to build a shape congruent to the box or even build a shape congruent to a layer. Develop again the algorithm "V = L X W X H" by following steps a to e of the previous activity but without actually building a shape similar to the box.

Test the algorithm "V = L X W X H" using a different cardboard box (#1) of dimensions 4 dm, 3 dm and 1 dm:

f. Find the length, width and height of the box in decimetres and multiply them.

g. Build a shape similar to the box with decimetre cubes and count the number of cubes.

h. Compare the results in f and g.

4. (6 min) <u>Application of the volume algorithm to diagrams of cuboids:</u>

Present a poster (#2.2) of diagrams of cuboids and apply the algorithm as directed.

a. Refer to the partitioned cuboid on the poster. Without actually using the blocks develop again the algorithm "V = L X W X H" by verbally following steps a to e of the previous activity.

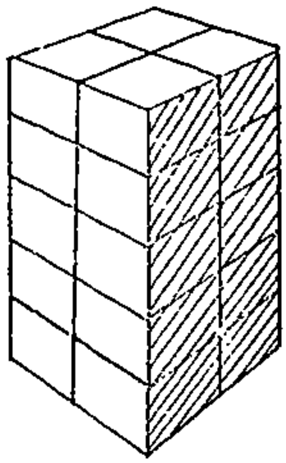
b. Refer to the partially partitioned and the non-partitioned cuboids. In each case determine the volume of each layer, the number of layers and the total volume. Use the algorithm "V = L X W X H" to compute the volume. Compare the two answers.

5. (7 min) <u>Worksheet:</u>

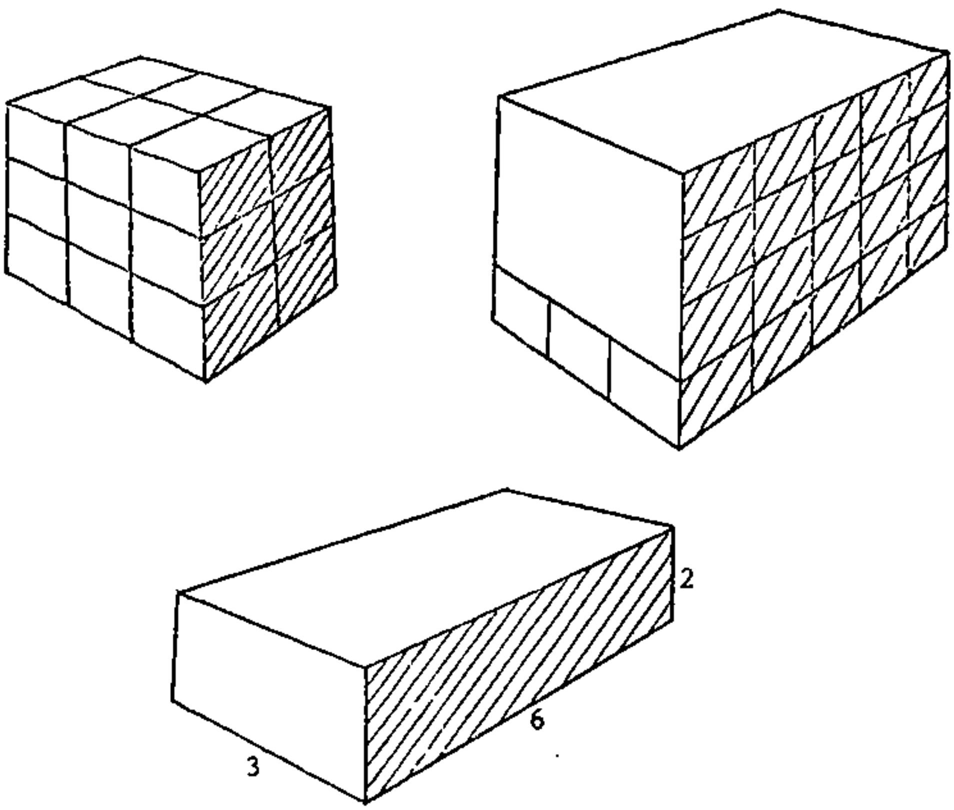
Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Posters of Lesson A2

No. 2.1



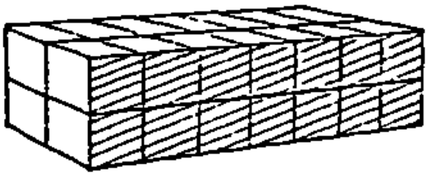
No. 2.2



Name: Last:______________, First:___________

Lesson A2 - Worksheet

1. Find the volume of each of the figures below. (You may use the cubes to build models of the figures).



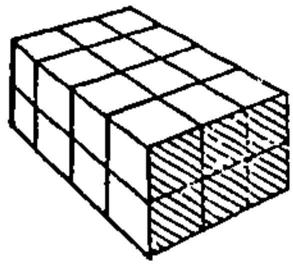


Figure D

Volume of a layer = _____

Number of layers = _____

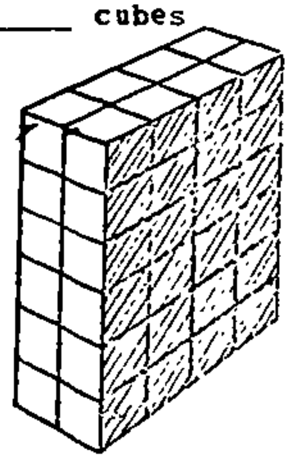
Volume =______ cubes

Figure F

Volume of a layer = _____

Number of layers = _____

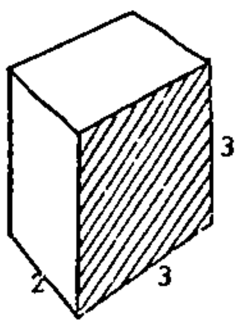
Volume = ______ cubes



2. Volume of the bottom layer = _______

Number of layers = _______

Volume = _______

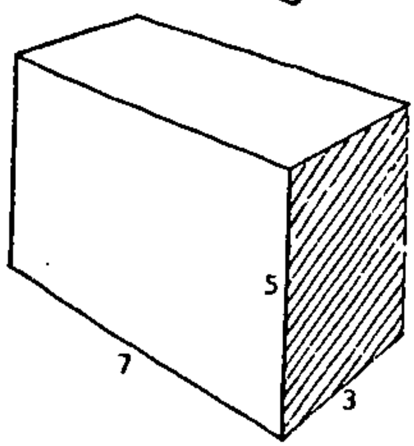


3. L = _______

W = _______

H = _______

V = _______



4. Volume of the box = _______

Treatment A

Lesson 3

Behavioral objectives:

1. Given either a diagram of a non-partitioned cuboid with its dimensions marked, or a word description of the dimensions of a cuboid without a diagram, the students will be able to use the volume algorithm in order to determine the volume of the cuboid.

2. Given a diagram of cuboids touching side by side and all of the required dimensions, the students will be able to use the volume algorithm in order to determine the total volume of the cuboids.

3. Given a diagram of a cuboid to which there are attached half cubes (rectangular parallelepipeds or triangular prisms) the students will be able to determine the total volume.

4. Given a diagram of a partitioned cuboid which is partially covered, the students will be able to determine the total volume of the cuboid.

Outline:

1. Follow up of the worksheet from the previous lesson.

2. Application of the volume algorithm to the following cases:
   a. Word description of cuboids.
   b. Cuboids touching side by side.
   c. Diagrams of cuboids with some unit cubes attached or removed.
   d. Attachments of half cubes to cuboids.
   e. Diagrams of partially covered cuboids.

3. Worksheet.

Materials:

1. Posters of cuboids touching side by side, cuboids with some units attached or removed and attachments of half cubes to cuboids.

Activities:

1. (7 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain that "V = L X W X H" helps us to compute the volume (the number of cubes needed to build a similar shape) of each of the shapes drawn on the worksheet.

For illustration explain, as each student follows on his own worksheet, that in #2 the volume of the top layer (of L = 4 and W = 2) is L X W = 4 X 2 = 8, that there are 6 layers and that the volume is the number of cubes in one layer (8) multiplied by the number of layers (6) i.e., V = 4 X 2 X 6 = 48.

2. (18 min) Application of the volume algorithm to the following cases:

a. Word description of cuboid:

Write on the board a verbal description of the dimensions of a rectangular box (L = 6 dm, W = 2 dm, H = 5 dm). Substitute the given numbers for L, W, and H in "V = L X W X H" and compute the volume. Draw a diagram on the board and illustrate that "L X W" gives the number of blocks in one layer and L X W X H is the total volume.

b. Cuboids touching side by side:

Present the poster (#3.1) of two cuboids touching side by side. Ask students to compute the volume of each cuboid and add to determine the total volume.

c. Diagrams of cuboids with some unit cubes attached or removed:

Present the poster (#3.2) of cuboids with some unit cubes attached or removed. Lead the students to compute the volume of the cuboid as if there were nothing attached or removed. Determine the volume of the unit blocks to be added or subtracted. Add or subtract to determine the volume.

d. Attachments of half cubes to cuboids:

Present the poster (#3.3) of the diagrams of the attachments of half cubes to cuboids. In each case ask the students to determine the volume of the cuboid, the volume of the attached half cubes and add to determine the total volume.

e. Diagrams of partially covered cuboids:

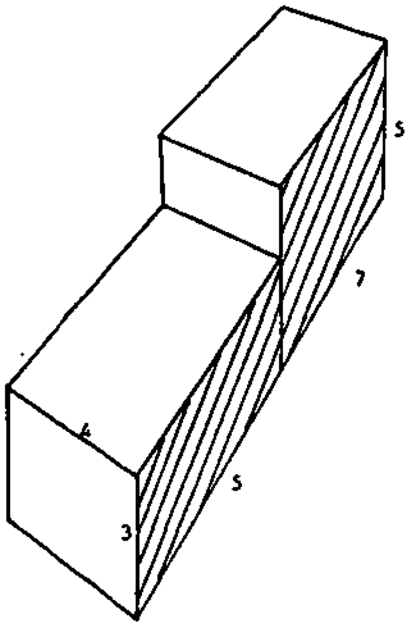
Present the poster (#3.4) of a partially covered cuboid (4X3X6). Point out that the blocks of the top layer are shown and there is a total of 6 layers. Determine the length, width and height of the cuboid and use the algorithm to compute the volume. Count the number of blocks in the top layer (12) and conclude that each of the 6 layers has 12 blocks. Determine the total volume (6x12). Compare the two answers.

3. (10 min) Worksheet:

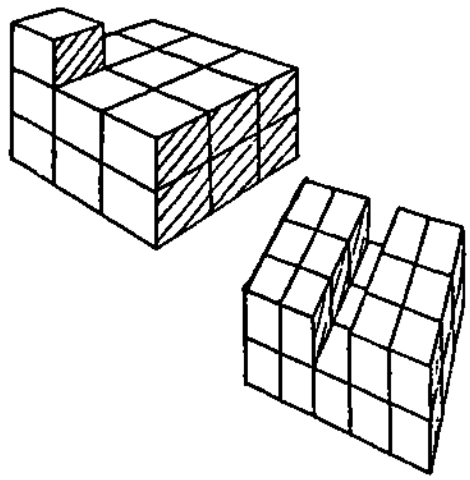
Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Posters of Lesson A3

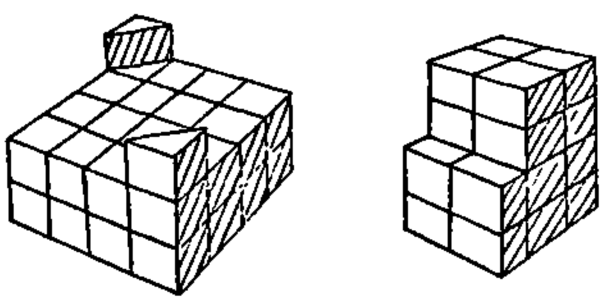
No. 3.1



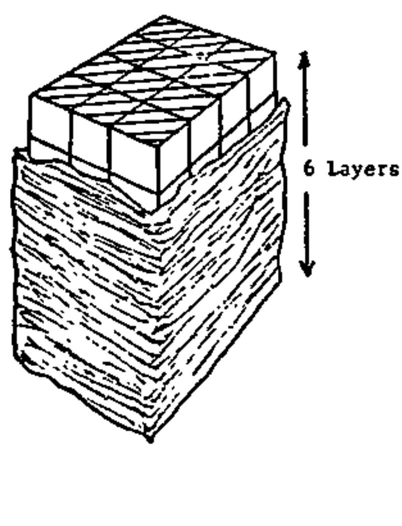
No. 3.2



No. 3.3



No. 3.4



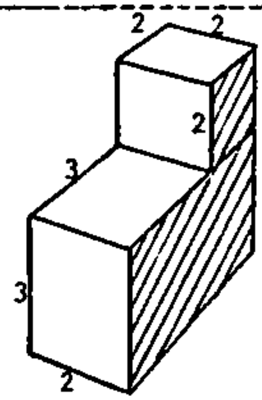
Name: Last:________________, First:____________

Lesson A3 - Worksheet

1. A box has a length of 14, a width of 11 and a height of 3.

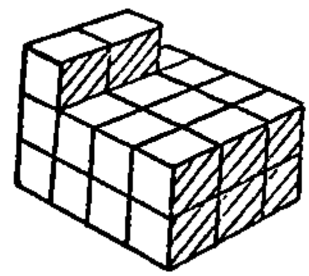
What is the volume of the box? ________

2.



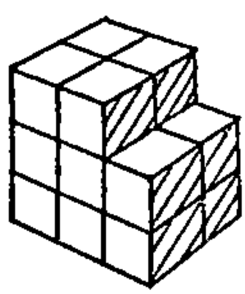
What is the volume ________

3.



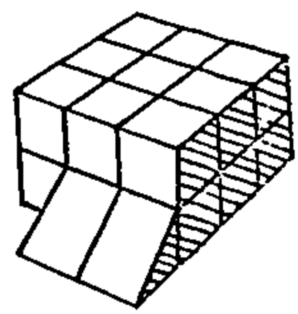
What is the volume ________

4.



What is the volume ________

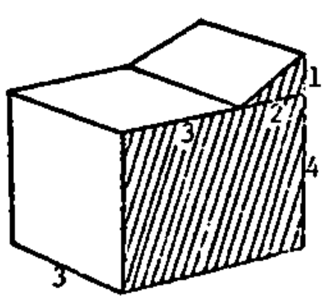
5.



A pile of cubes and half cubes

What is the volume ________

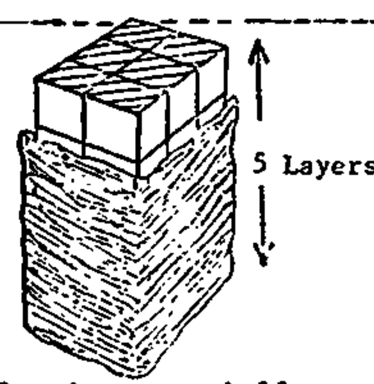
6.



A metal block and a half of a different block on top

What is the volume ________

7.



A pile of cubes partially covered

What is the volume ________

Treatment A

Lesson 4

Behavioral Objective:

1. Given a diagram or a word description of a cuboid of known dimensions and given a proposed additive or multiplicative dimensional transformation, the students will be able to state the volume of the cuboid that would result after the transformation.

Outline:

1. Follow up of the worksheet from the previous lesson.
2. Application of the volume algorithm to cuboids with proposed dimensional transformations.
3. Summary of generalizations.
4. Worksheet.

Materials:

1. A poster of a partitioned cuboid.

Activities:

1. (5 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain that in #2 for example, there are 2 half-cubes attached to a rectangular pile of cubes of L = 3, W = 3 and H = 2. The volume of the 2 half-cubes is 1, the volume of the pile is 3 X 3 X 2 = 18 (using V = L X W X H) and the total volume is therefore 18 + 1 = 19.

Similarly for #6, explain that the volume of the block is 5 X 3 X 4 = 60 (using V = L X W X H), the volume of the half block on top is (3 X 2 X 1)/2 = 3 and the total volume is therefore 60 + 3 = 63.

2. (18 min) Application of the volume algorithm to cuboids with proposed dimensional transformations:

Write on the board "V = L X W X H", display a poster (#4.1) of a partitioned cuboid of dimensions 6, 4 and 10 and replace L, W, and H by the numbers 6, 4 and 10. Compute the volume (240) and replace V by 240.

Apply the following changes to the factors (L, W, H) and observe the changes in the product (volume). Encourage the students to state and test conjectures about the effect on the volume when the dimensions are changed.

2.1. Additive (or multiplicative) increase in one, two or three of the dimensions produces additive (or multiplicative) increase in the volume.

2.1.1. Additive increase in one, two or three of the dimensions:

Ask the students to use the algorithm "V = L X W X H" to calculate the volume of the cuboid if its length increases to 7 units. Write "7 X 4 X 10 = 280" underneath "6 X 4 X 10 =240."

Continue by asking the students to predict what happens to the volume (240) if the width increases or the height increases. Allow time for responses, write on the board the two sentences "6 X 5 X 10 = ____" and "6 X 4 X 11 = ____", divide the students into two groups and ask each group to complete one of the statements. Solicit the answers (300,264) and complete the statements written on the board.

Ask the students to predict what happens to the volume (240) if any one of the dimensions increases. When conjectures are made test them using examples such as "8 X 4 X 10 = 320" and "6 X 4 X 12 = 288." Lead the discussion in order to conclude that the volume increases if any one of the dimensions increases.

Similarly, ask the students to predict what happens to the volume (240) if two of the dimensions increase. Allow time for responses and test them using examples such as "7 X 5 X 10 = 350" and "6 X 6 X 10 = 360." Lead the discussion in order to conclude that the volume increases if two of the dimensions increase.

Similarly, lead the discussion in order to conclude that the volume increases if all dimensions increase. Use "7 X 5 X 11 = 385" for illustration.

2.1.2. Multiplicative increase in one or two of the dimensions:

Ask the students to predict what happens to the volume (240) if any one of the dimensions is multiplied by "2" (doubled). Allow time for responses and ask three groups to test them using "12 X 4 X 10 = _____", "6 X 8 X 10 = _____" and "6 X 4 X 20 = _____" (answer: 480). Conclude that the volume is multiplied by 2 (doubled). Continue by asking the students to predict what happens to the volume (240) if any one of the dimensions is multiplied by 3 (tripled) or 4. Test the conjectures using:

6 X (3 X 4) X 10 = 720 = (3 X 240) and
6 X 4 X (4 X 10) = 960 = (4 X 240).

Lead the discussion in order to conclude that the volume is multiplied by 2 (doubled), 3 (tripled), ... if any one of the dimensions is multiplied by 2 (doubled), 3 (tripled), ...

Likewise, ask the students to predict what happens to the volume (240) if each of two dimensions is multiplied by 2; then if one is multiplied by 2 and another by 3, etc. Use examples such as the ones written below to establish that if one dimension is multiplied by a whole number and another dimension is also multiplied by a whole number, the volume will be multiplied by the product of the two numbers.

(5 X 6) X (2 X 4) X 10 = (5 X 2) X 240
30 X 8 X 10 = 2400

6 X (3 X 4) X (2 X 10) = (3 X 2) X 240
6 X 12 X 20 = 1440

2.2. Additive (or multiplicative) decrease in one, two or three of the dimensions produces additive (or multiplicative) decrease in the volume.

Model the discussion of this section after the discussion of the previous section (2.1). For each of the generalizations below (lettered a, b, ...) ask the students to predict what happens to the volume (240) if the proposed changes in the dimensions are applied, test the students' predictions using the given examples and conclude the generalization.

2.2.1. Additive decrease in one, two or three of the dimensions: (move quickly through this section)

a. The volume decreases if any one of the dimensions decreases.
Examples: 5 X 4 X 10 = 200; 6 X 3 X 10 = 180.

b. The volume decreases if any two of the dimensions decrease.
Examples: 5 X 3 X 10 = 150; 5 X 4 X 3 = 60

c. The volume decreases if all dimensions decrease.
Example: 4 X 3 X 8 = 96

2.2.2. Multiplicative decrease in one or two of the dimensions:

a. The volume will be divided by 2 (halved), 3, ... if any one of the dimensions is divided by 2, 3, ...

Examples:
6 X (4/2) X 10 = 240/2
6 X 2 X 10 = 120

6 X 4 X (10/5) = 240/5
6 X 4 X 2 = 48

b. If one of the dimensions is divided by a whole number and another dimension is divided by a whole number, the volume will be divided by the product of the two numbers.

Examples:
(6/2) X (4/2) X 10 = 240/(2 X 2) = 240/4
3 X 2 X 10 = 60

2.3. Additive increase in one or two of the dimensions and additive decrease in one or two can produce any one of the following:

a. Additive increase in the volume
b. Additive decrease in the volume
c. No change in the volume

Ask the students whether they can predict what happens to the volume if one of the dimensions increases and another decreases. Allow time for predictions and ask three groups to test them using the following examples:

6 X 5 X 9 = 270 (the volume increases)
7 X 4 X 3 = 84 (the volume decreases)
8 X 3 X 10 = 240 (the volume does not change)

Conclude that the volume may increase, decrease or stay the same if one dimension increases and another decreases; each example has to be examined individually.

3. (2 min) Summary of generalizations:

Summarize the generalizations made in this lesson by asking this series of questions and encourage students to answer them:

i. What happens to the volume if we increase one, two or three of the dimensions?

ii. What happens to the volume if we decrease one, two or three of the dimensions?

iii. What happens to the volume if we increase one of the dimensions and decrease another?

iv. a. What happens to the volume if we double only one dimension?
b. What happens to the volume if we double two dimensions?
c. What happens to the volume if we double all three dimensions?

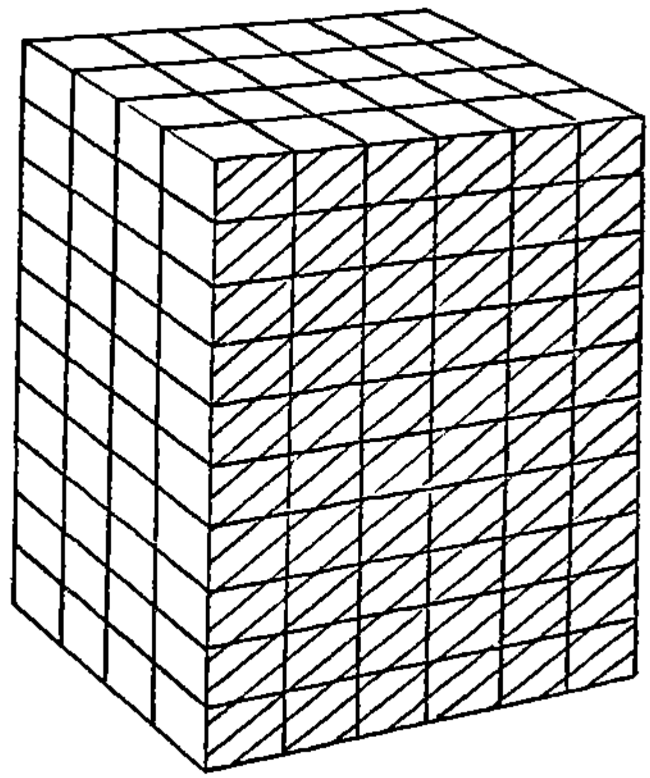
v. What happens to the volume if we double one dimension and halve another dimension?

4. (10 min) Worksheet:

Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Poster of Lesson A4

No. 4.1



Name:Last:________________, First:____________

Lesson A4-Worksheet

Complete the following:

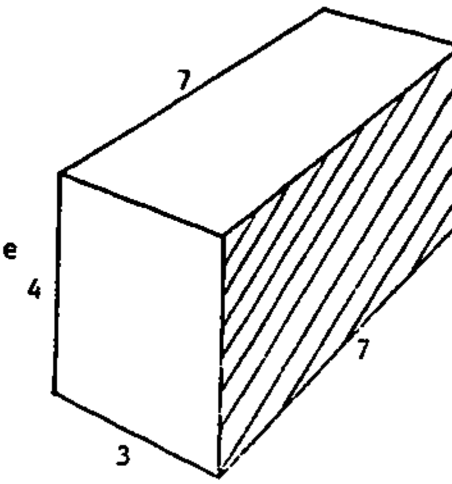
1. A plastic box.
   If we increased the width of this box by three units and the length and the height stayed the same, then in the new box what would be the

   Length? ___

   Width? ___

   Height? ___

   Volume? ______



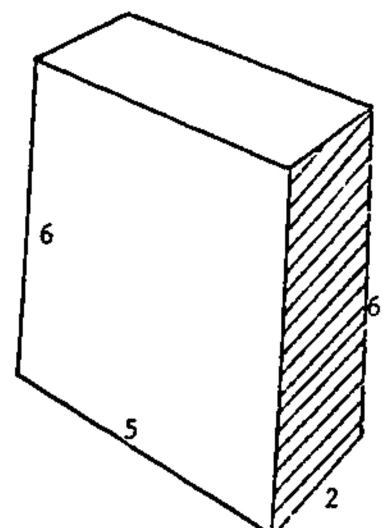
2. A metal box.
   If we decreased the height by 3 units and the the length by 2 units but the width stayed the same, then in the new box what would be the

   Length? ___

   Width? ___

   Height? ___

   Volume? ______



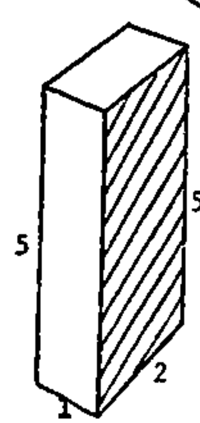
3. A wooden box.
   If we decreased the length by 1 unit and we increased the height by 2 units but the width stayed the same, then in the new box what would be the

   Length? ___

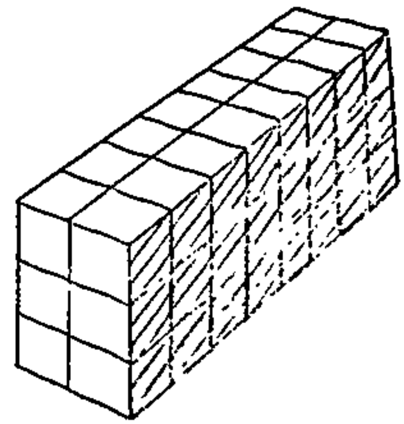
   Width? ___

   Height? ___

   Volume? ______



4. A rectangular pile of cubes.
If we tripled the height of this pile and the length and the width stayed the same, then in the new pile what would be the



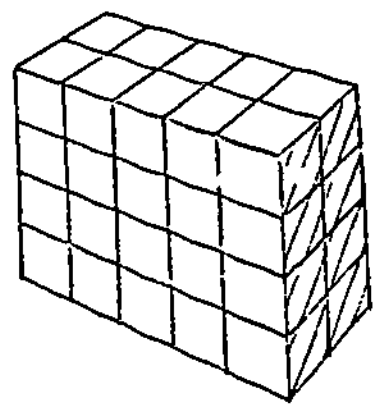
Length? ___

Width? ___

Height? ___

Volume? _____

5. A rectangular pile of cubes.
If we doubled the length and halved the width but the height stayed the same, then in the new pile what would be the



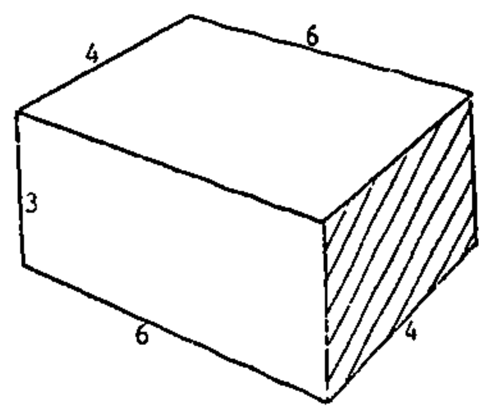
Length? ___

Width? ___

Height? ___

Volume? _____

6. A cardboard box full of cubes.
If we multiplied the height by 3 and divided the length by 6 and the width by 2, then in the new box what would be the



Length? ___

Width? ___

Height? ___

Volume? _____

Treatment B

Lesson 1

Behavioral Objectives: a, b and n denote natural numbers.

1. Given a multiplication equation with the same three factors on each side of the equal sign, such that the factors on both sides are not in the same order and that one or two factors of one side are missing, the students will be able to state the missing factors.

2. Given an assertion of the form $a \times b > a \times c$, $a \times b < a \times c$, $c \times a > b \times a$ or $c \times a < b \times a$ such that a is known, but only one of b or c is known, the students will be able to ascribe correct limits to the range of values for the unknown number.

3. Given an assertion of the form $a \times b \times n > a \times c \times n$ or $n \times c \times a > n \times b \times a$ such that n and a are known and only one of b or c is known, the students will able to ascribe correct limits to the range of values for the unknown number to make the assertion true.

Outline:

1. Review of the commutative and associative principles.

2. Prescribing the range for the missing factor in an inequality involving two factors at each of its sides.

3. Prescribing the range for the missing factor in an inequality involving three factors at each of its sides

4. Worksheet.

Materials:

1. A poster of 3×5 grid.

Activities:

1.a. (5 min) Review of the commutative principle:

Present the poster (#1.1) of 3 X 5 grid and ask the students to tell the number of rows (3) and the number of squares per row (5). Ask the students to give a multiplication sentence to describe the total number of squares (3 X 5 = 15). Write "3 X 5 = 15" on the board. Declare that the numbers 3 and 5 are called the factors while 15 is called the product.

Turn the poster 90 degrees and ask similar questions to the ones in the previous paragraph. Conclude that the sentence describing the total number of squares is now "5 X 3 = 15". Write "5 X 3 = 15" on the board underneath "3 X 5 = 15". Lead the discussion to illustrate that both "3 X 5" and "5 X 3" describe the same number of squares and are therefore equivalent. Write the statement "3 X 5 = 5 X 3" on the board.

Ask the students to make a generalization about the order of the factors based on the statement "3 X 5 = 5 X 3". Allow time for responses and emphasize the commutative principle (without mentioning the term "commutative") i.e., the order of multiplying the factors does not affect the product. Test this principle using 3 X 4 and 4 X 3 by determining the answer to 3 X 4 and 4 X 3. Confirm the commutative principle by asking the students to compute the answer (180) to each side of the sentence 12 X 15 = 15 X 12.

1.b. (5 min) Review of the associative principle:

Write "2 X 3 X 4" on the board and ask the students if it makes any difference in choosing the numbers to be multiplied first, 2 X 3 or 3 X 4. Then write (2 X 3) X 4 = ____ and 2 X (3 X 4) = ____ on the board. Ask the students to help in the steps for finding the answers and conclude that one may multiply the first two factors first or the last two first. Write on the board (2 X 3) X 4 = 2 X (3 X 4). Ask the students to make a generalization about the order of multiplying the factors based on the sentence written on the board. Allow time for responses and emphasize the associative principle (without mentioning the term "associative") for the product of any three factors i.e., the grouping of the factors does not affect the product. Test and confirm this principle using "5 X 3 X 8".

Write the following statements on the board and ask the students to find the numbers which complete the statements.

____ X 8 X 11 = 8 X 6 X 11;   7 X 12 X 4 = ____ X 7 X ____

2. (10 min) Prescribing the range for the missing factor in an inequality involving two factors at each of its sides.

Write on the board "3 X [ ] > 3 X 5" and follow the discussion below in order to lead the students to prescribe whole numbers that can fit in the [ ].

T: If we replace [ ] by 10, will it be true?
(Response: "Yes". Write 10 on the board underneath [ ])

T: Is there a number greater than 10 that would make it true? If so, tell me such a number?
(Write pupils' responses in ascending order. Leave spaces for contingencies)

T: What if we replace [ ] by 1, will it be true?
(Response: "No". Ask how we tell)

T: Are there any numbers less than 10 that would make it true? If so, which numbers?
(Write them with the sequence in order)

T: What is the least number we have been able to replace [ ] by, so far? Does that mean that 6 is the least whole number we can use? Can anyone name a smaller whole number than 6 to make it true? Why not?

T: (Point out the "gap" in the sequence on the board which probably looks like this:

Numbers that make it true: 6, 8, 10, 64, 100)

Are there any numbers between these two (point to 6 and 8) that make it true? How about here? (point to 10 and 64) etc.

T: Could we possibly list all of the numbers that make it true? Why not?

T: So without actually listing them all, what could we write that would clearly indicate all of the numbers that make it true? Could this do? (write on the board "Any number greater than 5" or "n > 5")

Similarly let the students prescribe answers to the following statements.

3 X 8 < [ ] X 8

12 X 15 > [ ] X 12

3. (5 min) <u>Prescribing the range for the missing factor in an inequality involving three factors at each of its sides:</u>

Write the statement "2 X 3 X 6 < 2 X [ ] X 6" on the board and follow the steps below in order to lead the students to prescribe numbers that can fit in the [ ]. (Answer: "Any number greater than 3" or "n > 3").

3.1. Determine a number, say 10, that makes the statement true.

3.2. Determine a number, say 2, that makes the statement false.

3.3. Establish a lower bound; Find all the numbers less than 10 which make the statement true.

3.4. Determine a larger number than 10, say 50, which makes the statement true.

3.5. Determine two or three numbers between 10 and 50 that make the statement true.

3.6. Establish that it is impossible to list all the numbers that make the statement true.

3.7. Write a sentence that prescribes all the numbers that make the statement true. ("Any number greater than 3" or "n > 3").

Similarly let the students prescribe numbers that fit in the [ ] in order to make the statement "6 X 2 X [ ] < 6 X 2 X 20" true.

4. (10 min) <u>Worksheet:</u>

Give each of the students a copy of the attached worksheet and explain it to them. Move around and help them to complete it. At the end of the period collect the worksheets.

Poster of Lesson B1

No. 1.1

Name: Last: _________________, First:____________

Lesson B1 - Worksheet

Complete the following:

1. 5 X 7 X 12 = 12 X 5 X ___

2. 9 X (14 X 10) = 10 X (___ X ___)

3. (23 X 7) X 13 = (___ X 13) X ___

4. 7 X 9 > 7 X [ ]
   What number or numbers could go in the [ ]?

   ____

5. 53 X 16 > 16 X [ ]
   What number or numbers could go in the [ ]?

   ____

6. 22 X 25 X 26 < 22 X [ ] X 26
   What number or numbers could go in the [ ]?

   ____

7. 213 X 19 X [ ] < 213 X 19 X 6
   What number or numbers could go in the [ ]?

   ____

In each of the following write: <, = or > in the ◯.

8. 5 X 4 ◯ 4 X 5

9. 6 X 4 ◯ 6 X 5

10. 9 X 3 ◯ 3 X 8

11. 4 X (2 X 3) ◯ (4 X 2) X 3

12. 9 X (8 X 13) ◯ (8 X 13) X 9

Treatment B

Lesson 2

Behavioral Objectives: a, b, c and P denote natural numbers.

1. Given an assertion of the form a X b = P where a, b and P are known, the students will be able to describe the effect on P when any one of the following changes is applied additively or multiplicatively:
   - i. Increase in a, or b, or both.
   - ii. Decrease in a, or b, or both.
   - iii. Increases in a together with decrease in b (or vice versa).

2. Given an assertion of the form a X b X c = P the students will be able to describe the effect on P when any one of the following changes is applied additively or multiplicatively to the factors.
   - i. Increase in a, b or c
   - ii. Decrease in a, b or c
   - iii. Increase in one or two of the factors a, b or c and decrease in two or one of the factors a, b or c

Outline:

1. Follow up of the worksheet from the previous lesson.

2. Effect on the product of two factors when these factors are changed additively or multiplicatively. (Note: "additively" and "multiplicatively" will subsume decrease as well as increase.)

3. Effect on the product of three factors when these factors are changed additively or multiplicatively. (Note: "additively" and "multiplicatively" will subsume decrease as well as increase.)

4. Worksheet

Materials:

(The activities of this lesson are mainly number manipulations and do not require physical materials)

Activities:

1. (4 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain that in #4, any number less than 9 can go in the [ ] and make the statement true. Any number greater than or equal to 9 will make the statement false. Similarly, in #7 any number less than 6 will make the statement true and any number greater than or equal to 6 will make the statement false.

2. (20 min) Effect on the product of two factors when these factors are changed additively or multiplicatively.

Write on the board the statement "6 X 4 = 24". Apply the following changes to the factors (4 and 6) and observe the changes in the product (24). Encourage the students to state and test conjectures about the effect on the product when the factors are changed.

2.1. Additive (or multiplicative) increase in one or two of the factors produces additive (or multiplicative) increase in the product.

2.1.1. Additive increase in one or two of the factors:

Ask the students to predict what happens to the product if the factor 6 is replaced by a larger number, say 7 or 8 (desirable response: "It increases"). Allow time for responses and write on the board "7 X 4 = 28" underneath "6 X 4 = 24."

Continue by asking the students to predict what happens to the product (24) if any one of the factors increases. When conjectures are made test them using examples such as "8 X 4 = 32" and "6 X 7 = 42." Lead the discussion in order to conclude that the product increases if any one of the factors increases. Write on the board

6	X	4	=	24
↓		↓		↓
increases		stays the same		increases

Similarly, ask the students to predict what happens to the product (24) if both of the factors increase. Allow time for responses and test them using examples such as "7 X 5 = 35" and "10 X 6 = 60." Lead the discussion in order to conclude that the product increases if both of the factors increase. Write on the board

```
              6   X   4    =    24
              ↓       ↓         ↓
     increases   increases   increases
```

2.1.2. <u>Multiplicative increase in one or two of the factors:</u>

Ask the students to predict what happens to the product (24) if any one of the factors is multiplied by 2 (doubled). Allow time for responses and ask two groups to test them using "12 X 4 = ____ (answer: 48)" and "6 X 8 = ____ (answer: 48)." Conclude that the product is multiplied by 2 (doubled). Continue by asking the students to predict what happens to the product (24) if any one of the factors is multiplied by 3 (tripled) or 4. Test the conjectures using:

6 X 4 = 24 (original statement)
6 X 12 = 72 = 3 X 24 and
24 X 4 = 96 = 4 X 24.

Lead the discussion in order to conclude that the product is multiplied by 2 (doubled), 3 (tripled), ... if any one of the factors is multiplied by 2 (doubled), 3 (tripled), ... Write on the board

```
              6   X   4    =    24
              ↓       ↓         ↓
         stays    is            is
      the same multiplied    multiplied
                  by 3          by 3
```

Likewise, ask the students to predict what happens to the product (24) if one of the two factors is multiplied by 2 and the other by 3, etc. Use examples such as the one written below to establish that if one factor is multiplied by a whole number and another by a whole number, the product will be multiplied by the product of the two numbers.

6 X 4 = 24 (original statement)
30 X 8 = 240 = 10 X 24

2.2. <u>Additive (or multiplicative) decrease in one or two of the factors produces additive (or multiplicative) decrease in the product.</u> (Move quickly through this section)

Model the discussion of this section after the discussion of the previous section (2.1). For each of the generalizations below (lettered a, b, ...) ask the students to predict what happens to the product (24) if the proposed changes in the factors are applied, test the students' predictions using the given examples and conclude the generalization.

2.2.1. <u>Additive decrease in one or two of the factors:</u> (move quickly through this section)

a. The product decreases if any one of the factors decreases.
Examples: 5 X 4 = 20; 6 X 3 = 18.

Write on the board

6 X 4 = 24
↓ ↓ ↓
stays the same — decreases — decreases

b. The product decreases if both of the factors decrease.
Example: 5 X 3 = 15.

Write on the board

6 X 4 = 24
↓ ↓ ↓
decreases — decreases — decreases

2.2.2. <u>Multiplicative decrease in one, two or three of the factors:</u> (move quickly through this section)

a. The product will be divided by 2 (halved), 3, ... if any one of the factors is divided by 2, 3, ...

Example:
6 X 2 = 12 = 24/2

Write on the board

6 X 4 = 24
↓ ↓ ↓
stays the same — is divided by 2 — is divided by 2

B. The product (24) will be divided by the product of two numbers if one of the factors is divided by one of the numbers and another factor is divided by the other number.

Examples:
3 X 2 = 6 = 24/4
2 X 1 = 2 = 24/12

write on the board

```
   6   X   4    =   24
   ↓       ↓        ↓
is       is       is
divided  divided  divided
by 3     by 4     by 12
```

2.3. Additive (or multiplicative) increase in one of the factors and additive (or multiplicative) decrease in the other factor can produce any one of the following:

a. Additive (or multiplicative) increase in the product
b. Additive (or multiplicative) decrease in the product
c. No change in the product

2.3.1. Additive increase in one of the factors and additive decrease in the other factor:

Ask the students whether they can predict what happens to the product if one of the factors increases and the other decreases. Allow time for predictions and ask three groups to test them using the following examples:

6 X 4 = 24 (original statement)
5 X 9 = _____ (45, the product increases)
7 X 3 = _____ (21, the product decreases)
8 X 3 = _____ (24, the product does not change)

Conclude that the product may increase, decrease or stay the same if one factor increases and the other decreases: each example has to be examined individually.

Write on the board

```
       6      X   4     =    24
       ↓          ↓          ↓
increases   decreases   increases, decreases
                        or stays the same
```

3. (4 min) Effect on the product of three factors when these factors are changed additively or multiplicatively: (move quickly through this section)

Write of the board the statement "4 X 6 X 10 = 240" and ask the students whether they can predict the change in the product (240) of three factors (4, 6, 10) when these factors vary. Ask this following series of questions and encourage students to answer them by making generalizations. When necessary test the generalizations using numerical examples.

i. What happens to the product if we increase one, two or three of the factors? (answer: it increases)

ii. What happens to the product if we decrease one, two or three of the factors? (answer: it decreases)

iii. What happens to the product if we increase one of the factors and decrease the other? (answer: we can't tell; it may increase, decrease or stay the same. Each example has to be examined individually)

iv. a. What happens to the product if we double only one factor?
b. What happens to the product if we double two factors?
c. What happens to the product if we double all three factors?

(answers: it will be multiplied by 2, 4 or 8).

v. What happens to the product if we multiply one factor by 6 and divide another by 2? (answer: it will be multiplied by 6 and divided by 2.

4. (7 min) <u>Worksheet:</u>

Give each of the students a copy of the attached worksheet and explain it to them. Warn the students that they may not have enough time to finish the work and ask them to do as much as they can. Move around and help them to complete it. At the end of the period collect the worksheets.

Name:Last:_________________, First:________________

Lesson B2-Worksheet

Complete the following:

1. 16 X 24 = 384

16 ↓ decreases; 24 ↓ stays the same; 384 ↓ ?

If 16 were replaced by a smaller number and 24 stayed the same, what should happen to the product?

____

2. 24 X 15 = 360

24 ↓ increases; 15 ↓ increases; 360 ↓ ?

If 24 were replaced by a larger number and 15 were replaced by a larger number, what should happen to the product?

____

3. 35 X 28 = 980

35 ↓ becomes twice as big; 28 ↓ becomes 3 times as big; 980 ↓ ?

If 35 were replaced by a number twice as big and 28 were replaced by a number three times as big, what should happen to the product?

____

4.

17	X	9	X	14	=	2142
↓		↓		↓		↓
stays the same		decreases		decreases		?

If 9 were replaced by a smaller number, 14 were replaced by a smaller number and 17 stayed the same, what should happen to the product?

5.

3	X	36	X	13	=	1404
↓		↓		↓		↓
become twice as big		becomes one-third as big		becomes 3 times as big		?

If 3 were replaced by a number twice as big, 13 were replaced by a number three times as big and 36 were replaced by a number one-third as big, what should happen to the product?

Treatment B

Lesson 3

Behavioral Objectives: a, b, c and P denote natural numbers.

1. Given an assertion of the form a X b = P where a and b are known and given a conditional statement that P remains fixed while one factor is replaced by one of its multiples or divisors, the students will be able to anticipate a suitable replacement for b.

2. Given an assertion of the form a X b X c = P such that all variables are known and given a conditional statement that P remains fixed while one or two of the factors are replaced by their multiples or divisors, the students will be able to anticipate suitable replacement for the remaining factor or factors.

Outline:

1. Follow up of the worksheet from the previous lesson.

2. Effect on one of two factors when the other factor is changed and the product is fixed

3. Effect on two (or one) of the three factors when one (or two) of these factors is (are) changed and the product is fixed.

4. Worksheet

Materials:

(The activities of this lesson are mainly number manipulations and do not require physical materials)

Activities:

1. (4 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain how one can obtain the correct answers to #2 and #5.

For #2 write "24 X 15 = 360" on the board and explain that if 24 and 15 were replaced by larger numbers the product would increase. Illustrate by writing "30 X 20 = 600" underneath "24 X 15 = 360."

Similarly for #5, write 3 X 36 X 13 = 1404 then 6 X 12 X 39 = ______. Explain that the product will be multiplied by 2 X 3 and divided by 3. Therefore the number to fill in the blank will be 2 X 1404 = 2808.

2. (8 min) Effect on one of two factors when the other factor is changed and the product is fixed:

Write on the board the statement "9 X 6 = 54" and underneath it write "18 X [ ] = 54" and tell the students that in the examples on the board, the product 54 was fixed while the factor 9 was doubled to 18. Ask the students to suggest a numbers that can go in the [ ] to make the statement true (answer: 3). Continue by telling the students that 9 was multiplied by 2 to get 18 and ask them to predict what happened to 6 to get 3. Help the students conclude that 6 was divided by 2 since 9 was multiplied by 2.

Similarly write on the board "5 X 8 = 40" and underneath it write "10 X [ ] = 40". Ask the students to guess how 10 was obtained from 5 (multiplied by 2 or doubled) and to predict what would happen to 8 in order to to get the same product 40 (divide it by 2 or halve it). Allow time for respouses and replace "[ ]" by "4".

Write on the board "4 X 6 = 24" and ask the students to predict what happens to one of the factors if the product (24) is fixed and the other factor is multiplied by 2. Test the students' predictions using "8 X [ ] = 24" and "[ ] X 12 = 24". Lead the students to conclude that if the product is fixed and one of the factors is multiplied by 2 (doubled) the other should be divided by 2 (halved).

Similarly, refer to "3 X 12 = 36" and ask the students to predict what happens to one of the factors if the product is fixed and one of the factors is divided by 3. Test the students' predictions using "1 X [ ] = 36" and "[ ] X 4 = 36". Lead the students to conclude that if the product is fixed and one of the factors is divided by 3 the other factor should be multiplied by 3.

Write on the board "9 X 8 = 72" and ask the students to predict what should happen to one of the factors if the product is fixed and the other factor is multiplied or divided by a number. Allow time for responses and test them using:

3 X [ ] = 72	(answer: 3 X [24] = 72)
18 X [ ] = 72	(answer: 18 X [4] = 72)
[ ] X 72 = 72	(answer: [1] X 72 = 72)
[ ] X 2 = 72	(answer: [36] X 2 = 72)

Help the students generalize that if the product is fixed and one of the factors is multiplied (divided) by a number the other factor should be divided (multiplied) by the same number.

3. (13 min) <u>Effect on two (or one) of the three factors when one (or two) of these factors is (are) changed multiplicatively and the product is fixed.</u>

3.1. <u>Multiplicative increase in one of the factors and multiplicative decrease in another and vice versa</u> (move quickly through this section).

Write of the board the statement "4 X 6 X 10 = 240" and underneath it write "8 X [ ] X 10 = 240". Tell the students that the product 240 is fixed and one of the factors 10 is also fixed while another factor 4 was multiplied by 2 to become 8. Ask the students to predict what should happen to 6 in order to find the number (3) that should go in [ ] and make the statement true. Test the students' prediction by replacing the suggested number in [ ] and complete the statement. Conclude that 6 was divided by 2.

Refer to the statement "4 X 6 X 10 = 240" and ask the students to predict what happens to one of the factors if the product and one of the factors are fixed while a factor is multiplied by a number. Ask the students to test their predictions using:

12 X [ ] X 10 = 240	(answer: 12 X [2] X 10 = 240)
[ ] X 6 X 20 = 240	(answer: [2] X 6 X 20 = 240)
4 X 1 X [ ] = 240	(answer: 4 X 1 X [60] = 240)

Help the students to conclude that in this case if one factor is multiplied by a number, the other should be divided by the same number. Similarly, ask the students to predict what happens to one of the factors if another factor is divided by a number while the third factor and the product are fixed. Ask the students to test their predictions using

2 X [ ] X 10 = 240	(answer: 2 X [12] X 10 = 240)
[ ] X 2 X 10 = 240	(answer: [12] X 2 X 10 = 240)
4 X 1 X [ ] = 240	(answer: 4 X 1 X [60] = 240)

3.2 Increase in two of the factors and decrease in one and vice versa

Refer to "4 X 6 X 10 = 240" and tell the students that the product 240 is fixed while 6 is multiplied by 2 and 10 is multiplied by 2. Write "[ ] X 12 X 20 = 240" underneath "4 X 6 X 10 = 240" and ask the students to predict what should happen to the factor 4. Test the students' prediction and lead the discussion to conclude that since two factors were multiplied by 2 each and the product is fixed the third factor should be divided by 2 X 2 or 4.

Similarly, ask the students to predict what should happen to the factor 6 if 10 was divided by 5 and 4 by 2. Test the predictions and help the students to conclude that the factor 6 should be multiplied by the product 5 X 2 (or 10). Test this conclusion using

2 X [ ] X 2 = 240 (answer:2 X [60] X 2 = 240)

Write "4 X 12 X 8 = 384" and tell the students that the product 384 is fixed and the factor 4 is multiplied by 4 to get 16. Write "16 X [ ] X ( ) = 384" underneath "4 X 12 X 8 = 384." Ask the students to predict what should happen to 12 or to 8 or to 12 <u>and</u> 8 in order to make the statement true. Test the predictions and lead the students to conclude that since one of the factors was multiplied by 4 any one of the following could be done:

a. divide any other factor by 4
b. divide each of the factors by 2.

Similarly, refer to "4 X 12 X 8 = 384" and tell the students that the product 384 is fixed and the factor 12 is divided by 6 to get 2. Write "[ ] X 2 X ( ) = 384" underneath "4 X 12 X 8 = 384." Ask the students to predict what should happen to 4 or to 8 or to 4 <u>and</u> 8 in order to make the statement true. Test the predictions and lead the students to conclude that since one of the factors was divided by 6 any one of the following could be done

a. multiply any other factor by 6
b. multiply one of the factors by 2 and the other by 3.

4. (10 min) <u>Worksheet</u>

Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Name: Last:________________, First:___________

Lesson B3 - Worksheet

Complete the following:

1. 8 X 5 = 40
   [ ] X 10 = 40
   The product 40 stayed the same; 5 was replaced by 10. What number should go in the [ ]? _____

2. 9 X 15 = 135
   27 X [ ] = 135
   The product 135 stayed the same; 9 was replaced by 27. What number should go in the [ ]? _____

3. 34 X 12 X 9 = 3672
   34 X [ ] X 36 = 3672
   The product 3672 stayed the same; 34 stayed the same; 9 was replaced by 36. What number should go in the [ ]?
   _____

4. 11 X 15 X 22 = 3630
   33 X [ ] X 22 = 3630
   The product 3630 stayed the same; 22 stayed the same; 11 was replaced by 33. What number should go in the [ ]?
   _____

5. 26 X 45 = 1170
   13 X [ ] = 1170
   If the product 1170 stays the same and 26 is replaced by 13, what number should replace 45? _____

6. 28 X 8 X 23 = 5152
   [ ] X 32 X 23 = 5152
   If the product 5152 stays the same, and 23 stays the same and 8 is replaced by 32, what number should replace 28?
   _____

7. 18 X 5 X 32 = 2880
   6 X [ ] X 16 = 2880
   If the product 2880 stays the same, and 18 is replaced by 6 and 32 is replaced by 16, what number should replace 5?
   _____

Treatment B

Lesson 4

Behavioral Objectives:

1. Given a cuboid or a diagram of a cuboid, which is non-partitioned, partially partitioned or completely partitioned into unit cubes, the students will be able to use the volume algorithm "V = L X W X H" in order to determine the volume

2. Given a diagram of a partitioned cuboid which is partially covered the students will be able to determine the total volume of the cuboid.

3. Given a diagram or a word description of a cuboid of known dimensions and given a proposed additive or multiplicative dimensional transformation the students will be able to state the volume of the cuboid that would result after the transformation.

Outline:

1. Follow up of the worksheet from the previous lesson.

2. Clarification of the concept of volume.

3. Algorithm for the volume of a cuboid "V = L X W X H."

4. Application of the volume algorithm to partitioned, partially partitioned and partially covered cuboids.

5. Application of the volume algorithm to cuboids with proposed dimensional transformations.

6. Worksheet.

Materials:

1. Cardboard boxes.
2. Decimetre cubes.

3. A poster of polyhedral models built from unit cubes.
4. Posters of partitioned, partially partitioned, non-partitioned and partially covered cuboids.

Activities:

1. (4 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain how one can obtain the answers to #5 and #7.

For #5, explain that 26 was divided by 2 in order to get 13. Therefore 45 should be multiplied by 2 in order to get the same product, 1170. Check by writing "13 X [90] = ______" and computing the answer (1170). Write "1170" in the blank.

For #7, explain that 18 was divided by 3 to get 6, 32 was divided by 2 to get 16. Therefore 5 should be multiplied by 3 X 2 = 6 in order to get the same product, 2880. Thus 5 should be multiplied by 30. Check by writing 6 X [30] X 16 = ______ and computing the answer (2880). Write "2880" in the blank.

2. (5 min) Clarification of the concept of volume:

Display the two closed cardboard boxes A and B of sizes 6 cm X 4 cm X 2 cm and 40 cm X 20 cm X 10 cm respectively. Ask the students to guess which is bigger, which occupies more space and which has the greater volume. Conclude that box B is bigger than box A and that any one of the following sentences describes this fact.

a. Box B is bigger than box A
b. Box B takes up more room than box A
c. Box B occupies more space than box A
d. Box B has a larger volume than box A.

Display a closed cardboard box in each of two distant locations of the classroom (use box #1 and box #2) and ask the students to compare the volumes of the boxes without moving them. Lead the discussion in order to conclude that perception may be deceiving; determine and compare the volumes using the following activities:

a. Use decimetre cubes as units to build a cuboid next to each box with the same shape and volume as that of the box.
b. Count the number of units and write the volumes of the boxes on the board.
c. Compare the volumes of the boxes using the numbers of units found in b.

Stress that in this process, any units of the same size (in this case decimetre cubes) could be used, but the same units must be used throughout.

3. (5 min) Algorithm for the volume of a cuboid "V = L X W X H":

Display a poster (#4.1) of a partitioned cuboid whose dimensions are 6, 3, and 4. Remind the students that the height is always the vertical dimension, the length is the longer of the two horizontal dimensions and the width is the shorter of the two horizontal dimensions. Discuss with the students how one can determine the volume of the cuboid. Lead the activities in order to determine:

a. The number of cubes along the length of the top layer. Write on the board "L(length) = 6 " i.e., 6 cubes fit along the length of the top layer.

b. The number of cubes along the width of the top layer. Write on the board "W(width) = 3 "

c. Conclude that the number of cubes that can fit in the top layer is given by L X W.

d. The number of cubes along the height of the box. Write on the board "H(height) = 4 ". This is the number of layers.

e. Conclude that the total volume is the volume of one layer (L X W) multiplied by the number of layers (H) i.e., V = L X W X H.

4. (5 min) Application of the volume algorithm to the following cases:

a. Partially partitioned and non-partitioned cuboids:

Refer to the partially partitioned and the non-partitioned cuboids on the poster (#4.2). In each case determine the volume of each layer, the number of layers and the total volume. Use the algorithm "V = L X W X H" to compute the volume. Compare the two answers.

b. Partially covered cuboids:

Present the poster (#4.3) of a partially covered cuboid (4X3X6). Point out that the blocks of the top layer are shown and there is a total of 6 layers. Determine the length, width and height of the cuboid and use the algorithm to compute the volume. Count the number of blocks in the top layer (12) and conclude that each of the 6 layers has 12 blocks. Determine the total volume (6x12). Compare the two answers.

5. (7 min) Application of the volume algorithm to cuboids with proposed dimensional transformations:

Write on the board the statement 6 X 4 X 10 = 240. Review with the students how some of the changes in the factors (6,4,10) affect the product (240) by asking this series of questions and encouraging the students to make generalizations.

i. What happens to the product if we increase one, two or three of the factors?

ii. What happens to the product if we decrease one, two or three of the factors?

iii. a. What happens to the product if we double only one factor?
b. What happens to the product if we double two factors?
c. What happens to the product if we double all three factors?

iv. What happens to the product if we double one factor and halve another factor?

Display a poster (#4.4) of a partitioned cuboid of dimensions 6, 4, and 10. Ask the students to state the algorithm for finding the volume of the cuboid and write on the board V = L X W X H. Replace L, W, and H by 6, 4 and 10 respectively and compute the volume (240). Write on the board 6 X 4 X 10 = 240 underneath V = L X W X H.

Ask the students to predict the changes in the volume (240) when L (6), W (4), and H (10) vary. Help the students make the generalizations by asking them this series of questions.

i. What happens to the volume if we increase one, two or three of the dimensions?

ii. What happens to the volume if we decrease one, two or three of the dimensions?

iii. What happens to the volume if we increase one of the dimensions and decrease another?

iv. a. What happens to the volume if we double only one dimension?
b. What happens to the volume if we double two dimensions?
c. What happens to the volume if we double all three dimensions?

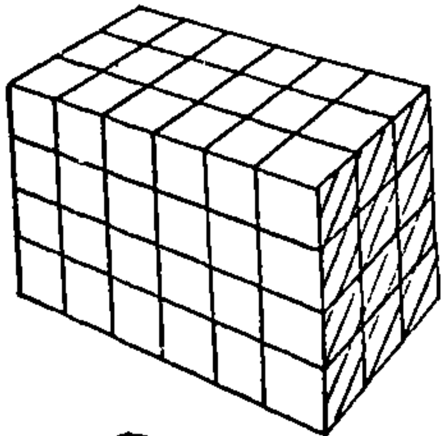
v. What happens to the volume if we double one dimension and halve another dimension?

6. (9 min) Worksheet:

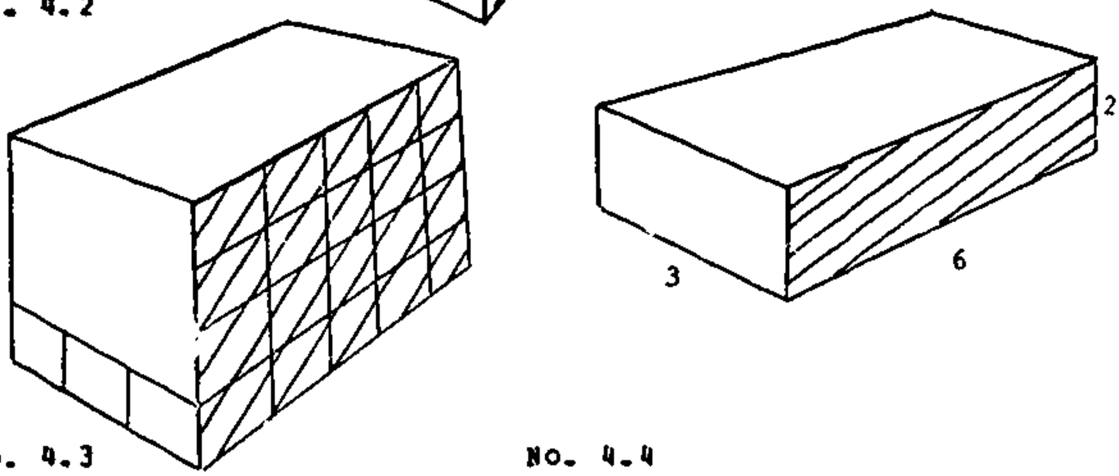
Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Posters of Lesson B-4

No. 4.1



No. 4.2



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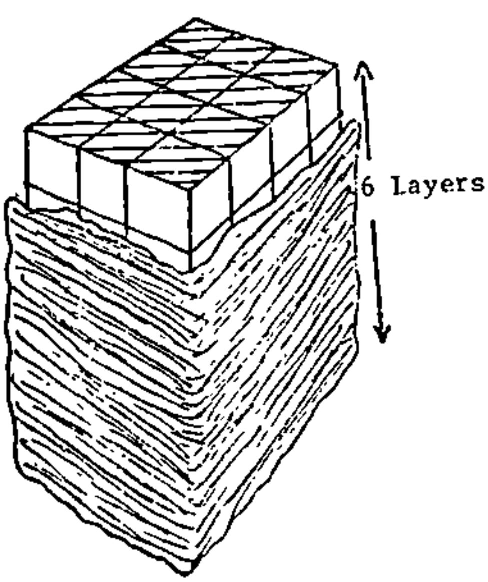
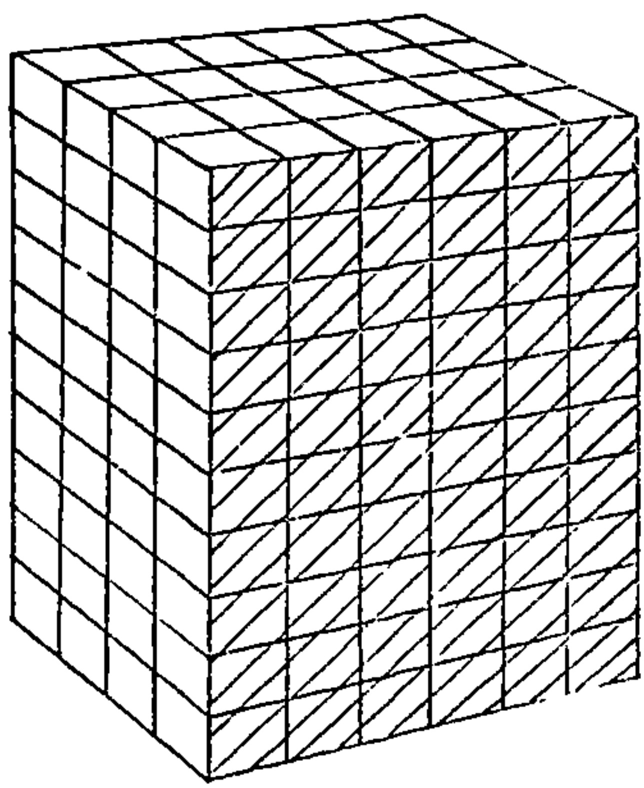


No. 4.4



Name:Last:_________________, First:___________

Lesson D4-Worksheet

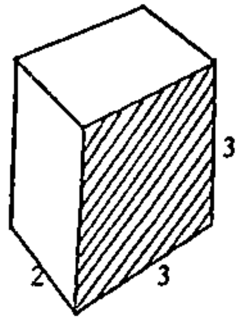
Complete the following:

1. L = ______

   W = ______

   H = ______

   V = ______

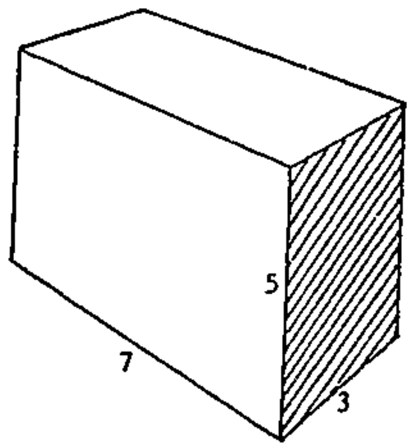


2. L = ______

   W = ______

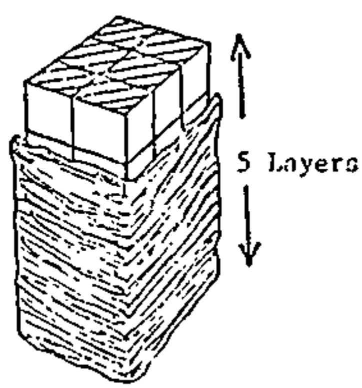
   H = ______

   V = ______

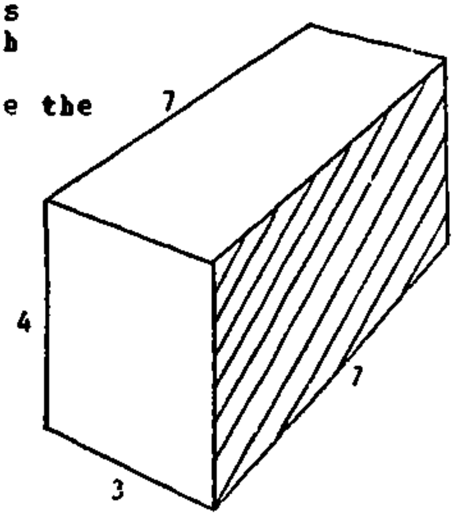


3. A pile of cubes partially covered.

   Total volume = ______



4. A plastic box.
If we increased the width of this box by three units and the length and the height stayed the same, then in the new box what would be the



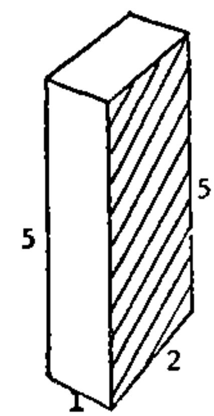
Length? ___

Width? ___

Height? ___

Volume? ______

5. A wooden box.
If we decreased the length by 1 unit and we increased the height by 2 units but the width stayed the same, then in the new box what would be the



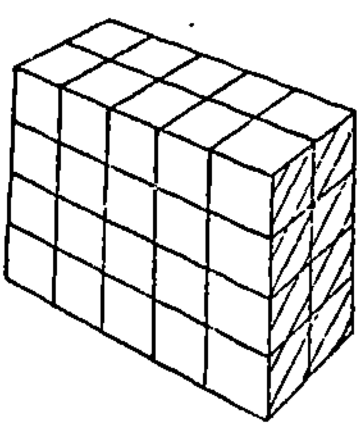
Length? ___

Width? ___

Height? ___

Volume? ______

6. A rectangular pile of cubes.
If we doubled the length and halved the width but the height stayed the same, then in the new pile what would be the



Length? ___

Width? ___

Height? ___

Volume? ______

Treatment C

Lesson 1

Behavioral Objectives:

1. Given a numeral in Base 10 for a number n ($n < 10^5$), the students will be able to write the numeral in expanded notation using either of the forms illustrated below:

a. 324 = 3 hundreds + 2 tens + 4 ones
b. 324 = 3 X 100 + 2 X 10 + 4

2. Given a numeral in Base 10 for a number n ($n \leq 124$), the students will be able, with manipulative aids, to convert the numeral to the equivalent numeral in Base 5.

Outline:

1. Review of Base 10 place value concepts

2. Bundling in fives and expressing numbers in Base 5

Materials:

1. Popsicle sticks
2. Twist ties
3. Base 10/Base 5 table
4. Base 10 & Base 5 mats: a paper on one side of which there are three large columns with headings: ten-tens, tens and ones; the other side has three large columns with headings: five-fives, fives and ones.

Activities:

Give each student 45 popsicle sticks, 5 twist ties, a Base 10 and a Base 5 mat and a copy of the Base 10/Base 5 table illustrated below. Ask the students to lay aside the handouts, the sticks and the twist ties because they will be used later in the period.

expanded form			short form	short form	expanded form		
ten-tens	tens	ones			five-fives	fives	ones

1. (15 min) Review of Base 10 place value concepts:

Write "111" on the board then ask the students to read this number. Give pupils opportunity to respond then ask about the relationship of the "1" in the tens place to the "1" in the ones place and the "1" in the hundreds place. Use popsicle sticks (1 stick, 1 bundle of 10, and 1 bundle of 10 tens) in order to illustrate that '1' in the tens place means 10 times as much as the '1' in the ones place; the '1' in the 100's place means 10 times as much as the '1' in the 10's place and 100 times as much as the '1' in the ones place. Conclude that "111 = (100) + (10) + 1".

Write "444 = ______ hundreds + _____ tens + _____ ones" and then ask the children to suggest numbers that will make the assertion true. Review with the children that:
444 = 4 hundreds + 4 tens + 4 ones.
444 = (4 x 100) + (4 x 10) + 4.

Repeat using "2795" in order to conclude that :
2795 = (2 x 1000) + (7 x 100) + (9 x 10) + 5.

2. (20 min) Bundling in fives and expressing numbers in Base 5:

Tell the students that after what they have just done the following may seem to be ridiculously easy. Give the students the following instructions and make sure that each student completes the work.

a. Count out 14 sticks.

b. Group the sticks in tens and ones. Use the twist ties to bundle each group of ten.

c. Place the bundles (1) and sticks (4) in the proper sections on the Base 10 mat.

d. Enter in the table on the left what the bundles show.

e. Enter in the table on the left the short form of what the bundles show.

f. Unbundle the sticks.

g. Regroup the sticks in fives and ones using the twist ties.

h. Place the bundles (2) and the sticks (4) in the proper sections on the Base 5 mat.

i. Enter in the table on the right what the bundles show.

j. Enter in the table on the right the short form of what the bundles show.

Repeat the above steps of instruction for "thirty-two" and "twenty- six"; in step g say that as soon as 5 bundles are made they should be bundled into a larger bundle of five-fives.

Tell the students that when they bundled in fives the written numbers were in Base 5, just as when they bundled in tens the numbers were in Base 10. Show the equivalence of the numerals by using the numerals written on the table and writing the following:

14 (Base 10) = 24 (Base 5)

22 (Base 10) = 42 (Base 5)

32 (Base 10) = 112 (Base 5)

26 (Base 10) = 101 (Base 5)

Treatment C

Lesson 2

Behavioral Objectives:

1. Given a numeral in Base 10 for a number n ($n \leq 124$), the students will be able, with and without manipulative aids, to convert the numeral to the equivalent numeral in Base 5.

2. Given a number no greater than 124 (Base 10) which is suggested by a given real life situation, the students will be able to write the equivalent Base 5 numeral for that given number.

Outline:

1. Review of section 2 of the previous lesson
2. Counting in Base 5
3. Converting numerals from Base 10 to Base 5
4. Worksheet

Materials:

1. Popsicle sticks
2. Twist ties
3. Base 10/Base 5 table
4. Base 10 & Base 5 mats: a paper on one side of which there are three large columns with headings: ten-tens, tens and ones; the other side has three large columns with headings: five-fives, fives and ones.

Activities:

1. (7 min) Review of section 2 of the previous lesson:

Hold up 22 sticks. Show the students that 4 bundles of 5 sticks each can be made and there will be 2 sticks left over.

Draw the Base 5 table on the board and write "4" under the "fives" and "2" under the "ones". Then write on the board the statement "22 (Base 10) = 42 (Base 5)".

2. (3 min) Counting in Base 5:

Ask the students to enter "1", "2", ..., "12" vertically in the left short form section of the table. For each of these numerals write the equivalent Base 5 numeral. Emphasize that in Base 5, fives must be grouped (bundled). Make a correspondence that in Base 5 there are ones, fives, five-fives, ... just as in Base 10 there are ones, tens, ten-tens, ... Also point out that in Base 5 the only digits needed are the five digits "0, 1, 2, 3 and 4." Contrast with Base 10 in which the ten digits "0, 1, ..., 9" are needed.

3. (10 min) Converting numerals from Base 10 to Base 5:

Tell the students to enter "38" in the left short form section of the table. Ask the students to think of 38 sticks and the number of bundles that could be made. Ask if 1 bundle of five could be made (Yes), 2 bundles (Yes), 3 bundles (Yes), 4 bundles (Yes) and 5 bundles (Yes). Ask what we would do with 5 bundles of five (Make 1 bundle of five-fives or twenty five). Ask the students to enter the number of bundles of twenty five (1) in the table.

Continue by asking the students to calculate the number of sticks that would be left over (13), the number of bundles of five that could be made (2) and the number of sticks left over (3). Ask the students to enter the numbers (2,3) representing the fives and the ones in the table and to also enter the short form (123). Write on the board "38 (Base 10) = 123 (Base 5)." Repeat for "56" and conclude that:

56 (Base 10) = 211 (Base 5)

4. (10 min) Worksheet:

Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. At the end of the period collect the worksheets.

Name: Last:_________________, First:___________

Lesson C2 - Worksheet

Do each calculation and write your answer in the space provided for it.

1. 938 = 9 hundreds + _____ tens + _____ ones

2. 4207=(___ x 1000) + (___ x 100) + (0 x 10) + 7

3. 279 = (2 x ____) + (7 x ____) + ___

4. 18 (Base 10) =

5's	ones

= _____ (Base 5)

5. 30 (Base 10) =

25's	5's	ones

= ______ (Base 5)

6. 86 (Base 10) =

25's	5's	ones

= ______ (Base 5)

7. 57 (Base 10) = ________ (Base 5)

8. The Base 5 number for the number of days in a week is ______ (Base 5)

9. The Base 5 number for the number of months in a year is ______ (Base 5)

10. The Base 5 number for the number of days in January is ______ (Base 5)

Treatment C

Lesson 3

Behavioral Objectives:

1. Given a numeral in Base 5 for a number n [$n \leq 444$(Base 5)], the students will be able to convert the numeral to the equivalent numeral in Base 10.

Outline:

1. Follow up of the worksheet from the previous lesson
2. Converting numerals from Base 5 to Base 10
3. Worksheet

Materials:

(the activities of this lesson consist mainly of number manipulation and do not require physical materials)

Activities:

1. (5 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain how one can obtain the answer to #6 and #10.

For #6 explain that from 86 sticks one can bundle 3 bundles of 25's, 2 bundles of 5's and have 1 stick left. Therefore "3", "2" and "1" should be written under "25's", "5's" and "ones" and thus "321" should be written in the space provided on the right.

Similarly, explain that in #10 the 31 days in January allow for writing "1" under "25's", "1" under "5's" and "1" under "ones". The answer is therefore 111 (Base 5).

2. (15 min) Converting numerals from Base 5 to Base 10:

Write the place values for Base 5 on the board as follows:

25's	5's	Ones

Write also "123(Base 5)" on the board and ask the students what "123(Base 5)" means (1 twenty-five, 2 fives and 3 ones). Write "1", "2" and "3" in the proper sections in the place value table on the board. Then write on the board
123 (Base 5) = (1 X   ) + (2 X   ) + 3 and ask the students to state numbers that will make the assertion true. Conclude that
123 (Base 5) = (1 X 25) + (2 X 5) + 3 = 25 + 10 + 3

123 (Base 5) = 38 (Base 10).

Likewise ask the students to state the usual way (in Base 10) of writing the numbers in the following exercises:

203 (Base 5) = (2 X   ) + (0 X   ) + 3 = _____ (Base 10)
334 (Base 5) = (3 X   ) + (3 X   ) + 4 = _____ (Base 10)

3. (15 min) Worksheet:

Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Name: Last:________________, First:___________

Lesson C3 - Worksheet

Do each calculation and write your answer in the space provided for it.

1. 23 (Base 5) = 5's | ones = ______ (Base 10)

5's	ones

2. 401 (Base 5) = 25's | 5's | ones = ______ (Base 10)

25's	5's	ones

3. 34 (Base 5) = _________ (Base 10)

4. 332 (Base 5) = _________ (Base 10)

5. 304 (Base 5) = _________ (Base 10)

6. 432 (Base 5) = _________ (Base 10)

Treatment C

Lesson 4

Behavioral Objectives:

1. Given two numerals in Base 5 which represent two numbers the sum of which is not greater than 444 (Base 5), the students will be able to calculate the sum of the numbers and express it by a numeral in Base 5 without translating the numerals into Base 10.

2. Given two numbers in Base 5, neither of which is greater than 444 (Base 5), the students will be able to calculate the difference of the two numbers and express it by a numeral in Base 5 without translating the numerals into Base 10.

Outline:

1. Follow up of the worksheet from the previous lesson
2. Addition in Base 5 with and witbout renaming
3. Subtraction in Base 5 with and without renaming
4. Worksheet

Materials:

1. Popsicle sticks
2. Twist ties
3. Base 5 table
4. Base 5 mat

Activities:

1. (3 min) Follow up of the worksheet from the previous lesson:

Give each student his corrected worksheet from the previous lesson and explain how one can obtain the correct answer to #6.

Explain that "4" means four 25's or 100, "3" means three fives or 15 and "2" means two ones. Therefore "432" in Base 5 is equivalent to "117" in Base 10 (100 + 15 + 2).

2. (12 min) Addition in Base 5 with and without renaming:

2.1. Addition without renaming (using the sticks):

Give each student 40 popsicle sticks, 5 twist ties, a Base 5 mat and a copy of the Base 5 table illustrated below.

short form	expanded form		
	five-fives	fives	ones

Hold up in one hand one bundle of five popsicle sticks and 3 sticks and ask the students to use sticks and twist ties in order to isolate a similar amount and place the bundle and the sticks in the proper places on the mat. Ask the students to record in the Base 5 table the expanded form of the numeral (13) representing all the sticks on the mat. Similarly, hold up in the other hand two bundles of five sticks and one stick. Ask the students to isolate a similar amount, place the sticks on the mat and record in the same table underneath "13" the numeral (21) representing all the sticks just placed on the mat. Ask the students to draw a horizontal line underneath "21" and add the two numbers step by step as follows:

a. Join the sticks (3 and 1) on the mat in order to get "4." Record the number of sticks (4) in the proper place in the table.

b. Join the bundles (1 and 2) and record the number of bundles (3) in the proper place in the table.

2.2. Addition with renaming (using the sticks):

Repeat activities similar to the ones described in the above section (2.1) using 2 bundles and 4 sticks (24) in one hand and 1 bundle and 3 sticks (13) in the other. Remind the students that in Base 5 we have to bundle by fives if we can. Ask the students to add 24 and 13 step by step as follows:

a. Join the sticks (4 and 3) on the mat, make a bundle of five sticks and place it with the bundles on the mat. Record in the table the resulting number of sticks (2).

b. Join the bundles (1, 2 and 1) on the mat and record in the table the resulting number of bundles.

Similarly, consider 3 bundles of fives and 2 sticks (32) and 3 bundles of fives and 4 sticks (34). Ask the students to place the sticks on the mat, record the numerals and add step by step as follows:

a. Join the sticks (2 and 4) on the mat, make a bundle of five sticks and place it with the bundles on the mat. Record in the table the resulting number of sticks (1).

b. Join the bundles (1, 3 and 3) on the mat, bundle five of them into a bundle of five-fives and record in the table the resulting number of fives (2).

c. Record the number of bundles of five-fives (1) on the table.

2.3. <u>Addition without using the sticks:</u>

Write on the board

```
  31 (Base 5)
+  2 (Base 5)
-----
_____(Base 5)
```

Ask the students to copy the numerals to the short form section on the Base 5 table and add. Help the students to calculate the number of ones and fives resulting from addition without actually manipulating the sticks.

Similarly, help the students perform the addition without the sticks for both of the following exercises:

```
  23 (Base 5)          124 (Base 5)
+  4 (Base 5)        + 133 (Base 5)
----                 -----
____ (Base 5)        _____ (Base 5)
```

3. (12 min) <u>Subtraction in Base 5 with and without</u>

renaming:

3.1. Subtraction without renaming (using the sticks):

Hold up three bundles of fives and two sticks and ask the students to isolate a similar amount, place the sticks on the mat and record in the Base 5 table the expanded form of the numeral (32) representing all the sticks. Ask the students to follow each of the steps below in order to subtract (take away, remove) 22 from the number represented in the table.

a. Record "22" in the table underneath "32" and draw a horizontal line underneath "22."

b. Remove 2 sticks and record the number of the resulting sticks (0).

c. Remove two bundles and record the number of the resulting bundles (1).

3.2. Subtraction with renaming (using the sticks):

Repeat activities similar to the one described in the above section (3.1) using 4 bundles, 2 sticks and subtract 2 bundles, 4 sticks. Ask the students to carry out the subtraction (take away) by following the steps below:

a. Place 4 bundles and two sticks in the proper columns on the mat. Record "42" in the expanded form section of the table. Write "24" underneath "42" and draw a horizontal line underneath "24."

b. In the ones column on the mat, "2 take away 4: can't do."

c. Regroup 1 bundle into sticks in order to have sufficient amount of sticks to allow the "take away" of 4. Place the sticks in the proper column on the mat.

d. Remove 4 sticks and record the resulting number (3) on the table.

e. Remove 2 bundles and record the resulting number (1) on the table.

Similarly, consider 1 bundle of five-fives, 1 bundle of fives and 2 sticks (112). Ask the students to place the bundles and sticks on the mat and record "112" in the expanded form of the table. Ask the students to record "34" on the table underneath "112", draw a horizontal line underneath "34" and subtract (take away) by following the steps below:

a. In the ones column on the mat, "2 take away 4: can't do."

b. Regroup 1 bundle into sticks in order to have sufficient amount of sticks to allow the "take away" of 4. Place the sticks in the proper column on the mat.

c. Remove 4 sticks and record the resulting number (3) on the table.

d. In the fives column on the mat, "0 take away 3: can't do."

e. Regroup the bundle of five-fives in order to get sufficient amount of fives to allow the "take away." Place the fives in the proper column on the mat.

f. Remove 3 fives and record the resulting number of fives (2) on the table.

3.3. Subtraction without using the sticks:

Write on the board

```
   44 (Base 5)
 - 32 (Base 5)
 -----
 _____(Base 5)
```

Ask the students to copy the numerals to the short form section on the Base 5 table and subtract. Help the students to calculate the number of ones and fives resulting from subtraction without actually manipulating the sticks.

Similarly, help the students perform the subtraction without the sticks for both of the following exercises:

```
    23 (Base 5)     6.    340 (Base 5)
 -   4 (Base 5)         - 243 (Base 5)
 ----                   -----
 ____ (Base 5)          _____ (Base 5)
```

4. (8 min) Worksheet:

Give each of the students a copy of the attached worksheet and explain it to them. Go around and help them to complete it. Collect the worksheets at the end of the period.

Name: Last:__________________, First:____________

Lesson C4 -- Worksheet

Do each calculation and write your answer in the space provided for it.

```
1.     21 (Base 5)       2.     34 (Base 5)       3.     213 (Base 5)
     +  4 (Base 5)            +  3 (Base 5)            + 144 (Base 5)
     -----                    -----                    -----
     _____(Base 5)            _____(Base 5)            _____(Base 5)


4.     34 (Base 5)       5.     23 (Base 5)       6.     340 (Base 5)
     - 32 (Base 5)            -  4 (Base 5)            - 143 (Base 5)
     -----                    -----                    ------
     _____(Base 5)            _____(Base 5)            ______(Base 5)
```

7. 143 (Base 5) + 31 (Base 5) = _____ (Base 5)

8. 201 (Base 5) - 32 (Base 5) = _____ (Base 5)

9. 244 (Base 5) + 40 (Base 5) = _____ (Base 5)

10. 300 (Base 5) - 223 (Base 5) = _____ (Base 5)

Appendix B

ITEM STATISTICS

Table B1
Volume Achievement Pretest Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	80.7	0.35	15	4.1	0.47
2	73.1	0.51	16	19.9	0.69
3	52.0	0.58	17	11.7	0.49
4	28.1	0.66	18	35.1	0.64
5	8.8	0.49	19	18.1	0.74
6	22.8	0.68	20	11.1	0.74
7	51.5	0.53	21	13.5	0.74
8	25.1	0.72	22	19.3	0.68
9	13.5	0.69	23	19.3	0.68
10	27.5	0.66	24	9.9	0.67
11	24.0	0.67	25	11.7	0.68
12	32.7	0.65	26	21.6	0.58
13	8.2	0.65	27	45.6	0.54
14	14.6	0.63			

Table B2
Volume Conservation Pretest Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	72.5	0.67	7	57.3	0.78
2	76.6	0.75	8	66.1	0.76
3	35.7	0.52	9	56.1	0.72
4	56.1	0.62	10	56.1	0.71
5	58.5	0.65	11	42.1	0.46
6	53.8	0.73			

Table B3
Volume Achievement Posttest Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	78.4	0.50	15	8.2	0.38
2	74.9	0.61	16	42.1	0.77
3	62.6	0.66	17	17.0	0.35
4	53.2	0.72	18	58.5	0.79
5	27.9	0.63	19	39.8	0.65
6	41.5	0.64	20	32.7	0.73
7	60.8	0.60	21	36.3	0.66
8	57.9	0.76	22	40.9	0.67
9	48.5	0.73	23	33.9	0.62
10	54.4	0.73	24	32.2	0.66
11	43.3	0.70	25	30.4	0.65
12	51.5	0.75	26	41.5	0.62
13	23.4	0.64	27	56.7	0.61
14	43.3	0.74			

Table B4
Volume Conservation Posttest Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	86.0	0.65	7	63.7	0.72
2	87.1	0.75	8	76.0	0.75
3	59.6	0.60	9	73.1	0.72
4	66.7	0.47	10	71.9	0.70
5	74.3	0.56	11	56.1	0.46
6	67.3	0.70			

Table B5
Multiplication Posttest Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	85.4	0.58	11	71.3	0.58
2	80.7	0.59	12	63.7	0.58
3	79.5	0.47	13	33.9	0.53
4	49.7	0.60	14	22.8	0.37
5	50.9	0.65	15	19.9	0.42
6	33.3	0.62	16	67.3	0.59
7	8.2	0.34	17	71.3	0.58
8	43.3	0.56	18	23.4	0.46
9	36.6	0.67	19	22.8	0.42
10	84.8	0.58	20	23.2	0.36

Table B6
Volume Achievement Retention Test Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	87.1	0.39	15	12.9	0.41
2	82.5	0.54	16	54.4	0.71
3	74.3	0.57	17	22.8	0.41
4	59.6	0.62	18	60.2	0.71
5	29.2	0.59	19	46.2	0.60
6	46.8	0.65	20	36.8	0.65
7	63.2	0.58	21	42.1	0.70
8	59.1	0.73	22	45.6	0.73
9	49.1	0.78	23	40.4	0.70
10	59.1	0.68	24	34.5	0.68
11	45.6	0.70	25	37.4	0.69
12	56.1	0.67	26	38.6	0.51
13	24.6	0.58	27	63.2	0.55
14	46.2	0.69			

Table B7
Volume Conservation Retention Test Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	90.1	0.54	7	71.9	0.77
2	91.2	0.59	8	82.5	0.70
3	68.4	0.60	9	76.0	0.72
4	81.9	0.54	10	75.4	0.60
5	79.5	0.60	11	59.1	0.33
6	71.9	0.65			

Table B8
Multiplication Retention Test Item Statistics

Item Number	Percent Correct	Point-biserial Coefficient	Item Number	Percent Correct	Point-biserial Coefficient
1	93.0	0.50	11	77.8	0.40
2	91.2	0.46	12	66.7	0.44
3	80.1	0.35	13	28.7	0.35
4	47.4	0.56	14	19.3	0.35
5	56.1	0.55	15	22.8	0.35
6	30.4	0.55	16	71.3	0.50
7	7.6	0.34	17	73.7	0.47
8	50.9	0.55	18	22.2	0.50
9	36.6	0.53	19	26.3	0.44
10	88.9	0.44	20	14.6	0.40

Appendix C

RAW DATA

Codes:

1 = Nonconserver, 2 = Partial Conserver, 3 = Conserver
Pre = Pretest, Post = Post Test, Ret = Retention Test
Vol = Volume, Mul = Multiplication, Ach = Achievement
SAT = Stanford Achievement Test, Tr = Treatment
V = Volume, M = Multiplication, C = Control

Subject Number	Conservation Level Pre	Post	Ret	Tr	SAT	Vol Ach Pre	Vol Ach Post	Mul Ach Post	Vol Ach Ret	Mul Ach Ret
002	3	2	2	V	30	4	18	10	17	14
004	2	1	'	V	33	4	12	5	8	9
006	1	1	1	M	29	4	2	10	2	8
007	1	2	1	C	25	1	4	7	5	7
C08	3	3	3	M	31	3	0	15	1	11
014	3	3	3	C	40	16	23	15	23	13
015	2	3	3	C	37	3	1	9	3	7
016	1	3	1	M	31	5	6	9	15	8
022	1	1	3	C	32	5	6	8	6	7
023	1	1	1	V	26	17	18	13	18	14
024	3	3	3	V	30	8	15	8	7	12
027	1	2	1	M	35	4	10	8	7	11
031	1	1	1	M	43	4	8	14	8	9
032	1	3	3	V	34	14	21	4	21	13
033	3	2	2	M	39	12	21	14	24	8
034	2	2	3	V	25	7	15	3	19	6
036	1	2	3	C	28	3	0	7	3	7
037	1	1	1	C	32	3	2	11	1	13
038	3	1	2	V	38	1	12	10	12	8
039	2	2	2	M	42	11	17	14	16	15
041	2	1	2	C	31	6	11	6	12	10
042	1	1	1	V	21	3	10	8	9	8
043	3	3	3	M	39	11	12	12	11	11
044	1	1	1	C	31	3	2	4	5	10
045	1	1	3	M	21	6	3	6	4	5
047	3	3	3	V	29	4	18	11	21	10
051	3	3	3	C	35	20	25	9	22	15
052	1	1	1	V	39	9	21	13	25	12
053	1	2	2	V	34	7	25	6	23	9
056	1	1	1	C	40	2	1	8	1	10
057	2	2	3	C	18	2	0	0	0	5
058	1	1	2	M	26	4	19	10	18	11
059	3	3	3	V	40	9	19	12	23	10
061	1	1	1	V	28	7	13	11	23	12
062	1	1	1	V	35	2	5	3	10	9
063	1	3	1	V	43	18	22	12	25	13
065	2	2	2	M	40	3	3	14	1	8
066	1	1	3	C	37	2	2	5	3	13
067	2	1	1	V	33	5	6	8	5	9
069	3	3	3	C	39	2	5	13	4	12
070	1	1	1	C	28	3	2	6	4	12
071	1	3	3	M	30	2	2	8	6	8

074	3	3	3	M	39	13	25	14	25	11
075	1	1	1	C	35	7	7	10	12	12
076	1	3	3	M	35	9	6	6	15	12
077	1	1	3	C	38	3	4	5	7	4
078	2	3	3	V	36	18	23	11	22	13
079	2	1	1	M	23	3	1	13	2	6
080	1	1	1	C	36	3	7	14	3	10
082	3	3	3	V	35	19	26	16	23	15
083	1	1	3	V	33	4	24	4	19	8
084	1	3	3	V	20	5	12	7	8	8
085	1	1	1	C	43	3	3	11	1	14
087	1	1	2	C	44	12	26	19	26	15
088	1	1	1	C	37	1	13	9	10	7
089	1	1	1	M	27	10	12	6	14	5
090	1	1	1	V	30	3	16	9	9	7
091	1	1	1	M	33	2	10	12	14	13
092	1	3	3	M	38	7	14	15	12	15
093	1	1	1	V	38	5	17	11	21	9
094	2	1	1	M	42	23	25	19	26	19
095	1	1	3	C	41	3	3	8	11	13
096	1	1	2	V	28	2	20	12	17	12
097	1	1	1	C	41	4	8	15	15	14
098	2	1	2	V	44	23	25	16	26	18
099	1	1	1	M	36	6	9	14	20	16
100	1	1	2	V	36	7	22	10	20	9
101	1	1	1	V	41	5	22	12	24	13
102	1	1	1	M	39	7	24	14	23	14
104	3	3	3	M	44	7	20	18	22	18
105	3	3	3	M	39	24	24	19	25	9
109	1	3	3	C	27	0	0	9	7	7
110	1	1	1	C	37	16	20	12	20	12
111	1	1	1	V	42	14	24	18	23	17
113	1	3	3	M	39	21	21	14	22	17
114	1	3	3	M	28	9	22	16	17	15
119	1	3	3	M	18	10	12	9	15	9
120	1	2	1	C	20	5	7	15	1	9
121	1	1	1	V	23	3	17	9	6	9
122	3	3	3	C	44	26	26	19	26	19
124	1	1	3	V	30	11	18	8	23	10
126	1	3	3	M	29	11	3	9	10	6
127	2	3	3	V	31	4	22	8	15	6
128	3	2	3	M	31	5	14	11	23	12
129	1	1	3	V	28	1	9	13	24	10
130	3	3	3	M	35	4	19	15	13	11
131	3	3	3	M	42	18	22	18	23	14
133	2	2	1	M	31	8	3	10	10	11
136	3	3	3	V	35	18	16	4	20	12
137	1	3	3	C	38	10	14	15	15	9
140	3	3	3	M	39	20	24	18	23	13
142	3	3	3	C	34	14	14	12	17	12
143	3	3	3	M	35	13	18	15	23	16
145	2	2	2	M	36	4	1	11	2	14
148	3	3	3	M	45	8	23	18	6	15
150	3	1	3	C	28	6	11	11	7	8

152	3	3	3	V	36	15	17	9	20	9
154	3	3	3	V	33	4	16	10	18	10
157	3	3	3	V	33	2	14	7	13	5
160	3	3	3	V	33	17	25	11	21	11
161	3	3	3	M	29	6	18	15	9	11
164	2	3	3	C	36	12	14	12	19	11
165	3	3	3	M	36	6	10	17	10	13
169	1	2	3	M	36	16	20	11	22	15
170	3	3	3	M	36	9	4	11	17	11

Number of subjects = 105

Appendix D

UNADJUSTED DESCRIPTIVE STATISTICS

Table D1
Unadjusted Means, Standard Deviations, and Group Sizes of Volume Achievement Posttest Scores for Treatments by Conservation Levels (Maximum Score = 27)

Conservation Level	Treatments Volume	Multiplication	Control	Total
Non-conservers	17.68 (5.63) 19	12.28 (7.27) 18	6.55 (6.87) 20	11.75 (6.58) 57
Partial-conservers	17.17 (7.41) 6	8.33 (10.17) 6	6.50 (7.05) 4	11.19 (8.36) 16
Conservers	17.82 (4.29) 11	16.93 (7.54) 15	17.33 (8.59) 6	17.31 (6.62) 32
Total	17.64 (5.52) 36	13.00 (7.82) 39	8.70 (7.24) 30	13.36 (6.86) 105

Regression coefficient = 0.66

Table D2
Unadjusted Means, Standard Deviations, and Group Sizes of Volume Achievement Retention Test Scores for Treatments by Conservation Levels (Maximum Score = 27)

Conservation Level	Treatments Volume	Multiplication	Control	Total
Non-conservers	18.32 (6.51) 19	13.56 (6.34) 18	7.81 (6.90) 20	13.13 (6.59) 57
Partial-conservers	15.83 (8.13) 6	9.50 (10.00) 6	8.50 (8.66) 4	11.62 (8.96) 16
Conservers	17.73 (5.08) 11	17.00 (7.96) 15	16.50 (9.05) 6	17.16 (7.17) 32
Total	17.72 (6.34) 36	14.26 (7.53) 39	9.63 (7.56) 30	14.12 (7.13) 105

Regression coefficient = 0.67

Appendix E

TESTS

Volume Conservation Test

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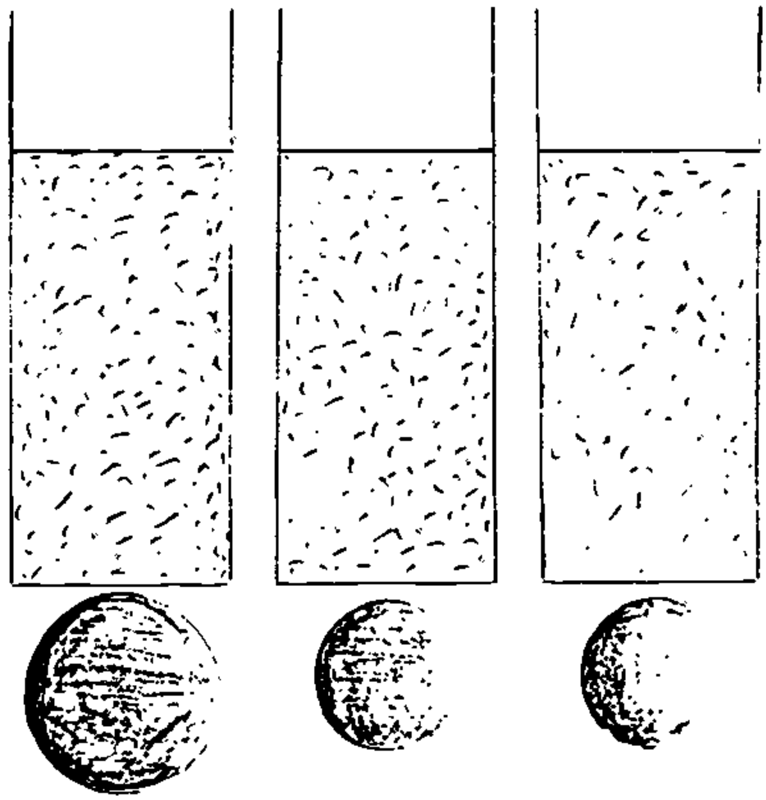

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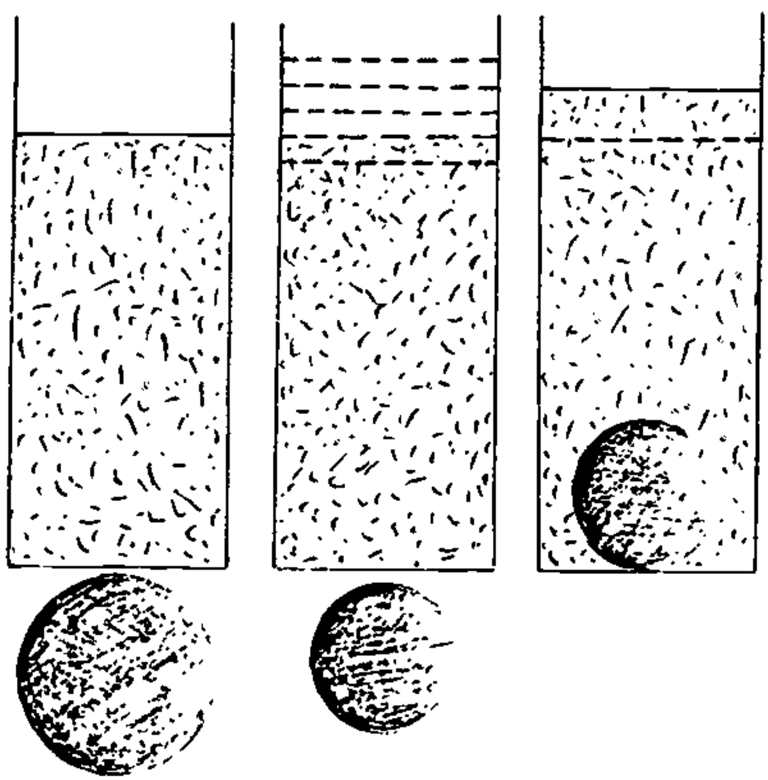

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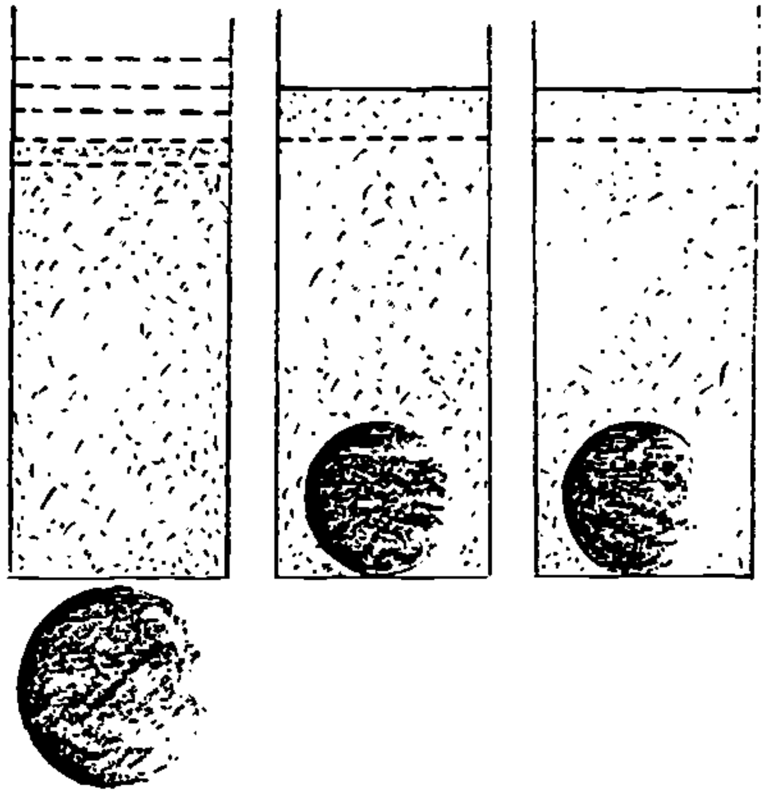

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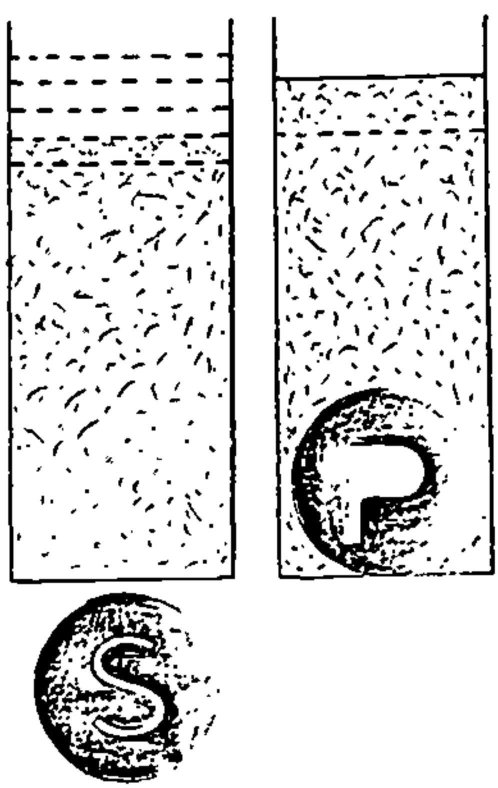

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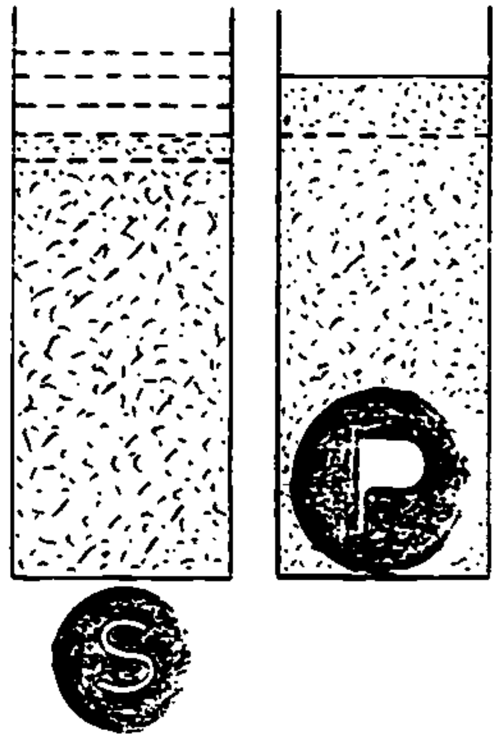

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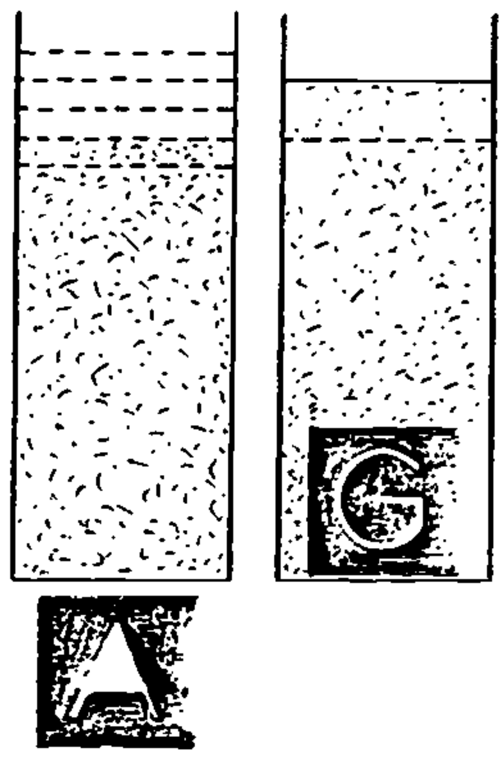

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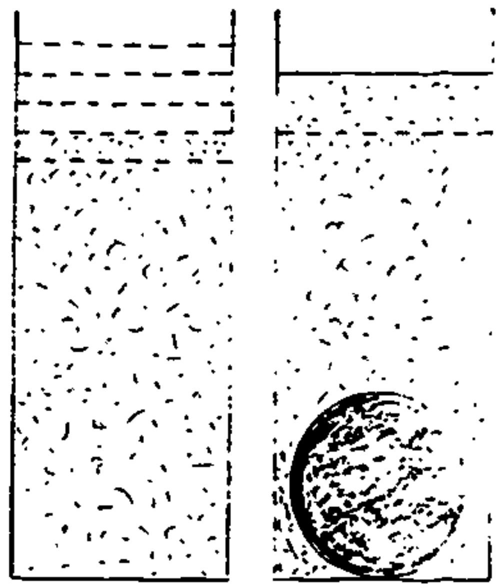

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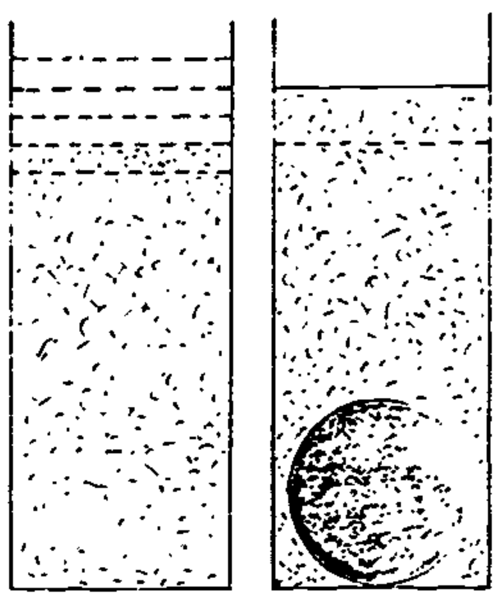

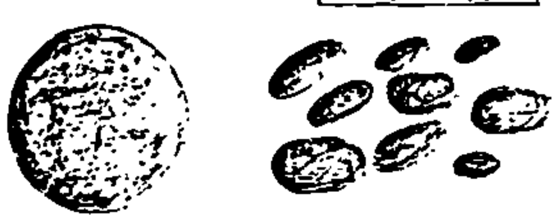

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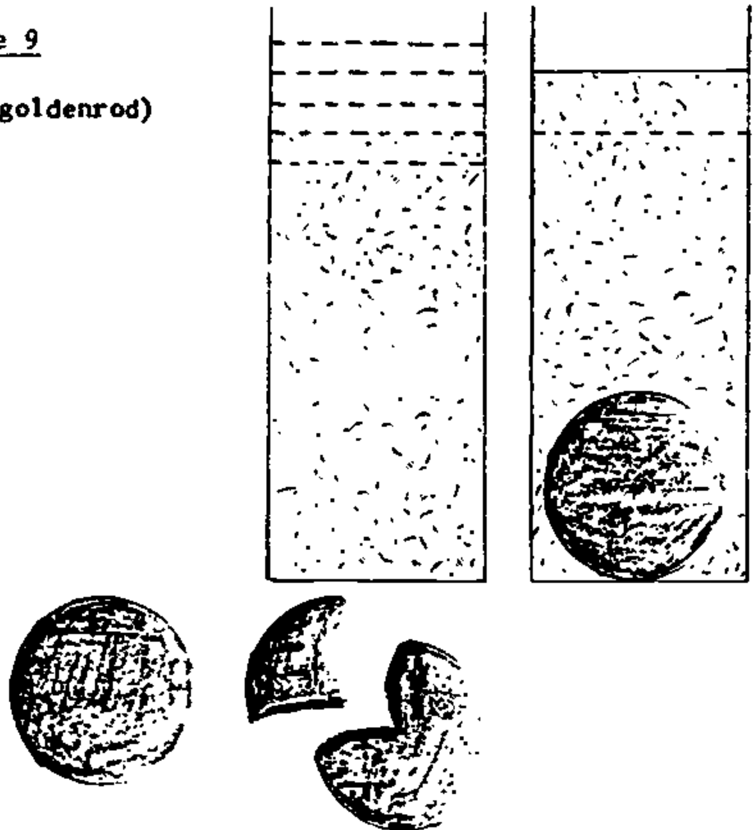

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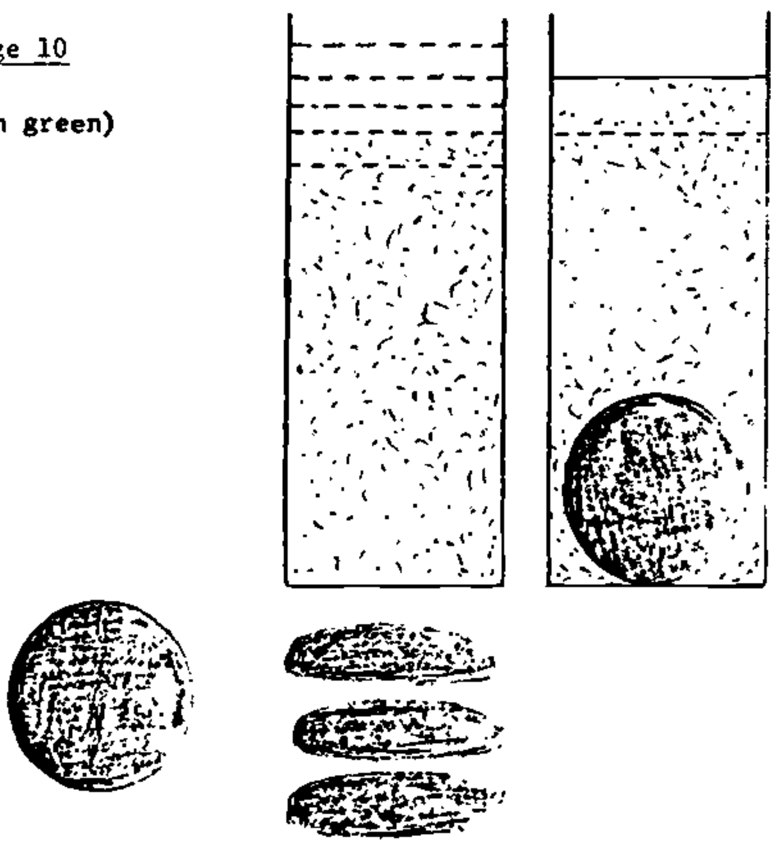

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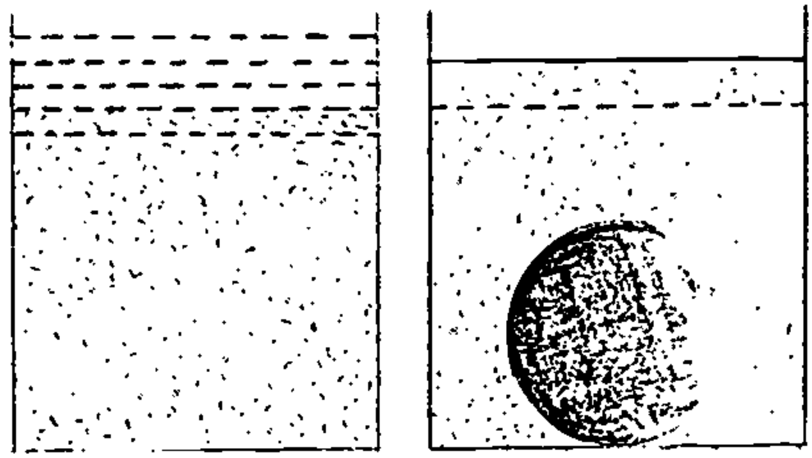

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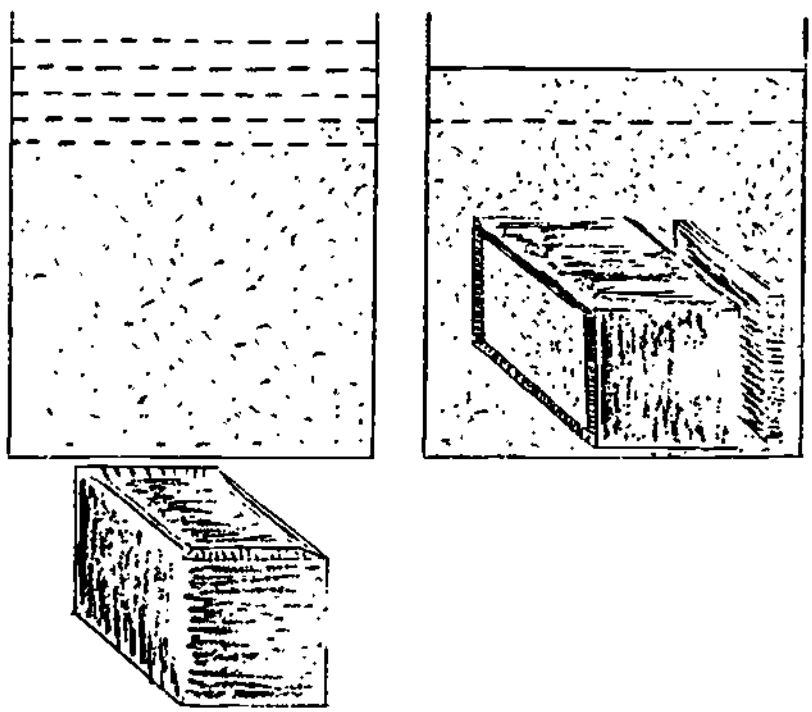
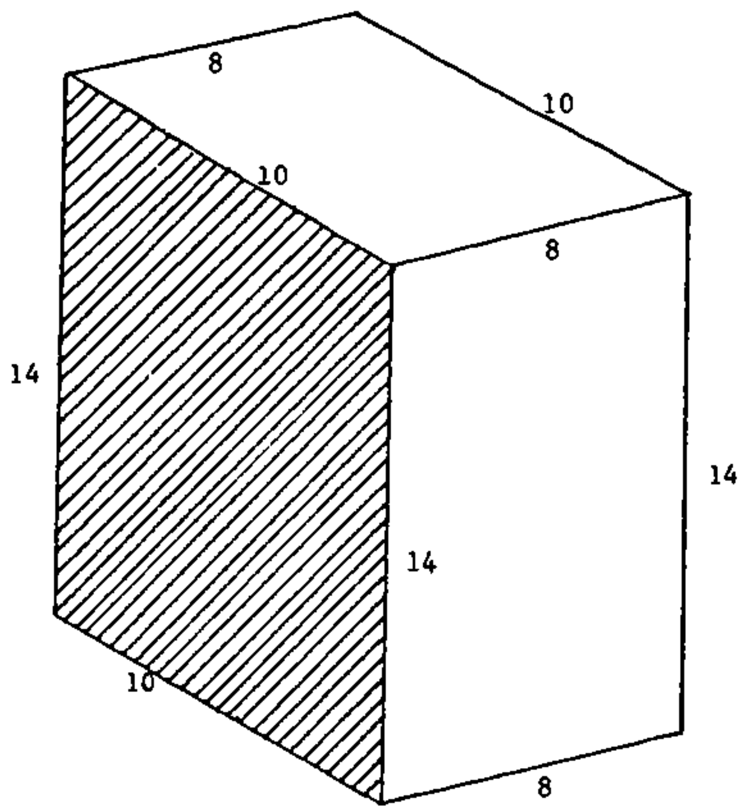


Name: Last:________________ First:______________

School:____

<u>**Volume Achievement Test**</u>



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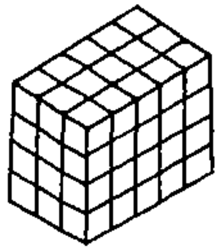
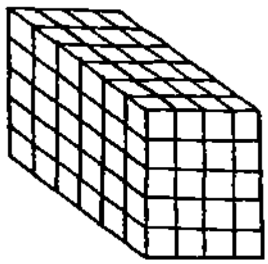


A rectangular pile of cubes.

Volume of the top layer = __________

Number of layers = __________

2.

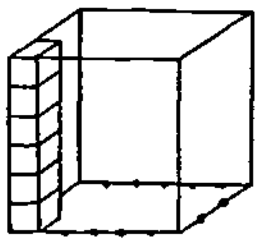


A rectangular pile of cubes.

Volume of a layer = __________

Total volume = __________

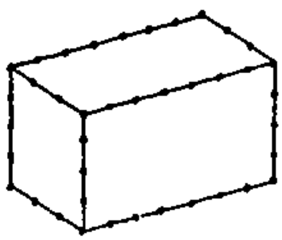
3.



A glass box with some cubes in it.

Volume of the box = __________

4.



A cardboard box full of cubes.

Volume of the bottom layer = __________

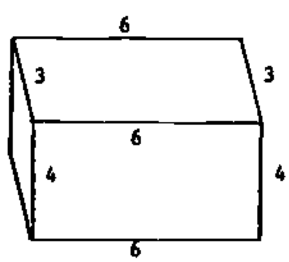
Number of layers = __________

5.

A box is 10 units long, 5 units wide and 2 units high.

What is the volume of the box? ____________________

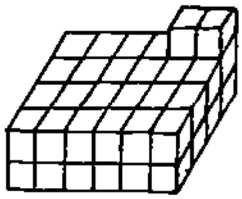
6.



A rectangular block of wood.

Volume = __________

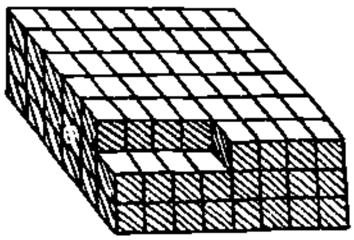
7.



A pile of cubes.

Total volume = ________

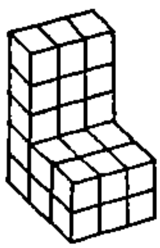
8.



A pile of cubes.

Volume = ________

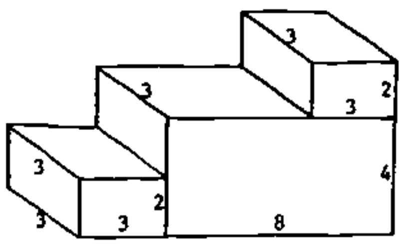
9.



A pile of cubes.

Volume = ________

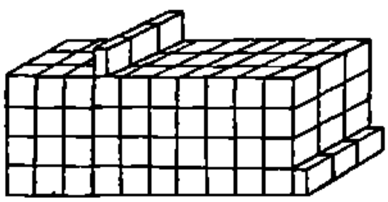
10.



Three rectangular wooden blocks.

Total volume = ________

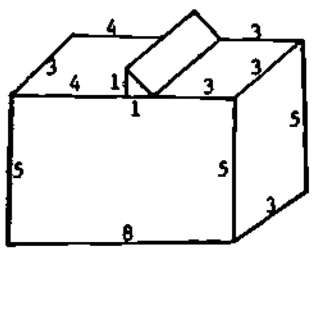
11.



A pile of cubes and half cubes.

Total volume = ________

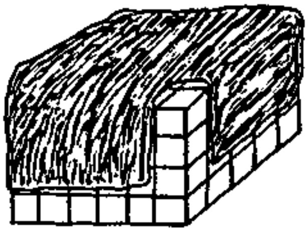
12.



A wooden block and half of a different block on top.

Total volume = ________

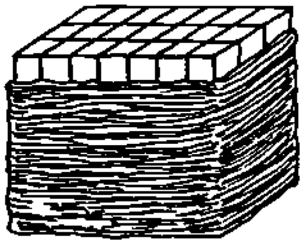
13.



A rectangular pile of cubes partially covered.

Total volume = ________

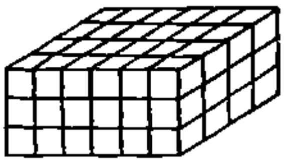
14.



A pile of cubes. 9 layers are covered.

Total volume = ________

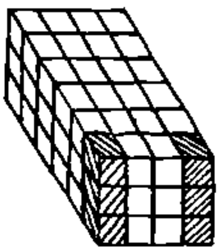
15.



A pile of cubes.

If we removed the top layer, what would be the volume of the part left? ________

16.

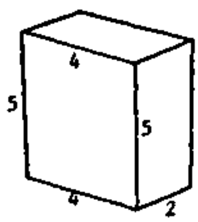


A pile of cubes.

If we removed both of the shaded portions, what would be the volume of the part left?

____

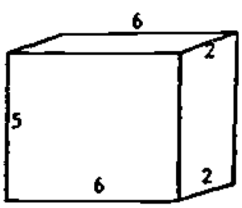
17.



A plastic box.

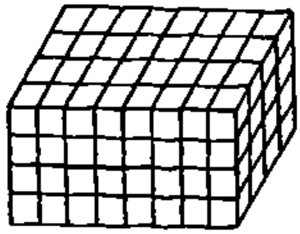
If we increased the width of this box by 1 unit but the length and height stayed the same, what would be the volume of the new box? ________

18.



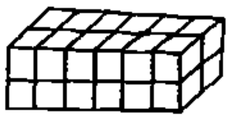
A metal box.

If we doubled the length of this box but the width and the height stayed the same, what would be the volume of the new box? ________

19. 

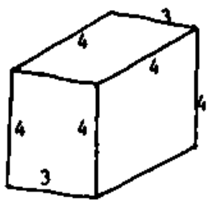
A rectangular pile of cubes.

If we halved the length and doubled the width but the height stayed the same, what would be the volume of the new shape? ________

20. 

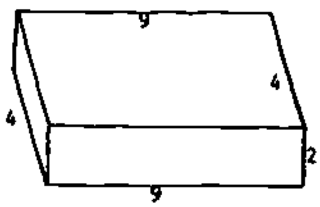
A rectangular pile of cubes.

If we doubled the length, tripled the height and halved the width, what would be the volume of the new shape? ________

21. 

A wooden box.

If we decreased the width by 1 unit and we increased the height by 2 units but the length stayed the same, what would be the volume of the new shape? ________

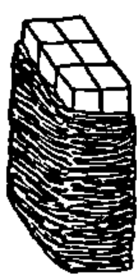
22. 

A metal box.

If we decreased the width by 2 units, decreased the length by 6 units and increased the height by 8 units, what would be the volume of the new shape?

____

23. A brick has a height of 5 units, a length of 6 units and a volume of 120 units. What is its width? ________

24. 

A rectangular pile of cubes partially covered.

Total volume = 42 cubes.

How many layers are there altogether? _____

Name:Last:................., First:..............

Multiplication Test

A. In each of the following write your answer in the box at the right.

1. (23 x 17) x 36 = 17 x ([ ] x 23)
What number should go in the [ ]?

2. 13 x (5 x 19) = ([ ] x ◯) x 5
What two numbers should go in the [ ] and in the ◯ ?

3. 6 X 6 = 9 X [ ]
What number should go in the [ ]?

4. 15 X [ ] = 30 X 25
What number should go in the [ ]?

5. 24 X 11 = [ ] X 33
What number should go in the [ ]?

6. 12 X 15 X 22 = 36 X [ ] X 22
What number should go in the [ ]?

7. 41 X 50 X 7 = [ ] X 5 X 35
What number should go in the [ ]?

8. 18 X 24 = 432

If 18 is replaced by 9 and if the product, 432, stays the same, then 24 must be replaced by what number?

9. 9 X 13 X 21 = 2457

If 9 is replaced by 27 and if the product, 2457, stays the same, then 21 must be replaced by what number?

8. <u>In each of the following write the letter corresponding to the correct answer in the box at the right.</u>

10. 18 X 27 = 486

If 27 is replaced by a larger number and if 18 stays the same, what happens to the product, 486?

a. It becomes smaller
b. It becomes larger
c. It stays the same

11. [ ] X 14 < 9 X 14

What number (or numbers) could go in the [ ]?

a. Any number greater than 9
b. Any number less than 9
c. Any number greater than 14
d. Any number less than 14
e. 9

12. [ ] X 27 > 41 X 27

What number (or numbers) could go in the [ ]?

a. Any number less than 27
b. Any number greater than 27
c. 27
d. Any number less than 41
e. Any number greater than 41

13. 38 X 53 = 2014

If 38 is replaced by a number twice as big and if 53 is replaced by a number three times as big, what happens to the product, 2014?

a. It increases to five times as much
b. It increases to six times as much
c. It decreases to one half as much
d. It decreases to one fifth as much
e. It changes but it is impossible to know how much it increases or decreases.

14. 72 X 35 = 2520

If 72 is replaced by a number half as big and
if 35 is replaced by a number one fifth as big,
what happens to the product, 2520?

a. It decreases to one seventh as much
b. It increases to ten times as much
c. It decreases to one tenth as much
d. It increases to seven times as much
e. It changes but it is impossible to know how much it increases or decreases.

15. 40 X 24 = 960

If 40 is replaced by a number one fourth as big and
if 24 is replaced by a number twice as big,
what happens to the product, 960?

a. It increases to six times as much
b. It decreases to one sixth as much
c. It increases to twice as much
d. It decreases to one half as much
e. It changes but it is impossible to know how much it increases or decreases.

16. 12 X 25 X 18 < 12 X [ ] X 18

What number (or numbers) could go in the [ ]?

a. Any number less than 25
b. Any number greater than 25
c. 12
d. Any number greater than 12
e. Any number less than 12

17. 14 X 9 X 13 = 1638

If 9 is replaced by a smaller number and
if 13 is replaced by a smaller number and
if 14 stays the same,
what happens to the product, 1638?

a. It decreases
b. It increases
c. It stays the same
d. It changes but it is impossible to know whether it increases or decreases
e. It could increase or it could decrease or it could stay the same

18. 21 X 18 X 25 = 9450

If 18 is replaced by a number half as big and
if 21 is replaced by a number one third as big and
if 25 stays the same,
what happens to the product, 9450?

a. It increases to six times as much
b. It increases to five times as much
c. It decreases to one fifth as much
d. It decreases to one sixth as much
e. It changes but it is impossible to know how much it increases or decreases.

19. 4 X 27 X 17 = 1836

If 4 is replaced by a number twice as big and
if 17 is replaced by a number three times as big and
if 27 is replaced by a number one third as big,
what happens to the product, 1836?

a. It increases to three times as much
b. It decreases to one third as much
c. It increases to twice as much
d. It decreases to one half as much
e. It changes but it is impossible to know how much it increases or decreases.

20. 16 X 154 X 2 = 4928

If 16 is replaced by a number twice as big and
if 2 is replaced by a number seven times as big and
if the product, 4928, stays the same,
what happens to 154?

a. It increases to nine times as much
b. It increases to fourteen times as much
c. It decreases to one ninth as much
d. It decreases to one fourteenth as much
e. It changes but it is impossible to know how much it increases or decreases.

***************** End of Test ****************