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ABSTRACT

This book is the teacher's manual to a text designed to meet the particular needs of those children who have "accumulated discouragements" in learning mathematics. It is a manual of suggested teaching strategies and additional materials aimed at compensating for past student failures to understand mathematical concepts. This book, L, is designed for the fourth-grade mathematics program. Individual chapter titles are: Properties: Geometry: Fractions: and Techniques of Addition and Subtraction of Whole Numbers. (MP)

Southwest Educational Development Laboratory

MATHEMATICS Book L

Teacher Guide

MATHEMATICS

BOOK L

TEACHER GUIDE

PRELIMINARY EDITION

SOUTHWEST EDUCATIONAL DEVELOPMENT LABORATORY

These instructional materials were developed and published by the Southwest Educational Development Laboratory using funds from:

- a grant from the National Science Foundation,
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- a contract with the Texas Education Agency.

PREFACE

The teaching strategies and mathematics materials suggested in this teacher's manual and the accompanying mathematics books for children are part of the Southwest Educational Development Laboratory's Mathematics/ Science Program.

Users of these adapted materials have the opportunity to revise and improve them in the light of experience and evaluation of results of their effectiveness in the classroom. This interaction of program designers and writers with teachers and pupils is consistent with the process of educational development -- the continuous improvement of materials and techniques. As these materials are pilot tested, the teachers' experiences with them have almost instant impact on their continuing revision and improvement.

Designed to compensate for pupils' past failures to understand mathematical concepts, the Southwest Educational Development Laboratory's Mathematics/Science Education Program takes into account the social and cultural background and cognitive skills a student brings to the learning situation.

This book, Mathematics, Book L, includes adaptations of the mathematics program commonly experienced in the fourth year of school. It is one of four books, K, L, M, and N, for this level. The adaptations are designed to meet the particular needs of those children who have accumulated discouragements in learning mathematics. With this in mind, the reading level required of the pupils has been reduced. More importantly, meaningful mathematical experiences are presented in ways which give the pupil many opportunities for success.

As in any sound educational program, the role of the teacher is critical. A teacher's interest and enthusiasm are contagious to students, but interest and enthusiasm are dependent upon the teacher's assessment of his own competence. This guide is designed to assist the teacher in directing classroom activity and in developing an understanding and appreciation of the mathematical concepts and skills to be taught.

The following premises guided the team of teachers and mathematicians who adapted and wrote these materials:

- Unnecessary use of vocabulary which has no meaning for children can be avoided.
- Teaching mathematics requires patience, purposeful planning, and opportunity for learning.
- Mathematical experiences can be adapted to children rather than adapting children to mathematical experiences.

The Laboratory's Mathematics Program has been expanded to include Science. Long range plans include adapting science materials to meet the needs of pupils who have failed to respond to traditional materials and teaching approaches.

Edwin Hindsman
Executive Director

ACKNOWLEDGMENTS

These materials were prepared by the Southwest Educational Development Laboratory's Mathematics Education Program during two summer writing conferences. The 1968 Summer Mathematics Writing Conference participated in the initial adaptation of these materials, and the 1969 Summer Mathematics Writing Conference participated in their revision.

The 1969 Summer Mathematics Writing Conference, held in Austin, Texas, was coordinated by Floyd Vest, Professor of Mathematics Education, North Texas State University, Denton, Texas. He was assisted by James Hodge, Professor of Mathematics, North Texas State University, and Palma Lynn Ross, Department of Mathematics, University of Texas at El Paso.

Participants for the 1969 writing conference included: Carmen Montes, Santiago Peregrino, Rebecca Rankin, Rudolph Sanchez, and Flora Ann Sanford, El Paso Independent School District, El Paso, Texas; Jimmye Blackmon, J. Leslie Fauntleroy, Barbara Graham, and Sophie Louise White, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; Lawrence A. Couvillon and James Keisler, Louisiana State University, Baton Rouge, Louisiana and Socorro Lujan, Mathematics Education, Southwest Educational Development Laboratory, Austin, Texas.

Consultants for this conference included: Sam Adams, Louisiana State University, Baton Rouge, Louisiana; James Anderson, New Mexico State University, Las Cruces, New Mexico; R. D. Anderson, Louisiana State University, Baton Rouge, Louisiana; Robert Cranford, North Texas State University, Denton, Texas; William T. Guy, Jr., University of Texas at Austin, Austin, Texas;

Lenore John, School Mathematics Study Group, Stanford, California; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana, and B. G. Nunley, North Texas State University, Denton, Texas.

The 1968 Summer Mathematics Writing Conference was coordinated by James Keisler, Professor of Mathematics, Louisiana State University. Participants for this conference included: Stanley E. Ball, University of Texas at El Paso, El Paso, Texas; Lawrence A. Couvillon, Louisiana State University, Baton Rouge, Louisiana; Rosalie Espy, Alamo Heights Independent School District, San Antonio, Texas; J. Leslie Fauntleroy, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; Norma Hernandez, University of Texas at Austin, Austin, Texas; Glenda Hunt, University of Texas at Austin, Austin, Texas; Carmen Montes, El Paso Independent School District, El Paso, Texas; Santiago Peregrino, El Paso Independent School District, El Paso, Texas; Rebecca Rankin, El Paso Independent School District, El Paso, Texas; Ida Slaughter, East Baton Rouge Parish Schools, Baton Rouge, Louisiana; and Sister Gloria Ann Fielder, CDP, Our Lady of the Lake College, San Antonio, Texas.

Consultants for this conference included: R. D. Anderson, Louisiana State University, Baton Rouge, Louisiana; William DeVenney, School Mathematics Study Group, Stanford, California; Sister Claude Marie Faust, Incarnate Word College, San Antonio, Texas; Mary Folsom, University of Miami, Coral Gables, Florida; William T. Guy, Jr., University of Texas at Austin, Austin, Texas; Houston T. Karnes, Louisiana State University, Baton Rouge, Louisiana; William McNabb, St. Marks School, Dallas, Texas; Sheldon Myers, Educational Testing Service, Princeton, New Jersey; and Ann Tinsley, East Baton Rouge Parish Schools, Baton Rouge, Louisiana.

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UNIT 4

Properties

OBJECTIVES:

1. To study mathematical sentences that use several number operations.
2. To develop skill in solving story problems.
3. To develop the concept of operation.
4. To introduce the meaning of a property of an operation.
5. To discover some properties the basic operations have.
   a. Addition and multiplication have the commutative property.
   b. Addition and multiplication have the associative property.
   c. Multiplication is distributive over addition.
   d. Zero and one have special properties under addition and multiplication.

BACKGROUND INFORMATION FOR TEACHERS:

The purpose of this unit is to help the pupils better understand what is meant by an operation on numbers. The pupils have been adding, subtracting, multiplying, and dividing numbers for some time. These processes are called operations. Now we look at the general meaning of an operation on numbers so that we can discover some "properties" of an operation.

A property of an operation is a special characteristic that the operation has. A property of iron is its hardness and a property of birds is their ability to fly. We study the properties of the operations so that we can base all the techniques of the operations on their properties.

The operations that pupils first use are called binary operations. For example, addition is a binary operation because it assigns to each pair of whole numbers something -- in this case, one whole number. Addition is an operation that

assigns to the pair 3 and 5 the whole number 8,
assigns to the pair 7 and 2 the whole number 9,

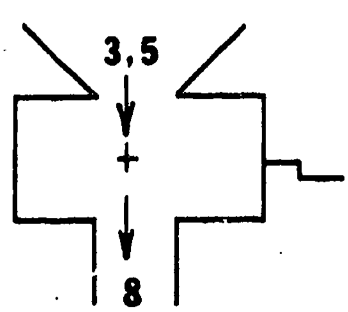
and so on; for every pair of whole numbers, addition assigns one whole number.

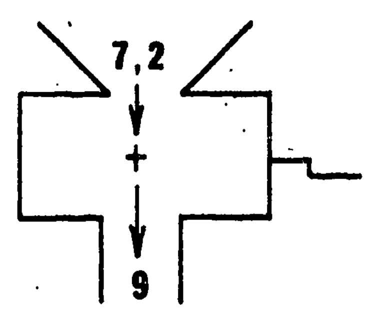
We can write this in many ways. For example:

(3,5) ⟶ 8

(7,2) ⟶ 9

Or we can think of the addition operation as a machine into which we put a pair of numbers and out of which comes one number, the sum.





Of course, the usual way to write the fact that addition assigns to the pair (3,5) the number 8 is as follows:

$3 + 5 = 8$

Later the pupils will use operations on a single number. An example of such an operation is the squaring operation, which assigns to each single whole number its square. Squaring assigns to the number 3 the number 9; squaring assigns to the number 5 the number 25; and so on. For example:

3 ⟶ 9

5 ⟶ 25

When we use the word "operation", we will mean binary operation.

We try to help a pupil understand that there are many kinds of operations other than the four they have already studied. By making up new operations, the pupils begin to see that some operations have certain properties and that others do not have these same properties.

For example, they find that addition and multiplication are commutative operations but that subtraction and division are not.

Commutative property. An operation is commutative if it assigns the same number to the pair a, b as it assigns to the pair b, a. In our work here, a and b are any whole numbers. For example,

$2 + 3 = 3 + 2$ and $5 + 1 = 1 + 5.$

Similarly, for any two whole numbers. Also,

$2 \times 4 = 4 \times 2$ and $3 \times 7 = 7 \times 3.$

Similarly, for any two whole numbers. Thus, addition and multiplication are commutative for whole numbers because $a + b = b + a$ and $a \times b = b \times a$ are true sentences for any whole number a and any whole number b.

But some sentences of the following kind are <u>not</u> true:

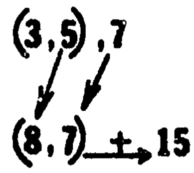
$$6 \div 3 = 3 \div 6 \qquad 9 - 2 = 2 - 9$$

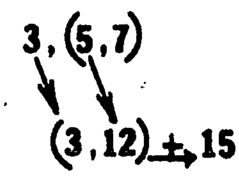
$$8 \div 2 = 2 \div 8 \qquad 7 - 4 = 4 - 7$$

This means that subtraction and division do <u>not</u> have the commutative property.

Another property of an operation that the pupils study is the associative property. They discover that addition and multiplication are associative operations but that subtraction and division are not.

<u>Associative property</u>. All binary operations assign something (in our case, numbers) to pairs of numbers. When three numbers are to be added, we first must add two of them and then add their sum and the third number. This can be done in two ways. For example:





That is:

$$(3 + 5) + 7 = 8 + 7 \quad \text{and} \quad 3 + (5 + 7) = 3 + 12$$

$$= 15 \qquad\qquad = 15$$

In both addition and multiplication, the pupils see that the result is the same no matter how the three numbers are grouped when the operation is applied. Thus, we see that addition and multiplication are associative because

$$(a + b) + c = a + (b + c) \text{ and } (a \times b) \times c = a \times (b \times c)$$

for any whole number a, any whole number b, and any whole number c.

GRADE 4

The following revised order of units is recommended. More time spent on the early units will help children develop a sound foundation; thus less teaching time will be required for later units.

UNIT	TITLE	BOOK
1	Sets and Numbers	K
2	Operations on Sets and Numbers	K
2½	Introduction to Multiplication	K
3	Multiplication and Division of Whole Numbers	K
4	Properties	L
9	Techniques of Multiplication and Division	M
5	Geometry	L
6	Fractions	L
7	Techniques of Addition and Subtraction of Whole Numbers	L
12	Techniques of Multiplication and Division II	N
10	Operations on Fractions	M
8	Measurement	M
14	Techniques of Multiplication and Division III	N
11	Geometry II	M
13	Integers	N

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But subtraction and division are not associative operations. These sentences are not true:

$(8 - 4) - 2 = 8 - (4 - 2)$; $(8 \div 4) \div 2 = 8 \div (4 \div 2)$;

$(12 - 6) - 2 = 12 - (6 - 2)$; $(12 \div 6) \div 2 = 12 \div (6 \div 2)$

Zero and one. In Unit 3 the pupils found that certain numbers have special properties when used in addition or multiplication. We call these properties:

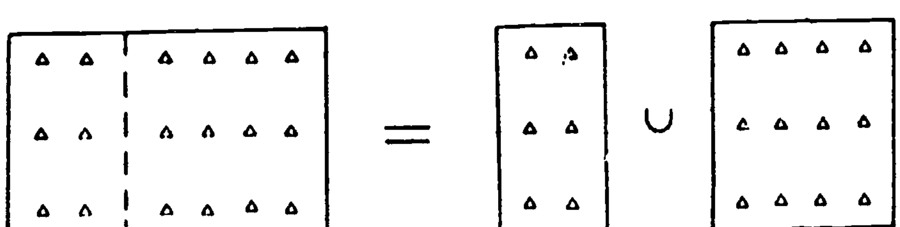
Addition property of 0. For any whole number a, the sentence $0 + a = a$ is true. (For example, $0 + 7 = 7$; $0 + 81 = 81$.)

Multiplication property of 1. For any whole number a, the sentence $1 \times a = a$ is true. (For example, $1 \times 6 = 6$; $1 \times 37 = 37$.)

These properties of 0 and 1 are used over and over again in the development of arithmetic.

Distributive property. The children will discover a property that connects the two basic operations, addition and multiplication. It is called a distributive property. They find that multiplication is distributive over addition, but that addition is not distributive over multiplication.

When a rectangular array is separated into two rectangular arrays, the operations of multiplication and addition are suggested.



$$3 \times (2 + 4) = (3 \times 2) + (3 \times 4)$$

In the same way, the pupils find that both of the following sentences are true.

$$4 \times (1 + 6) = (4 \times 1) + (4 \times 6)$$

$$3 \times (5 + 2) = (3 \times 5) + (3 \times 2)$$

That is, multiplying the sum of two numbers by a number gives the same result as multiplying the first addend by that number and then the second addend by that number and then finding the sum of these products. We say:

> Multiplication is distributive over addition using whole numbers because
>
> $$a \times (b + c) = (a \times b) + (a \times c)$$
>
> is a true sentence for any whole number a, any whole number b and any whole number c.

But when the pupils try to distribute addition over multiplication, they find that these sentences are not true:

$$3 + (5 \times 2) = (3 + 5) \times (3 + 2)$$

$$4 + (2 \times 3) = (4 + 2) \times (4 + 3)$$

Thus, addition is not distributive over multiplication.

In all the arithmetic and later mathematics that the pupils will study, the property that multiplication is distributive over addition will be called simply the distributive property. It is the most powerful and useful property available to the pupil in all he discovers about the techniques of arithmetic.

The pupils should begin to understand what these properties are and how they can be used, but not much emphasis need be placed on the names of the properties. The words "commutative", "associative" and "distributive" should be mentioned, but the pupils should not be drilled on the words. The emphasis should be on the understanding of meanings of the properties rather than on the names.

In this unit there are two minor ideas that are important:

1. The pupils need to learn the use of symbols for grouping. For example, the parentheses in the sentence

   $3 \times (4 + 2) = n$

   tell us that the numerals inside the parentheses name one number. Then

   $3 \times (4 + 2)$ means 3×6.

   As another example,

   $(3 \times 4) + 2$ means $12 + 2$.

2. The children must understand that when the same letter occurs several times in a sentence, that letter refers to the same number each time it occurs in the sentence. For example the sentence $n + (2 + n) = 6$ becomes the false sentence

   $3 + (2 + 3) = 6$

   if we learn that n is a numeral for 3.

   If $n = 2$, then the sentence $n + (2 + n) = 6$ becomes

   $2 + (2 + 2) = 6$ (a true sentence).

The emphasis in this unit is on understanding the meaning of these properties and not on the names of the properties. In the activities, strange operations will be used to extend the pupils' understanding of the properties discussed.

TOPIC I: MATHEMATICAL SENTENCES

OBJECTIVES:

1. To study mathematical sentences that use several number operations.
2. To develop skill in solving story problems.

VOCABULARY: none

SYMBOLS: < , >

MATERIALS: at least 30 counters for each child; pupil pages 1 - 13

Activity 1: (Oral) Regrouping sets and renaming numbers

Objective:

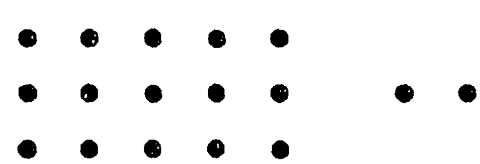
Children can write mathematical sentences for different arrangements of counters.

Materials: 20 counters per child

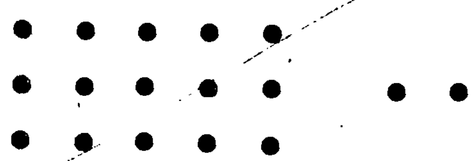
Teaching Procedure:

Note: The use of counters and arrays to establish mathematical ideas is important. It may be necessary to use counters for several days or at intervals throughout the unit.

Let each pupil place a set of exactly 17 counters on his desk. Ask the pupils to arrange some of their counters in a rectangular array and to place the remaining counters in a group beside it. On the board, draw a picture of one possible arrangement for the pupils to see.



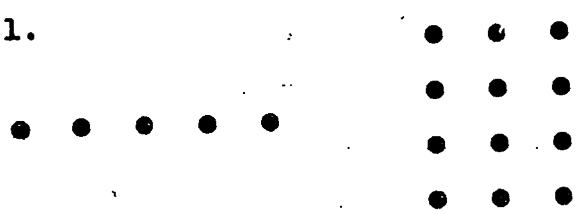
Ask the pupils what product is shown by the dots in the rectangular array. They must say, (3 x 5). Ask, "How many are left over?" (2) Help the pupils make a mathematical sentence that describes the arrangement of the dots on the board. Write the mathematical sentence below the drawing on the board:



17 = (3 x 5) + 2

Ask the pupils to think of other ways of arranging the same number of counters, putting some of the counters in a rectangular array. Let each pupil make a mathematical sentence to describe his arrangement and write the sentence in his book. Let the pupils work in pairs and check each other's work. If they have difficulty, draw pictures on the board of other possible arrangements of the 17 counters. Ask the pupils in turn to write a mathematical sentence below each of the arrangements.

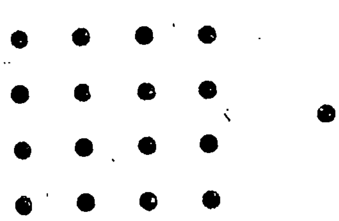
1.



5 + (4 x 3) = 17

or 17 = 5 + (4 x 3)

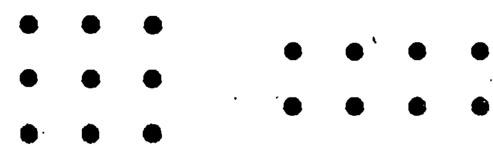
2.



(4 x 4) + 1 = 17

or 17 = (4 x 4) + 1

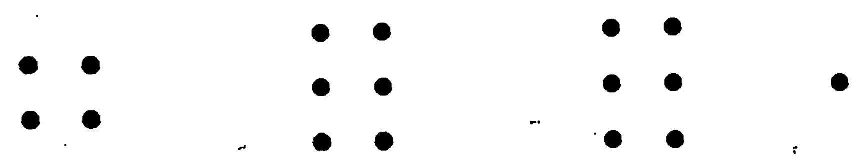
3.



(3 x 3) + (2 x 4) = 17

or 17 = (3 x 3) + (2 x 4)

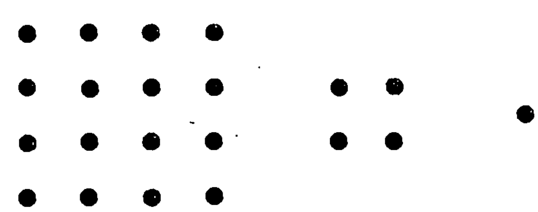
4.



(2 x 2) + (3 x 2) + (3 x 2) + 1 = 17

or 17 = (2 x 2) + (3 x 2) + (3 x 2) + 1

Note: Make sure that the pupils use the parentheses correctly to show which operations are performed first. For example, the arrangement



is properly described by the sentence:

(4 x 4) + (2 x 2) + 1 = 21,

but the arrangement is not described by the sentence

4 x 4 + 2 x 2 + 1 = 21.

The latter sentence does not tell which operations are done first, and a pupil might think it means

4 x (4 + 2) x (2 + 1) = 21,

which is a false sentence.

Let each pupil get 3 more counters so they each have 20. Let them arrange their 20 counters in different rectangular arrays as they did with the 17 counters. Ask, "Can all of the counters be put in a rectangular array? Can some of the counters be put in an array, with some left over?" Let the pupils arrange the counters in many ways and write a mathematical sentence for each arrangement. Some of the sentences they may write are these:

$10 \times 2 = 20$ or $2 \times 10 = 20$

$20 = 5 \times 4$ or $4 \times 5 = 20$

$(4 \times 4) + 4 = 20$ or $20 = 4 + (4 \times 4)$

$20 = (5 \times 2) + (2 \times 5)$ or $(2 \times 5) + (5 \times 2) = 20$

$(3 \times 5) + 5 = 20$ or $20 = 5 + (5 \times 3)$

$20 = (4 \times 2) + 2 + (4 \times 2) + 2$ or $2 + (2 \times 4) + 2 + (4 \times 2) = 20$

$(6 \times 3) + 2 = 20$ or $2 + (3 \times 6) = 20$

$20 = (9 \times 2) + 2$ or $2 + (9 \times 2) = 20$

Encourage each pupil to show at least three different arrangements and write the mathematical sentence for each.

Allow the pupils to choose any number of counters. For example, one pupil may choose 36. Let him arrange his 36 counters in arrays in different ways and write sentences such as these:

$12 \times 3 = 36$ or $(6 \times 2) + (6 \times 2) + (6 \times 2) = 36$ or $36 = (3 \times 6) + (6 \times 3)$

Activity 2: Solving Mathematical Sentences

Pupil pages 1, 2

Objectives:

1. Children can solve a mathematical sentence using counters when needed.
2. Given an incomplete sentence containing parentheses and involving one operation, children can choose a symbol (< , > , or =) that makes the sentence true.

Materials: 36 counters or flannel board and 36 figures for flannel board, 56 counters per pupil (if necessary)

Teaching Procedure:

On pupil page 1, the pupils work with sentences in which there are several operations. Use some or all of the seven examples suggested below to help the pupils before they begin work in their books. Write each example on the board as it is used.

Example 1: Ask the pupils to find the number that makes this sentence true:

$(6 \times 3) + n = 20$

$18 + n = 20$

n is ______.

Let the pupils use counters or figures on a flannel board to help them solve the problem. They begin with 20 counters. Since 6 x 3 is 18, they know that 18 counters plus n counters must be 20 counters. That is, they must add 2 to 18 to get 20. Therefore, n is 2.

<u>Example 2</u>: Ask, "What number makes this sentence true?"

$15 = (b \times b) + 6$

b is _____.

The pupils must know that b is the same number each time it is in the sentence.

Let the pupils talk about the sentence. They might say, "We have 15 counters. We should arrange the counters into two groups. One group contains 6 counters. We must arrange the other group into an array with b rows and b members in each row," as below:

Write 3 x 3 under the first array. Therefore, b is 3.

Or they might say, "The sentence tells us that we must add (b x b) to 6 to get 15. That means that b x b is 9, b x b is 3 x 3 and b is 3."

They should decide that if b is 3, then the sentence

$15 = (b \times b) + 6$ becomes $15 = (3 \times 3) + 6$,

which is a true sentence. Thus, the number that makes this sentence true is 3.

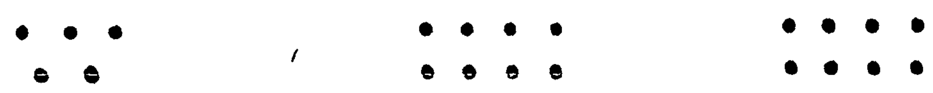
<u>Example 3</u>: Ask the pupils to find the number that makes this sentence true:

$(n + 5) + n = 21$

n is ______.

In this sentence, both n's are the same number.

Pupils might say, "We have 21 counters to begin with. We arrange them into three groups to represent each addend. Two addends are the same number n. Therefore, two groups must have the same number of members." Pupils might arrange counters as below:



Let the pupils guess what number n is and try it. A pupil might think and say, "If n is 5 then

$$(n + 5) + n = 21 \text{ becomes } (5 + 5) + 5 = 21,$$

which is not true because (5 + 5) + 5 = 15. n is not 5." Another pupil might then try a greater number, 9. He may say, "If n is 9, then

$$(n + 5) + n = 21 \text{ becomes } (9 + 5) + 9 = 21,$$

which is not true because (9 + 5) + 9 is 23. n is not 9."

Another pupil might try a number less than 9. He may say, "If n is 8, then

$$(n + 5) + n = 21 \text{ becomes } (8 + 5) + 8 = 21,$$

which is true. n is 8."

In this way, the children find that a number, n, which makes that sentence true, is 8.

$$(n + 5) + n = 21$$

n is 8.

Examples 4 - 7: Write the following examples on the board and let pupils in turn solve the sentences:

$28 = (k \times k) + 3$	$(d + 7) + d = 17$
k is **5**.	d is **5**.
$(6 \times h) + (3 \times h) = 36.$	$13 + (n \times 5) = 18$
h is **4**.	n is **1**.

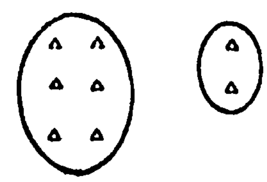
Let the pupils talk about ways of solving sentences. Encourage the use of counters if pupils have trouble solving the problem verbally.

Ask the pupils to turn to page 1 and find the numbers that make each sentence true. Under each sentence let them draw dots in arrays to show what they did to find the answer.

For example, the first exercise will look like this:

$(3 \times 2) + a = 8$

$a = 2.$



(Let the pupils use the figure at the top of page 1 as an example.)

Give the pupils help in using parentheses. Ask them to tell which of the symbols < or > or = makes the following sentence true:

$(3 \times 2) \times 2$ __________ $3 \times (2 \times 2)$

In this sentence the symbol = makes a true sentence

because $(3 \times 2) \times 2 = 6 \times 2 = 12$

and $3 \times (2 \times 2) = 3 \times 4 = 12$.

Write this sentence on the board and ask the pupils to make it true:

$4 + (5 \times 3)$ __________ $(3 \times 4) + 6$

The pupils find the number named on the left and the number named on the right. They choose a symbol (< or > or =) that makes a true sentence. In this example, they choose the symbol > because

$4 + (5 \times 3)$ is 19,

and $(3 \times 4) + 6$ is 18.

and $19 > 18$, which is true.

Let the pupils turn to page 2 and write a symbol in each blank space to make the sentences true. Ask the pupils to work all the examples on the left, then all the examples on the right, then to choose and write the symbol of comparison.

Activity 3: Adding and multiplying

Pupil pages 3, 4

Objective:

Children can write the simplest numeral for a numeral containing parentheses and the signs for different operations.

Teaching Procedure:

In this activity the pupils will use addition and multiplication in the same exercise. They will use parentheses to group several numerals together to represent one number.

Write on the board these two numerals:

(3 + 5) x 4 3 + (5 x 4)

Ask the pupils to read each. (The first is read "The number 3 + 5 times 4"; the second is read "3 plus the number 5 times 4".) Ask what number is named by the first numeral. (8 x 4, or 32) Say, "32 is the simplest name for (3 + 5) x 4." "What number is named by the second numeral?" (3 + 20, or 23) Say, "23 is the simplest name for 3 + (5 x 4)."

This reviews the meaning of the parentheses as signs of grouping. Remind the pupils that these two numerals written on the board have the same symbols in them, but the numerals are not grouped the same way. Help the pupils understand the importance of the grouping.

Let the class turn to page 3, and write a simpler name for each numeral. This will give the pupils practice in using the grouping signs (parentheses). Page 4 is a supplementary page.

Activity 4: Mathematical Sentences

Pupil page 5

Objective:

Given an incomplete sentence containing parentheses and involving different operations, children can choose a symbol (<, > , or =) that makes the sentence true.

Teaching Procedure:

Write on the board: $4 \times (5 + 2)$ ____ $(4 \times 5) + 2$. Point out that the line shows that a symbol is missing in the sentence. Ask, "What symbol can you write to make the sentence true?" Guide the children to decide that the missing symbol is > because

$$4 \times (5 + 2) = 4 \times 7 \quad \text{and} \quad (4 \times 5) + 2 = 20 + 2$$

$$= 28 \qquad\qquad = 22$$

$$\text{and } 28 > 22$$

Emphasize the importance of first finding the numbers shown in the parentheses. Get a child to complete the sentence.

$4 \times (5 + 2)$ ____ $(4 \times 5) + 2$

Write another sentence such as:

$8 + (14 \div 2)$ ____ $(14 \div 2) \times 8$

Guide the children to see that the missing symbol is < because:

$$8 + (14 \div 2) = 8 + 7 \quad \text{and} \quad (14 \div 2) \times 8 = 7 \times 8$$

$$= 15 \qquad\qquad = 56$$

$$\text{and } 15 < 56$$

Get a child to complete the sentence:

$$8 + (14 \div 2) \_\_\_\_ (14 \div 2) \times 8$$

Use other sentences to help the children understand that the numeral inside the parentheses is a name for one number. Emphasize again the importance of first finding the number shown inside the parentheses and then doing the other operations.

Ask the children to complete the sentences on pupil page 5 making the sentences true by writing the missing symbols.

It may be well to suggest that the pupils work the examples on the left first, then those on the right, then choose which symbol makes the sentence true.

Activity 5: Solving problems

Pupil pages 6 - 10

Objective:

Given a story problem involving several different operations, children can write a number sentence for the problem, can find the number that makes the sentence true, and can answer the question asked by the problem.

Teaching Procedure:

Note: In this mathematics program there is a definite plan for teaching the solving of word problems. It has the following steps for pupils to follow:

1. Know what the problem is about and the question asked in the problem.
2. Write a mathematical sentence for the problem, with a letter representing the missing number.

3. Find a number that makes the mathematical sentence true.

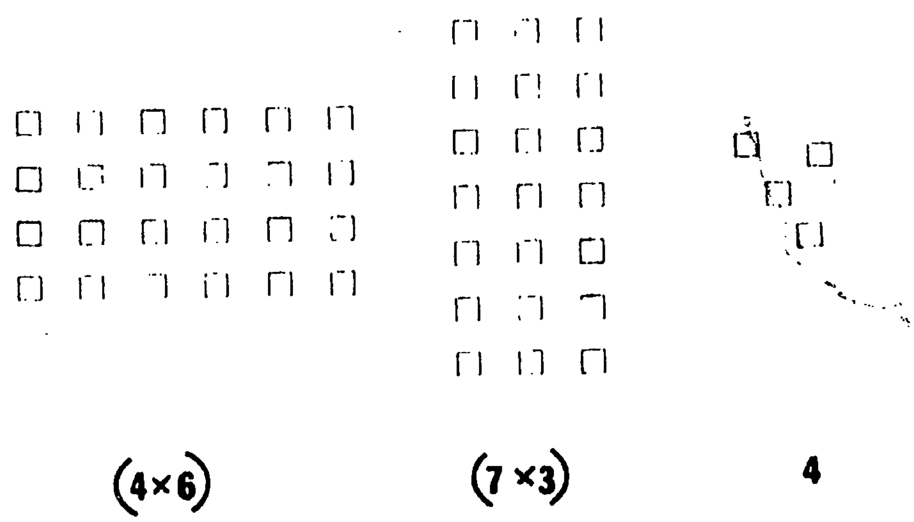
4. Use the number to answer the question in the problem.

Follow this plan as you help the pupils solve word problems. It is the plan that is used in all levels of mathematics in solving problems.

The purpose of this activity is to help the pupils learn to solve problems stated in words. The problems use several different operations. Before the pupils begin the exercises, help them solve some sample problems. Tell this problem to the pupils and let them talk about it:

Problem 1: Ronald collected stamps. Four girls in his class each gave him 6 stamps. Seven boys in his class each gave him 3 stamps. The teacher gave him 4 stamps. How many stamps did Ronald collect?

Let the pupils talk about the problem. Get them to say what question it asks. Write the question on the board. As they talk about the problem they may say Ronald got stamps from 4 girls (4 x 6), from 7 boys (7 x 3) and from the teacher (4). They may suggest drawing a picture like this:



Guide the pupils to then make the mathematical sentence:

$(4 \times 6) + (7 \times 3) + 4 = s$

Let them talk about what each part (4 x 6; 7 x 3; 4; and s) of the mathematical sentence means. Get them to rename the numbers in the mathematical sentence and say

s is 49.

Ronald collected 49 stamps.

Let the pupils talk about how they answered the question in Problem 1. (We knew the problem and the question it asked; we wrote a mathematical sentence; we found a number to make the mathematical sentence true; and we answered the question asked in the problem.)

Go on in a similar way with Problem 2. The work that you and the pupils may write on the board is given.

Problem 2: One morning a storekeeper had many eggs in his basket for sale. Five men each bought 8 of the eggs. Four women each bought 7 of the eggs. That night the storekeeper looked in his basket. He saw 12 eggs left. How many eggs were in his basket in the morning?

How many eggs were in the basket in the morning?

$(5 \times 8) + (4 \times 7) + 12 = e$

e is 80.

There were 80 eggs in the basket.

Let the pupils open their books to pages 6 - 10. There are ten word problems on these pages. Each problem has questions to be answered. Let the pupils tell what numbers to write in the blank spaces to make the sentences true. Let them write the following for each of the exercises: the mathematical sentence; a statement telling what the missing number in the mathematical sentence is; and an answer to the question asked in the problem. This is the way their page may look:

1. $3 \times 6 = e$

e is 18.

Mary has 18 eggs.

2. $(3 \times 6) + 7 = a$

a is 25.

Now Mary has 25 eggs.

3. $s = (5 \times 10) - 8$

s is 42.

John has 42 oranges left.

4. $b = (7 \times 5) - 8$

b is 27.

There are 27 birds left.

5. $(4 \times 3) + (2 \times 4) = n$

n is 20.

There are 20 sides.

6. $(4 \times 8) + (2 \times 5) = n$

n is 42.

They caught 42 fish.

7. $e = (5 \times 7) + (3 \times 3)$

e is 44.

There are 44 small squares.

8. $(7 \times 4) + (4 \times 2) = u$

u is 36.

The animals have 36 legs.

9. $(24 \div 4) - 2 = s$

s is 4.

John has 4 marbles left.

10. $g = (24 - 6) \div 2$

g is 9.

Each boy has 9 fish.

<u>Activity 6</u>: Story problems

Pupil pages 11 - 13

<u>Objective</u>:

Given a story problem involving several different operations, children can write a number sentence for the problem, can find the number that makes the sentence true, and can answer the question asked by the problem.

Materials: one piece of paper for each child

Teaching Procedure:

Before doing the exercises on pupil pages 11 - 13, give a few problems as examples and discuss ways to solve them.

Example 1: Ken bought 2 bags of oranges. Each bag contained 6 oranges. He shared these oranges equally among 4 boys. How many oranges did each boy get?

Ask, "What question does the problem ask?" (How many oranges did each boy get?)

Guide the children to write a mathematical sentence for the problem by helping them: decide how many oranges Ken bought (Two bags of 6 oranges are shown as (2 x 6)); think about what Ken did with oranges (shared them equally among 4 boys); and discuss the problem and its mathematical sentence until a sentence is written.

$$(2 \times 6) \div 4 = b$$

Let the pupils find what number b makes the sentence true. (Since $(2 \times 6) \div 4 = 3$, b is 3.) Ask, "What is the answer to the question in problem?" (Each boy gets 3 oranges.)

Example 2: Nina's family invited Amy's family and Herman's family for a birthday party. There were 22 people in all. If 10 people were members of Nina's family and 8 were members of Amy's family, how many were members of Herman's family?

Ask the children what question the problem asks. They can let s be the number of people in Herman's family. Guide the pupils to write the mathematical sentence by helping them decide that at the party there were $(10 + 8 + s)$ people in all. Let someone write the sentence:

$$(10 + 8) + s = 22$$

Ask, "What is (10 + 8)? (18) What number s makes the sentence true? (4) "What is the answer to the question in the problem?" (There were 4 people from Herman's family at the party.)

Let the pupils open their books to pupil pages 11 - 13 and answer the questions about the story problems. Get the pupils to write the following on another worksheet for each exercise: the mathematical sentence, the missing number, and the answer to the question asked in the problem. This is the way their paper may look for pupil pages 11 - 13:

1. $(7 \times 4) - 6 = n$

   n is 22

   22 animals were put back on the shelf.

*2. $(n + 7) \div 13 = 2$

   n is 19

   There were 19 peaches on the ground at first.

3. $(32 \div 4) + v = 15$

   v is 7

   Each friend had 7 marbles at first.

4. $(n \times 8) - 14 = 42$ or
   $n \times 8 = 14 + 42$

   n is 7

   There were 7 boys.

5. $(r \div 2) - 2 = 8$

   r is 20

   The boys have 20 beans together.

*6. $(n \times 4) + (n \times 3) = 56$

   n is 8

   The teacher drew 8 squares and 8 triangles.

*Starred problems are challenge problems.

TOPIC II: OPERATIONS

OBJECTIVES:

1. To develop the concept of operation.
2. To make new operations by use of an operation machine.

VOCABULARY: operation machine, number pairs, operation

SYMBOLS: +, −, x, ÷

MATERIALS: cardboard operation machine; thirty-seven 3" by 3" numeral cards (from 0 to 36); four 3" by 3" symbol cards (+, −, x, ÷); pupil pages 14 - 17

Activity I: Recognizing operations and making new operations (oral)

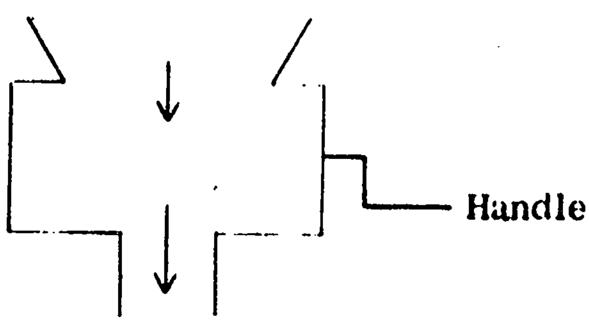
Objectives:

1. Given a pair of whole numbers and the corresponding answer, children can identify the operation (addition or subtraction) the machine is performing.
2. Given a pair of whole numbers and the operation (addition or subtraction), children can find the answer for the operation machine.

Materials: cardboard operation machine if desired; thirty-seven 3" by 3" numeral cards from 0-36; four 3" by 3" symbol cards (+, -, x, :)

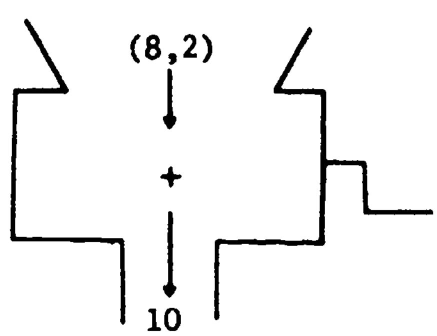
Teaching Procedure:

On the board draw a box like this:



Say, "Pretend that this is a machine. It is an operation machine. See the handle. We put two numbers into the top of the machine." Write the numerals (8, 2) above the machine. (See the drawing below.) "When we turn the handle, the machine does something to the numbers, and one number pops out of the bottom." Pretend you are turning the handle and write 10 below the box. (See drawing below.) "What did the machine do to our pair of numbers to get the number 10?" If no one says "add", or even if they do, erase the numerals. Then write (4, 3) above the box. Say, "When we turn the handle the machine takes the numbers 4 and 3 and out pops the number 7." Write "7" at the bottom. "What did the machine do to our pair of numbers to get the number 7?" Continue putting pairs of numbers at the top and their sum at the bottom.

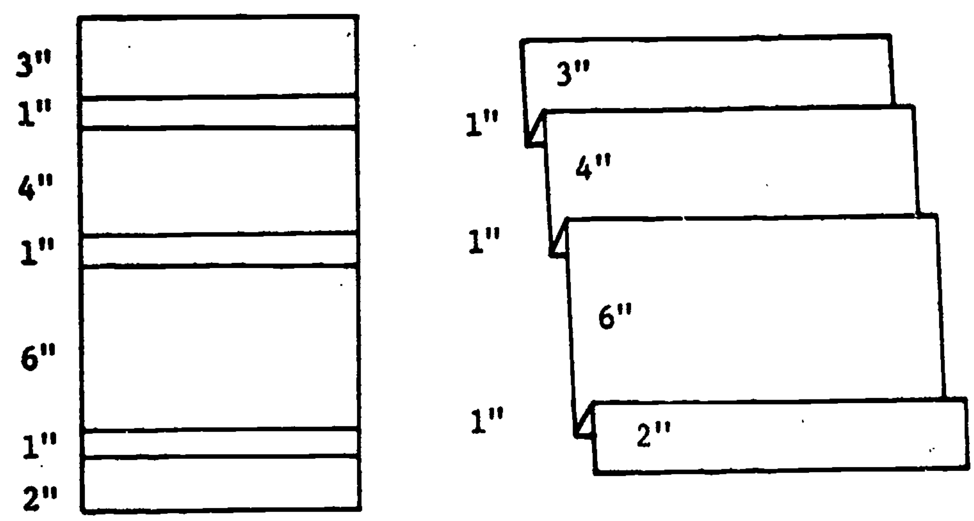
Let the class think about and discuss these questions until they decide that the machine adds the pairs of numbers. Say, "Yes, this is an addition machine." Write a plus sign on the middle of the machine. Now the board looks like this for the first example:



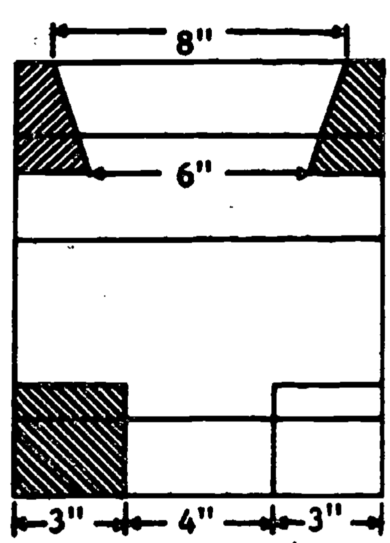
Erase the numerals and write the pair (6, 3) above the machine. Pretend you are again turning the handle. Ask, "What number will pop out of the addition machine?" "9" Continue this with other number pairs. Each time, ask what number will come out of the addition machine. Let the pupils in turn write the numerals for number pairs they want to put into the machine and for the number that pops out.

If the teacher desires to have an operation machine, directions for making one are as follows:

To make an operation machine for the pupils, use a 10" by 18" piece of cardboard. Mark these points along the 18" sides: 3", 1", 4", 1", 6", 1", 2", as shown at the left below. Then fold as shown at the right.



Draw a picture of an operation machine on the folded cardboard and cut off the regions that are shaded in the figure below:



Make 3" by 3" numeral (from 0 to 36) and symbol (+, -, x, :) cards to be used with the machine. The cards fit into the folds of the machine.

Fasten the cardboard operation machine to the wall. Place the card marked "x" in the fold for the operation symbol and say, "This is a multiplication machine." Put two numeral cards, for example, 3 and 6, in the top fold and say, "If we put the pair (3, 6) into the multiplication machine and turn the handle, what number comes out?" When the pupils say "18", put a numeral card for 18 in the bottom fold.

Continue this type of activity with the multiplication machine. Let a pupil put a pair of numeral cards at the top of the machine and ask another pupil to pretend he is turning the handle. Let still another pupil tell what number pops out and place the numeral card at the bottom.

After the pupils have used the multiplication machine for a while, remove the "x" card and replace it with the "-" card. Say, "Now we have a subtraction machine. We will put these two numbers into the subtraction machine. Put the numeral cards (7, 2) at the top of the machine. (If you use any other pair of numbers, make sure that the first is greater than or equal to the second.) Ask someone to come up and pretend to turn the handle of the machine. Let a pupil tell what number pops out and let him come up and put the numeral for the number at the bottom of the machine. No matter what numeral he puts at the bottom, let the rest of the class decide whether he chose the correct card. Get pupils to tell why the result is correct or incorrect (5 pops out because 7 minus 2 equals 5, or 5 must be added to 2 to get 7) When a pupil puts a number pair like (2, 7) into the machine, the pupils should say, "No number pops out." You say, "Our subtraction machine is a whole number machine and, therefore, will not work for this pair of numbers."

Continue this activity until the class has some experience in using the subtraction machine.

Activity 2: Finding the operation

Pupil pages 14 - 17

Objectives:

1. Given a pair of whole numbers and the corresponding answer, children can identify the operation (addition, subtraction, multiplication, or division) the machine is performing.
2. Given a pair of whole numbers and the operation (addition, subtraction, multiplication, or division), children can find the answer for the operation machine.
3. Given the operation (addition, subtraction, multiplication, or division) and the answer, children can find a pair of whole numbers the operation machine is using.

Teaching Procedure:

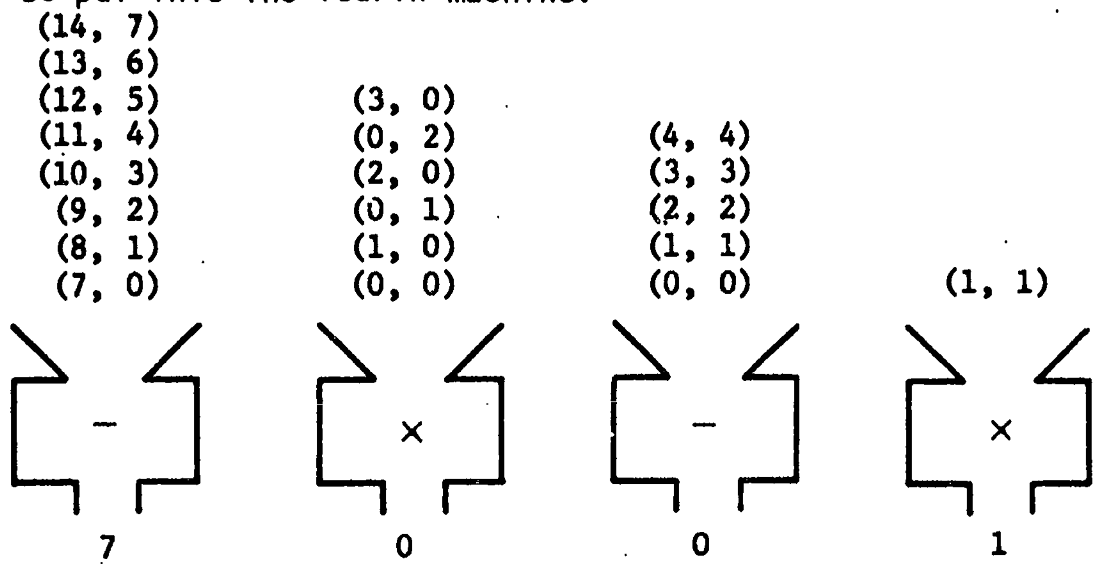
Let the pupils use the operation machine as they did in the previous class activity.

Remove the "-" card from the operation machine. Say, "I put a number pair (8, 2) into the operation machine. "Do we know what operation the machine will do?" (No, there is no symbol to tell us.) Say, "We will see what happens." Get a pupil to turn the handle. Say, "A number is popping out." Put a numeral card for 4 in the bottom of the machine. "The machine gave us 4. What kind of operation machine is this?" Let the pupils discuss this question. If they cannot answer, use other number pairs. Put (8, 4) in the top of the machine and 2 in the bottom. Again ask what kind of machine it is. Continue until the pupils decide that it is a division machine.

Place a symbol card for ÷ in the proper place. Go on putting different pairs of numeral cards in the top and asking what number pops out of the bottom. For example, put (6, 2), (9, 3), (12, 3) in the top of the machine. Also try the number pair (2, 6). Guide the pupils to say, "No number pops out." You say, "The division machine will not work for this number pair because the answer is not a whole number."

Vary the activity. Sometimes put a pair of numerals in the top and a numeral at the bottom and ask what operation the machine is performing. Other times, put an operation card in the middle and a pair of numeral cards in the top and ask what number pops out at the bottom. Choose the cards so the pupils find all of the four operations. Ask the pupils to turn to pages 14 - 17 in their books and look at the pictures of the operation machines shown there. In some of the pictures of the machines the numeral at the bottom is missing. In some the operation symbol is missing. In others the pair of numerals at the top is missing. Let the pupils talk about these operation machines and decide what the missing numerals or operation symbols are.

The pupils will decide that exactly one numeral is needed at the bottom of each machine. Notice that there is one exception. The last machine on page 16 will not give a whole number because 6 ÷ 4 is not a whole number. When the numeral at the bottom of the machine is given, there may be many different pairs of numbers that can be put at the top to give that numeral at the bottom. For example, look at the fourth row of machines shown on page 14. There are many pairs, such as those shown below that can be put into the first three machines, but only one pair can be put into the fourth machine.



In the machines with the operation signs missing, the pupils will decide what the operation is. In some cases, there may be two possibilities. For example, on page 14, third row, the last machine can be either a subtraction or a division machine.

Let the pupils do the exercise and put in all the numerals and symbols. If there are several number pairs, get them to choose one. Let the pupils talk about these exercises.

Make sure the pupi' do all the exercises on pages 14 - 17 over a period of several days. The exercises help to develop an idea of what an operation is. They also are good review exercises.

Activity 3: Finding a new operation

Objective:

- Given a pair of whole numbers and the corresponding answer, children can identify the new operation the machine is performing.

Materials: Cardboard operation machine used previously

Teaching Procedure:

Say, "Here is a new operation machine." Put the following pairs of numeral cards at the top of the cardboard operation machine, with corresponding numeral cards at the bottom:

(4, 3)	(5, 2)	(1, 8)	(7, 2)	(3, 4)
4	5	1	7	3

Tell the class that in order to keep a record of the numbers that pop out of the machine for this operation, we write:

(4, 3) ⟶ 4

(5, 2) ⟶ 5

(1, 8) ⟶ 1

(7, 2) ⟶ 7

(3, 4) ⟶ 3 and so on.

Each time you put a number pair at the top and a number at the bottom write the result on the board as follows:

(5, 7) ⟶ 5

The pupils will be puzzled by this operation machine. It uses an operation strange to them. They may doubt that the machine is really performing an operation. Ask, "Does the machine give us a number for each pair of numbers we put into it?" (Yes) Say, "Then the machine is doing an operation. Our operation gives us one number for each pair of numbers." Review the addition and multiplication operations to show that with these operations we get one number for each pair of numbers.

Talk to the pupils again about the new operation. Put more pairs of numbers into the machine:

$(0, 5) \longrightarrow 0$

$(4, 1) \longrightarrow 4$

Ask, "If I put the pair (3, 6) into the machine, what number will come out?" If someone says "3" write $(3, 6) \longrightarrow 3$. Ask the pupil who answered "3" to keep the operation a secret until the others find it.

If no one says "3" write in "3". Say and write the number pair (6, 3). Ask for a number for this pair. If someone says "6" write this numeral. Continue until the class sees that this operation machine always gives the first of the pair of numbers that is put into the machine.

Since this operation is new to the pupils, let someone give it a name and a symbol. A pupil may suggest that it be called "Firsting" because it always gives the first of the numbers in the pair and give it a symbol as F. Then write:

$(3, 6) \xrightarrow{F} 3$

$(2, 7) \xrightarrow{F} 2$

$(6, 3) \xrightarrow{F} 6$ and so on.

Let the pupils use this machine until they see how it works. Ask the pupils to use the name and symbol they made up.

Note: This operation and the others to follow are not important. The pupils do not need to remember them. The operations are used here only to help pupils learn about operations.

Say, "I am thinking of still another operation." This time you do not need to use the cardboard operation machine. Write the pairs and the resulting numbers as follows:

$(1, 2) \longrightarrow 2$

$(3, 2) \longrightarrow 3$

$(7, 3) \longrightarrow 7$

$(4, 6) \longrightarrow 6$

Ask a pupil to give a pair of numbers. If he gives (3, 3), write

(3, 3)

and ask what number is given by the operation. If a pupil says "3", write it after the pair. Go on in this way until the class discovers that this new operation always gives the greater of the two numbers if they are different and the number if they are the same. Again you can let the class give the operation a name and a symbol. They may call it the "greater" operation and use the symbol G. Then

$(5, 2) \xrightarrow{G} 5$

$(6, 6) \xrightarrow{G} 6$

$(1, 3) \xrightarrow{G} 3$ and so on.

Note: If the pupils enjoy this activity, continue with some other new operations for them to discover. For example:

1. $(1, 2) \xrightarrow{\triangle} 4$
   $(3, 1) \xrightarrow{\triangle} 4$
   $(4, 2) \xrightarrow{\triangle} 10$
   $(2, 6) \xrightarrow{\triangle} 18$

   (The result of this operation is always the product of the two numbers plus the second number:
   $(4, 3) \xrightarrow{\triangle} (4 \times 3) + 3$.)

2. $(0, 2) \xrightarrow{*} 3$
   $(4, 1) \xrightarrow{*} 2$
   $(3, 7) \xrightarrow{*} 8$
   $(5, 0) \xrightarrow{*} 1$

   (The result of this operation is always one more than the second number.)

3. $(4, 3) \xrightarrow{\odot} 11$
   $(0, 1) \xrightarrow{\odot} 1$
   $(1, 0) \xrightarrow{\odot} 2$
   $(5, 5) \xrightarrow{\odot} 15$
   $(3, 7) \xrightarrow{\odot} 13$

   (The result of this operation is always 2 times the first number plus the second number. For example,
   $(3, 4) \xrightarrow{\odot} (2 \times 3) + 4$.)

Encourage some of the pupils to make up their own operations. Play the game called: What is the Operation? It is played as follows:

Teacher: Tell me a pair of numbers both less than 6.

John: 2 and 4

Teacher writes on the board: $(2, 4) \longrightarrow 12$

Joseph: 0 and 5

Teacher writes: $(0, 5) \longrightarrow 10$.

Mary: I think I know the operation. If I say (1, 1), you will write 4.

Teacher: That is right, Mary. We have a secret. You take my place and see if other pupils can find the operation.

Mary: Tell me another pair.

Bob: 3 and 2

Mary writes: (3, 2) ⟶ 10

Bob: I know the operation. If I say 2 and 1, you will write 6.

Mary: How do you get 6 from 2 and 1?

Bob: I find 2 times the sum of 2 and 1.

The game can be placed from the beginning with a pupil in the place of the teacher. Let a pupil think of an operation and write some examples on the board.

TOPIC III: PROPERTIES OF OPERATIONS

OBJECTIVES:

1. To develop the meaning of property of an operation.
2. To discover some properties the basic operations have:
   a. Addition and multiplication have the commutative property.
   b. Addition and multiplication have the associative property.
   c. Multiplication is distributive over addition.
   d. Zero and one have special properties under addition and multiplication.

VOCABULARY: commutative property, property, associative property, distributive property

MATERIALS: Six cardboard operation machines, flannel board, 25 figures for flannel board, yarn or string, 25 counters for each pupil, a piece of string for each pupil, pupil pages 18 - 26

Activity 1: Commutative properties of addition and multiplication

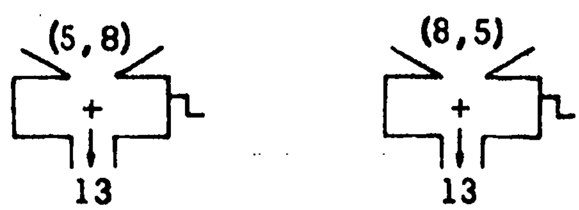
Pupil page 18

Objective:

Children can use the commutative property for addition and multiplication to determine if a sentence is true or false and to find a missing number.

Teaching Procedure:

Draw an addition machine on the board. Ask a pupil to name a pair of whole numbers, say (5, 8). Write the numerals for the numbers on the picture. Say, "I put in the number 5, then the number 8." Pretend to turn the handle. Ask, "What number pops out of the machine?" (13) Draw another addition machine. Get a child to put the same numbers in this machine but change the order. (The 8 goes in first, then the 5.) Let another child pretend to turn the handle and show the number that pops out. (13) The board will look like this:



Say, "You changed the order of the addends, but the same number popped out. (13) Is this true for every pair of numbers you add?" (Yes) Let the children try other pairs of numbers. Guide the children to show this idea in mathematical sentences. For example:

$8 + 5 = 5 + 8$

Repeat the activity with other pairs of whole numbers, if necessary, and finally lead them to see that for any two whole numbers _a_ and _b_:

$$a + b = b + a$$

Remind the children that this is the commutative property of addition. That is, the sum of two whole numbers is the same no matter in which order you add them.

Repeat the above activity but use a multiplication machine. In a similar way lead the children to see that for any two whole numbers _a_ and _b_,

$$a \times b = b \times a$$

Remind them that this is a property of multiplication and is called the commutative property of multiplication.

Let the pupils turn to pupil page 18. Let them talk about a few of the exercises. Guide the pupils to see that you do not have to do the operations each time to find the missing number. Let them use the commutative property to find the missing number by inspection. For example, in $9 + 7 = 7 + b$, if _b_ is 9, the sentence is true. (Commutative property of addition)

After the children have finished the exercises, let them talk about their answers and look for patterns. Get them to make true sentences for those sentences that are false. Guide them to use their knowledge of the properties.

Activity 2: The commutative property

Pupil page 19

Objectives:

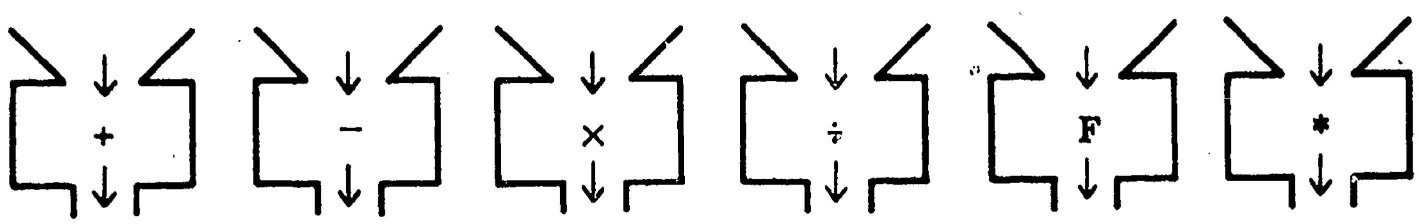
1. Children can determine if operations like subtraction, division, and firsting have the commutative property.
2. Children can use operation machines to show what it means for an operation to have the commutative property.

Materials: six cardboard operation machines

Teaching Procedure:

The purpose of the activity is to help the pupils identify a property of operations called the commutative property. Let the pupils again think about many operations as they use their operation machines.

Use several cardboard operation machines or draw them on the board as shown below. (Using the * operation is optional.)



Say to the class, "Let us put the same pair of numbers into each one of these operation machines." Get a pupil to suggest a pair, such as (4, 2). Then let the class tell what numbers come out of the machines. Write the results below the machines:

$(4, 2) \xrightarrow{+} 6$ $(4, 2) \xrightarrow{-} 2$ $(4, 2) \xrightarrow{\times} 8$ $(4, 2) \xrightarrow{\div} 2$ $(4, 2) \xrightarrow{F} 4$ $(4, 2) \xrightarrow{*} 3$

Ask, "What happens if we put the same two numbers into the operation machines, but this time put 2 in first and 4 in second?" Let the pupils tell what numbers come out of the machines, and write the results below the others:

$(4, 2) \xrightarrow{+} 6$ $(4, 2) \xrightarrow{-} 2$ $(4, 2) \xrightarrow{\times} 8$ $(4, 2) \xrightarrow{\div} 2$ $(4, 2) \xrightarrow{F} 4$ $(4, 2) \xrightarrow{*} 3$

$(2, 4) \xrightarrow{+} 6$ $(2, 4) \xrightarrow{-}$ $(2, 4) \xrightarrow{\times} 8$ $(2, 4) \xrightarrow{\div}$ $(2, 4) \xrightarrow{F} 2$ $(2, 4) \xrightarrow{*} 5$

The pupils will tell you that the subtraction machine and the division machine will not work when (2, 4) is put into the machines. (4 cannot be subtracted from 2 to give a whole number; 2 cannot be divided by 4 to give a whole number.)

Ask, "Which machine gives the same number for (2, 4) as for (4, 2)?" (The addition and multiplication machines give the same number for (2, 4) as for (4, 2).) Try more pairs in the addition and multiplication machines to see whether the numbers that pop out are changed when the order of the numbers of the pair is changed.

$(4, 7) \xrightarrow{+} 11$ $(4, 7) \xrightarrow{\times} 28$

$(7, 4) \xrightarrow{+} 11$ $(7, 4) \xrightarrow{\times} 28$

Note: We usually write $(4, 7) \xrightarrow{+} 11$ as the true sentence $4 + 7 = 11$ and $(4, 7) \xrightarrow{\times} 28$ as the true sentence $4 \times 7 = 28$.

Let the pupils try many number pairs in this way. Guide them to tell about the order of the numbers of a number pair and the addition and multiplication operations. Encourage them to say something like this: "The addition operation gives the same number for a pair no matter which one of the number pair is first." There are many ways to say this. Accept any statement that gives the idea.

Explain to the class that they have found something about addition and multiplication that is always true. They have found a property of addition and a property of multiplication. The name of this property is the commutative property.

Say, "You found that addition and multiplication have the commutative property. Does subtraction have the commutative property?" (No) Ask a pupil to show this with a subtraction machine. Let him put in a pair such as (7, 4) in which the first number is greater than the second. Guide the pupils to say, "We do not get the same number for (4, 7) as we do for (7, 4) in the subtraction machine. Subtraction does not have the commutative property."

In the same way, guide the class to decide that division does not have the commutative property.

Let them discover that the "firsting" and the "star" operations do not have the commutative property. Do this by showing other examples in which we use pairs written in two different orders:

$(3, 6) \xrightarrow{F} 3$ $(3, 6) \xrightarrow{*} 7$

$(6, 3) \xrightarrow{F} 6$ $(6, 3) \xrightarrow{*} 4$

(Note: * operation is optional. If necessary, look again at some of the other operations that were used in Activity 3 and let the class decide whether they are commutative. For example, the triangle operation does not have the commutative property.

$(5, 2) \xrightarrow{\Delta} 12$ $(2, 5) \xrightarrow{\Delta} 15$

(These sentences may also be written $5 \triangle 2 = 12$ and $2 \triangle 5 = 15$.) The G operation has the commutative property. For example:

$(2, 1) \xrightarrow{G} 2$ $(5, 0) \xrightarrow{G} 5$ $(7, 6) \xrightarrow{G} 7$

$(1, 2) \xrightarrow{G} 2$ $(0, 5) \xrightarrow{G} 5$ $(6, 7) \xrightarrow{G} 7$

But the circle dot operation does not have the commutative property. For example:

$(3, 4) \xrightarrow{\odot} 10$ $(4, 3) \xrightarrow{\odot} 11$

(These sentences may also be written $3 \odot 4 = 10$ and $4 \odot 3 = 11$.)

Guide the pupils by questions to again say that addition and multiplication have the commutative property. Let them discuss what this means. Ask them to use their operation machines to show what it means for an operation to have the commutative property.

Let the pupils turn to page 19 in their books and talk about the exercises. Get them to find the numbers that make the first few sentences true. Ask a pupil whether he can find one of the missing numbers without adding or multiplying. Let the pupil explain why. For example, a pupil will say, "I can answer the first exercise without multiplying the numbers; n is 3. We get the same product when we multiply 3 and 4 as when we multiply 4 and 3." Or he may say, "I know n is 3 because $3 \times 4 = 4 \times 3$. Guide the pupils to see that you do not have to do the operations each time to find the missing number. Ask the pupils to tell which other sentences can be made true without adding numbers or multiplying numbers. (Sentences 1, 2, 3, 4, 6, 8, 9, 11, 12) Each time, let the pupil explain why he can do this and tell the number that makes the sentences true. Do not require that he say "because of the commutative property."

Activity 3: Adding and multiplying three numbers

Objective:

Children can tell what it means for an operation to have the associative property.

Teaching Procedure:

Review the process of adding three addends and multiplying three factors.

Remind the pupils that addition and multiplication are operations on pairs of numbers. We always put two numbers in the operation machine and get one number out. Ask, "How can we add three numbers if we can put only two at a time into the addition machine?"

Let the pupils discuss this, using three numbers 3, 5, 2. Guide them to decide that two of the numbers can be added first, and then the sum can be added to the third. Write this on the board:

(3 + 5) + 2

Let the pupils tell you that it means 8 + 2, or 10. Ask whether they could do the addition another way. Guide them to decide that the second and third numbers could be added, and then add the first number to the sum. Write this on the board:

3 + (5 + 2)

Let the pupils tell you that this means 3 + 7, or 10. Write this sentence on the board and let the pupils decide whether it true and why:

(3 + 5) + 2 = 3 + (5 + 2)

Repeat this activity with three other numbers. Let the pupils put the addends in different groups of two numbers to add them. Then get the pupils to follow the same plan in multiplying the same three numbers. For example:

4, 6, 2: $(4 + 6) + 2 = 10 + 2 = 12$
$4 + (6 + 2) = 4 + 8 = 12$ $\quad (4 + 6) + 2 = 4 + (6 + 2)$

4, 6, 2: $(4 \times 6) \times 2 = 24 \times 2 = 48$
$4 \times (6 \times 2) = 4 \times 12 = 48$ $\quad (4 \times 6) \times 2 = 4 \times (6 \times 2)$

Repeat for several other sets of three numbers. Use, for example,

$2 + 5 + 7$; $2 \times 5 \times 7$; $4 + 6 + 5$; $4 \times 6 \times 5$; $8 + 2 + 4$; $8 \times 2 \times 4$

Ask, "When we add three numbers, do we get the same sum no matter which two of the numbers we add first?" (Yes) "When we multiply three numbers, do we get the same product no matter which two of the numbers we multiply first?" (Yes) "We have found another property of addition and multiplication. The name we give to this property is the associative property."

Let a pupil tell about this property of adding or multiplying three numbers. Do not require the pupils to use the word "associative".

Ask whether subtraction has the associative property. Check this by trying to subtract three numbers, for example, 8, 4, 2. Let the pupils subtract the first two, then the third:

$(8 - 4) - 2$

Then subtract the last two, and subtract this number from the first:

$8 - (4 - 2)$

Ask these questions: "What is $(8 - 4) - 2$?" ($4 - 2$ or 2) "What is $8 - (4 - 2)$?" ($8 - 2$ or 6) "Do you get the same number when you subtract these three numbers in different groups of the two numbers?"

(One way we get 2, but the other way we get 6.) "Does subtraction have the associative property?" (No)

Ask whether division has the associative property. Check with the three numbers 8, 4, 2. "What is $(8 \div 4) \div 2$?" (1) "What is $8 \div (4 \div 2)$?" (4) Does $(8 \div 4) \div 2$ equal $8 \div (4 \div 2)$?" (No) "Does division have the associative property?" (No)

Activity 4: Associative properties of addition and multiplication

Pupil page 20

Objective:

Children can use the associative property for addition and multiplication to determine if a sentence is true or false and to find a missing number.

Teaching Procedure:

Write on the board:

(1) $(4 + 3) + 5 = n$

ask, "What number n makes the sentence true?" (12)

Then say, "Suppose we group the numbers another way." Write:

(2) $4 + (3 + 5) = n$

Say, "The addends in the two sentences are the same numbers but the parentheses show a different grouping. What number n makes the second sentence true?" (12) Guide the children to see that they begin by finding the number named in the parentheses.

Ask, "Was the sum the same as in the first sentence?" (Yes) (In both cases it was 12.) Use other examples to decide that no matter which of the two ways we group three whole numbers the sum is the same. Say, "If we have any three whole numbers a, b, and c, then:

$(a + b) + c = a + (b + c)$."

Remind the children that this is another property of addition. It is called the associative property.

Ask, "Do you think multiplication has the associative property? Is this sentence always true for any three whole numbers?"

$$(a \times b) \times c = a \times (b \times c)$$

Use the same procedure with several other examples until the children decide that multiplication is associative. Ask the pupils to do the exercises on pupil page 20. Guide them to use the associative property to help them answer the questions and to see that you do not have to do the operations each time to find the missing number.

Activity 5: Separating arrays and writing sentences | Pupil page 21

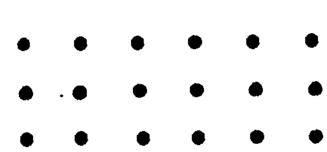
Objectives:

1. Children can write mathematical sentences for separated arrays of dots which illustrate the distributive property.
2. Children can separate arrays to represent mathematical sentences which illustrate the distributive property.

Materials: flannel board, 25 figures for flannel board, yarn or string, 25 counters for each pupil, a piece of string for each pupil

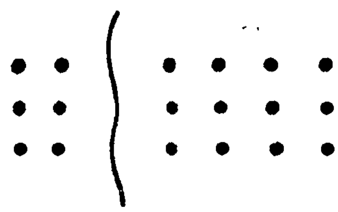
Teaching Procedure:

Make a rectangular array on a flannel board such as the array shown below.



Ask the class to describe this array. Guide them to say that this array shows 3 x 6. Write on the board 3 x 6.

Separate the array into two arrays using a string or yarn as shown below.



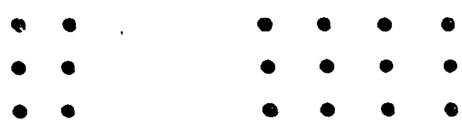
Ask the class to describe this array. They may say that there are still 3 rows but now each row has 2 members and 4 members. Write on the board

3 x (2 + 4).

Ask, "Are 3 x 6 and 3 x (2 + 4) represented by the same number of dots? (Yes) Therefore, we can write 3 x 6 = 3 x (2 + 4)." Write on the board

3 x 6 = 3 x (2 + 4)

Separate the array into two arrays as shown below.



Ask the class to describe the array. (3 x 2 and 3 x 4) Write on the board

(3 x 2) + (3 x 4)

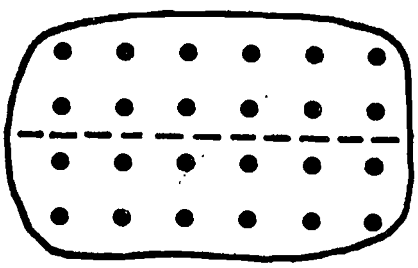
Ask, "What happens if we put the arrays together? (We'll get the original array.) Are 3 x 6 and (3 x 2) + (3 x 4) represented by the same array?" (Yes) Write on the board

3 x 6 = (3 x 2) + (3 x 4)

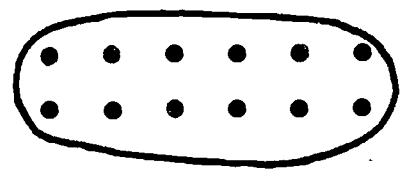
Ask, "Are 3 x (2 + 4) and (3 x 2) + (3 x 4) represented by the same number of dots?" (Yes) Write on the board

3 x (2 + 4) = (3 x 2) + (3 x 4)

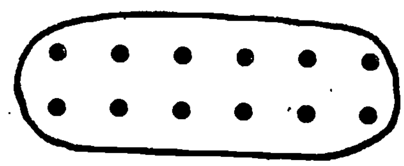
Continue this activity with other arrays and separate them in several ways. For example,



(2 + 2) x 6



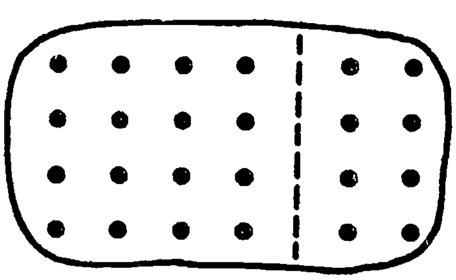
(2 x 6)



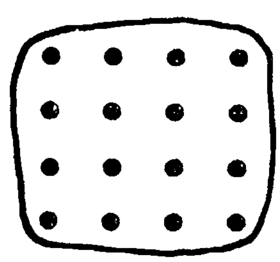
(2 x 6)

(2 + 2) x 6 = (2 x 6) + (2 x 6)

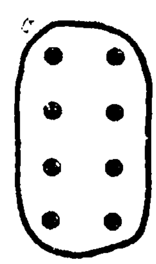
Note: You will observe from the examples that if the multiplier is on the right (2 + 2) x 6, the array is split horizontally. If the multiplier is on the left 4 x (4 + 2), the array is split vertically.



4 x (4 + 2)



(4 x 4)



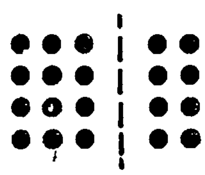
(4 x 2)

4 x (4 + 2) = (4 x 4) + (4 x 2)

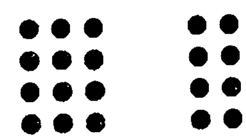
Give each pupil 25 counters and a piece of string. Write the following sentence on the board.

$$4 \times (3 + 2) = (4 \times 3) + (4 \times 2)$$

Ask, "How many counters do we need to represent this sentence?" (20) Ask pupils to make a 4 by 5 array. Let them arrange their counters to represent $4 \times (3 + 2)$ using string. Then let them arrange their counters to represent $(4 \times 3) + (4 \times 2)$.



$4 \times (3 + 2)$



$(4 \times 3) + (4 \times 2)$

Continue this activity with the following:

$$4 \times (2 + 3) = (4 \times 2) + (4 \times 3)$$

$$3 \times (3 + 4) = (3 \times 3) + (3 \times 4)$$

Ask the pupils to turn to pupil page 21 and write sentences in their books for each of the separated arrays of dots.

Activity 6: A pattern in multiplication

Pupil page 22

Objective:

Children can write two mathematical sentences for a story problem which illustrates the distributive property.

Teaching Procedure:

Give the students the following story problem. Peter collects money on Monday and Tuesday for a boys' club. He collects five cents from 2 boys on Monday and from three boys on Tuesday. How much does Peter collect altogether? Let the pupils talk about the two ways in which Peter can count his total collection:

(2×5)	+	(3×5)	=	n
Number of cents collected on Monday		number of cents collected on Tuesday		total number of cents collected

or

$(2 + 3)$	$\times$	5	=	n
number of pupils who paid on Monday and Tuesday		number of cents collected from each pupil		total number of cents collected

Let the class decide again that these two sentences tell about the same problem and the same number makes the sentences true.

Say, "We would like to find a property of multiplication and addition together. Let us solve more story problems and try to find such a property." Let the pupils turn to page 22 and read the first word problem. Help them think about the problem and the two mathematical sentences that tell about the problem. Let the pupils give the two sentences. Then write them on the board as follows:

(3×6)	+	(3×4)	=	n
number of nuts in first box		number of nuts in second box		total number of nuts

3	$\times$	$(6 + 4)$	=	n
number of rows		number of nuts in each row		total number of nuts

As you point to each part of the sentences, let the pupils tell what it represents. Let the class decide that these two sentences tell the same thing about the problem. Finally, write this sentence:

$$(3 \times 6) + (3 \times 4) = 3 \times (6 + 4)$$

Let the pupils decide that it is a true sentence.

Remind the class that the sentence you wrote has both multiplication and addition in it, in a very special way. Let the class give sentences for the other three problems on page 22. Write them all on the board.

1. $(3 \times 6) + (3 \times 4) = 3 \times (6 + 4)$
2. $(2 \times 5) + (2 \times 5) = 2 \times (5 + 5)$
3. $(5 \times 3) + (5 \times 4) = 5 \times (3 + 4)$
4. $(2 \times 4) + (2 \times 5) = 2 \times (4 + 5)$

In all these true sentences there is a pattern. Let the pupils discover the pattern if they can. (If a number is multiplied by the sum of two addends, the result is the same as multiplying the number by each of the two addends and adding the products.)

Activity 7: The distributive property

Pupil page 23

Objective:

Children can use the distributive property to determine if a sentence is true or false and to find a missing number.

Teaching Procedure:

Write the following story problem on the board:

Carl bought 3 cans of biscuits on Monday. He bought 4 cans on Tuesday. Each can contained 5 biscuits. How many biscuits did Carl buy on Monday and Tuesday together?

Let the pupils talk about the story problem. Ask, "How many biscuits did Carl buy on Monday? (15) How do you know it is 15?" The children may say something like, "There are 5 biscuits in each can. Carl bought 3 cans. He bought 3 x 5 or 15 biscuits." Guide them to find in a similar way the number of biscuits Carl bought on Tuesday. (4 x 5) Ask, "How many biscuits did Carl buy altogethor?" Help them to make the sentence:

$$n = (3 \times 5) + (4 \times 5)$$

Let the children say what number $\underline{n}$ is. (35)

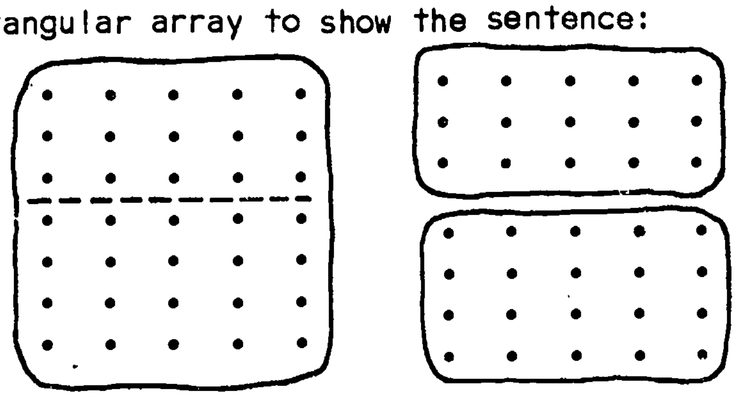
Ask, "Can you think of another way of finding how many biscuits Carl bought?" Help the pupils by asking questions such as, "How many cans did Carl buy altogether? (3 + 4 or 7) How many biscuits were in each of these cans?" (5) Guide them to write:

$$n = (3 + 4) \times 5$$

Help the children discover that in both sentences $\underline{n}$ is the same number. Guide them to write:

$$(3 + 4) \times 5 = (3 \times 5) + (4 \times 5)$$

Draw a rectangular array to show the sentence:



$(3 + 4) \times 5$ $(3 \times 5) + (4 \times 5)$

Write on the board:

$$3 \times (5 + 2) = (3 \times 5) + (3 \times 2)$$

Ask, "Is this sentence true?" (Yes) Let the children tell why. Do other similar examples until the children are convinced that for any three whole numbers, <u>a</u>, <u>b</u>, <u>c</u>,

$$a \times (b + c) = (a \times b) + (a \times c)$$

Remind the pupils that this is called the distributive property of multiplication over addition. We also say that multiplication is distributive over addition.

Let the children do the exercises on pupil page 23. Guide them to use the distributive property in working out the exercises. For example, in Exercise 1 they will see by the distributive property that <u>a</u> is 8 without doing the operations.

<u>Activity 8</u>: Properties of 0 and 1

Pupil page 24

<u>Objectives</u>:

1. Children can identify the identity properties of 0 and of 1 when they are used.
2. Children can use the identity properties of 0 and of 1 to find a missing number.

<u>Teaching Procedure</u>:

Review these addition facts:

0 + 1 = 1; 0 + 2 = 2; 0 + 3 = 3; 4 + 0 = 4; 0 + 5 = 5; 6 + 0 = 6; and so on.

Write these sentences on the board and ask what number makes them true

$0 + 732 = n$ $\qquad$ $a + 350 = 350$

The pupils will say that n is 732 and a is 0. You and the pupils write and solve other sentences with 0 as an addend.

Ask, "When a number and 0 are added, what is the sum?" (The sum is the same as the number.) Ask what number makes these sentences true:

$0 + x = x \qquad x + 0 = x$

Many numbers will be named. Let the class decide that the sentences are true for each of the numbers. Guide the pupils to say that $0 + x = x$ is true no matter what number x is. Say, "We have found a property of the number 0. When 0 is an addend, the sum is always the other addend."

Now review these multiplication facts:

$1 \times 0 = 0$; $1 \times 1 = 1$; $1 \times 2 = 2$; $1 \times 3 = 3$; $4 \times 1 = 4$; $5 \times 1 = 5$; $1 \times 6 = 6$; and so on.

Let the pupils write other multiplication sentences using the factor 1.

Ask the class to tell you a property of the number 1. If no one can do this, ask for the number that makes these sentences true:

$1 \times 312 = n \qquad b \times 350 = 350$

The pupils will say that n is 312 and b is 1. You and the pupils write and solve other sentences with 1 as a factor.

Ask, "When a number and 1 are multiplied what is the product?" (The product is the same as the number.) Ask, "What number makes these sentences true?"

$1 \times y = y \qquad y \times 1 = y$

The class will decide that the sentences are true no matter what number y is. Say, "We have found a property of the number 1. If 1 is a factor, the product is always the same as the other factor."

Ask the children to turn to pupil page 24 and answer the questions in their books. Read over the questions with the class. In exercises 7 to 12 ask the pupils to decide what numbers make the sentences true.

<u>Activity 9</u>: Properties of zero and one

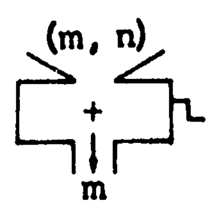
Pupil pages 25, 26

<u>Objective</u>:

Children can use the commutative, associative, and identity properties for addition and multiplication and the distributive property to determine if a sentence is true or false and to find a missing number.

<u>Teaching Procedure</u>:

Draw an operation machine for addition on the board with <u>m</u> and <u>n</u> as the ordered pair of numbers put into the machine. (See diagram below.)



Say, "Let us choose any number <u>m</u>. Imagine that same number <u>m</u> is assigned by addition to the ordered pair of numbers (m, n). Do we then know what number <u>n</u> is?" If no one can tell, suggest that they try a number for <u>m</u> and see what <u>n</u> must be. If a child says, "Let <u>m</u> be 5", then 5 is assigned to the pair (5, n). Ask someone to show a sentence for this:

$$5 + n = 5$$

Let children suggest other numbers and write the resulting sentences. Continue until there are four or five examples, such as:

$5 + n = 5$, $8 + n = 8$, $3 + n = 3$, $19 + n = 19$

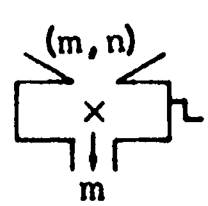
Ask, "What number n makes these sentences true?" They should see that n is 0 in each sentence. Guide the children to say, "When 0 is added to a number, the sum is that number." Tell them this is called the identity property of zero. Ask, "If 0 is an addend what can you say about the sum?" (It is the other addend.)

Put several sentences on the board to illustrate the identity property of zero.

$32 + 0 = a$	$n = 195 + 0$	$513 + 0 = s$
$17 + 0 = x$	$0 + c = 62$	$m = 0 + 12$

Let the children tell what numbers make the sentences true.

Draw a multiplication machine on the board and say, "Here is another operation machine. It is a multiplication machine. When you put in the pair of numbers (m, n), m comes out. What number do you think n is? Try a few examples and see."



Again, let us suggest different numbers for m. If a child says "eleven", then pretend to turn the handle and write "11" as the number assigned to (11, n). Let the child write a sentence to show this:

$11 \times n = 11$

After a few more examples ask, "What number is n? (One) This is called the identity property of one. What is the property of one? (When any number and 1 are multiplied the product is the same as the number.) You can say this another way. If 1 is a factor, then the product is the same as the other factor."

Write several sentences to illustrate the multiplication property of one. Let the children tell what numbers make the sentences true:

$19 \times 1 = n$ $43 \times 1 = m$ $r = 1 \times 613$

$m = 1 \times 367$ $930 \times 1 = w$ $s = 20 \times 1$

Let the pupils turn to page 25 and decide which sentences are true. Ask pupils if they can decide which sentences are true without working them out. Guide pupils to look for patterns illustrating properties and patterns not illustrating properties to decide which sentences are true.

Let the pupils decide what numbers make the sentences true in the exercises on page 26. Remind the children that many times they do not have to do the operations to find the missing number. Let them tell how they found their answers.

UNIT 5
Geometry

OBJECTIVES:

1. To review the use of the words "beside", "to the right of", "to the left of", "between".
2. To review the use of the words "inside", "outside".
3. To introduce the concepts of point, line segment, and end-point.
4. To introduce the idea of a simple closed curve.
5. To introduce the notion of extending a line segment to a line and to a ray.
6. To introduce the ideas of plane region and polygonal region.
7. To introduce the idea of angle.
8. To introduce the idea of intersection of lines with lines and planes with lines.
9. To extend the ideas of angle, triangle, and polygon.

BACKGROUND INFORMATION FOR TEACHER:

This unit is designed to give the pupils pleasure while discovering and experimenting with geometric concepts.

In this unit the ideas of points, space, line segment, end-points, rays, planes, regions, angles, polygons, and paths will be introduced. Relations such as between, beside, above, on, and inside are studied as "nearby relations". (They are called "nearby" because they are relations between objects which are usually near each other.)

The word "point" in the dictionary, is defined by other words. One of the words used in the definition, if also looked up, will be defined by "point" or some other word previously used.

In mathematics some words are not defined, such as "point" and "line". However, other words in mathematics are defined using such words as "set", "point", and "line".

Pupils give meaning to these words from the descriptions that are supplied them. For example, a point is described as an exact location. A point cannot be seen or felt and has no size. Note that this is a description of a point, not a definition.

A small dot may be used to picture a point; the smaller the dot the better the picture. Under a microscope, however, even a small dot can be seen to cover many locations.

We think of a point as denoting a fixed location. For example, the point indicated by the corner of a desk would remain unchanged even though the desk were moved, only now it is no longer represented by the corner of the desk.

The tip of a pin would represent a point. If the pin were moved, the point would remain--the location is fixed but the tip that represented it was moved.

A string, whether held loosely or stretched tightly, shows a geometric figure. The name "curve" is given to all figures shown on a string. A curve can be thought of as the path (whether "straight" or not) traced in going from one location to another.

Other examples are a "curve" drawn on a sheet of paper, a route taken from one city to another, telephone wires between two poles, etc. Thus we see that a curve contains more points than we can count.

A special curve represented by a string stretched tightly between two points, say A and B, is called a line segment, denoted $\overline{AB}$, or $\overline{BA}$, 

. Another line segment is represented by the edge of a desk, or by a path drawn with a pencil and ruler connecting the end-points of the line segment. If the string is removed, the line segment remains since the string only represented a set of locations.

A <u>line</u> may be thought of as an extension of a line segment in both directions. A line is shown as 

, denoted $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$ with arrows showing that the line is considered to be extended infinitely.

A <u>ray</u>, denoted $\overrightarrow{BA}$, 

, is defined as that part of a line consisting of a point called the end-point, and all points of the line in one direction from the end-point. Our example shows a ray with end-point B and the part of the line to the left of B. A flashlight may be used to help students visualize a ray; the rays of light emanate from the source which would correspond to the end-point. Note that the first letter mentioned denotes the end-point.

We now return to the concept of <u>curve</u> and we will restrict our discussion to curves in a plane. A curve may be represented by a set of points traced without lifting the pencil from the paper.

Thus we see that line segments, lines, and rays are all curves. In a simple <u>closed</u> <u>curve</u>, it is possible to start at any point, trace the entire path without lifting the pencil and come back to the starting point without intersecting (or touching) the curve.

Thus, a circle, a triangle, and a square are good illustrations of a simple closed curve. Each of these, in a plane (flat) surface, has what might be called an inside and an outside. Indeed, for any simple closed curve in the plane, three sets are distinguished: the inside of the curve, the outside of the curve, and the curve itself. The points of the curve are neither on the outside nor the inside. The union of the set of points of a simple closed curve and the set of points in the interior of the curve is called a plane region. The top of a desk would represent a plane region and so would the floor in the classroom, the bottom of a waste-paper basket, a wall surface, the surface of a window-pane, etc.

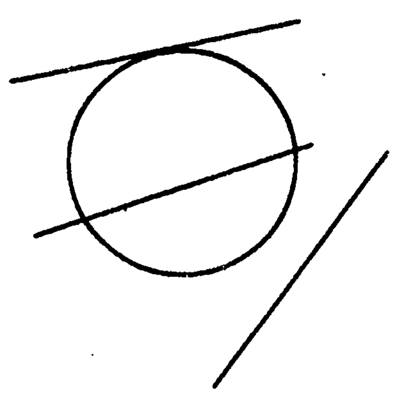
A triangle is a simple closed curve which is the union of three line segments. The three segments are the sides of the triangle. A quadrilateral is a simple closed curve which is the union of four line segments no two of which are on the same line. The four segments are the sides of the quadrilateral.

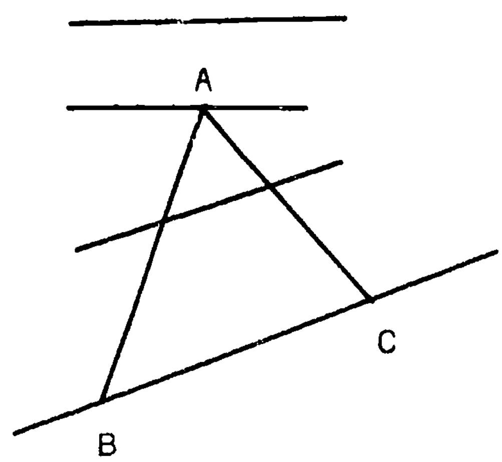
In everyday conversation, we often use words like "circle", "triangle", and "rectangle" in two ways. They sometimes are used to mean a plane simple closed curve and sometimes are used to mean a plane region, the curve together with its inside. For example, people say that the rim of a wheel and the top of a tin can both represent circles. However, the rim of a wheel represents a circle, while the top of a can represents the region bounded by a circle.

In this book, "circle", "triangle", and "rectangle" will refer to the curves. We will say "circular region", "triangular region", and "rectangular region" when we refer to the regions. In summary, names of plane figures usually refer to the curves, not the plane regions.

An <u>angle</u> is defined as the union of two rays which are not on the same line, and have the same end-point, called the <u>vertex</u>.

Intersections of streets, corners of boxes and buildings, knife cuts in cheese, and folds and wrinkles in paper are examples of the geometric idea of intersection of figures. In geometry, a place where two figures meet is sometimes a point. The point lies on both figures. Thus, we say the figures have the point in common. In some cases, two figures meet at more than one point. The figures shown by the intersection of the floor and a wall of a room have a line segment in common; they intersect in a line segment. A circle and a line may intersect in two points or one point or no points. A triangle and a line may intersect in no points, one point, two points, or in a line segment. Pictures of these intersections are shown below.





This entire unit should be taught with the objective in mind that the students have fun trying out the activities suggested. The student should be encouraged to do much free-hand drawing of lines and circles as the class discussion centers around these ideas. The student should not feel that he must draw lines with a straight-edge in order to understand the notions presented. If the students have difficulty in drawing circles then they may draw triangles for some of the activities that involve simple closed curves.

The students should be given every opportunity possible to explore the notions suggested before generalizations are made. The students should also be given opportunity to verbalize their ideas; they should be encouraged, even urged, to talk about what they think about the ideas. The teacher should not expect that formal, precise language be used, but that the students have the opportunity to discuss their ideas about geometry as they see it. As the discussions progress, students will be able to sharpen their perceptions by comparing them to those of their classmates.

TOPIC I: GEOMETRIC OBJECTS AND IDEAS OF RELATION

OBJECTIVES:

1. To provide interesting objects that demonstrate geometric figures.
2. To review the use of the words: "beside", "to the right of", "to the left of", "between", "inside", and "outside".

VOCABULARY: beside, to the right of, to the left of, between, above, below, on, inside, outside, curve

SYMBOLS: None

MATERIALS: ball, box, pencil, bottle top, rock; pupil pages 27 - 29

Activity I: (Oral) Collecting objects that represent geometric figures

Objective:

Children can collect objects which show the following figures: points, line segments, angles, right angles, triangles, circles, circular regions, rectangles, rectangular regions, squares, 4-sided figures with no right angle, 5-sided figures, and regions.

Teaching Procedure:

Let the pupils help you collect, make, and identify different geometric shapes. Many things which suggest shapes are in the classroom or can be obtained easily. The children may volunteer to bring some things from home. Bring some of the objects yourself, and have materials available for making some others.

Collect objects that show the following figures: points, line segments, angles, right angles, triangles, triangles with two sides of same length, right triangles, circles, circular regions or discs, rectangles, squares, four-sided figures with no right angles, five-sided figures and regions, etc.

Listed on the following page are some objects that show these shapes. You will think of other objects. You and the children collect or make as many of these objects as you think desirable.

1. Points are shown by dots on the board, tips of corners of books, ends of sharpened pencils, ends of pins, pin holes in paper, intersections of folds in paper.

2. Line segments are shown by edges of books, stretched strings and rubber bands, straight sticks, pencils, grass, straw, some shadows, folds in paper, edges of wooden board, chalk and pencil traces of straight edges.

3. Lines are imagined as the extensions of line segments shown by objects like those listed above. The extensions are indicated by drawing arrowheads on the ends of pictures of line segments.

4. Angles are shown by corners of objects; right angles are shown by square corners.

5. Triangles, four-sided figures, squares, rectangles, and circles are shown by string, by bent wire, by sticks fastened together, by the edges of cutouts, by drawings on the board.

Pupils may enjoy making a flannel board display of the above (points, line segments, lines, angles, triangles, four-sided figures. squares rectangles, and circles) by using yarn and felt scraps. It may be a continuing activity, growing as new geometric ideas are introduced, such as rays, curves, intersections.

<u>Activity 2</u>: Using the words "beside", "to the right of", "to the left of", and "between"

Pupil pages 27, 28

Objective:

Given a picture showing a series of objects, children can identify their placement in the picture using terms as to the right of, to the left of, beside, between, above, and below.

Materials: ball, box, pencil, bottle top

Teaching Procedure:

Get the pupils to show their right hands and then their left hands. Help those who do not show the proper hands to decide which hands are right hands and which are left hands.

Say, "I am going to tell you about John and Bob. John went looking for his friend Bob. Bob lived in a house beside a tree." Draw a house with a tree beside it on the board.

"John called to Bob, 'Come out and play.' Bob came out of the house and stood beside the house. The house was between Bob and the tree." Draw a picture of Bob like this:

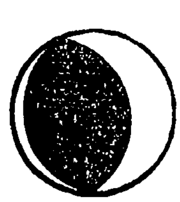
"John called, 'Bob, I can see you to the right of your house. The house is between you and the tree. The tree is to the left of your house'."

Let the children be John and Bob and go through the story. Let John tell about where Bob is standing. Let Bob say things like, "My house is to the right of me. The tree is to the right of me."

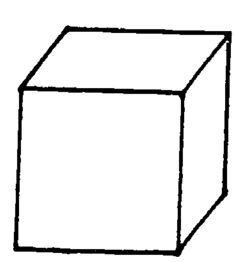
Place a box and a ball on a table at the front of the room. Ask a child to place the ball to the left of the box. Say, "The ball is beside the box. The ball is to the left of the box. Say, "The box is to the right of the ball." Ask a child, "Where is the ball?" (The ball is beside the box; the ball is to the left of the box.) Ask a child to face the class from the other side of the table. Ask, "Where is the ball?" (To the right of the box.)

Let the pupils discuss the fact that the ball is to the right of the box for the child facing the class and to the left of the box for the other children in the room. Let children go to the front of the room, face the class, and decide whether this is true.

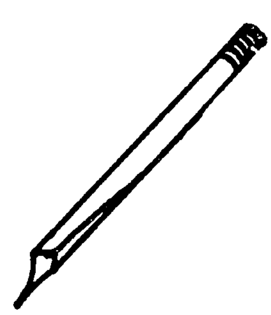
Give a child a pencil, and ask him to put the pencil to the right of the box. Give another child a bottle top and ask him to put it to the right of the pencil.



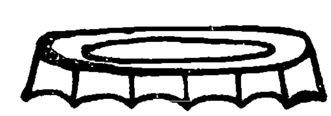
Ball



Box



Pencil



Bottle Top

Let the pupils tell you the location of the pencil. For example, they may say, "The pencil is between the box and the bottle top. The pencil is to the left of the bottle top. The pencil is to the right of the box and to the right of the ball."

Ask a child to face the class as he stands on the other side of the table. Ask him to tell where the pencil is. He will say, among other things, "The pencil is to the right of the bottle top. The pencil is to the left of the box. The pencil is to the left of the ball."

Let the pupils talk about why their statements are different. Get several pupils to make the statement, "The pencil is to the right of the box." Then let them move to the other side of the table and say, "The pencil is to the left of the box."

Repeat this activity, with the pupils talking about the location of the other objects on the table.

Let each pupil place his book and a pencil on his desk. Say things like, "Put your right hand to the right of the book. Put your right hand between the pencil and the book. Put your left hand to the left of the pencil. Put your left hand to the right of the book." Move about the room as you say these things, helping children who need help. Let some of the children give similar directions to the class.

Tell the pupils to look at pupil page 27. Ask a child to read the first sentence in exercise I and tell whether the sentence is a true sentence or a false sentence. Let the class decide whether his answer is correct. Ask other children to read sentences and tell whether they are true or false sentences. If a sentence is false, ask the child to make a true sentence. After all the sentences are read, say, "Make other statements like these about the rectangle, circle, and triangle."

Tell the pupils to read the sentences in Exercise 2. Ask a child, "What word do you put in the blank in the first sentence to make it true?" Let him read his sentence. Ask the other children to decide whether the sentence is true. Ask similar questions for the other sentences.

Tell the pupils to look at the number line in Exercise 3. You make some statements about the numerals on the number line using the words "beside", "between", "to the right of", and "to the left of". Then ask the children to make other statements about the numerals.

If the pupils already understand the ideas of this activity or if they learn them very quickly, shorten or omit several of the suggested activities. However, be sure to let the pupils use the ideas with the number line of Exercise 3 on page 27.

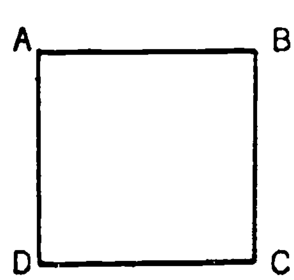
Say, "Hold your right hand above your head. Place your left hand below the top of the desk." If all the pupils do this correctly, they understand "above" and "below", and this activity is then very short. Tell the children to do a few other things, such as the ones listed below. You will think of others.

> Hold a pencil below your book; hold your book below your pencil.
>
> Place your right hand below your left hand.
>
> Put your book above your left hand; place your right hand below your book.

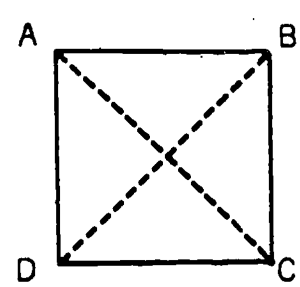
Tell the pupils to look at Exercise 1 on page 28. Ask questions like, "Is the triangle above the square?" (Yes) "Is the circle below the triangle?" (Yes) Let the children say other things about these figures, using "above", "below", and "between".

Ask similar questions about the numerals on the number line in Exercise 2 on pupil page 28. Ask, for example, "Is the numeral 3 above the numeral 1? Is the numeral 4 between the numeral 5 and the numeral 2?" Get the pupils to make complete statements when they answer. Let them ask other questions.

Ask the pupils to look at Exercise 3. Let them describe the location of the barn, bird, plane, and cloud in relation to each other using the words "above", "below", or "between". (Continue with activities of the following type.) Draw this figure on the board: Ask, "Is there a point on the board between A and C and also between B and D? How would you find it? Is the point inside the square or outside the square?"

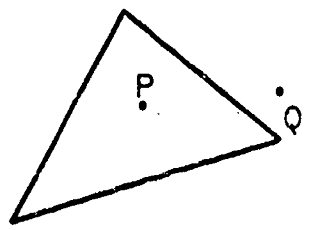


Ans.

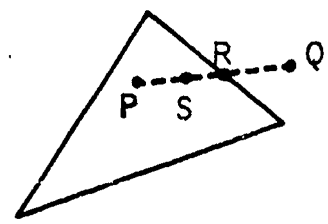


Consider a triangle and a point P inside and a point Q outside the triangle as indicated. Is there a point of the triangle between P and

Q? (Yes) Can you find two points of the triangle between points P and Q? (No)



Ans.



Point R is the only point of the <u>triangle</u> that is between P and Q. (Points like S are points of the <u>triangular region</u> which are between P and Q.)

<u>Activity 3</u>: Extending the use of "inside and "outside" (oral)

Pupil page 29

<u>Objective</u>:

Given a picture showing a series of figures one within another, children can identify their placement using such terms as: inside, outside.

<u>Materials</u>: box, rock, pencil

<u>Teaching Procedure</u>:

Say, "Name something you see inside this classroom. Name something outside the classroom." (John is inside the classroom. John's mother is outside the classroom. The tree is outside. The table is inside.) Continue until many of the children use the words.

Place a rock inside a box. Put a lid on the box. Put the box on a table beside a pencil. Write on the board, "The rock is inside the box.

The pencil is outside the box." Ask a pupil to read the sentences. Make sure the pupils can read and know the meaning of each word.

Tell the pupils to look at page 29, Exercise 1. Ask a child to read the sentences. Ask another child whether they are true sentences.

Tell the pupils to look at Exercise 2. Ask a child to read the sentences and put the words in the blanks to make true sentences. (The circle is inside the square, and so on.) Tell the pupils to look at Exercise 3. Ask them to make statements about the figures, using the words "inside" and "outside". Guide them to include statements like "The square is inside the circle. The square is outside the triangle." Let the pupils make similar statements about the figures in Exercise 4. An arrow and a cross and two points, M and L, have been included for variety.

TOPIC II: POINTS AND CURVES

OBJECTIVES:

1. To introduce the concept of point.
2. To introduce the concept of a simple closed curve.

VOCABULARY: points, curves

MATERIALS: none; pupil pages - none

Activity I: (Oral) Points

Objective:

Children can describe a point as a location in space.

Teaching Procedure:

The children may be asked to answer the following questions: What is a point? Is it the tip of your pencil? Is it the sharp tip of a pin?

Is it a dot made with a pencil on your papor? Is it a dot on your paper made with a crayon? (Have students draw the dots with the pencil and crayon.) Is it a dot on the board made with chalk? with a pencil? No, none of these. Dots are used to help us think of points. We use a letter to name the point. Thus we have points

. B

. A

. C

Show some dots on the board named A, B, C.

In geometry, we think of a point as an exact location. A point cannot be seen or felt. It is so small it has no size. How can we see germs? If we were to see a germ under a microscope, it would appear very large and cover many locations. Any object, not matter how small, covers many locations and is not a point, only a picture of a point--such as a dot.

Hold your pencil over your desk with the eraser tip down. Ask, "Could the eraser tip show a point? (Yes) Could the sharpened tip show a point?" (Yes) Move the sharp tip of the pencil to another place on the desk. Tell the students "I am now indicating another point, a different location."

Suppose we have three points and we wish to name them points P, Q, R. Capital letters are usually used to name points. Can you describe a set of four points in your room? Answers will vary, and they might suggest something like four corners on the top of their desks, the four points where their chair touches the floor, etc.

Many interesting ideas can be formed using sets of points.

Draw the following dots on the board and ask the children which of these points shows the best picture of a point. Why? • ● •

Ask the children how many points a dot drawn with a pencil on a piece of paper covers. Have them draw a dot on their paper. Does it cover one point, several points, a thousand, more than we can count? (More than we can count.)

What describes a point best? A chalk mark on the board, a dot made with a very sharp pencil, a dot made with a sharp pin, or an exact location in space?

<u>Activity 2</u>: (Oral) Curves and points

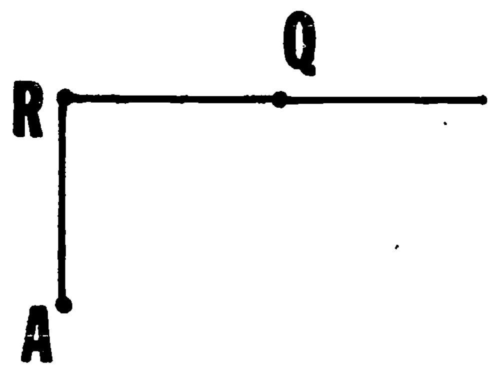
<u>Objective</u>:

Children can identify a curve as a path.

<u>Teaching Procedure</u>:

Tell a story like this: "Once upon a time there was a little ant, named Speedy, who wanted to see the world. He said good-bye to all his friends and started out over the flat ground. After a while he came to a rock. He thought, 'I will not climb over this rock. I will turn to the right.' Soon he met his friend Quickfoot. They touched feelers. 'I am going to see the world,' said Speedy. 'The world is big,' replied Quickfoot. Speedy went on."

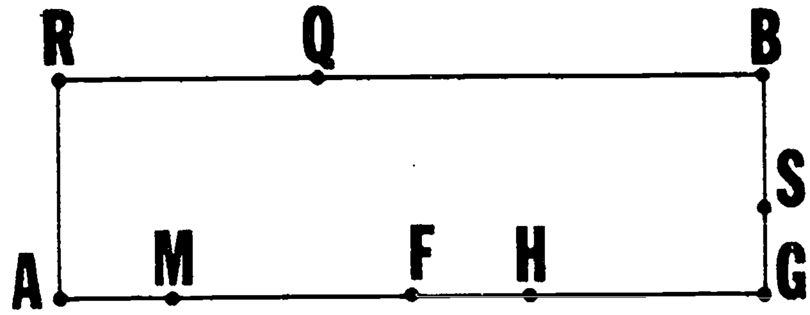
Say, "Let us draw a picture of Speedy's path. Where did he start and where did he go?" Let the children tell you what happened. Draw a picture like this on the board:



Say, "A shows the ant hill where Speedy started. R shows the rock. Q shows the place where he met his friend Quickfoot. Our picture is not his real path. We call the picture of a path a curve. Our picture shows a curve. Pictures of places along the path, like R and Q, we call points. The picture of Speedy's path contains line segments AR and RQ."

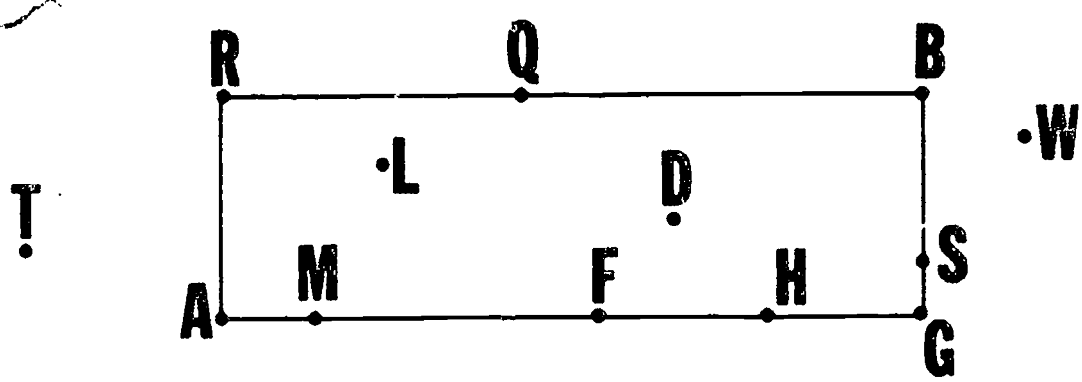
Go on with the story. "Speedy next met a beetle. Speedy was afraid of the beetle, so he turned to the right again. A little farther on he met a scout from the ant hill. 'Speedy, what are you doing so far from home?' asked the scout. 'I am going to see the world,' Speedy replied. 'Find good things,' said the scout, 'and you may be a scout some day.' Speedy soon found a big pile of very tasty young grass. He thought, 'We have not had tasty grass like this to eat for a long, long time. I must tell the other ants.' He took a piece in his mouth and started home by turning right again. He met three friends, Happy, Fairy, and Muddy. To each he said, 'Look at the very good grass I found.' Muddy ran on ahead. He called to the other ants at the ant hill. 'Here comes Speedy. He has found a pile of fresh new grass.' The other ants ran to Speedy. They asked, 'Where is the grass?' Speedy told them. Soon the ants had eaten all the tasty grass they could eat. The chief of the ants sent for Speedy and said, 'Thank you, Speedy, for finding the delicious grass for us. You are now one of our Chief Scouts.' Speedy was very happy to be a scout."

While telling the story, draw the picture of Speedy's path to look like this:



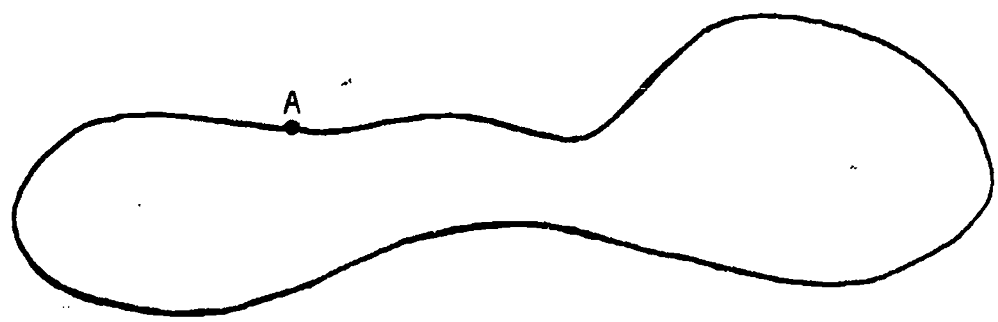
Let the pupils talk about the picture. Get them to put their fingers on the points and say things like, "At point S, Speedy met the scout. At point G, he found the tasty grass. At point R, he came to the rock." Say, "In the picture, points show places."

Go on. "The paths that Scout Speedy took were over a flat field. As he did his scouting, he found many things. One day he found a leaf, a piece of wire, a dog, and a tree." Mark points to show the places where Speedy found these things, as shown in the picture below.



Say, "Points L, W, D, and T show the places where Scout Speedy found the leaf, the piece of wire, the dog, and the tree. Speedy's path is called a curve. We say that point F is on the curve, point D is inside the curve, and point W is outside the curve." Ask questions like, "Is point H on the curve? Is point G inside the curve? Is point W inside the curve? Point L? Point A?" If an answer is "No", ask pupils where the point is. Let them tell where other points are; for example, points S, T, and R.

Say, "The curve showing Speedy's first path is a rectangle. One day Speedy's path was different. It was like this."



Mark some points on, inside, and outside this curve. Ask questions like, "Is this point inside the curve? Is this point outside the curve? Is this point on the curve?" Let pupils mark some points and tell where they are.

TOPIC III: LINE SEGMENTS; LINES; AND RAYS

OBJECTIVES:

1. To introduce the notion of extending a line segment to a line and to a ray.
2. To extend the idea that the number of points on a line segment is infinite.

VOCABULARY: line segment, end-point, ray, line

MATERIALS: I piece of string (about 2 feet long); I brightly colored piece of paper; I piece of paper for each child; a flashlight; one 30- or 40-foot string; pupil page 30

Activity I: (Oral) Line segments and end-points

Objectives:

1. Children can draw a picture of a line segment.
2. Children can identify a line segment as a special type of curve which happens to be straight.
3. Children can identify the points at either end of a line segment as endpoints.

Materials: a piece of string, I brightly colored piece of paper, I piece of paper for each child

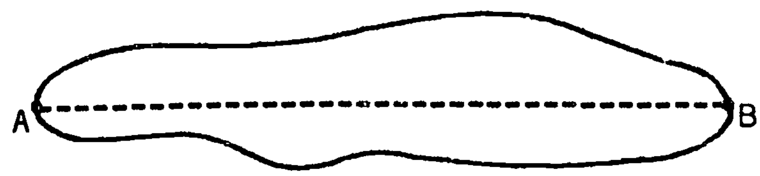
Teaching Procedure:

Point to objects that have edges that are straight, such as tables, books, paper, and walls. Run your finger along the edges that are straight and say, "This is a straight edge." Hold a string stretched tightly between your hands and say, "This is straight. We use a stretched string to see if an edge is straight." Hold the stretched string along some edges to test their straightness.

Trace some of the straight edges on the board. Tell the pupils that when a straight edge is traced on the board, or on paper, "We drew a line segment." Let a few of the children trace straight edges on the board and ask them what figure they have drawn. (Line segment) Test these for straightness with a stretched string.

Ask the children to find some objects on their desks that have straight edges. Let them trace these on their papers. Ask them to say, "This is a line segment."

Cut out a brightly colored piece of paper in a shape something like this, but larger.



Put two dots on it as in the diagram. Ask the children to trace different paths in order to go from A to B. The children may follow a path that touches the paper only on A and B. How many paths are there? (More than we could count) Which path is the most direct to get from A to B? This direct path shows a special type of curve called a line segment. A line segment is a set of points. It is the set of points that were marked as we moved from A to B. It also includes the points A and B. We sometimes show line segments like this: $\overline{AB}$. One of the best ways to show a line segment is to draw a picture of one using a ruler and a pencil and using dots at the end to show points A and B. Points A and B are called the <u>end-points</u> because the line segment ends at points A and B.

Have the students fold a piece of paper to make a straight edge. Say, "I do not have a square corner. I have one straight edge." Run your finger along the straight edge and ask, "What is this?" (A straight edge.) Trace the straight edge on the board and ask, "What have I drawn?" (A line segment.)

Ask, "Does the straight edge have corners?" Let the children talk about this. Try to get them to say that at each end of the paper straight edge there is a corner and the corner has a tip. Hold the paper straight edge in position against the line segment, put your finger at one end of the straight edge, and ask, "Can I draw a picture of the tip of this corner?" Draw this dot.

Remove the paper and repeat for the other corner. Say, "I have drawn a picture of the straight edge and a picture of each corner tip." Run your finger along the line segment and ask, "What is this?" (A line segment.) Point to each dot, say, "At this end of our line segment is a ________." Get the children to say the word "point". Repeat for the other end. Say, "At <u>each</u> end of our line segment there is a point."

Repeat the above, tracing the edge of a book. Say, "When we draw a line segment it has a point at each end. These points have names. They are called ________ of line segments. (end-points)

Let the children in turn, choose objects with straight edges and trace the edges on the board. Ask them what they have drawn. (Line segments) Ask them what are the ends of their line segments. (End-points) Have them draw these points.

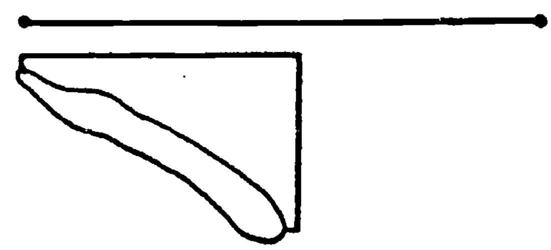
<u>Activity 2</u>: (Oral) Other points on a line segment

<u>Objective</u>:

Children can mark many points on a line segment.

<u>Teaching Procedure</u>:

Using the paper straight-edge you used in Activity I, draw a line segment on the board. Draw its end-points. Say, as you point, "This line segment has two end-points, one here and one here." Holding the paper in place on the board, fold it over to make a square corner. The children will see this:

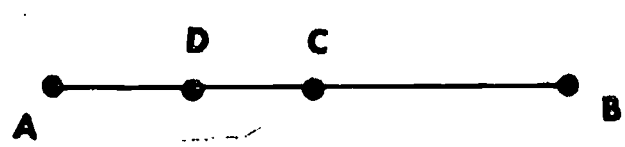


Point to the square corner and ask if it has a tip. Say, "I am drawing a picture of the tip. Draw the picture, making a dot that is <u>on</u> the line segment. Ask, "What is the dot?" (It is a picture of the tip of the square corner. It is a point.) "Is it on the line segment or off the line segment?" (On) "Is it one of the end-points of the line segment?" (No)

Ask, "How many points have we marked on the line segment?" (Three) Label the points A, B, C. "What can we say about point C? (It is between points A and B.)

Unfold the square corner and ask, "Can I mark any more points on the line segment?" (Yes. Make a different square corner with the paper.) Fold the paper over to make a square corner at a different place on the paper straight-edge.

Draw the picture of the tip of this corner. Say, "This dot gives another point on the line segment. How many points have we marked on the line segment now?" (Four) Label the point D. What can you say about these points? The figure could now look like this:



Ask, "What can you say about point D? C? (Point D is between A and C and also between A and B. Point C is between D and B and also between A and B.) Ask, "Can you find any more points on the line segment?" (Yes) Let the children draw several more points. Let them talk about how many points they could mark on the line. (More than we could count.)

Let the children discuss the idea of betweeness of the points they locate. They may even generalize that between any two points on the line segment another point may be found.

Activity 3: Extending a line segment to lines and rays

Pupil page 30

Objectives:

1. Children can indicate a ray by extending a line segment in one direction, shown by an arrowhead.
2. Children can indicate a line by extending a line segment in both directions, shown by arrowheads at the extremities.
3. Children can mark many points on a line.
4. Children can indicate that there are more points on the line than can be marked or counted.

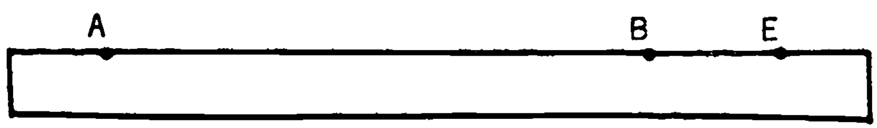
Materials: 1 30- or 40-foot string; 1 flashlight

Teaching Procedure:

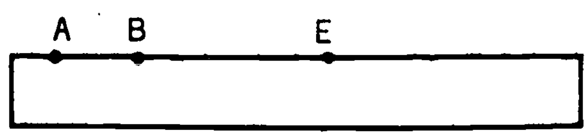
Draw a line segment on the board and name the two end-points. Use a book or similar object to trace the line segment.

Say, "This is the line segment AB. It is a curve. It has two end-points, A and B."

Ask, "Can you make a line segment longer than AB?" Help a child place a long straight-edge or a yardstrick along the line segment AB like this:

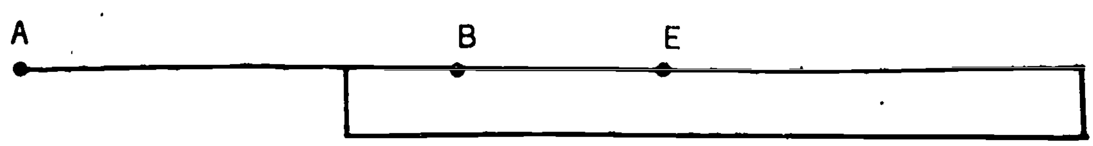


Draw a line segment along the straight-edge to a point marked E.

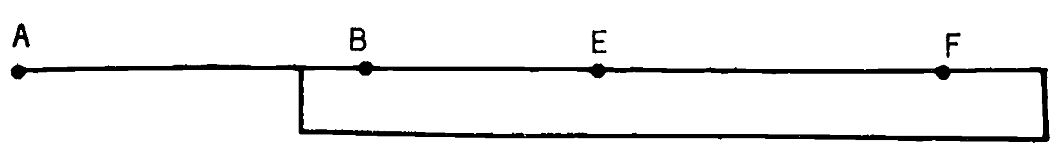


Say, "We had the line segment AB. Now we have a new line segment AE. The line segment AB was extended to make the line segment AE." Ask, "Is the point B an end-point of line segment AE?" (No) Ask, "Is point B on line segment AE?" (Yes)

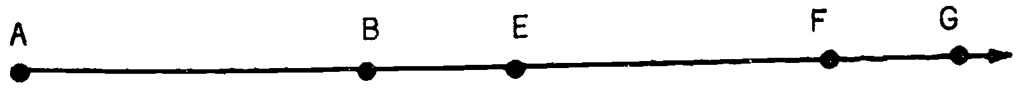
As you hold the straight-edge as shown below, ask, "Can we draw a line segment longer than line segment AE?" (Yes)



Help a child draw line segment AF.



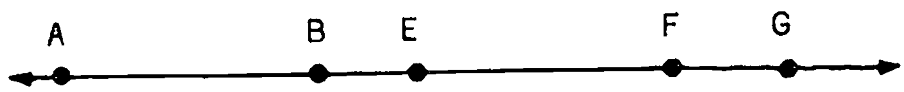
Extend the line segment several more times, almost to the edge of the board. Let the pupils talk about each extension. Get them to say, "The board is not long enough for any more extension. If the board were longer, the line segment could be extended on and on." Show this possible extension by putting an arrowhead on the right end of the segment. Guide the children to talk about what the arrowhead means.



The line was extended in only one direction; the new figure formed by extending a line segment in one direction only is called a <u>ray</u>; it has only one end-point. A ray is denoted by $\overrightarrow{AB}$, where A is the end-point and B is any other point on the extended line segment.

At this point a flashlight can be used to demonstrate the notion of a ray. Ask the students, "How is this flashlight (turned on) like a ray?"

Let the pupils discuss whether the line segment can be extended to the left. Get them to decide that it can be extended to the left as far as they please. Let a child show this. (He should draw an arrowhead on the left end of the line.)



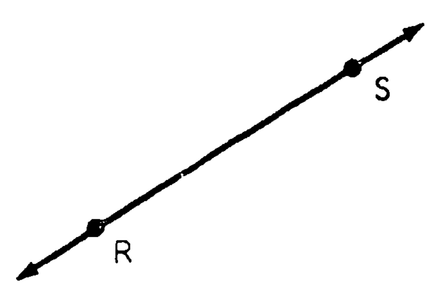
Say things like, "We can imagine the line segment is a magic string. It can be stretched and stretched. The magic string is stretched to the tree outside and then to the rock and then on and on."

Say, as you point to line segment AB, "This figure is a line segment. We imagine it extends on and on as far as we please in both directions. We drew arrowheads to show this." Point to line AG and say, "We call this figure a line."

Ask questions and make statements that get the pupils to say things like, "Points B and E are on the line. Point F is between points A and G. Point E is to the left of point F."

A line, which extends as far as we please in both directions, is named by giving two points on it. For example, the line AB is also named AE. It is the same line as line AF and line BG. We may also give it other names.

Draw another line on the board as shown below. Mark points R and S.



Ask a pupil, "Can you mark another point on the line RS?" Let him mark a point and name it with his own first initial. Ask, "Can another point be marked on line RS?" Let another pupil mark his point. Let several pupils mark points. Guide them to say that every pupil in the class can mark a point. Ask, "Could each of you mark two points?" (Yes) "Could all these points be different?" (Yes) (It may be necessary to say that the smaller the chalk mark is made the better the point is shown.) Help the pupils understand that they can mark as many points as they please on the line and then more.

On the board, draw a line segment about a foot long. Take a 30- or 40-foot length of string. Tell the pupils you are going to show the extension of the line segment. Hold a part of the string along the line segment with your hands at the end-points of the line segment. Then hold a longer part of the string along the line segment. Guide the children to say, "Your arms are not long enough." Ask two pupils to hold the string along the line segment, holding longer and longer parts between them. Let the children decide they could go on and on extending the line segment if the string were longer and longer.

Ask the pupils to imagine a line segment made of magic rubber. Imagine two magic birds each taking an end-point in his beak and flying away forever in opposite directions. Let the pupils talk about the rubber stretching and stretching to show a line.

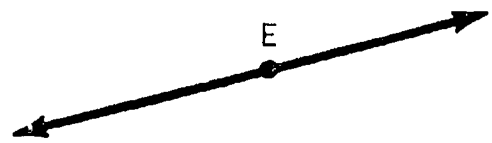
Tell the pupils to look at pupil page 30. Tell them to hold a string along line segment AB. Then let them imagine holding longer and longer parts of the string along AB. Tell them, "Imagine your string is very long. What will it show? (A line) How far would the line go?" Let the pupils use their imaginations and talk about the line going on and on.

Ask the pupils to place a small dot on the line LM. Tell them the dot represents a point. Then let them place another small dot on the line. Then another small dot. Ask, "If the dots were very small, how many could be put on the line?" The children may begin by saying "5", then "10" and then larger and larger numbers. Some child may finally say something like, "As many as I want and then more and more." This is the idea this activity should develop.

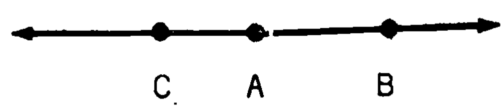
Summarize the differences between line segment, ray, and line. Ask, "How are a line segment, a ray, and a line alike? How would you show a line segment on a paper? A ray? A line? (Let the children draw them as you discuss.) "How is a line segment different from a ray? (Line segment has two end-points and a ray has only one.) A ray and a line? (A ray has one end-point but a line has no end-points.)

Additional Exercises:

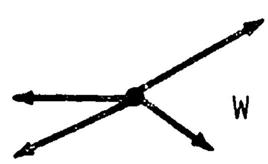
1. Draw a line on a sheet of paper. Label a point on the line E. How many rays do you see with E as an end-point? (Two)



2. Draw a picture of a line on your paper. Let A be a point on the line. Choose a point on the line different from A and call it B. Choose another point on the line different from A in a different direction from A and label it C. Name two rays with end-point A which are part of the line.



3. Label a point on your paper as W. Draw a ray on your paper with end-point W. Draw another ray on your paper with end-point W. Draw two more rays on your paper with end-point W. How many rays can be drawn with W as end-point? (More than we can count.)



4. On your paper draw ray VW. What is its end-point? How many rays are there with end-point V and containing point W? (only one)

TOPIC IV: PLANES AND REGIONS

OBJECTIVE:

To introduce the idea of plane region and polygonal region.

VOCABULARY: plane region, rectangle, circle, triangle, curve

MATERIALS: one crayon and one piece of paper for each child; one piece of string; one foot piece of string for each child; one circular piece of cardboard; one rectangular piece of cardboard, one box, pupil page 31

Activity I: Flat surfaces or planes

Objectives:

1. Children can identify objects which represent part of a plane.
2. Children can indicate that a plane contains more points and more lines than can be counted.
3. Given a representation of a plane, children can show rays that are on the plane and that are not on the plane.

Materials: one crayon and one piece of paper for each child

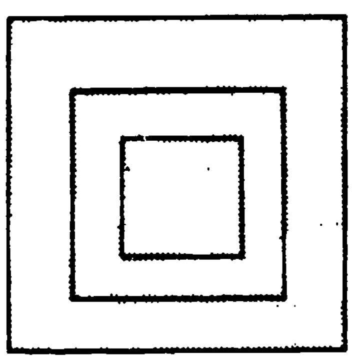
Teaching Procedure:

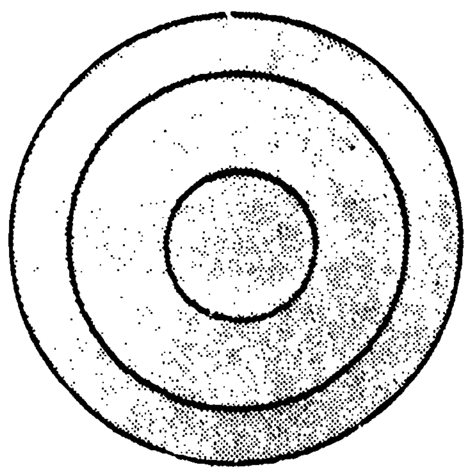
Ask the pupils to find some flat surfaces in the classroom. (desk top, floor, chalkboard, windowpane, wall, side of a book, etc.) "Do you know the name for the set of points suggested by a flat surface?" (A plane) The floor, the desk top, the sheet of paper, all represent part of a plane; they help you think of a part of a plane.

Place a tip of a pencil on the top of the desk. Next put it on a different place. Ask the students "How many points do you think there are on the flat top of this desk? (More than we can count) How many do you think there are in a plane?"

Have the students draw a triangle, circle, or rectangle on a sheet of paper. Have them trace the figure with a colored crayon and have them color the inside of the figure with the same color. Ask, "Does this colored figure help us think of a part of a plane? (Yes) Have the students draw a larger figure that encloses the colored region. Have the students color this region with the same color. Does this new colored region help you think of a part of a plane? (Yes) Have the students enclose the region with another figure, and another, each time coloring the new region. After they have done this several times, ask, "Can you draw a picture of a whole plane?" (No, a plane extends on and on without end.)

The students' figures might look like this:





"Just as we think of a line containing more and more line segments, so shall we think of a whole plane as containing larger and larger flat surfaces. Imagine that your desk top is stretching and getting longer and longer and wider and wider. If you put a pencil on your desk, it could now roll as far as you wished in any direction."

"Often we shall use a sheet of paper on the desk to help us think of a part of a plane. Draw a line on your sheet of paper. Can we draw two lines? (Yes) More lines? (Yes) How many lines? (More than we can count)

Have a student draw a ray on his sheet of paper. Have him describe a ray that is not on this plane. Have the students describe a ray whose endpoint is on the plane, but not the rest of the ray. (A pencil with tip on the sheet of paper and perpendicular to the plane of the paper. It can also be at any angle to the paper. Answers will vary.) Have the students describe a ray whose end-point is not on the plane. Ask, "Is the ray on the plane?" (No) Ask, "If the end-point of a ray and one other point are on the plane, must the ray be on the plane?" (Yes) Let the students discuss this.

<u>Activity 2</u>: Learning about regions

Pupil page 31

<u>Objectives</u>:

1. Children can identify curves, closed curves, and plane regions.
2. Children can mark points inside, outside, and on plane figures.

<u>Materials</u>: one piece of string, a one foot piece of string for each child, one circular piece of cardboard, one rectangular piece of cardboard, one box

Teaching Procedure:

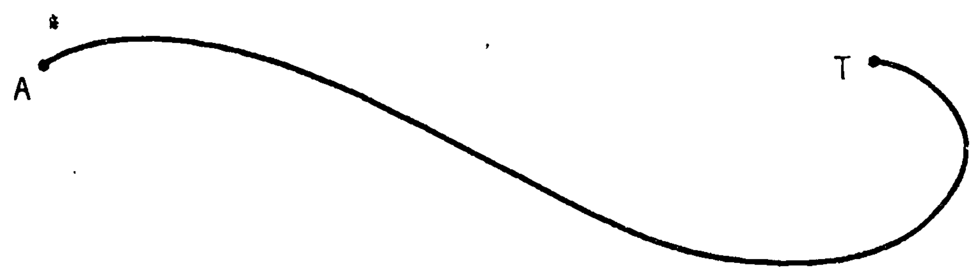
Draw a circle on the board. Mark points inside, outside, and on the circle. Ask questions like this about each point: "Is this point inside the circle, outside the circle, or on the circle?" Run your finger along the circle and say, "This is a curve." Mark other points and let the pupils tell about their locations. Let pupils mark points and tell where they are.

Say, "A circle is a closed curve. A closed curve starts and ends at the same point. An ant like Speedy can start at any point, move along the path shown by the curve and he will return to the point where he started." Ask, "Is a rectangle a closed curve? Can an ant move along the path shown by the rectangle and return to his starting point?" (Yes. A rectangle is a closed curve.)

Ask, "Is a triangle a closed curve?" (Yes) "Is a line segment a picture of a path?" (Yes) "Is a line segment a curve?" (Yes)

Note: The children may need to talk about these last two questions. Guide them to call a path a curve. A path may be straight or it may not be straight. If a path is straight, it is a line segment. Some people speak of "straight lines" and "curved lines". In later work in mathematics this leads to confusion. In this book, all lines are straight. What some people call "curved lines", we call "curves".

Draw a curve like this on the board:



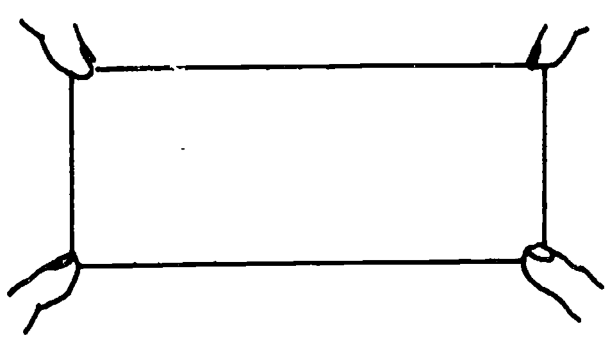
Ask, "Is this a curve?" (Yes) "Is it a closed curve?" (No) "Why?" (We cannot follow the path and come back to the point where we started.)

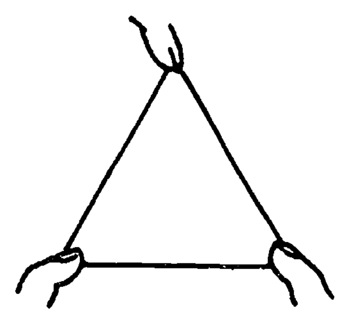
Draw a line segment on the board. Ask, "Is this a curve?" (Yes)

Hold up a piece of string so that it is not straight. Ask, "Does this show a curve?" (Yes) "Does the curve have end-points?" (Yes) Get a child to show the end-points. Tighten the string to show a line segment. Let the children use the string to show many different curves. Guide them to say that the string shows a curve in any position they put it.

Let each pupil put a piece of string about one foot long on his desk to show a curve. Walk about the room to give help as needed. Ask the pupils to stretch their strings to show curves which are straight. Let them say, "The strings show line segments."

Tell the pupils to tie the ends of their strings together. You do the same with a string. Ask, "Does each string with ends tied together show a curve?" (Yes) "What do we call such a curve?" (Closed curve) Let the children show many closed curves with their string loops. Guide them to form rectangles, triangles, and circles. Two children may work together when more fingers are needed to hold the string. Let some pupils show the class the figures they made.





Vary the whole group activity by allowing two pupils to demonstrate forming closed curves to the class. A flannel board, yarn, and pins may be used. This is an appropriate activity for students who need successful experiences.

Note: Only closed curves which do not intersect themselves are discussed with the children. If a child forms a closed curve like the numeral 8 (or 8), just say, "These are closed curves, but not simple ones." Do not emphasize this type of closed curve.

Say, "Do you remember Speedy, our ant friend? Speedy's paths were in a flat field. The board is flat." Point to a curve on the board. "This curve is on a flat board. A circle is a curve on something flat. A rectangle is a curve on something flat."

Go on. "A rectangle is a closed curve on a flat surface. It has an inside. The inside of a rectangle is flat and is called a plane region." Place your hand on the inside of the rectangle as you say this. "The inside of a circle is also a plane region." Place your hand on the inside of the circle as you say this.

Hold up a rectangular piece of cardboard. Say, "This shows a plane region. It is the plane region of a rectangle." Pass your fingers over the face of the figure as you say "region". Trace the edge of the cardboard with your finger as you say "rectangle".

Hold up a circular piece of cardboard. Say, "This shows a plane region. It is the plane region of a circle." Run your fingers over the face of the cardboard as you say this.

Hold up a box. Say, "The side of the box is a plane region." Let the children show plane regions on objects in the classroom.

Tell the children to look at pupil page 31. Let them talk about the closed curves they see and the curves that are not closed. Ask the pupils to name the curves. Note that some curves are named by a single letter. We will use small dots to show points. Say, "Think of these dots as very small. Place a dot on point A. Show B by placing a dot on point B. Show a point on the line segment BC. Show a point inside the circle D. Show a point on the curve EF." Tell the pupils to show other points on, inside, and outside each curve. Say, "Show a point inside curve EF." If there is some hesitation, ask questions like, "Can't you find an inside? Is EF a closed curve?" Guide the children to say such things as, "EF has no inside." Or "EF is not a closed curve."

Ask the pupils to run a hand over the inside of the triangle, over the inside of curve G, and along curve G. Get them to show plane regions in the drawings.

TOPIC V: ANGLES AND INTERSECTIONS

OBJECTIVES:

1. To introduce the idea of angle.
2. To introduce the idea of intersection of lines and planes with lines.

VOCABULARY: angle; vertex; intersection (of paths, curves, and line segments); arrowheads

MATERIALS: Two pieces of paper for each child; a one foot string for each child; some small seeds or beans for each child; a straight edge (yardstick); two large pieces of cardboard; a ruler for each child; pupil pages 32 - 35

Activity I: (Oral) Angles

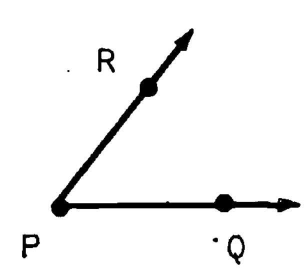
Objectives:

1. Children can identify an angle.
2. Children can identify the vertex of the angle.
3. Children can draw a picture of an angle.

Materials: one piece of paper for each child

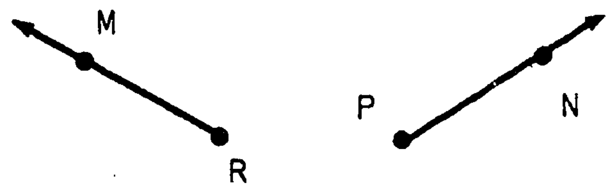
Teaching Procedure:

Several representations of angles by various objects in the classroom may be pointed out to the children. The parts of the angles can be mentioned. The teacher may begin by asking the following questions while the children do the work at their seats. "Mark a point P on your paper. Draw a ray with end-point at P and mark another point on the ray and label it Q. Now, draw a second ray with end-point at P. (See that the children do not draw it on the line PQ. If it were, this would put both rays on the same line and an angle, as we have defined it, would not be formed.) Mark a point on this new ray R. Your figure should look something like this."



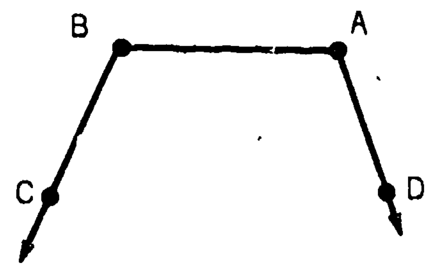
"This figure represents an angle. We can say that an angle is the union of two rays that have a common end-point and not on the same line. We called P the vertex of the angle. It is the end-point of both rays."

"If we draw these two rays like this, do we form an angle? Why?"



(No, an angle is not formed because the two rays do not have a common end-point.)

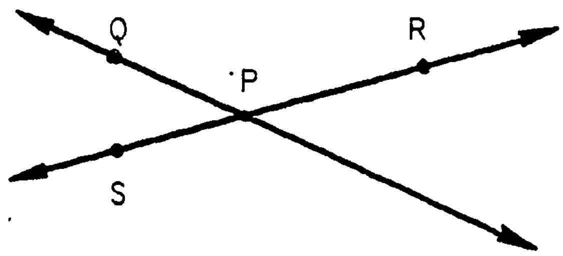
"Is this figure an angle?"



(No, this figure is not an angle because it contains two rays and a line segment and an angle is the union of two rays.)

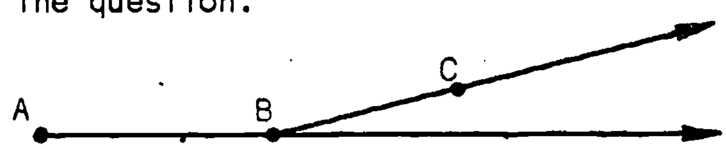
Have the students look around the room and point out objects that represent angles such as the two walls that meet to form an angle or the two edges of the student's desk that meet at a corner. Ask, "What represents the vertex of the angle?" (The corner where the edges meet.) "What represents the rays of the angle?" (The edges of the desk.) "Why is only part of an angle represented by the two edges meeting at the corner of the desk?" (The rays extend without ending, but the edges of the desk do end.)

Draw this figure on the board and have students answer questions about it.



Ray PQ and ray PR are subsets of different lines. Does their union form an angle? (Yes) Why? (The rays have common end-point P and are not on the same line.) Does the union of ray PR and ray PS form an angle? (No. Ray PR and ray PS have a common end-point, but they are on the same line.)

Ask, "If ray AB and ray BC are subsets of two different lines, is their union an angle? Why?" Have the students draw a figure like the following to help them answer the question.



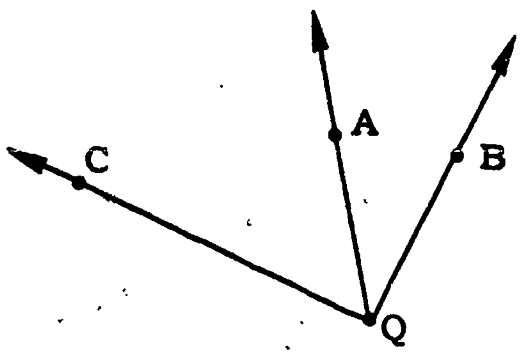
(No, because the rays do not have a common end-point.) Point out to the students that in our notation for rays the first letter mentioned always corresponds to the end-point.

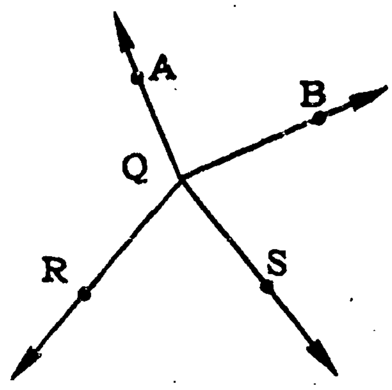
Allow time for drill in the new vocabulary and notation by having the students draw the following:

1. Draw an angle. Label the vertex A.
2. Draw an angle and label the angle PQR. What is the label for the vertex? (Q)
3. Draw an angle by drawing rays MN and M .

4. Draw an angle. Label its sides PS and PR.

5. Draw two angles with a common vertex. How many angles do you see? (Answers may vary)





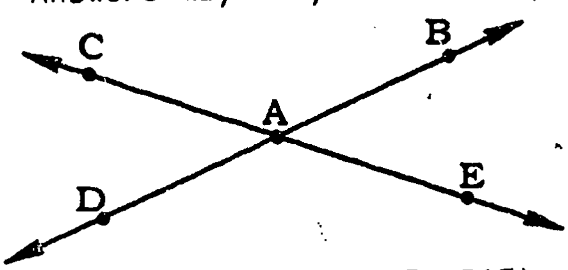
The two angles were CQB and CQA. We also see the angle AQB.

The two angles were AQB and RQS. We see here 6 angles: AQB, AQS, AQR, BQS, BQR, SQR

There are other possibilities. If AQ and QS were on the same line, then AQS is not an angle. Depending upon the two angles chosen, the number of angles exhibited may be 3, 4, 5, or 6.

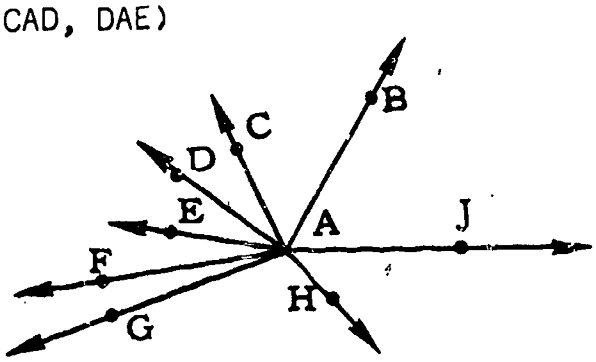
6. Draw four angles with a common vertex.

Answers may vary. The "simplest" possible answer is:



(Angles EAB, BAC, CAD, DAE)

Another answer:



(Angles BAC, DAE, FAG, HAJ)

Activity 2: Intersections of paths, curves, and line segments

Pupil page 32

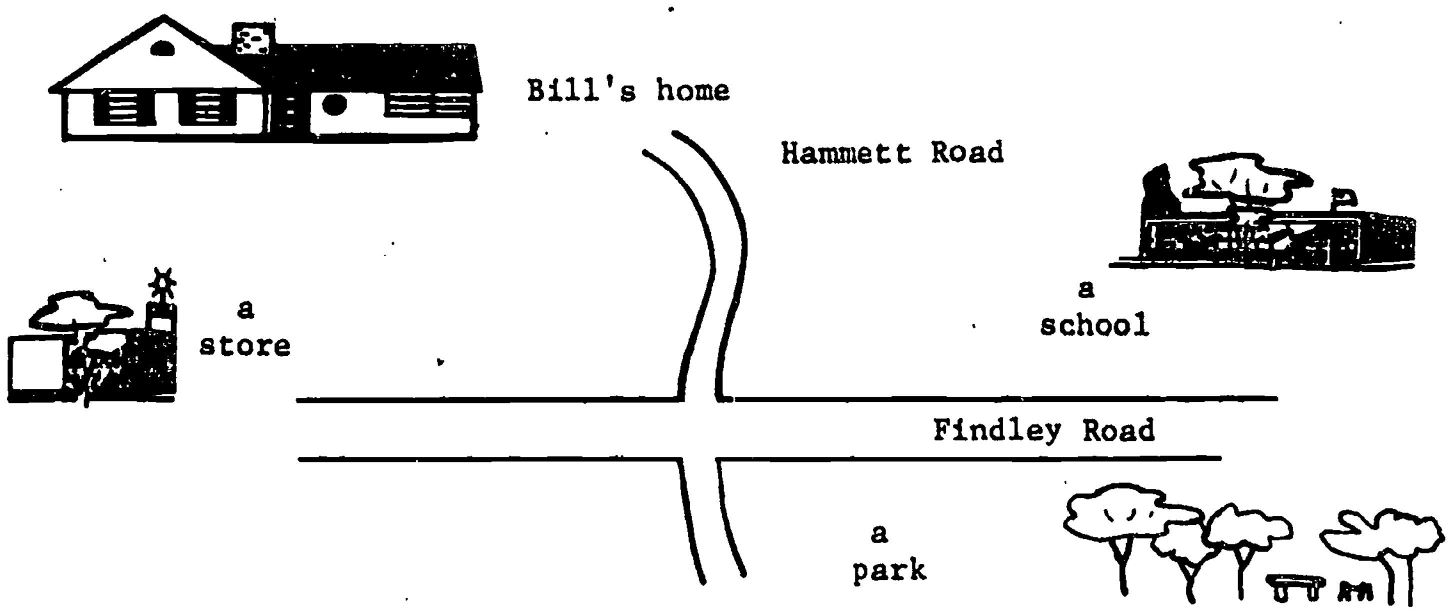
Objective:

Children can identify the point of intersection of two line segments.

Teaching Procedure:

In telling the following story, you may wish to substitute names of children in the class and names of streets on which they live to build interest.

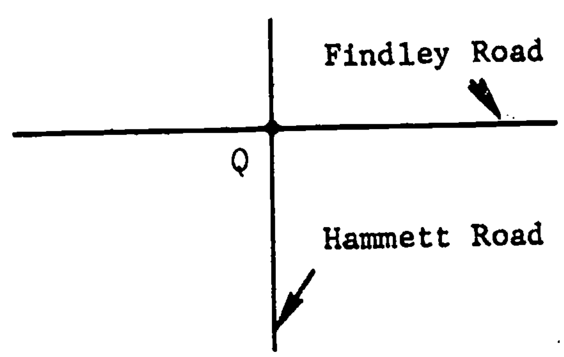
"Bill was walking along Hammett Road from his home to a park. He met his friend Peter, who was walking along Findley Road." While you are telling the story, draw the picture below on the board.



"'What are you doing here on Findley Road?' asked Bill. 'You are wrong, Bill,' said Peter. 'We are on Hammett Road.' They talked and talked. Bill said they were on Findley Road and Peter said they were on Hammett Road."

Ask the class, "Which boy was right?" Let the pupils talk about the question. If no child says, "They both were right," ask a child to come to the board. Get him to put his finger at the point in the picture where the boys met. Ask which road they were on. The pupil will say that the boys were on Findley Road and Hammett Road at the same time. Say, "The place where they met is common to both roads. It is the place where the roads intersect."

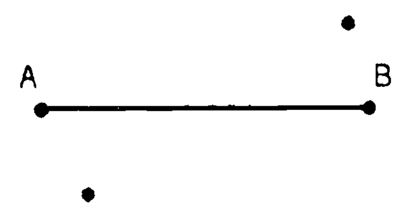
Say, "Let us show the roads with line segments. Where is the place the boys met?" Draw a picture like this:



Let a pupil mark the point where the line segments intersect, say Q. Ask, "On which line segment is point Q?" (Both) Say, "The line segments have the point Q in common. Q is the intersection of the line segments. If two line segments have a point in common, they intersect at that point." Ask questions and let the pupils talk of these same ideas in their own words.

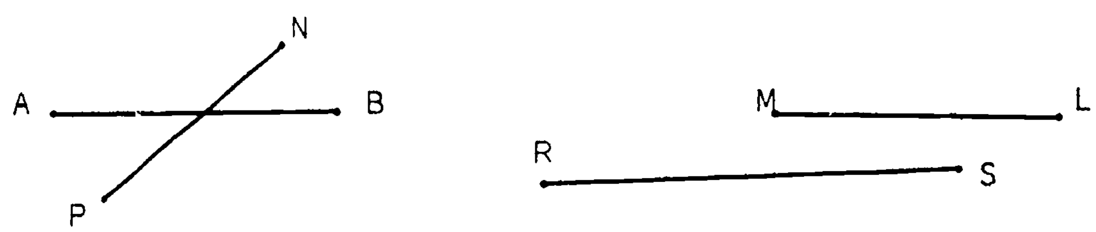
Mark two points on the board about one foot apart. Ask, "Can you draw a line segment that has these two points as end-points?" Let the children decide they can. Give a child a straight edge to use, and let him draw the line. Let another child name the line segment, say $\overline{AB}$.

Mark two other points on the board like this:

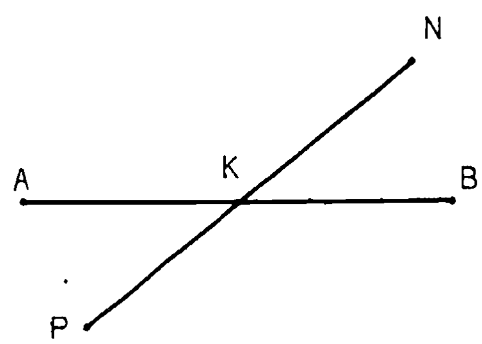


Ask, "Can we draw a line segment with these two points as end-points?" Give a child the straight-edge and let him draw the line segment and name it, say $\overline{PN}$. (Note that these two line segments intersect.)

Draw two other line segments which do not intersect. The board will look like this:



Get the pupils to talk about the two pairs of line segments. Get them to say that line segments ML and RS do not intersect and line segments AB and PN do intersect. Ask a pupil to point to the place where the line segments AB and PN intersect. Ask another pupil to mark the point and name the point, say K.



Guide the pupils to say, "Point K is on line segment AB. Point K is on line segment PN. Line segment AB intersects line segment PN. Point K is common to both line segments. K is the point of intersection of the line segments."

Let a child draw another line segment on the board. Ask another child to draw a second line segment that intersects the first. Ask, "Do these two line segments intersect?" (Yes) Let one of the children mark and name the point where the two line segments intersect, say J. Say, "These are intersecting line segments."

Let another child name the end-points of the intersecting line segments, say $\overline{MH}$ and $\overline{XY}$. Let the children tell all the marked points on each of the two line segments. Guide them to say that J is a point on $\overline{MH}$ and also on $\overline{XY}$.

Tell the pupils to turn to page 32 in their books. Ask a child to give the names of two intersecting line segments. Let the other children talk about the line segments named and decide whether they do intersect. Get them to fin other intersecting line segments. Ask for the names of two line segments that do not intersect. Ask a child for the names of the points marked on $\overline{AB}$; on $\overline{CD}$; on $\overline{MQ}$; and so on. Each answer should be talked about; the children should decide whether it is correct and why or why not.

Ask, "If $\overline{WX}$ is extended, will it intersect line segment SV? How can we decide?" Guide a child to suggest holding a string along line segment WX. Ask the pupils to talk about this and decide whether the line segments intersect. Continue with similar questions.

Activity 3: Points, lines, and intersections

Pupil page 33

Objective:

Children can find and mark points of intersection.

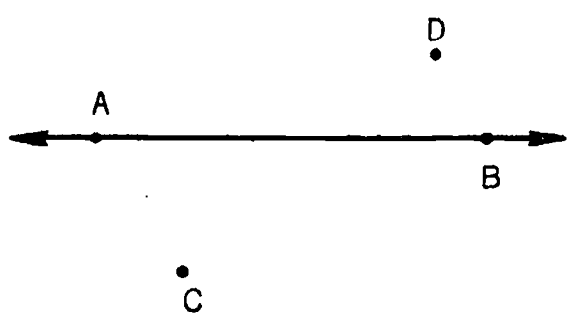
Materials: a one-foot string for each child; small seeds or beans for each child

Teaching Procedure:

Ask a child to draw a picture of a line on the board. The picture you hope the child will draw is like this:

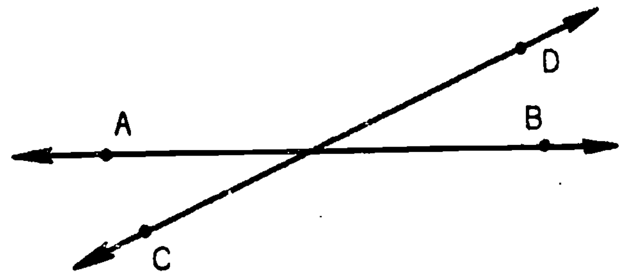
If he does not draw the arrowheads, ask, "Does the line keep going to the right? How do you show the line keeps going to the right?" Ask similar questions about extending the line to the left. Let a child draw two points on the line and name them A and B. Say, "The line goes through the points A and B."

Draw two more points, C and D, which the line does not go through. The board may look like this:

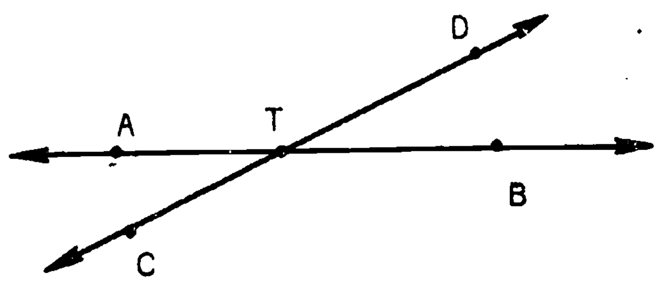


Ask a child to draw a line through the points C and D. If needed, ask questions like, "Is there a line segment with C and D as end-points?"

(Yes) "Can the line segment be extended to a line?" (Yes)



Ask, "What can we say about these two lines?" If no pupil says something like "They cross", or "They have a point in common", ask "Do they have a point in common? Why?" Let a child show the point where the lines intersect and name it.



Ask a child to show line AB with a string. Get him to hold the string along the line AB. Ask another child if the string shows the line AB. Ask another child if the line stops at B? At the end of the arrow? At the place the string is held? Get the children to say the line goes on and on.

Let another child show the line CD with a string. Ask another child to place his fingers at the point where the lines intersect. Ask what the two strings show. Guide the children to say something like, "The strings show two lines. The lines intersect."

Give each child a piece of string about one foot long and some small seeds. Tell the children to turn to pupil page 33. Ask, "What geometric objects are drawn on this page?" (Points) "How many points are marked?" (16) "Do the points have names?" (Yes, A, B, C, to P)

The list of directions and questions that follows will help the children to think about points, lines, and intersections. You will think of others.

Stretch a piece of string to show the line that points B and O are on.

Are any of the other points on the line? (No)

Stretch a piece of string to show the line that points B and G are on.

Are any of the other points on the line? (No)

Find other pairs of points that are on lines. Let the children say which pairs of points they think lie along lines. (All pairs)

Is point K on the same line as points M and G? (No)

Is point K on the same line as points D and P? (Yes)

Is point I on the same line as points F and O? (No)

Is point O on the same line as points G and K? (No)

Is point B on the same line as points H and D? (Yes)

Can you stretch the piece of string along the line that has five points drawn on it? (A, E, F, K, and N)

How many lines can you find on the page that have exactly three of the points on them? (Three: DP; IP; HB)

Find a point that is on the same line that point B is on. (There are 15 such lines.)

Place a seed so that it will lie along the same line as points M and D.

Find a line that has the points D and M on it. Find the line that has the points F and G on it. Can you place a seed at the point where these two lines intersect? (Give children help if they need it. They must imagine where these two lines intersect. Let them guess where the point is first. Then they can test by stretching two pieces of string between the proper points.) Let the children practice finding the intersection of lines on the page. Let the children work in pairs, one holding his string along the line DM, the other along the line FG.

Activity 4: (Oral) Many lines through one point; only one line through two points

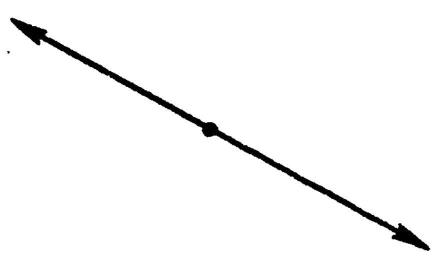
Objectives:

1. Children can mark many lines through a given point.
2. Children can mark only one line through two given points.

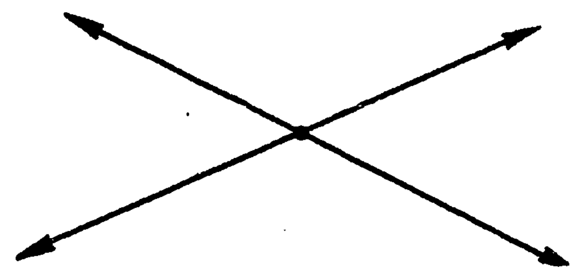
Materials: a straight edge; 2 large pieces of cardboard

Teaching Procedure:

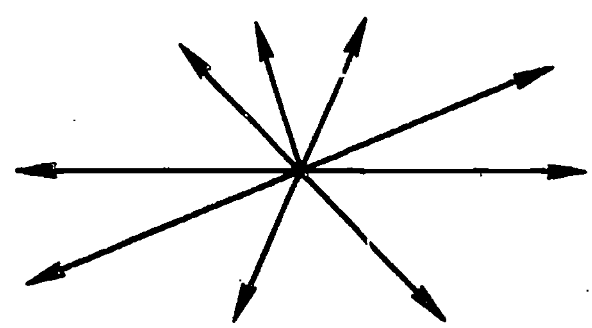
Mark a point on the board. Ask whether a line can be drawn through the point. Let the children talk about this and let one of them come to the board and draw a line. Let another child help him hold the straight edge up to the board.



Ask, "Did he draw a line through the point?" , "Can another line be drawn through the point?" Let the children talk about this. Get another child to draw a second line through the point.

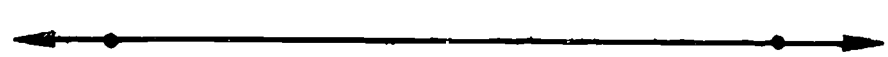


Ask, "Can a third line be drawn through the point? a fourth?" Continue as before, with children deciding that another and still another line can be drawn through the point. Let them draw many lines through the point.

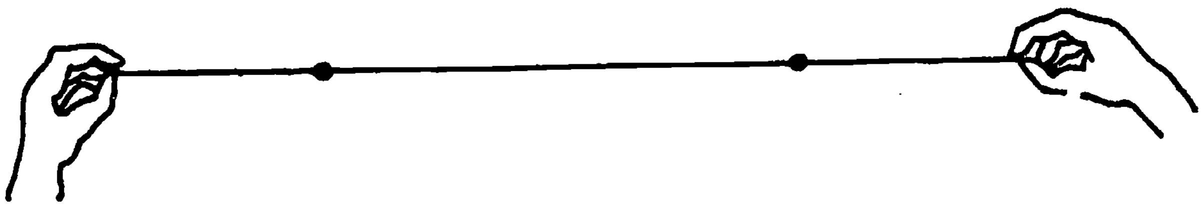


Ask, "How many lines have you drawn through the point? Can more lines be drawn through the point?" Get the children to say that many, many lines can be drawn through the point.

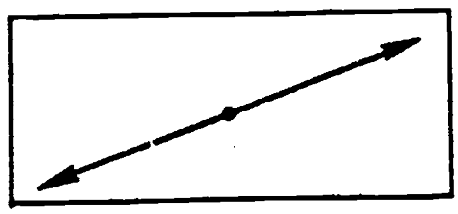
Let a child draw two points on the board. Ask, "Can you draw a line that goes through both of these points?" Let the children talk about this and let one of them draw the line. Let him choose some one to hold the straight edge up to the board.



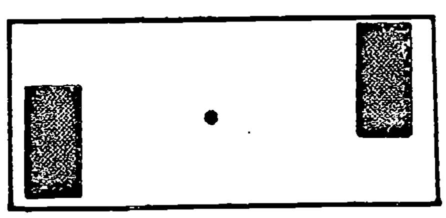
Ask, "Can you draw another line through these two points?" Let the children try. Guide them to discover that only one line can be drawn through two points. Get other children to stretch a string through both of these points, extending it to the edges of the board, then stretching it to the walls. Point again to the two points and ask, "How many lines can we draw through these two points?" (Only one)



Mark a point on the board. Help a child to draw a line through the point extending from one side of the board to the other. (Do not draw a horizontal line.) The diagram below is the picture of an entire section of board.



Quickly erase the left part of the line. Leave the arrowhead. Erase the rest of the line except for the original point and the other arrowhead. Cover the two arrowheads with pieces of cardboard. Leave only the original point for the children to see. Erase the line well so that children cannot see where it was drawn.

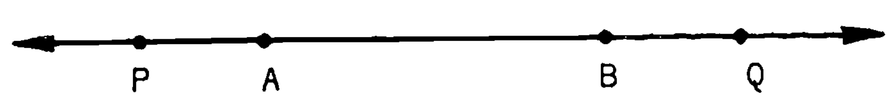


Say, "The line is erased but the point is still there. Will someone put the line back?" Let a child try to redraw the line. Be sure he cannot see the arrowheads under the cardboards. Ask, "Is this new line in the place where the erased line was?" If they do not agree, let them draw lines where they think the erased line was.

Remove the cardboards to let the children see whether the rubbed-out line has been replaced. Repeat this exercise several times. Get the children to see that it is difficult to redraw an erased line when only one point is known. They may say that if they could see one of the arrowheads they could put the line back.

Change the activity. This time mark two points on the board instead of one. Again let a child draw a line through the points and mark arrowheads at the ends. Again erase the line except for the arrowheads and the two points. Cover the arrowheads with large pieces of cardboard. Then ask the same question, "Can someone put the line back?" Get the children to see that it is easy to put the line back, because now they see two points that were on the line. Only one line can be drawn through two points.

Let a child draw a line on the board. Get another child to draw two points on it, say A, B. Ask still another child to mark two other points on this line, say P and Q, one near one arrow, the other near the other arrow.



Ask a fourth child to take a string and show the line through P and Q.

Ask, "Does the line through P and Q go through A and B? Is the line through P and Q the same as the line through A and B? (Yes) "Is the line through P and B the same as the line through A and Q?" (Yes) Repeat this exercise with other lines and points on the lines.

The children should understand that through any two points of a particular line only that line can be drawn. The children may not be able to say this in words. You try to help them understand the ideas.

Activity 5: Intersection of line segments

Pupil page 34

Objective:

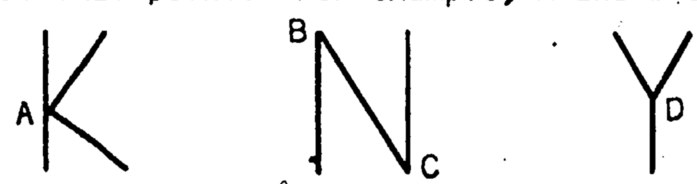
Children can mark points of intersection of line segments.

Teaching Procedure:

Note: Block letters of the alphabet exhibit geometric figures. All of the letters except "C", "U", "S", and "O" contain at least one line segment. Many types of intersection of line segments are shown in these letters. For example, the letter "X" shows two line segments that intersect at a point that is not an end-point. The letter "L" shows two line segments which intersect at an end-point. The letter "T" shows two line segments intersecting at a point that is an end-point of one line segment and not an end-point of the other line segment.

Tell the pupils to turn to page 34 in their books. Ask, "Do you see any line segments on this page?" Remember that the edge of the page shows line segments and that the rectangles in which letters are contained also show line segments. Let the pupils talk about the letters and whether line segments make the letters. Let them also talk about the right angles shown by some of the letters and about the intersections of the line segments to form letters.

Let the pupils count the number of line segments in each of the letters. Let them tell which letters use at least one, two, three, and four line segments. Let them place dots at the points of intersection. Let them talk of the different ways the line segments intersect. Help them recall that if two line segments have a point in common they intersect at that point. For example, A and D are points of intersection



of three line segments; two line segments intersect at B and at C. (A may also be seen as the intersection of four segments.)

Activity 6: Circles

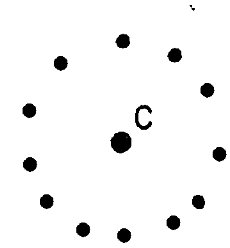
Objective:

Children can draw a picture of a circle by marking and connecting a series of points the same distance from a given point.

Materials: one piece of paper for each child; a ruler for each child

Teaching Procedure:

Have the children mark a point on their paper and label it C. Mark another point two inches from point C. Have them mark many other points that are two inches from C. Be sure that the points are marked in all directions from C. Ask the children to draw a closed curve suggested by the set of points marked. Point C is not used.



Now have the children find some more points that are two inches from point C. Where are these new points located? Does this special closed curve have a special name? (Yes) The name of this curve is circle.

Have the children draw other circles by having them mark a point D on their paper and finding many points one inch from D; then another point E and finding many points one and a half inches from E, etc.

Activity 7: Intersection of curves and line segments

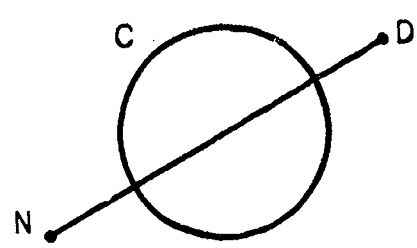
Pupil page 35

Objective:

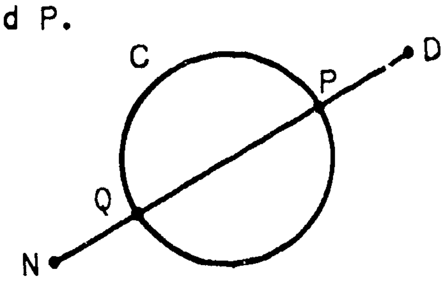
Children can identify points of intersection of plane figures.

Teaching Procedure:

Draw a circle and line segment that intersect like this:

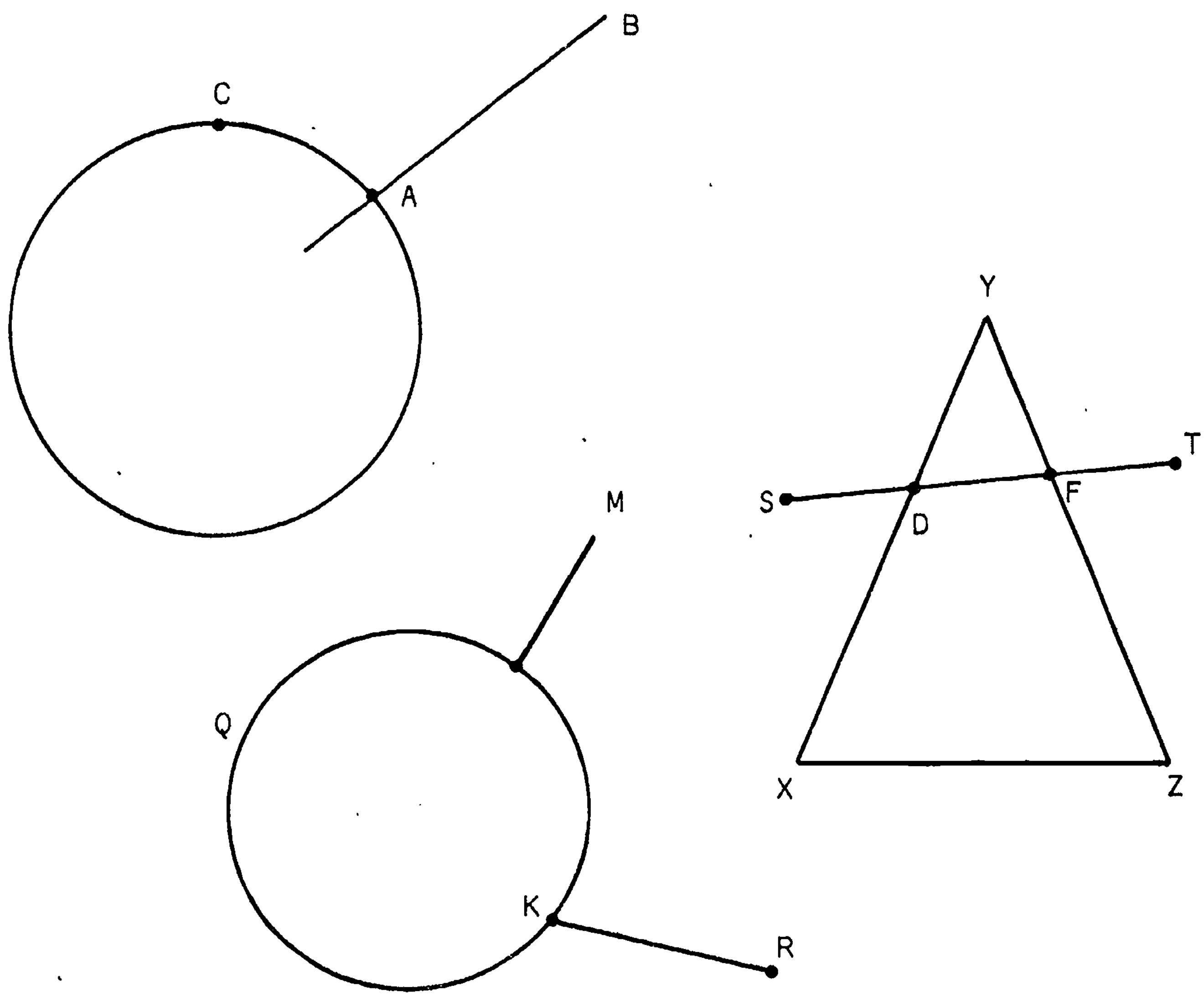


Let a pupil name the line segment, say ND, and the circle, say C. (You may need to say that the circle may be named with a single letter.) Let the pupils talk about the figure. Get them to say that there are two points that are common to both the circle and the line segment. Let a pupil name them, say Q and P.



Ask questions like, "Do the circle and the line segment intersect?" (Yes) "Why do you say they intersect?" (The line segment and circle intersect because a point on the line segment is also a point on the circle.) "Where do they intersect?" Help them to trace along the circle and line segment and name both point P and point Q.

Make figures on the board like the following ones, and talk about them in a similar way.



Points A, V, K, D, F, X, Y, and Z are all points of intersection.

Let the pupils look at page 35. Ask questions like, "Do you see a circle that intersects a line segment?" (Pictures 1, 3, 6, and 8) "In which pictures are there circles that intersect circles?" (5) Let the pupils ask each other questions. Make sure the intersection shown in each picture is discussed.

Get the pupils to place dots on the points where a circle intersects a triangle. Ask, "What does picture 7 show? Which picture shows a triangle intersecting a square?" (None)

TOPIC VI: REVIEW OF ANGLES, TRIANGLES, AND POLYGONS

OBJECTIVES:

1. To explore the idea of angles and of triangles.
2. To introduce the idea of plane regions and polygonal regions.
3. To develop the understanding that a polygon is a simple closed curve that is the union of line segments.

VOCABULARY: angle, vertex, vetices, joining two points, congruent angles

MATERIALS: pencils or several sticks to make demonstration models of a triangle; three pencils or sticks for each two children; pupil pages 36 - 38

Activity I: (Oral) Review of angles and of triangles

Pupil pages 33, 36, 37

Objectives:

1. Children can identify, name, and mark angles and triangles.
2. Children can identify and name the angles determined by a triangle.

3. Children can draw representations of the rays of the angles determined by a triangle.

Materials: three pencils or sticks for each two children

Teaching Procedure:

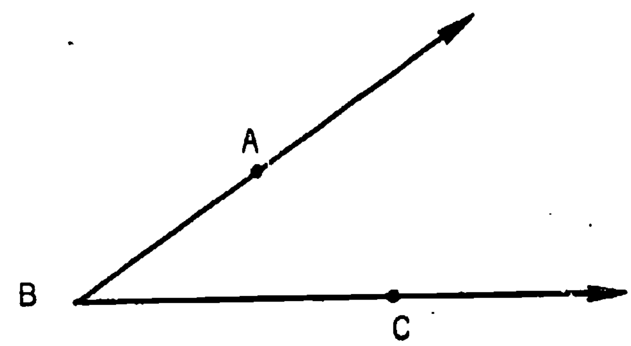
Hold up a book. Run two of your fingers along the two edges that show an angle. Let your fingers meet at the corner. Ask, "What do these two edges show?" You might get several answers, such as two rays, edges of the book, a corner, and an angle. Agree with these answers. If some child says, "An angle", ask a child to draw a picture of the angle on the board. If no child says angle, ask a child to draw a picture of the corner of a book on the board.

Ask questions to get children to name the end-points of the two line segments that make the figure. The picture may look like this:

Let the children talk about the figure. Get them to say things like: $\overline{PN}$ is a line segment; $\overline{NR}$ is a line segment; the line segments intersect; the point N is on both line segments. Point out that the line segments NP and NR might be extended to rays. Indicate the rays by arrows, as in the picture. Remind the children that this is an angle. It is made from two rays that have the same end-point and are not on the same line. Point N is called the vertex. Let them continue to talk about the figure. Guide them to say things like: the two rays form an angle; the angle is named angle PNR or angle

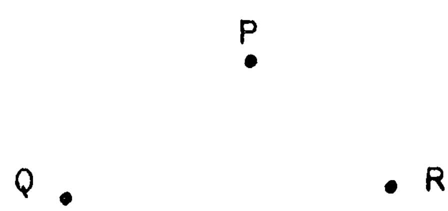
RNP; the point N is the vertex of the angle. Two line segments meeting at a common end-point indicate an angle formed by extending the line segments to rays.

Mark three points on the board. Say, "Who will mark an angle using these three points?" If no child volunteers, choose a child to draw a ray through two of the points. Let another child name the points. Ask a third child to draw a ray through two different points. The rays may form a figure like this.

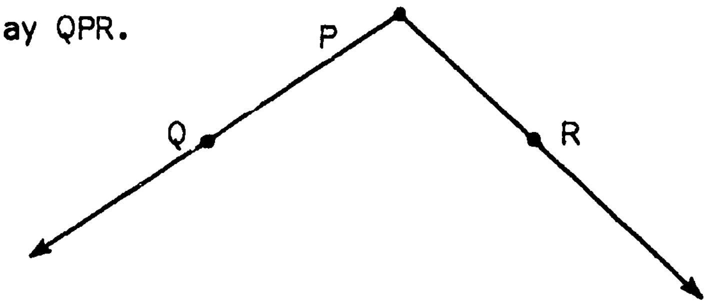


Get the children to agree that: the figure is an angle; it is formed by rays BA and BC; the rays intersect at point B; point B is called the vertex.

Mark three points in positions similar to A, B, and C at a different part of the board. (Be sure the points are so arranged that the angles formed will be congruent.) Say, "Here are three other points like A, B, and C." Ask, "Can we draw an angle using these three points?"

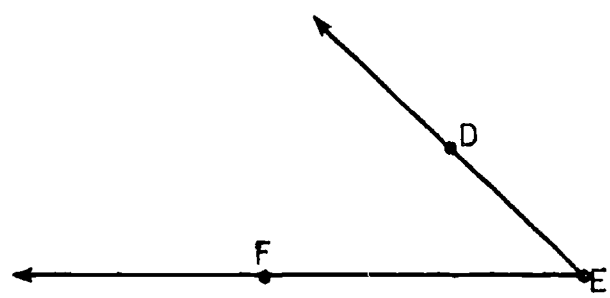


Let a child do so. If he draws angle PQR, say "John has drawn an angle using P, Q, and R. It looks like angle ABC which we just drew. Can a different angle be drawn using P, Q, and R?" Then rub out the picture of the angle just drawn. With the same thre points get the children to draw another angle, say QPR.

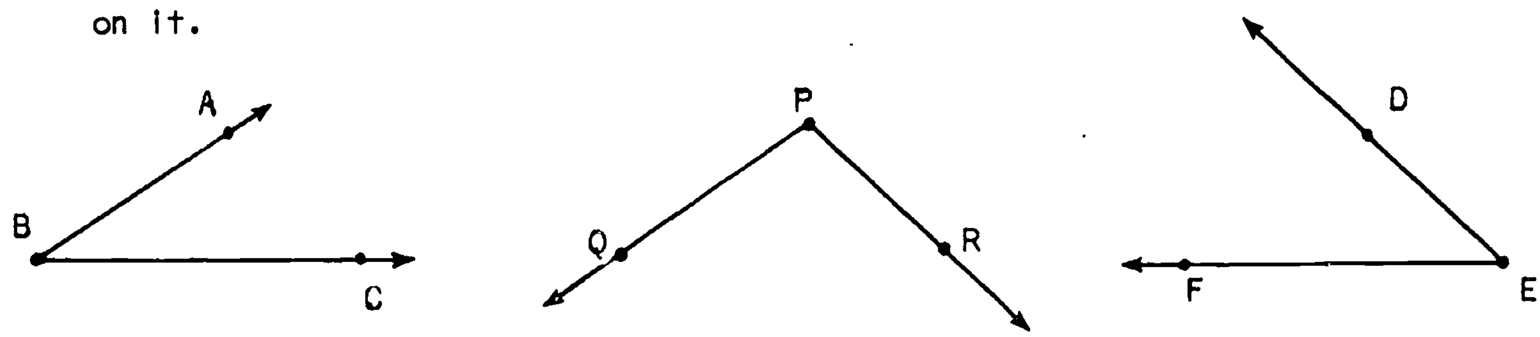


Say, "When we draw a line segment with two points as end-points, we say we join the two points or connect the two points." Let a child say things like, "Points P and Q are joined by the line segment PQ."

Draw three more points in positions similar to A, B, and C at another part of the board. Say, "We have made two angles by joining points; can we draw a third?" Get a child to name the points, say D, E, F.

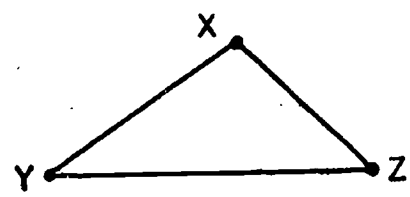


Let another child draw angle DEF. The board will now have three figures on it.



Ask the pupils to name the vertex of each angle. (B, P, E) Then ask them to name the angles on the board. As they give their answers, review the idea of naming the vertex between the other points. (ABC or CBA, QPR or RPQ, DEF or FED)

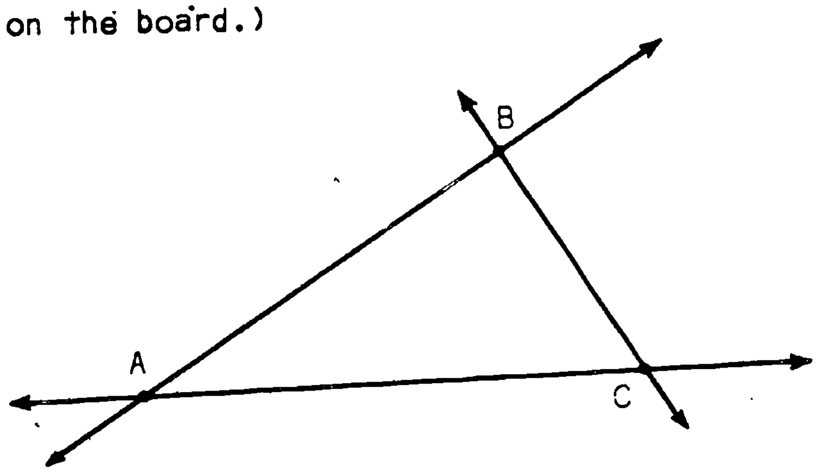
Draw three more points like A, B, C. Let children draw all three line segments to form a triangle. The board will now include this figure:



Let the children talk about the triangle. Let them say things like: the triangle has three sides; the sides are line segments; the triangle is a closed curve.

Ask the pupils to turn to pupil page 36 and to look at the points labeled A, B, and C. Say the following:

1. Draw $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, $\overline{BA}$, $\overline{CB}$, $\overline{CA}$.
2. Does your drawing look something like this? (Draw this figure on the board.)



3. Write in the blanks the words that complete these sentences.

   a) Line segments AB, AC, and BC form a (triangle).

   b) The angle with vertex A which contains B and C is called (BAC or CAB).

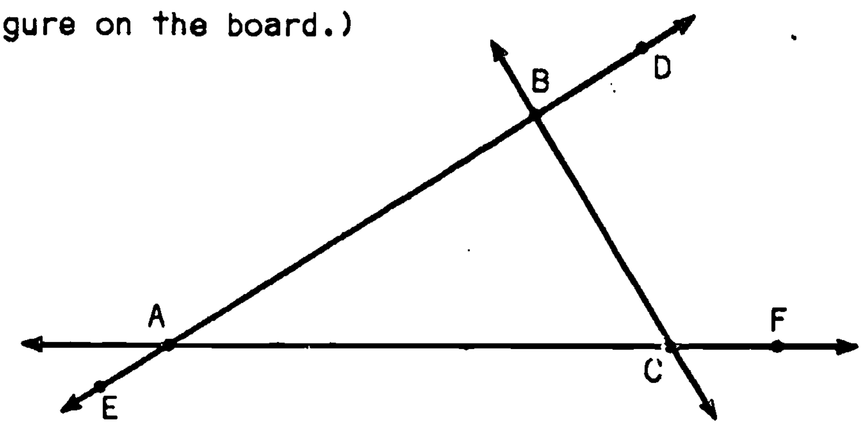
   c) The angle with vertex B which contains A and C is called (ABC or CBA).

   d) The angle with vertex C which contains A and B is called (ACB or BCA).

4. Mark a point of ray AB which is not a point of line segment AB. Label it D.

5. Mark a point of ray BA which is not a point of line segment AB. Label it E.

6. Mark a point of ray AC which is not a point of line segment AC. Label it F. Does your drawing look like this now? (Draw this figure on the board.)



7. Are D, E, and F points of the rays of the angles you named on your paper? (Yes)

8. a) Are D, E, and F points of the triangle? (No)

   b) Are D, E, and F points of the inside the triangle? (No)

   c) Are D, E, and F points outside the triangle? (Yes)

Say, "Our figure shows that a triangle suggests or determines three angles. These angles are not part of the triangle. This is true because a triangle is made up of segments and an angle is made up of rays.

"Remember when we studied circles we spoke of the center of a circle. The center is not part of the circle. In the same way we say angles ABC, BCA, and CAB are angles of the triangle although they are not part of the triangle. We call the vertices of these angles the vertices of the triangle."

9. Draw a triangle on the bottom of your page. Label its vertices E, A, T.

   a) Write the names of the three angles determined by the triangle in the blanks at the bottom of the page. (EAT, TEA, and ATE)

   b) The three angles of a triangle suggest how many rays? (6)

Let the children draw other triangles and name the angles. The vertices may be named P, Q, and R, or L, N, and D.

Look at page 33 again. Ask them to join points O, N and C with pencil lines. Say, "What is the name of the figure you have drawn?" (triangle) Ask children to join the following sets of points: J, F, and H; M, G, and C; and A, E, and N. The last three points lie on the same line; so the drawing will not show a triangle. Let the children discover this for themselves and tell about it.

Additional Exercises on Triangles and Triangular Regions

The exercises on page 37 can be used to help the students explore the properties of triangles further and to strengthen their vocabulary of geometric terms.

Activity 2: Drawing of four-sided figures (quadrilaterals) | Pupil page 38

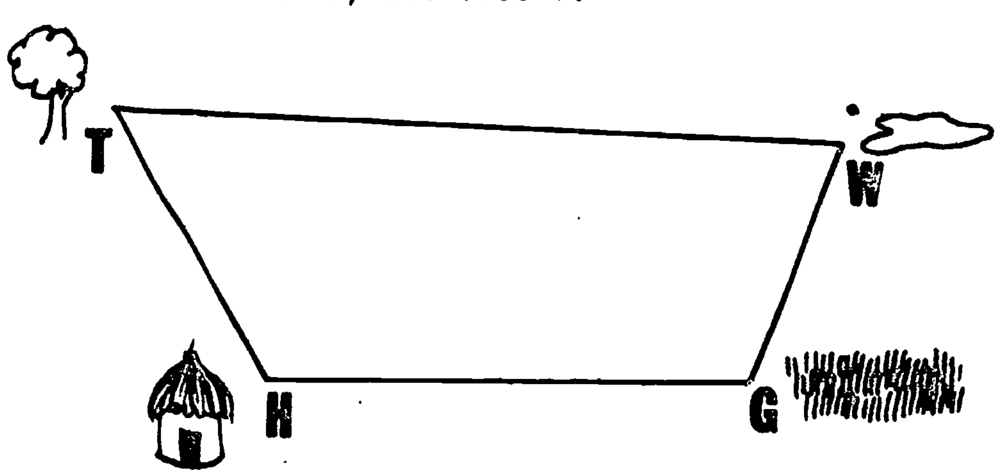
Objectives:

1. Children can identify the vertices of four-sided figures.
2. Given the vertices, children can draw a picture of a quadrilateral.

Teaching Procedure:

Tell this story: "Babar was a little elephant with big ears who lived in the shade of a tall tree. One morning he awakened early and flapped his ears and ran straight to a waterhole. After drinking, he was hungry. He ran directly to a field of fresh grass. As he was eating, he heard a boy crying. 'What is the matter, little boy?' asked Babar. 'Boo, hoo', cried the little boy. 'I have been walking and walking and cannot find my way home.' 'Never mind', said Babar. 'You hop on my back and we will see what we can do.' The boy hopped on Babar's back. Babar flapped his ears. Before the little boy knew it, they reached his home. The boy's mother was very happy. She gave Babar three bunches of bananas, four coconuts, and five loads of grass. Babar ran happily straight to his home by the tree."

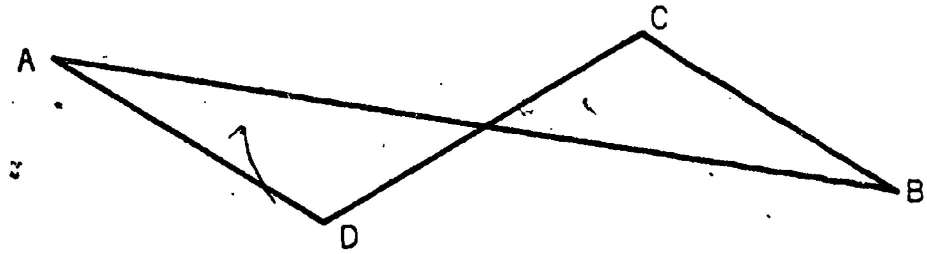
As you tell the story, draw a picture of Babar's path from the tree T, to the waterhole W, to the grassy field G, to the boy's home, H, and back to Babar's home by the tree T.



Ask questions like, "What geometric figure does Babar's path show?" (4-sided figure) "How many line segments are there?" (at least 4) "What are the names of the line segments? How many angles are suggested by the figure?" (4) "T is the vertex of what suggested angle?" (Angle HTW) Say, "T is also called a vertex of the 4-sided figure TWGH."

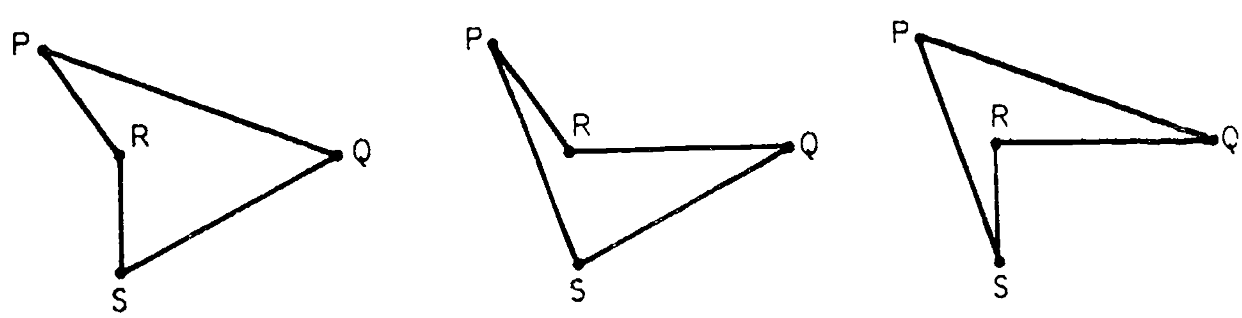
Let a child draw 4 other points on the board. Ask another child to name them A, B, C, and D. Let another child draw a 4-sided figure using these points as vertices.

Note: If someone connects the points as shown,



tell them that even though these are very interesting, we want to "save some things for high school." We consider only four-sided figures which do not intersect themselves. Likewise, we do not consider four-sided figures with any three of the points on a line.

Let them look at pupil page 38. Ask the children to draw a four-sided figure using points A, B, C, and D as vertices. Then ask them to draw a four-sided figure using points P, Q, R, and S as vertices. Children will see that these points can be joined to form several different four-sided figures by observing the work of others. In section 3, ask pupils to draw a different figure using the same vertices as were used for section 2. Their drawings may resemble those below:



UNIT 6

Fractions

OBJECTIVES:

1. To review the meaning of the fractions one-half, one-fourth, one-eighth, one-third, and one-sixth.
2. To extend the concept of fractions to include one-fifth, one-tenth, and one-twelfth and to learn their names and numerals.
3. To introduce order of fractions and to compare pairs of fractions that are not equal; one fraction may have many names.
4. To introduce multiples of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$.
5. To help students discover that if both the numerator and denominator for a fraction are multiplied by the same number, the result is another name for the same number.

BACKGROUND INFORMATION FOR TEACHERS:

We use the whole numbers 0, 1, 2, 3, 4 . . . when we talk about the number of members in a set. But often we want to talk about parts of objects or about separating sets into subsets of the same size. When we talk about parts of objects or parts of sets or regions, we use another kind of number. These numbers are called fractions.

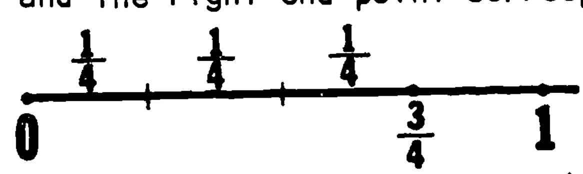
If we cut an orange into three parts of the same size, we say we have separated it into thirds. If we consider two of the three parts, we are thinking of two-thirds of the orange. If we think of a rectangular region separated into three congruent parts, then two of the parts are two-thirds of the rectangular region. If we have a set of three balls, then the number of members in the subset of two balls is two-thirds of the number of the members of the set. The fraction two-thirds is a number that describes all such parts of a whole which are two of three parts of the same size.

Fractions have numerals (names). The usual numeral for a fraction uses a pair of numerals. For example, for the fraction two-thirds we may use the symbol $\frac{2}{3}$. The "3" means that we have three parts of the same size; the "2" means that we are thinking of two of the parts.

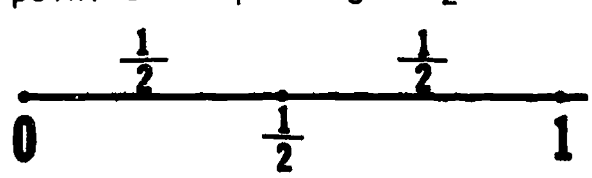
In the interest of uniformity, it may be pointed out here that even though either form of the symbols for fractions is good usage ($\frac{1}{2}$ or 1/2), it will be less confusing for the children if the teacher will use the vertical form ($\frac{1}{2}$) that is presented in the pupil's book. It is not necessary to mention this to the pupils unless one asks; then, he may be told that either form is correct.

We associate the whole numbers with points on the number line. The number line helps us show the order of the whole numbers. If a point is to the left of a second point then the number associated with the first point is less than the number associated with the second. If a point is to the right of a second point, then the number associated with the first point is greater than the number associated with the second. On the number line, 5 is to the left of 7 and we say $5 < 7$. Also, 7 is to the right of 5, and we say $7 > 5$.

We may also associate fractions with points on the number line and thereby set up an order among fractions. Suppose we want to compare the number $\frac{1}{2}$ with the number $\frac{3}{4}$. We locate the point that corresponds to each of these numbers on the number line. To find the point on the number line which corresponds to $\frac{3}{4}$, the line segment from 0 to 1 is divided into 4 equal segments. Each of these 4 parts has length which corresponds to $\frac{1}{4}$ unit. The union of three of the equal parts laid end to end, is a line segment whose length is $\frac{3}{4}$ unit; its left end-point corresponds to zero and the right end-point corresponds to the point $\frac{3}{4}$.



Similarly, the point corresponding to $\frac{1}{2}$ is located.



After having located the points corresponding to $\frac{3}{4}$ and $\frac{1}{2}$, we find that the point for $\frac{3}{4}$ is to the right of the point for $\frac{1}{2}$. We say that $\frac{3}{4} > \frac{1}{2}$. Likewise we notice that the point for $\frac{1}{2}$ is to the left of the point for $\frac{3}{4}$, that is $\frac{1}{2} < \frac{3}{4}$. If two numerals, such as $\frac{1}{2}$ and $\frac{2}{4}$ name the same point on the line they are names for the same fraction. Thus, the comparison of fractions is related to the comparison of line segments. Consider the line segment from the point for 0 to the point for 1. If we compare one-half of that line segment to three fourths of that line segment, the part described by the number $\frac{3}{4}$ (three of four equal parts) is longer than the part described by the number $\frac{1}{2}$ (one of two equal parts). We say that $\frac{3}{4} > \frac{1}{2}$.

If the pupils have had no introduction to fractions in the third grade, the teacher will find that pupils will make more progress if they are taken slowly through the unit, with additional duplicated materials provided for practice when needed. It will be helpful to study S.E.C.L. units on fractions for grades 2 and 3.

TOPIC I: REVIEWING THE MEANING OF FRACTIONS AS NUMBERS

OBJECTIVES:

1. To review the meaning of the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{6}$.
2. To develop the idea of using fractions for subsets and parts of regions.

VOCABULARY: whole, part, one-half, one-third, one-fourth, one-sixth, one-eighth, halves, thirds, fourths, sixths, eighths, fraction, numerator, denominator

MATERIALS: five 2" by 18" strips of paper for each child; several 1-inch squares of paper for each child; small objects; counters; three cutouts of circles, rectangles, squares, hexagons, and triangles for each child; five 5" by 7" numeral ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$) cards; five boxes (or baskets); pupil pages 39 - 43

Activity I: Showing parts of things

Objective:

Children can fold strips of paper into parts showing the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{6}$.

Materials: five 2" by 18" strips of paper for each child

Teaching Procedure:

Give each pupil five 2" by 18" strips of paper. (Pupils may work in pairs.) Let them fold one of the strips of paper to make the ends meet, then unfold the paper and tell the number of parts into which the whole strip has been separated. Ask whether the parts are the same size. Guide the pupils to say, "Each part is one-half of the paper." Let a pupil write the numeral for one-half on the board. ($\frac{1}{2}$) Ask the pupils to write the numeral for one-half on each part of their folded paper. Ask, "How many halves are in the strip?" (Two; there are two halves) Guide the pupils to say, "One half is one of two parts of the same size."

Let the pupils point to parts of objects in the room that show one-half; for example, one-half of the door, one-half of a book, one-half of a window.

Ask the pupils to fold another 2" by 18" strip of paper into three parts of the same size. This is more difficult and will require trial and error in folding. The teacher should demonstrate with 2" by 18" strip by folding both ends toward each other, one overlapping the other until three parts of the same size are obtained. Ask someone to show that the three parts are the same size. Let the pupils decide on the fraction that describes each of the parts. (One-third) Ask a pupil to write the numeral for the fraction on the board. ($\frac{1}{3}$) Let each pupil write the numeral for one-third on each part of his folded strip. Say, "You have folded your strip into thirds. How many thirds are in the whole strip?" (Three; there are three thirds)

Repeat the activity by letting the pupils fold strips into fourths, sixths and eighths. Each time, let them check to see if the parts are the same size, name the fraction that describes one of the parts and write the numeral for the fraction on each part of the strip. Ask one pupil to write the numeral on the board.

Tell the pupils that each of the numerals on the board names a number. These numbers have a special name. They are called fractions.

Point to the numeral for one-fourth ($\frac{1}{4}$). Say, "This is the name for the number one-fourth. We call 1 the numerator for the fraction. It tells how many parts we are thinking of. For example, we can show one fourth of a strip of paper. We call 4 the denominator for the fraction one-fourth. It tells into how many parts of the same size we have separated the whole. For example, we folded this strip into four parts of the same size.

Say, "Tell the numerator and denominator for the other fractions." Let the pupils tell what each fraction they showed with folded paper means. Guide them to tell about each fraction as you did for one-fourth. Then let them name fractions they know other than the ones named.

Activity 2: Fractions for parts of regions

Pupil page 39

Objective:

Given regions one part of which is shaded, children can name the fractions which are associated with the shaded parts and can write the numerals for the fractions. ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$)

Materials: several 1-inch square cards for each child

Teaching Procedure:

Ask the pupils to turn to page 39 in their books. Tell them to point to the first picture at the top of the page. Ask, "What is the name of this figure?" (Rectangle) Let the pupils talk about what they know about rectangles. Hold your book up and move your finger along the edges of one of the rectangles pictured. As you do this say, "This is a rectangle." The following questions may be used to guide the discussion. You will think of others.

How many sides does a rectangle have? (4)

What do you know about line segment BC and line segment AD?

(They are the same length; they fit exactly.)

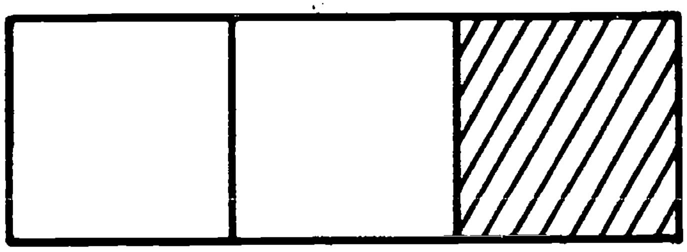
What do you know about line segment BA and line segment CD?

(They fit exactly.)

What do you know about the angles suggested? (They are all right angles.)

Move your hand over the inside of the rectangle. As you do this say, "This is the inside of the rectangle. It is the region of the rectangle."

Point to the second picture. Ask, "What do we call this figure? Into how many parts of the same size has the region been separated?" (2) "How many parts are shaded?" (1) "What number describes the shaded part?" ($\frac{1}{2}$) Let the pupils answer similar questions about the shaded parts of the other regions on the page. Give each pupil several one-inch square cards. Ask them to write on a card the numeral for the fraction describing the shaded region of each figure and to place the card beside the figure. Draw the figure below on the board as an example:



$\frac{1}{3}$

Go around the classroom, giving help as needed. Ask questions so the pupils use the ideas of fractions, numerator and denominator.

Activity 3: (Oral) Fractions for subsets

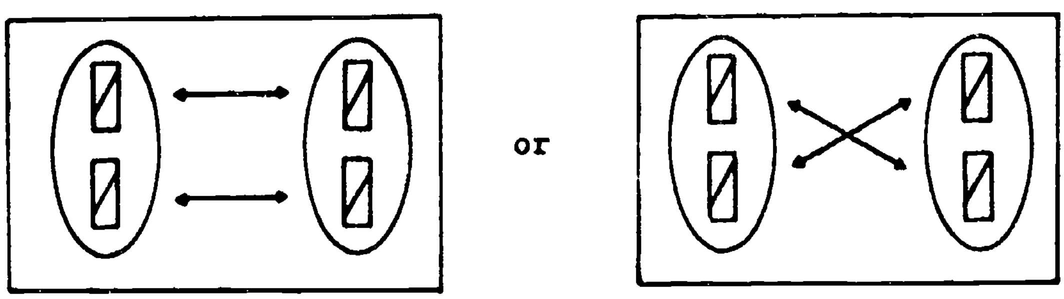
Objective:

Given a set of objects, children can separate it into equivalent subsets, can describe the subset by using a fraction, and can write the numeral for the fraction. ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$)

Materials: small objects, counters, five 5" by 7" numeral cards

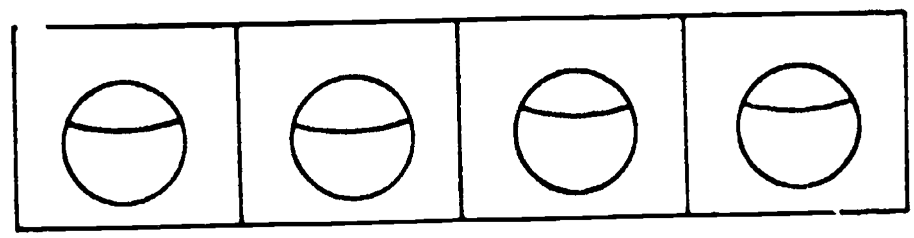
Teaching Procedure:

Place a set of four objects on your table. Ask a pupil to separate the set into two subsets so that each subset has the same number of members. Check this by asking another pupil to match the members of the subsets. Ask, "How many members are in each subset?" (2) The members may be matched exactly one to one in this way:



Ask a pupil to point to one subset and tell what fraction describes the part the subset is of the whole set. (We made two subsets with the same number of members. Each is one-half of the set.)

Select a pupil to separate a set of four things into four subsets, each with the same number of members. Ask, "What number describes one subset as part of the whole set?" (One-fourth) Let a pupil write the numeral ($\frac{1}{4}$) on the board.

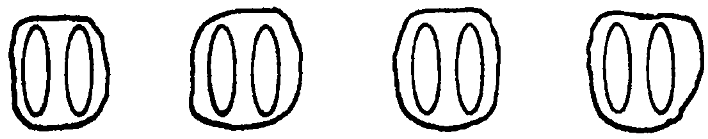


Ask the pupils to put a set of eight counters on their desks. The following directions may be used to guide the pupils' activity and discussion:

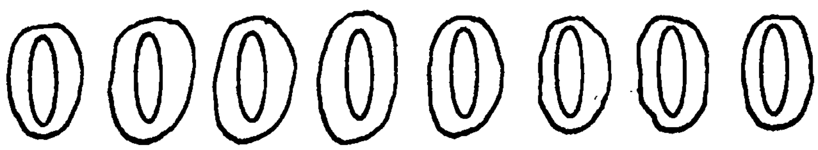
1. Separate your set of counters into two subsets so that each subset has the same number of members. Point to one of the subsets. Tell what fraction describes this part of the whole set. (One-half) Tell whether one-half is a number. (Yes) On a card, write the numeral for the fraction. ($\frac{1}{2}$) Place the card by a subset.

2. Separate your set of eight counters into four subsets so that each subset has the same number of members. Point to one subset. Tell what number describes this part of the whole set. (One-fourth)

Is the fraction a number? (Yes) On a card, write the numeral for the number. ($\frac{1}{4}$) Place the card by a subset.



3. Separate your set of eight counters into eight subsets so that each subset has the same number of members. Tell how many members are in each subset. Tell what number describes each subset. (One-eighth) On a card, write the number. ($\frac{1}{8}$) Place the card by a subset.



Ask the pupils to remove two counters from their desks so that their set of counters now has six members. Repeat activities similar to those above with the sets of six counters. Ask the pupils to separate their sets into two, three, and six subsets so that each subset has the same number of members. Each time, let the pupils tell what fraction describes one subset and write the numeral for the fraction.

Activity 4: Fractions for Subsets (continued)

Pupil page 40

Objective:

Given a set, children can name the fraction associated with the shaded subset and can write the numeral for the fraction.

Teaching Procedure:

Ask the pupils to open their books to page 40. Let a pupil read the question at the top of the page. Say, "Each of the sets of beads in the picture has a subset of black beads. There is a fraction that tells us what part the subset is of the whole set." (Point to picture A.) "A subset of set A has black beads. What part of the whole set is the subset of black beads?"

Guide the pupils to say that if they think of the black beads as one subset, then that subset is one of two subsets with the same number of members. One subset is one-half of set A.

The following questions may be used to guide the discussion:

How many members has the subset of black beads? (Two)

Is there a subset of white beads in picture A that matches exactly the subset of black beads? (Yes)

Into how many subsets of the same number has the set of beads been separated? (Two)

What part is the subset of black beads of the whole set? What fraction describes this? ($\frac{1}{2}$)

Ask the pupils to write beside each letter the numeral for the number that describes the subset of black beads in that picture.

A $\frac{1}{2}$ (or $\frac{2}{4}$)

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{6}$

E $\frac{1}{2}$ (or $\frac{3}{6}$)

F $\frac{1}{4}$ (or $\frac{2}{8}$)

G $\frac{1}{2}$ (or $\frac{4}{8}$)

H $\frac{1}{3}$ (or $\frac{2}{6}$)

I $\frac{1}{8}$

J $\frac{1}{2}$ (or $\frac{6}{12}$)

K $\frac{1}{2}$ (or $\frac{5}{10}$)

L $\frac{1}{4}$ (or $\frac{3}{12}$)

M $\frac{1}{3}$ (or $\frac{4}{12}$)

Activity 5: Fractions for regions (oral)

Pupil pages 41 - 43

Objective:

Given different shaped regions separated into fractional parts, children can shade one fractional part, can name the fraction associated with the shaded part, and can write the numeral for the fraction.

Materials: cutouts of circles, triangles, squares, rectangles and hexagons sufficient to give three shapes to each child.

Teaching Procedure:

Let the pupils select from the pupil pages 41 - 43 three shapes, each showing a different fractional division. Ask them to color one part of each shape. Let them tell about the shaded parts, tell what fraction describes the shaded part, and write the numeral for the fraction on the chalkboard. If the teacher wishes, she may prepare many cutouts of heavy paper in the shape of circles, triangles, squares, rectangles, and hexagons, drawing lines to separate the region of each cutout into parts of the same size representing halves, fourths, sixths, thirds, and eighths, and giving each pupil at least three shapes showing different fractions. Save the cutouts for use with Activity 6.

For the remaining shapes on the pupil pages, discuss each shape in the following manner:

Into how many parts is this region divided?

What is the name we give to one part of this region?

Color one part of this region.

Write the fraction that describes the part you have colored.

At a later time, if pupils have difficulty understanding that $\frac{2}{4}$ and $\frac{3}{6}$ are other names for $\frac{1}{2}$, ask them to return to these pages, coloring $\frac{2}{4}$ of of a region or $\frac{3}{6}$ of a region. They will see that they have colored $\frac{1}{2}$ of the region. This activity may be extended to cover $\frac{2}{6}$, $\frac{4}{6}$, $\frac{2}{8}$, $\frac{6}{8}$, and so on. The purpose of these activities is to help the child learn to color a part of a region, given a numeral, or to write a numeral for a shaded part of the region.

<u>Activity 6</u>: Matching fractions to parts of regions (oral)

<u>Objective</u>:

Given a numeral for a fraction and different shaped regions, children can identify the regions whose shaded parts are described by the fraction.

<u>Materials</u>: 5 boxes (or baskets); five 5" by 7" numeral cards ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$)

<u>Teaching Procedure</u>:

Place the 5" by 7" cards with the numerals $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$ written on them, each against an empty box. (Shoe boxes or baskets may be used.) Place the cards so that each pupil in the class can see the numerals.

Give out the cutouts that the pupils shaded in Activity 5. Make sure each pupil has three cutouts with shaded parts that are described by different fractions.

Point to the numeral card for $\frac{1}{2}$ and ask a pupil what number it names. Let the pupils look at their cutouts and decide which cutouts have one-half of their regions shaded. Ask all the pupils who have a cutout with one-half the region shaded to place the cutout in the box marked "$\frac{1}{2}$"

Display the cutouts from the "$\frac{1}{2}$" box for the pupils to see. Ask whether all the half-regions are the same size and the same shape. (No) Emphasize the idea that not all halves of regions are the same size and shape but that each part is described as one-half of its own region. The pupils should notice that if two regions are not the same size and shape, then the halves of those regions will not be the same size and shape. Lead the pupils to understand that two halves of the same region make one whole.

Point to the numeral card for $\frac{1}{3}$. Let a pupil tell the name of the fraction. Ask all who have cutouts with one-third of the region shaded to place them in the box marked "$\frac{1}{3}$". Display all these cutouts and let the pupils talk about the set of all the regions of cutouts that we describe as one-third. Emphasize the fact that the shaded thirds of different regions may be of different sizes and shapes but that each is one-third of its whole region. Guide the pupils to say, "Three thirds of the same region make a whole."

Continue in a similar way with the other boxes and cutouts showing fourths, sixths, and eighths.

TOPIC II: ORDERING FRACTIONS

OBJECTIVES:

1. To introduce order of fractions and to compare pairs of fractions that are not equal.
2. To extend the idea of ordering fractions on a number line and to provide experiences in naming points on a number line.
3. To extend the concept of fractions to include one-fifth, one-tenth, and one-twelfth.

VOCABULARY: one-fifth, one-tenth, one-twelfth

MATERIALS: eight 3" by 30" strips of paper, a number line, pupil pages 44 - 45

Activity I: Naming points on the number line (oral)

Objective:

Children can fold paper strips to find and name points for the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$ on a number line.

Materials: two 3" by 30" strips of paper

Teaching Procedure:

Place two of the 3" by 30" strips of paper on your desk. Hold them up for the class to see. Let the pupils show that the strips are of the same size.

Draw a line segment on the board, or put it on paper taped to the board. It will be used in the next activity. It should be more than 60 inches long. Mark a point near, but not on the left end of the line segment and write a numeral 0 at the point.

Say, "This is the number line. We can mark and name points on the line with numbers. Here is one point. Let us name it 0. Let us mark another point."

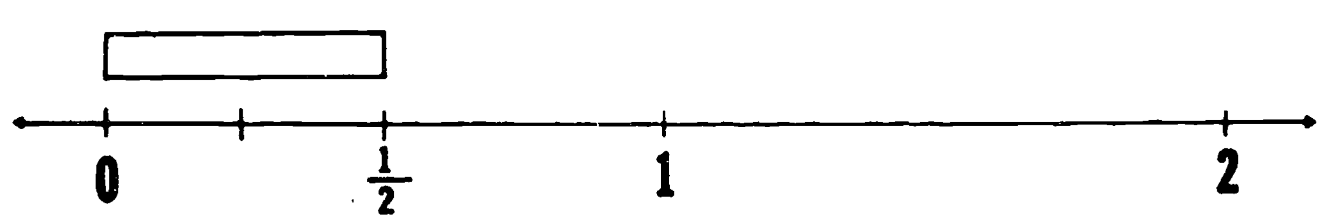
Place one of the strips of paper along the number line, with the left end at the point named 0. Mark a point at the right end of the strip and name it 1. Ask the pupils to tell what this point is named. (1)

Ask a pupil to put his finger where he thinks the point for 2 will be. After he guesses, help him to check his answer. Let him put the strip of paper along the line segment, with the left end of the strip at the point marked 1. Let him mark a point at the right end of the strip and name it 2. Let the class say whether the pupil's guess about the point for 2 was a good guess. Stress the idea that the line segment from 0 to 1 and the line segment from 1 to 2 are congruent. The edge of the paper strip fits both exactly.

Ask a pupil to fold one paper strip into halves. Ask him to show the class how he folded the strip, to unfold it and to tell into how many parts the strip is separated. (2) Ask, "Are the two parts the same size?" Let the pupils tell the name of the fraction which describes one part. ($\frac{1}{2}$)

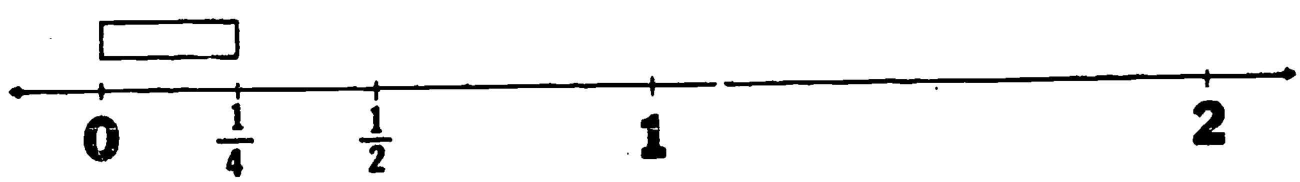
Hold the strip up and say, "We know that the edge of the paper strip fits the line segment from the point for 0 to the point for 1. If we fold the strip, we know that one of the two parts we formed is one-half of the strip." Ask a pupil to use the folded strip to find the point on the number line that we can name $\frac{1}{2}$.

Help a pupil to place the left end of the folded strip at 0 on the number line and mark a point at the right end. Say, "We can name this point $\frac{1}{2}$." Write the numeral for $\frac{1}{2}$ below the marked point. Guide the pupil to make statements such as the following: The $\frac{1}{2}$ point is between the point named 0 and the point named 1; the line segment from 0 to $\frac{1}{2}$ is one of two parts of the same size; the two parts make the line segment from 0 to 1.



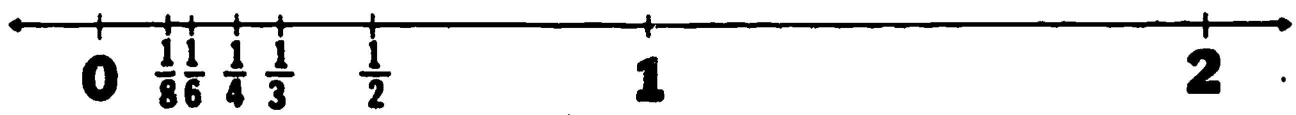
Point to each of the two parts of the strip which $\frac{1}{2}$ describes. Say as you fold each part, "I am separating each of these two parts into two parts of the same size." Unfold the paper and show it to the pupils. Ask, "Into how many parts is the paper separated by the folds?" (4) "Are the four parts all the same size?" (Yes) Let a pupil show one-fourth of the strip. Guide the pupils to say that this part is one of four parts of the same size.

Let another pupil fold the paper so that only one-fourth of the folded paper shows. Let still another pupil use the folded paper to mark the point for $\frac{1}{4}$ on the number line. Let another pupil write the numeral for $\frac{1}{4}$ below the marked point. (As shown in diagram below.)



Ask, "How do we find the point for $\frac{1}{8}$?" If a pupil suggests that we fold each part of the strip again so that there are eight parts of the same size, let him do this. Let another pupil use the folded paper to mark the $\frac{1}{8}$ point on the line. Ask another pupil to write the numeral for $\frac{1}{8}$ below the mark.

Ask, "Where do you think the point for $\frac{1}{3}$ should be?" Get the pupils to talk about this. Let several pupils put their fingers on the places where they think the point should be. Ask the class how they can decide where the $\frac{1}{3}$ point is. Guide the pupils to say that they can use a paper strip folded into three parts of the same size. Let a pupil fold a strip and use it to mark a point for $\frac{1}{3}$. Write the numeral for the fraction below the mark. Let the pupils tell who made the best guess. Using the same paper strip, follow the plan above to mark and name the point for $\frac{1}{6}$. (Note: Save the number line, as it is marked, for use in the next activity.)



Note: If pupils have trouble relating fractions to the number line, let them make their own number lines on strips of paper, using a piece of string as their unit. String is easily folded into halves, fourths, eighths, thirds, sixths, and so on. It can be marked with crayon or ink at the fold before marking the paper number line.

Activity 2: Reviewing the order of fractions

Pupil page 44

Objectives:

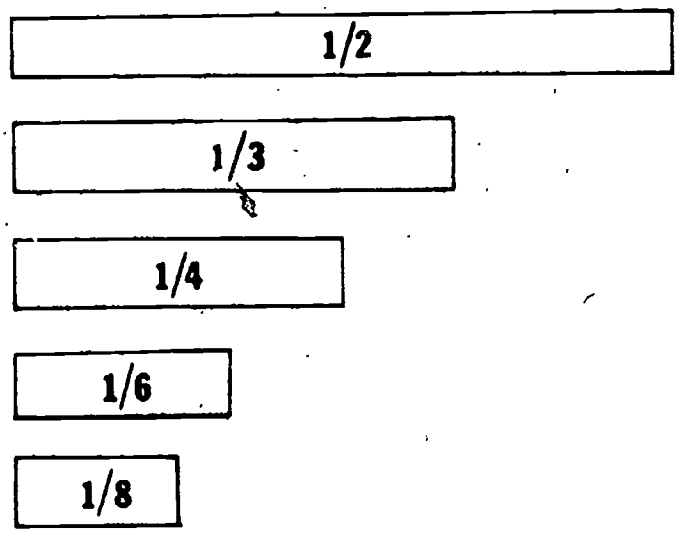
1. Given strips of paper to be folded into parts of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, children can find and mark the corresponding points on the number line.
2. Children can compare fractions by locating the points named by the fractions on the number line, by noticing which point is to the right of the other (to the left of the other), and therefore conclude which fraction is greater than (less than) the other.

Materials: five 3" by 30" strips of paper

Teaching Procedure:

For this activity, use the number line drawn and marked on the board for Activity 1. Put out five of the 3" by 30" strips of paper.

Let five pupils fold the strips so that one-half of one strip shows, one-fourth of another strip shows and one-eighth of another strip shows, one-third of another strip shows and one-sixth of another strip shows. Ask each pupil to write the numeral for the fraction that describes that part on one part of his strip. Then let the pupils compare the parts they show and decide which part is longest, which is shortest and so on. Ask them to place all the parts of the folded strips on a table in order, from the longest to the shortest.



Ask, "Which part of the strip is longer, the part that shows one-half or the part that shows one-sixth?" (One-half) "We say that the number $\frac{1}{2}$ is greater than the number $\frac{1}{6}$."

Place the part of a strip that shows one-half along the number line to remind the pupils that the edge is congruent to the line segment from the point for 0 to the point for $\frac{1}{2}$. Show in the same way that the one-sixth part of a strip has an edge that is congruent to the line segment from the point for 0 to the point for $\frac{1}{6}$. Ask, "Which line segment is longer, the line segment that is one-half the whole line segment from 0 to 1 or the segment that is one-sixth the line segment from 0 to 1?" (The line segment that is one-half)

Say, "The point for $\frac{1}{2}$ is to the right of the point for $\frac{1}{6}$. One-half is greater than one-sixth." Write the following sentence on the board and ask the children to read it:

$$\frac{1}{2} > \frac{1}{6}$$

Get the pupils to compare the one-third part of a strip and the one-fourth part of a strip. (It should be emphasized that the whole strips are all of the same size.) Ask which of the two parts is greater. (One-third) Say, "One-third is greater than one-fourth." Let two pupils find the points for $\frac{1}{3}$ and $\frac{1}{4}$ on the number line. Ask, "Which is at the right?" ($\frac{1}{3}$) Let a pupil write this sentence:

$$\frac{1}{3} > \frac{1}{4}$$

Say, "We can also ask which number is less than another. If $\frac{1}{3}$ is greater than $\frac{1}{4}$, we can say that $\frac{1}{4}$ is less than $\frac{1}{3}$. One-fourth is to the left of $\frac{1}{3}$ on the number line." Let a pupil write the sentence on the board to show which of the numbers is less:

$$\frac{1}{4} < \frac{1}{3}$$

Guide the pupils to use their folded strips and the number line to make statements such as the following: "If $\frac{1}{3}$ is less than $\frac{1}{2}$, then the point for $\frac{1}{3}$ is to the left of the point for $\frac{1}{2}$ on the number line; and if $\frac{1}{2}$ is greater than $\frac{1}{3}$, then the point for $\frac{1}{2}$ is to the right of the point for $\frac{1}{3}$."

Compare several pairs of numbers on the number line. Get the pupils to decide which number of the pair is less and which is greater by noting which point is to the left and which point is to the right. Let them write sentences like these on the board:

$\frac{1}{8} < \frac{1}{4}$ $\frac{1}{4} < \frac{1}{2}$ $\frac{1}{3} > \frac{1}{8}$ $\frac{1}{2} > \frac{1}{3}$ $\frac{1}{6} < \frac{1}{4}$

Ask the pupils to turn to page 44 in their books. Let them write in the correct symbol, > or < or = , to make each sentence true. Say, "In each sentence, one number is either greater than, less than, or equal to the other. Make true sentences by writing the proper symbol." Complete one sentence on the board as an example.

$$\frac{1}{3} > \frac{1}{6}$$

Activity 3: Naming points on the number line (oral)

Objective:

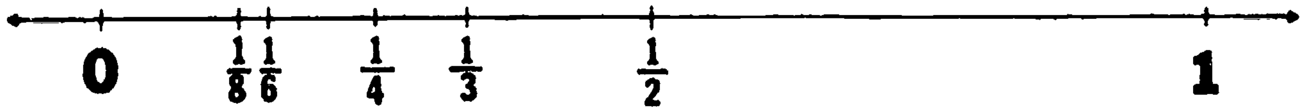
Children can order the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$ on the number line by using folded strips of paper.

Materials: Three 3" by 30" strips of paper and the strips used in Activities 1 and 2

Teaching Procedure:

Get out three unused 3" by 30" strips of paper and the strips used in Activities 1 and 2.

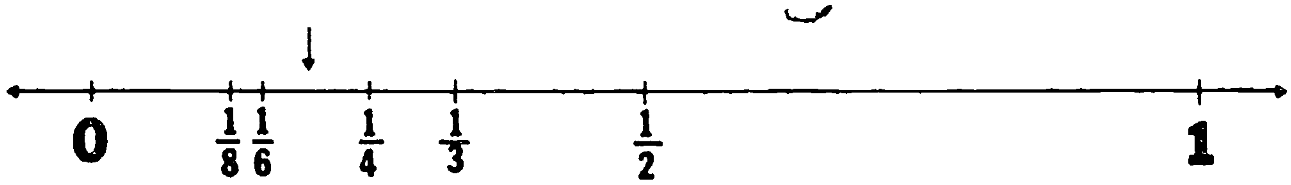
Draw the number line on the board, using a strip of paper to mark the segment from 0 to 1. Let the pupils use the folded strips from previous activities to mark and name the $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{8}$ points.



Let the pupils discuss the order of the fractions. Help them discover patterns in the order by noticing that $\frac{1}{8}$ is less than $\frac{1}{6}$, that $\frac{1}{6}$ is less than $\frac{1}{4}$, that $\frac{1}{4}$ is less than $\frac{1}{3}$ and so on. They should notice that the point for $\frac{1}{3}$ is between the points for $\frac{1}{4}$ and $\frac{1}{2}$, that the point for $\frac{1}{6}$ is between the points for $\frac{1}{8}$ and $\frac{1}{4}$ and so on.

Write the numeral $\frac{1}{5}$ on the board in a place apart from the number line. Say, "This is another fraction. Name the fraction." (One-fifth) "Look at the number line. Can someone guess where the one-fifth point is?"

Let a pupil put his finger on the place on the number line where he guesses the point is. Mark it with a small arrow above the line to show that it is a guess.



Ask whether any pupils agree. If some disagree, let them indicate their guesses by marking other arrows. Let them discuss why they think a point for $\frac{1}{5}$ should or should not be where suggested. Help them to discover that there are several ways to think about where the $\frac{1}{5}$ point is. They may see a pattern. For example, a pupil may say, "The denominators go 8, 6, 4, 3, 2. I think the denominator 5 goes between 6 and 4. I think the point for $\frac{1}{5}$ goes between the point for $\frac{1}{6}$ and the point for $\frac{1}{4}$."

They may think of the line segment from 0 to 1 as separated into five congruent line segments. They may think of the paper strip folded into five parts of the same size and try to imagine the size of one of them. They know that if the line segment from 0 to 1 is separated into five congruent line segments, then the endpoint of the line segment beginning at point 0 will be named $\frac{1}{5}$. Say, "These are ways that help us guess where the point is. How can we find and mark the $\frac{1}{5}$ point?"

Use another strip of paper folded into five parts of the same size to locate the one-fifth point and decide whether the pupils' guesses were good guesses. (Prepare the strip before class. Mark a place on the strip six inches from one end. Since six inches is one-fifth of thirty inches, this will help you to fold the strip quickly into five parts of the same size.)

Let a pupil unfold the strip and count the number of parts of the same size. Get another pupil to use the folded strip to mark the point for $\frac{1}{5}$. Let him write the numeral $\frac{1}{5}$ below the point marked.

Get the pupils to notice that $\frac{1}{5}$ follows the pattern of the numbers on the number line. There is an order in the denominators. The point for $\frac{1}{5}$ is between the points for $\frac{1}{6}$ and $\frac{1}{4}$. Help the pupils to notice also that to locate $\frac{1}{5}$ we separate the segment from 0 to 1 into more parts of the same size than we did to locate $\frac{1}{4}$ and that each part is shorter.

Ask, "Where do you think we should put the point for one-tenth? Is $\frac{1}{10}$ greater than, or less than $\frac{1}{8}$?" (Less) "Will the point be to the right or to the left, of the point for $\frac{1}{8}$?" Let the pupils guess the position of the point and mark their guesses with small arrows. Then ask, "How can you find the position of the point for $\frac{1}{10}$?" Get the pupils to suggest that the strip folded into five parts of the same size be folded again to show ten parts of the same size. One of the ten parts of the same size will represent one-tenth of the paper. Let a pupil fold the strip, mark the point on the number line and name the point $\frac{1}{10}$.

Use the same plan to locate the point for one-twelfth. The strip folded and used to find the point for $\frac{1}{6}$ can be folded again into twelfths.

Activity 4: Naming points and comparing fractions   Pupil page 45

Objective:

Children can order the fractions $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{10}, \frac{1}{12}$ on the number line by comparing denominators.

Teaching Procedure:

The purpose of this activity is to guide the pupils toward realizing that as long as the numerator remains the same, the larger the denominator, the smaller the fractional part.

Write the following numerals on the board:

$$\frac{1}{4} \quad \frac{1}{10} \quad \frac{1}{8} \quad \frac{1}{3} \quad \frac{1}{6} \quad \frac{1}{2} \quad \frac{1}{5} \quad \frac{1}{12}$$

Let the pupils talk about what the numerals mean and whether they are in order. Draw a vertical number line on the chalkboard. Using strips of paper or string, ask the pupils to help you mark and label the positions of the numerals named above. This exercise prepares them for using the vertical number line on pupil page 45. (A horizontal number line in the text would show units too small to mark with ease.)

Ask the pupils to open their books to page 45 and read the sentence at the top of the page. Ask them to find the point for 0 on the number line. Then ask them to find the point for 1. Say, "There are other points marked on the number line, but the numeral for the number is not always written. Can you find a numeral for point H?" ($\frac{1}{12}$) Write this sentence on the board:

Point H is named $\frac{1}{12}$.

Say, "You complete each of the other sentences by choosing one numeral from the list on the board. Choose the numeral that can be written in the box to make a true sentence. For example, point A has a number name. It is the name of a fraction. Write its numeral in the box. Fill in the other boxes with names of fractions to make true sentences." (Some children may need to fold string or strips of paper to decide which fraction to use.)

Ask the pupils to look at the incomplete sentences on the right side of the page. Go over one sentence with the class. Let a pupil read this sentence:

$$\frac{1}{3} < \square$$

Say, "One-third is less than a number. Can you find its name on the list?" Get the pupils to notice that there is more than one number that will make the sentence true. They can choose any number that will make the sentence true. One pupil might write "$\frac{1}{2}$", another might write "1". Say, "Make the sentence true by writing a numeral in each box." Give help as it is needed.

TOPIC III: COUNTING FRACTIONS WITH NUMERATOR 1

OBJECTIVES:

1. To introduce multiples of $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{5}, \frac{1}{10}, \frac{1}{12}$.
2. To give practice in finding and naming fractions with numerator 1 on the number line.

VOCABULARY: no new vocabulary

MATERIALS: a rectangular piece of paper about 5" by 7" for each child; one square piece of paper about 6" by 6"; three strips of paper about 2" by 18"; three strips of paper about 3" by 30"; blank numeral cards; string; pupil pages 46 - 48

Activity 1: Reviewing multiples of fractions | Pupil page 46

Objectives:

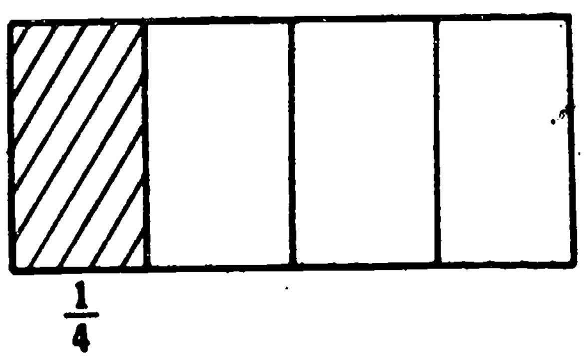
1. Given paper to fold, the children can use it to find multiples of $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{1}{6}, \frac{1}{5}, \frac{1}{10}, \frac{1}{12}$ by shading various parts of the regions and by identifying the fractions illustrated.
2. Given pictures of circles separated into fractional parts, children can write the numerals for the fractions associated with the shaded parts.

Materials: piece of paper 5" by 7" for each child; one 6" by 6" square of paper

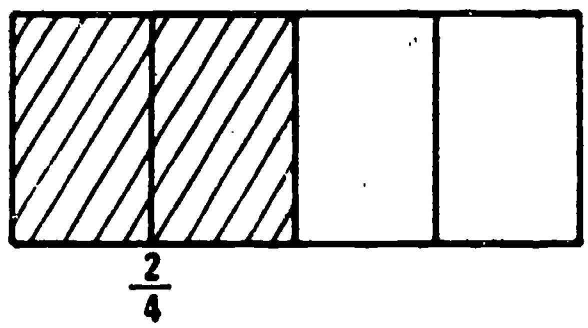
Teaching Procedure:

Give each child a piece of paper 5" by 7". Ask them to fold the papers into four parts of the same size. (Suggest they fold the papers as shown in the picture below.) Let them unfold the papers and count the fourths. Hold up one paper and point to each part in turn as you count, "One, two, three, four." Say, as you outline the whole, "This shows four fourths." Let the children talk about this.

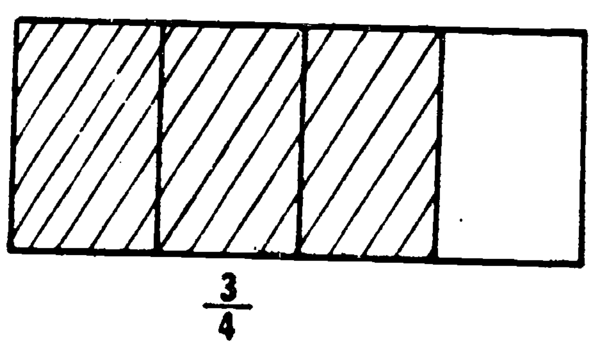
Draw around one of the pieces of paper on the board and draw lines to separate the region shown into four regions of the same size. Shade one of the four regions. Ask, "What part of the whole region is shaded?" (One-fourth) Let the children move their fingers about a region of one-fourth of their papers. Then let a child write the numeral "$\frac{1}{4}$" below the shaded part of the region on the board.



Draw on the board another rectangle exactly like the first. Shade two of the four parts one at a time. Say, "I shaded a part of the whole rectangular region. Count the fourths in the shaded part. What fraction does the shaded part show?" (Two-fourths) Let the children move their fingers about the part of their papers that shows two-fourths. Then write the numeral "$\frac{2}{4}$" just below the middle of the shaded part. Get the children to fold their papers into halves. Guide them to say, "The part we call two-fourths is the same as the part we call one-half. Two-fourths and one-half are names for the same number."

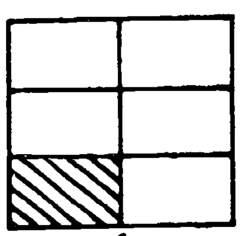


Draw another rectangle like the others on the board and shade three of the four parts. Continue as before. Ask the children to count the fourths as they are shaded. Let them find that the shaded region is three-fourths the whole rectangular region. Ask a child to write the numeral for the fraction below the shaded part.

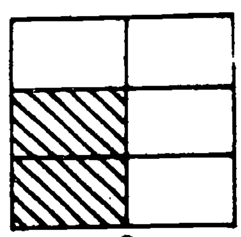


Draw another rectangle and continue in the same way as you shade all four of the parts. Ask the children the name of the fraction that the region shows. Let a child write a numeral for the fraction. Get the children to say, "All of the region is shaded. We counted four fourths. Four fourths is the same as one whole." Write the sentence $\frac{4}{4} = 1$ on the board to express this idea.

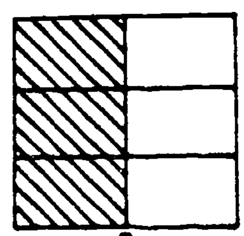
Guide a child to fold a 6" by 6" square of paper to show sixths. Help him first fold the paper into three parts of the same size and then to fold the paper again to make six parts of the same size. See the pictures below. Draw around the paper on the board and then draw lines to separate the rectangular region into six parts of the same size. Use the same plan to talk with the children about sixths that you used to talk about fourths. Shade one of the regions at a time and ask, "What part of the region is shaded?" Get the children to tell the name of the fraction that describes the shaded part and write the numeral for the fraction.



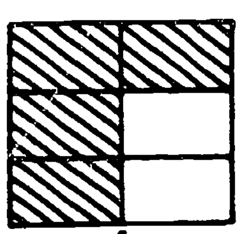
$\frac{1}{6}$



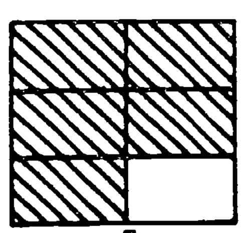
$\frac{2}{6}$



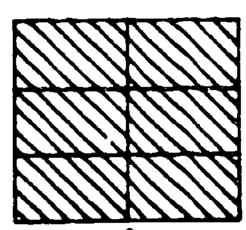
$\frac{3}{6}$



$\frac{4}{6}$



$\frac{5}{6}$



$\frac{6}{6}$

As you discuss sixths with the children, emphasize the different names for the same fraction. For example, the children can look at the region shaded to represent three-sixths and see that one-half the rectangular region is shaded. This way they discover that $\frac{3}{6}$ and $\frac{1}{2}$ are names for the same number. Guide the children to write the sentence that expresses this:

$$\frac{3}{6} = \frac{1}{2}$$

Let the children say that in the picture for six-sixths, the whole region is shaded. Thus they see that $\frac{6}{6}$ and 1 are different names for the same number. Let them write:

$$\frac{6}{6} = 1$$

Use the same plan to develop the idea of fractions with other denominators, such as 5, 3, 8, and 10.

Ask the children to point to the Circle A on page 46 of their books. Say, "The region of the circle is separated into parts. Are these parts the same size? Into how many parts has the circular region been separated?" (8) "Some of the parts are shaded. What is the name of the fraction that tells how much of the whole region is shaded?" (Three-eighths) Discuss the fact that this number, three-eighths, refers to three of eight parts of the same size. Let someone write the numeral on the board next to the letter A.

Direct the children to write next to each letter the numeral for the fraction that describes the shaded region of that figure.

A. $\frac{3}{8}$ B. $\frac{4}{8}$ C. $\frac{7}{8}$ D. $\frac{5}{8}$ E. $\frac{2}{3}$ F. $\frac{3}{4}$ G. $\frac{3}{6}$ H. $\frac{2}{6}$

Activity 2: Reinforcing that fractions are numbers

Pupil page 47

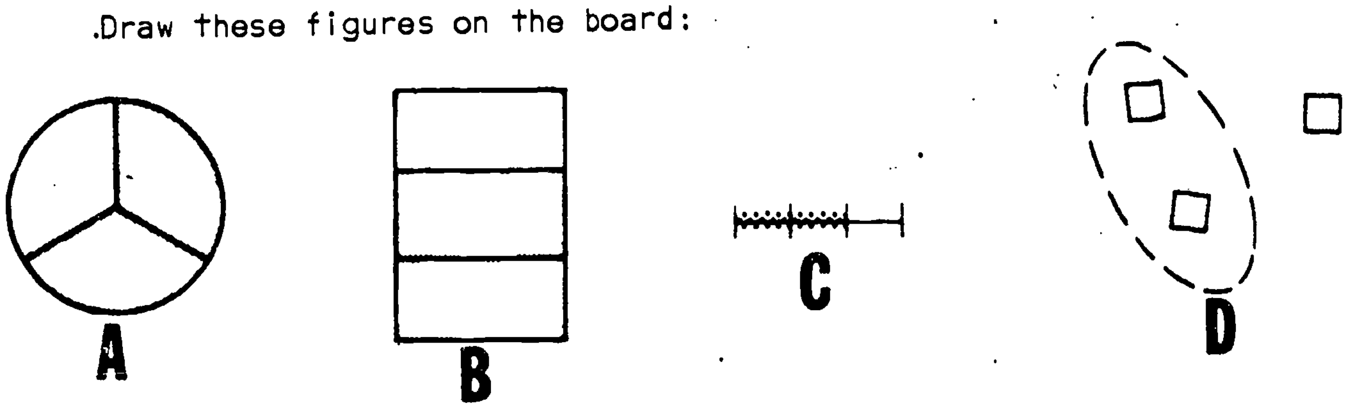
Objective:

Children can name the fractions associated with shaded parts of regions, line segments, and sets and can write the numerals for the fractions.

Teaching Procedure:

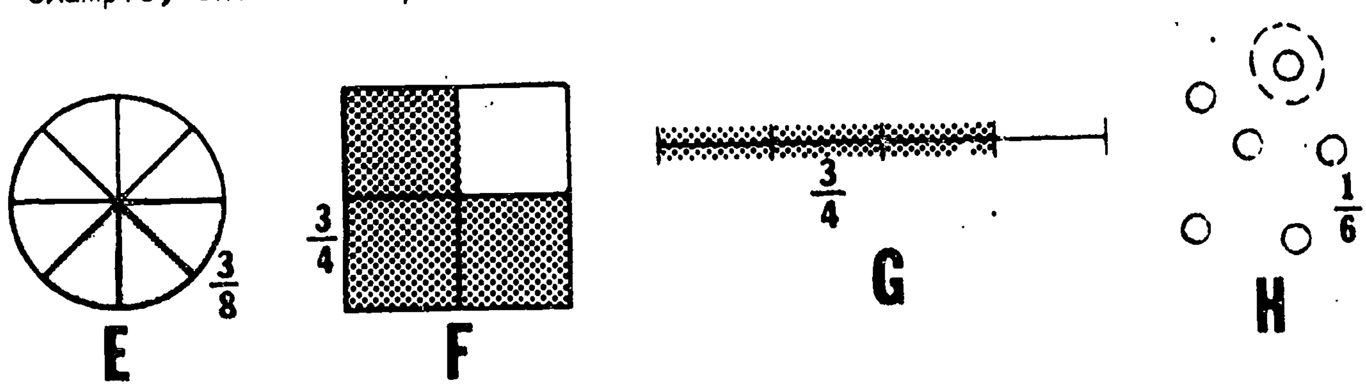
Remind the children that we use numbers called fractions when we talk about parts of whole objects or parts of sets.

Draw these figures on the board:



Say, "Use a fraction to describe the part of A, of B, and of C that is shaded. ($\frac{2}{3}$ of the region A is shaded. $\frac{2}{3}$ of region B is shaded. $\frac{2}{3}$ of line segment C is shaded.) What fraction describes the part of set D that has a ring around it?" ($\frac{2}{3}$)

Let the children draw regions, line segments and sets on the board, show parts of each thing drawn and assign a fraction to each part. For example, children may draw these:



Review some (or all) of the numbers that the children have used to describe parts of things. Ask, "What number did you use to describe the shaded part of circle A? Part of region B?" and so on.

$\frac{2}{3}$ of region A was shaded;

$\frac{2}{3}$ of line segment C was darkened;

We darkened $\frac{3}{4}$ of G:

We draw a ring around $\frac{1}{6}$ of set H; and so on.

Ask, "What numbers did you use? ($\frac{2}{3}, \frac{3}{8}, \ldots, \frac{3}{4}, \frac{1}{6}$) What do we call numbers with these names? (Fractions) Fractions are numbers that we can use to describe a part of a whole thing or part of a set. What does the 3 mean in the statement, $\frac{2}{3}$ of region A is shaded? (It is the number of congruent parts into which we separated region A.) What does the 2 mean in the statement '$\frac{2}{3}$ of region A is shaded'?" (It is the number of congruent parts of region A that are shaded.)

Continue to have the children draw regions, line segments, and sets to show fractions such as $\frac{1}{5}$, $\frac{[illegible]}{8}$, and $\frac{2}{6}$.

If more practice is needed, let the children tell what parts of their figures are unshaded. Emphasize that they are using numbers called fractions.

Ask the pupils to do the exercises on pupil page 47. While the pupils work or after they finish, talk with them about the exercises. Exercises 10 and 11 help the children summarize the lesson.

Activity 3: Showing multiples of fractions with numerator 1 on the number line

Pupil page 48

Objectives:

1. Children can find and name multiples of a given fraction on a number line by folding paper strips into congruent parts.

2. Given pictures of number lines marked to show different fractions, children can write the numerals for the fractions which name the indicated points.

Materials: 2" by 18" strips

Teaching Procedure:

Draw a line segment on the board. Give one child a 2" by 18" strip of paper and help him to mark points for 0, 1, 2, and 3 to make a number line. Let him begin by marking a point for 0 near the left end of the line segment. Then use the strip to mark the points for 1, 2, and 3 by placing the left end of the strip on each point and marking the next point to the right end as shown on the following page.

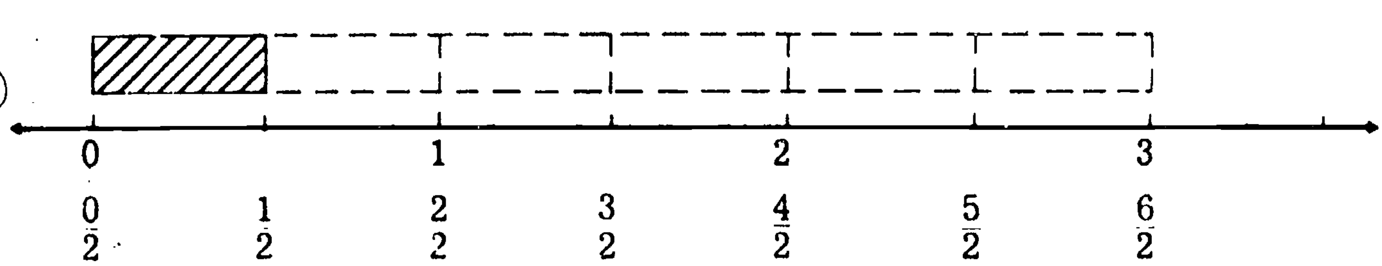
Ask another child to fold the strip in half. Let him use the folded strip, holding one end at 0 and marking a point at the other end to show one-half. (See the following figure.) Let him write the numeral "$\frac{1}{2}$" just below the mark. Ask another child to use the same folded strip to count and mark the point for two-halves. They will discover that this is a point that is marked with the label "1". Let the children talk about this and help them to say that they have found another name for the point. It is "two-halves". Ask, "Can you write a numeral for this fraction?" Let a child write "$\frac{2}{2}$" below the point and below the numeral for one.

Write the following sentence on the board and say, "This sentence tells us that 'two-halves' and 'one' are names for the same number."

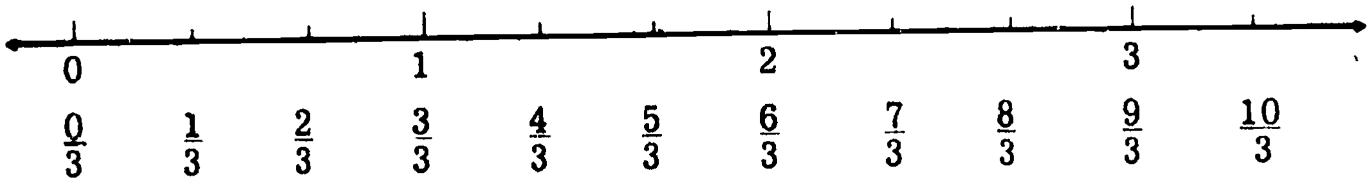
$$\frac{2}{2} = 1$$

Let a child continue marking points, using the strip folded in half, as far as the line is drawn. Let the children count the halves and mark the points: $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$ and so on. Explain that some fractions name the same points that the whole numbers on the line name, by writing sentences such as $\frac{4}{2} = 2$ and $\frac{6}{2} = 3$.

Point to the mark labeled "0" and say, "Can you think of another name for this number? How many halves are described by the number 0?" Help the children to conclude that the point is for zero-halves. Let a child write the numeral "$\frac{0}{2}$" below the numeral "0".



Draw another number line on the board and use another 2" by 18" strip folded into three parts of the same size to mark thirds. Let the children mark the points and count the thirds. Follow the steps used with the number line for halves.



Construct a number line for fourths, for fifths, for sixths, and for other fractions as needed. Continue to label all points, including the point for zero, with names for the fractions.

Ask the children to open their books to page 48. Get the children to notice that each picture of the number line on the page is marked to show different fractions. Discuss the first picture. Ask, "Into how many segments is the segment from 0 to 1 separated?" (2) Let the children find a point that has a numeral box beneath it. Say, "What will you label this point?" ($\frac{1}{2}$) "Write a numeral in the box." Check to see that the children have written "$\frac{1}{2}$" in the proper box. Let them continue to fill all the boxes with numerals.

If needed, the children can use paper strips to help in finding the fractions for the points.

TOPIC IV: DIFFERENT NAMES FOR A FRACTION

OBJECTIVE:

To help students discover that if both the numerator and denominator of a fraction are multiplied by the same number, we get another name for the same number.

VOCABULARY: no new vocabulary

MATERIALS: three 2" by 30" strips of paper; a string (1 yard); pupil pages 49 - 54

Activity 1: Different names for a fraction

Pupil pages 49, 50

Objectives:

1. Given number lines marked in the same scale (halves, thirds, fourths, fifths, sixths, eighths), children can use string to find different names for fractions.

2. Given number lines marked in the same scale (halves, thirds, fourths, fifths, sixths, eighths), children can use string to compare fractions.

Materials: three 2" by 30" strips of paper; a string (1 yard)

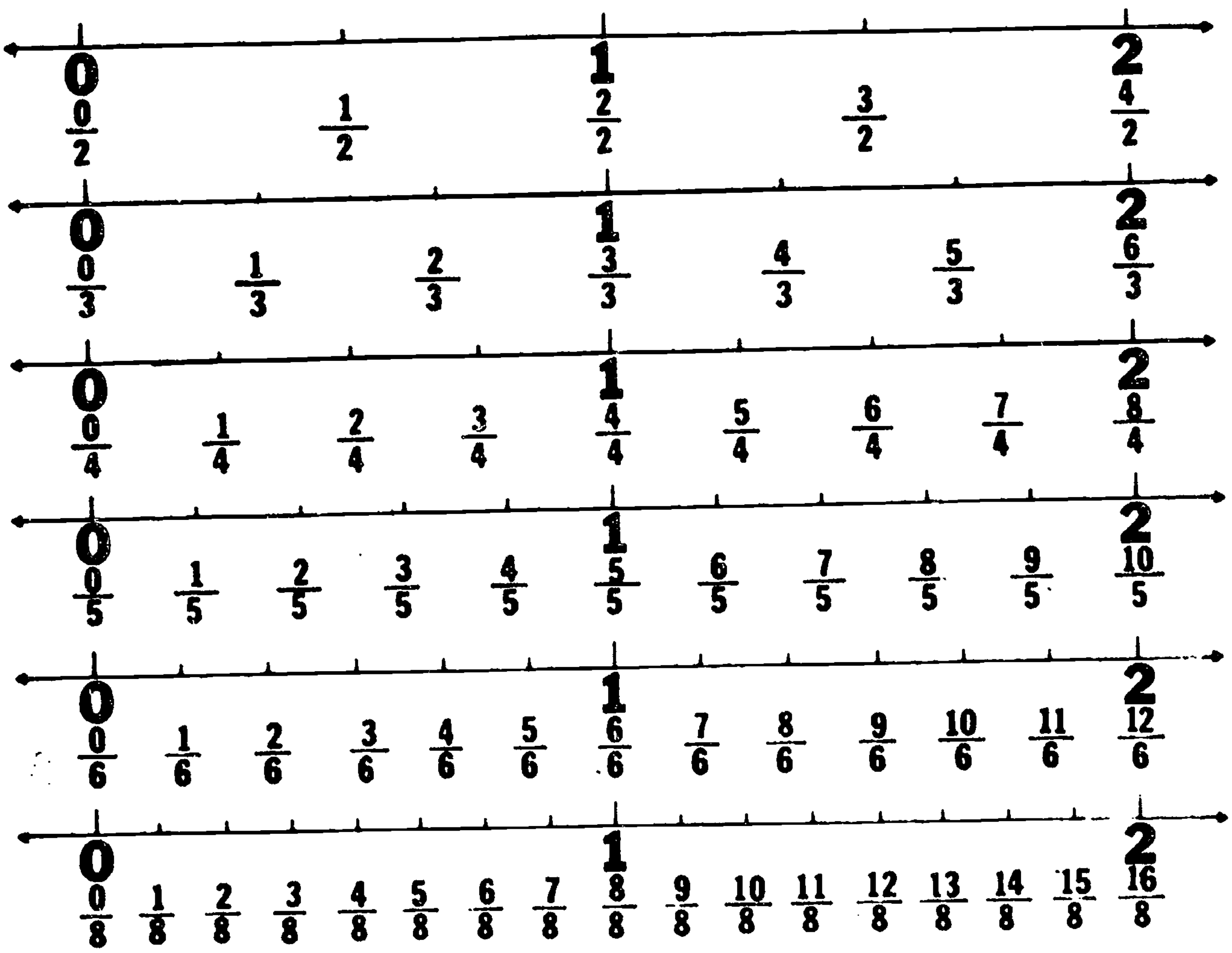
Teaching Procedure:

Draw six line segments on the board one below the other. Leave enough space between the lines so that numerals for fractions can be written below each line. Use 2" by 30" strips to mark the 0, 1, and 2 points. The points for 0, 1, and 2 on each line should be just below those points on the line above.

Let the children fold the paper strips and mark points for halves, thirds, fourths, fifths, sixths and eighths as shown on the next page. After all the numerals have been written, let the class check whether the segments that are halves are the same size, whether the segments that are thirds are the same size and so on. Let them also check to see that the numerals are in sequence.

Point to the figure and say, "These are all pictures of the number line, the points for halves are marked on one picture, the points for thirds are marked on another picture and so on." Let the children tell about the idea that 0 in each of the pictures labels the same point on the number line, that 1 in each of the pictures labels the same point on the number line, and 2 in each of the pictures labels the same point on the number line. You can make this clear by stretching a string vertically from the 0 mark of the top picture to the 0 mark of the bottom picture so that the string touches the 0 mark of each picture. Do the same for the point labeled 1. Let them say that this point has many different names. Ask the children to tell some of the names. Write the numerals on the board as they name the fractions. Write the symbol = between each pair to show that the fractions are equal.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{8}{8}$$



Follow the same plan to show different names for the number 2.

$$2 = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = \frac{10}{5} = \frac{12}{6} = \frac{16}{8}$$

Ask a child to choose another fraction and find several different names for the fraction. If he chooses $\frac{1}{2}$, for example, help him to stretch the string tightly from the top line to the bottom line to see what points it touches. In this way he can find different names for one-half. Let another child write the numerals on the board as the names are called.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$$

Continue with this procedure, asking the children to choose different points and test with the string to find the different names for the points. This plan may be extended to tenths and twelfths by drawing two more number lines.

Let the children open their books to page 49 and read the instructions. They are asked to write numerals in the spaces to make true sentences. They are asked to make true sentences in the second part by using one of the symbols, =, > or <, in each sentence. Pupil page 50 provides a copy of the chart made by the children on the chalkboard. They may find other names for the numerals given if they lay a piece of string vertically across the chart. If they have difficulty in understanding how to use the chart, you may make a transparency of the chart, showing it on the overhead projector with pieces of string to help pupils understand.

Activity 2: Discovery of the idea $\frac{a}{b} = \frac{a \times c}{b \times c}$

Pupil pages 51, 52

Objective:

Given a fraction, children can find another name for it by multiplying both the numerator and denominator by the same number.

Teaching Procedure:

Write the following sentences on the board and say, "Here are some true sentences you made." Review briefly the idea that the sentences show different names for the same number.

$\frac{1}{4} = \frac{2}{8}$	$\frac{1}{4} = \frac{3}{12}$	$\frac{1}{5} = \frac{2}{10}$
$\frac{1}{2} = \frac{2}{4}$	$\frac{2}{4} = \frac{6}{12}$	$\frac{1}{3} = \frac{2}{6}$
$\frac{3}{5} = \frac{6}{10}$	$\frac{2}{3} = \frac{6}{9}$	$\frac{2}{3} = \frac{4}{6}$
$\frac{2}{4} = \frac{4}{8}$	$\frac{1}{5} = \frac{3}{15}$	$\frac{3}{4} = \frac{6}{8}$

Discuss these sentences. Guide the children to look for a pattern or to look for similarities in the pairs of fractional numerals. For example, ask them to look at all the pairs in the first column. Ask, "Is there anything you notice about the pairs of numerators?" Help them to discover that in every pair the second numerator is two times the first numerator. Go on, "Is there anything you notice about the pairs of denominators? Help them to discover that in every pair the second denominator is two times the first denominator.

Ask them to look at the pairs in the second column. Again, guide them to see the relationship between the pairs of numerators and the pairs of denominators. In these examples, they find that the numerator and denominator of the second fractional numeral are three times the numerator and denominator of the first. Go on to the third column and compare the pairs in the same way.

In discussing the sentences, help the children to conclude that if both the numerator and denominator of a fractional numeral are multiplied by the same number we get another name for the same fraction.

Ask, "Do you think this will be true for any fraction we choose?" Let the children suggest other fractions and test their conclusion. In many cases they can use the number lines from Activity I to check the result.

Choose some of the sentences from the board and write sentences like these to help the pupils understand the idea.

$$\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$$

$$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Ask the children to open their books to page 51 and complete the sentences. The first sentence is completed as an example.

If pupils experience no difficulty with this page, ask them to use the same pattern to find another name for the fractions on pupil page 52. Accept any correct answers for questions 6 and 11, but give special notice to pupils who name these numbers 2 and 1.

For children having trouble with pupil page 51, provide them with additional practice work before letting them find another name without using the written pattern.

Activity 3: Fractions and the number line

Pupil pages 53, 54

Objectives:

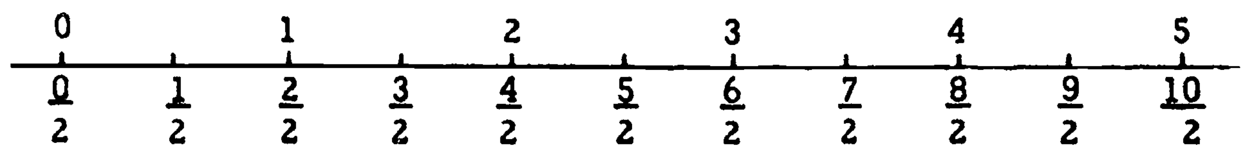
1. Children can count by halves, thirds, fourths, sixths, tenths, and twelfths from 0 to 2 using a number line.
2. Given a fraction, children can find other names for it by using a number line.

Teaching Procedure:

Review the meaning of the words denominator and numerator. Ask, "What are the denominators of $\frac{0}{2}$, $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, ...? (2) What does the 2 mean in $\frac{3}{2}$ and $\frac{9}{2}$? (It tells the number of congruent parts of the unit segment.) What are the numerators? (0, 1, 2, ...) What does the numerator 3 mean in $\frac{3}{2}$?" (It tells the number of congruent parts we counted.)

Draw on the board the number line as shown below. As you draw the line, get the children to say that each line segment between consecutive whole numbers is separated into halves:

Put a finger on the first point to the right of the 0-point that marks a line segment of unit length. Ask, "What number should we assign to this point?" ($\frac{1}{2}$) Let a child write $\frac{1}{2}$ under this point. Say as the child writes $\frac{1}{2}$, "He is assigning a number to the point. Its name is $\frac{1}{2}$." Put a finger on the next point, the 1-point. Say, "This is the 1-point. What fraction can you assign to the point?" ($\frac{2}{2}$) Continue in this way until all the marked points to the right of the 0-point have fractions assigned to them. Ask, "How many halves is 0?" (Zero halves) Write $\frac{0}{2}$ under the 0-point. The number line now looks like this:



Ask, "Can you count by $\frac{1}{2}$'s?" ($\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$,...) Let the children imagine the number line continuing to the right as they count. Vary the activity by letting the pupils skip count backwards from say $\frac{8}{2}$ to $\frac{1}{2}$, by $\frac{1}{2}$'s and then by $\frac{2}{2}$'s, etc. Sometimes let them use their eyes, and other times suggest that the children not look at the number line as they count.

Ask the children to count by thirds, fourths, sixths, tenths and twelfths.

Have one pupil labeling a number line on the board as they count.

Ask the pupils to turn to pupil page 53 and work the exercises. Ask the children to see if they can work the challenge problems on pupil page 54.

UNIT 7

Techniques of Addition and Subtraction of Whole Numbers

OBJECTIVES:

1. To review techniques of addition of whole numbers.
2. To extend techniques of addition to addition of three addends.
3. To review techniques of subtraction of whole numbers.
4. To achieve mastery of techniques of addition and subtraction of whole numbers.
5. To develop the ability to express the mathematical relationships in a word problem as a mathematical sentence; to find a number that makes the mathematical sentence true; to answer the question asked in the problem.

BACKGROUND INFORMATION FOR TEACHERS:

This unit, for your convenience, contains extra problem pages labeled "Supplementary Pages". These pages can be used for review or for more practice by those students who need more work. Furthermore, the teacher commentary for certain activities has been divided into two sections: One for those students who are having difficulty with the concept (Group I) and one for those students who are having little or no difficulty (Group II). Two complete activities (Activities 4 and 5) are suggested for Group I only.

We will discuss briefly the ideas on which the techniques of addition and subtraction are based.

Addition The expanded form of numerals together with the properties of addition are used in a short form for addition. For example:

475 400 + 70 + 5 (Expanded form)

367 300 + 60 + 7 (Expanded form)

700 + 130 + 12 (add ones; add tens; add hundreds)

700 + (100 + 30) + (10 + 2) (Rename 130 and 12)

(700 + 100) + (30 + 10) + 2 (Associative property)

800 + 40 + 2 (add hundreds, add tens)

842 (Rename)

The process used in the short form may be described as follows:

475 1. Think of 475 as 4 hundreds, 7 tens and 5. Think of 367
367 as 3 hundreds, 6 tens and 7.

2. Add the ones. 5 + 7 = 12. Rename 12 as 1 ten and 2 ones. Write 2 in the ones' place of the sum. Remember 1 ten.

3. Add the tens. (1 + 7 + 6) tens = 14 tens. Rename 14 tens as 1 hundred and 4 tens. Write 4 in the tens' place of the sum. Remember 1 hundred.

4. Add the hundreds. (1 + 4 + 3) hundreds = 8 hundreds. Write 8 in the hundreds' place of the sum.

5. The sum is 842.

Subtraction As an example, find the number that makes this sentence true:

475 - 367 = n

Using the expanded form we have

475 400 + 70 + 5

367 300 + 60 + 7

We first find the ones' digit in the numeral for n. There are not enough ones in the ones' place of 475 to subtract 7. So we rename 475 to give:

475	400 + 60 + 15
367	300 + 60 + 7
	100 + 0 + 8
	108

The process used in the short form may be described as follows:

475 1. Think of 475 as 4 hundreds, 7 tens and 5. Think of 367 as
367 3 hundreds, 6 tens and 7.
108 2. There are fewer ones in the ones' place of 475 than in the ones' place of 367. Rename 475 as 4 hundreds, 6 tens and 15 ones. Subtract the ones. 15 - 7 = 8. Write 8 in the ones' place of the missing addend.

3. Subtract the tens. (6 - 6) tens = 0 tens. Write 0 in the tens' place of the missing addend.
4. Subtract the hundreds. (4 - 3) hundreds = 1 hundred. Write 1 in the hundreds' place of the missing addend.
5. The missing addend is 108.

TOPIC I: ADDITION

OBJECTIVES:

1. To review the meaning of addition.
2. To review techniques of addition using expanded form.
3. To review renaming in addition.
4. To extend addition to three, four, and more numbers.

VOCABULARY: estimating

MATERIALS: pupil pages 55 - 77

Activity I: Review of meaning of addition

Pupil pages 55 - 57

Objective:

Children can add two numbers, each less than 100, without regrouping and without using expanded form.

Teaching Procedure:

Tell the pupils the following word problem:

Mr. Ball has 62 cows. Mr. Green has 35 cows. How many cows have they altogether?

Ask questions such as the following to help the pupils understand the problem:

What does the problem tell us? (Mr. Ball has 62 cows, and Mr. Green has 35 cows.)

What question is asked in the problem? (How many cows have they altogether?)

How can we find the number of cows they have altogether? (By adding the number of Mr. Green's cows and the number of Mr. Ball's cows.)

Guide the pupils to suggest the mathematical sentence below. Let a pupil write it on the board.

$$62 + 35 = n$$

GROUP I: Ask, "What is missing in this sentence, an addend, or the sum?" (Sum) "How can you find the sum?" (By adding 62 and 35) Say, "It will help us to add if we think of these numbers in expanded form." Ask the pupils to rename each number in expanded form. As they tell you the expanded forms, rewrite the mathematical sentence on the board as follows:

60 + 2 + 30 + 5 = n

Ask the pupils if this mathematical sentence is the same as the first sentence they wrote. (Yes) Let them tell why. (The same numbers are in both mathematical sentences. The numbers just have different names.)

Say, "Who can remember how we added two numbers in Unit 2?" Guide children to tell about the steps. (Sentences at right are the steps; they are for your use and should not be written on the board.)

[illegible]:

62 +35	60 + 2 30 + 5	(Rename each addend in expanded form.)
	90 + 7	(Add ones, then tens.)
	97	(Rename the sum in its simplest form.)

Write the vertical form for the process of addition as shown below. Say, "Let us show the addition of 62 and 35 another way."

62
+ 35

Guide the pupils to think as described to the center and right below. Write only what is shown on the left below. (Tell them to first add the number of ones and then the number of tens.)

62 + 35	2 ones + 5 ones = 7 ones. Write 7 as shown.
7 + 90	6 tens + 3 tens = 9 tens. Write 9 tens (90) as shown.
97	7 ones + 9 tens = 97. Write 97 as shown.

Say, "You found the sum, 97. Can you make your mathematical sentence true?" Guide a pupil to write the following sentence:

62 + 35 = 97

Say, "You added numbers to find out something about Mr. Ball's and Mr. Green's cows. What question did the problem ask about the cows?" (How many cows do the men have altogether?) "Can you answer the question asked by the problem?" (They have 97 cows altogether.) Write the following incomplete sentence on the board. Let a pupil complete it.

They have <u>97</u> cows altogether.

Note: If any of the pupils need more help in addition, let them show the exercise with bundles of sticks in a place value box.

Write the following sentence on the board:

46 + 12 = p

Use the vertical form for the process of addition. Let the pupils write the exercise as shown on the left below. Guide the pupils to think as described on the right.

46	6 ones + 2 ones = 8 ones. Write 8 as shown.
+<u>12</u>	4 tens + 1 ten = 5 tens. Write 5 tens (50) as shown.
8	
+<u>50</u>	
58	8 ones + 5 tens = 58. Write 58 as shown.

Now help the pupils to see that the form used above may be shortened in the following way. (Emphasize that the numbers of ones are added first, then the numbers of tens.)

46	Add the number of ones: 6 + 2 = 8. Write 8 in the
+<u>12</u>	ones' place.
58	Add the number of tens: 4 + 1 = 5. Write 5 in the
	tens' place
	46 + 12 = 58

Get the pupils to talk about the two forms they have just used to show the addition of 46 and 12. Help them to decide that the last form saves time. Tell them to use it if they understand it.

GROUP II: Ask, "What is missing in this sentence, an addend or the sum?" (sum) "How can you find the sum?" (By adding 62 and 35.)

Use the vertical form of the process of addition. Review the short technique for addition by guiding the pupils to think as described on the right. Write only what is shown on the left below. (Emphasize adding the number of ones first, then the number of tens.)

62	Add the number of ones: 2 + 5 = 7
+ 35	Write 7 in the ones' place.
97	Add the number of tens: 6 + 3 = 9
	Write the 9 in the tens' place.

GROUPS I AND II: Ask the pupils to open their books to page 55. Help them work one exercise on the board. Then let them work the rest of the exercises in their books. Give help as needed. As you help, get the pupils to tell you what they think as they add.

Pages 56 - 57 are supplementary pages. They may be used as needed by the pupils now or later. Some will not need to work any of the exercises; some will need to work part of the exercises; and some will need to work all of the exercises.

Activity 2: Addition of three numbers

Pupil pages 58 - 60

Objective:

Children can add three numbers, each less than 100, without using expanded form.

Teaching Procedure:

Tell the pupils this problem:

John gave me 23 nuts, Jose gave me 34 nuts and Mary gave me 32 nuts. How many nuts did they all give me?

Ask questions like the following to help the pupils understand the problem:

What does the problem tell us? (John gave me 23 nuts, Jose gave me 34 nuts and Mary gave me 32 nuts.) What question does the problem ask? (How many nuts did they all give me?) How do we find the number of nuts they gave me? (We add the numbers.) How do we show this in a mathematical sentence?

Guide the pupils to suggest the mathematical sentence below. Write the sentence on the board.

$$23 + 34 + 32 = c$$

Say, "This problem is about three sets of nuts. We could join these three sets in many ways and the result would always be the same. We would always have the same number of nuts. How can we add the three numbers?" Guide the pupils to suggest, "We add two numbers at a time. We can add 23 and 34 first. Then to their sum we add 32." As they say this, write $(23 + 34) + 32 = c$. Guide them to say, "We could add 34 and 32 first and then add this sum to 23." As they say this, write $23 + (34 + 32) = c$. Ask, "Are you sure that each of these ways of grouping the numbers gives the same sum?" (Yes) Say, "We can group numbers in either of these ways because of the associative property." If any of the pupils are unsure, let them try both ways. (If needed, let the pupils form sets and join them to show the problem.)

Point to this sentence on the board. (Erase the other sentence.) Say, "We wi.l use this sentence."

(23 + 34) + 32 = c

Guide the pupils to add the first two addends as shown below to the left. The sum is 57. Then help them to add this sum and the third addend as shown below on the right.

```
 23     57
+34    +32
 57     89
```

Ask, "What is the sum of the three numbers?" (89) Write this sentence:

c is 89.

Ask, "Could we put these steps together into one step? Let us try?" Write the addends as shown:

```
23
34
32
```

Note: The pupils may group the addends together in either an upward or downward direction. The sum is the same. Choose one way (downward is used is this book) and get the pupils to follow it. Later they will learn to group first in one direction (up or down) and then to group in the other direction to check their work.

Guide the pupils to first add the number of ones and then add the number of tens. They will write the exercise as shown on the left below. Their thinking will be somewhat as described on the right.

23	3 ones + 4 ones = 7 ones; 7 ones + 2 ones = 9 ones
34	Write 9 in ones' place, as shown.
+ <u>32</u>	
89	2 tens + 3 tens = 5 tens; 5 tens + 3 tens = 8 tens.
	Write 8 in tens' place, as shown.

Point to the sentence and ask, "Have we found the number that makes our addition sentence true?" (Yes, the number is 89.) Write

23 + 34 + 32 = c

c is 89.

Say, "Our problem was about nuts. What question did it ask?" (How many nuts did they all give me?) If the pupils are unable to tell the question, tell them the story again and let them tell the question. Then ask for an answer to the question. (They all together gave me 89 nuts.) Write the following incomplete sentence on the board. Ask a pupil to complete the sentence.

They gave me <u>89</u> nuts.

Ask the pupils to open their books to page 58. With the help of the pupils, work several of the exercises on the board. Guide the pupils as they work the other exercises in their books. Make sure they understand the process of addition.

You and the pupils may use pages 59 and 60 as needed. You may wish to use them later as review.

Activity 3: Review of the techniques of addition using expanded form

Pupil pages 61, 62

Objective:

Children can use the expanded form to add two numbers, each less than 100, with regrouping for the sum.

Teaching Procedure:

Write this problem on the board:

Tom's father bought bricks for a new house. He bought 316 bricks in the first week, 248 bricks in the second week, and 122 bricks in the third week. How many bricks did he buy in the three weeks?

Ask, "What question is asked in the problem? (How many bricks did Tom's father buy in the three weeks?) What do you know about the problem?" (We know the number of bricks Tom's father bought in each of the three weeks.) Help the children to make a mathematical sentence for the problem. Let a child write this sentence on the board:

$$b = 316 + 248 + 122$$

GROUP I: Guide the children to write the numerals in vertical form to find the sum of the numbers. Get them to write the expanded form of the numerals and then to add the ones, the tens, and then the hundreds. The work on the board will look like this:

316	300 + 10 + 6
248	200 + 40 + 8
+ 122	100 + 20 + 2
	600 + 70 + 16
	600 + 70 + 10 + 6
	600 + 80 + 6
	686

Point to the mathematical sentence and ask, "What is b? (b is 686) Can you answer the question in the problem?" (Tom's father bought 686 bricks in the three weeks.)

Ask the pupils to open their books to pupil pages 61-62 and do the exercises. Encourage students to do problems 1 and 2 without renaming. If needed, help the children work one or two exercises on the board. Let the pupils, who can, add the numbers without writing the numerals in expanded form.

GROUP II: Write the addends in vertical form, as shown on the left below. Guide the pupils to think and write as described on the right. Suggestions are given to you in parentheses:

$$\begin{array}{r} 316 \\ 248 \\ +\ \underline{122} \\ 686 \end{array}$$

6 ones + 8 ones = 14 ones. 14 ones + 2 ones = 16 ones. (Say, "Think of 16 ones in its expanded form as 1 ten + 6 ones. Remember the 1 ten. You will add it with the other tens.") Write 6 in the ones' place. 1 ten + 1 ten = 2 tens; 2 tens + 4 tens = 6 tens; 6 tens + 2 tens = 8 tens. Write 8 in the tens' place. Add the number of hundreds. 3 + 2 = 5; 5 + 1 = 6. Write 6 in the hundreds' place.

Point to the mathematical sentence, b = 316 + 248 + 122 and ask, "Do you know the number that makes the mathematical sentence true?" (Yes, the number 686 makes the sentence true.) "Can you answer the question in the problem?" (Tom's father bought 686 bricks in the three weeks.)

Ask the pupils to open their books to pupil pages 61 - 62 and do the exercises. If needed, help the children work one or two exercises on the board.

Activity 4: Renaming in addition (Group I only) Pupil page 63

Objective:

Children can add two or three numbers, each less than 1,000, with regrouping for the sum, using the short form.

Teaching Procedure:

Write this sentence on the board:

35 + 27 = p

Write the addends in vertical form, as shown on the left below. Guide the pupils to think and write as described on the right. Suggestions are given to you in parentheses.

35	5 ones + 7 ones = 12 ones. (Say, "Think of 12 ones in its
+27	expanded form as 1 ten + 2 ones.
62	Remember the 1 ten. You will add it with the other tens.")
	Write 2 in the ones' place. 1 ten + 3 tens = 4 tens;
	4 tens + 2 tens = 6 tens. Write 6 in the tens' place.

Let the pupils tell how they add 35 and 27. They will say something like this: "We add 5 ones and 7 ones. The sum is 12. We think of 12 ones as 1 ten and 2 ones. We remember 1 ten and add it to the other tens. We write 2 in the ones' place. We add 1 ten, 3 tens, and 2 tens. The sum is 6 tens. We write 6 in the tens' place. The sum of 35 and 27 is 62." Point to the mathematical sentence 35 + 27 = p and ask, "Have we found the number that makes this sentence true?" (Yes, 62 is the number.) Write

35 + 27 = p

p is 62.

Tell the pupils this word problem:

There are 513 boys and 429 girls in Merrydale School. How many pupils attend Merrydale School?

It is very important that the pupils learn to express the relationships of the numbers in a problem as a mathematical sentence. The following questions will help the pupils to think of a mathematical sentence. Answers the pupils may give are suggested.

What does the problem tell us? (There are 513 boys and 429 girls in Merrydale School.) What question does the problem ask? (How many pupils attend Merrydale School?) How can you find the number of pupils at Merrydale School? (By adding the number of boys and the number of girls. Add 513 and 429.) What mathematical sentence shows this?

By questions of this type, guide the pupils to suggest the sentence below. Write the sentence on the board.

513 + 429 = p

Ask, "How do you find p?" (By adding the numbers) Write the addends in vertical form, as shown below at the left. Guide the pupils to think about the addition in the way shown at the right below. The writing on the board will be like that at the left below.

$$\begin{array}{r} 513 \\ +\underline{429} \\ 942 \end{array}$$

First add the number of ones. 3 + 9 = 12. Rename 12 ones as 1 ten and 2 ones. Remember 1 ten and write 2 in the ones' place. Add the number of tens. 1 + 1 = 2; 2 + 2 = 4. Write 4 in the tens' place. Add the number of hundreds. 5 + 4 = 9.

Write 9 in the hundreds' place.

Point to the mathematical sentence 513 + 429 = p and ask, "Do you know the number that makes this sentence true?" (Yes, p is 942.) Write

513 + 429 = p

p is 942.

"We made this sentence for a problem about Merrydale School. What question did the problem ask?" (How many pupils attend Merrydale School?) Ask, "Can you answer the question asked by the problem?" (942 pupils attend Merrydale School.) Write the following incomplete sentence on the board. Ask a pupil to complete it.

942 pupils attend Merrydale School.

Get the pupils to open their books to page 63. Help them to work one or more of the exercises on the board. Then guide them as they work the rest of the exercises in their books.

Activity 5: Review of techniques of addition (Group I only)

Pupil page 64

Objective:

Children can add two or three numbers, each less than 1000, with regrouping for the sum using the short form.

Teaching Procedure:

Write the following sentence on the board:

771 + 649 = a

The pupils will suggest adding the numbers and writing the addends in vertical form. Guide the pupils to think and write in this way:

771	First add the number of ones. 1 + 9 = 10. Rename 10 ones as 1 ten. Remember 1 ten and write 0 in the ones' place.
+ 649	
1420	Add the number of tens. 1 + 7 = 8; 8 + 4 = 12. Rename 12 tens as 1 hundred and 2 tens. Remember 1 hundred and write 2 in the tens' place. Add the number of hundreds. 1 + 7 = 8; 8 + 6 = 14. Write 14 hundreds as 1 thousand and 4 hundreds.

Point to the mathematical sentence 771 + 649 = a and ask, "Do you know the number that makes the mathematical sentence true?" (Yes, the number 1420 makes the sentence true.) Write

771 + 649 = a

a is 1420 .

Tell the pupils to open their books to page 64. Help them to work one or more exercises. Ask the pupils to work the rest of the exercises in their books. Go around the class giving help as needed. Ask the pupils to tell you what they think as they add.

<u>Activity 6</u>: Extending addition of three numbers

Pupil pages 65, 66

<u>Objective</u>:

Children can add two or more numbers, each less than 10,000, with regrouping for the sum using the short form.

<u>Teaching Procedure</u>:

Tell the pupils the following word problem:

From January to April Mr. Miller, a salesman, traveled 1952 miles. From May to August he traveled 2763 miles, and from August to December he traveled 3124 miles. How far did Mr. Miller travel in the year?

Ask the pupils to think of a mathematical sentence for the problem. Ask questions to help them think and write this sentence:

1952 + 2763 + 3124 = n

Ask, "How can you find n?" (By adding the numbers) Let a pupil write the addends as shown on the left. Help the pupils to add numbers. Guide them to think about addition as shown on the right.

1952 2763 + 3124 7839	Add the number of ones. 2 + 3 = 5; 5 + 4 = 9. Write 9 in the ones' place. Add the number of tens. 5 + 6 = 11; 11 + 2 = 13. Rename 13 tens as 1 hundred and 3 tens. Remember 1 hundred and write 3 in the tens' place. Add the number of hundreds. 1 + 9 = 10; 10 + 7 = 17; 17 + 1 = 18. Rename 18 hundreds as 1 thousand and 8 hundreds. Remember 1 thousand and write 8 in the hundreds' place. Add the number of thousands. 1 + 1 = 2; 2 + 2 = 4; 4 + 3 = 7. Write 7 in the thousands' place.

Point to the mathematical sentence 1952 + 2763 + 3124 = n and ask, "Do you know the number that makes the sentence true?" (Yes, the number is 7839.) Write

1952 + 2763 + 3124 = n

n is 7839.

Say, "This sentence came from a problem about Mr. Miller and how far he traveled. What did the problem ask?" (How far did Mr. Miller travel in the year?) "Can you answer the question?" (Mr. Miller traveled 7839 miles in the year.) Write the following incomplete sentence on the board. Ask a pupil to complete it.

Mr. Miller traveled 7839 miles in the year.

Tell the pupils to open their books to page 65. On the board work one or more exercises with the class. Tell the pupils to work the other exercises in their books. Go around the classroom. Guide the pupils as they write mathematical sentences. Listen to them tell what they think as they add. This will help you help them develop good habits of work.

Pupil page 66 may be used by those pupils who need more exercises.

Activity 7: Review of the techniques of addition using the short form

Pupil pages 67 - 71

Objective:

Children can add two or more numbers, each less than 10,000, with regrouping for the sum using the short form.

Teaching Procedure:

Write the following on the board:

There are four schools in a town. In the first school the total number of pupils is 684, the second has 360 pupils, the third has 844 pupils, and the fourth has 285 pupils. How many pupils are there altogether in the four schools?

Ask, "What question does the problem ask?" (How many pupils are in all four schools?) Guide the children to see that they can find the answer by adding the numbers of pupils enrolled in the schools. Let a child write this mathematical sentence on the board:

$684 + 360 + 844 + 285 = m$

Say, "It is easier to add these numbers if you write the numerals in vertical form." Write the numerals in the form to the left below and guide the work as shown to the right:

684	1. Think of 684 as 6 hundreds, 8 tens and 4 ones; think of 360 as 3 hundreds and 6 tens; and so on.
360	
844	2. Add the ones. 4 + 0 + 4 + 5 = 13. Rename 13 as 1 ten and 3. Write 3 in the ones' place of the sum. Remember 1 ten.
+ 285	
2173	
	3. Add the tens. (1 + 8 + 6 + 4 + 8) tens = 27 tens. Rename 27 tens as 2 hundreds and 7 tens. Write 7 in the tens' place of the sum. Remember 2 hundreds.

4. Add the hundreds. (2 + 6 + 3 + 8 + 2) hundreds = 21 hundreds. Rename 21 hundreds as 2 thousands and 1 hundred. Write 2 in the thousands' place and 1 in the hundreds' place of the sum.

5. The sum is 2173.

Point to the mathematical sentence and ask, "What is m? (m is 2173) Can you answer the question in the problem? (There are 2173 pupils in the four schools.)

Let the pupils do the exercises on pupil pages 67 - 71.

Activity 8: Adding three or four numbers

Pupil pages 72, 73

Objective:

Children can add three or four numbers in the thousands using the short form.

Teaching Procedure:

In this activity, the children add three or four numbers, some with sums that exceed 10,000. Encourage the use of the vertical form in writing the exercises. Review often the ideas of place value in decimal names and renaming. Addition in this activity includes renaming in two places. (Note that because of the use of renaming, the word "carrying" is not necessary. The words used in this program help to develop meaning: the word "carrying", we feel, does not.)

Write this story problem on the board:

Jamestown is divided into three sections. In the first section, there are 1134 people; in the second, there are 1376 people; and in the third, there are 2244 people. How many people are there in Jamestown?

Let the pupils talk about the problem and tell what it asks. Say, "How can you find the number of people in Jamestown?" (Add the numbers of people in the three sections.) Let the children make a sentence about the number of people in the town and let a child write:

1134 + 1376 + 2244 = n

Guide the children to find n by writing the exercise in the form to the left below and thinking as to the right:

1134 1376 + 2244 4754	1. Think of 1134 as 1 thousand, 1 hundred, 3 tens and 4; of 1376 as 1 thousand, 3 hundreds, 7 tens and 6; and so on. 2. Add the ones. 4 + 6 + 4 = 14. Rename 14 as 1 ten and 4. Write 4 in the ones' place of the sum. Remember 1 ten. 3. Add the tens. (1 + 3 + 7 + 4) tens = 15 tens. Rename 15 tens as 1 hundred and 5 tens. Write 5 in the tens' place of the sum. Remember 1 hundred. 4. Add the hundreds. (1 + 1 + 3 + 2) hundreds = 7 hundreds. Write 7 in the hundreds' place of the sum. 5. Add the thousands. (1 + 1 + 2) thousands = 4 thousands. Write 4 in the thousands' place of the sum. 6. The sum is 4754.

Guide the class to see that to add they always start by adding the number of ones, then the tens, then the hundreds, and so on. Renaming is done whenever necessary. Go over the process again, adding up if the children added down. Ask them to do the renaming and to give the sums.

Guide them to see that this change in the order of adding is a check. When the addition has been checked, ask, "What is n? (n is 4754.) Can you answer the question in the problem?" (Yes. There are 4754 people in Jamestown.) Do another example with the children in a similar way. You or the children may make up a problem which gives a sentence like this:

$$4823 + 3406 + 1143 + 2952 = n$$

Let the pupils turn to pupil page 72. Say, "In exercises 1 through 6, some of the numbers are shown in code. A letter of the alphabet is the code name for a number. If the same letter is used more than once in the same problem, it represents the same number. Look at Exercise 2. The letter N is used twice, but both N's represent the same number. If you study the exercises carefully, you will be able to tell what number each code letter stands for."

Let the pupils do the exercises on pupil pages 72 and 73. Encourage them to write the numerals in each exercise in vertical form before adding. If they have difficulty, help them do a few more examples.

Activity 9: Adding more than four numbers | Pupil pages 74 - 77

Objective:

Children can add three or more numbers, each less than one million, using the short form.

Teaching Procedure:

In this activity only the vertical form is used in addition. Renaming may be needed in any or all places. If more help is needed use examples with three addends in which renaming is done in only one place, then two places, and so on. Some examples you may use for review are:

1. 14763	2. 7466	3. 21376
11004	6120	4174
+ 4114	+ 3349	+ 7212
29881	16935	32762

Write the following story problem on the board:

A farmer has five farms. At harvest time his children picked the fruits. In the first farm they picked 5423 fruits; in the second farm they picked 4916 fruits; in the third farm they picked 7013 fruits, in the fourth they picked 11,246; and in the fifth they picked 8129 fruits. How many fruits did they pick altogether?

Ask, "What does the problem ask? (How many fruits did the children pick altogether?) Let n be that number. How do you find n? (Add the number picked in each farm.) Let a child write a sentence to show this:

$$5423 + 4916 + 7013 + 11{,}246 + 8129 = n$$

Guide the children to see that to add so many numbers it helps to arrange the numerals in vertical form as shown below:

```
  5423
  4916
  7013
 11246
+ 8129
 36727
```

Let a pupil show and explain the work at the board. Ask, "What is the sum?" (36,727) Encourage the children to check the sum by adding the other direction also.

Let the children answer the question in the word problem. (n is 36,727. Since n is the total number of fruits picked by the children, the answer to the question is this: 36,727 fruits were picked by the children.)

Ask the children to do the exercises on pages 74 and through 77. These exercises may be done in several different lessons.

When a change of activity is indicated, allow problems to be solved in this manner:

Ask several children to write problems from the lesson (a different one for each child) on the board. Before working, each child chooses a "checker" who will check his work by working the same problem at his desk and comparing the sums. Repeat the process until all have had a turn working at the board or checking at their desks.

In story problems, encourage the children to begin by writing a mathematical sentence about the story. Then they add the number with the numerals arranged in vertical form. The final step is to answer the question asked in the problem.

TOPIC II: INEQUALITIES AND MAGIC SQUARES

OBJECTIVES:

1. To review the meaning of < and > using addition.
2. To review the properties of addition and subtraction.

VOCABULARY: magic square

MATERIALS: pupil pages 78, 79

Activity I: Using the symbols < and >

Pupil page 78

Objective:

Children can compare a sum and a number using the symbols >, <, or =.

Teaching Procedure:

Tell the pupils the following word problem:

A school ordered 2 gross of exercise books (1 gross = 144). The school received 4 boxes with 60 exercise books in each box. Did the school receive all the exercise books that were ordered?

Let the pupils talk about the problem. Guide them to say, among other things:

We need to know how many exercise books were ordered. We need to know how many exercise books were received. We need to decide if as many books were received as were ordered.

Help the pupils to think in a way similar to this:

The school ordered some books (144 + 144). The school received some books (60 + 60 + 60 + 60). We need to find whether the number of books ordered is the same as the number of books received. We can write a mathematical sentence that is incomplete:

$$144 + 144 \ \_\_\_\_\_\_ \ 60 + 60 + 60 + 0$$

We must decide which of the following goes in the sentence to make it true:

$=, >, <$.

Guide them to decide that $288 > 240$ so $144 + 144 > 60 + 60 + 60 + 60$ and all the exercise books were not received.

Write this incomplete mathematical sentence on the board:

$$37 + 73 \ \_\_\_\_\_ \ 100$$

Ask, "What symbol, =, > or < can you put in this sentence to make it true?" Guide the pupils to say, "We must add 37 and 73." Let them add the numbers and decide that 110 is greater than 100 and write

$$110 > 100.$$

Ask the pupils to open their books to pupil page 78. With pupils' help, work one or more of the exercises on the board. Let the pupils work the other examples in their books.

Activity 2: Magic squares

Pupil page 79

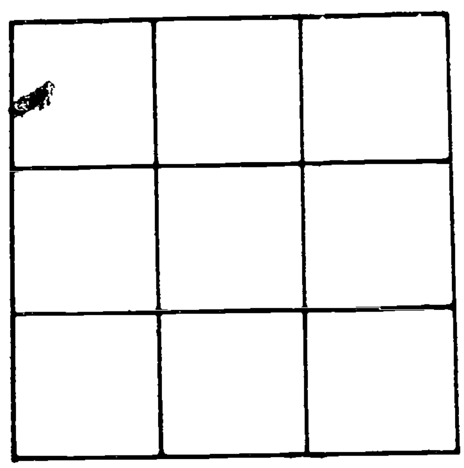
Objective:

Children can add and subtract to find the missing numbers in a magic square.

Teaching Procedure:

Magic squares are used in this activity to give the pupils practice in adding and subtracting numbers.

Draw the frame for a 3 by 3 magic square on the board as shown below:



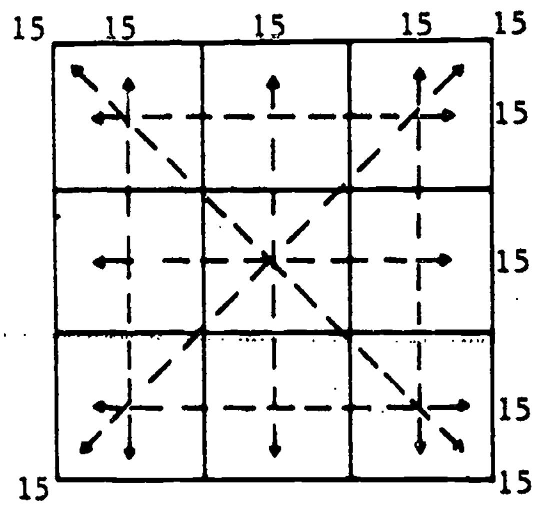
Sum: 15

Say, "This is a magic square. It is not complete. We will finish it in this way." Then give the directions below:

1. Each of the nine numerals 1-9 goes in one of the nine regions of the frame. We will find where they go and write them there. When a numeral is written in one region, we may not write it in any other region.

2. We will write the numerals in such a way that the sum of the three numbers represented by the numerals in any one direction is the same. The three numerals will be in rows, in columns, or on a diagonal.

3. In this magic square the sum of the three numbers is 15. The sum of the numbers represented by the numerals in each row is 15. The sum of the numbers represented by the numerals in each column is 15. The sum of the numbers represented by the numerals on each diagonal is 15.



Sum: 15

Say as you write in the numerals shown below, "I will write some of the numerals in the magic square. You find the correct numerals for the other regions."

	3	
9		1
	7	

Sum: 15

Say, "Let us first find the numeral to write in the center region. What numeral goes in the center of the frame? How can we find it? Look at the center row. What addition sentence can you make?

$9 + n + 1 = 15$

Get the pupils to look at the center column, and write

3 + n + 7 = 15

Ask, "What number will make each of these sentences true?" (5) If pupils have difficulty in discovering the number 5 in the first sentence, guide their thinking in this way:

9 + n + 1 = 15 or

9 + 1 + n = 15

Add 9 and 1:

10 + n = 15

Write this subtraction sentence:

n = 15 - 10

Subtract 10 from 15:

n is 5

The pupils should think in a similar way about the second sentence:

3 + n + 7 = 15

3 + 7 + n = 15

Add 3 and 7:

10 + n = 15

Write this subtraction sentence:

n = 15 - 10

Subtract 10 from 15:

n is 5

After the pupils understand that the numeral 5 goes in the center region, they will need to experiment with the other rows and columns. Let them try numerals in regions and decide whether the numerals are in the correct region. If they follow the rules of the game, they will soon find where to write the other numerals. The answers are shown below but let the pupils discover the answers for themselves.

4	3	8
9	5	1
2	7	6

Ask pupils in turn to write on the board true addition sentences that the magic square shows.

The following sentences are true. (The sentences may be written in other forms.)

Rows	Columns	Diagonals
(4 + 3) + 8 = 15	(4 + 9) + 2 = 15	(4 + 5) + 6 = 15
(9 + 5) + 1 = 15	(3 + 5) + 7 = 15	(2 + 5) + 8 = 15
(2 + 7) + 6 = 15	(8 + 1) + 6 = 15	

Encourage the pupils to use parentheses to show which two numbers they add first (associative property).

If the pupils enjoy this activity, give them other magic squares to solve. In the one that appears on the next page, the rules are the same as those stated for the magic square above except for these: the sum is 45; and any numbers the pupils know may be used in the magic square.

18		6
	15	
24		12

Say to the pupils, "Which region will you fill first?" A pupil may suggest finding the missing number in the top row. If so, he should write the sentence and proceed as shown below:

18 + a + 6 = 45

18 + 6 + a = 45

Add 18 and 6.

24 + a = 45

Write this as a subtraction sentence:

a = 45 - 24

Subtract 24 from 45.

a is 21.

The answers are shown below. Let the pupils discover the answer for themselves.

18	21	6
3	15	27
24	9	12

When the pupils understand how magic squares are made, let them open their books to pupil page 79 and fill the regions with the correct numerals.

TOPIC III: SUBTRACTION

OBJECTIVES:

1. To review the meaning of subtraction.

2. To review the techniques of subtraction using expanded form.

3. To review renaming in subtraction.

VOCABULARY: none

MATERIALS: pupil pages 80 - 87

Activity I: Review of subtraction

Pupil page 80

Objective:

Children can use the short form of subtraction without regrouping for sums less than 1,000.

Teaching Procedure:

Tell the pupils the following word problem:

289 pupils attend Dawson School. 136 of them are girls. How many boys attend Dawson School?

Let the pupils talk about the problem. Guide them to express the relationship of the numbers in the problem as a mathematical sentence. The following questions should help them:

What is the problem about? (There are 289 pupils in Dawson School, and 136 of these pupils are girls.) What does the problem ask? (How many boys attend Dawson School?)

Guide the pupils to suggest <u>either</u> of the sentences below. When one is given, help them write the other and to say, "The sentences mean the same thing."

$136 + c = 289$

$289 - 136 = c$

Let the pupils talk about the sentences they made and say, "c is a missing addend." Get them to tell how they can find the missing addend. Write the known addend and the sum as shown below and say, "Let's find the missing addend this way."

```
  136
+ ___
  289
```

Ask, "6 ones plus how many ones equals 9 ones?" (3 ones) Write 3 in ones' place. Ask, "3 tens plus how many tens equals 8 tens?" (5 tens) Write 5 in tens' place. Ask, "one hundred plus how many hundreds equals 2 hundreds?" (1 hundred) Write 1 in hundreds' place. The board will look like this:

```
  136
+ 153
  289
```

Say "You are subtracting. We will write the numerals in subtraction form and subtract." Guide the pupils to find the missing addend again as they use subtraction language. (Write only what is shown on the left on the following page.)

289 First subtract the number of ones. 9 - 6 = 3. Write 3 in the
- <u>136</u> ones' place.
153 Subtract the number of tens. 8 - 3 = 5. Write 5 in the tens' place. Subtract the number of hundreds. 2 - 1 = 1. Write 1 in the hundreds' place.

Let the pupils say that 153 will make their sentences true. Write:

136 + 153 = 289 and

289 - 136 = 153

Guide the pupils to see that if one of the sentences is true, the other sentence is true. Let the pupils add 136 and 153 to make sure the sentence is true.

Say, "We are solving a word problem." Ask, "What question did the problem ask?" (How many boys attend Dawson School?) Ask, "Can you answer the question?" (153 boys attend Dawson School.) On the board, write the following incomplete sentence. Choose a pupil to help you complete it as shown.

<u>153</u> boys attend Dawson School.

Note: If the pupils need more help in understanding subtraction, let them show the exercises with bundles of sticks in a place value box.

Note: In some problems that require subtraction, the answer may be called the <u>difference</u>. For example, in this problem the difference between 289 pupils and 136 pupils is 153.

Ask the pupils to open their books to page 80. With the pupils' help work one or more exercises on the board. Then let the pupils work the other exercises in their books. Go around the classroom, giving help as needed.

Activity 2: Further study of subtraction

Pupil page 81

Objective:

Children can use the short form of subtraction with regrouping required for sums less than 1,000.

Teaching Procedure:

Tell the pupils the following word problem:

Mr. Williams had 83 bags of flour. He sold 67 bags. How many bags were not sold?

Ask questions to help the pupils express the relationship of the numbers in the problem as a mathematical sentence. The pupils may suggest either of the following sentences:

$67 + y = 83$ or

$83 - 67 = y$

Guide the pupils to say, "y is the missing addend. We subtract numbers to find the missing addend. We subtract the known addend, 67, from the sum, 83." Write the subtraction exercise as shown below:

$$\begin{array}{r} 83 \\ -\underline{67} \end{array}$$

Tell the pupils that the subtraction process depends upon renaming the sum and the known addend. Ask them to give different names for the number 83. They may mention

8 tens and 3 ones (80 + 3)

7 tens and 13 ones (70 + 13)

On the board, write the forms shown below:

Subtract	Subtract
8 tens and 3 ones	7 tens and 13 ones
6 tens and 7 ones	6 tens and 7 ones

Ask, "Which of the above forms makes it easier for you to subtract 67 from 83? Which name shall we use for 83?" (7 tens and 13 ones) "Why is it easier to subtract when 83 is renamed as 7 tens and 13 ones?" (We need more than 3 ones to subtract 7 ones)

Guide the pupils to think and write in the following way:

83 - 67 16	First subtract the number of ones. There are not enough ones to subtract 7 ones from 83 when we think of 83 as 8 tens and 3 ones. Rename 83 as 7 tens and 13 ones. 13 - 7 = 6. Write 6 in the ones' place. Subtract the number of tens. 7 - 6 = 1. Write 1 in the tens' place.

Point to the sentence 83 - 67 = y. Ask, "Do you know the number that makes this sentence true?" (Yes, 16) Write the following:

16 + 67 = 83 and

83 - 67 = 16

Ask the pupils to add 16 and 67 to make sure the addition sentence is true.

Get the pupils to think again of the word problem. Ask, "What question did the problem ask?" (How many bags of flour were not sold?) Say, "Answer the question asked by the problem." (16 bags of flour were not sold.) Write the following incomplete sentence. Get a pupil to complete it.

16 bags were not sold.

Ask this question: "What is the <u>difference</u> between 526 dollars and 351 dollars?" Get the pupils to write a mathematical sentence for the question. They may suggest one of the following sentences:

n + 351 = 526 <u>or</u> 351 + n = 526 <u>or</u>

526 - 351 = n

Let the pupils talk about how they can make the sentence true. Guide them to decide to subtract 351 from 526. Write the numerals as shown below:

```
  526
- 351
```

Say, "We first subtract the number of ones, then the number of tens and then the number of hundreds. The subtraction process depends upon renaming the sum." Guide the pupils to study the exercise and to choose the most convenient name for the sum. Ask, "Do you need more ones in the name of the sum?" (No. There are enough ones. 6 ones - 1 one = 5 ones.) "Do you need more tens in the name of the sum?" (Yes. To subtract 5 tens from 526 we need at least 5 tens in the name of 526.) "How can you rename 526 to name more tens?" (4 hundreds, 12 tens and 6 ones)

Write these forms on the board:

<u>Subtract</u>

5 hundreds 2 tens 6 ones

<u>3 hundreds 5 tens 1 one</u>

<u>Subtract</u>

5 hundreds 1 ten 16 ones

<u>3 hundreds 5 tens 1 one</u>

<u>Subtract</u>

4 hundreds 12 tens 6 ones

<u>3 hundreds 5 tens 1 one</u>

Let the pupils examine each of the subtraction exercises and decide that each shows 526 - 351. Get the pupils to try to subtract, using the forms as given. Ask, "Which of these forms makes it easier to subtract?" (The last one) "Which name for 526 is the best one for you to use to subtract?" (4 hundreds, 12 tens and 6 ones)

Help the pupils to think and write as shown below:

526	First subtract the number of ones. 6 - 1 = 5.
- 351	Write 5 in ones' place.
175	Subtract tens. There are not enough tens to subtract 5 tens from 526 when we think of 526 as 5 hundreds, 2 tens and 6 ones. Rename 526 as 4 hundreds, 12 tens, 6 ones. 12 - 5 = 7. Write 7 in tens' place.
	Subtract hundreds. 4 - 3 = 1. Write 1 in hundreds' place.

Guide the pupils to say, "175 is the missing addend." Point to the sentence 351 + n = 526 and ask, "Have you found the number that will make the sentence true?" (Yes, 175) Write the following:

526 - 351 = 175 <u>and</u> 351 + 175 = 526

Let the pupils add 351 and 175 to make sure the addition sentence is true.

Ask the pupils to open their books to pupil page 81. You and the pupils work one or more of the exercises on the board. Get the pupils to work the other exercises in their books. Give help as needed. Get the pupils to tell you how they name the sum to subtract. Help them to choose the best name.

Activity 3: Review of the techniques of subtraction | Pupil pages 82 - 84

Objective:

Children can subtract, regrouping when required for sums less than 10,000.

Teaching Procedure: (Group I)

Write this on the board:

```
  325
- 148
```

Ask the children to work together to subtract the numbers. Say, "Think of the numbers in expanded form and then subtract the number of ones and then tens and then hundreds." Get the children to write the expanded form of the two numerals:

```
  325   300 + 20 + 5
- 148   100 + 40 + 8
```

Ask, "Are there enough ones in the ones' place of 325 to subtract 8? (No) What can you do?" (Rename the sum)

```
  325   300 + 20 + 5    300 + 10 + 15
- 148   100 + 40 + 8    100 + 40 +  8
```

Let a child subtract the ones (15 - 8 = 7) and write 7 in the ones' place. Ask, "Are there enough tens in the tens' place of the sum to subtract 40? (No) What can you do?" Rename the sum.

```
  325   300 + 10 +15    200 + 110 + 15
- 148   100 + 40 + 8    100 +  40 +  8
```

"Can you subtract the tens?" (Yes. (11 - 4) tens = 7 tens.) Let the child who answers write 70 in the missing addend. Ask, "Can you subtract the hundreds?" (Yes. (2 - 1) hundreds = 1 hundred.) Write 100 in the missing addend. Say, "The missing addend is 100 + 70 + 7. What is its simplest name?" (177) Tell the children that there is a shorter form for writing their work. Let the pupils go over the example again. As they do so, guide them to write the form to the left and to explain the work to the right as shown in the following example. Point to the proper numerals as the children talk.

325	1.	Think of 325 as 3 hundreds, 2 tens and 5. Think of 148 as 1 hundred, 4 tens, and 8.
- 148		
177	2.	There are not enough ones in the ones' place of the sum to subtract 8. Rename 325 as 3 hundreds, 1 ten and 15. Subtract the ones. 15 - 8 = 7. Write 7 in the ones' place.
	3.	There are not enough tens in the tens' place of the sum to subtract 4 tens. Rename 3 hundreds and 1 ten as 2 hundreds and 11 tens. Subtract the tens. (11 - 4) tens = 7 tens. Write 7 in the tens' place.
	4.	Subtract the hundreds. (2 - 1) hundreds = 1 hundred. Write 1 in the hundreds' place.
	5.	The missing addend is 177.

For the next example, write the following story problem on the board:*

In two days, trains carried 9724 tons of copper to the coast. If 2428 tons were carried on the second day, how many tons were carried on the first day?

*Group II begins here. Group I should also use this material.

Ask, "What question does the problem ask? (How many tons were carried on the first day?) What do you know? (9724 tons were carried altogether and 2428 of these tons were carried the second day.) How do you find how many tons were carried the first day?" Help the children to see that they subtract to find the answer. Guide the children to write a mathematical sentence for the problem. They may write either of the following sentences:

$$9724 - 2428 = d$$

$$d + 2428 = 9724$$

Before talking about the method of subtraction, let the children make guesses as to <u>about</u> what number d makes the sentence true. They may think: the sum is about 9000; one addend is about 2000; the missing addend is about 7000.

Say, "You guessed that d is about 7000. Now subtract the numbers and find what d is." Write the numerals for the numbers on the board. Let the children go through the steps of renaming and then subtracting. You point to the numerals and write the digits in the answer as they talk.

$$\begin{array}{r} 9724 \\ -\ 2428 \\ \hline 7296 \end{array}$$

Let the children say that d is 7296, and that 7296 tons of copper were carried the first day. Ask, "Did you make a good guess? (Yes. We guessed 7000.) Can you check the answer?" (Yes. We can add 7296 and 2428. The sum should be 9724. There were 9724 tons of copper carried in two days.) Let them check their answers.

Let the children do the exercises on pupil pages 82 - 84. It is not necessary to do all three pages during one lesson. Spend only as much time on these pages as the children need.

Activity 4: Using subtraction | Pupil page 85 |

Objective:

Children can subtract, regrouping sums represented by 3 or 4 digit numerals containing one or more zeros.

Teaching Procedure:

Tell the pupils the following word problem:

Mr. Warren had 636 cans in his shop on Monday morning. When he closed his shop on Saturday, he had 379 cans left. He had sold the rest. How many cans had he sold?

Guide the pupils to suggest one of the following mathematical sentences:

$s + 379 = 636$ or $379 + s = 636$ or

$636 - 379 = s$

Write the exercise in vertical form. Guide the pupils to think about the exercise as shown at the right below.

636	Subtract the number of ones. There are not enough ones to
- 379	subtract 9 ones when we think of 636 as 6 hundreds, 3 tens
257	and 6 ones. Rename 636 as 6 hundreds, 2 tens and 16 ones.
	16 - 9 = 7. Write 7 in the ones' place.
	Subtract the number of tens. There are not enough tens to subtract 7 tens when we think of 636 as 6 hundreds, 2 tens and 16 ones. Rename 636 as 5 hundreds, 12 tens and 16 ones. 12 - 7 = 5. Write 5 in the tens' place.

Subtract the number of hundreds. 5 - 3 = 2. Write 2 in the hundreds' place.

Point to the mathematical sentence 379 + s = 636. Get the pupils to say, "257 is the missing addend. 257 makes the sentence true." Write the following:

379 + 257 = 636 and

636 - 379 = 257

Let the pupils check their work by adding 379 and 257.

Tell the pupils the word problem again about the cans Mr. Warren sold. Ask, "What question did the problem ask?" (How many cans had Mr. Warren sold?) Say, "Answer the question asked by the problem." (He had sold 257 cans.) Write the following incomplete sentence on the board and get the pupil to complete it:

He had sold 257 cans.

Write the following on the board.

900 - 532 = n

Write the exercise in vertical form and guide the pupils to think and write as shown.

900	First subtract the number of ones. There are not enough ones
- 532	to subtract 2 ones from 900 when we think of 900 as 9 hundreds.
368	Rename 900 as 90 tens and then as 89 tens and 10 ones.
	10 - 2 = 8. Write 8 in ones' place.
	89 tens is 8 hundreds and 9 tens.
	Subtract the number of tens. 9 - 3 = 6. Write 6 in tens' place.
	Subtract the number of hundreds. 8 - 5 = 3. Write 3 in hundreds place.

Point to the sentence 900 - 532 = n and let the pupils say, "368 makes the sentence true."

Go over the subtraction of this exercise several times with the pupils. Help them to understand the renaming of 900 as 90 tens and 0 ones and then as 89 tens and 10 ones and then as 8 hundreds and 9 tens and 10 ones. Write the following on the board:

1009 - 781 = n

Write the exercise in vertical form and guide the pupils to think as shown below on the right:

1009 - 781 228	First subtract the number of ones. 9 - 1 = 8. Write 8 in ones' place. Subtract the tens. There are not enough tens to subtract 8 tens from 1009 when we think of 1009 as 1 thousand and 9. Rename 1009 as 9 hundred, 10 tens and 9. 10 - 8 = 2. Write 2 in tens' place. Subtract the hundreds. 9 - 7 = 2. Write 2 in hundreds' place.

Point to the sentence 1009 - 781 = n and ask whether the pupils can now make the sentence true. Get them to write:

1009 - 781 = 228

Ask the pupils to open their books to pupil page 85. With the pupils' help, work one or more of the exercises on the board. Then get the pupils to work the other exercises in their books. Give help as needed. Go around the class and get the pupils to tell you what they think as they subtract.

Activity 5: Reviewing renaming in subtraction

Pupil page 86

Objective:

Children can subtract, regrouping when required for sums less than 10,000.

Teaching Procedure:

In this activity review quickly the idea of renaming in subtraction and give practice in subtracting large numbers.

Before asking the pupils to work the exercises in their books, go over a few examples to illustrate the use of expanded form and renaming.

Write the following example on the board. Do it only in short form but discuss each step with the children. Let them tell what the steps in renaming are but write the numerals yourself to keep the activity moving as rapidly as possible. Let the thinking of the children be similar to that described to the right below:

```
  5211
- 2489
  2722
```

1. Think of 5211 as 5 thousands, 2 hundreds, 1 ten and 1. Think of 2489 as 2 thousands, 4 hundreds, 8 tens and 9.
2. There are not enough ones in the ones' place of the sum to subtract 9. Rename 5211 as 5 thousands, 2 hundreds, 0 tens, and 11. Subtract the ones. 11 - 9 = 2. Write 2 in the ones' place.
3. There are not enough tens in the tens' place of the sum to subtract 8 tens. Rename 5 thousands, 2 hundreds, 0 tens as 5 thousands, 1 hundred and 10 tens. Subtract the tens. (10 - 8) = 2 tens. Write 2 in the tens' place.

4. There are not enough hundreds in the hundreds' place of the sum to subtract 4 hundreds. Rename 5 thousands and 1 hundred as 4 thousands and 11 hundreds. Subtract the hundreds. (11 - 4) hundreds = 7 hundreds. Write 7 in the hundreds' place.

5. Subtract the thousands. (4 - 2) thousands = 2 thousands. Write 2 in the thousands' place.

6. The missing addend is 2722.

Do several other subtraction examples and go over the steps as before. Some you may use are these:

$$\begin{array}{r} 9514 \\ - \underline{1855} \end{array} \qquad \begin{array}{r} 6013 \\ - \underline{3489} \end{array}$$

After the subtraction in the second example, say, "Remember that subtraction is finding the missing addend. You can check your answer by thinking of the addition sentence for the example." Get the children to write the sentence 3489 + n = 6013 for the second example. Go on, "By subtracting you found that the missing addend is 2524. If n is 2524, is the addition sentence true?" Let the children add (3489 + 2524) to show that the sum is 6013. Say, "We call this 'checking our answer.'"

Write on the board

5000 - 1,999 = n

Say, "How can we find what n is?" Guide pupils to think of a quick way of solving for n other than the usual process of subtraction. The following questions may direct their thinking:

1999 is close to what number? (2000)

Would it be easier to subtract 2000 from 5000 or to subtract 1999 from 5000? (2000 from 5000)

2000 is how many more than 1999? (One) Now, how close do you think 3,000 is to the difference between 5000 and 1999? (One) Is the difference between 5000 and 1999 more than 3000 or less than 3000? (More than 3000) Why do you think it is more than 3000? (We added one to 1999 to make it 2000 for easy subtraction, so we can add one to the difference) What is the difference? (3001)

Say, "Let's write a mathematical sentence for what we did." Write

$$5000 - 1999 = (5000 - 2000) + 1$$

Ask a child to work both parts of the equation so that students can see that each expresses the same number, 3001.

Work several problems in this way. You may use such problems as

$5000 - 2001$ $\quad\quad$ $5000 - 1995$

Note: If necessary, use 500 - 199 or 50 - 19 and use counters or dots. It may prove helpful to demonstrate the use of this skill in problems involving money, a very practical application, such as:

$\$5.00 - \$1.99 = m$ or $\$50.00 - \$19.99 = n$

Ask the children to work the exercises on pupil page 86. Ask them to work the exercises at the bottom of the page using the "short cut".

Activity 6: Extending subtraction to numbers greater than 10,000 — Pupil page 87

Objective:

Children can subtract, regrouping when required, for sums less than 100,000.

Teaching Procedure:

The ideas and methods used in this activity are the same as in the last activity. Here the ideas are extended to numbers greater than 10,000 so that renaming and subtracting in the ten thousands' place are included.

Begin with an example like this:

```
  32,921
- 18,365
```

Ask the children to subtract. When they have completed the exercise, work it on the board and discuss each step.

After the example is completed, guide the children to see that they can check the subtraction by adding:

```
  18,365
+ 14,556
  32,921
```

Do a few other examples as needed and then let the children do the exercises on pupil page 87. Encourage them to use addition to check their answers. If some have difficulty, go over several exercises step by step with them.

TOPIC IV: REVIEWING ADDITION AND SUBTRACTION OF LARGE NUMBERS

OBJECTIVES:

1. To review addition and subtraction using inequalities.
2. To review techniques using story problems.

VOCABULARY: dozen

MATERIALS: pupil pages 88 - 93

Activity 1: Using < and >

Pupil page 88

Objective:

Children can compare a difference and a number using the symbols <, >, or =.

Teaching Procedure:

Note: If the pupils are not familiar with the word dozen, teach the meaning of the word before beginning this activity.

Tell the pupils the following word problem:

Mr. Purnell had 90 oranges. He sold 3 dozen of them. Did he have enough oranges left to sell 4 dozen more?

Guide the pupils to think:

Mr. Purnell had 90 oranges. He sold 3 dozen. Did he have enough left to sell 4 dozen more? He sold 3 dozen, or 12 + 12 + 12 = 36.

He had left (90 - 36) oranges, 90 - 36 = 54.

He wishes to sell 4 dozen, which is 12 + 12 + 12 + 12 = 48.

Is 54 less than, greater than, or equal to 48? (54 > 48) Get the pupils to answer the question asked in the problem. (Mr. Purnell has enough oranges left to sell 4 dozen more oranges.)

Get the pupils to open their books to page 88. Let them select one exercise and with their help work the exercise on the board. Get the pupils to work the other exercises in their books. Go around the classroom. Observe the pupils and listen to them tell how they subtract. Give help as needed.

Activity 2: Inequalities using addition and subtraction

Pupil page 89

Objective:

Children can compare sums and/or differences using the symbols <, > , or =.

Teaching Procedure:

Write the following problem on the board.

There were 213 people waiting for buses. Five buses arrived. Each bus seats 43 people. No one is allowed to stand. Can everyone be seated? If so, are there any seats not used?

Let the pupils talk about the problem. Guide the children to say, "We find which of two numbers is larger. One number 213 tells how many people there are. The other number, 43 + 43 + 43 + 43 + 43, tells how many seats there are." In sentence form, this will be:

213 ____ 43 + 43 + 43 + 43 + 43

Get the pupils to decide which symbol, < , > , or =, makes the sentence true. Ask one of the pupils to add five 43's. The sum is 215. (If a pupil notices that 43 + 43 + 43 + 43 + 43 = 5 x 43, let him do the multiplication.) Guide the children to decide: the correct symbol is <; there are enough seats for everyone; two seats are not used.

Below are other examples of this kind of exercise. You or the pupil may make up story problems to go with the sentences. In each exercise get the pupils to decide which symbol, < , >, or =, makes the sentence true:

342 + 574 ____ 1000 - 81

876 + 754 ____ 726 + 871

948 + 385 ____ (474 + 117) - 28

8 + (728 - 432) ____ 654 - 349

Let the pupils open their books to pupil page 89 and do the exercises.

Activity 3: Story problems with large numbers

Pupil pages 90 - 92

Objective:

Children can solve story problems involving addition and subtraction of numbers less than one million.

Teaching Procedure:

Ask the children to turn to pupil page 90. Let a child read the first problem aloud. Go over the procedure for solving the problem. The procedure for problem 1 is:

a. Compare the three numbers telling the height of the three mountains. Which is the largest number? (14,162) Therefore, Mount Shasta is the highest.

b. Compare the other two numbers. Which is larger, 14,110 or 8,751? (14,110) Therefore, Pike's Peak is higher than Guadalupe Peak.

c. Answer the questions: How can we find how much higher Pike's Peak is than Guadalupe Peak? (We subtract the height of Guadalupe Peak from the height of Pike's Peak.)

d. Show the mathematical sentences for these.

e. Answer the questions after doing the subtractions.

Read through the other problems with the class and let the children do the work. Tell them to write the mathematical sentences for the questions asked, solve the sentences and write the answers to the questions.

Notice that in the second problem they are asked to show a mathematical sentence comparing a greater number with a lesser number, using population figures. They may show this as:

938,219 > 679,684

If there is difficulty with reading the problem let the pupils help each other read the problems one at a time, giving the children sufficient time to do each before reading the next. Follow the same procedure for the problems on pupil pages 91 and 92.

Activity 4: Addition and subtraction of large numbers

Pupil page 93

Objective:

- Children can add and subtract numbers less than one million.

Teaching Procedure:

Ask the pupils to turn to page 93 and do the exercises. Remind the children to read the exercises carefully because there are both addition and subtraction exercises. Say, "Remember that it usually helps in solving word problems to write a mathematical sentence. Then you can solve the sentence to find the answer to the question asked."

TOPIC V: ENRICHMENT

OBJECTIVE:

To provide challenge problems in addition and subtraction.

VOCABULARY: none

MATERIALS: pupil pages 94 - 97

Activity I: Using properties

Pupil page 94

Objective:

Children can use the commutative and identity properties of addition to complete an addition chart.

Teaching Procedure:

In Unit 6, operation machines were used to get the pupils to discover the properties of addition. This method may be used again to start the discussion.

Get the pupils to recall the meaning of the commutative property of addition: We may add two numbers in either order and we always get the same sum. (In symbols, $a + b = b + a$, where $\underline{a}$ and $\underline{b}$ are any whole numbers.) Get the pupils to recall also that the sum of any number and zero is that number. (In symbols, $a + 0 = a$, for any whole number $\underline{a}$.)

Put an addition chart like this on the board. (Use any five numbers.)

+	5	19	32	6	11
5					
19					
32					
6					
11					

First ask the pupils for the sums that go in the boxes that are filled in the following example. Remind them that the sums are obtained by adding the number to the left in a row and the top number in a column. For example, $5 + 32 = 37$.

+	5	19	32	6	11
5	10	24	37	11	16
19		38	51	25	30
32			64	38	43
6				12	17
11					22

Note: Answers are not obtained by adding preceding answers. For example, in the table on the previous page, $10 + 24 \neq 37$.

Ask, "Can you fill in the other boxes without doing any other addition?" Guide the children to say that because of the commutative property, they know all the sums. Let the children fill the other boxes.

Let the pupils open their books to pupil page 94. Chart 1 can be completed without any additions. Also use this exercise to review the property of zero. Chart 2 requires two additions and one subtraction.

In Exercise 3, Frank's way uses both the commutative property and the associative property. The commutative property is used several times. This exercise may be too difficult for some pupils. It is a challenge problem and is marked with an *.

Exercise 4 is solved in this way:

$$(2 + 23) + (5 + 20) + (8 + 17) + (11 + 14) = m$$

$$4 \times 25 = m$$

m is 100

You may give this sentence to some of the children to solve if they find Exercises 3 and 4 a challenge. If you wish to give other sentences, be sure the numbers are arranged in sequential order.

$$n = 5 + 10 + 15 + 20 + 25 + 30$$

Activity 2: Review of operation machine — Pupil page 95

Objective:

Given an operation machine and a pair of numbers, children can find the missing operation, the missing sum, or the missing addend.

Teaching Procedure:

Let the pupils open their books to pupil page 95 and discuss the operation machines on the page. They will notice that a part of each exercise is missing. Let them find the missing part.

Activity 3: Associative property of addition

Pupil pages 96, 97

Objective:

Children can use the associative property to find short cuts for addition.

Teaching Procedure:

Again, operation machines may be used to start the discussion. Get the pupils to recall the meaning of the associative property: If three numbers are added, the sum is the same no matter which pair is added first. (In symbols, $a + (b + c) = (a + b) + c$, where a, b, and c are whole numbers.)

You may want the children to review one of the operations introduced in Unit 6 for which this property did not hold. One operation used was: "Take half the sum." It was called the "$\triangle$" operation. For example,

$$(2 \triangle 6) \triangle 10 = 4 \triangle 10 = 7$$

$$2 \triangle (6 \triangle 10) = 2 \triangle 8 = 5$$

The operation $\triangle$ is not associative. You may want to use it for contrast with addition. (Choose carefully the numbers you use with operation $\triangle$ so the children can find $\frac{1}{2}$ of all the sums.)

Let the pupils open their books to pupil page 96. Let them read and think about Calvin's way of adding. Help them to decide why it works. They should tell where the associative property is used. Let the children use Calvin's method to work these easier examples:

a. $19 + 12 = 19 + (1 + 11)$
$= (19 + 1) + 11$
$= 20 + 11$
$= 31$

b. $57 + 17 = 57 + (3 + 14)$
$= (57 + 3) + 14$
$= 60 + 14$
$= 74$

Get the children to solve Exercises 1 to 3 on pupil pages 96 and 97. Then discuss Calvin's way of subtracting. Below are more examples you and the children may work together. The work may look like this:

c. $49 + n = 145$
$(45 + 4) + n = 145$
$45 + (4 + n) = 145$
$4 + n$ is 100
n is 96

d. $19 + n = 64$
$(14 + 5) + n = 64$
$14 + (5 + n) = 64$
$5 + n$ is 50
n is 45

Then get the pupils to solve Exercises 4 and 5.