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ABSTRACT

This investigation used a clinical interview technique to identify the difficulties of kindergarten children who are unable to develop models or representations of simple arithmetic story problems. It is hypothesized that effective human problem solvers first generate some type of "physical model" and use this model to create a mathematical one. Further, the development of a model requires children to make three types of abstractions from a story. These are: making an abstraction of the (1) objects described, (2) locations or possessors of the sets, and (3) operations or relationships described. The type of physical model involved in the study is "operations on sets." The focus of the research is on student attempts to answer addition and substraction problems. Four components identified in tasks of abstraction are: set identity, set numerosity, operation on sets, and identity of answer set. The study results review each identified component, identify some pupil difficulties, and support several specific conclusions concerning pupil success with each aspect of abstraction. Also included are suggested steps for aiding students experiencing specific difficulties. (Author/MP)

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The solution of many types of problems encountered by humans involves the use of some type of model, a model which serves to represent the essential features of the problem situation in some simplified form and which can be manipulated to solve the problem. With many problems the most effective, or even the necessary, model is a mathematical model. When we work with arithmetic story problems, we are basically concerned with problems where the ultimate model for solution is mathematical, for example, some type of number sentence or some more complex formula or equation.

Skemp (1971) discusses the development and use of mathematical models and identifies three levels or realms of thought that are involved as one develops a mathematical model for solving a specific problem. He describes these realms in terms of the operations that are involved.

Realm 1: operations on (mental representations of) physical objects.

Realm 2: operations on their physical qualities.

Reaml 3: mathematical operations. (Skemp, 1971, p. 183)

Skemp points out that going from realm 1 to realms 2 and 3 involves processes of abstraction, abstracting from the actual problem to those qualities that are essential to problem solution, and then abstracting from these qualities to their mathematical representation. He describes this as a method "of abstracting, manipulating the abstractions instead of manipulating physical objects, and then re-embodying the result in the situation from which the abstractions were taken" (Skemp, 1971, p. 179). He then goes on to give a simple illustration of this in a situation where the immediate problem is that of how many places to set at a table when guests are expected.

> We may be expecting a visit from friends. 'There are four of us, and they will be two grown-ups and three children' the first stage of abstraction is that verbalized above, using primary concepts. For a particular purpose we are not interested in age, sex, or whether resident or visitor. So we abstract still further: 4, 2, 3. [the abstraction is from the actual people to the physical quality of numerosity.] In the general situation of having tea together, we concentrate on the combining aspect and represent this by the mathematical operation of addition: 4 + 2 + 3. (Skemp, 1971, p. 179)

As this type of problem is solved, one goes from the actual problem to some abstracted representation or model that permits one to work with the quantities that are essential to problem solution (e.g., the problem could be modelled and solved by using one's fingers to represent the quantities involved) and then to the more efficient model represented by a number sentence. The focus of the work reported in this paper is on how kindergarten children carry out the processes of abstraction that are involved in going from the problem as represented by the original story to a reformulation and solution of the problem through the use of some type of physical model.

There is considerable logic and quite a bit of empirical evidence to support the view that effective human problem solvers, including primary grade children, when attempting to solve a problem that requires a quantitative answer, first generate some type of physical representation or model that embodies the physical qualities essential to solving the problem and then use this physical model to write the necessary mathematical model. (What we are calling a "physical model" here may take the form of a representation with actual objects, may involve some type of marks or drawings on paper, or may be only in the way the person "thinks about" the essential elements of the story.) This analysis of what takes place in certain types of human problem solving has been supported by the work of a number of persons (Larkin, 1977; Simon and Simon, 1978; Heller and Greeno, 1978b). Earlier research by the writers provides some support for the idea that this is what primary grade pupils do when they solve simple arithmetic story problems. This study (Lindvall and Ibarra, 1980) provided correlational evidence that the ability to develop a physical model of a story was prerequisite to the the ability to write the correct number sentence for it. Also, a number of studies (Hebbeler, 1977; Ibarra and Lindvall, 1979) have indicated that providing physical aids, which might help the modelling process, results in improved performance on story problems. If this ability to develop a physical model is indeed a prerequisite to writing the correct number sentence, then knowledge concerning the nature of such models and how we can help children learn to develop them would seem to be essential.

It may be useful to emphasize here that a "model", in the sense in which the term is used in this paper (and as we understand Skemp's use of the term), is the unique representation that the problem solver builds for each story. It is to be distinguished from a general schema or mental structure that may be carried in the problem solver's head and be useful in suggesting the model for many problems of a given type. That is, a pupil may have a "joining sets" schema or an "addition" schema that can be employed in developing a solution for a story requiring the combining of groups of objects, but the "model" employed by the student would be a physical representation of sets of a specific size (e.g., 4 blocks joined with 3 blocks) or a specific addition sentence (e.g., 4 + 3 = 7). The model is a model for the specific problem. Of course, as will be discussed later, there may be "types" of models of which a given model is a specific example.

If the three "realms" identified by Skemp (1971) as associated with the problem solving process are sequential stages in model development, it would suggest that effective problem solvers do not develop a number sentence for the story as such. Rather, they develop a number sentence for their physical model of the story. If this is true, it would imply that teaching pupils to solve story problems by writing a number sentence involves teaching two distinct steps (1) developing the proper physical model of the story and (2) writing the correct number sentence for this physical model. This is not intended to imply that pupils are then going to go through life solving quantitative problems using this very deliberate two-step process. Certainly with relatively simple problems, the first step, the physical modelling, becomes something that is done mentally and is done so quickly and automatically that it is not even recognizable as a separate step. However, this step is always present in some form when the truly effective problem solver is solving the story.

The Process of Developing a Model

As suggested by Skemp (1971), the process of developing a physical model and then a mathematical model for a given story in arithmetic can be described as a process of identifying the essential information in the story and representing this in an abstracted form that can then be manipulated to solve the problem. Figure 1 represents an effort to indicate the essential components of the information needed to solve a simple addition or subtraction story problem and to suggest the abstracted representation of that information that would be used in the physical model and the mathematical model for the story. Since the work described in this paper is focused on how children develop a physical model, it is concerned with the tasks of abstraction described in the second column of Figure 1. These tasks can be exemplified by using the following simple story problem.

> Joe had 3 apples
>
> Tom had 5 apples
>
> How many apples did Joe and Tom have together?

The child who uses counting cubes or blocks to build a physical model of this story might start by counting out a set of 3 blocks to represent Joe's apples. Note that this set of blocks is a representation of both the identity of the set (Joe's) and the numerosity (3) of the set. Of course, the numerosity is represented in concrete form but the identity is something that the child must have in memory or must indicate in some more concrete form (e.g., tag with a picture). The child would also build a set of 5 blocks to represent Tom's apples. To model the problem correctly the child would then have to interpret the story question as having the abstracted meaning that the two sets must be joined, that this union set then has the identity of "the set that Joe and Tom have together," and that the answer is obtained by counting the number in this set. In our work we have been concerned with how children carry out all of these tasks of abstraction and with some typical difficulties that they have.

Method

This study employed a clinical interview procedure to obtain information on how kindergarten children proceed when they are asked to develop physical models to represent and solve simple addition and subtraction story problems. The data obtained and analyzed consisted of the notes taken to record exactly what children did as they attempted each step in the model building process and of tape recordings of what pupils said in response to questions or in giving directions or explanations for each step. The specific procedures followed are described below. Subjects used in the study were 20 kindergarten children, with IQs ranging from 105 to 139, enrolled in a campus laboratory school.

Type of Story Problem

An assumption underlying our current work is that there is probably quite a variety in story problem types if these types are categorized in terms of the general form of physical model that is most appropriate for modelling each type. Our present work, then, is with story problems that deal with groups (or sets) of elements that are to be combined or subdivided and where the problem requires that one find either the result of such operations or find the size of one of the constituent groups if the result is known. The type of physical model involved here is that of operations on sets (joining sets, removing a sub-set). This type would, for example, be different from one involving the comparions of sets, or one involving increments or decrements on a number line.

If we are confining our attention to story problems that can be modelled by the union of two sets or the removal of a subset from a given set, we are concerned with a type of story that could be modeled mathematically by either an addition sentence or a subtraction sentence. The basis for the use of an addition sentence in solving such stories is the definition of addition that equates it with the symbolic description of the union of two disjoint sets.

> Definition. The addition of the natural numbers a and b is the assignment of the natural number c to the ordered pair (a, b), such that $N(A) + N(B) = N(A \cup B)$, where A and B are nonempty, finite sets, $A \cap B = 0$, and $N(A)=a$, $N(B)=b$, and $N(A \cup B)=c$. That is, $a + b = c$. (McFarland and Lewis 1966, p. B7)

This definition of addition makes it clear that students who write an addition sentence for stories of this general type must understand the story as describing the union of two disjoint sets. That is, students who write such an addition sentence, with a clear and correct understanding of what they are doing, must first model the story as an operation on sets and then write the number sentence for that set operation. (Note here that we are not making this assertion concerning all story problems where addition might be involved but only concerning those stories that describe the combining of groups (sets) of things).

The types of story problems used in this study are presented in Figure 2 where an abbreviated version of each sentence in the story is paired with a typical example of the correct response that a pupil should make in terms of building and manipulating the physical model. As indicated by the footnote, the steps in physical modelling as presented in this figure are based on the assumption that when children hear or read a sentence describing a set with "some," they build a set of an arbitrary size. Our experience is that many children actually do this. On the other hand, many children do nothing in response to such a sentence and merely wait for more information. For these latter children a modified version of Figure 2 is needed to described what they do. A further discussion of this matter is provided in our presentation of results.

In developing and categorizing the stories involved we have used the categories and definitions presented by Heller and Greeno (1978a). It will be noted that the "combine" stories are those identified as "part-part-whole" by Carpenter and Moser (1979) and as "static" by others (Nesher, 1979; Ibarra and Lindvall, 1979) while the "change" stories are referred to as "joining" and "separating" by Carpenter and Moser (1979) and as "transformation" by Steffe (1970) and LeBlanc (1971).

Procedure

Data used in this study were obtained by interviewing each child on an individual basis. In all interviews two adults were present, one to read the story and conduct the interview session, the other to record the child's comments and actions. To aid in this latter task all sessions were recorded on an audio tape. Each child was interviewed on at least two occassions and several were interviewed three or four times. The number of interviews needed was determined by the rate of progress of the child.

In each session the child was told that an arithmetic story problem would be read, line by line[1], and that he/she was to use the blocks (one-inch counting cubes) to show what the sentence said and to answer the question asked in the story. The recorder then noted what the child did in response to each sentence, using the actions described in Figure 2 as a guide. That is, the recorder noted whether the child took these hypothesized correct actions or took some specific alternative actions. Since the general investigative procedure involved here was that of the clinical interview, the tester, and the recorder, used follow-up questions and supplementary activities to investigate any action, or lack of action, that appeared to have special significance.

With several students, and on selected stories, the interview procedure was modified. This involved having a third adult play the role of a "poor student." That is, the child being interviewed was told by this third adult "Pretend that I am a student in your class who does not know how to show what these problems mean. When each sentence is read, you tell me exactly what I should do." This usually resulted in the child playing the teacher role quite well and saying such things as "Get 3 blocks," "Put these together," "Count these," etc.

Results

The results from this study are contained in the protocols reporting the students actions and verbalizations as they attempted to demonstrate the meaning of each story and to arrive at a correct solution. These results have been analyzed in terms of what they reveal about students' capabilities and difficulties in translating the information provided in the story into appropriate manipulations of a physical model using counting cubes. The outline of this task as presented in Figure 1 may be viewed as proposing the hypothesis that this over-all task involves the students ability to make four essential types of abstractions. Moser (1971) has suggested that there are three components of an addition or subtraction problem, (1) the given sets (2) the described relationships or actions, and (3) the problem question. Our analyses is somewhat parallel. However we have found that when kindergarten children solve addition or subtraction story problems, there are two different aspects of set representation that many cause difficulty, set identity and numerosity. Hence, for our purposes it appears useful to be concerned with four components.

1. Set Identity: abstracting from a specific identification of the possessor (or container, etc.) of a set in the story to an identity based on an arbitrary location (e.g., area on table) of a constructed set.

2. Set Numerosity: abstracting from a collection of a specific number of named objects (e.g., apples) to a representation of the numerosity of this collection by a set of the proper number of blocks.

3. Operation on Sets: abstracting from some specific operation or relationship described in the story to one of a limited number of operations on the set(s) of blocks.

4. Identity of Answer Set: abstracting from story description of what answer is desired to the identification of one specific set of blocks that must be counted to find the answer.

The results will be presented and discussed in terms of their relationship to each of these tasks of abstraction.

Set Identity

The problem of set identity in the use of a physical model is both one of establishing initial identity as sets are built or are created by joining or separating and of maintaining this identity as operations are carried out.

Difficulties with initial identity were not common with our group of kindergarten children. However two children displayed some lack of ability in this task. Student B, one of these children, did nothing when asked to use the blocks to show that "Jim has 3 fish." In an effort to explore B's difficulty the tester produced some plastic cut-outs of fish, emphasized the point that these were fish, and then repeated the story. B still did not respond. The tester then made use of two pictures, one of "Jim," the other of "Amy." The following is a record of what transpired when these pictures were used.

Tester: "This is Jim." (Places picture of boy on table)
"This is Amy." (Places picture of girl on table)
"Pretend that these blocks are fish and show me what this story says."
Tester: "Jim has 3 fish."
B: Placed 3 blocks on picture of Jim.
Tester: "Amy has 2 fish."
B: Placed 2 blocks on picture of Amy.
Tester: "How many fish do Jim and Amy have altogether?"
B: Counted the total and answered "5".

The presence of the two characters from the story, in the form of their pictures, appeared to make the story real enough to B so that she had no difficulty in comprehending and solving it. She could not work with the abstract idea that Jim and Amy could be represented by locations on the table. The characters had to be present in some more tangible form.

Providing pictures of the characters also appeared to be clarifying in the case of certain stories for student C. C had solved addition problems correctly but had difficulty with the subtraction stories. He was then tested as follows.

Tester: "This is Sam." (Places picture of boy on table)
"This is his sister." (Places picture of girl on table)
"Let these blocks stand for apples."
"Sam had 6 apples. Can you show me this?"
C: Placed 6 apples on Sam's picture.
Tester: "He gave 2 apples to his sister."
C: Moved 2 of the 6 apples to the sister's picture.
Tester: "How many apples does Sam have left?"
C: "One, two, three, four" (counted the 4 remaining). Answered "4".

C also used this same procedure to solve a second problem of the same type.

Of course, some difficulties with establishing the correct initial identity of a set may be the result of story wording that is less than completely clear. Consider the following story.

Together, Tom and Joe have 8 apples.
Tom has 3 apples. How many does Joe have?"

With a story worded exactly in this form we found that some of our students built a set of 8 in response to the first sentence and then built a new and separate set of 3 in response to the second sentence. Evidently they interpreted the situation as one where Tom and Joe share one set of apples but where Tom also has a separate set of 3 that is entirely his own. This misconception was corrected when the story was reworded to read as follows:

Together, Tom and Joe have 8 apples.
3 of these apples belong to Tom.
How many of them belong to Joe?

With this modification the students correctly modelled the story by building a set of 8 and removing a subset of 3.

It might be argued that students who build a set of arbitrary size when they encounter a sentence with the word "some" in it are using this as a device for establishing the identity of this set. The performance of student D, when he was giving instructions to a "poor student," seems to suggest this.

Tester:	"Frank has 6 fish."
D:	"Take out 6 blocks."
Tester:	"Tom has some fish."
D:	"Take out 3." (with little hesitancy)
Tester:	"Together they have 10 fish."
D:	"Take 1 more out." (indicates that it should be added to arbitrary set of 3.)
Tester:	"How many does Tom have?"
D:	(pointing) "Count the 4 blocks."

Student D apparently was not using the blocks for computational purposes. At least, he was not overtly counting the blocks. The "arbitrary" set appeared only to represent a set of unknown size, the size of which was determined by some type of mental counting or computation after D had all the information he needed.

We also interpreted the behavior of student E as evidence that he thought of an arbitrary set as a device for establishing and maintaining set identity. When E solved a story by himself he did not build an arbitrary set. He appeared able to retain the set identity in his head. However when he played the role of teacher for our hypothetical "poor" student, E told the student to "take some, any number" when instructing the student on how to respond to the sentence "Tom had some apples." E apparently conceived of the arbitrary set as good way of helping the student remember that he had this set of unknown size that had to be considered in further steps in problem solution.

Set Numerosity

Certainly a key and essential step in the modelling of any arithmetic story problem is the correct representation of the quantities involved. That is, in response to a sentence such as "Joe had 8 pieces of candy" the student who is using blocks to model the story must build a set of exactly 8 blocks. In our study we defined the ability to construct a set of a given size, up to 10, as a prerequisite to participation in the study. Hence, the only real test for a given student attempting to model a story was whether he or she had any difficulty with the idea of using 8 blocks to represent 8 pieces of candy. With our subjects, no one had any difficult with this type of abstraction of the basic quality of numerosity. Evidently, children are so accustomed to playing games involving "Pretend that these blocks are ..." that this is a very natural task of representation.

In some earlier work that was preliminary to our main study we did some investigating of the pupils' use of their fingers and of tally marks that they made on paper as a means of representing and manipulating the sets.

Here we found that some physical problems of "holding down" the proper number of fingers and of keeping one set separated from another as well as actually counting the number in a set caused some problems when fingers were used. With the use of tallies, difficulties were evidenced in such things as showing a sub-set that was to be removed or in counting a remainder set. It was because of these difficulties with other manipulatives that the physical modelling in our actual study always involved the use of blocks.

Operations on Sets

Since the stories used in the present study were limited to the "combine" and "change" stories, as categorized in Figure 2, the operations on sets that the students had to use were those of (1) joining two sets and (2) removing a subset (or the related operations of increasing or decreasing the size of a set by a counting process.)

The students in our study had little difficulty representing the correct set operation when the story involved finding the number in the union set for two known sets or finding the number in the remaining set when a subset of certain size had to be removed from a known set. As is obvious, the language of our stories was simple, and children had little difficulty in comprehending the operation described. However, some insights concerning their understanding of the set operations was obtained from the procedures they used in solving stories that had an unknown set (i.e., a "some" sentence) in other than the final position.

As mentioned previously, the students in our study appeared to use one of two different strategies in their modelling of stories involving a representation of a set with "some" in it, (1) to merely retain this set in memory as an "unknown set," or (2) to construct a set of "arbitrary" size. The use of an unknown set is illustrated by the instructions given by student F (playing the role of teacher).

Tester: "Joe has some cards"
F: "Take zero out." (Her version of an unknown set)
Tester: "Frank has 4 cards."
F: "Take 4 out."
Tester: "Together Joe and Frank have 7 cards."
F: "Take 3 more out" (Evidently knowing that she must add 3 to the 4 to get a total of 7)
Tester: "How many cards does Frank have?"
F: "Count this." (pointing to the set of 3)

From her response to the sentence "Together Joe and Frank have 7 cards" it was quite obvious that F understood this as meaning that she had to combine Joe's (unknown) set with Frank's set of 4 to get the total of 7. As was the case with most of the problem solving efforts of our subjects, F did not use the blocks as a computational aid. She only used them to represent the sets and the operations, that is, to "model" the story problem. Computational procedures were not a focus of our study, but the quickness of F's response in this situation would be compatible with the assumption that she "knew" that 4 plus 3 equals 7.

Student F's responses in the following dialogue appear to provide further evidence that, although she used blocks to model the sets and the relationship involved, she did not use them for purposes of computation (at least not overtly)

Tester: "Julie had 8 flowers."
F: "Put 8 out"
Tester: "She lost some of her flowers."
F: "Read the rest."
Tester: "Then she had 2 flowers left."
F: "Take 6 out." (pointing to the set of 8)
Tester: "How many did she lose?"
F: "Count these," (pointing to the 6)

That F was quite flexible in her approach to representing the operation described in a story is suggested by the procedure exemplified in the following dialogue, where she made temporary use of an unknown set and then used an arbitrary size set when she had more information.

Tester: "Jim has some marbles."
F: "Read the rest."
Tester: "Then he lost 4 of them."
F: "Take out 5." "Take 4 away" (from the 5)
Tester: "Then he had 5 left."
F: "Take out 5" (from the supply box)
Tester: "How many marbles did Jim have to begin with?"
F: "Count these." (pointing to the 4 "taken away" plus the new set of 5)

The performance of student E in giving instructions to the "poor student" was informative in suggesting how this one student solved simple change problems.

Tester: "Jane had 6 buttons."
E: "Take 6 blocks."
Tester: "Then she found some more buttons."
E: "Take some."
Poor Student: "How many shall I take."
E: "Take some; any number" ("Student" took 5)
Tester: "Jane then had 9 buttons altogether."
E: "Count the 6 and then count more up to 9." (indicating that the blocks counted should come from the arbitrary set.). "Put the rest back in the pile." (in the supply box)
Tester: "How many buttons did Jane find?"
E: "Count these." (the set of 3 added to the 6)

It appeared that E had a clear understanding of the operation that had to be carried out to determine the number "found."

Identity of Answer Set

Some children, after carrying out the proper operation on sets of the correct size still had difficulty in identifying the set that represented the answer. In some cases this appeared to be only a momentary confusion that was cleared up when the story was re-read. The following is an example of this.

Tester: "John had 6 baseball cards."
G: "Put out 6 blocks."
Tester: "Greg also had some baseball cards."
G: "Put out 2 more blocks."
Tester: "Together, John and Greg had 9 baseball cards."
G: "Count them" (referring to all 8 blocks). (Subject counted 8) "Put 1 more out." (indicating that it should be placed with the 2 added)
Tester: "How many baseball cards did Greg have?"
G: "Count these." (pointing to the 6) (She lost the identity of the sets. When the story was re-read for her, she pointed to the correct set of 3 as the one to be counted.)

In the case of another story, however, G could not identify the correct answer set even when the story was re-read.

Tester: "Chuck had 5 toys."
G: "Put out 5."
Tester: "Chuck then found some more toys."
G: "Put out 3"
Tester: "Then he had 9 toys altogether."
G: "Count all of them" (Subject counted the 8) "Put out 1 more, here." (Pointing to set of 3)
Tester: "How many toys did Chuck find?"
G: Count both." (Subject counted the 9)

In this case, re-reading the story did not lead to a correct solution.

Another example of the incorrect identification of the answer set is shown in the following performance of student H.

Tester: "Joan has some candy."
H: "Don't do anything." (H did not use an arbitrary set)
Tester: "Elaine has 4 pieces of candy."
H: "Get 4 out."
Tester: "Together Joan and Elaine have 7 pieces of candy."

H: (after some counting on his fingers)
"Get 3 more blocks."
Tester: "How many did Joan have?"
H: "4" (pointing to Elaine's set)

With combine stories, involving two characters, it was our finding that using pictures was usually all that was needed to clarify the identity of the sets and enable the student to identify the correct answer. This was true in the case of Student E. In the initial presentation of one version of a combine story with a subset unknown E responded as follows:

Tester: "Frank had some marbles."
E: "Take a handful." (an arbitrary set)
Tester: "Tom had 4 marbles."
E: "Take out 4."
Tester: "Together Frank and Tom had 7 marbles."
E: "Count out 7." (by adding to the 4)
Tester: "How many did Frank have?"
E: "Count the 7." (what they had altogether)

E had lost the identity of the sets. The story was then repeated with pictures of the characters, Frank and Tom, provided. The use of the pictures permitted E to be more systematic in developing and manipulating the sets and in identifying the set to count to get the answer.

It would appear that the task of "tagging" each set, whether the set is one built from blocks in the central store or one generated through an operation on a set previously built, is a task that causes difficulty for many children. Students such as I, above, profit from the rather specific tagging represented by pictures. The use of pictures appears helpful both in identifying the sets when they are originally generated and in identifying the answer set.

Discussion

The clinical observation data obtained in this study provided a number of insights concerning how kindergarten children proceed when asked to develop a physical representation or model of simple story problems and concerning some of the difficulties certain students have in carrying out this task. Here the modelling task was viewed as a matter of abstracting, from the actual story, those elements that had to be represented in the model if it was to be useful for solving the problem. The essential elements were described as (1) set identity, (2) set numerosity, (3) the operations on the set(s), and (4) identification of the answer set. Results from the study may be discussed in terms of each of these tasks of abstraction.

Set Identity. A few children apparently have difficulty in developing a model for a story because they are at a loss to show that a set belongs to a certain story character (e.g., "Joe had 3 apples."). This difficulty is seen both in the initial representation of sets and in the identification of a set that is produced as a result of some operation described in the story. The problem of set identity is usually clarified if the student is provided with pictures of the story characters and can place "Joe's 3 apples" on Joe's picture. It can be assumed that this makes the story less abstract in that the character (Joe) is now present in a relatively realistic form. This would appear to suggest that a useful instructional strategy to explore would be to provide pictures of the characters in a story for those students who were having difficulty.

Set Numerosity. Kindergarten children appear to have little difficulty in using various countable elements (fingers, blocks, tallies on paper) to build the sets described in a story. However, some children are more successful in solving stories when they use blocks (rather than fingers or tallies) because any necessary manipulations can be carried out more easily and correctly using blocks. Providing the actual objects described in the story (e.g., pencils, apples, etc.) rather than having the children use blocks to represent the objects does not result in any greater success in modelling a story. Evidently children of this age have had so much experience in "pretending that" one object is something else that this type of abstraction poses no difficulty for them.

Operations on the Sets. The task of using the modelled sets to carry out any operations described in the story (e.g., "lost," "found," "gave," "altogether," etc.) appears to involve some of the same difficulties that have been identified when older children use number sentences and arithmetic operations to solve stories or when they attempt to complete open sentence. For example, stories are modelled much more successfully when the missing term is the third term presented in the story rather than the first or second. When representing and operating on these sets of unknown size (e.g., "Joe had some marbles."), some students merely keep the existence of this set in memory while others construct, and operate on, a set that is of some arbitrary size. In most cases, with stories of this type, the actual computations (e.g., finding the unknown amount that had to be added to reach a given sum) appear to be carried out independently of the block representation. That is, the students do the necessary counting or computation mentally or with their fingers and use the blocks to organize, or model, the problem and to check computations. Of course, in cases where the problem involves finding the number in the union of two given sets or the number in the remaining set when a subset is removed the students do appear to use the model for computing the answer.

Identification of the Answer Set. A rather common cause of failure to give the right answer for a story problem, as displayed in this study, was a lack of ability to select the correct set to count. Repeating the story clarified this difficulty in many cases. This, of course, suggests that this difficulty should not be present at any age level where children read the story for themselves and hence have it available for re-reading and study. (Assuming that they develop this habit). With students who still could not identify the correct set when the story was re-read, most of them were able to respond correctly if pictures are provided as an aid to set identification. It is assumed that this not only gives the student a rather concrete tag for each set but also clarifies what operations are to be carried out and where the answer set is to be found.

Some Instructional Implications

An essential step in the solution of mathematical story problems is the development of some type of physical model or representation of the problem as an intermediate step between the initial comprehension of the problem and the application of a mathematical operation to arrive at the answer. The development of this ability to use physical representations to model and solve stories starts at the kindergarten age or earlier (Hebbeler, 1977; Ibarra and Lindvall, 1979). Although the present study provided additional evidence that many kindergarten children could model and solve a variety of simple story problems, it also identified some of the difficulties that children encounter in carrying out this task. If the ability to do this type of modelling is indeed a prerequisite to being able to solve a story problem using a mathematical model (e.g., a number sentence), then it is important that all children become proficient in this modelling task. This study provides some guidance for how a diagnosis of individual pupil difficulties might be carried out. It also has certain implications for some procedures (the use of pictures of story characters, the use of arbitrary size sets to establish identity of an unknown set) that could be used as intermediate steps in instructing students on the modelling process.

Some Needed Research

As has been indicated, the present study was based on the assumption that ability to develop a physical model for a story problem is a prerequisite to being able to write a number sentence for it. Although this assumption has been supported by what is essentially correlational evidence (Lindvall and Ibarra, 1980) it should be important to investigate this relationship through some type of experimental study. Of course, this latter study would have to be preceded by some type of study which investigated the possibility of teaching modelling ability and identified effective procedures for this type of instruction.

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Footnotes

[1]An initial procedure of first reading the entire story without pausing was abandoned when it was found that almost all children made no meaningful response to this presentation.

Figure 1. Abstractions Made for Essential Problem Components as Child Develops Physical Model and Mathematical Model for Arithmetic Story Problem

Essential Problem Component as Described in Story	Abstract Representation of Component as Used in Model	
	Physical Model	Mathematical Model
Identity of sets involved. (e.g., Tom's apples, candies that Sue and Bob had together, etc.)	Sets "tagged" in some manner. (e.g., remembering that the set in this spot represents Tom's apples)	Numerals "tagged" in some manner. (e.g., This number represents Tom's apples)
Size of set of specific elements. (e.g., 5 apples)	Correct number of any countable and manipulable elements. (e.g., 5 blocks)	Correct numeral (e.g., 5)
Operation on sets of specific elements described in story. (e.g., combining Tom's and Joe's apples)	Operation on abstracted sets (e.g., joining two sets of blocks)	Arithmetic operation (e.g., 5 + 3)
Identifying result of operation that gives required answer. Count number. (e.g., count total in set after combining)	Identifying set that gives answer. Counting number in this set. (e.g., count number of blocks in union set)	Result of operation (e.g., 5 + 3 = 8)

Figure 2. Examples of Types of "Change" and "Combine" Stories Used in the Study Showing Correct Operation on Physical Model for Each Sentence of Story

Combine Stories

(Subset + Subset = Combined Value)

A. Combined Value Unknown

Story	Physical Model
Joe had 3	Build set of 3
Tom had 5	Build set of 5
Altogether?	Join: count

(Combined Value - Subset = Subset)

B. Subset Unknown

Story	Physical Model
Joe and Tom, 8	Build set of 8
Tom had 5	Separate 5
Joe, how many?	Count subset

Change Stories

(Start + Change = Result)

C. Result Unknown

Story	Physical Model
Joe had 3	Build set of 3
Found 5	Increase by 5
Then?	Count total

(Start - Change = Result)

D. Result Unknown

Story	Physical Model
Joe had 8	Build set of 8
Lost 5	Decrease by 5
Then?	Count remainder

E. Change Unknown

Story	Physical Model
Joe had 3	Build set of 3
Found some	Incr. by arb. amt[1]
Then had 8	Adjust arb. amt.
Joe found?	Count arb. amt.

F. Change Unknown

Story	Physical Model
Joe had 8	Build set, 8
Lost some	Decrease by arb.
Then had 3	Adjust arb. amt.
Joe lost?	Count arb. amt.

G. Start Unknown

Story	Physical Model
Joe had some	Build arbitrary set
Found 3	Increase by 3
Then had 8	Adjust arb. set
In beginning?	Count arb. set

H. Start Unknown

Story	Physical Model
Joe had some	Build arb. set
Lost 5	Decrease by 5
Then had 3	Adjust arb. set
In beginning?	Count arb. set

[1]The modelling procedure outlined in this figure assumes that the child always uses a set of an arbitrary size to represent "some" and then adjusts the size of this set. Of course, not all children do this (although they may do it mentally).