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ABSTRACT

This book is designed as a report and an in-service tool for teachers of grades K-6. It is built around the idea that the standard textbook treatment of metric measurement gives rise to many misconceptions among primary grade children. This report is a record of the development of alternative methods for introducing measuring activities to first-grade children. Sections of the book are organized by clusters of related problem-solving activities and strategies. The major sections are titled: Size Discrimination: Non-Standard Units of Measurement; and Measuring with Standard Units. The objectives are: (1) to be able to use a unit of measure with reasonable accuracy; (2) to record results of observations in several forms; (3) to estimate with increasing accuracy; (4) to give reasons to support estimates; (5) to choose an appropriate instrument for the measuring task; and (6) to transfer learnings to the system of imperial measures. (MP)

AN INTRODUCTION TO LINEAR MEASUREMENT WITH THE METRIC SYSTEM

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READ ON AND FIND OUT...

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PROJECT ON ELEMENTARY SCHOOL MATHEMATICS AND SCIENCE

DIRECTOR: PETER B. SHOREMAN, COLLEGE OF EDUCATION

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Dear Colleague:

The Project on Elementary School Mathematics and Science at the Booker T. Washington School in Champaign, Illinois, is a cooperative venture of the University of Illinois and Champaign Community Unit School District No. 4. The full energies of the Project are presently being devoted to the development of curriculum materials for use in open classroom settings.

University staff members and Champaign teachers are working together on these development efforts. Opportunities are available daily for working both with individual children and with large and small groups in developing materials. This interaction of staff, children, and materials has constituted the heart of the development process.

Some units were initiated in the 1970-71 school year. Approximately a dozen units were begun during the 1971 Summer Planning and Writing Conference. Still other units were started during the 1971-72 school year. Currently, these units are being put into the form of preliminary editions. This booklet, and those that are forthcoming, represents an early stage of curriculum development. Consequently, teachers using these materials for the first time should consider their initial attempts exploratory.

It might also be noted that many of these booklets have been written to emphasize the use of resources in the Champaign-Urbana area. While it is hoped that the ideas contained can be generalized and adapted to fit your local environment, we would appreciate hearing of any difficulties you encountered in attempting to use this unit in your own community.

A COURSE CONTENT IMPROVEMENT PROJECT SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

Because of the experimental nature of our materials, we are earnestly soliciting feedback so that future revisions may benefit from actual classroom use. To facilitate this feedback, a brief Teacher Reaction Form has been included for your use. We would appreciate the return of this form as soon as possible.

We hope that you will freely explore this unit with your children and let us know how you think our materials might be improved. Our project staff looks forward to hearing--and learning--from you.

Sincerely,

Peter B. Shoresman

Peter B. Shoresman
Professor of Science Education

AN INTRODUCTION TO LINEAR MEASUREMENT WITH THE METRIC SYSTEM

Preliminary Edition

Sylvia J. Pattison

and

Richard W. Griffiths

Project on Elementary School Mathematics and Science
University of Illinois at Urbana-Champaign

Prepared under a grant from the National Science Foundation

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The authors also wish to recognize Professor Peter B. Shoresman, Project Director, who provided an atmosphere conducive to exploration and creative activity.

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INTRODUCTION

Rationale and Objectives

Measurement is an application of mathematics that is used daily by most adults. Yet the treatment of measurement in elementary school texts is often sterile and divorced from the real world. Children are rarely given the opportunity to use non-standard units to measure anything more exciting than a picture in a textbook. All too often, they are introduced to standard rulers as early as possible in their school careers and asked to measure numerous line segments on a worksheet.

Because the authors feel that the standard textbook presentation gives rise to many misconceptions concerning the nature of linear measurement, an alternate method has been devised for introducing initial measurement activities to primary grade children. This booklet is a record of the development of this method with a group of first graders.

Children have been "measuring" long before they enter first grade; but most of their measuring has been either <u>direct</u> comparisons between objects, distances, and weights, or <u>indirect</u> comparisons through the use of their own bodies or strength (e.g., stating the width of a desk in terms of hand spans or comparing the effort necessary to lift different objects). All that is lacking when they are measuring in this way is a <u>unit</u>

of measure by which fine comparisons can be made--a unit of measure which serves as a reference, a standard in terms of which all subsequent measurements are performed.

Listed below are the basic ideas which we felt were important for children to develop during their work with linear measurement. These ideas do not depend in any way on the measurement system used.

1. In order to measure, there must be some unit of measure.

2. Units of measure are arbitrary.

3. A selected unit of measure may become standardized for the convenience of public use. (The most widely used unit of measure for small distances is the centimetre.*)

4. After a unit of measure has been selected for a given task, it should be used consistently throughout the task. (This may necessitate involvement with multiples and fractions of the unit.)

5. The measurement should be labeled to indicate what unit has been used.

6. All actual measurements are approximate. The degree of accuracy depends on the refinement of the measurement tool and the skill with which the tool is used.

7. The goal of measuring is to be as accurate as possible--to find an "answer" that is close enough for practical purposes.

*The authors have used the International Standard spelling of the various metric units (e.g., "centimetre," "metre") for the following reasons: First, this is the spelling in international use. Second, it is also the spelling which the Center for Metric Education (Western Michigan University, Kalamazoo, Michigan) will recommend to the Congress for official adoption in the United States. And, third, it was noted that the children seemed to have less difficulty in spelling "-metre"--perhaps because of its similarity to "metric"--than "-meter." For these reasons, the authors suggest that teachers use the "-metre" spelling when implementing the activities in this booklet.

8. Many measurements of the same thing tend to produce a clustering of measures and a range of reasonable results.

The authors decided to use the metric system in this development work for several reasons: (1) There is a growing interest in the metric system in the United States and a distinct possibility, at the time of this writing, that this country may "go metric" in the foreseeable future; (2) metric measures based on multiples of ten correlate measurement work with computational work in the decimal system and are more easily handled by first graders than the mixed-base system of imperial (English) measures (see Appendix A); (3) many of the new elementary school science programs expect children to use metric measurement; and (4) all children are able to start work on standard measures on an equal footing since any advantage due to familiarity with imperial units is eliminated.

Also, before commencing their work, the authors established as objectives the following six measurement skills:

1. The ability to use a unit of measure with reasonable accuracy.
2. The ability to record results of investigations in several forms:
   a. Number of units used, labeled with name of unit.
   b. Charts and tables.
   c. Graphs.
3. The ability to estimate with increasing accuracy.
4. The ability to give reasons to support estimates.
5. The ability to choose an appropriate instrument for the measuring task at hand.
6. The ability to transfer learnings to the system of imperial measures.

By participating in activities designed to aid them in acquiring the skills mentioned above, and through group discussions following each activity, it was hoped that the children would develop a basic understanding of mensuration.

Organization of Activities

The activities in this booklet are organized with the following general sequence in mind:

1. Size discrimination
2. Non-standard units of measurement
3. Standard units of measurement

The children were first presented with situations where they had to differentiate between objects of different sizes. This procedure was intended to provide them with experiences in verbalizing size differences ("larger than"/"smaller than"). It was also designed to assess their ability to discriminate objects by size. A typical activity was to give each child a number of washers of different sizes to place in order.

The next series of activities provided the children with experiences with non-standard units of measurement. They first measured a variety of things in their classroom (desks, chairs, chalkboards, length and width of room, etc.) using body units (their hands, their feet, etc.). The use of these non-standard units gave them experience in measuring, recording, and expressing the size of things. It soon became apparent to the children that it was not possible to make exact comparisons of their measurements of the same objects since their measuring instruments (e.g., their feet) were all different sizes. This, however, provided the basis for comparison of a different sort. For example, the children became aware of the fact that if they all measured the width of the hall, they could predict who would take the fewest steps (person with biggest feet) and who would take the most (person with smallest feet).

After using non-standard units, the children moved to "standard non-standard" units. These were measuring sticks of uniform lengths made from cardboard. Four different lengths were used. There were no unit markings of any kind on them. The sticks of different lengths can be designated as "A-sticks," "B-sticks," "C-sticks," and so on, or by painting them different colors. With one class the authors used sticks of four different lengths--70, 50, 30, and 10 centimetres. They were designated as A-, B-, C-, and D-sticks, respectively. The children were not told how long each of the sticks was. This, they later discovered for themselves.

With a second group of students, the measuring sticks were painted yellow, blue, and red and were 10, 40, and 60 centimetres in length, respectively. Again the children were not told how long the sticks were.

Both methods of designating the sticks (by lettering them or by painting them different colors) worked equally well to distinguish sticks of different lengths. It was later determined, however, that using sticks of 10, 20, 40, and 80 centimetres was especially convenient since many students became involved in expressing fractional parts.

After working with various non-standard units of measurement, the children were gradually introduced to standard metric units. The first unit with which they worked was the metre. An unmarked metre stick was given to each child. This stick contained no centimetre or millimetre markings. The children were told that the stick was called a "metre stick" and that a "metre" was a unit widely used for measuring objects and distances. They were then provided with a wide variety of experiences in measuring. As experience was gained, more stress was given to accuracy of measurement. The children quickly realized that, although they all measured the same things, different people often got different answers. The reasons for these differences and the ways in which the children became aware of their inaccuracy in measuring are discussed in the section entitled "Using Calibrated Metre Sticks."

After gaining experience with the unmarked metre stick, the children were given a metre stick calibrated in centimetres. Again, a variety of activities was provided for the purpose of measuring objects and distances.

To encourage them to think in terms of the various units of measurement used (non-standard and standard), the children were asked to estimate the length of a given dimension before they actually measured it. Thus, the feeling for the size of a metre and/or a centimetre was gradually developed. At first, the estimates made by the children were generally not too accurate. It became apparent, however, that as they gained experience the accuracy of their estimates improved considerably.

SIZE DISCRIMINATION

This initial series of activities was designed to give the children experience in three skill areas: (1) Differentiating between the sizes of pictured objects, (2) verbalizing the observed differences, and (3) ordering objects by size.

Who Likes Ice Cream Cones?

A series of five cards with drawings of ice cream cones of different sizes (see Figure 1) was used in this set of activities. In the actual development work, each cone had ice cream of a different color. The cards were shown to the

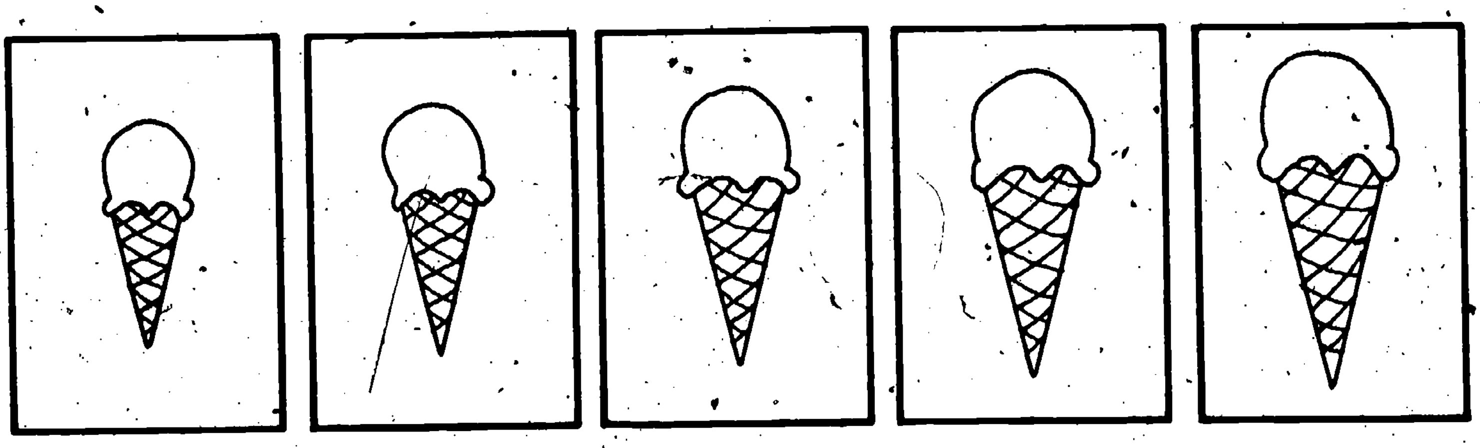


Fig. 1--Size Discrimination Using Drawings of Ice Cream Cones

children two cards at a time. First, the largest ice cream cone and the smallest ice cream cone were paired and displayed. The children were asked to choose the cone that they would select if the cones were real. (The assumption was that they would choose the card showing the largest ice cream cone.)

The first child who responded expressed a desire for the smaller cone. When asked why, he replied: "Because I like raspberry better than lemon." (Apparently our affluent society has produced some children who are swayed more by taste preference than by quantity!) When the other children were asked the same question, they all chose the larger (lemon) cone, giving the reason that it was the larger of the two. Other combinations of ice cream cone pictures were then shown to the children. Some discriminated readily while others had difficulty at first. Since the children were in the first grade, some problems were to be expected. (The chance that a child might choose cones by flavor rather than by size can be eliminated by making all of the ice cream the same color.)

When the combinations of two were exhausted, the children were asked to place the ice cream cone cards in order by size. Again, some could perform this task readily while others encountered some difficulty.

Let's Play Ball

A series of five cards, each picturing a ball of a different size, was presented to the children (see Figure 2). The

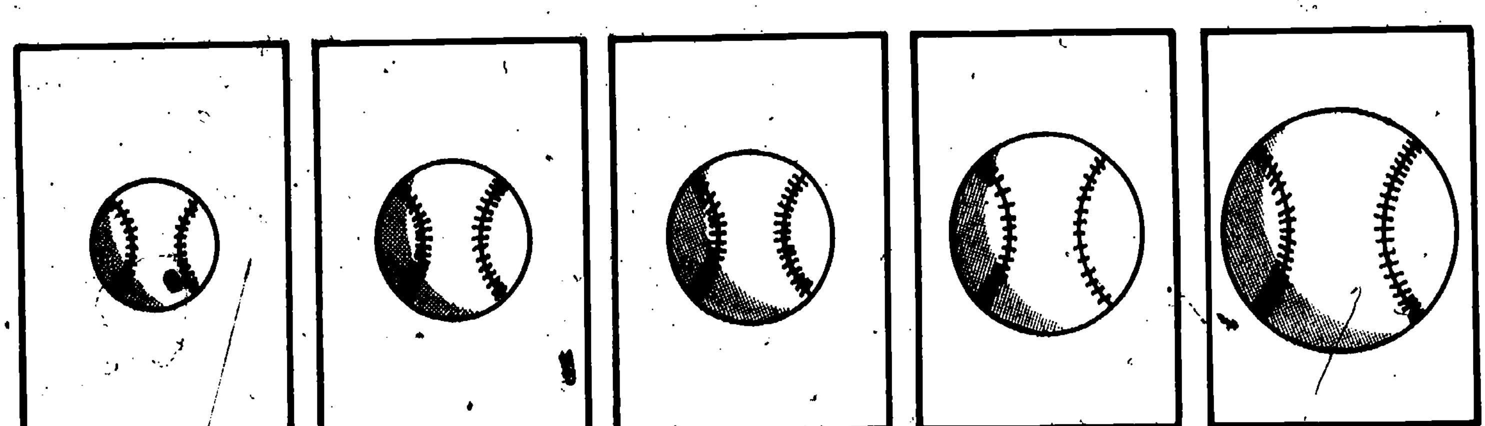


Fig. 2--Size Discrimination Using Drawings of Balls

children were asked to place the cards in order by size of ball. Having had experience with the ice cream cone cards, most children easily accomplished this task. When the cards were in order, a number of questions similar to the following were asked: "Which ball is smaller than two other balls and larger than two other balls?"

The Birthday Presents

A series of five cards showing presents of different sizes (see Figure 3) was used for those children who needed additional practice. The children were expected first to put them in order and then to describe each present as being either larger or smaller than each of the other presents.

The same procedure was followed with washers of different sizes.

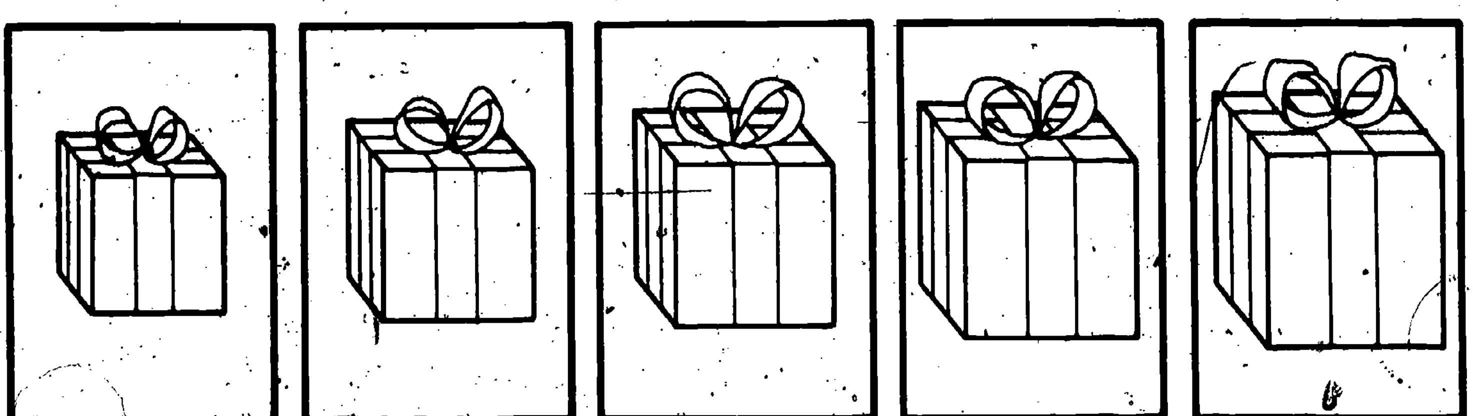


Fig. 3--Size Discrimination Using Drawings of Presents

I'm Taller Than . . .

An interesting activity for the children was for them to stand in a row and arrange themselves in order by height. This afforded an opportunity for gross comparison of size, a precursor of measurement.

It was necessary, first of all, for the children to devise a method for ascertaining who was tallest, next tallest, and so on, so that a descending order by height could be established. This method can be as simple as merely standing back to back, two at a time; or the children can be encouraged to suggest a more elaborate means for determining differences in height. Once they are in order, they should be able to describe their position in the line in a manner similar to the following: "I am taller than two others and shorter than four others." The point is that the children should be able to see a hierarchy and be able to describe the uniqueness of each position in that hierarchy.

Sticks of different lengths were then placed randomly on the floor to be put in order. Again, gross comparisons of length were needed. The children quickly found, by placing side by side what they thought were the two longest sticks, that they could determine the longer one and correctly position it in the sequence. In general, this procedure was followed until all sticks were in their proper order. Thus, they demonstrated in a simple way a systematic method of measuring.

The children in one class developed their own comparison game using Cuisenaire rods which were available in the room. One child was chosen as the "leader." Each of the other children had a set of the ten different Cuisenaire rods. The following excerpts from the anecdotal record of the authors describe the game:

Ruth: "I am thinking of a rod that is bigger than four [rods] and smaller than five [rods]."

[The children looked at the rods and picked up the appropriate one (the yellow rod).]

Dawn: "Which rod is smaller than three others and bigger than six?"

[Again the children selected the appropriate rod (the black one).]

Ann: "I am thinking of a rod that is bigger than eight and smaller than two."

[The children looked at their rods, picked up the blue one, put it down, picked up the brown rod, put it down, and then started re-counting.]

Dawn: "We can't do it. Eight and two is ten, and the numbers are only supposed to add up to nine. It's one of these two [holding up the blue and brown rods], but we can't tell which."

This activity was valuable in that the children gained experience in describing the relative positions of each rod. Because children may have memorized the color-length sequence of the Cuisenaire rods, such rods would not be appropriate for the stick activity described on the preceding page.

NON-STANDARD UNITS OF MEASUREMENT

Measuring with Your Own Body

In order to demonstrate to the children how their own bodies could be used to measure a variety of things, a "Hands Tall" chart was constructed from tri-wall (three-layer cardboard) (see Figure 4). Using the chart, the children were

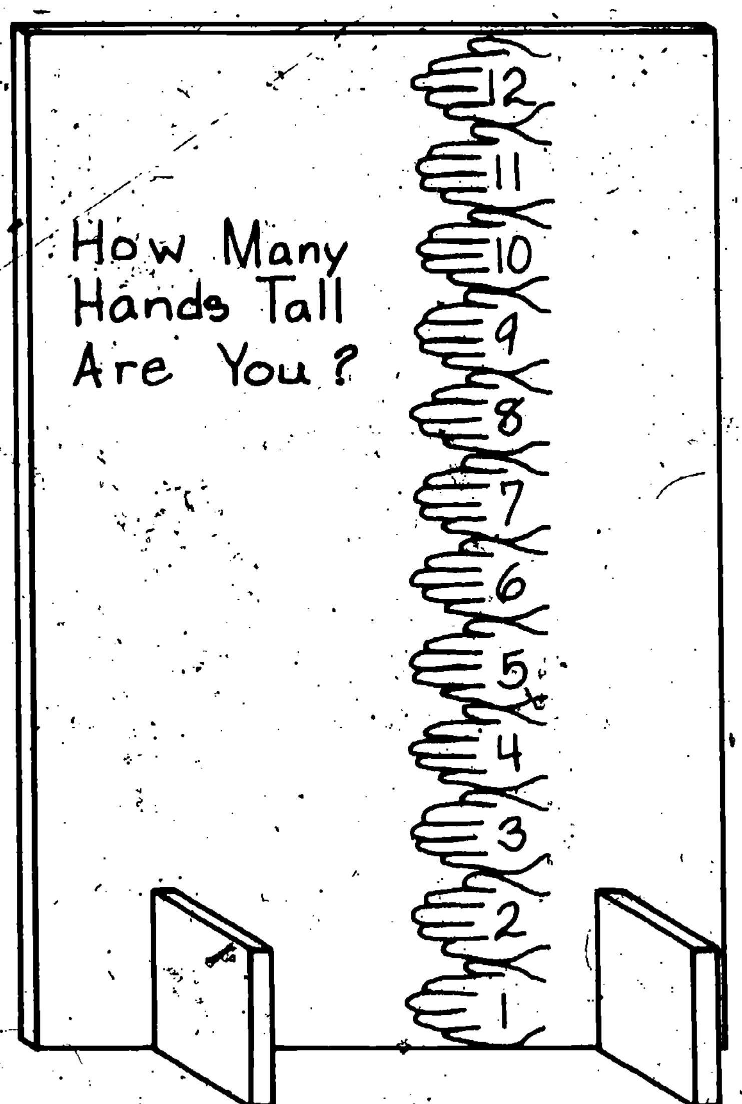


Fig. 4--Hands Tall Chart

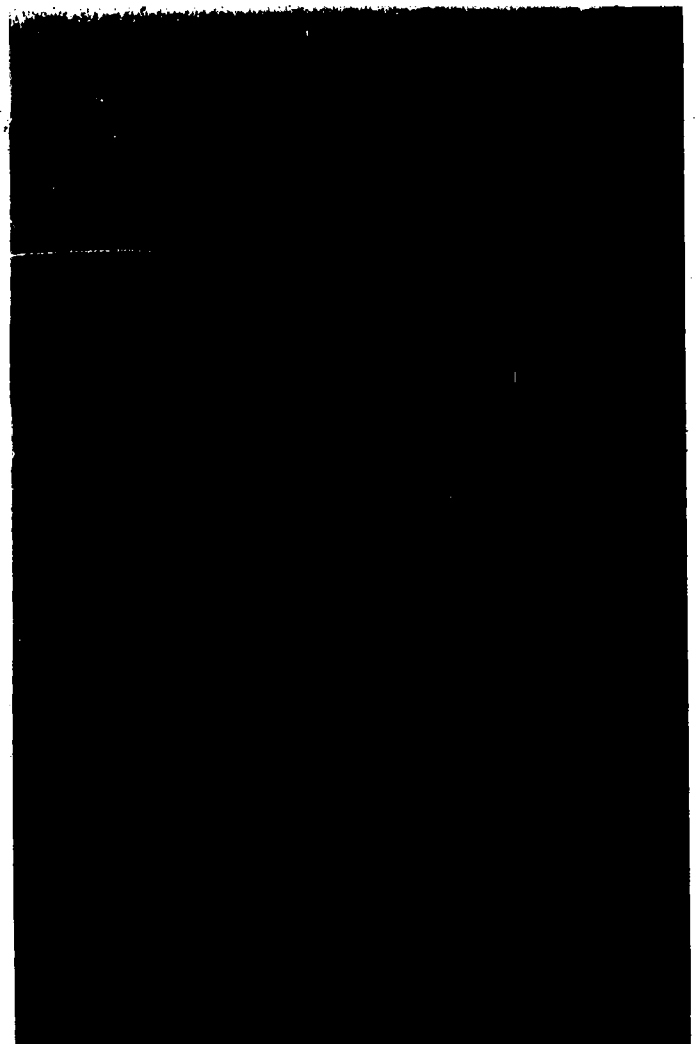
able to measure their heights in "hands" and express those heights. They were shown how the chart could be made by placing a hand on the cardboard and drawing its outline enough times so that the total height of the "stack" of hands exceeded the height of the tallest child in the group. (The chart depicted in Figure 4 was made by tracing an adult's hand which was four inches wide. A child's hand could be used as well.)

While using the "Hands Tall" chart, the children made some interesting discoveries. After two boys had measured themselves, they compared their heights. One boy was 11 hands tall and the other was 12 hands tall. They agreed that the taller boy was one hand taller than the shorter boy.

Occasionally, a fractional part of a hand was noted in measuring someone's height. For example, if a child's height was between 10 and 11 hands, it could be expressed as 10 1/2 hands. It was found that many first grade children do this spontaneously--some with and some without an understanding of a fractional part of a whole. Another way to handle this is to express a quantity between 10 and 11 hands as "10 hands and a little more," which the children suggested could be written as "10+". Given the opportunity, children will create a number of such notations for expressing lengths that happen to be "a little more than" or "a little less than."

The children's own hands became their first measuring instruments. They were encouraged to measure a wide variety of objects in the room (chairs, desks, bookcases, etc.) using the width of their hands as the unit of measurement. In succeeding sessions other "body units" were used to measure distances and objects in and outside of the classroom. Feet were used to measure the length and width of the room, while fingers were used to measure small objects such as pencils, pens, and books.

When the children were asked to use their feet to measure something, such as the width of the room, the answers from child to child were not the same. Attention should be focused



Measuring the width of the hall
using a body unit

on these differences early in the activity. The various results of a given measurement can be written on the chalkboard for the children to compare. The answers will almost certainly be discrepant. The major cause is that their feet are of different sizes. Of course, some of the differences could also be due to errors in measurement. At any rate, the children should be encouraged to consider the possible reasons for the observed variations. Some of the reasons given by the children were as follows:

> "Their answers are different because Becky put her feet close and Kate didn't." [Error caused by measuring technique.]
>
> "There should be a difference because our feet are not the same size."
>
> [Sometimes the reasons were more generalized.] "We just goofed."

Following a discussion of reasons, it is a good idea to ask: "According to our measurements, who should have the biggest feet?" Children should understand that the person who took the fewest steps must have the biggest feet and the person who took the greatest number of steps must have the smallest feet. Some children readily see this inverse relationship while others do not.

A group of students was asked to measure the widths of their desks using their hands as measuring instruments. The results of their measurements (see Figure 5) and excerpts from the ensuing discussion follow:

Tesha	10	hands	wide
Ragan	9	hands	wide
Kate	5	hands	wide
Mark	8	hands	wide
Todd	9	hands	wide

Fig. 5--A Simple Table to Record the Results of a Measurement

Teacher: "Why do you suppose that we have many different results?"

Ragan: "Todd and me have the same."

Harry: [Who was absent the day the measurements were made.] "Some people have bigger hands and some have smaller hands."

Teacher: "Let's look at the results again. Kate seems to have a much smaller number than anyone else. Why do you suppose that is?"

Becky: "Maybe her hands is bigger."

Teacher: "Let's find out. Mark, will you use your hand to find out if your hand is smaller than Kate's, larger than Kate's, or about the same?"

Mark: "Mine's bigger."

Teacher: "Kate, your hands are smaller than Mark's, but you reported that your desk was only five hands wide. Mark reported that his desk was eight hands wide. Can we see how you measured your desk?"

Kate went to her desk and used her hands to measure the width--and the problem was solved. Instead of using the width of her hand, Kate had used the length from her wrist to the tip of her middle finger.

Mark: "If I did it that way, my answer would be smaller, too."

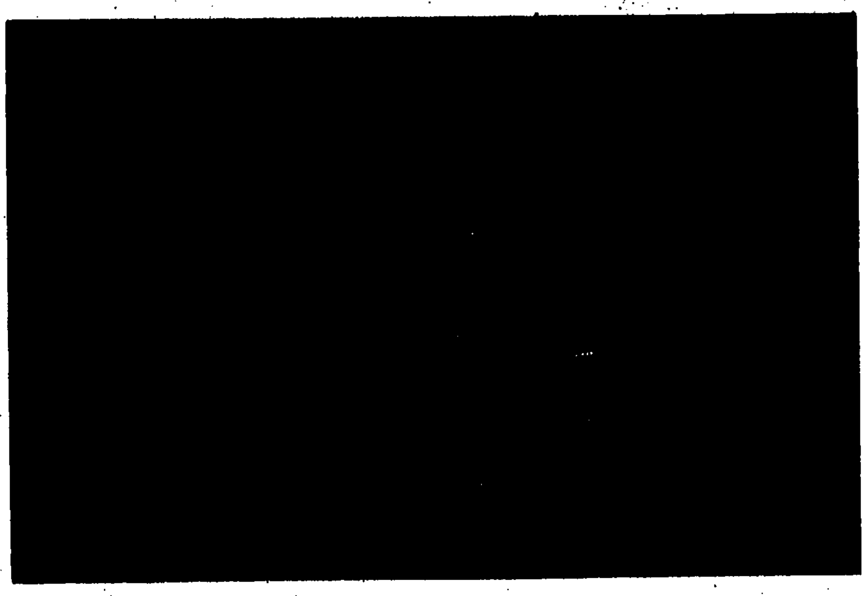
Teacher: "If someone had large hands, would he have a larger number than you, Harry?"

Harry: "No, it would be smaller."

Measuring the width of a desk
using hand lengths

Each child also made a "hand-span tape" by marking off on paper tape units equal to the width of his hand with the fingers spread. (The paper tape used was "teletypesetter" tape, obtained at no cost from a local newspaper office.) A wide variety of objects and distances was then measured and recorded. The worksheet entitled "Measuring with the 10-Span Tape" (see Appendix B) indicates a few of the things that can be measured in this way.

In a later activity the children used cardboard sticks of different colors to measure a variety of objects (see page 25). Several students decided that they could measure faster and more easily if they marked off a piece of paper tape in "yellow-stick lengths" (i.e., in 10-centimetre lengths). Some used sticks of other colors. All of the



Marking off a paper tape in
"yellow-stick lengths"

children made their tapes longer than the original 10-span tape, enabling them to measure longer distances more conveniently.

Estimation and Accuracy

As soon as the children had gained some experience in measuring with non-standard units (fingers, feet, hand spans, etc.), they were asked to begin estimating the various dimensions before actually measuring them. A typical activity of this type could be initiated by posing the following question:

"If I measured the length of the room with my feet, how many 'steps' would it take to reach from one end of the room to the other?" The children should first establish that the teacher, because he has larger feet, would take fewer steps than a child.

This procedure was followed with one group of children who had reported measurements for the length of the room ranging from 42 to 55 of their own feet. When asked to guess how many of the teacher's feet it would take to span the room, some children thought that it would take 21 while the rest guessed 22. As the teacher stepped off the distance, the children counted, finding the length of the room to be 27 "steps." These initial estimates were reasonably close. As the children gained experience in estimating distances, their accuracy improved markedly--both in estimation and in measurement.

There seems to be merit in having children estimate a distance before making a measurement of it. Before long they start to use previous measurements as a basis for making succeeding estimates. Practice in estimating the magnitude of a dimension tends to eliminate the implausible estimates that some children make early in their measurement activities.

A game of estimation (*Estimo*) is described in Appendix C. This game can be introduced when the teacher feels that the children have had sufficient experience measuring with the metric system.

Another technique that seemed to be effective in improving skill and accuracy in measurement was to have the children compare independent measurements of the same things. In this way, errors in their measurements were aired and corrected by the group. Figure 6 presents a table showing measurements made by five children of five different objects. The desk, chair, and table were measured with their hands. The room (length) and hall (width) were measured with their feet.

	Desk	Chair	Table	Room	Hall
Nancy	11	7	30	44	16
Brendan	9	9	22	40	14
Michele	7	5	20	55	19
Wendy	11	11	13	43	17
Todd	9	11	22	40	14

Fig. 6--Comparative Measurements Made with Hands and Feet

Obvious discrepancies in their results can be noted by inspecting the table. The following dialogue ensued regarding these measurements:

Teacher: "Let's look at your measurements of the desk. What is the largest number of hands that anyone found for the width of the desk?"

Wendy: "Nancy and me got eleven."

Teacher: "Who found the smallest number of hands for the width of the desk?"

Michele: "I did. I got seven."

Teacher: "Do you think all of these answers are correct?"

Michele: "Yes."

Teacher: "Why do you think they are all correct, Michele?"

Michele: "Because our hands are different [sizes]." [Although this is true, it does not account for the nearly 60% difference between the largest and smallest measurements.]

Teacher: "Could you tell whose hand is biggest and whose hand is smallest from these answers?"

Michele: "Wendy's and Nancy's are the biggest."

Teacher: "Would you say that the person who had the largest answer has the biggest hands?"

Michele: "Yes." [She has not assimilated the inverse relationship at this point, viz., the bigger the hand, the <u>smaller</u> the number of hands needed to measure a given distance.]

Teacher: "What do the rest of you think?"

[Uncertain response from others.]

Teacher: "My hand is bigger than any of yours. If I measured the width of the desk, would I get more hands than any of you?"

Michele: "No!" [She has, of course, contradicted herself.]

Teacher: "If I measured the desk, how many hands do you think it would be?"

Wendy: "A smaller number." [She seems to understand the inverse relationship.]

Teacher: "From that answer, whose hand do you think is biggest?"

Michele: "Wendy's and Nancy's." [She is still not clear.]

[At this point Todd was shaking his head.]

Todd: "I think Michele's hand is biggest." [The table indicates this to be true, even though in reality her hand was not the biggest.]

Wendy: "Yes, Michele, because it's a less [smaller] number."

[Both Todd and Wendy had made a valid interpretation of the information presented in the table. However, the accuracy of the table information had not yet been established.]

During the discussion--which continued for some time--the students compared their hands and found them to be almost identical in size. From this information they quickly realized that there were errors in their measurements. A remeasurement confirmed that errors were indeed present. The results of the measurement of the other things were also compared and evaluated by the students. Several errors were found and corrected. Thus, interpretation of data, validation of measurements, and correction of errors were all accomplished through this discussion.

Using Standard Non-Standard Measuring Instruments

Each child was provided with some narrow strips of triwall that had been cut to standard lengths of 10, 20, 40, and 80 centimetres. The children were not aware of how long the sticks were. Figure 7 on the following page depicts the relative lengths of the sticks that each child received. In the actual development work, to make identification easier when measurements were being discussed, the sticks were painted the colors indicated in the parentheses.

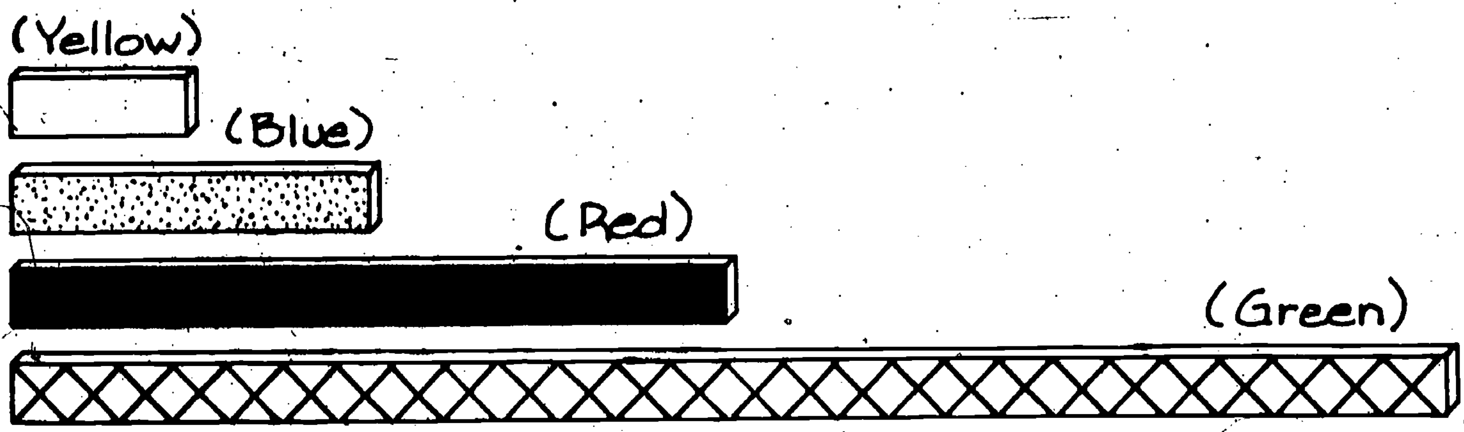


Fig. 7--Cardboard Measuring Sticks

The children were given a variety of opportunities to measure many things with the different measuring sticks. They were asked, for example, to estimate the width of their classroom and then measure it with one of the cardboard instruments to see how close their estimates were. Their results are shown in Figure 8.

	Estimates	Measurements
Todd	12	17½
Kate	24	24
Tesha	110	20½
Ragan	12	17
Harry	20	20
Becky	16	16
Mark	20	17
	The room was actually 17½ blue sticks wide.	

Fig. 8--Room Width in Blue-Stick Lengths

As can be seen, most (but not all) estimates were plausible, and some measurements were very close to the actual width of the room. As the children gained more experience in estimating and in measuring, they became increasingly more proficient at both.

A similar activity involved using a variety of measuring instruments to determine the width of the hall. The results of the children's measurements are presented in Figure 9.

How Wide Is the Hall?

	Yellow Sticks	Blue Sticks	Red Sticks	Feet	Hands
Becky	30	9	5	16	48
Harry	30½	9	6	15	44
Kate	27½	7½	5	15	27
Mark	34	8	7	15	46
Ragan	32	9	6	17	22
Tesha	29	9	20	14	5
Todd	25	9½	5	16	47
Actual Widths	29	7.6	5.1		30.5 of Teacher's Hands

Fig. 9--Hall Widths in Non-Standard Units

There is a great deal of agreement in the measurements depicted; however, some discrepancies caused by carelessness are also evident. For example, in the "Yellow Sticks" column, the measurements ranged from 25 to 34 units. The magnitude of this variation suggests carelessness of some sort during the measuring process.

Two details should be noted with respect to the measurements in Figure 9. First, some of the children began to express their measurements using fractions, for example, 7 1/2. They were not encouraged to do this, but neither were they discouraged. Although this use of fractions was unanticipated, it was noticed that the children seemed to gain a rather sophisticated understanding of the meaning of a fraction as this series of activities progressed.

Second, the children were aware that all of their measurements with the yellow sticks should be the same, just as all of their measurements with the blue sticks or the red sticks should be the same. When asked why their measurements with a stick of a given color were not the same, they replied: "Some of us goofed." At the same time, they were aware that the measurements with their hands or feet should be different since these appendages differed in size from child to child.

Because there were some discrepancies in their measurements with the yellow, blue, and red sticks, the children were asked how they could determine what the width of the hall actually was. A large number of the colored sticks were made available. One child suggested taking enough of the yellow sticks to lay them end to end across the hall. Then the number of sticks could be counted to ascertain the correct width. This was done first with the yellow sticks, then with the blue sticks, and finally with the red sticks.

Measuring the width of a hall
using yellow sticks

The length of the hall was investigated next. In this activity, the children were asked first to estimate the length and then to measure it. The sheet on page 30 (Figure 10) was used with this activity and presents the estimates and measurements made. It was noted that some children preferred to measure first and then estimate. This seems to reflect, at least in part, the notion held by many children that it is bad to make a mistake. An attempt should be made to dispel this fear by encouraging the children to use their prior experiences with measuring to make reasonable estimates.

How Long Is It?

	Yellow Sticks		Blue Sticks		Red Sticks		Feet		Hands	
	My Guess	My Measurement	My Guess	My Measurement	My Guess	My Measurement	My Guess	My Measurement	My Guess	My Measurement
Kate	34	34	12	12	8	8	29	25	36	36
Mark	50	60	10	14½	10	9½	20	24	*	71
Ragan	39	44	66	13½	9	9	24	26	64	64
Tesha	28	45	24	61½	8	8½	26	37	*	*
Todd	100	51	10	10	9	*	100	22	100	*
Becky	38½	48½	21½	15	31	8½	33½	28	78	78
Harry	34	34	24	13	10	10	22	22	58	58

*Child did not complete activity.

42

Fig. 10--Hall Lengths in Non-Standard Units

Guess What I'm Describing

A simple game was devised to provide further experiences in estimating. It is played by having one child look around the room and choose an object. He then gives his estimate of one or more of its dimensions in terms of any of the non-standard units he has used, in a manner similar to the following: "I'm thinking of something that is two blue sticks long," or "I am thinking of an object that is one red stick high and ten hands wide." From the dimensions given, other children must try to determine what is being described. Considerable interest and excitement were generated by this game.

MEASURING WITH STANDARD UNITS

Using Uncalibrated Metre Sticks

The children were asked to measure the distance between two yellow lines painted on the playground blacktop using uncalibrated metre sticks. Although chalk was available to them, their initial decision was to use a pencil to mark the position of the end of the metre stick. For most of them, this "marking" was carried out in a rather crude fashion. Scott did not use his pencil to make a mark at the end of the stick. Instead, he placed the pencil on the ground at the end of the stick as shown in Figure 11. Next, he picked up the metre stick and placed it on the other side of the pencil. This process was repeated until the distance was completely spanned. Scott's method failed to take into account the width of the pencil. Furthermore, the wind rolled the pencil forward a few centimetres each time he moved the stick. Paul attempted to compensate for the width of the pencil by placing it as shown in Figure 12. Only

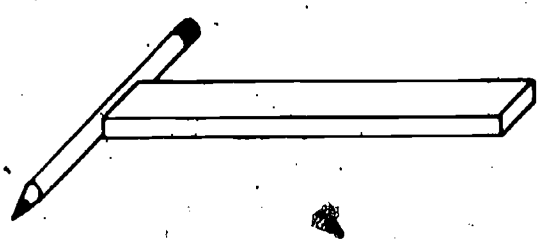


Fig. 11--Scott's Method

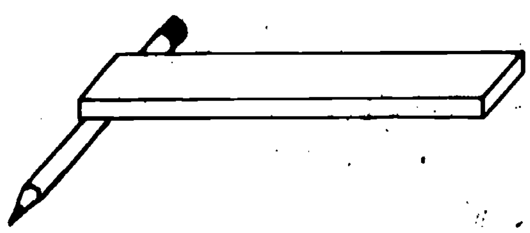


Fig. 12--Paul's Method

Mary used the point of the pencil to make a mark on the blacktop to indicate the end of the metre stick.

The range of results reported for the first measurement was 17 1/2 metres to 25 metres. During the comparison of these results, Jane attempted to explain the observed discrepancies by suggesting that the units used were not of the same size. (This explanation was probably based on her prior experience with non-standard units where the units--for example, feet--actually were of different sizes.) However, Paul demonstrated that the metre sticks were the same length by placing two of them together.

The children then decided to pair up and remeasure. This time they asked for chalk to mark the end of the metre stick. Halfway down the blacktop Jane lost count and wanted to go back to the beginning to start again, but her partner Mary said, "It doesn't matter; you can just count the marks when you're through." The remeasuring produced a range of 17 1/2 metres to 18 metres--a result with which the children were satisfied.

Using the Trundle Wheel

After measuring the distance between the two yellow lines, the children were asked to use their metre sticks to measure the width of the blacktop. They paired themselves up to perform this task. Paul and Scott reported a measurement of 18 1/2 metres. Jane and Mary reported a measurement of 18 1/4 metres. Jane offered the following explanation for this discrepancy: "It's awful hard to keep straight when you have to measure something long. Mary and me measured right on top of the yellow line so we could be sure."

When the children were asked to measure the same distance with a trundle wheel, Jane was not at all convinced that they would come up with the same measure. Paul picked up the wheel

Measuring with a trundle wheel

and pushed it along the entire length of a metre stick. "Look," he said, "one turn is the same as a metre stick." In spite of this, Paul and Scott reported their first measurement as 19 1/2 metres. When this report was challenged in view of their prior measurement with the metre stick, they remeasured. This time they employed the technique of using one of the yellow lines to be sure that they measured along a straight line. The result of this measurement was reported to be "18 full turns and about a half a turn more."

Using Calibrated Metre Sticks

Next, the children were introduced to a calibrated metre stick and given the task of measuring the objects pictured on a worksheet similar to the one included as Appendix D. They were asked to estimate the length of each object and record these estimates before making their measurements. The estimates made at first were generally not very close to the actual measure; but, as the session proceeded and as the children gained more experience using centimetres, their estimates became much more reasonable--the errors ranging from 0 to 23% of the actual lengths.

During the next session, the children were presented with a summary chart of their measures (see Figure 13), and they discussed this chart with some insight. They recognized the discrepancies and offered the following hypotheses in explanation:

"Maybe we didn't all measure the same hammer." [True.]

"I measured a longer handsaw than Eric." [True.]

"I bet she wrote them down backwards [in the wrong place]." [Referring to Jane's reported measures for short and long screwdrivers.]

Another explanation for certain discrepancies was suggested when it was brought out that Paul and Eric had employed slightly different methods of rounding off their measurements. While Paul had stated that the length of the short screwdriver was "a little more than 8 centimetres," Eric insisted that it was "a little less than 9 centimetres."

Our Measurements — Centimetres

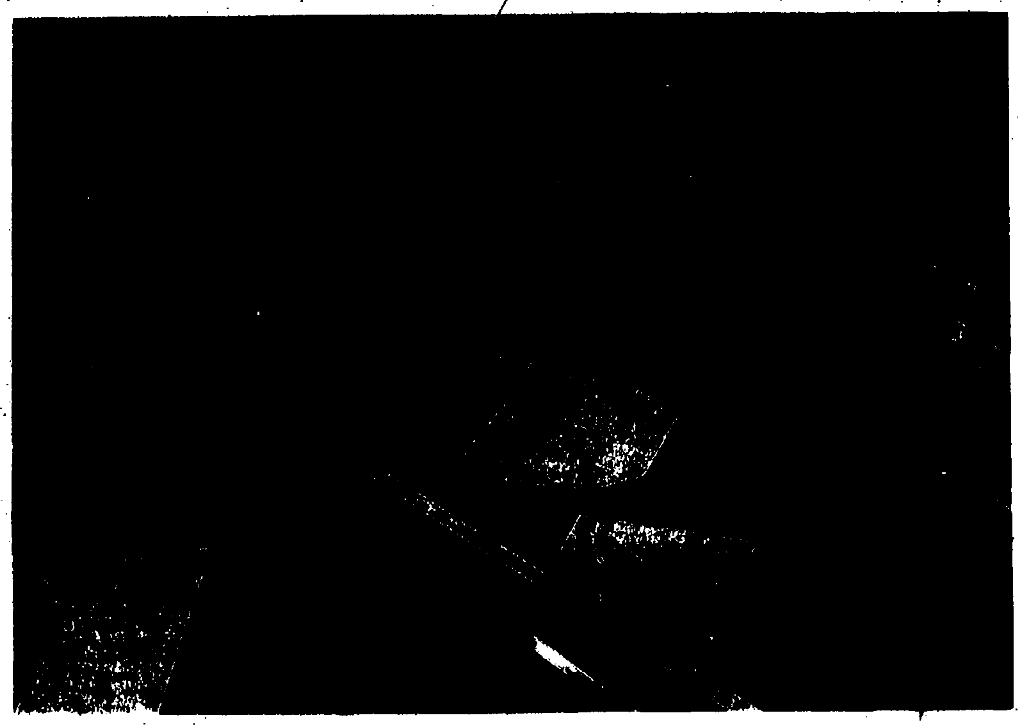
	Eric	Paul	Scott	Jane	Mary
Screwdriver	9	8	9	20	20
Screwdriver	26	20	26	9	26
A-Stick	70	70	30	70	70
B-Stick	30	30	*	30	30
Saw	57	73	53	72	71
Hammer	33	33	32	31	33
Coping Saw	47	42	*	43	40
Ruler	61	31	*	61	61
Table	203	200	203	203	203

*Child did not complete activity.

Fig. 13--Summary Chart of Lengths

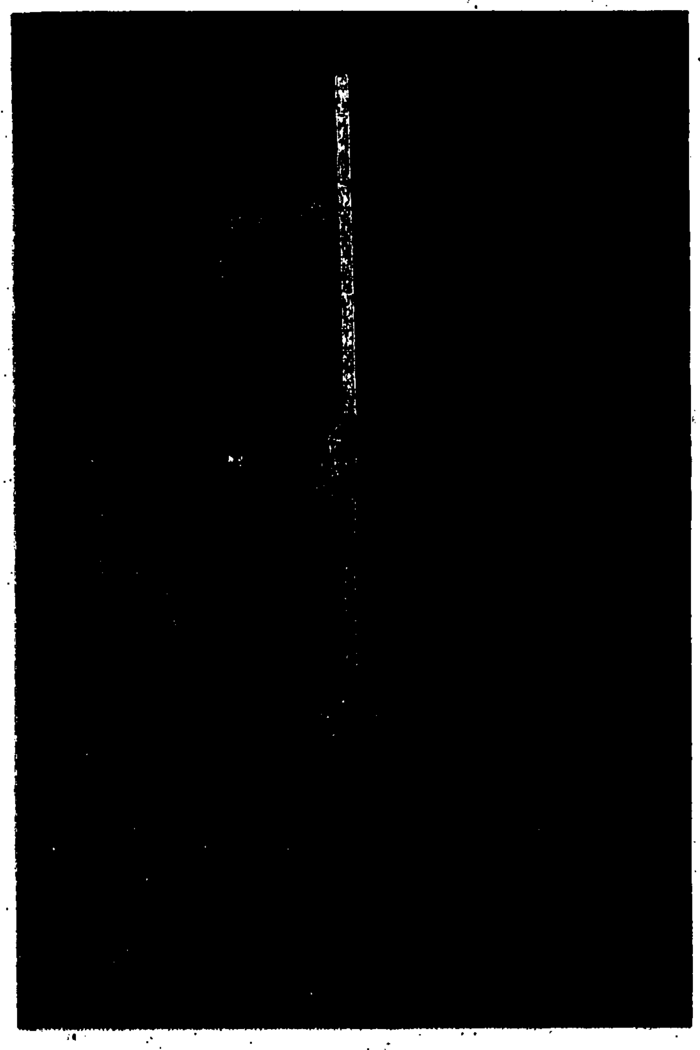
Interestingly, when Paul was questioned about the measure that he reported for the table (200 centimetres), he replied: "I measured it. It took two metre sticks, that's 200, and there was just about an inch [III] left over."

There was much discussion about the range of measures for the coping saw. The children decided that the spread of 7 centimetres was just too large to be caused by anything but careless error. After insisting on and completing remeasurement, they came to a consensus that the actual length was somewhere between 42 and 43 centimetres.



Measuring lengths

The children were next given a worksheet similar to the one included as Appendix E and were asked to make estimates of height, record them, and then measure the objects pictured. In working with height, the children's estimates assumed the same pattern that had been evident in the previous activities with length. Their first estimates tended to bear little relationship to the reported measures, but the percentage of error dropped markedly during the course of their work.



Measuring heights

During the second session devoted to this activity, the children looked at the summary chart of their measurements (see Figure 14 on page 41) and discussed it at length:

Paul: "Hey, we got better!" [When pressed for an explanation, he went on.] "Well, on the table and on the wood box we all got the same, and we mostly got the same on the trash barrel."

Mary: [After remeasuring the trash barrel.] "I got 89 this time. I don't know how I got 81 last time. Paul got a different one on the set of tool drawers. How come?"

Eric: "We all got 28 and Paul got 29. That's only this much." [He indicated a one-centimetre span on the metre stick.] "You could look at it wrong and miss that much."

Eric: [Commenting on the 16-centimetre range of measures for the shelves.] "You couldn't make that much of a mistake by just looking at it wrong. Somebody really goofed."

Mary: [Defensively.] "It's hard measuring something that tall. The metre stick can slip down real easy."

Jane: "We did better on the door. There's only this much difference." [Indicating a little less than a 4-centimetre span on the metre stick.]

The activities above proved extremely useful in encouraging children to look for discrepant data and in helping them to think seriously about the need for accuracy in measurement. Through these experiences the children came to accept a clustering of measurement results, although at the same time they became increasingly insistent that the range of these results be a small one.

After measuring the height of a door, another group of children reported measures ranging from 203 to 205 centimetres. The following discussion then arose:

Teacher: "Is this a reasonable range?"

Children: "Yes."

Teacher: "Are the measures accurate enough?"

Kara: "Doesn't it depend on what you want to do with them?"

Our Measurements — Centimetres

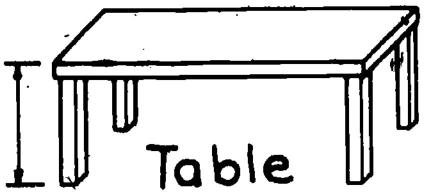
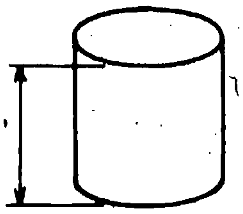
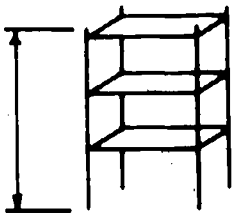
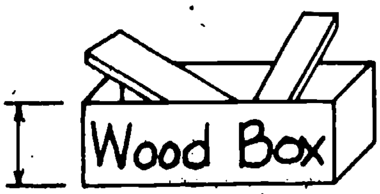
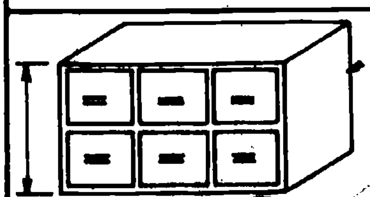
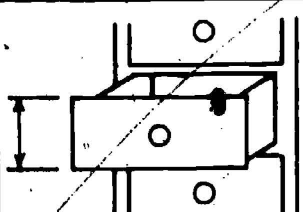
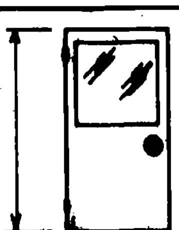
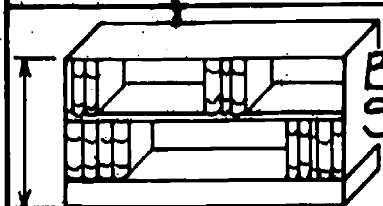
	Eric	Paul	Scott	Mary	Jane
 Table	55	55	55	55	55
 Trash Barrel	89	89	89	81	89
 Shelves	216	200	215½	205	216
 Wood Box	41	41	41	41	41
 Tool Drawers	28	29	28	28	28
 Drawer	13½	10	12	13½	14
 Door	200½	200½	203	204	204
 Book Shelf	113	100½	113	113	102

Fig. 14--Summary Chart of Heights

Teacher: "What do you mean?"

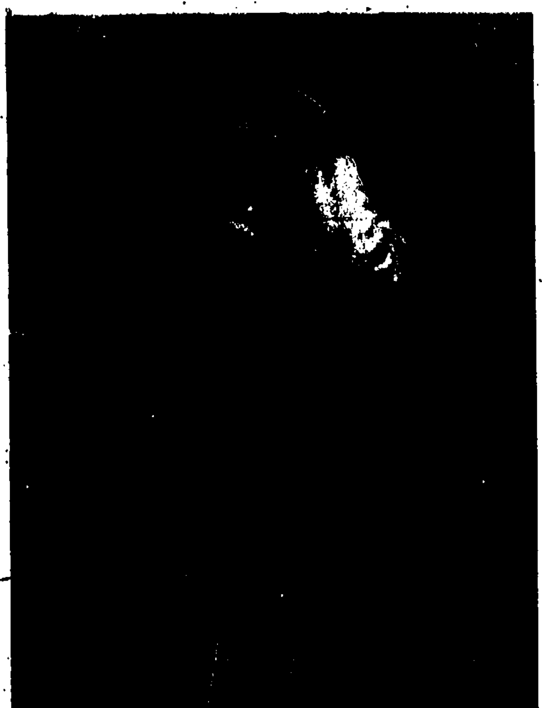
Kara: "Well, if I just wanted to find out whether or not I could get something tall through the door--like those shelves--it might be good enough, but if Mr. Salz was ordering a new door, he'd have to really know how tall the door is and the range would be too big then."

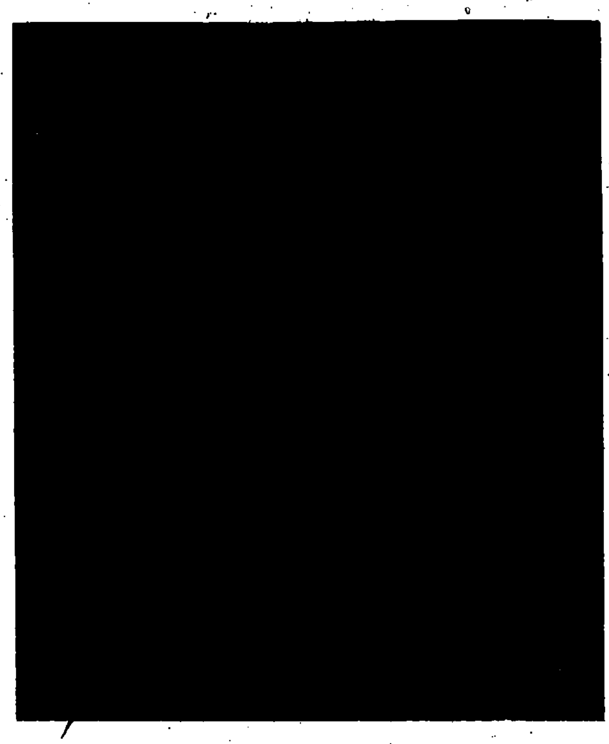
Measuring the Body in Centimetres

The activities described below were introduced by asking the children to find a way to measure the distance around the teacher's head. They began by using a series of metre sticks, placing one against the side of the head and starting a second at the point where, because of the curvature, the first stick no longer touched the head. Continuing in this manner, the children managed to work their way completely around the circumference. However, they became so engrossed in the mechanics of their procedure that they lost track of the various small measures and were not able to determine the total distance sought. In looking for another way to solve this problem, Eric came up with the idea of wrapping a strip of tape around the head, and Paul suggested that the length of the tape could then be found by measuring it with a metre stick.

Once the above task had been accomplished, the children were given a copy of the worksheet "All About Me" (see Appendix F). The rest of the session was then spent working on this sheet. Although some of the measurements could be made individually, many of them necessitated peer cooperation.

Two sessions were required to enable the children to finish making their measurements. At the end of the second session,





Taking measurements of body parts

the children were asked to perform two additional tasks: First, they were asked to make a tape just as long as they were tall and, second, they were asked to record on a summary chart certain selected body measurements. These tasks were designed to prepare materials to be used in subsequent activities (see below).

Graphing Measurements

The children began this activity by making certain that their height tapes accurately reflected the actual order of their heights. To determine this, they lined up from tallest to shortest. Two errors were found and corrected before they proceeded further.

The task was then posed to find a way to communicate their results to their classmates. Someone suggested that the tapes be pasted on a large sheet of paper. Mary objected, saying the tapes should be colored so that they might be seen more easily. Scott suggested using colored strips with gummed backs. Mary argued that the 50-centimetre strips available were not long enough, but Scott proposed that each person use more than one and overlap them.

It was finally decided to record the measurements on a large sheet of centimetre graph paper. Also, each child was to choose a gummed strip of a different color to represent himself. After the axes of the graph had been drawn and calibrated, the end of each height tape was laid on the base line and its length marked off on the vertical axis. Colored strips of appropriate lengths were then pasted onto the graph. The children themselves suggested that the strips be mounted in order of height.

The teacher then asked, "How tall would you be if you were just half as tall as you are now?" Many different ways of finding out were suggested and tried. Eric, after much thought, came up with the idea of folding his height tape in half. The children were unsure about who would be tallest if they were all half size, so they checked by Eric's method. They then asked if they could graph these results. Making a record of quarter-size tapes was also suggested. The graph made during these activities is shown in Figure 15. The children commented on this graph at some length and insisted that Scott remeasure and correct his errors in the one-half and one-fourth size tapes. It was Scott who suggested the title for the graph.

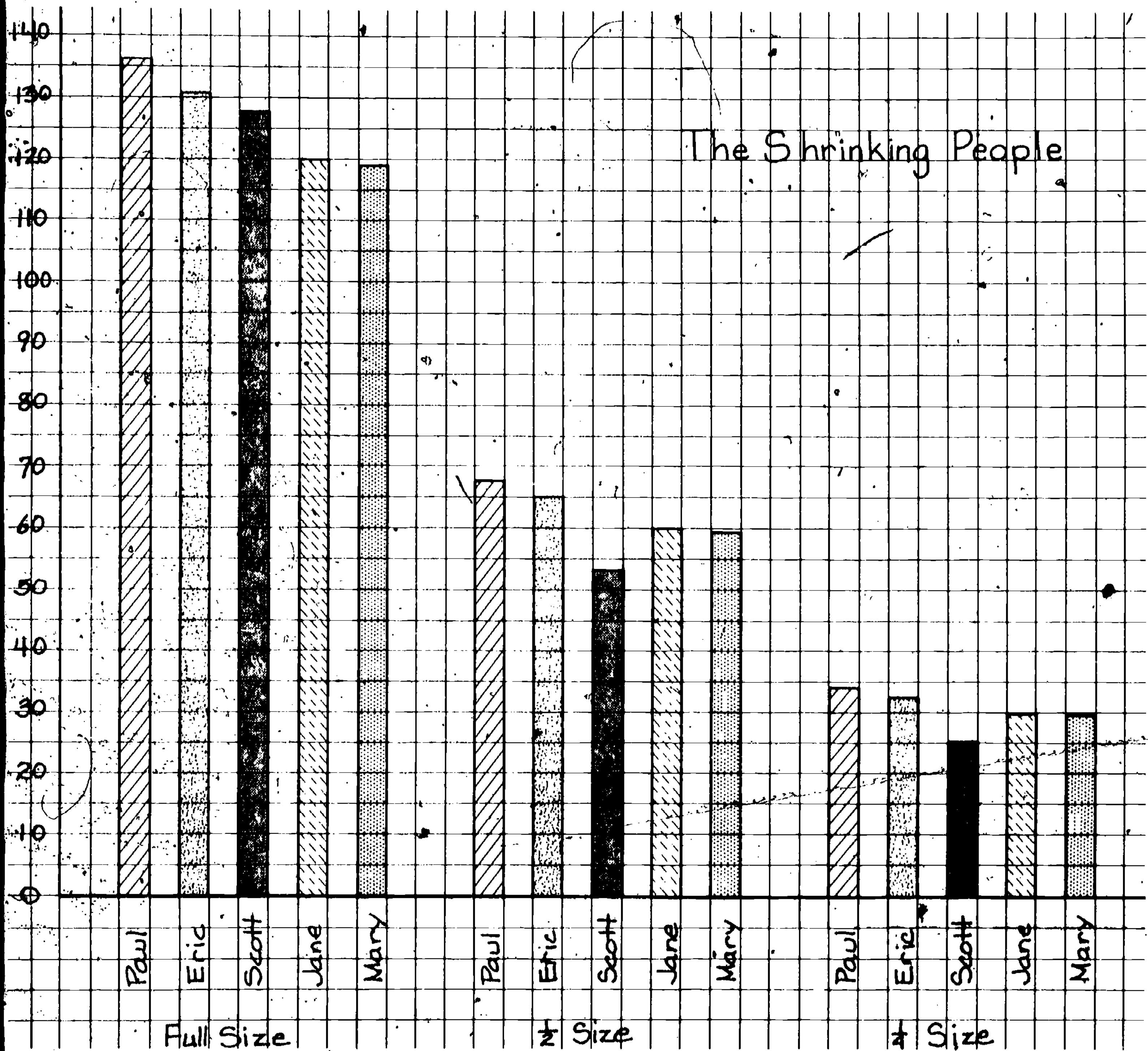


Fig. 15--A Graph of Heights
(The full-scale graph was originally prepared
on a large sheet of centimetre graph paper.)

The construction of one-eighth size graphs of selected body measurements was then initiated. First, each child was given a copy of the summary chart appearing in Figure 16.

Our Measurements in Centimetres

	Height	Waist	Chest	Head	Neck	Wrist
Eric	131	61½	41	55	26	13
Jane	120	62	59+	52	27	14
Mary	119	45	60	53	27	11
Paul	136	57	66	54½	30	14½
Scott	128	56	63	55	29	13

Fig. 16--Summary of Selected Body Measurements

Next, the children were asked if they would like to make individual graphs of their own body measurements. They indicated that they would. The teacher then asked how such graphs could be made <u>without</u> using large sheets of graph paper. Jane suggested that they just fold their tapes again and make charts "half of quarter size." The children agreed that one-eighth scale graphs would be a good solution and proceeded to make them. During this activity, the children compared graphs and asked each other many questions. A brief excerpt of their discussion follows:

Eric: "Look, my waist is just about half my height. Is that true for everyone?"

Jane: "My wrist is half my neck. Does anyone else have that?"

Mary: "I guess height doesn't matter. Paul's tallest, but Jane has the biggest waist. Eric and Scott have the biggest heads. Paul's tallest, but he's not the biggest all over. I'm shortest, but I'm not smallest all over."

After their discussion was concluded, a check was made for "reversibility," that is, the ability of the children to construct a full-size tape from a quarter-size tape. The children were shown a piece of tape and were told that it represented a quarter-size tape of a little girl. They were asked if they could think of a way to find out how tall the little girl really was. Mary suggested an easy way: She laid the quarter-size tape on a long strip of blank tape and marked it off four times. Upon completing her work, she proudly held up the full-size tape.

The children then took the corrected "Shrinking People" graph into the classroom to explain it to their classmates. Examples of their explanations follow:

Paul: "This shows us all full size, and this is half size, and this is how tall we would be if we were quarter size. I'm 136 centimetres tall full size and 68 centimetres half size, and 34 centimetres quarter size."

Scott: "That says 'The Shrinking People.' We are really this size, but we shrunk to half size first and then to quarter size. We did one-eighth size too, but that's in our desks."

Eric: "You can tell who is who by using the colors. Paul has orange, I have purple, Scott has blue, Jane is yellow, and Mary is green."

Mary: "We're different sizes here [indicating full size], and we're still different here [half size] and here [quarter size]. Paul's tallest in all of them, and I'm the smallest."

When the classroom teacher told Mary that she and Jane looked the same in quarter size, Mary responded, "We're really not, but there wasn't much difference full size, so when we shrank you can't see it so good."

Introducing Perimeter

The following experiences were designed as an introduction to the idea of perimeter. Provision was made on a worksheet (see Appendix G) for recording length, width, and perimeter, but no stress was laid on trying to have the children discover the relationship between these quantities.

The children devised a method in which tape was used to find the perimeter of an object or a drawing. Sides were measured one at a time and laid off on a strip of tape. When all four sides had been laid off, the children measured the length of the tape to arrive at the perimeter. Two of them became aware that they could find the answer by adding, but were unable (as first graders) to handle the computation necessary.

In another activity, the children measured the perimeter of a room which had an irregular shape. Appendix H contains the worksheet used in implementing this activity.

CONCLUDING REMARKS

The authors do not consider this booklet to constitute a complete or exhaustive treatment of linear measurement. Rather, they view the ideas it contains as a beginning. It is hoped that the activities discussed will stimulate you to create additional activities of your own.

Above all, it is recommended that you do not push your students through a single, narrow sequence of experiences. They should be given time to consider ideas, go off on tangents, discuss their results, try their own adaptations, consider alternatives, and . . . think.

A brief excursion into mass measurement has also been written. It is entitled "An Introduction to Mass Measurement with the Metric System", and will be available in a "sampler" to be published by the Project on Elementary School Mathematics and Science. The materials it contains are appropriate for the latter part of the first grade and for the second or third grade.

APPENDIXES

APPENDIX A

Background Information

on the Metric System

Every major country in the world is either required by law or permitted for convenience to use the metric system of weights and measures. Although the United States ordinarily does not use the metric system, except for scientific purposes, the metre and the kilogram are employed as important standards even in this country. The metre is the standard unit for length; and, from an exact comparison with this standard, the United States derives its standard yard, which is defined as 0.9144 metres.

The metric system was introduced by the French in 1790, during the French Revolution. It was part of the plan to start afresh the whole social and economic life of the French without any ties to the past. The metre, the fundamental unit of length, was originally intended to be one ten-millionth of the distance between the North Pole and the equator as measured along the meridian which passes through Paris. But, in the eighteenth century, instruments were not as accurate as they are today, and somewhere an error was made in measurement. By the time scientists discovered this mistake, the length of the metre was so well established that it was decided to keep it as originally determined.

Because of the more precise measurements that are possible today, a new standard for the official length of the metre was set in 1960. This standard, which is used primarily by scientists, defines the metre as 1,650,763.73 wavelengths of the orange-red light given off by Krypton 86--a rare gas found in the atmosphere. But people other than scientists still continue to think of the metre as being equivalent to 39.37 inches.

The metric system devised by the French spread rapidly throughout Europe and to the colonies of most European nations, although France itself did not adopt the system officially until 1830. At that time, however, Britain and the United States were satisfied with the system of imperial units (inches, feet, pounds, quarts, etc.) which they were already using and felt no need to change. In 1866 the United States Congress made the use of the metric system legal in situations where it was more convenient, but the system has never come into general use in this country. Britain, however, has begun the process of converting to the metric system and will be totally metric by 1975. Pressure is currently being brought to bear on our government to make a similar conversion, and it appears that such a conversion will be made in the near future.

One thing that makes the metric system simpler to use than the imperial system is that it is a decimal system--a system based on multiples of ten. Unlike the imperial system, in which every unit of length has a separate name that gives no clue to its value, the name of each unit in the metric system indicates its magnitude. There are six major prefixes that can be attached to any of the units, and each of these prefixes denotes a number (see Figure 17 below). The prefixes for the fractional parts of the units come from Latin, while the prefixes for the multiples of the units come from Greek. Any of these prefixes attached to "-metre" tells you exactly what the value is; for example, a *centi*metre is one hundredth of a metre. For the purposes of this booklet, only the *centi*- prefix has been stressed.

milli-	=	one thousandth	=	1/1000
centi-	=	one hundredth	=	1/100
deci-	=	one tenth	=	1/10
deca-	=	ten	=	10
hecto-	=	one hundred	=	100
kilo-	=	one thousand	=	1000

Fig. 17--Some Prefixes Used in the Metric System

Another advantage of the metric system is that simple relationships exist between the various types of measurement, i.e., length, capacity, and mass. Specifically, a cube with a side 10 centimetres in length has a capacity of 1 litre; and the mass of 1 litre of water at a barometer reading of 760 mm of mercury and a temperature of 4° centigrade is 1 kilogram.

In countries where the metric system has been adopted, the metre is employed for measuring all kinds of dry goods and materials, for building and engineering, and for any purpose where the yard or foot might be used in the United States. In measuring longer distances than can be conveniently measured with metres, the kilometre is utilized instead of the mile. Centimetres and millimetres are used instead of inches in constructing machinery, in making furniture, and in measuring book, paper, and film sizes.

Area is calculated by multiplying length by width. Whereas the imperial system would use square yards and square inches, the metric system employs square metres and square centimetres.

In measuring volume, or what a space will hold, the appropriate metric unit is the cubic metre. This unit is used for reckoning the tonnage of ships and the contents of tanks or reservoirs, for measuring timber, and for calculating fill for land. The cubic centimetre (cc) is a unit of volume utilized by doctors and pharmacists.

The conversion table below (see Figure 18) is included for the information of the teacher only. The activities in this booklet make exclusive use of the metric system with no

Units of Length	
Imperial to Metric	Metric to Imperial
1 inch = 25.4 millimetres (mm) 1 inch = 2.54 centimetres (cm) 1 foot = 0.3048 metres (m) 1 yard = 0.9144 metres 1 mile = 1,609.347 metres 1 mile = 1.6093 kilometres (km)	1 millimetre = 0.03937 inches 1 centimetre = 0.3937 inches 1 metre = 39.37 inches 1 kilometre = 0.62137 miles

Fig. 18--Conversion Table

mention of conversion to the imperial system of measures. This is by deliberate design, not because of oversight. The authors feel that many of the problems connected with the teaching of the metric system in the past have arisen because of the attempt to combine an introduction to the system with the use of conversion tables. With young children, this can be disastrous. The systems are different, and it is felt that they should be introduced separately.

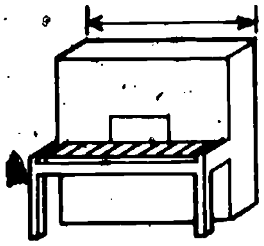
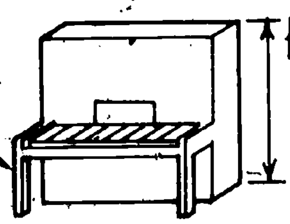
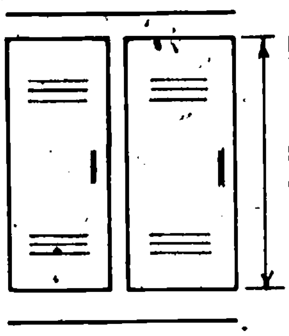
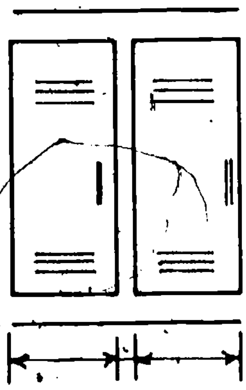
APPENDIX B

Measuring with

the 10-Span Tape

Name ______________________

Measuring with the 10-Span Tape

	My Guess	My Measurement
 Width of Piano		
 Height of Piano		
 Height of My Locker		
 Width of My Locker		
Width of Hall		
Length of Hall		

APPENDIX C

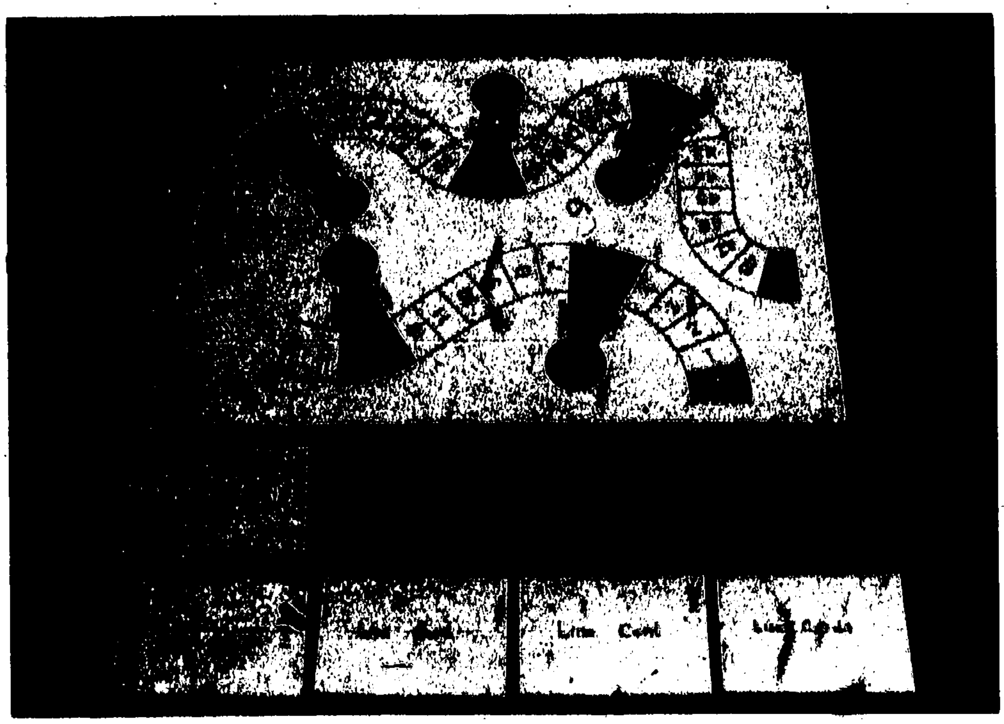
ESTIMO: A Game with

Metric Measurement

Assembling the Game

A colored playing board has been included with this booklet. Before play is initiated, the board should be mounted on a rigid piece of material (e.g., cardboard or plywood).

The game also requires five markers. Buttons, small toy animals, geometric shapes, or any other small objects may be used.



Estimo: Game parts

"Line cards" and "question cards" (described below) will also need to be prepared.

Object of the Game

Each player selects a marker. Markers are advanced on the board by correctly identifying lines of specified lengths on the line cards. The first player to reach the "End" space is declared the winner.

The Playing Board

The playing board contains 48 numbered spaces between "Start" and "End." This includes 10 numbered spaces that are colored red: 5 and 6, 13 and 14, 25 and 26, 33 and 34, and 40 and 41. The procedure to be followed when a player lands on one of these red spaces will be described below in the section entitled "Playing the Game."

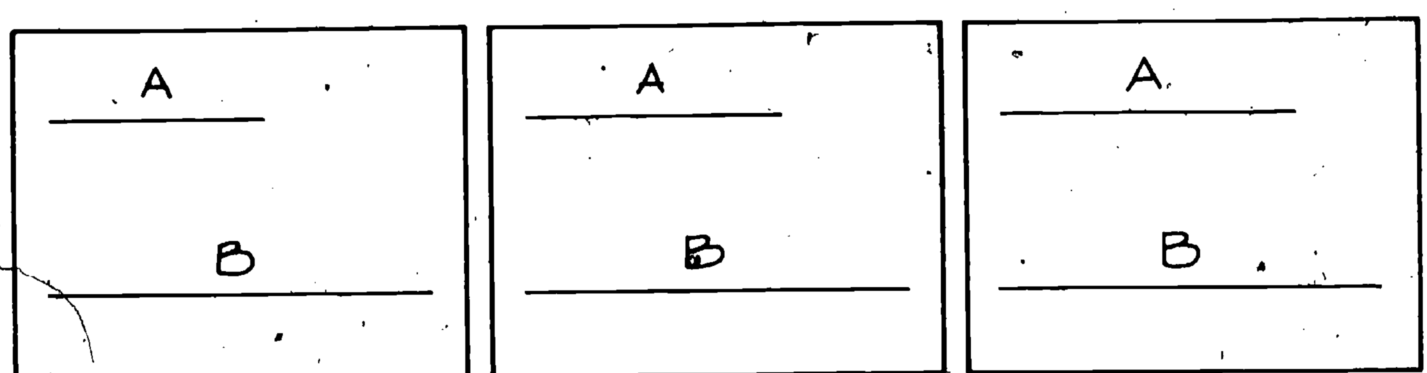
Line Cards

There are four different kinds of line cards. They should be placed in four separate piles. Either 2, 3, 4, or 5 lines are drawn on each card. The greater the number of lines on a card, the more difficult it is to answer. But, by correctly answering a more difficult card, a player is entitled to advance more spaces. The cards and their "values" are as follows:

> Card with 2 lines--answer correctly and advance 2 spaces.
>
> Card with 3 lines--answer correctly and advance 3 spaces.
>
> Card with 4 lines--answer correctly and advance 4 spaces.
>
> Card with 5 lines--answer correctly and advance 5 spaces.

If a given line card is not answered correctly, the player must move <u>backward</u> the number of spaces indicated above. (However, regardless of the number of spaces a marker is to be moved backward, it may be moved no further backward than the "Start" space.)

The authors made sets of line cards at three different levels of difficulty. The difficulty was determined by the relative lengths of the lines on each card. Examples are illustrated in Figure 19. It is suggested that the teacher first make up a set of cards that is relatively easy for the children to answer. As the skill of the class develops, more difficult cards can be devised to replace the easier ones.



Easiest Most Difficult

Fig. 19--Line Cards of Varying Difficulty

<u>Question Cards</u>

The questions on these cards are intended to apply to any of the line cards. At the bottom of each card is a line which is equal in length to the length of the line specified.

Using this line, the players can check their estimates. An example of a typical question card follows:

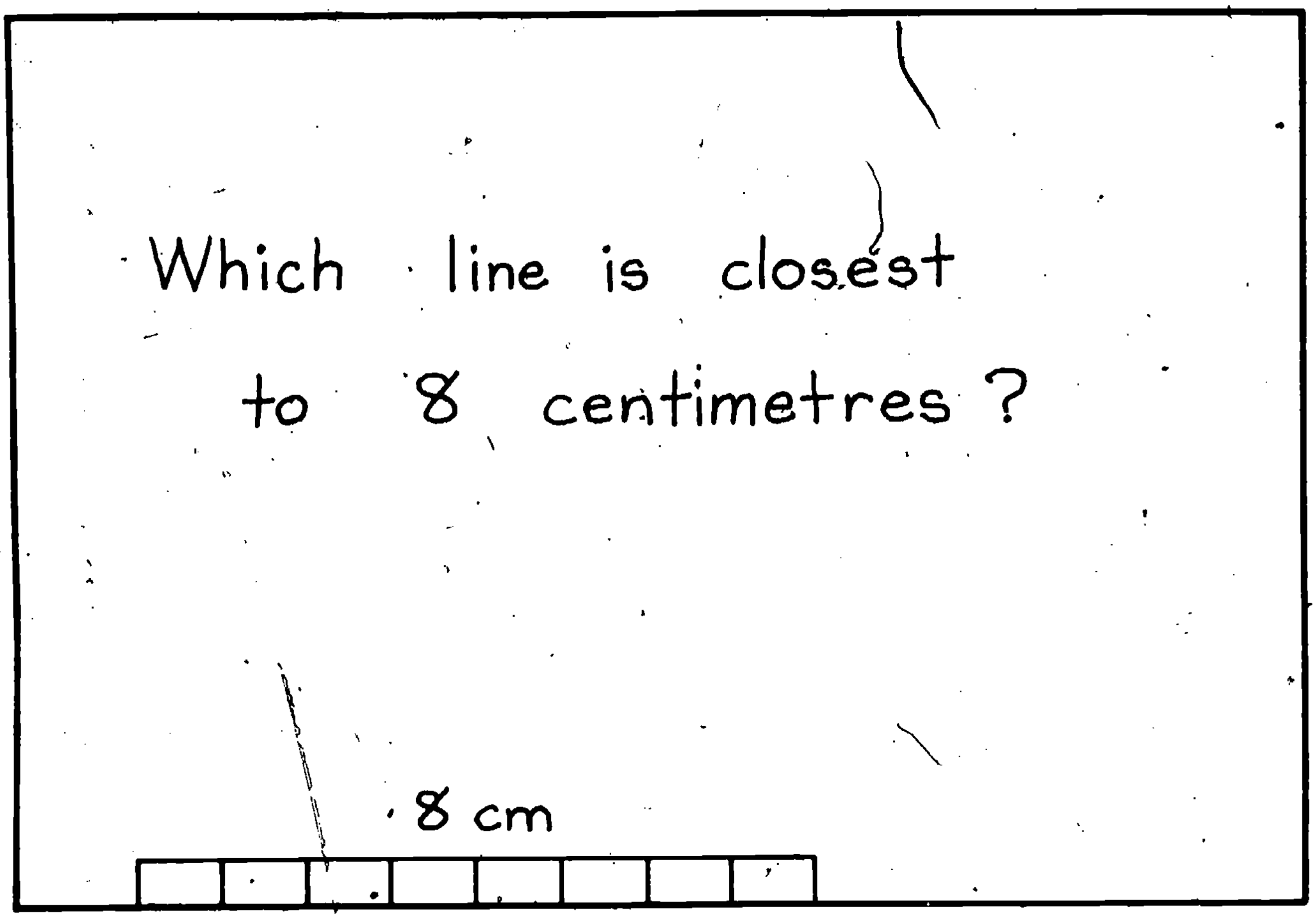


Fig. 20--Sample Question Card

Experience has demonstrated that the game proceeds faster if one person takes the job of reading all the question cards.

Playing the Game

The game is recommended for two to five players. Each player selects a marker. Play can proceed in any previously agreed upon order. (The order might be determined by rolling dice, guessing numbers, or any other convenient method.)

To begin, a player decides how difficult a line card he wants to try. He then takes a card from the top of the appropriate pile. The questioner takes a card from the top of the question card pile and reads it to the player. (Both line cards and question cards are placed face down in their respective piles.)

Let us assume, for example, that the player chooses a line card with four lines. This card might look like the one shown in Figure 21. The question card selected might ask

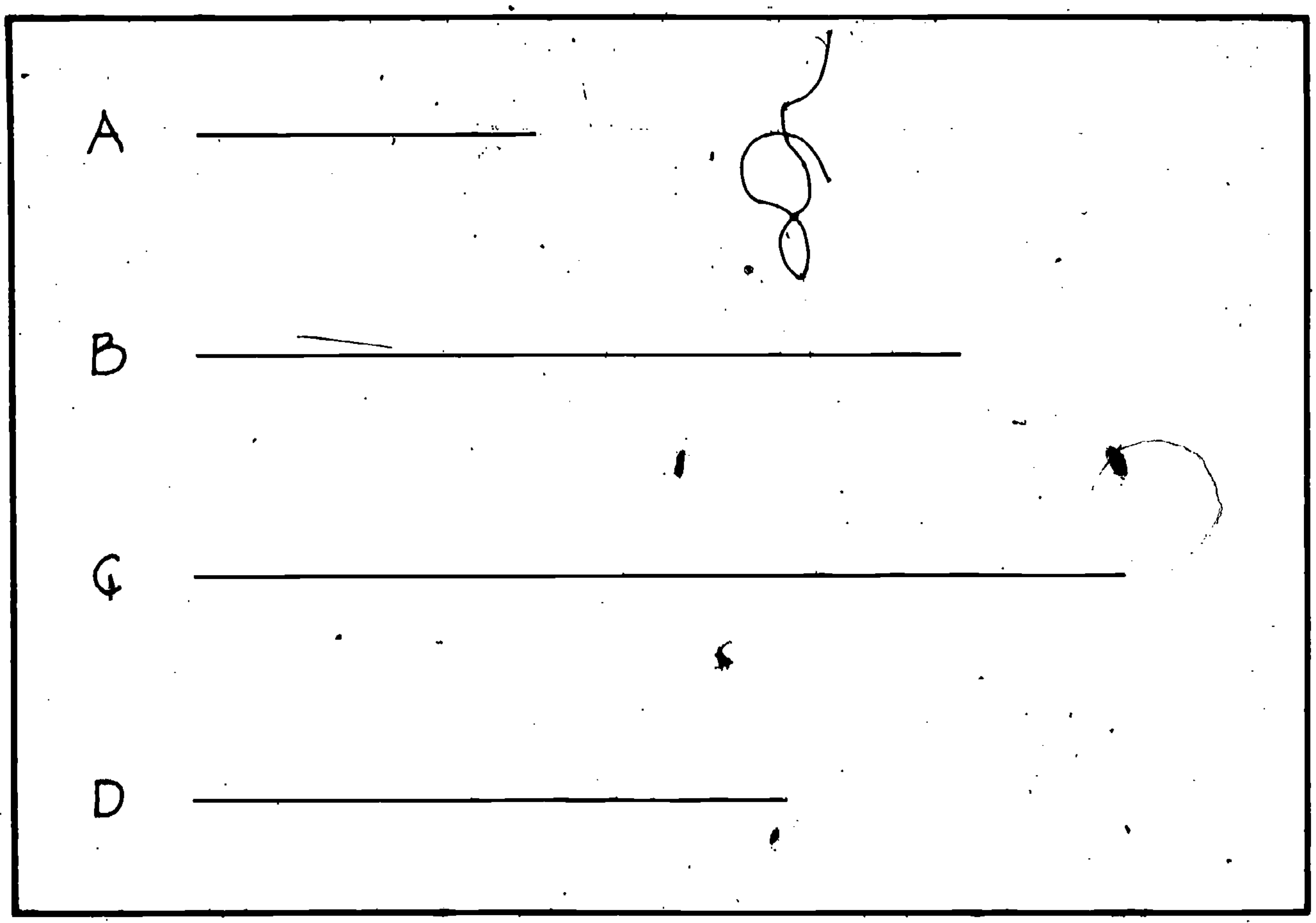


Fig. 21--Sample Line Card

him to estimate "Which line is closest to 8 centimetres?" If the player is successful in selecting the line closest to 8 centimetres, he <u>must</u> move forward four spaces, since he attempted a four-line card. It is recommended that the questioner serve as the final authority in determining whether or not an answer is correct.

It is possible that none of the four lines is exactly 8 centimetres long. Suppose, for example, that line B is 9 centimetres long and Line D is 7 centimetres long. In this case, choosing either one of the lines would constitute a correct answer since both are one centimetre from the length specified (viz., 8 centimetres).

If a player lands on any of the red spaces, he must move his marker into the round space of the area called the "Rabbit Cage" and wait until his next turn to try to get out. To get out, the player must correctly answer his next question. This question must be on a line card with a value of 4 or 5 so that he can count his way out as shown in Figure 22.

The game continues with each player proceeding in turn until the first person reaches the "End" space. It is also possible to continue play until additional players have reached the "End" space.

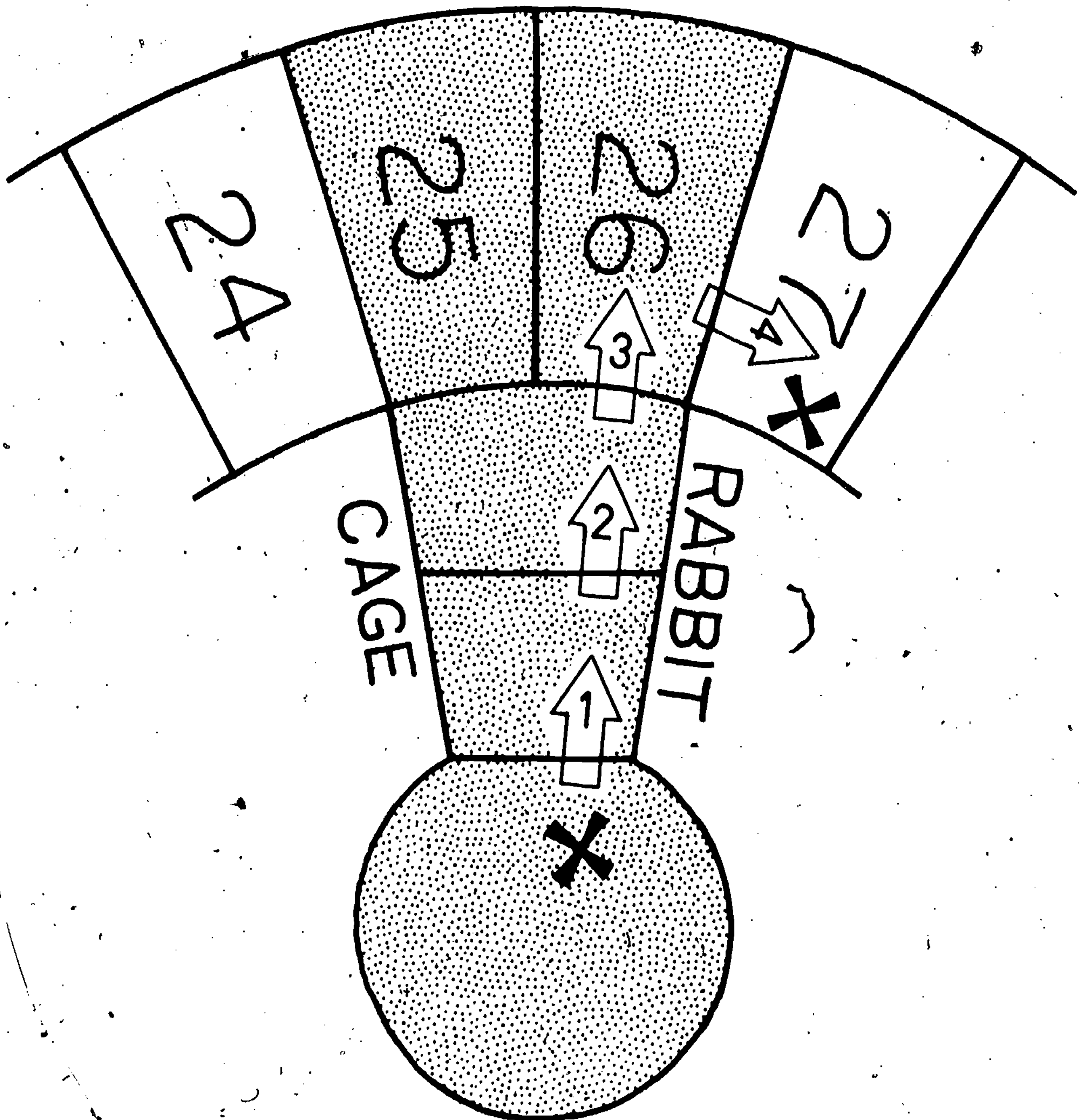


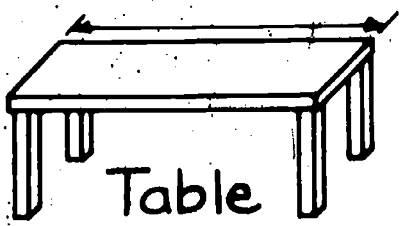
Fig. 22--Counting Out of
a Rabbit Cage

APPENDIX D

How Long?

Name ____________

How Long?

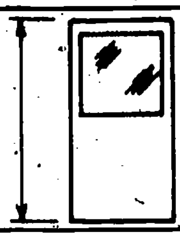
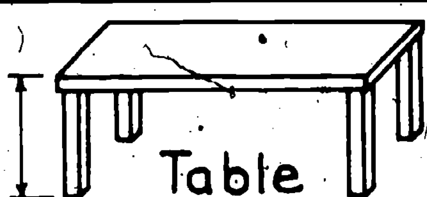
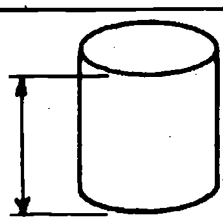
	My Guess	My Measurement
 Pencil		
 Table		
Cardboard Measuring Stick		

APPENDIX E

How High?

Name ____________

How High?

	My Guess	My Measurement
 Door		
 Table		
 Trash Barrel		

APPENDIX F

All About Me

ALL ABOUT ME

Head_____cm

Eye_____cm

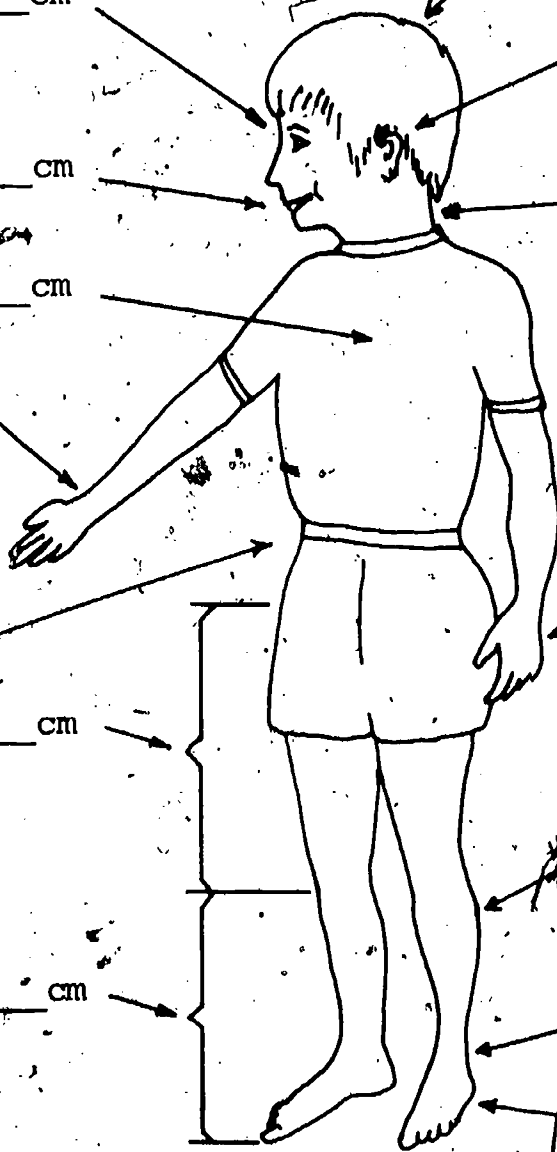
Ear_____cm

Mouth_____cm

Neck_____cm

Chest_____cm

Wrist_____cm



Waist_____cm

Hand_____cm

Hip to Knee_____cm

Calf_____cm

Knee to Floor_____cm

Ankle_____cm

Foot_____cm

I am _______cm tall

APPENDIX G

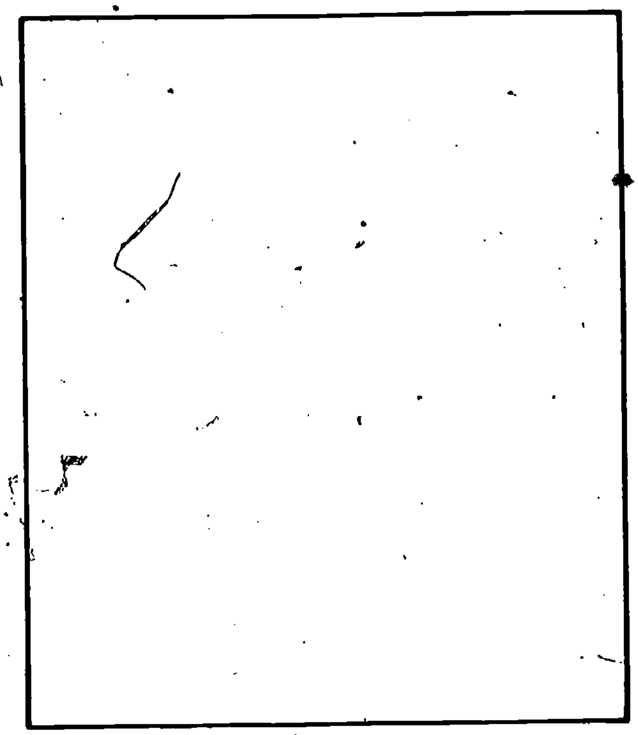
Exploring Perimeter

Name ______________

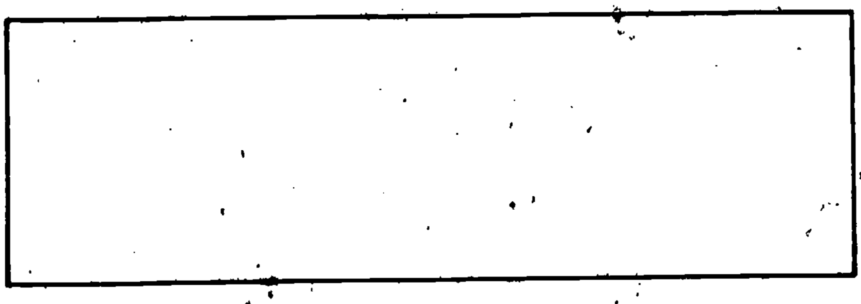
Exploring Perimeter

	Length		Width		Perimeter	
	My Guess	My Measurement	My Guess	My Measurement	My Guess	My Measurement
Book						
Paper						
Table Top						
Figure A						
Figure B						

A



B

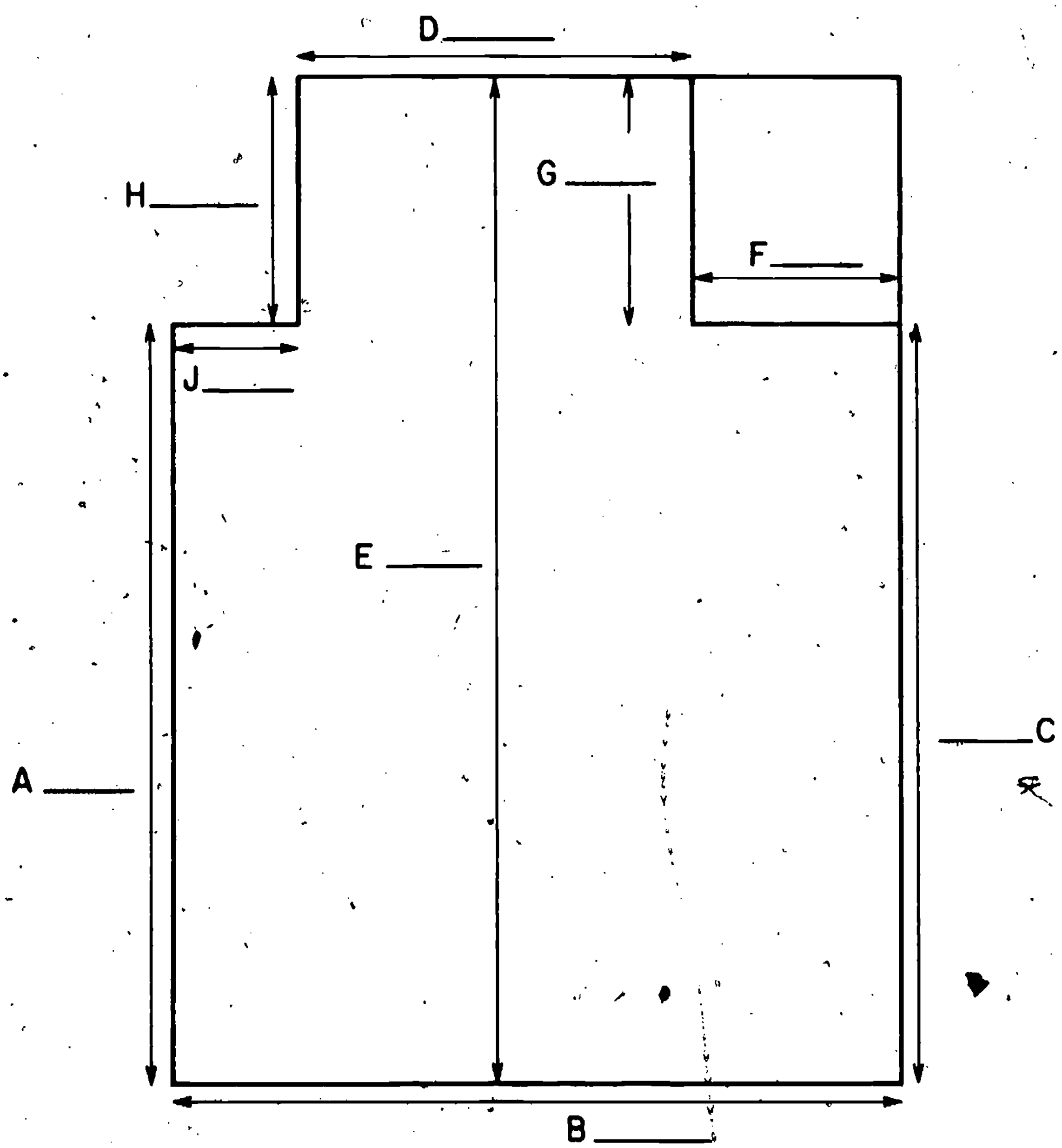


APPENDIX H

Measuring the Perimeter of a Room

Name ____________________

Measure in metres. Fill in the blanks.



APPENDIX I

Checking Up on Measurement

At the conclusion of the activities described in this booklet, the authors administered the three worksheets in this appendix to assess what the children had learned. The summary below illustrates some of the information that might be gained from such an evaluation.

The first two worksheets deal with estimation. The twelve children working on these sheets were given no measuring instruments.

Item 1: Only one child had an accurate mental picture of the length of a 10-centimetre segment. Three of the children thought that the 15-centimetre segment (E) was 10 centimetres long.

Item 2: The tapes made by the children ranged from 90 centimetres to 152 centimetres. One child's tape was within 4 centimetres of a metre, but most of the children overestimated by a large amount.

Item 3: There were very mixed results for this item. One child seemed to have the ability to estimate quite closely the length of segments less than 10 centimetres, usually on the side of underestimation. (It appears that not enough time was spent in having the children estimate lengths from 1 to 20 centimetres.) However, with one exception, most of the errors in estimation appeared to fall within the range expected for children of this age. (It should be noted that, to aid in estimation, the length of line C was given as 10 centimetres. The children were asked to guess the lengths of only lines A, B, D, and E.)

The last worksheet deals with both estimation and measurement.

Item 4: While the estimates of the width of the "Lab" ranged from 5 metres to 20 metres, the reported measures were accurate, for the most part, within a 5% measurement error (which may have been caused by the rounding off of a fractional metre).

Item 5: The estimates of perimeter ranged from 5 to 30 centimetres; but, with the exception of one child (who added a centimetre to each side of the figure), all the children were quite accurate in their reported measures.

Item 6: Given a quarter-size tape, the children were asked first to make a full-size tape and then to measure it. With one exception, the children were accurate within one centimetre. The exception came about because one of the children thought that there were eight quarters in a whole.

Name ____________________

(1) Which line is 10 cm long? Circle the letter.

A ________________________________

B ______________

C ______________________

D ____________________________

E ____

(2) Cut a piece of tape that you think is 1 metre long.

Staple the tape to this paper.

Name______________________

(3) Guess how long each line is.

Write your guess in front of the letter.

_____ A ________________________

_____ B ____________

10cm C ____

_____ D ____

_____ E ____________________

Name ________________

(4) Width of the Lab in metres.

Estimate ________________

Measure ________________

(5) Distance around this figure.

Estimate ________________

Measure ________________

(6) Look at the 1/4-size tape below.
Make a full-size tape and measure it.

Write your measure here: ____________ cm