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ABSTRACT

This instructor's manual presents lesson plans for a unit of mathematics within the Biomedical Interdisciplinary Curriculum Project (BICP), a two-year interdisciplinary precollege curriculum aimed at preparing high school students for entry into college and vocational programs leading to a career in the health field. Lessons concentrate on biomedical applications in the mathematical area of quadratics, relating mathematical concepts to chemical concepts of pH, scientific notation, molarity, equilibrium and solubility, dissociation constants, and chemical equilibrium. Designed to accompany the student text, lesson plans include objectives, recommended teaching time, and remarks. Keys to problem sets are also included. (CS)

BIOMEDICAL MATHEMATICS

UNIT III

QUADRATICS

INSTRUCTOR'S MANUAL
REVISED VERSION, 1976

THE BIOMEDICAL INTERDISCIPLINARY CURRICULUM PROJECT
SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

TABLE OF CONTENTS

LESSON 1: QUADRATIC FUNCTIONS

OBJECTIVES:

The student will:

- recognize quadratic functions.
- put quadratic functions in standard form.
- correctly use the terms vertex, axis of symmetry, y-intercept, parabola and opens upward as they apply to the graph of $y = x^2$

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This lesson begins a sequence on quadratic functions. The first nine lessons cover the theoretical aspects of the subject, discussing graphing, square completion and the quadratic formula. These lessons have been developed with the following purposes in mind.

1. The lessons are intended to provide some general background for college-entrance examinations.

2. The graphing material is intended to provide some preparation for the polynomial graphs encountered in the second year of this program.

3. The quadratic formula material is intended to prepare students for the applied quadratics material which follows immediately.

The applied quadratics sequence, beginning with Lesson 10, concerns chemical equilibrium in the digestive tract. Quadratic equations of various levels of difficulty are used in exploring the following subjects.

1. The behavior of antacids in the digestive tract.

2. The behavior of certain food additives in the digestive tract.

3. The behavior of heavy-metal paint pigments after ingestion.

In preparation for the applied quadratics sequence, it is essential that Lesson 6 and Lesson 8 be covered. If you wish to omit any of the remaining material in the first nine lessons, it will not intefere with the applied development.

Unit III ends with a discussion of complex numbers. This subject arises naturally from imaginary roots of quadratic equations.

KEY--PROBLEM SET 1:

1. a. quadratic b. linear c. linear d. quadratic
2. a. $y = 12x^2 - 2x + 3$ b. $y = -x^2 + 2$ c. $y = 2x^2 + 5x$ d. $y = 10x^2$
3. 4.6 months
4. $y = -x^2 + 8$
5. $s = 3t^2 + t$
6. $y = x^2 + 2x$
7. $v = u^2 - 1$
8. $y = \frac{1}{4}x^2 - 2x + 4$
9. $s = 2t^2 + 4t + 1$
10. $z = 4w^2 - 20w + 24$
11. $y = \frac{1}{2}x^2 - 2x + 7$
12. $y = 2t^2 + 2$
13. a. 1 b. 0 c. 0
14. parabola
15. upward
16. axis of symmetry
17. 0
18. vertex

LESSON 2: GRAPHING FUNCTIONS OF THE FORM $y = ax^2 + c$

OBJECTIVES:

The student will:

- draw graphs of functions of the form $y = ax^2 + c$.
- correctly apply the terms vertex, axis of symmetry, y-intercept, parabola, opens upward, opens downward and congruent to functions of the form $y = ax^2 + c$.

PERIODS RECOMMENDED:

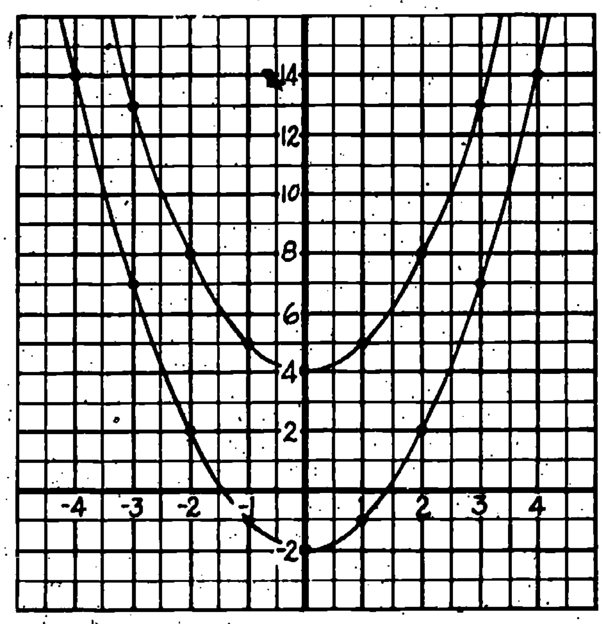
One

KEY--PROBLEM SET 2:

NOTE: STUDENT SCALES NEED NOT BE THE SAME AS THOSE FOUND HERE.

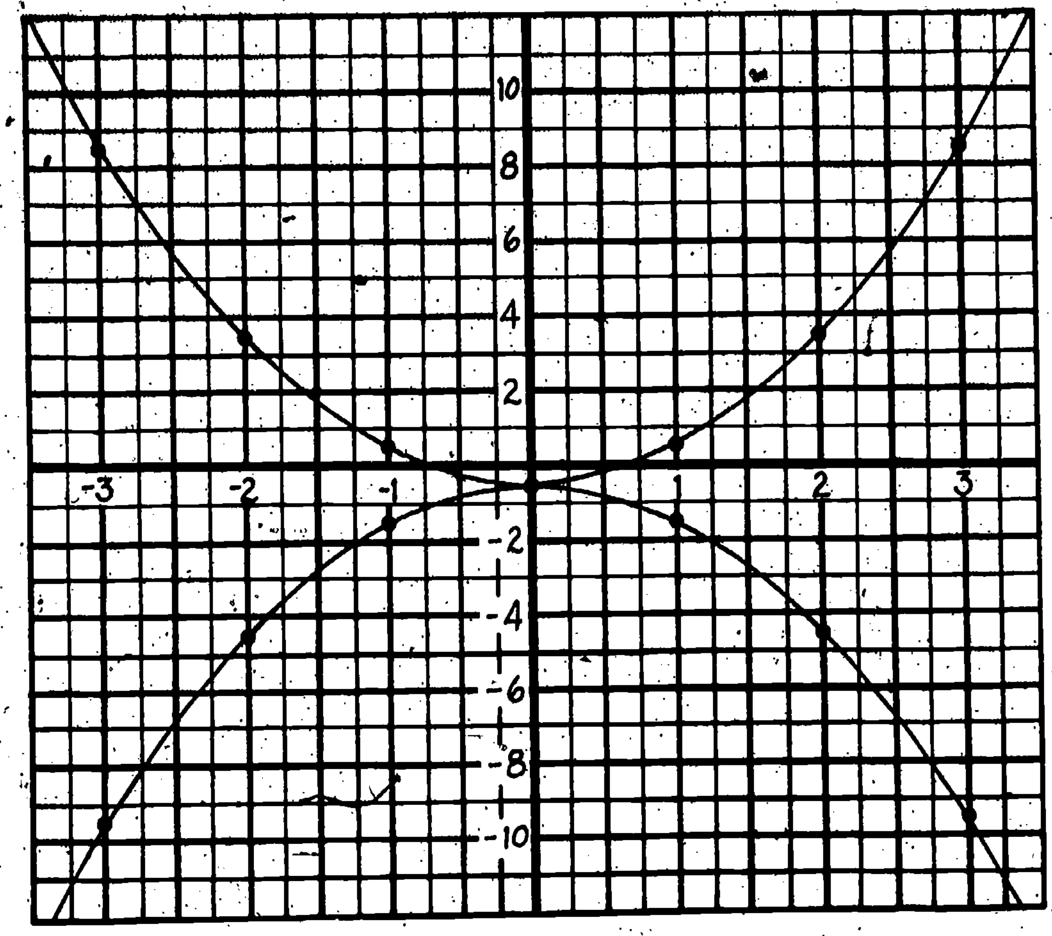
1.

x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2 + 4$	20	13	8	5	4	5	8	13	20
$y = x^2 - 2$	14	7	2	-1	-2	-1	2	7	14



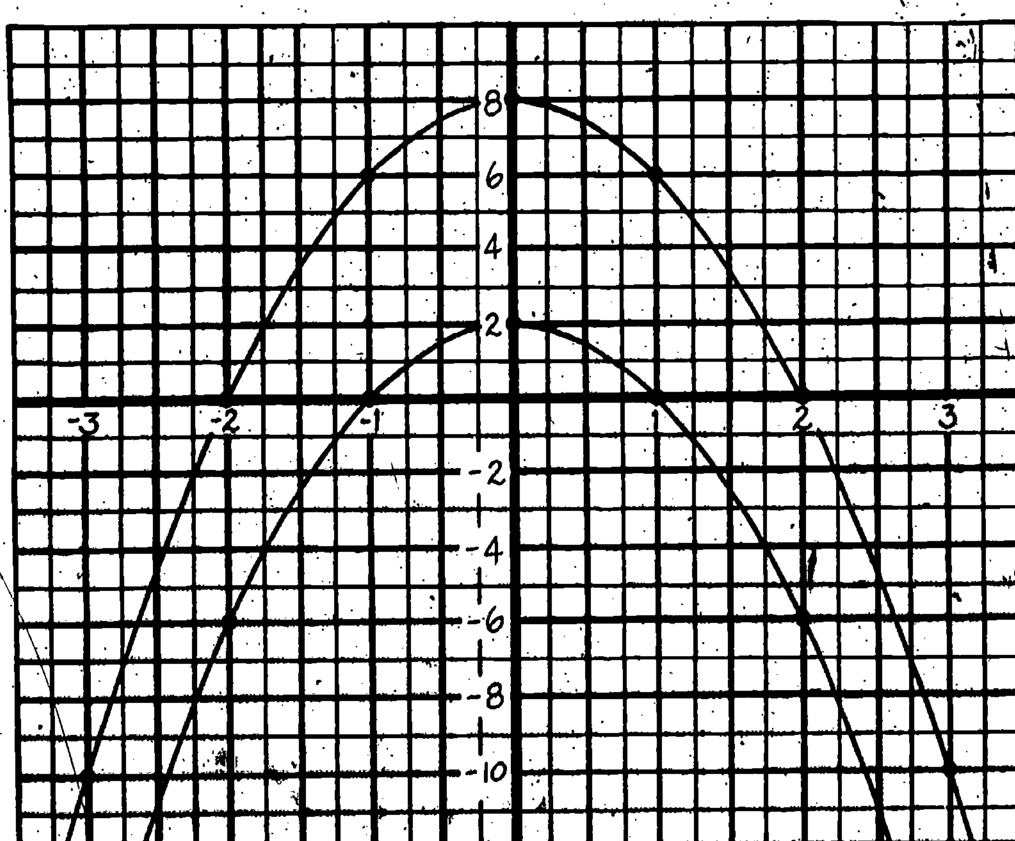
2.

x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2 - \frac{1}{2}$	$15\frac{1}{2}$	$8\frac{1}{2}$	$3\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$	$8\frac{1}{2}$	$15\frac{1}{2}$
$y = -x^2 - \frac{1}{2}$	$-16\frac{1}{2}$	$-9\frac{1}{2}$	$-4\frac{1}{2}$	$-1\frac{1}{2}$	$-\frac{1}{2}$	$-1\frac{1}{2}$	$-4\frac{1}{2}$	$-9\frac{1}{2}$	$-16\frac{1}{2}$



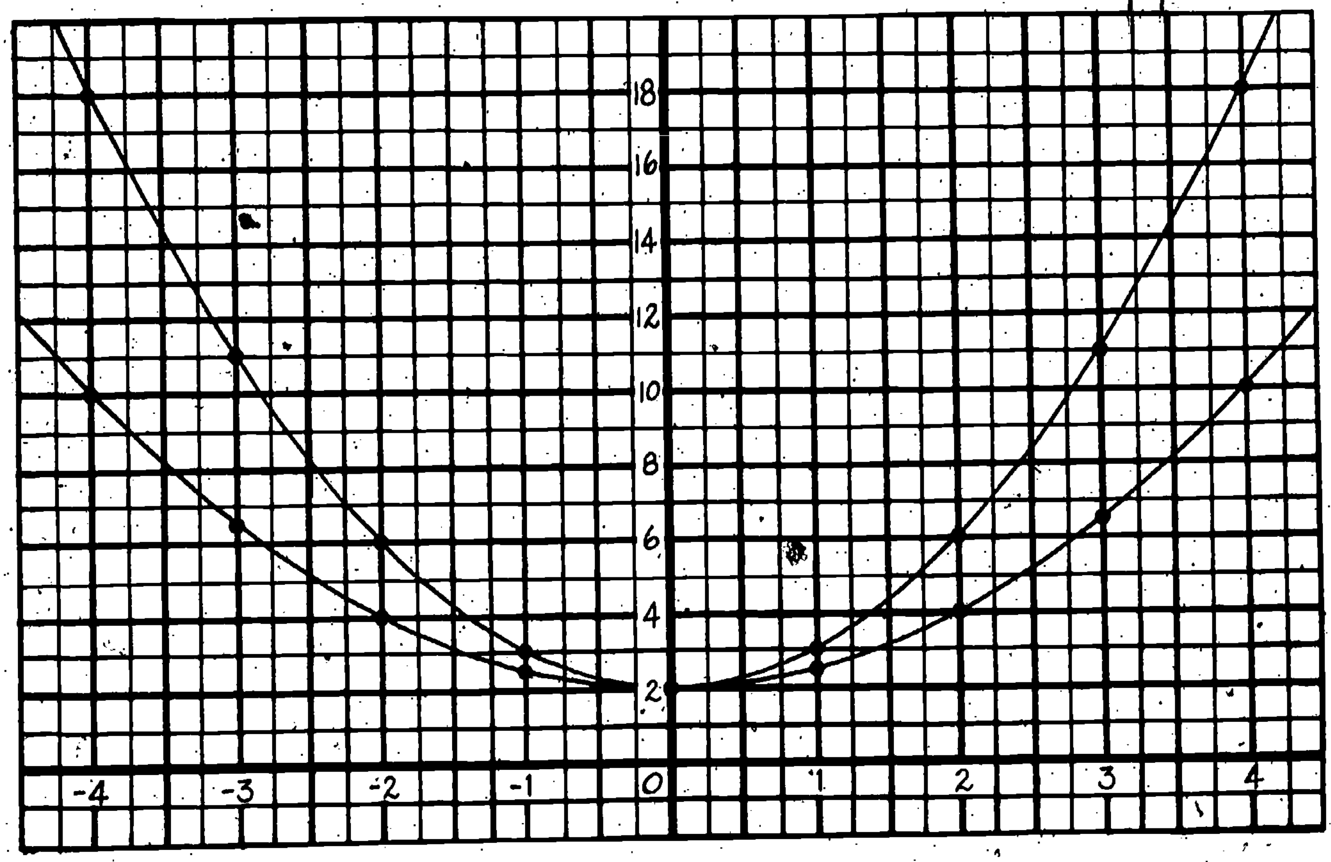
3.

x	-3	-2	-1	0	1	2	3
$y = -2x^2 + 8$	-10	0	6	8	6	0	-10
$y = -2x^2 + 2$	-16	-6	0	2	0	-6	-16



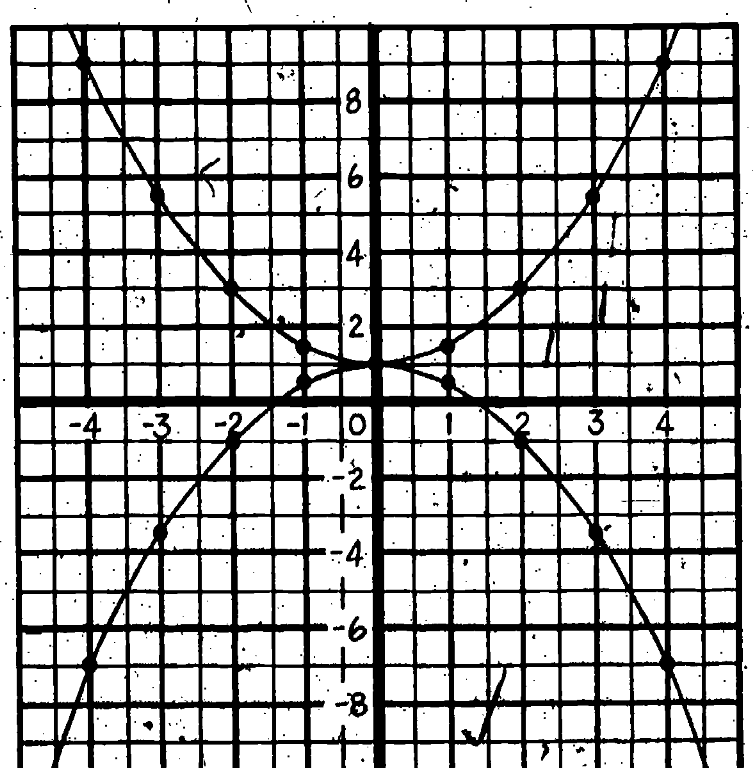
4.

x	-4	-3	-2	-1	0	1	2	3	4
$y = x^2 + 2$	18	11	6	3	2	3	6	11	18
$y = \frac{1}{2}x^2 + 2$	10	$6\frac{1}{2}$	4	$2\frac{1}{2}$	2	$2\frac{1}{2}$	4	$6\frac{1}{2}$	10



5.

x	-4	-3	-2	-1	0	1	2	3	4
$y = \frac{1}{2}x^2 + 1$	9	$5\frac{1}{2}$	3	$1\frac{1}{2}$	1	$1\frac{1}{2}$	3	$5\frac{1}{2}$	9
$y = -\frac{1}{2}x^2 + 1$	-7	$-3\frac{1}{2}$	-1	$\frac{1}{2}$	1	$\frac{1}{2}$	-1	$-3\frac{1}{2}$	-7



6. y-intercept: 0 vertex: (0, 0)

7. 1 (0, 1)

8. −5 (0, −5)

9. $-\frac{1}{6}$ $(0, -\frac{1}{6})$

10. 55 (0, 55)

11. 81 (0, 81)

12. $-\frac{1}{10}$ $(0, -\frac{1}{10})$

13. downward

14. b

15. a

16. $y = -8x^2 + 7$

LESSON 3: GRAPHING FUNCTIONS OF THE FORM $y = a(x - k)^2 + p$

OBJECTIVES:

The student will:

- draw graphs of functions of the form $y = a(x - k)^2 + p$.
- correctly apply the terms vertex, axis of symmetry, parabola, opens upward, opens downward and congruent to functions of the form $y = a(x - k)^2 + p$.

PERIODS RECOMMENDED:

One or Two

KEY--PROBLEM SET 3:

1. k
2. p
3. a
4. k

5. a = 1 k = 1 p = 1 vertex: (1, 1) axis: x = 1

6. $a = -\frac{1}{2}$ k = 3 p = 2 vertex: (3, 2) axis: x = 3

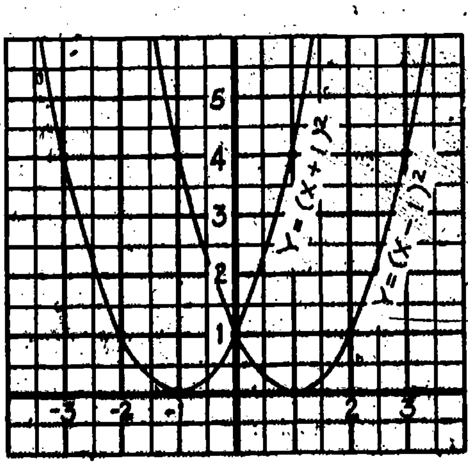
7. a = −2 k = 8 p = 5 vertex: (8, 5) axis: x = 8

8. a = 6 k = −5 p = −1 vertex: (−5, −1) axis: x = −5

9. a = −1 k = −7 p = −4 vertex: (−7, −4) axis: x = −7

10. a = −2 k = −10 p = 5 vertex: (−10, 5) axis: x = −10

11.



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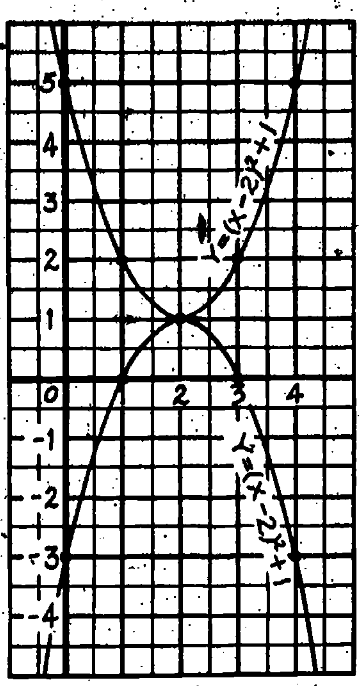

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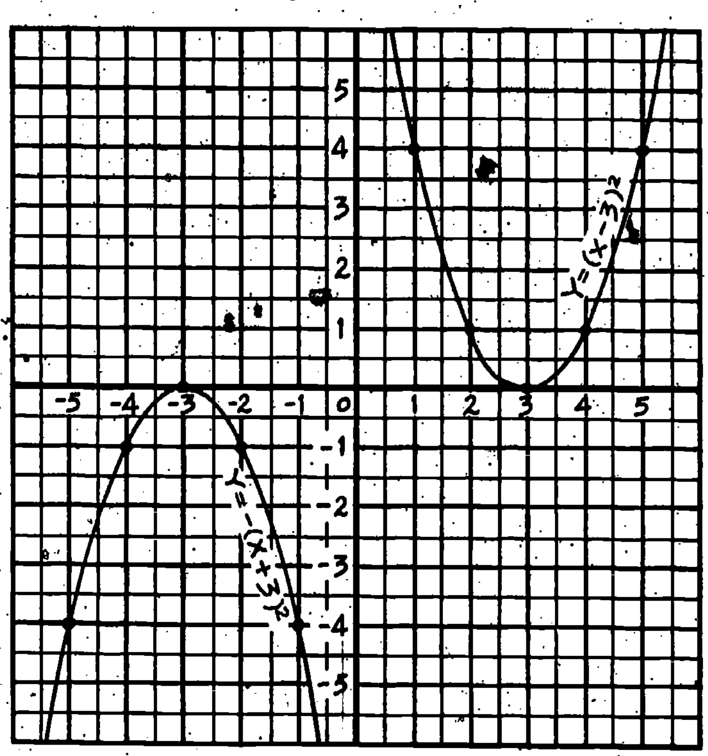

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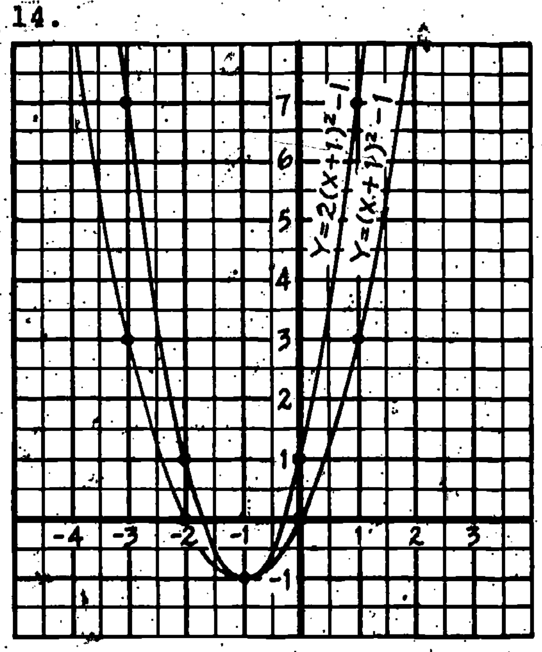

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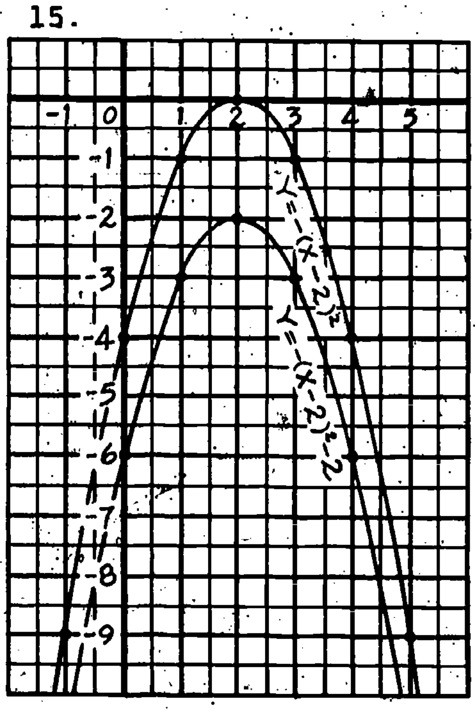

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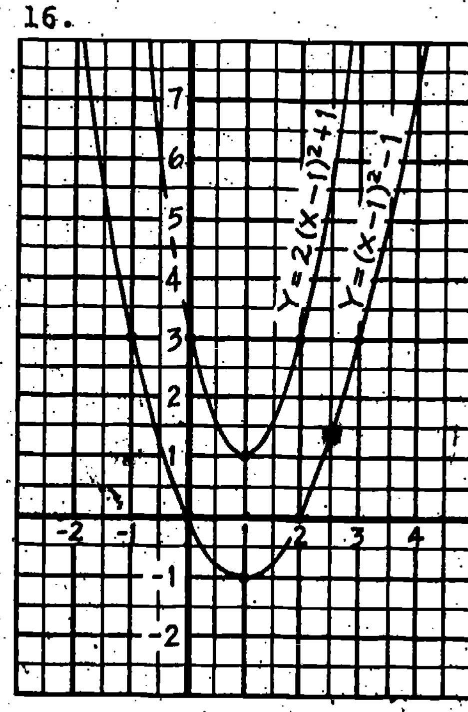


17. $y = x^2 - 1$

18. $y = (x - 2)^2 + 1$

19. $y = (x - 2)^2 - 1$

20. $y = (x + 1)^2 + 1$

21. $y = -2(x - 4)^2 - 3$

22. $y = 3x^2 - 60x + 307$

LESSON 4: COMPLETING THE SQUARE (PART I)

<u>OBJECTIVE:</u>

The student will put functions of the form

$$y = x^2 + bx + c$$

into the form

$$y = (x - k)^2 + p$$

<u>PERIODS RECOMMENDED:</u>

One

KEY--PROBLEM SET 4:

1. 49
2. c
3. -14
4. b
5. -7
6. 49
7. c
8. a. $y = (x + 9)^2$
   b. $y = x^2 + 18x + 81$
   c. 81
   d. 81
   e. yes
9. $y = (x - 4)^2$
10. $y = (x + 8)^2$
11. no
12. $y = (x + 25)^2$
13. no
14. no
15. no
16. no
17. 9
18. 36
19. 400
20. 25
21. $\frac{1}{100}$
22. $\frac{25}{4}$
23. $y = (x - 5)^2 - 22$
24. $y = (x + 11)^2 - 131$
25. $y = (x + 8)^2 - 64$
26. $y = (x - 15)^2 + 5$
27. $y = (x - \frac{3}{2})^2 - \frac{9}{4}$
28. $y = (x + \frac{7}{2})^2 - \frac{53}{4}$
29. $y = (x + \frac{11}{2})^2 - \frac{121}{4}$
30. $y = (x + \frac{5}{4})^2 + 1$

LESSON 5: COMPLETING THE SQUARE (PART II)

OBJECTIVE:

The student will put functions of the form

$$y = ax^2 + bx + c$$

into the form

$$y = a(x - k)^2 + p$$

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 5:

1. $12(x^2 + x) + 1$
2. $-3(x^2 + 3x) + 11$
3. $2(x^2 - 11x) + 13$
4. $-4(x^2 - \frac{7}{2}x) - 6$
5. $6(x^2 - \frac{1}{6}x)$
6. $5(x^2 + \frac{2}{5}x) + 7$
7. $8(x^2 + \frac{3}{2}x) - 100$
8. $-1000(x^2 + \frac{1}{10}x) - 10$
9. $2(x^2 + 2x + 1) - 2 + 6$
10. $-4(x^2 - 8x + 16) + 64 - 11$
11. $5(x^2 - 4x + 4) - 20 + 66$
12. $-3(x^2 + 20x + 100) + 300 + 200$
13. $7(x^2 - 10x + 25) - 175 - 60$
14. $-10(x^2 + 18x + 81) + 810$
15. $y = 3(x + 1)^2 - 2$
16. $y = -2(x - 2)^2 + 8$
17. $y = 4(x - 5)^2 + 19$
18. $y = -(x - 1)^2 - 1$
19. $y = 5(x + 3)^2 + 2$
20. $y = 2(x - 6)^2$
21. $y = 2(x - \frac{1}{2})^2$
22. $y = -3(x + \frac{1}{3})^2 + \frac{1}{3}$
23. $y = 4(x + \frac{1}{2})^2 + 2$
24. $y = \frac{1}{2}(x - 2)^2 - 2$
25. $y = -\frac{1}{3}(x - \frac{1}{2})^2 + \frac{1}{6}$

LESSON 6: MANIPULATION OF RADICALS

OBJECTIVE:

The student will use a square root table and the properties

$$\sqrt{s^2 r} = s\sqrt{r}$$

$$\sqrt{\frac{r}{s^2}} = \frac{\sqrt{r}}{s}$$

to find the square root of a number with three digits or less.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The ideas introduced in this lesson will be expanded in Lesson 13, which treats square roots of numbers expressed in scientific notation.

KEY--PROBLEM SET 6:

1.	3.606	13.	100	26.	61.81
2.	6.325	14.	100	27.	195.4
3.	28.57	15.	100	28.	1.944
4.	18.76	16.	4	29.	8.025
[illegible]	23.56	17.	5	30.	301.8
6.	45.93	18.	10	31.	6.496
7.	67.08	19.	10	32.	2.054
8.	85.56	20.	2.17	33.	.6496
9.	99.90	21.	6	34.	2.943
10.	35.36	22.	31	35.	.2943
11.	100	23.	5	36.	4.5 months
12.	100	24.	10	37.	3.3 months
		25.	10	38.	2.6 months

LESSON 7: QUADRATIC EQUATIONS

OBJECTIVE:

The student will solve quadratic equations by the method of square completion.

PERIODS RECOMMENDED:

One or Two

KEY--PROBLEM SET 7:

1.	c	10.	3, -2	19.	.697, 4.303
2.	T	11.	5	20.	-4.550, .550
3.	F	12.	NR	21.	$x = \frac{-1 \pm \sqrt{1 - 4c}}{2}$
4.	F	13.	1, -2	22.	$x = \frac{1 \pm \sqrt{1 + 4c}}{2}$
5.	F	14.	-2, -4	23.	$x = \frac{-b \pm \sqrt{b^2 - 4}}{2}$
6.	2, -4	15.	2, 5	24.	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
7.	3	16.	-6		
8.	1, 5	17.	.172, 5.828		
9.	NR	18.	-3.449, 1.449		

13

LESSON 8: THE QUADRATIC FORMULA

OBJECTIVE:

The student will:

solve quadratic equations by use of the quadratic formula.

use the discriminant to decide how many roots a given quadratic equation has.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 8:

1.	$b^2 - 4ac = 1$	2 roots	11.	4, -1
2.	-44	no roots	12.	-5, -8
3.	-3	no roots	13.	no real roots
4.	0	1 root	14.	$\frac{1}{3} = 0.333$ $\frac{1}{2} = 0.5$
5.	0	1 root	15.	-1, -0.8
6.	113	2 roots	16.	-4.792, -0.208
7.	40	2 roots	17.	-3.236, 1.236
8.	0	1 root	18.	-7.140, 0.140
9.	-35	no roots	19.	-5.76, 0.26
10.	48	2 roots		

LESSON 9: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 1 through 8.

PERIODS RECOMMENDED:

One or Two

OVERVIEW AND REMARKS:

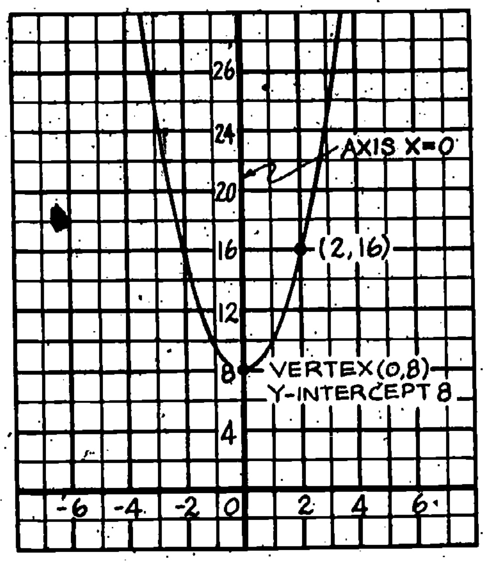
Before the students begin working on the problem set, you may want to review the following terms.

1. parabola
2. congruent parabolas
3. vertex
4. axis of symmetry
5. x-intercept, y-intercept
6. completing the square
7. quadratic equation
8. quadratic formula

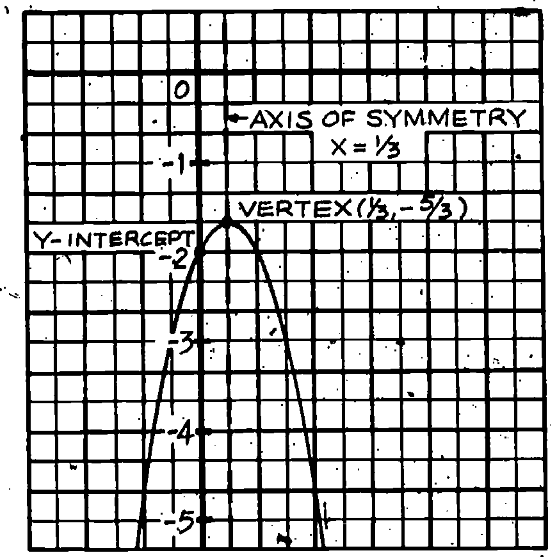
KEY--REVIEW PROBLEM SET 9:

1. a. $y = 2x^2 + 8$
   b. $y = -3(x - \frac{1}{3})^2 - \frac{5}{3}$
   c. $y = (x - \frac{7}{4})^2 - \frac{1}{16}$
   d. $y = -2(x + \frac{5}{4})^2 + \frac{17}{8}$

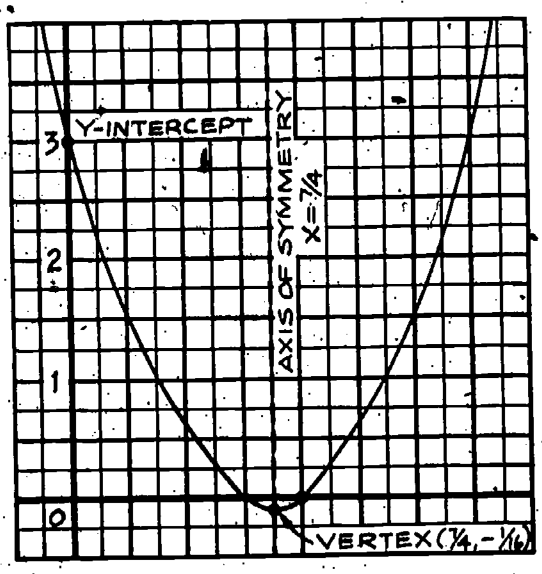
2. a.



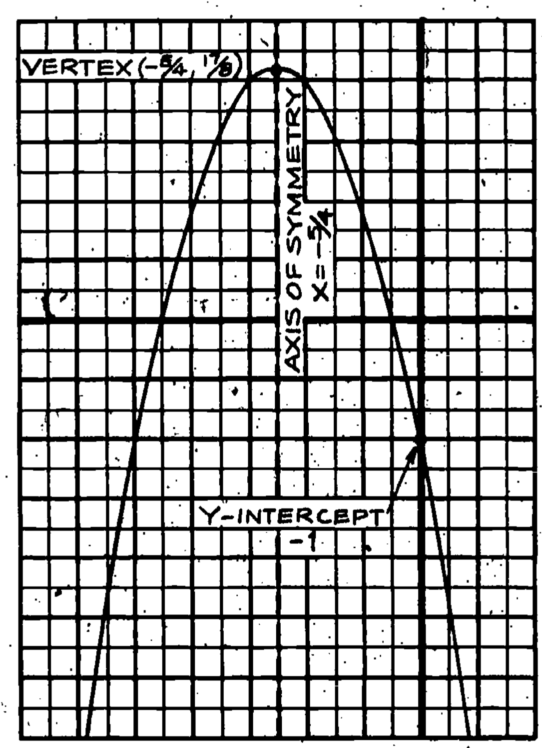
b.



c.



d.



3. a. $y = -6(x + 1)^2 - 9$
   $= -6x^2 - 12x - 15$
   b. none c. $y = -5x^2 + 11$

4. a. 25.81 b. 35.78 c. 70.71 d. .8832 e. 3.050 f. 9.132

5. a. 6, -3 b. 11, -1 c. 2, -1 d. none e. -4 f. 5.123, -3.123

6. a. $1, -\frac{1}{5}$ b. 1.236, -3.236 c. 6, -2

7. a. two b. none c. one

LESSON 10: SCIENTIFIC NOTATION AND MOLARITY

OBJECTIVES:

The student will:

- convert decimal numbers into scientific notation.
- convert numbers in scientific notation into decimal form.
- interpret concentration symbology (i.e., $[H^+]$).

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This lesson initiates the applied sequence of this unit. Lessons 10 through 19 will deal with chemical equilibria. However there will be no quadratic equations until Lesson 13. If the students are disappointed at having to wait so long to solve more quadratic equations, explain that all intermediate material must be mastered before the application can be understood.

Each lesson in the sequence 10 through 15 reviews one mathematics skill and one chemical application of the mathematics skill. All skills in this sequence are cummulative. For example, pH is reviewed early and then used in the balance of the sequence.

Throughout this chemistry-related sequence it would be very beneficial to be in day-to-day contact with the science teacher. For example, the method of determining a gram molecular weight is reviewed, although it is not an objective of the lesson. The science teacher, on the other hand, might want to take the opportunity to review this technique during science class on the day you teach this lesson.

Science Unit III deals with blood and the circulatory system. Mathematics Unit III is applied mainly to chemistry and the digestive system with only occasional forays into topics related to the circulatory system. We have focused on the digestive system because the equilibria are generally simpler and more immediately relevant.

KEY--PROBLEM SET 10:

(Answers in equivalent forms are also correct)

1. 6×10^{9}
2. 9×10^{-10}
3. 1.0004×10^{-4}
4. $6.473296734 \times 10^{9}$
5. 4.5×10^{1}
6. 4.03×10^{14}
7. 1.0000007×10^{-3}
8. 1.047×10^{4}
9. 6.34×10^{7}
10. $6.402 \quad 10^{-6}$
11. 160,000,000,000
12. .0000000000004
13. 3,141,592.654
14. 314.1592654
15. 6,023,000,000,000
16. .0078
17. .00000048
18. 50,000,000,000
19. .00000000007
20. 700,000

21. F

22. T

23. F

24. T

25. F

26. F

27. F

28. F

29. 6.023×10^{23}

30. The concentration of carbonate ions is three-tenths molar.

31. First, determine the molecular mass of the substance. Then, weigh out a mass in grams which is numerically equal to the molecular weight.

33. 1.25×10^{14}

34. 5.4×10^{9}

LESSON 11: A REVIEW OF MULTIPLICATION AND DIVISION IN SCIENTIFIC NOTATION AND THE pH SCALE

OBJECTIVES:

The student will:

- multiply and divide numbers in scientific notation.
- convert pH values to molarity values and the reverse.

PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

None of the material in this lesson is new; however, experience has shown that a review is worthwhile at this time. The material first appeared in Unit I and many students may have forgotten it by now.

It is especially important that the conversion from pH to $[H^+]$ and the reverse be mastered. This is an important skill for both the mathematics and science courses.

Finally we want to call your attention to Problems 41 through 47. These problems deal specifically with blood and should not be skipped even though they are at the end of the problem set.

KEY--PROBLEM SET 11:

1. a. $(x^a)(x^b) = x^{a+b}$ b. $\frac{x^a}{x^b} = x^{a-b}$

The answers need not be in the same form as given below.

2. 18×10^{16}
3. 24×10^{15}
4. 12×10^{-19}
5. 35×10^{26}
6. 8×10^{-20}
7. 4.8×10^{1}
8. 5×10^{14}
9. 8×10^{12}
10. 9×10^{-3}
11. 5×10^{4}
12. 4×10^{4}
13. a. 24×10^{17}
    b. 36×10^{9}
    c. 36×10^{6}
14. 6×10^{23}
15. 30×10^{21}
16. 18×10^{19}
17. 12×10^{18}
18. 6×10^{16}
19. 24×10^{15}
20. 48×10^{13}

21. 1×10^{-3} M	28. 10	33. 6.3×10^{-2}
22. 1×10^{-4} M	29. 2	34. 7.9×10^{-4}
23. 1×10^{-7} M	30. 1	35. 2×10^{-5}
24. 1×10^{-9} M	31. a. acidic	36. 7.9×10^{-6}
25. 1×10^{-11} M	b. basic	37. 1.6×10^{-6}
26. 6	32. a. -4.3 c. 5	38. 3.2×10^{-8}
27. 8	b. .7 d. 5×10^{-5} M	39. 1.25×10^{-9}
		40. 5×10^{-11}

41. $5.4 \times 10^{9}\ \frac{\text{erythrocytes}}{\text{ml blood}} \cdot 10^{3}\ \frac{\text{ml blood}}{\text{liter blood}} \cdot 5.5\ \frac{\text{liters blood}}{\text{adult male}} = 2.97 \times 10^{13}\ \frac{\text{erythrocytes}}{\text{adult male}}$

42. $2.7 \times 10^{8}\ \frac{\text{hemoglobin molecules}}{\text{erythrocyte}}$

43. $2.7 \times 10^{8}\ \frac{\text{hemoglobin molecules}}{\text{erythrocyte}} \cdot 2.97 \times 10^{13}\ \frac{\text{erythrocytes}}{\text{adult male}}$

$= 8.019 \times 10^{21}\ \frac{\text{hemoglobins}}{\text{adult male}}$

44. $2.3 \times 10^{6}\ \frac{\text{deaths}}{\text{second}}$

45. $2.3 \times 10^{6}\ \frac{\text{deaths}}{\text{second}} \cdot 60\ \frac{\text{sec}}{\text{min}} \cdot 60\ \frac{\text{min}}{\text{hr}} \cdot 24\ \frac{\text{hr}}{\text{day}} \cdot 1.8 \times 10^{4}\ \frac{\text{day}}{\text{50 yr}} \approx 3.6 \times 10^{15}\ \frac{\text{deaths}}{\text{50 yr}}$

46. $7\ \text{micrometers}\ \frac{1\ \text{mm}}{10^{3}\ \text{micrometers}} = 7 \times 10^{-3}\ \text{mm}$

47. $10\ \text{nanometers} \cdot \frac{1\ \text{micrometer}}{10^{3}\ \text{nanometers}} \cdot \frac{1\ \text{mm}}{10^{3}\ \text{micrometers}} = 10^{-5}\ \text{mm}$

LESSON 12: CHEMICAL EQUILIBRIUM AND SOLUBILITY PRODUCT

OBJECTIVES:

The student will:

- relate the concept of dynamic equilibrium to solubility products.
- determine whether or not a precipitate will form when given initial concentrations of reactants.
- identify the larger (or smaller) of two numbers written in scientific notation.

PERIODS RECOMMENDED:

One or Two

OVERVIEW AND REMARKS:

Once again, close cooperation with the science teacher would be very beneficial. For example, in Section 12-4 there is a list of issues that quite naturally arise in connection with solubility products and chemical equilibrium. Since these issues are beyond the ken of typical math teachers, it would be very helpful to you if you could get the science teacher to take a few minutes to discuss them.

Once again the problems which deal with the primary objective, predicting whether a precipitate forms, occur at the very end of the problem set. Do not overlook them.

A note of caution: the procedure described in this section for predicting the formation of precipitates applies only to binary ionic compounds. If a compound forms three or more ions when it dissolves, then a more elaborate approach is needed. For example, lead chloride dissolves as described by the equilibrium

$$PbCl_2 \rightleftharpoons Pb^{++} + Cl^- + Cl^-$$

and the concentrations at equilibrium are governed by the solubility product equation

$$[Pb^{++}] \times [Cl^-]^2 = K_{sp}$$

Notice that the $[Cl^-]$ is squared. Since we do not wish to deal with the mathematical implications of this complicating feature, we will treat only compounds that form two ions upon dissociation. Those teachers who wish to generate more problems must keep this restriction in mind. More problems may be generated by referring to the "solubility product" table in the CRC Handbook of Chemistry and Physics.

KEY--PROBLEM SET 12:

1. False
2. False
3. True
4. False
5. True
6. True
7. False

Note: There are other ways to correct statements #8 through #15.

8. The symbol $[Ca^{++}]$ is read, "The concentration of calcium ion."

9. If the product of $[Ca^{++}]$ and $[SO_4^{--}]$ is less than the solubility product, then all of the $CaSO_4$ is dissolved.

10. The product $[Ca^{++}] \times [SO_4^{--}]$ is an example of solubility product.

11. The symbol K_{sp} is an abbreviation for solubility product.

12. A saturated solution is in equilibrium.

13. The chemical equation

$$CaSO_4\downarrow \rightleftharpoons Ca^{++} + SO_4^{--}$$

signifies that the reaction is reversible.

14. $(2 \times 10^{-1})(6 \times 10^{-2}) = 12 \times 10^{-3}$

15. The interpretation of $[Ba^{++}] = .1\ M$ is, "The concentration of barium ions is one-tenth of a mole per liter of solution."

16. A mole of water has approximately 6.023×10^{23} molecules.

17. $.18 \times 10^{-7} < .9 \times 10^{-7}$

18. $.47 \times 10^{-8} > 4.1 \times 10^{-8}$

19. $.061 \times 10^{-3} < 18 \times 10^{-3}$

20. $.07 \times 10^{-8} < .8 \times 10^{-8}$

21. $27 \times 10^{-8} < 30 \times 10^{-8}$

22. $.38 \times 10^{-8} < 9 \times 10^{-8}$

23. $3 \times 10^{-3} > .005 \times 10^{-3}$

24. $60 \times 10^{-6} < 140 \times 10^{-6}$

25. $210 \times 10^{-4} > 1.9 \times 10^{-4}$

26. $430 \times 10^{-11} > .2 \times 10^{-11}$

27. Problems 19, 23, 25, 26

28. a. .1 M b. .2 M c. $.02 = 2 \times 10^{-2}$ d. greater e. yes

29. a. 1×10^{-8} M b. .1 M c. 1×10^{-9} d. yes

30. $[Ag^+] \times [Br^-] = 6 \times 10^{-15}$

$$6 \times 10^{-15} < K_{sp}$$

or $$6 \times 10^{-15} < 4 \times 10^{-13}$$

LESSON 13: APPLIED QUADRATIC EQUATIONS

OBJECTIVE:

The student will calculate the concentrations of ions in solutions of slightly soluble compounds.

PERIODS RECOMMENDED:

Two

SUPPLEMENTARY REFERENCE:

Mary E. Clark: Contemporary Biology, W.B. Saunders, Philadelphia, 1975

OVERVIEW AND REMARKS:

In this lesson the mathematics takes a back seat to the chemistry. Perhaps the most important idea in it is lead poisoning. Even though lead compounds are only slightly soluble, the amount that dissolves (i.e., the part that dissociates into Pb^{++} and anions) is dangerous. Typically lead poisoning develops after a long period of low-level exposure. The body absorbs a small amount of the lead to which it is exposed. Once absorbed it tends to stay put; therefore, a dangerous level of lead in the body can be built up slowly over a long period of low-level exposure.

Ingestion of lead-based paint flakes is a serious public health problem in many of the nation's central cities. This is the major cause of lead poisoning by ingestion. In addition to ingestion, much lead is absorbed by way of the lungs. It is in the air we breathe and comes mainly from leaded gasoline. Lead from gasoline additives also finds its way into our food and water, thus compounding the problem.

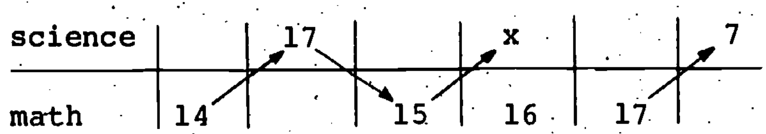
If you or your students want to know more about this topic the supplementary reference has a great deal of good information.

Starting with Section 13, we use "z" to represent an unknown concentration instead of the more traditional "x". This is done to eliminate any confusion with the multiplication symbol "x" used in scientific notation.

PREPARATION FOR FUTURE LESSONS:

Lessons 15 and 17 are designed to be taught in coordination with Science Lessons X and Y. These science lessons provide concrete experiences relating to the material in the relevant mathematics lessons. These science lessons are of the "floating" variety. They do not fit within the flow of science material, but are designed to be inserted at the appropriate time for mathematics. You should alert the science teacher that you are approaching the time of need and determine together how to coordinate your classes to achieve optimal sequencing.

Science Lessons 17, 18, X and Y deal with acids and buffers. An ideal sequencing would be approximately as follows.



The important thing will be for Science Lesson X to be used right after Mathematics Lesson 15 and Science Lesson Y to follow Mathematics Lesson 17.

KEY--PROBLEM SET 13:

1. 4×10^{-1}
2. $16 \times 10 = 1.6 \times 10^{2}$
3. $\sqrt{10} \times 10^{-1} \approx 3.162 \times 10^{-1}$
4. 5×10^{8}
5. $\sqrt{2} \times 10^{-2} \approx 1.414 \times 10^{-2}$
6. 9×10^{4}
7. 13×10^{-5}
8. $20 \times 10 = 2 \times 10^{2}$
9. 7×10^{-12}
10. 11×10^{-39}
11. 3×10^{-77}
12. a. one
    b. z
    c. $z \approx 8.8 \times 10^{-4}$
    d. $[Tar^{--}] \approx 8.8 \times 10^{-4}$ M

Answers in any equivalent form are acceptable for Problems 13 through 23.

13. $[Pb^{++}] \approx 13.3 \times 10^{-8}$ M
14. $[Mg^{++}] \approx 5.1 \times 10^{-3}$ M
15. $Ca^{++} \approx 8.9 \times 10^{-5}$ M
16. 18×10^{-15} M
17. 6.3×10^{-27} M (about one molecule in 300 liters)
18. 6.0×10^{-15} M
19. 13×10^{-6} M
20. 9.3×10^{-6} M
21. 1.7×10^{-13} M
22. 18×10^{-8} M
23. 1×10^{-4} M
24. a. less
    b. small, small
    c. vermillion, H_gS
    d. $PbSO_4$
    e. lead
25. Any four carbonated beverages
26. a. >
    b. $PbSO_4$ will precipitate
    c. It will tend to lower the $[Pb^{++}]$.

LESSON 14: WEAK ACIDS AND DISSOCIATIONS CONSTANTS

OBJECTIVES:

The student will:

- calculate the dissociation constant for a particular weak acid when given the equilibrium pH and the initial concentration.
- add and subtract numbers expressed in scientific notation.

PERIODS RECOMMENDED:

Two or Three

OVERVIEW AND REMARKS:

This lesson and Lesson 15 prepare for Lesson 16 in which equilibrium problems will be considered that require the quadratic formula for their solution. If you think Lesson 14 will be frustratingly difficult for your students, Lessons 14 through 18 may be omitted. No subsequent mathematics or science material depends on them. Furthermore, quadratic equations have actually been applied in Lesson 13, although the quadratic formula has not yet been required for the solution of an equilibrium problem.

We have included the Lessons 14 through 18 to demonstrate actual applications of the quadratic formula to real biomedical situations. However, we realize that the level of the material may be above capabilities of some students. If this is true for your students, you may still want to spend a class period or two scanning and discussing the sequence--the objective being to convince them that the quadratic formula is actually required for the solution of certain kinds of real problems.

If you elect to delete part or all of the lessons, be sure to notify your colleague in Biomedical Science since it will affect the student's mastery of equilibrium concepts needed for Science Lessons X and Y.

KEY--PROBLEM SET 14: All answers may be in equivalent forms.

1. $K = \dfrac{(3 \times 10^{-5})(3 \times 10^{-5})}{5 \times 10^{-5}} = 1.8 \times 10^{-5}$

2. $K = \dfrac{(3 \times 10^{-4})(3 \times 10^{-4})}{5 \times 10^{-3}} = 1.8 \times 10^{-5}$

3. $K = \dfrac{(3 \times 10^{-3})(3 \times 10^{-3})}{5 \times 10^{-1}} = 1.8 \times 10^{-5}$

4. $K = \dfrac{(1.4 \times 10^{-2})(1.4 \times 10^{-2})}{1.4} = 1.4 \times 10^{-4}$

5. $K = \dfrac{(1.4 \times 10^{-3})(1.4 \times 10^{-3})}{1.4 \times 10^{-2}}$

6. $K = \dfrac{(1.4 \times 10^{-4})(1.4 \times 10^{-4})}{1.4 \times 10^{-4}} = 1.4 \times 10^{-4}$

7. $K = \dfrac{(9 \times 10^{-2})(9 \times 10^{-2})}{1.8} = 4.5 \times 10^{-3}$

8. $K = \dfrac{(3 \times 10^{-2})(3 \times 10^{-2})}{2 \times 10^{-1}} = 4.5 \times 10^{-3}$

9. $K = \dfrac{(6 \times 10^{-3})(6 \times 10^{-3})}{8 \times 10^{-3}} = 4.5 \times 10^{-3}$

10. $K = \dfrac{(1.2 \times 10^{-3})(1.2 \times 10^{-3})}{3.2 \times 10^{-4}} \doteq 4.5 \times 10^{-3}$

11. -8×10^{-4}

12. 2.6×10^{-6}

13. 3.21×10^{-11}

14. 350.4×10^{-5}

15. 2.02×10^{-1}

16. 1.47×10^{-5}

17. 1×10^{-13}

18. -4.6×10^{-9}

19. -7×10^{-9}

20. 5×10^{-5}

21. a. see graph

b. see graph

c. 1×10^{-4} M

d. 85×10^{-5} M

e. 18.5×10^{-5} M

f. 13×10^{-5} M

g. 94×10^{-5} M

h. 4×10^{-5} M

*22. a. see graph

b. see graph

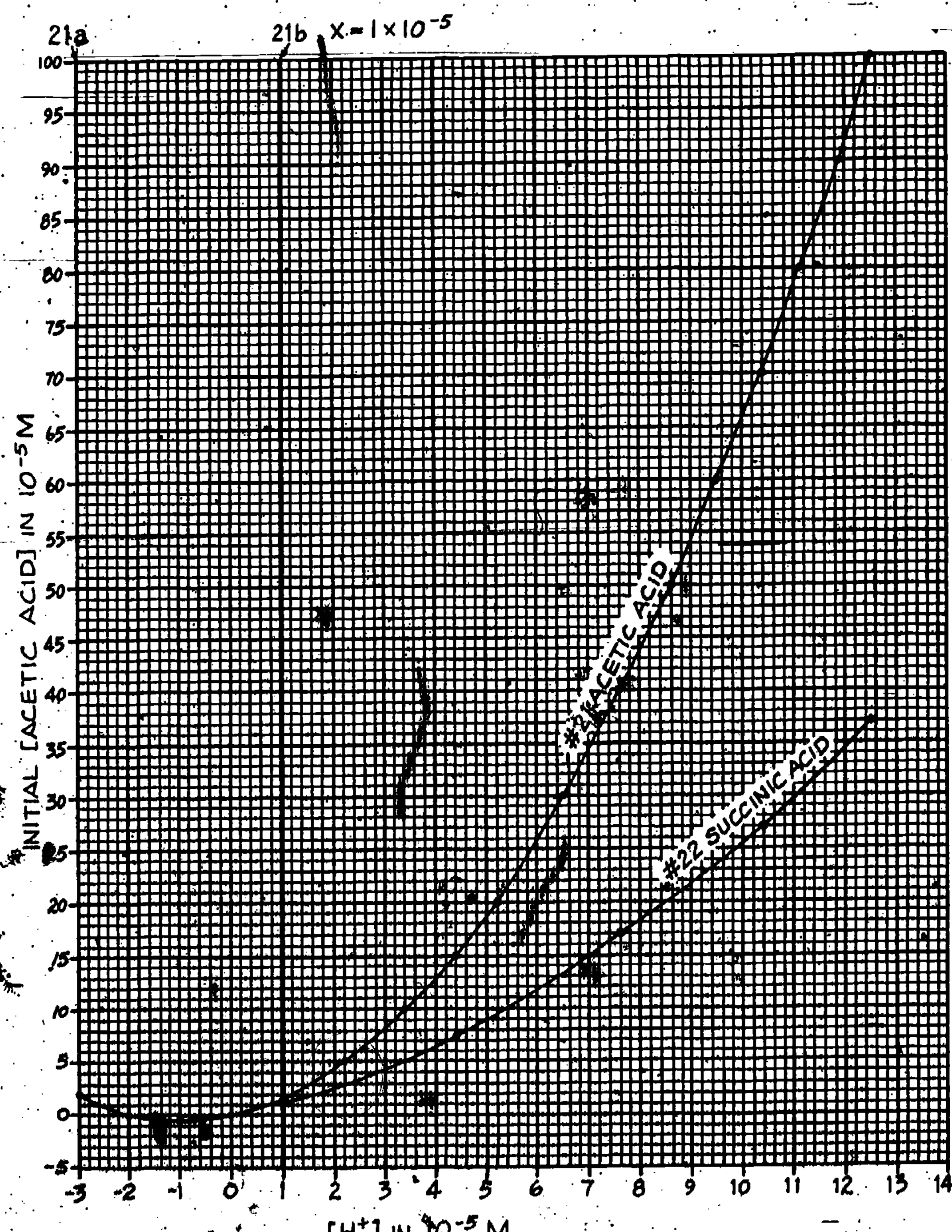
c. less

d. less

e. succinic

f. stronger

g. smaller, more easily



23. a. $[H^+] \approx 4 \times 10^{-4}$ M

b. same, $[NO_2^-] = 4 \times 10^{-4}$ M

c. initial

d. 4×10^{-4} M

e. $K = 4 \times 10^{-4}$

24. a. $[H^+] \approx 2.5 \times 10^{-3}$

b. $[Sac^-] \approx 2.5 \times 10^{-3}$

c. $[HSac] = 2.5 \times 10^{-4}$

d. $K = 2.5 \times 10^{-2}$

25. $K = 1.6 \times 10^{-5}$

26. $K = 3.2 \times 10^{-5}$

LESSON 15: CALCULATING THE INITIAL CONCENTRATION FOR A PARTICULAR $[H^+]$

OBJECTIVE:

The student will calculate the initial concentration required to achieve a particular $[H^+]$.

PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

The pattern of the problems in this section differs in one important detail from those found in Lesson 14. The initial [Acid] is an unknown instead of a given. To find the initial [Acid] it is necessary to solve a linear equation.

The reasoning required to fill the tables is parallel with the reasoning required in Lesson 14. It rests on two main ideas:

1. Equilibrium $[H^+]$ = Equilibrium $[Anion^-]$
2. Initial [HAnion] = Equilibrium $[Anion^-]$ + Equilibrium [HAnion]

Science Lesson X (Unit III) should be coordinated with this lesson. It will provide direct laboratory support for the concepts taught in this lesson. Students will calculate the initial concentrations required to achieve certain pH's and then actually make up the solutions. For maximum effectiveness Science Lesson X should occur shortly after work on this lesson has begun in mathematics.

KEY--PROBLEM SET 15:

1. a.

Concentrations (M)		H^+	NO_2^-	HNO_2
	Initial	0	0	$10^{-3} + z$
	Equilibrium	10^{-3}	10^{-3}	z

b. $z = 2.5 \times 10^{-3}$ M

c. Initial $[HNO_2] = 3.5 \times 10^{-3}$ M

2\. a.

		H^+	NO_2^-	HNO_2
Concentrations (M)	Initial	0	0	$10^{-2} + z$
	Equilibrium	10^{-2}	10^{-2}	z

b. $z = .25$ M

c. Initial $[HNO_2] = .26$ M

$= 2.6 \times 10^{-1}$ M

3\. Initial $[HNO_2] = 6.75 \times 10^{-2}$ M

4\. Initial $[H_2Mal] = 4.04$ M

5\. Initial $[HUr] = 5.226$ M

6\. Initial $[H_2Tar] = 7.5 \times 10^{-4}$ M

7\. Initial $[HBenz] = 1.26 \times 10^{-4}$ M

8\. Initial $[HBut] \simeq 2.17 \times 10^{-4}$ M

LESSON 16: CHEMICAL EQUILIBRIA AND THE QUADRATIC FORMULA

OBJECTIVE:

The student will determine the coefficients for the quadratic equation that results when the equilibrium concentrations are calculated from the initial [HAnion].

PERIODS RECOMMENDED:

Two or Three

OVERVIEW AND REMARKS:

The answers to Problems 4 through 10 in Problem Set 16 are used in Problem Set 17. The corresponding problem number in Problem Set 17 completes the solution of the problem begun in this lesson.

KEY--PROBLEM SET 16:

1\. a.

		H^+	H_2Cit^-	H_3Cit
Concentrations (M)	Initial	0	0	10^{-3}
	Equilibrium	z	z	$10^{-3} - z$

b. $9 \times 10^{-4} = \dfrac{z \cdot z}{10^{-3} - z}$

c. $0 = z^2 + (9 \times 10^{-4})z - (9 \times 10^{-7})$

d. $a = 1$, $b = 9 \times 10^{-4}$, $c = -9 \times 10^{-7}$

25

2. a.

		H^+	H_2Cit^-	H_3Cit
Concentrations (M)	Initial	0	0	5×10^{-3}
	Equilibrium	z	z	$5 \times 10^{-3} - z$

b. $9 \times 10^{-4} = \frac{z \cdot z}{(5 \times 10^{-3}) - z}$

c. $0 = z^2 + (9 \times 10^{-4})z - (4.5 \times 10^{-6})$

d. $a = 1,\ b = 9 \times 10^{-4},\ c = -4.5 \times 10^{-6}$

3. a.

		H^+	H_2Cit^-	H_3Cit
Concentrations (M)	Initial	0	0	10^{-2}
	Equilibrium	z	z	$10^{-2} - z$

b. $0 = z^2 + (9 \times 10^{-4})z - (9 \times 10^{-6})$

c. $a = 1,\ b = 9 \times 10^{-4},\ c = -9 \times 10^{-6}$

4. $a = 1,\ b = 8 \times 10^{-5},\ c = -8 \times 10^{-9}$

5. $a = 1,\ b = 3.2 \times 10^{-3},\ c = -3.2 \times 10^{-5}$

6. $a = 1,\ b = 6.4 \times 10^{-5},\ c = -6.4 \times 10^{-9}$

7. $a = 1,\ b = 4 \times 10^{-4},\ c = -4 \times 10^{-8}$

8. $a = 1,\ b = 4.6 \times 10^{-3},\ c = -2.3 \times 10^{-5}$

9. $a = 1,\ b = 2.5 \times 10^{-2},\ c = -6.25 \times 10^{-4}$

10. $a = 1,\ b = 1.4 \times 10^{-4},\ c = -4.2 \times 10^{-6}$

LESSON 17: THE SOLUTION OF QUADRATIC EQUATIONS RELATING TO CHEMICAL EQUILIBRIA

OBJECTIVE:

The student will complete the solution of quadratic equations that were developed in Lesson 16. These equations occur when the equilibrium concentrations are calculated from initial concentrations.

PERIODS RECOMMENDED:

Two or Three

OVERVIEW AND REMARKS:

Science Lesson Y (Unit III) should be coordinated with this lesson. It provides direct laboratory support of the concepts developed here. Students will confirm their theoretically calculated predictions by actually making solutions and then checking the pH. For maximum effectiveness Science Lesson Y should begin shortly after work on this lesson is begun.

Since Problem Set 17 uses the answers to Problems 4 through 10 of Problem Set 16, be sure that the correct answers to these problems are available to all students before work on Problem Set 17 begins. Otherwise students with the wrong answers to Problem Set 16 will waste their time in Problem Set 17.

You might wish to give the students the following set of rules that may be used to test the reasonableness of their answers.

1. If the initial [HAnion] = 2K, then the acid will be 50% dissociated at equilibrium. Therefore, at equilibrium

$$[H^+] = K$$
$$[Anion^-] = K$$
$$[HAnion] = K$$

This may be easily checked.

		H^+	$Anion^-$	HAnion
Concentration	Initial	0	0	2K
	Equilibrium	K	K	2K - K = K

and

$$\frac{[H^+][Anion]}{[HAnion]} = K$$

$$\frac{K \cdot K}{K} = K$$

2. If the initial [HAnion] < 2K, then the acid will be more than 50% dissociated at equilibrium.

3. If the initial [HAnion] > 2K, then the acid will be less than 50% dissociated at equilibrium.

KEY--PROBLEM SET 17

1. a. Each nitrite ion in solution used to be paired with an H^+; therefore, $[H^+] = [NO_2^-]$.

b. Each NO_2^- in solution comes from former HNO_2 molecule; therefore

$$[NO_2^-] + [HNO_2] = \text{Initial } [HNO_2]$$

or, equivalently,

$$[HNO_2] = 10^{-3} - [NO_2^-]$$

c.

		H^+	NO_2^-	HNO_2
Concentrations (M)	Initial	0	0	10^{-3}
	Equilibrium	z	z	$10^{-3} - z$

d. $4.6 \times 10^{-4} = \frac{z \cdot z}{10^{-3} - z}$

e. $0 = z^2 + (4.6 \times 10^{-4})z - (4.6 \times 10^{-7})$

f. $a = 1,\ b = 4.6 \times 10^{-4},\ c = -4.6 \times 10^{-7}$

g. $z \approx 4.9 \times 10^{-4}$

$z \approx -9.5 \times 10^{-4}$

h. Negative concentrations are positively meaningless.

i. $[H^+] \approx 4.9 \times 10^{-4}$

$[NO_2^-] \approx 4.9 \times 10^{-4}$

$[HNO_2] \approx 5.1 \times 10^{-4}$

2\. a.

		H^+	$HTar^-$	H_2Tar
Concentrations (M)	Initial	0	0	2×10^{-2}
	Equilibrium	z	z	$2 \times 10^{-2} - z$

b. $0 = z^2 + 10^{-3}z - (2 \times 10^{-5})$

c. $z = 4 \times 10^{-3}$

d. $[H^+] = 4.0 \times 10^{-3}$ M

$[HTar^-] = 4.0 \times 10^{-3}$ M

$[H_2Tar] = 1.6 \times 10^{-2}$ M

3\. a.

		H^+	$HTar^-$	H_2Tar
Concentrations (M)	Initial	0	0	.2
	Equilibrium	z	z	.2 - z

b. $0 = z^2 + (10^{-3})z - (2 \times 10^{-4})$

c. $z = 1.4 \times 10^{-2}$ M

d. $[H^+] = 1.4 \times 10^{-2}$ M

$[HTar^-] = 1.4 \times 10^{-2}$ M

$[H_2Tar] = .186$ M

4\. $[H^+] \approx 5.8 \times 10^{-5}$ M

$[HAsc^-] \approx 5.8 \times 10^{-5}$ M

$[H_2Asc] \approx 4.2 \times 10^{-5}$ M

5\. $[H^+] \approx 4.3 \times 10^{-3}$ M

$[Pyr^-] \approx 4.3 \times 10^{-3}$ M

$[HPyr] \approx 5.7 \times 10^{-3}$ M

6\. $[H^+] \approx 5.4 \times 10^{-5}$ M

$[HSuc^-] \approx 5.4 \times 10^{-5}$ M

$[H_2Suc] \approx 4.6 \times 10^{-5}$ M

7\. $[H^+] \approx 8.3 \times 10^{-5}$ M

$[HMal^-] \approx 8.3 \times 10^{-5}$ M

$[H_2Mal] \approx 1.7 \times 10^{-5}$ M

8\. $[H^+] \approx 3.0 \times 10^{-3}$ M

$[Gly^-] \approx 3.0 \times 10^{-3}$ M

$[HGly] \approx 2.0 \times 10^{-3}$ M

9\. $[H^+] \approx 1.5 \times 10^{-2}$ M

$[Sac^-] \approx 1.5 \times 10^{-2}$ M

$HSac^-$ 1.0 10^{-2} M

10\. $[H^+] \approx 2.0 \times 10^{-3}$ M

$[Lac^-] \approx 2.0 \times 10^{-3}$ M

$[HLac] \approx 2.8 \times 10^{-2}$ M

11. a.

		H^+	HCO_3^-	CO_2
Concentrations (M)	Initial	0	0	.15
	Equilibrium	z	z	.15 - z

b. $0 = z^2 + (4 \times 10^{-7})z - (6 \times 10^{-8})$

c. $b^2 = 1.6 \times 10^{-13}$

d. $-4ac = 2.4 \times 10^{-7}$

e. $-4ac = 2.4 \times 10^{-7}$ is more than a million times larger than 1.6×10^{-13}.

f. $\sqrt{2.4 \times 10^{-7}} \approx 4.90 \times 10^{-4}$

g. $[H^+] \approx 2.4 \times 10^{-4}$ M

$[HCO_3^-] \approx 2.4 \times 10^{-4}$ M

$[CO_2] \approx .14976$ M

LESSON 18: DISSOCIATION CONSTANTS AND THE BIOCHEMICAL BREW

OBJECTIVES:

The student will calculate the equilibrium $[H^+]$, [Anion$^-$] and [HAnion] when given the initial $[H^+]$ and [HAnion].

PERIODS RECOMMENDED:

Two or Three

OVERVIEW AND REMARKS:

If your students had trouble with Lesson 17, you may wish to omit this lesson and move on to Lesson 19, the review lesson. The situation dealt with in this lesson is a little more realistic regarding equilibria, but also a little more complicated.

In this lesson and earlier ones we have dealt with certain acids of the type H_2Anion and H_3Anion. We have always assumed that only one H^+ dissociated from a given molecule. This is not strictly true, but the contribution to the total $[H^+]$ made by the second and third H^+ dissociating is generally very insignificant in the kinds of situations that we have been analyzing. However, in solutions of the salts of these anions, this effect becomes important.

KEY--PROBLEM SET 18:

1. a.

		H^+	H_2Cit^-	H_3Cit
Concentrations (M)	Initial	1.3×10^{-3}	0	5×10^{-2}
	Equilibrium	$(1.3 \times 10^{-3}) + z$	z	$(5 \times 10^{-2}) - z$

1. b. $7 \times 10^{-4} = \frac{\{(1.3 \times 10^{-3}) + z\}(z)}{(5 \times 10^{-2}) - z}$

c. $(7 \times 10^{-4})\{(5 \times 10^{-2}) - z\} = z(1.3 \times 10^{-3}) + z^2$

$(35 \times 10^{-6}) - z(7 \times 10^{-4}) = z^2 + z(1.3 \times 10^{-3})$

$0 = z^2 + z(2 \times 10^{-3}) - (3.5 \times 10^{-5})$

d. $z = \frac{-(2 \times 10^{-3}) \pm \sqrt{(4 \times 10^{-6}) - 4(-3.5 \times 10^{-5})}}{2}$

$= \frac{-(2 \times 10^{-3}) \pm \sqrt{(4 \times 10^{-6}) + (14 \times 10^{-5})}}{2}$

$= \frac{-(2 \times 10^{-3}) \pm \sqrt{(4 \times 10^{-6}) + (140 \times 10^{-6})}}{2}$

$= \frac{-(2 \times 10^{-3}) \pm \sqrt{144 \times 10^{-6}}}{2}$

$= \frac{-(2 \times 10^{-3}) \pm (12 \times 10^{-3})}{2}$

$z = 5 \times 10^{-3}$

e. $[H^+] = 6.3 \times 10^{-3}$ M

$[H_2Cit^-] = 5 \times 10^{-3}$ M

$[H_3Cit] = 4.5 \times 10^{-2}$ M

f. pH = 2.2

It decreased by .7

2. a.

		H^+	H_2Cit^-	H_3Cit
Concentrations (M)	Initial	$.93 \times 10^{-2}$	0	5×10^{-2}
	Equilibrium	$(.93 \times 10^{-2}) + z$	z	$(5 \times 10^{-2}) - z$

b. $7 \times 10^{-4} = \frac{\{(.93 \times 10^{-2}) + z\}(z)}{(5 \times 10^{-2}) - z}$

c. $(7 \times 10^{-4})\{(5 \times 10^{-2}) - z\} = (.93 \times 10^{-2})z + z^2$

$(35 \times 10^{-6}) - z(7 \times 10^{-4}) = z^2 + z(.93 \times 10^{-2})$

$0 = z^2 + z\{(.93 \times 10^{-2}) + (7 \times 10^{-4})\} - (3.5 \times 10^{-5})$

$0 = z^2 + z(1 \times 10^{-2}) - (3.5 \times 10^{-5})$

2. d. $z = \frac{-(1 \times 10^{-2}) \pm \sqrt{(1 \times 10^{-2})^2 - 4(-3.5 \times 10^{-5})}}{2}$

$= \frac{-(1 \times 10^{-2}) \pm \sqrt{(1 \times 10^{-4}) + (1.4 \times 10^{-4})}}{2}$

$= \frac{-(1 \times 10^{-2}) \pm \sqrt{2.4 \times 10^{-4}}}{2}$

$\simeq \frac{-(1 \times 10^{-2}) \pm (1.549 \times 10^{-2})}{2}$

$z \simeq 2.745 \times 10^{-3}$

e. $[H^+] \simeq 1.2045 \times 10^{-2}$ M

$[H_2Cit^-] \simeq 2.745 \times 10^{-3}$ M

$[H_3Cit] \simeq 4.7255 \times 10^{-2}$ M

f. pH $\simeq$ 1.9

It decreased by about .1

3. $[H^+] = 1.2 \times 10^{-4}$ M

$[HAsc^-] = 2 \times 10^{-5}$ M

$[H_2Asc] = 3 \times 10^{-5}$ M

4. $[H^+] = 5 \times 10^{-3}$ M

$[Gly^-] = 4 \times 10^{-3}$ M

$[HGly] = 4 \times 10^{-3}$ M

5. $[H^+] = 2.8 \times 10^{-3}$ M

$[Pyr^-] = 2 \times 10^{-3}$ M

$[HPyr] = 1.75 \times 10^{-3}$ M

6. $[H^+] = 4 \times 10^{-4}$ M

$[HMal^-] = 2 \times 10^{-4}$ M

$[H_2Mal] = 2 \times 10^{-4}$ M

7. $[H^+] = 1 \times 10^{-3}$ M

$[NO_2^-] = 1 \times 10^{-4}$ M

$[HNO_2] = 2.5 \times 10^{-4}$ M

8. $[H^+] = 4 \times 10^{-2}$ M

$[Sac^-] = 2.5 \times 10^{-2}$ M

$[HSac] = 4 \times 10^{-2}$ M

9. $[H^+] = 2.9 \times 10^{-2}$ M

$[HTar^-] = 2 \times 10^{-2}$ M

$[H_2Tar] = 5.8 \times 10^{-1}$ M

10. a. $z \simeq 4.9 \times 10^{-3}$

b. $[H^+] \simeq 5.15 \times 10^{-3}$ M

$[H_2PO_4^-] \simeq 4.9 \times 10^{-3}$ M

$[H_3PO_4] \simeq 3.1 \times 10^{-3}$ M

c. pH $\simeq$ 2.3

11. a. $[K^+][HTar^-] = (2 \times 10^{-2})(1.6 \times 10^{-3})$

$= 3.2 \times 10^{-5}$

Since $3.2 \times 10^{-5} < K_{sp}$, a precipitate will not form.

b. The same amount of H_2Tar is dissolved in one tenth the water. This causes a tenfold increase in the initial $[H_2Tar]$.

11. c.

		H^+	$HTar^-$	H_2Tar
Concentrations (M)	Initial		0	.2
	Equilibrium	2.5×10^{-5}	z	.2 - z

d. $$10^{-3} = \frac{(2.5 \times 10^{-5})z}{.2 - z}$$

$$(.2 \times 10^{-3}) - 10^{-3}z = (2.5 \times 10^{-5})z$$

$$2 \times 10^{-4} = (1 \times 10^{-3})z + (.025 \times 10^{-3})z$$

$$2 \times 10^{-4} = (1.025 \times 10^{-3})z$$

$$\frac{2 \times 10^{-4}}{1.025 \times 10^{-3}} = z$$

$$1.95 \times 10^{-1} \simeq z$$

Equilibrium $[HTar^-] \simeq 1.95 \times 10^{-1}$ M

e. $[K^+][HTar^-] \simeq (10^{-1})(1.95 \times 10^{-1})$

$\simeq 1.95 \times 10^{-2}$

Since $1.95 \times 10^{-2} > 4 \times 10^{-4}$ a precipitate will form.

f. Yes

g. Probably

LESSON 19: REVIEW

OBJECTIVE:

The student will solve problems relating to the objectives of Lessons 10 through 18.

PERIODS RECOMMENDED:

One or Two

OVERVIEW AND REMARKS:

If some lessons were omitted and you want to use this review, be sure not to assign problems based on omitted lessons. The following table shows the correspondence between problem numbers and lesson numbers.

PROBLEM NUMBERS	LESSON NUMBERS
1-10	10
11-16	11
17-19	11, 12
20, 21	13
22	16, 17
23	18
24	15
25	18
26	15
27	16, 17

KEY--PROBLEM SET 19:

Answers to Problems 1 through 5 that are in equivalent forms are also correct.

1. 1.49×10^{8}
2. 4.6×10^{9}
3. 1.06×10^{12}
4. 3×10^{-23}
5. 3×10^{13}
6. 60,000,000,000
7. 3,000,000,000
8. 14,000,000,000,000,000
9. 15,000,000,000,000,000,000,000
10. .0000000317
11. pH = 2.9
12. pH = 5.1
13. pH = 8.3
14. 4×10^{-7} M
15. 1.6×10^{-10} M
16. .4 M
17. $[Pb^{++}][SO_4^{--}] = (10^{-3})(2 \times 10^{-4})$

    $= 2 \times 10^{-7}$

    $2 \times 10^{-7} > 1 \times 10^{-8}$

    precipitate forms
18. $[Ba^{++}][SO_4^{--}] = (3 \times 10^{-6})(8 \times 10^{-5})$

    $= 24 \times 10^{-11}$

    $= 2.4 \times 10^{-10}$

    $2.4 \times 10^{-10} > 1.6 \times 10^{-10}$

    precipitate forms
19. $[K^{+}][HTar^{-}] = (.1 \times 10^{-2})(7 \times 10^{-2})$

    $= .7 \times 10^{-4}$

    $.7 \times 10^{-4} < 4 \times 10^{-4}$

    precipitate does <u>not</u> form
20. $[Ag^{+}] = [OH^{-}] \approx 1.2 \times 10^{-4}$ M
21. $[Mg^{++}] = [CO_3^{--}] \approx 5 \times 10^{-3}$ M
22. $[H^{+}] = 1 \times 10^{-3}$ M

    $[HTar^{-}] = 1 \times 10^{-3}$ M

    $[H_2Tar] = 1 \times 10^{-3}$ M
23. $[H^{+}] = 9.6 \times 10^{-5}$ M

    $[HSuc^{-}] = 4 \times 10^{-5}$ M

    $[H_2Suc] = 6 \times 10^{-5}$ M
24. Initial $[H_2Mal] = 3.5 \times 10^{-3}$ M
25. $[H^{+}] = 3 \times 10^{-3}$ M

    $[HSal^{-}] = 1 \times 10^{-3}$ M

    $[H_2Sal] = 3 \times 10^{-3}$ M
26. Initial $[H_2Sal] = 1.1 \times 10^{-1}$ M
27. $[H^{+}] = 1.6 \times 10^{-3}$ M

    $[Pyr^{-}] = 1.6 \times 10^{-3}$ M

    $[HPyr] = .8 \times 10^{-3}$ M

LESSON 20: COMPLEX NUMBERS

OBJECTIVES:

The student will

- graph complex numbers.
- solve quadratic equations having complex roots.

PERIODS RECOMMENDED:

One

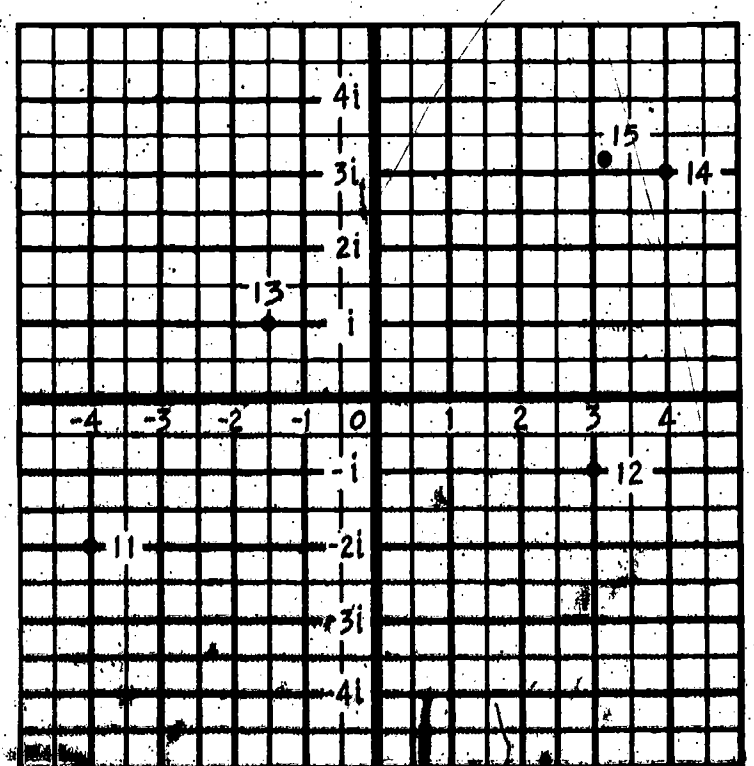
OVERVIEW AND REMARKS:

Lessons 20 through 22 treat the subject of complex numbers. This subject arises naturally in the process of solving quadratic equations. The basic arithmetic of complex numbers is the principal topic.

KEY--PROBLEM SET 20:

1. F
2. F
3. F
4. T
5. T
6. F
7. T
8. F
9. T
10. T

11-15.



16. $x = -1 \pm i$
17. $3 \pm 2i$
18. $-1 \pm 2i$
19. $2 \pm i$
20. $-2 \pm 2i$
21. $-3 \pm i$

LESSON 21: OPERATIONS WITH COMPLEX NUMBERS

OBJECTIVES:

The student will

- add complex numbers.
- subtract complex numbers.
- multiply complex numbers.
- compute powers of i.

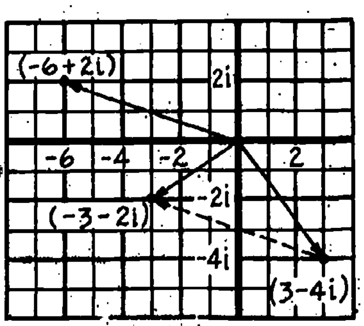
PERIODS RECOMMENDED:

One

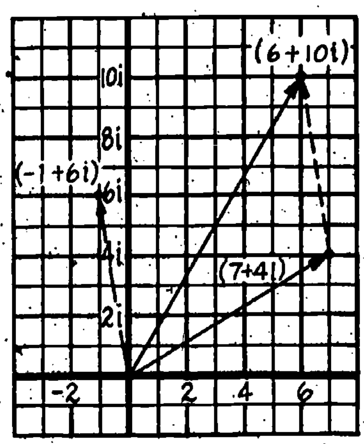
KEY--PROBLEM SET 21:

1. $8 - 9i$	7. $-8 + 6i$	13. $35 - 20i$
2. $5 + 4i$	8. $11 - 4i$	14. $-14 + 22i$
3. $1 - 8i$	9. $1 + 2i$	15. 13
4. $2 - i$	10. $-6 + 22i$	16. 17
5. $1 + 9i$	11. $-5 + i$	17. -10
6. $-2 + 2i$	12. $3 + 81i$	

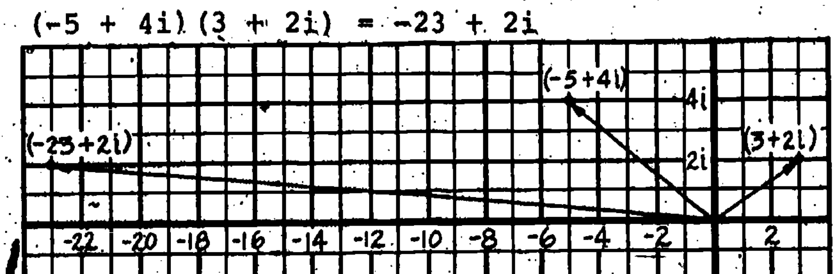
18.



19.



20. $(-5 + 4i)(3 + 2i) = -23 + 2i$



21. $i^3 = (i^2)i$
$= (-1)i$
$= i$

22. $i^4 = (i^2)(i^2)$
$= (-1)(-1)$
$= i$

23. $i^5 = (i^4)(i)$
$= i$

24. $i^7 = (i^4)(i^3)$
$= (1)(-i)$
$= -i$

25. $i^{24} = (i^4)^6$
$= (1)^6$
$= 1$

26. $i^{17} = (i^{16})i$
$= (i^4)^4 i$
$= i$

27. $i^{14} = (i^{12})(i^2)$
$= (i^4)^3(-1)$
$= -1$

LESSON 22. DIVISION OF COMPLEX NUMBERS

OBJECTIVES:

The student will:

- find the conjugate of a complex number.
- find the quotient of two complex numbers.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

If you want to discuss the long division example on the first page of Section 22, you will probably first need to review the technique for the students.

KEY--PROBLEM SET 22:

1. $3 + 2i$
2. $8 - 6i$
3. $100 + 30i$
4. $-10 - 7i$
5. $-15 + 19i$
6. $-6i$
7. $\frac{2}{5} + \frac{1}{5}i$
8. $\frac{1}{10} - \frac{3}{10}i$
9. $\frac{1}{2} + \frac{1}{2}i$
10. $\frac{1}{5} - \frac{2}{5}i$
11. $\frac{1}{4} - \frac{1}{4}i$
12. $\frac{1}{5} + \frac{2}{5}i$
13. $\frac{1}{6} - \frac{1}{6}i$
14. $-\frac{1}{6}i$
15. $2 - i$
16. $1 + 3i$
17. $1 - i$
18. $1 + 4i$
19. $1 - 3i$
20. $1 + 5i$
21. $2 + 2i$
22. $-1 + 10i$